UNIVERSITY OF HOUSTON
Cullen College of Engineering

Fundamentals of Engineering
Spring 2005 Review

Dynamics

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slides reordered just before presentation.

problem distribution

- morning: 8% (9 out of 120 problems)
- afternoon: 7% (4 out of 60 problems)

– The two "pendulum" examples are moved closer to their logical placement.

subject areas

- 23% vibrations (3)
- 23% constant acceleration (3)
- 15% velocity/acceleration (2)
- 15% force/motion (
- 15% simple mechanisms (2)
- 8% work/energy (spring) (1)

– some solutions are "corrected"
– sign errors

Engineering application of Newton's laws of motion

free body diagram

1st $\underline{a} = 0$ unless ext. force

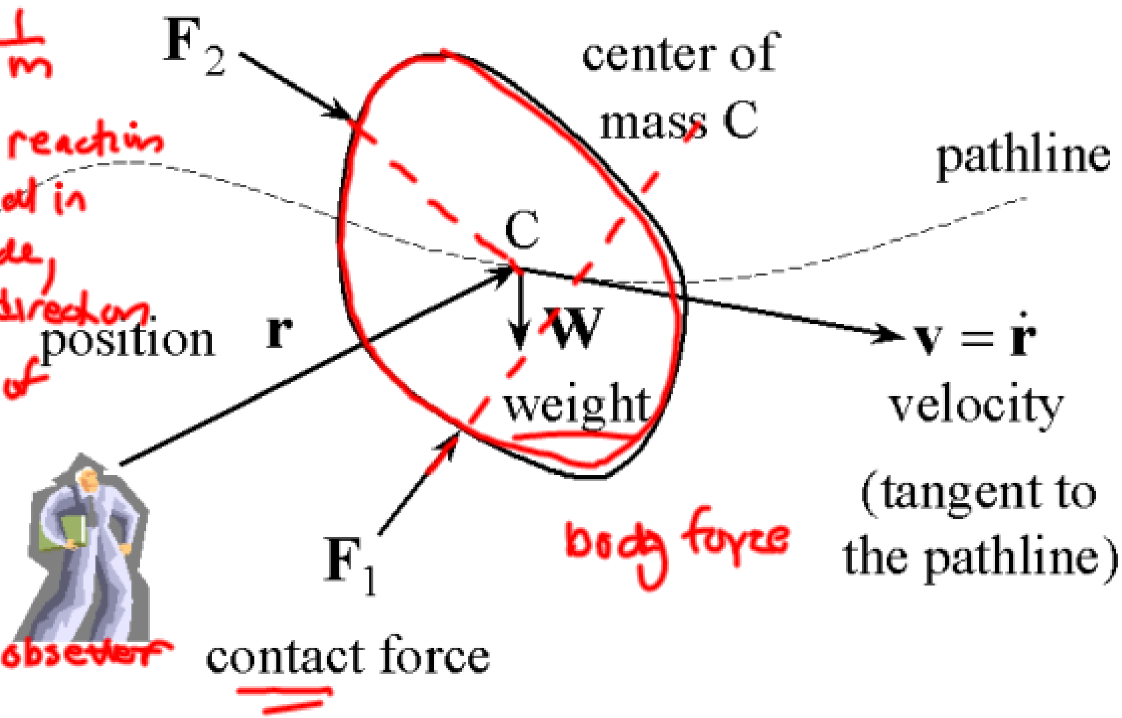
2ND $\underline{a} \propto F$

$\underline{\alpha} \propto \frac{1}{m}$

3rd action & reaction are equal in magnitude, opposite direction, same line of action

$F_g \propto m_1 m_2$

$F_g \propto \frac{1}{r^2}$



inertial reference $\underline{a} = 0$ for observer

$\Sigma F_x = m a_x$
$\Sigma F_y = m a_y$
$\Sigma F_z = m a_z$

equations of motion

total force → $\mathbf{F} = m\mathbf{a}$ ← mass

acceleration of the body's center of mass

- equation only valid in an inertial frame
- force obtained using a free body diagram
  - body forces (weight)
  - surface or contact forces

$\underline{r}$ is position vector

$\underline{v} = \frac{d}{dt}(\underline{r}) = \dot{\underline{r}} = \dot{x}\underline{i} + \dot{y}\underline{j} + \dot{z}\underline{k}$

$\dot{x} = \frac{dx}{dt}$

$\underline{a} = \frac{d}{dt}(\underline{v}) = \dot{\underline{v}} = \ddot{\underline{r}}$

acceleration

$\ddot{x} = \frac{dV_x}{dt} = \frac{d^2x}{dt^2}$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}}$$

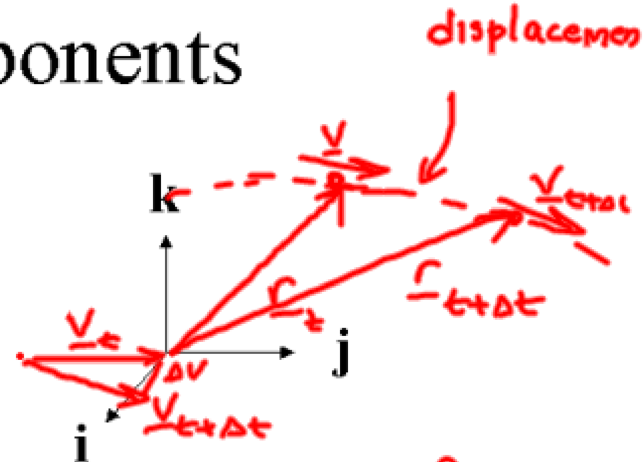
velocity position

Cartesian components

$$\mathbf{r} = \mathrm{x}\mathbf{i} + \mathrm{y}\mathbf{j} + \mathrm{z}\mathbf{k}$$

$$\mathbf{v} = \dot{\mathrm{x}}\mathbf{i} + \dot{\mathrm{y}}\mathbf{j} + \dot{\mathrm{z}}\mathbf{k}$$

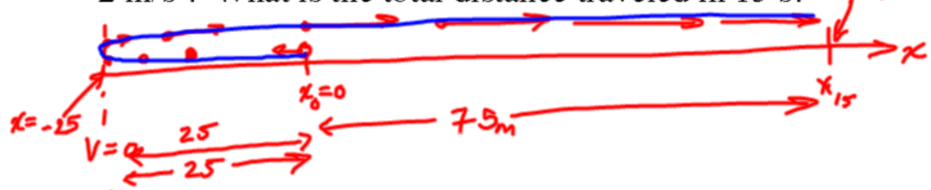
$$\mathbf{a} = \ddot{\mathrm{x}}\mathbf{i} + \ddot{\mathrm{y}}\mathbf{j} + \ddot{\mathrm{z}}\mathbf{k}$$



$\text{disp} = \underline{r}_{t+\Delta t} - \underline{r}_t$

$\lim_{\Delta t \to 0} \frac{\underline{r}_{t+\Delta t} - \underline{r}_t}{\Delta t} = \frac{dr}{dt}$

92. An object with initial velocity −10 m/s accelerates at 2 m/s². What is the total distance traveled in 15 s?



2 parts.

time from x_0 until $V=0$, → distance

time remaining until 15sec → distance

$$a=\ddot{x}$$

$$\int a\,dt = \int d\dot{x}$$

$$\int at + c_1 = \int \dot{x}$$

$$\frac{1}{2}at^2 + c_1 t + c_2 = x(t)$$

$$x_0 = x(0) = c_2$$

$$\cancel{\frac{1}{2}at^2 + c_1 t + x_0 = x(t)}$$

$$\cancel{x(0) = x_0 =}$$

$$at + c_1 = \dot{x} = V_0 \ (\text{at } t=0)$$

$$c_1 = V_0$$

$$\frac{1}{2}at^2 + V_0 t + x_0 = x(t)$$

- rectilinear motion
- constant acceleration

① $x = \frac{1}{2}at^2 + V_0 t + x_0$

$x_0 = 0$

$V_0 = -10\,m/s$ (given)

$a = 2\,m/s^2$ (given)

$\dot{x} = at + V_0$ for $\dot{x} = 0$ (when $V = 0$)

$0 = at + V_0$

$$\frac{0 - V_0}{a} = t = \frac{0 - (-10\,m/s)}{2\,m/s^2} = 5\,sec.$$

Distance in 5 sec.

$x(5) = \frac{1}{2}(2\,m/s^2)(5\,sec)^2 + (-10\,m/s)(5\,sec) + 0$

$= -25$

$x(15) = \frac{1}{2}(2)(15)^2 - 10(15) + 0 = 75$ meters

Total distance

25 m	$x=0,\ x=-25$	5 sec
25 m	$x=-25,\ x=0$	10 sec
75 m	$x=0,\ x=75$	
125 m		

constant acceleration

initial velocity

$$a = a_0$$

$$v = a_0 t + v_0 \; ; \; t = \frac{v - v_0}{a_0}$$

t_1 t_0

$$s - s_0 = \frac{a_0 t^2}{2} + v_0 t = \frac{v^2 - v_0^2}{2a_0} = \frac{(v + v_0)}{2} t$$

initial position

mean velocity

$s(t) = s_0 + \frac{1}{2} a_0 t^2 + v_0 t$

$s(t) - s_0 = \frac{1}{2} a t^2 + v_0 t$

reindex location of s_0 in mutipart problem (to handle negative displacements)

locations – solve directly

displacements – from difference in locations.

An object is launched upward at an initial velocity of 40 m/s. How high will it go?

sketch

$$y = \frac{1}{2} a_y t^2 + v_{y_0} t + y_0 \qquad a = -g$$

$$\dot{y} = a_y t + v_{y_0}$$ solve for t

$$0 = (-9.8\,m/s^2)(t) + 40\,m/s$$

$$\underbrace{\frac{40}{9.8} = t}_{\text{"precise"}} \qquad \underbrace{\left(\frac{40}{10}\right) = 4\,sec}_{\text{close approx}}$$

$$y = \frac{1}{2}(-9.8)(4)^2 + (40)(4) + 0$$

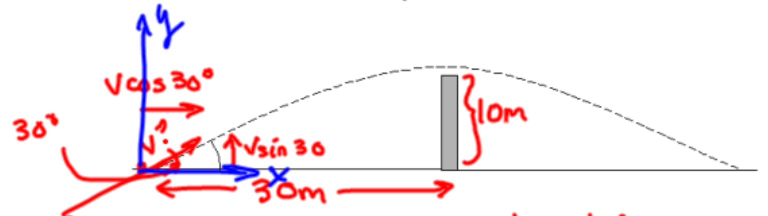
$$\approx \frac{1}{2}(-10)(16) + (160)$$

$$= -80 + 160 = 2\cancel{40\ meters}$$

80 meters

"minus sign"
$-a_y$ is down

93. A cannon is fired at 30° above the horizon. What is the minimum exit velocity needed to clear a 10 m wall?



$v_0 = V\cos 30$

$V_0 = V\sin 30$

2 equations

x-direction

y-direction

$$x = \tfrac{1}{2}\cancel{a_x}^{=0} t^2 + v_0 t + \cancel{x_0}^{0} = 30 \Rightarrow V\cos 30\, t = 30$$

$$V = \frac{30}{\cos 30\, t}$$

$$y = \tfrac{1}{2}a_y t^2 + V_0 t + y_0 = 10 \Rightarrow$$

$$(9.8) \rightarrow \frac{-10\,m/s^2}{2}t^2 + V\sin 30\, t + \cancel{y_0}^{=0} = 10$$

$$10 = -5t^2 + \frac{30\sin 30}{\cos 30\, t}t = -5t^2 + 30\tan 30$$

$$t^2 = \frac{10 - 30\tan 30}{-5} = 1.464$$

$$t = 1.21$$

$$V = \frac{30}{\cos 30\,(1.21\,sec)} = 28.6\,m/sec$$

polar components–
relative to a cartesion system

angular velocity (ω)

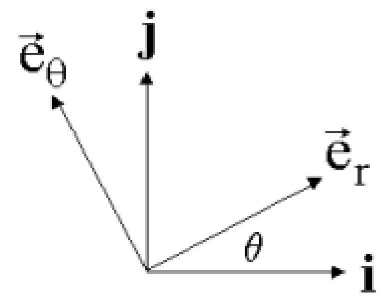
$$\mathbf{r} = r\mathbf{e}_r$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

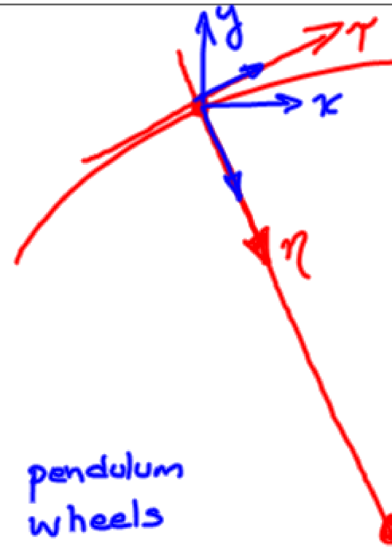
$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

radial Coriolis

centripetal angular acceleration (α)



- normal & tangential coordinates (path is known)
- radial & transverse (path unknown, angular velocities & accelerations are specified



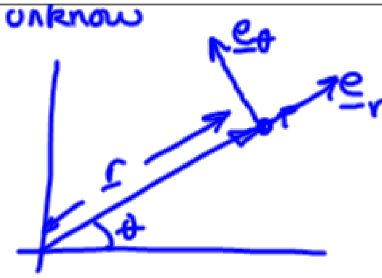
path known (i.e. circle, roller coaster parabola)

instantaneous rotation center

pendulum
wheels
pulleys
gears

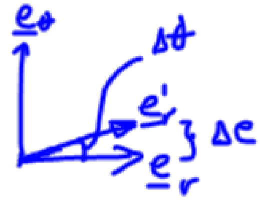
$\left\{\begin{array}{l}\text{roller coaster} \\ \text{curves on roads (radius specified)}\end{array}\right.$

path unknow



$$\underline{r} = r\,\underline{e}_r$$

$$\underline{v} = \dot{\underline{r}} = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt}$$



$$\Delta e = \sin\Delta\theta \simeq \Delta\theta\,\underline{e}_\theta$$

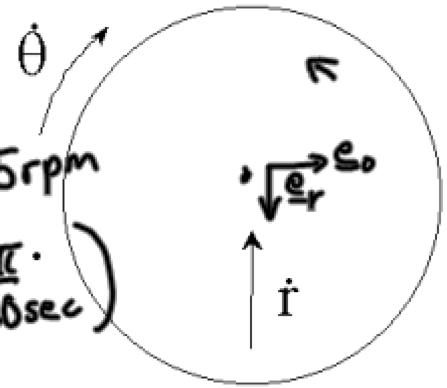
$$\Delta e_\theta = -\sin\Delta\theta = -\Delta\theta\,\underline{e}_r$$

$$\underline{v} = \dot{\underline{r}} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$\underline{a} = \dot{\underline{v}} = \ddot{\underline{r}} = \ddot{r}\underline{e}_r + \dot{r}\dot{\underline{e}}_r + \dot{r}\dot{\theta}\underline{e}_\theta + r\ddot{\theta}\underline{e}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

A boy walks at 4 m/s toward the center of a merry-go-round rotating clockwise at 5 rpm. Find his acceleration.



$= 5\left(\frac{2\pi}{60\,\text{sec}}\right)$

$\dot{r} = -4\,\text{m/s}\ \underline{e}_r$

$\ddot{r} = 0$ (constant speed towards center)

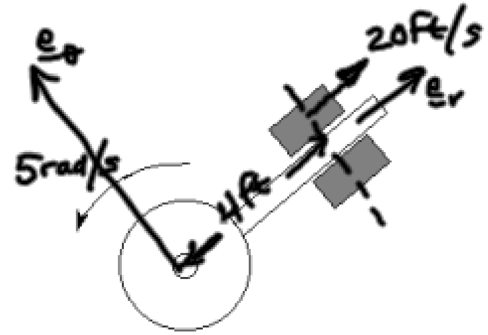
$\dot{\theta} = -\frac{5(2\pi)}{60\,\text{sec}}$ (rad/sec)

$\ddot{\theta} = 0$ (merry go-round has constant angular velocity)

$$\underline{a} = \left(0 - r\left(-\frac{10\pi}{60}\right)^2\right)\underline{e}_r + \left(r0 + 2(-4\,\text{m/s})\left(-\frac{10\pi}{60}\right)\right)\underline{e}_\theta$$

$$\underline{a} = -r\omega^2\,\underline{e}_r + 2\dot{r}\omega\,\underline{e}_\theta$$

A slider moves at 20 ft/s along a rod rotating about a pivot at 5 rad/s. Find acceleration 4 ft from the pivot?



$\dot{r} = 20\,\text{ft/s}$ $\qquad \ddot{r} = 0$ (?)

$r = 4\,\text{ft}$

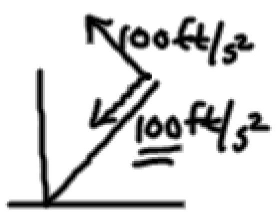
$\omega = 5\,\text{rad/s} = \dot{\theta}$

$\dot{\omega} = \ddot{\theta} = 0$ (constant rpm)

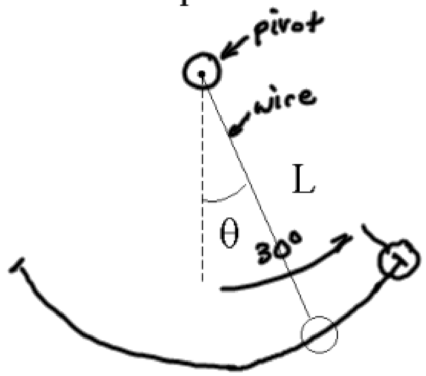
$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

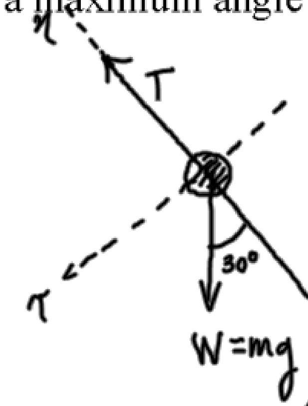
$$\underline{a} = \left(-\underline{4\,\text{ft}}\left(5\,\text{rad/s}\right)^2\right)\underline{e}_r + \left((4\,\text{ft})(0) + 2(20\,\text{ft/s})\left(5\,\tfrac{\text{rad}}{\text{s}}\right)\right)\underline{e}_\theta$$

$$= -100\,\text{ft/s}^2\,\underline{e}_r + 100\,\text{ft/s}\,\underline{e}_\theta$$



97. The maximum acceleration of a simple pendulum occurs at the top of a swing. Find this acceleration for a 2 m pendulum with a maximum angle of 30°.





$$\Sigma F_n = T - W\cos 30 = m a_n$$

$$\Sigma F_\tau = W\sin 30 = m a_\tau$$

$$\underbrace{\cancel{m} g \sin 30 = \cancel{m} a_\tau}$$

$$g \sin 30 = a_\tau$$

$$a_\tau = 10 \sin 30$$
$$= 5\ m/s^2$$

$$T - W\cos 30 = m a_n = \cancel{\frac{mv^2}{r}}^{\,0}$$

$$T = W\cos 30$$

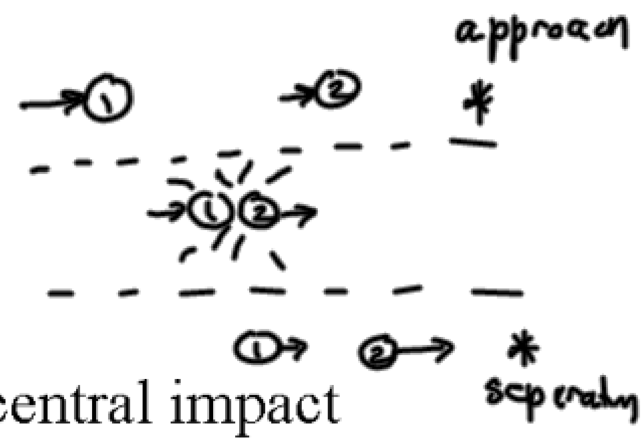
$$g \sin 30 = r\alpha = L\alpha$$

$$\alpha = \frac{g}{L}\sin 30$$

L gets large α gets small

Conservation of linear momentum

impact



- momentum constant for a central impact

$$\underbrace{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}_{\text{system momentum}} = \underbrace{m_1\mathbf{v}_1' + m_2\mathbf{v}_2'}_{\text{system momentum after impact}} + \Delta E$$

- dissipation affects the approach velocity

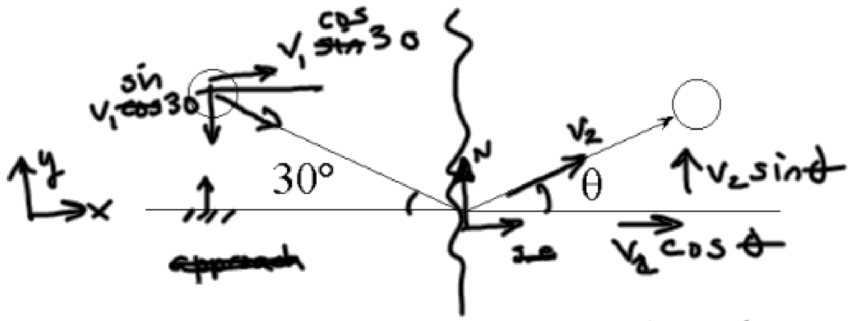
$$\overbrace{v_{1n}' - v_{2n}'}^{\text{separation}} = -e\overbrace{\left(v_{1n} - v_{2n}\right)}^{\text{approach}}$$

coefficient of restitution

$e = 1$

$e = 0$

99. A ball strikes a flat, horizontal surface at 30°. Find the reflection angle if the coefficient of restitution is 0.8.



x direction

$$v_1 \cos 30 = v_2 \cos\theta$$

$$v_1 = \frac{v_2 \cos\theta}{\cos 30}$$

y direction

direction

$$v_2 \sin\theta = -(0.8)(-v_1 \sin 30)$$

formula

$$\cancel{v_2} \sin\theta = 0.8\, \cancel{v_2} \frac{\cos\theta \sin 30}{\cos 30} = 0.8 v_2 \cos\theta \tan 30$$

$$\tan\theta = 0.8 \tan 30$$

$$\theta = \tan^{-1}(0.8 \tan 30) = 24.8°$$

Work done by/against
gravity – potential energy ; electrical fields
momentum – kinetic
springs – } potential
pressures –

$F \cdot d = W$

$F \cdot \frac{d}{t} = \frac{W}{t} = P$

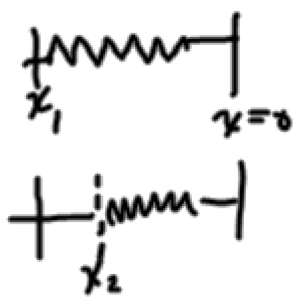
$\int P\,dt = W$

work and kinetic energy

$$W(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v}\, dt \quad \text{work}$$

examples: $\boxed{W = -mg\Delta h}$ $\quad W = -\frac{k}{2}\left(x_2{}^2 - x_1{}^2\right)$

gravitational $\qquad$ spring



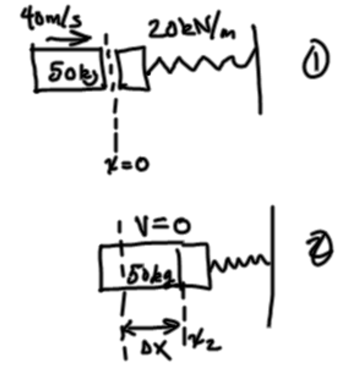
$$\boxed{KE(t) = \frac{1}{2} m\, \mathbf{v} \cdot \mathbf{v} \quad \text{kinetic energy}}$$

$$\underline{W(t_1, t_2)} = \underline{KE(t_2)} - \underline{KE(t_1)} \quad \text{work-energy relation}$$

98. A 50 kg object moving at 40 m/s strikes a spring (k = 20kN/m). Determine the maximum deflection.

K.E. system just before impact.

K.E. at full compression



ΔK.E. = W done on spring

from $W = -\frac{k}{2}(x_2^2 - x_1^2)$

$$KE_1 = \frac{1}{2}(50\,kg)(40\,m/s)^2$$
$$= 40{,}000\,N\cdot m$$

$$KE_2 = \emptyset$$

$$W_{1\to 2} = \cancel{-}40{,}000\,N\cdot m = \cancel{-}\underbrace{\frac{k}{2}(x_2^2 - x_1^2)}_{\text{spring}}$$

$$40{,}000\,N\cdot m = \frac{\left(\frac{20kN}{m}\right)}{2}(x_2^2)$$
$$= 10{,}000\,\frac{N}{m}(x_2^2)$$

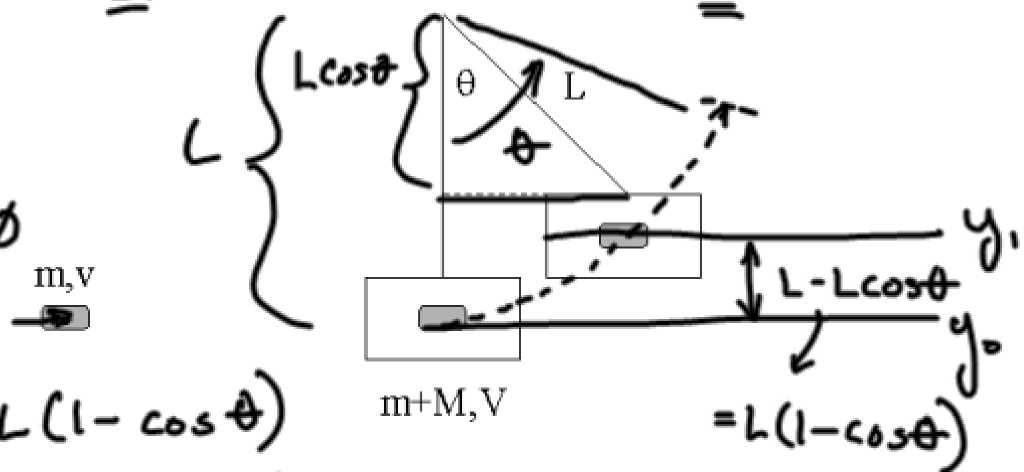
$$\frac{40{,}000\,N\cdot m\, m}{10{,}000\,N} = 4m^2 = x_2^2 \qquad x = 2m$$

A bullet of mass m strikes a stationary pendulum of mass M. Find the bullet velocity v in terms of the pendulum length L and the maximum angle θ.

K.E ~~before~~ at impact

K.E. at apogee = ∅

P.E. at apogee

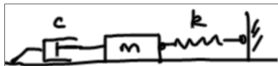


$$\frac{1}{2}mv^2 = (m+M)gL(1-\cos\theta)$$

$$v^2 = \frac{2(m+M)gL(1-\cos\theta)}{m}$$

$$v = \sqrt{2gL\left(\frac{m+M}{m}\right)(1-\cos\theta)}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)V^2$$



damping – friction
explicit damping

free vibrations

Forced vibration = f(t)

damping ratio ($0 \le \zeta < 1$)

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \ddot{x} + 2\zeta\omega_n\dot{x} + \boxed{\omega_n^2 x} = 0$$

damping, mass, spring, natural frequency

Fourier analysis & undet. coeff

$$x(t) = \exp(-\zeta\omega_n t)\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$\boxed{\omega_n = \sqrt{k/m}}\ ;\ \boxed{\zeta = \frac{c}{2\sqrt{mk}}}\ ;\ \boxed{\omega = \omega_n\sqrt{1-\zeta^2}}$$

$\zeta=1$ critically damped, $\zeta>1$ over-damped

periodic motion stops rapidly

periodic motion impossible

No damping $c = 0$

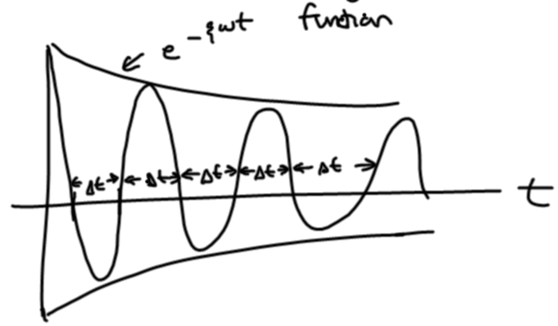
$\zeta = 0$

$x(t) = A\cos\omega t + B\sin\omega t$

If no spring, then no vibration

$$m\ddot{x} + c\dot{x} + kx = \underbrace{F_0 \sin\omega t}_{\text{Forcing function}}$$

at forcing frequency



8. Find the damping ratio and natural frequency of a system described by the following equation.

$$\ddot{y} + 8\dot{y} + 25y = \underline{\underline{16\sin(\Omega t)}}$$

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = 16\sin\Omega t$$

$$2\xi\omega_n = 8 \qquad \xi = \frac{8}{2\omega_n} = \frac{4}{\omega_n} = \frac{4}{5}$$

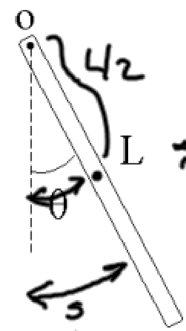
$$\cancel{\omega_n = \sqrt{\frac{k}{m}} =}$$

$$\omega_n^2 = 25; \quad \omega_n = 5 = \sqrt{\frac{k}{m}}$$

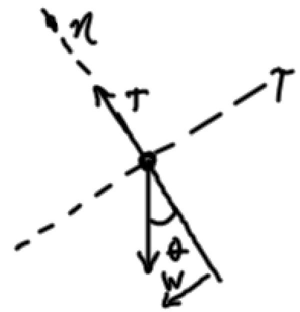
$$\sqrt{k} = 5$$

$$\therefore \quad k = 25$$

What is the natural frequency of a slender rod of length L and mass m that is pinned at one end?



$\approx$ pendulum



$$\Sigma F_t = -mg\sin\theta = m a_t$$

$$s = \frac{L}{2}\theta \qquad a_t = \frac{d^2 s}{dt^2}$$

$$\dot{s} = \frac{L}{2}\dot{\theta}$$

$$\ddot{s} = \frac{L}{2}\ddot{\theta}$$

$$-\cancel{m}g\sin\theta = \cancel{m}\frac{L}{2}\ddot{\theta}$$

$$\frac{L}{2}\ddot{\theta} + g\sin\theta = 0$$

$$\ddot{\theta} + \frac{2g}{L}\sin\theta = 0 \qquad \text{for small displacements } \sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{2g}{L}\theta = 0$$

$$m = 1, \quad c = 0, \quad k = \frac{2g}{L} \qquad \omega_n = \sqrt{\frac{2g}{L}}$$

planar rigid body motion

$\Sigma \underline{F} = m\underline{a}$

$\Sigma \underline{M} = I \underline{\alpha}$

mass moment of inertia

mass

8.2 p 344 review manual

moment of inertia

$$M_c = I_c \alpha \; ; \; \alpha = \dot{\omega}$$

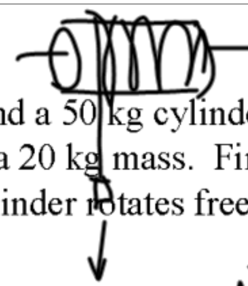
moment　　angular acceleration

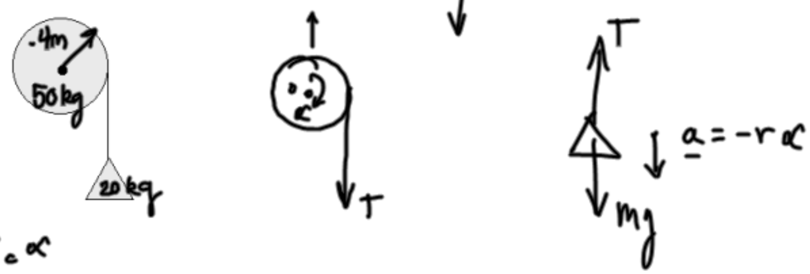
$M_c = I_c \alpha$

This equation applies at the center of mass ~~or a fixed pivot.~~

$I = I_c + md^2$

96. A string is wrapped around a 50 kg cylinder of radius 0.4 m and attached to a 20 kg mass. Find the tension in the string if the cylinder rotates freely?





$+\circlearrowleft \; \Sigma M_o = I_o \alpha$

$-T(0.4) = -\frac{mr^2}{2}\alpha$

$= -\frac{50(0.4)^2}{2}\alpha$

$T = \frac{50(.4)}{2}\alpha$

$\Sigma \underline{F} = m\underline{a}$

$T - mg = -r\alpha \cdot m$

$T - (20)(10) = -(0.4)\alpha(20)$

9.8

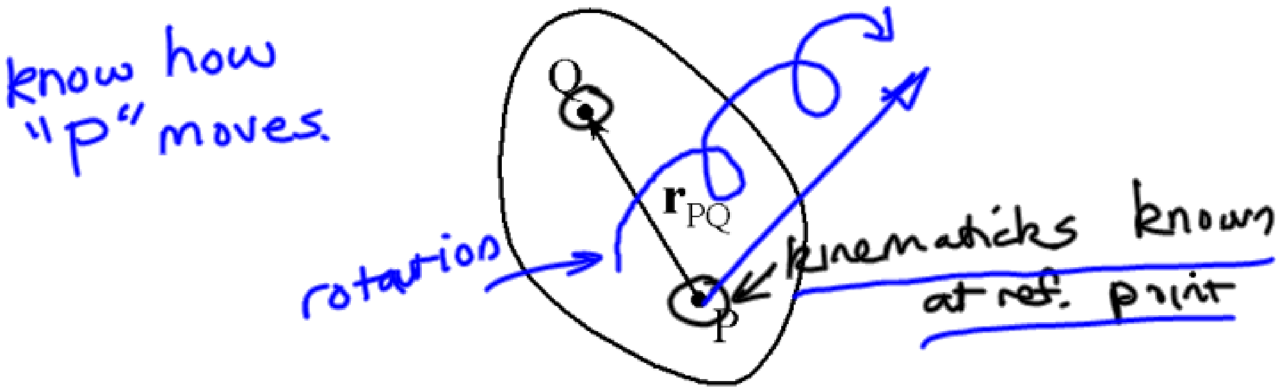
$\frac{50(.4)}{2}\alpha - 200 + 8\alpha = 0$

$18\alpha = 200$

$\alpha = 11.11 \text{ rad/s}^2$

$T = \frac{50(.4)}{2}(11.11 \text{ rad/s}^2) = \underline{\underline{111.1\text{N}}}$

velocity of a rigid body



position vector

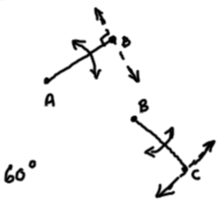
$$\mathbf{v}_Q = \mathbf{v}_P + \omega\times\mathbf{r}_{PQ}$$

velocity of Q = displacement of entire body + angular displacement of Q relative to P.

angular velocity

kinematics & trigonometry

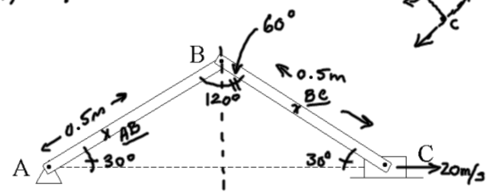
94. Find the angular velocity of link AB if the angles are both 30°. Both links are 50 cm long, and the slider moves at velocity 20 m/s to the right.



$V_{B/A} = \perp \underline{AB}$

$V_{C/B} = \perp \underline{BC}$

V_C = horizontal, 20m/s

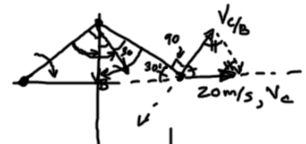


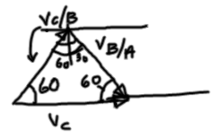
$V_B = \cancel{V_A}^{=0} + V_{B/A}$ (fixed)

$V_{B/A} = r\omega_{AB}$

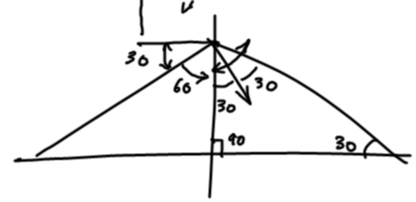
$V_C = V_B + V_{C/B}$

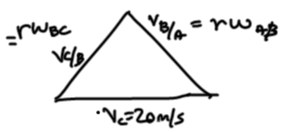
$V_{C/B} = r\omega_{BC}$





equilateral triangle





$\therefore$ 20 m/s $= r\omega_{BC}$

$\omega_{BC} = \dfrac{20}{(0.5m)} = 40\,\text{rad/sec}$

$r\,\omega_{AB} = 20\text{m/s}$

$\omega_{ab} = \dfrac{20\text{ m/s}}{(0.5m)} = 40\,\text{rad/sec}$

28. Find the slider velocity and the angular velocity of the 50 cm link BC. The 30 cm drive link is vertical with an angular velocity of 10 rad/s.

$V_B = V_A + V_{B/A}$

$V_A = \phi$ (A is fixed)

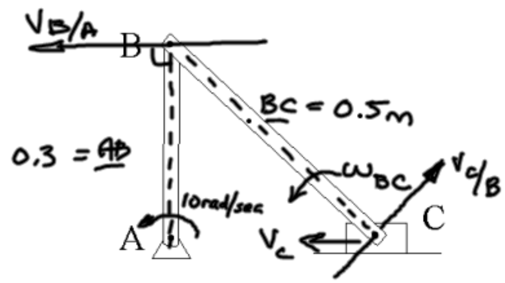
$V_C = V_B + V_{C/B}$

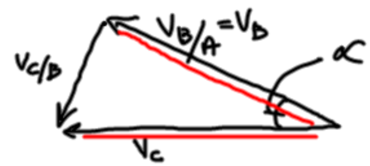
$= -(0.3)(10\,rad/s) + V_{C/B}$

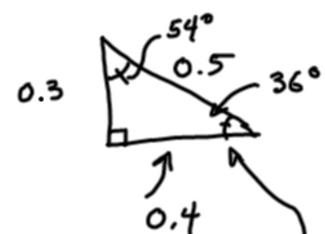
$-r\omega_{AB}$

$V_{C/B} = r\omega_{BC}$

$= (0.5)\omega_{BC}$







$\tan^{-1}\left(\frac{3}{4}\right) = \phi$

$V_{B/A}$ & V_C must be horizontal

$\alpha = \phi$

$V_{C/B} = \emptyset$ ∴ $\omega_{BC} = 0$