CIVE 4311 Engineering Design
Engineering Economics
Lecture 010

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Engineering economics on the FE will likely focus on cash-flow issues (present worth and future worth) , interest rates for buying money, and depreciation. Taxation, internal rate-of-return and break-even cost analysis are also possible. Larger economic issues regarding the creation of wealth (not the same as cash), flow of goods and materials at national scales are very unlikely.

Present worth and Future Value

Cash value is time dependent - the rate of interest charged reflects the lost opportunity cost of the cash to the lender. Engineering projects consume huge amounts of cash - they are rarely financed without OPM (Other People's Money); furthermore that cash is never available at one instant in time - Engineering as a business functions on borrowed money.

The value of an increment of cash today at some time in the future is called the future value or future worth. Typically FV is the future value, PV is the present value. Moving values forward and backward in time is fundamental in engineering economics.

FV of a single PV increment of cash

Table 1 illustrates a simple lump sum future worth equivalence. It can be worked in either direction (forward or backward) in time.

Table 1: Simple Table of Future Worth

Date	Beginning Value	Interest this Period	Ending Value
Year 1	P	$P \times i$	$P + P \times i = P(1+i)$
Year 2	$P(1+i)$	$P(1+i) \times i$	$P(1+i) + P(1+i) \times i = P(1+i)^2$
Year 2	$P(1+i)^2$	$P(1+i)^2 \times i$	$P(1+i)^2 + P(1+i)^2 \times i = P(1+i)^3$

As a practical tool, only the generic equation is needed.

$$FV = PV \times (1+i)^n \tag{1}$$

where FV and PV are the future- and present-value, i is the interest rate (rate of growth per counting period - typically one year), and n is the number of counting periods.

Training exercises:

1. What is the FV of \$ 1000 invested at 8% interest after 20 years?

2. What amount of cash PV must be invested today to accumulate a $FV = \$10,000$ in 5 years with a 6% interest rate?

Annuity (Stream of Payments) and Future Worth

An annuity is a uniform (all same amount - like a house note) series of payments or receipts of a set amount and frequency.

If FV is the future value, and A is the individual payment amount then

$$FV = A \times \frac{(1+i)^n - 1}{i} \tag{2}$$

and

$$A = FV \times \frac{i}{(1+i)^n - 1} \tag{3}$$

Training exercises:

1. You deposit \$2000 at the end of the year towards your retirement fund. How much will you have accumulated at the end of 10 years if the growth rate is 6% per year?

2. You deposit \$100 every month for 2 years at 12% annual interest compounded monthly. How much will you have saved in 2 years?

3. To save up to \$10,000 at the end of 5 years, how much should be deposited every month in your bank account if the bank pays 3.2% per year, compounded monthly?

Annuity (Stream of Payments) and Present Worth

An annuity is a uniform (all same amount - like a house note) series of payments or receipts of a set amount and frequency.

If PV is the present value, and A is the individual payment amount then

$$PV = A \times \frac{(1+i)^n - 1}{i(1+i)^n} \tag{4}$$

and

$$A = PV \times \frac{i(1+i)^n}{(1+i)^n - 1} \tag{5}$$

Training exercises:

1. How much must be deposited today to withdraw \$500 every year for 10 years, assuming an 8%interest rate ?

2. If you take out a \$100,000 payable monthly over 5 years with monthly installments of \$2225 each month, what is the nominal interest rate for the loan?

3. What is the cash value of a used car purchased under the following terms: \$500 today and \$250 every month for 2 years with an interest rate of 1% per month?

Gradient Worth

Gradient worth is similar to an annuity except the payment or expense stream decreases or increases linearly over time.

If PV is the present value, and A_G is the accumulated gradient value, G is the individual payment increment amount then

$$A_G = G \times (\frac{1}{i} - \frac{n}{(1+i)^n - 1}) \tag{6}$$

where $A_T = A_G$ and

$$PV = G \times (\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}) \tag{7}$$

Training exercise:

A service plan for a machine costs \$100 after the first year, \$120 after the second year, \$140 after the third year, and so on. The machine will last 14 years. Assuming a 10% interest rate, what is the present worth of the machine's service plan?