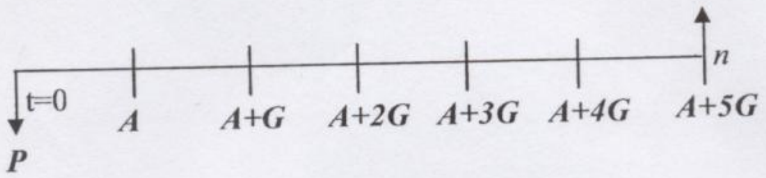
1.4 Gradient Worth Equivalence

$$A_G = G\left(\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right) \text{--}> A_G = G\left(A/G, i\%, n\right)$$
$$A_T = A + A_G$$
$$P = G\left(\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}\right) \text{--}> P = G\left(P/G, i\%, n\right)$$

Cash Flow Diagram For Gradient Equivalence:



✓ The gradient starts at $t=2$. However, the table makes the adjustment for it starting at the end of the second year.

Example 1.4.1

A maintenance plan for piece of machinery costs \$100 after the first year, \$120 after the second year, \$140 after the third year, and so on. The machinery lasts 14 years. A 10% interest rate is assumed. What is the present worth of the maintenance costs?

Example 1.4.2

For the above problem, what is the equivalent annual cost of the maintenance plan?

1.5 Nominal Interest Rate versus Effective Interest Rate

The nominal interest rate *(r)* is the annual interest rate not taking into consideration the effects of compounding. The effective interest rate (i_e) is the annual interest rate taking into consideration the effects of compounding. The number of compounding periods within a year is *m*.

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Tips:

✓ The nominal rate is equal to the effective interest rate when the compounding period is a year; otherwise, if compounding is done more frequently, the effective interest rate is higher than the nominal rate.

Example 1.5.1

A bank has a 6% interest rate compounded monthly. Calculate the interest rate per month, and the effective interest rate.

Example 1.5.1

The bank pays 2% per a quarter. What is the nominal rate?

1.6 Bonds

Bonds are used by the government and corporations to raise money. The Face Value of the bond is the principle to be paid back to the hold at the end of the maturity period. For some bonds, dividends are paid out during the maturity period, and are usually paid out on a semi-annually basis.

The present worth of a bond is equal to the present worth of the face value at the maturity period plus the present worth of any dividends paid out during the maturity period.

Example 1.6.1

A $10,000 bond matures in 10 years and has a rate of return of 6%. What is the purchase price of the bond?

1.7 Capitalized Cost

Capitalized costs are the present worth of the costs of a project with an infinite or perpetual time period.

$$Capitalized\ Costs = I + \frac{A}{i}$$

Example 1.7.1

A scholarship fund for an annual payment of $10,000 per year. How much must be invested today, assuming an 8% interest rate?

Example 1.7.2

The city is building bridge that will cost $1.2 million dollars today and $25,000 per year to maintain. How much money needs to be invested today at a 10% interest rate to ensure the city has the payments to begin and maintain the project?

1.8 Depreciation

Important Variables to Know when Calculating Depreciation:

C → Initial cost of asset
D_i → Depreciation for a specific year
BV_i → Book value at the end of a specific year
S_n → Asset value at the end of its useful life
n → Estimated useful life of an asset.

Two common methods for depreciation that are tested on the EIT.

1) Straight Line Depreciation - The asset will lose the same value each year.

2) Modified Accelerated Cost Recovery System (ACRS) Depreciation – The asset will lose a percentage of its initial cost each year. The salvage value is always zero. The depreciation factors must be read from the modified ACRS table in the FE reference handbook.

	Straight Line	***ACRS***
Initial Cost	C	C
Depreciation	$D_i = \frac{C - SV}{n}$	$D_i = f_i C$ *(f_i is the factor obtained from table)*
Book Value	$BV_i = C - (t)(D_i)$	$BV_i = C - (D_1 + D_2 + \ldots + D_i)$
Salvage Value	$SV = C - (n)(D_i)$	*0*
Estimated useful life	n	*3, 5, 7, or 10 year recovery period*

Example 1.8.1

A company purchased a machine for $250,000 with an expected 5-year useful life and a $40,000 salvage value. Which depreciation method, straight line or ACRS, will give a higher book value at the end of the 3rd year?

1.9 Taxes

Depreciation is calculated per each year and is used for tax purposes. IRS allows depreciation to be tax-deductible. Taxes are completed on a yearly basis.

Year	*Net Income Before Taxes*	*Depreciation*	*Taxable Income*	*Income Taxes*	*Net Income After Taxes*
i	I_b = *Revenues – Expenses*	D_i	$T = I_b - D_i$	$taxes = T * tax\ rate$	$I_a = I_b - taxes$

Example 1.9.1

The initial investment of a machine is \$150,000. The machine has operating costs of \$9,000 per a year. The company expects to receive revenues of \$35,000 each year as a result of the investment. The salvage value is \$30,000 for the machine, and its useful life is 10 years. The income tax rate is 35%. What is the after-tax net income for the 3rd year using straight line depreciation?

1.10 Net Present Value

Net Present Value (NPV) can be used to evaluate one project or compare multiple projects. The net present value is computed by finding the present worth of all the benefits through the lifetime of the project and subtracting the present worth of all the costs.

$$NPV = PW_{Benefits} - PW_{Costs}$$

Tips:

- ✓ To determine if a project is economically feasible, the NPV of the project should be equal to or greater than 0.
- ✓ When comparing projects using present worth, the time period must be the same for the projects.

Example 1.10.1

A company will purchase a machine for \$50,000, and it lasts 10 years. The machine is expected to sell for \$10,000 at the end of its useful life. The machine is expected to give an annual benefit of \$5000. The annual interest rate is 8%. What is the net present value of the investment?

1.11 Rate of Return

The rate of return is the interest rate that makes the present worth of the costs equal to the present worth of the benefits, or in other words, the NPV is 0.

Example 1.11.1

An $8000 bond has a maturity period of 6 years. The price of the bond is $4000. What is the rate of return for a bond holder?

1.12 Benefit / Cost Analysis

A benefit / cost ratio is the worth of the benefits divided by the worth of the costs. A project is economically good when the ratio is equal or greater than 1.

$$\frac{B}{C} = \frac{PW_{Benefits}}{PW_{Costs}}$$

Example 1.12.1

A proposal is being considered by the city government to purchase a building that will cost $500,000 and will require an annual maintenance fee of $40,000. The building will save the city $150,000 in rent fees each year. The building is expected to be used for 20 years. Should the city buy the building, assuming a 10% interest rate?

1.13 Equivalent Uniform Annual Cost

The annual cost analysis compares a project by looking at its costs minus benefits over an annual basis. EUAC can be used to compare projects with unequal time periods.

Example 1.13.1

A computer has an initial cost of $1000, an annual maintenance cost of $40 per a year, and a salvage value of $150. The useful life of the computer is 5 years. What is the equivalent uniform annual cost? Assume an interest rate of 6%.

Example 1.13.2

The maintenance costs for a machine that has a useful life of 6 years are: Year 1=$150, Year 2=$200, Year 3=$250, and so on. What is the equivalent uniform annual cost? Assume an interest rate of 6%.

Example 1.13.3

A company will purchase a machine for $50,000, and it lasts 10 years. The machine is expected to sell for $10,000 at the end of its useful life. The machine has an annual maintenance cost of $5000. The annual interest rate is 8%. What is its EUAC?

Example 1.13.4

A company is considering two investments: Investment A has an initial cost of $10000, generates $3,500 in revenues each year, and has a service life of 6 years. Investment B has an initial cost of $13,000, generates $5,300 in revenues each year, and has a service life of 4 years. Which investment should be selected? Assume an interest rate of 8%..

1.14 Breakeven Analysis for Economic Functions

The breakeven point for a function that has a dependent variable is where the costs of the function equal the benefits of the function. The breakeven point when comparing two functions that have the same dependent variable is where the costs for both functions are identical.

Example 1.14.1

A company invests $155,000 in manufacturing equipment that has a useful life of 6 years. The machine generates a profit of $5.50 per unit. How many units need to be produced to break even? Assumed an interest rate of 10%.

Example 1.14.2

The sales director is considering two options for his sales team. The company can lease a car for each salesperson for $1500 and pay 0.20 cents/mile each year. The lease lasts 3 years. The second option is to have each salesperson use their personal car and pay out 0.27 cents/mile. How many miles per a year will make the options equivalent?

1.15 Breakeven Analysis for Manufacturing

The breakeven point for a manufacturing facility for a certain item is where:

Total Revenues = Total Costs

(Selling Price) (Number of Items Sold) = Fixed Costs + (Variable Cost per Item) (Number of Items Sold)

Fixed costs do not change even if the production level does, while variable costs vary at a constant rate based on the production level.

Example 1.15.1

The fixed costs are $8000 per a month, and the variable costs are $10 per a unit. The revenues earned are $20 per a unit. The manufacturing facility has a production capacity of 3,000 units per month. Determine the level of production at which the facility will breakeven.

Example 1.15.2

At what level of production must the facility operate at to make a profit of $3000?

1.16 Inflation

Inflation deflates the present value of future sums of money. To take into consideration the effects of inflation, a combined rate d can be calculated using the interest rate i and inflation rate f.

$$\boxed{d = i + f + (i \times f)}$$

Example 1.16.1

If the interest rate is 4% and the inflation rate is 3%, what is the present value of $10,000 due five years from now?