problem distribution

- morning: 8% (9 out of 120 problems)
- afternoon: 7% (4 out of 60 problems)

slides reordered just before presentation. The two "pendulum" examples are moved closer to their logical placement.

subject areas

• 23%	vibrations (3)
• 23%	constant acceleration (3)
• 15%	velocity/acceleration (2)
• 15%	force/motion (2)
• 15%	simple mechanisms (2)
• 8%	work/energy (spring) (1)

- some solutions are "corrected"
- sign errors

Engineering application of Newton's laws of motion

free body diagram

1^{st} $\underline{a} = 0$ unless ext. force

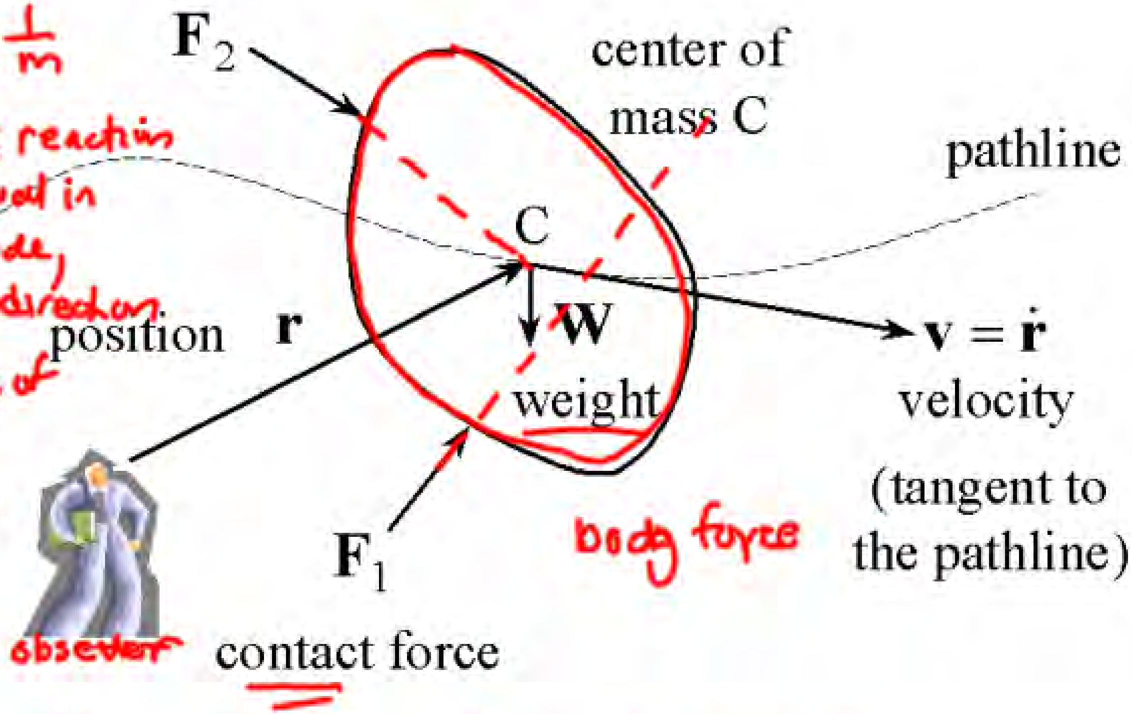
2^{ND} $\underline{a} \propto F$

$\underline{\alpha} \propto \frac{1}{m}$

3^{rd} action & reaction are equal in magnitude, opposite direction, same line of action

$F_g \propto m_1 m_2$

$F_g \propto \frac{1}{r^2}$



inertial reference $\underline{a} = 0$ for observer

$\Sigma F_x = m a_x$
$\Sigma F_y = m a_y$
$\Sigma F_z = m a_z$

equations of motion

total force — mass

$$\mathbf{F} = m\mathbf{a}$$

acceleration of the body's center of mass

- equation only valid in an inertial frame
- force obtained using a free body diagram
  - body forces (weight)
  - surface or contact forces

$\underline{r}$ is position vector

$\underline{v} = \frac{d}{dt}(\underline{r}) = \dot{\underline{r}} = \dot{x}\underline{i} + \dot{y}\underline{j} + \dot{z}\underline{k}$ $\qquad \dot{x} = \frac{dx}{dt}$

$\underline{a} = \frac{d}{dt}(\underline{v}) = \dot{\underline{v}} = \ddot{\underline{r}}$

acceleration

$$\ddot{x} = \frac{dV_x}{dt} = \frac{d^2x}{dt^2}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}}$$

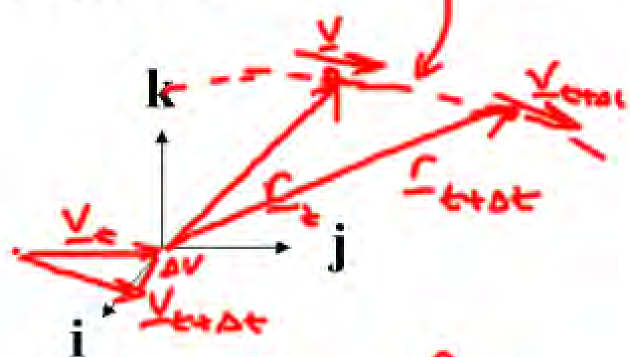
velocity position

Cartesian components

$$\mathbf{r} = \underline{x}\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$

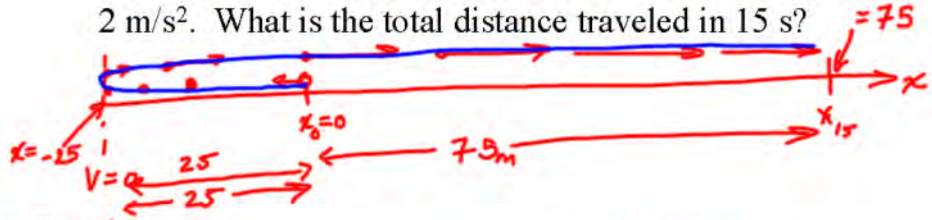
$$\mathbf{a} = \underline{\ddot{x}}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$



$disp = \underline{r}_{t+\Delta t} - \underline{r}_t$

$$\lim_{\Delta t \to 0} \frac{\underline{r}_{t+\Delta t} - \underline{r}_t}{\Delta t} = \frac{dr}{dt}$$

92. An object with initial velocity −10 m/s accelerates at 2 m/s^2. What is the total distance traveled in 15 s?



2 parts.

time from x_0 until V = 0, → distance

time remaining until 15 sec → distance

$$a = \ddot{x}$$

$$\int a\,dt = \int d\dot{x}$$

$$\int at + c_1 = \int \dot{x}$$

$$\frac{1}{2}at^2 + c_1 t + c_2 = x(t)$$

$$x_0 = x(0) = c_2$$

$$\cancel{\frac{1}{2}at^2 + c_1 t + x_0 = x(t)}$$

$$\cancel{x(0) = x_0 =}$$

$$at + c_1 = \dot{x} = V_0 \quad (\text{at } t=0)$$

$$c_1 = V_0$$

$$\frac{1}{2}at^2 + V_0 t + x_0 = x(t)$$

- rectilinear motion
- constant acceleration

① $x = \frac{1}{2}at^2 + V_0 t + x_0$

$x_0 = 0$

$V_0 = -10\,m/s$ (given)

$a = 2\,m/s^2$ (given)

$\dot{x} = at + V_0$ for $\dot{x} = 0$ (when $V=0$)

$0 = at + V_0$

$\frac{0 - V_0}{a} = t = \frac{0 - (-10\,m/s)}{2\,m/s^2} = 5\,sec.$

Distance in 5 sec.

$x(5) = \frac{1}{2}(2\,m/s^2)(5\,sec)^2 + (-10\,m/s)(5\,sec) + 0$

$= -25$

$x(15) = \frac{1}{2}(2)(15)^2 - 10(15) + 0 = 75\,meters$

Total distance

25m	x=0, x=-25	5 sec
25m	x=-25, x=0	10 sec
75m	x=0, x=75	
125m		

constant acceleration

initial velocity

$$a = a_0$$

$$v = a_0 t + v_0 \;;\; t = \frac{v - v_0}{a_0}$$

t_1 t_0

$$s - s_0 = \frac{a_0 t^2}{2} + v_0 t = \frac{v^2 - v_0{}^2}{2a_0} = \frac{(v + v_0)}{2} t$$

initial position

mean velocity

$s(t) = s_0 + \frac{1}{2} a_0 t^2 + V_0 t$

$s(t) - s_0 = \frac{1}{2} a t^2 + v_0 t$

reindex location of s_0 in multipart problem (to handle negative displacements)

locations – solve directly

displacements – from difference in locations.

$v_y=0$ here

An object is launched upward at an initial velocity of 40 m/s. How high will it go?

sketch

$$y=\frac{1}{2}a_y t^2 + v_{y_0} t + y_0 \qquad a=-g$$

$$\dot{y} = a_y t + v_{y_0} \quad \text{solve for } t$$

$$0 = (-9.8\,m/s^2)(t) + 40\,m/s$$

$$\underbrace{\frac{40}{9.8} = t}_{\text{"precise"}} \qquad \underbrace{\left(\frac{40}{10}\right) = 4\,sec}_{\text{close approx}}$$

$$y = \frac{1}{2}(-9.8)(4)^2 + (40)(4) + 0$$

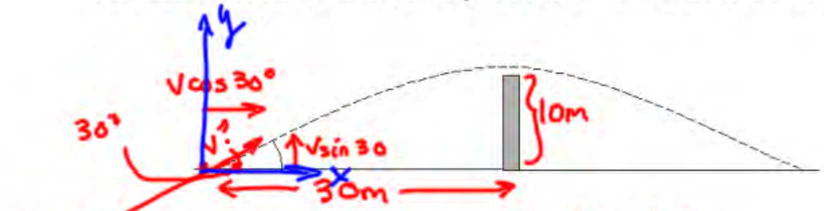
$$\approx \frac{1}{2}(-10)(16) + (160)$$

$$= -80 + 160 = \cancel{240\ meters}$$

80 meters

"minus sign" $-a_y$ is down

93. A cannon is fired at 30° above the horizon. What is the minimum exit velocity needed to clear a 10 m wall?



$u_0 = V\cos 30$

$V_0 = V\sin 30$

2 equations x-direction y-direction

$$x = \tfrac{1}{2}\cancel{a_x}^{=0}t^2 + u_0 t + \cancel{x_0}^{0} = 30 \Rightarrow V\cos 30\, t = 30$$

$$y = \tfrac{1}{2}a_y t^2 + V_0 t + y_0 = 10 \Rightarrow \qquad V = \frac{30}{\cos 30\, t}$$

$$(9.8) \rightarrow \frac{-10\,m/s^2}{2}t^2 + V\sin 30\, t + \cancel{y_0}^{=0} = 10$$

$$10 = -5t^2 + \frac{30\sin 30}{\cos 30\, t}t = -5t^2 + 30\tan 30$$

$$t^2 = \frac{10 - 30\tan 30}{-5} = 1.464$$

$$\underline{\underline{t = 1.21}}$$

$$V = \frac{30}{\cos 30\,(1.21\,sec)} = 28.6\,m/sec$$

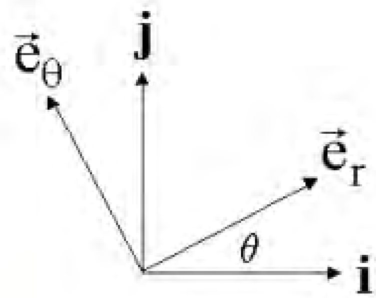
polar components– relative to a cartesian system

angular velocity (ω)

$$\mathbf{r} = r\mathbf{e}_r$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\overbrace{\dot{\theta}}^{\text{angular velocity }(\omega)}\mathbf{e}_\theta$$

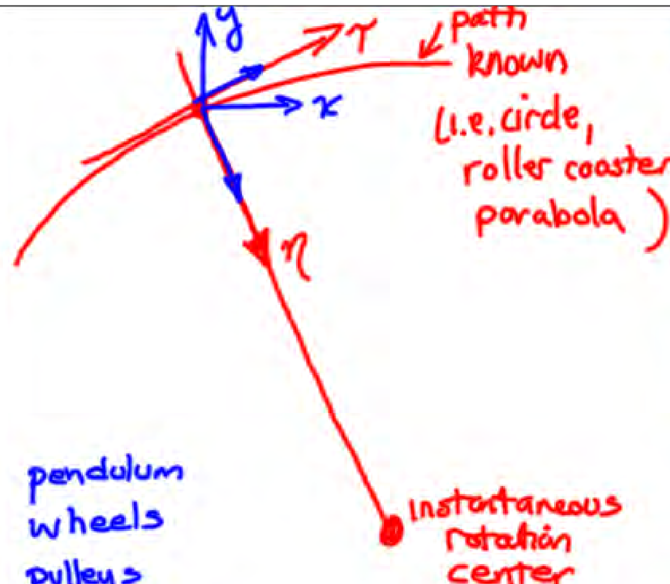
$$\mathbf{a} = \Big(\underbrace{\ddot{r}}_{\text{radial}} - \underbrace{r\dot{\theta}^2}_{\text{centripetal}}\Big)\mathbf{e}_r + \Big(\underbrace{r\ddot{\theta}}_{\text{angular acceleration }(\alpha)} + \underbrace{2\dot{r}\dot{\theta}}_{\text{Coriolis}}\Big)\mathbf{e}_\theta$$



radial Coriolis

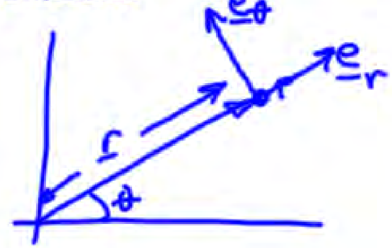
centripetal angular acceleration (α)

- normal & tangential coordinates (path is known)
- radial & transverse (path unknown, angular velocities & accelerations are specified



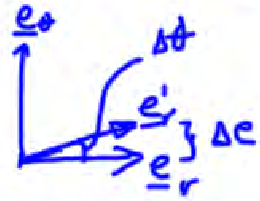
pendulum
wheels
pulleys
gears
{roller coaster
curves on roads (radius specified)

path unknown

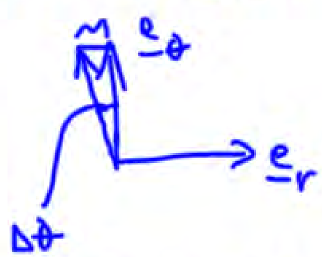


$\underline{r} = r\,\underline{e}_r$

$\underline{v} = \dot{\underline{r}} = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt}$



$\Delta e = \sin\Delta\theta \simeq \Delta\theta\,\underline{e}_\theta$



$\Delta e_\theta = -\sin\Delta\theta = -\Delta\theta\,\underline{e}_r$

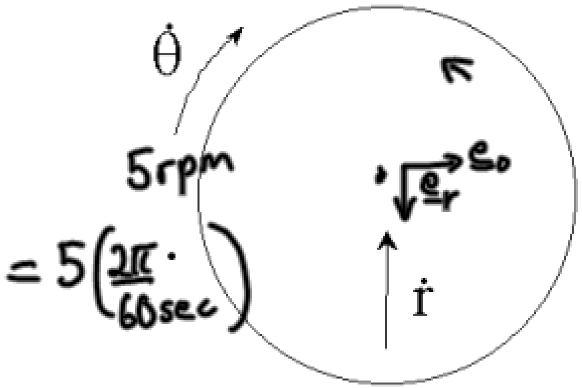
$\underline{v} = \dot{\underline{r}} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

$\underline{a} = \dot{\underline{v}} = \ddot{\underline{r}} = \ddot{r}\underline{e}_r + \dot{r}\dot{\underline{e}}_r + \dot{r}\dot{\theta}\underline{e}_\theta$

$+\, r\ddot{\theta}\underline{e}_\theta + r\dot{\theta}\dot{\underline{e}}_\theta$

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

A boy walks at 4 m/s toward the center of a merry-go-round rotating clockwise at 5 rpm. Find his acceleration.



$\dot{r} = -4 m/s\ \underline{e}_r$

$\ddot{r} = 0$ (constant speed towards center)

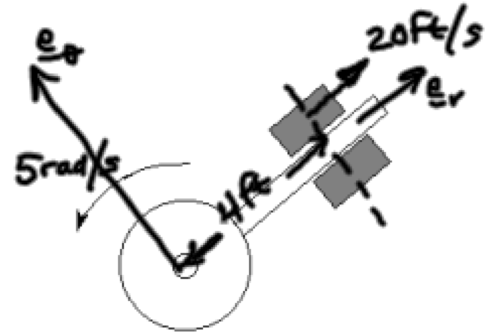
$\dot{\theta} = -\frac{5(2\pi)}{60sec}$ (rad/sec)

$\ddot{\theta} = 0$ (merry go-round has constant angular velocity)

$\underline{a} = \left(0 - r\left(-\frac{10\pi}{60}\right)^2\right)\underline{e}_r + \left(r0 + 2(-4m/s)\left(-\frac{10\pi}{60}\right)\right)\underline{e}_\theta$

$\underline{a} = -r\omega^2\underline{e}_r + 2\dot{r}\omega\underline{e}_\theta$

A slider moves at 20 ft/s along a rod rotating about a pivot at 5 rad/s. Find acceleration 4 ft from the pivot?



$\dot{r} = 20\,\text{ft/s}$ $\quad\quad$ $\ddot{r} = 0$ (?)

$r = 4\,\text{ft}$

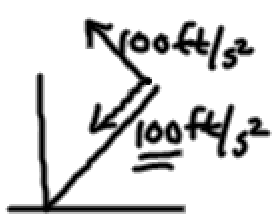
$\omega = 5\,\text{rad/s} = \dot{\theta}$

$\dot{\omega} = \ddot{\theta} = 0$ (constant rpm)

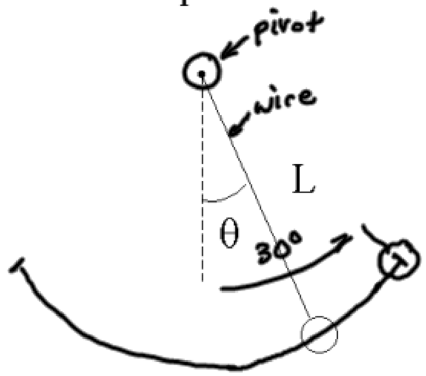
$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underline{e}_\theta$$

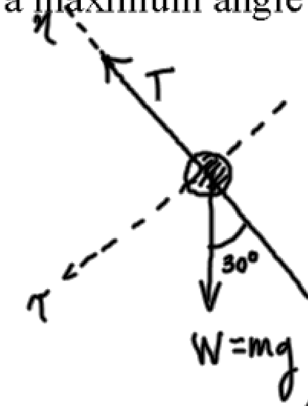
$$\underline{a} = \left(-4\,\text{ft}\,(5\,\text{rad/s})^2\right)\underline{e}_r + \left((4\,\text{ft})(0) + 2(20\,\text{ft/s})(5\,\tfrac{\text{rad}}{\text{s}})\right)\underline{e}_\theta$$

$$= -100\,\text{ft/s}^2\,\underline{e}_r + 100\,\text{ft/s}\,\underline{e}_\theta$$



97. The maximum acceleration of a simple pendulum occurs at the top of a swing. Find this acceleration for a 2 m pendulum with a maximum angle of 30°.





$\Sigma F_N = T - W\cos 30 = m a_n$

$\Sigma F_\tau = W \sin 30 = m a_\tau$

$\cancel{m} g \sin 30 = \cancel{m} a_\tau$

$g \sin 30 = a_\tau$

$a_\tau = 10 \sin 30$
$= 5 \, m/s^2$

$T - W\cos 30 = m a_n = \cancel{m \frac{v^2}{r}} \; 0$

$T = W \cos 30$

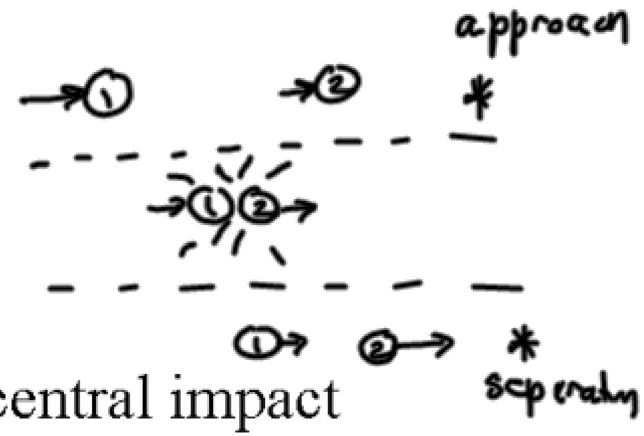
$g \sin 30 = r\alpha = L\alpha$

$\alpha = \frac{g}{L} \sin 30$

L gets large α gets small

Conservation of linear momentum

impact



- momentum constant for a central impact

$$\underbrace{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}_{\text{system momentum}} = \underbrace{m_1\mathbf{v}_1' + m_2\mathbf{v}_2'}_{\text{system momentum after impact}} + \Delta E$$

- dissipation affects the approach velocity

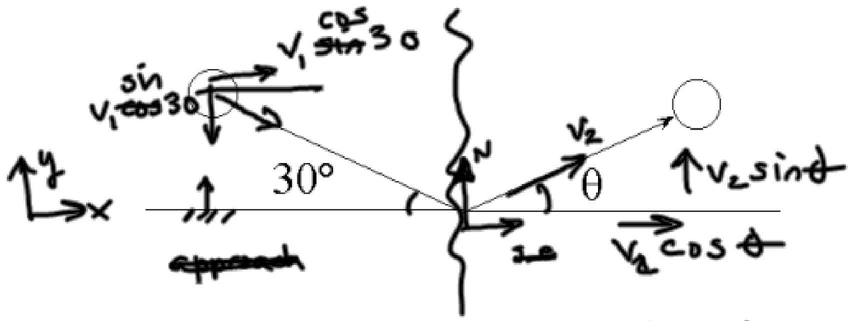
$$\underbrace{v_{1n}' - v_{2n}'}_{\text{separation}} = -e\,\underbrace{\left(v_{1n} - v_{2n}\right)}_{\text{approach}}$$

e: coefficient of restitution

$e = 1$

$e = 0$

99. A ball strikes a flat, horizontal surface at 30°. Find the reflection angle if the coefficient of restitution is 0.8.



x direction

$$v_1 \cos 30 = v_2 \cos\theta$$

$$v_1 = \frac{v_2 \cos\theta}{\cos 30}$$

y direction

direction

$$v_2 \sin\theta = -(0.8)(-v_1 \sin 30)$$

formula

$$\cancel{v_2} \sin\theta = 0.8\, \frac{\cancel{v_2} \cos\theta \sin 30}{\cos 30} = 0.8\, v_2 \cos\theta \tan 30$$

$$\tan\theta = 0.8 \tan 30$$

$$\theta = \tan^{-1}(0.8 \tan 30) = 24.8°$$

Work done by/against
gravity – potential energy; electrical fields
momentum – kinetic
springs – } potential
pressures – }

work and kinetic energy

$F \cdot d = W$

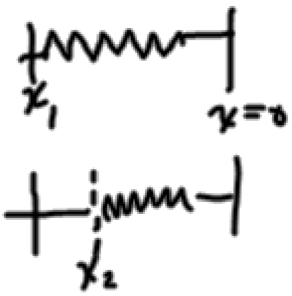
$F \cdot \frac{d}{t} = \frac{W}{t} = P$

$\int P\,dt = W$

$$W(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v}\,dt \quad \text{work}$$

examples: $\boxed{W = -mg\Delta h}$ $W = -\frac{k}{2}\left(x_2{}^2 - x_1{}^2\right)$

gravitational spring

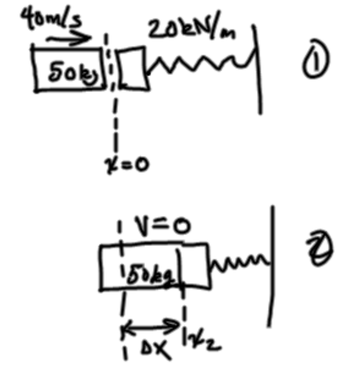


$$\boxed{KE(t) = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} \quad \text{kinetic energy}}$$

$$\underline{W(t_1, t_2)} = \underline{KE(t_2)} - \underline{KE(t_1)} \quad \text{work-energy relation}$$

98. A 50 kg object moving at 40 m/s strikes a spring (k = 20kN/m). Determine the maximum deflection.

K.E. system just before impact.



K.E. at full compression

$\Delta K.E. = W$ done on spring

from $W = -\frac{k}{2}(x_2^2 - x_1^2)$

$$KE_1 = \frac{1}{2}(50\,kg)(40\,m/s)^2$$
$$= 40{,}000\,N\cdot m$$

$$KE_2 = \emptyset$$

$$W_{1\to2} = \cancel{-}40{,}000\,N\cdot m = \cancel{-}\underbrace{\frac{k}{2}(x_2^2 - x_1^2)}_{\text{spring}}$$

$$40{,}000\,N\cdot m = \frac{\left(\frac{20kN}{m}\right)}{2}(x_2^2)$$
$$= 10{,}000\,\frac{N}{m}(x_2^2)$$

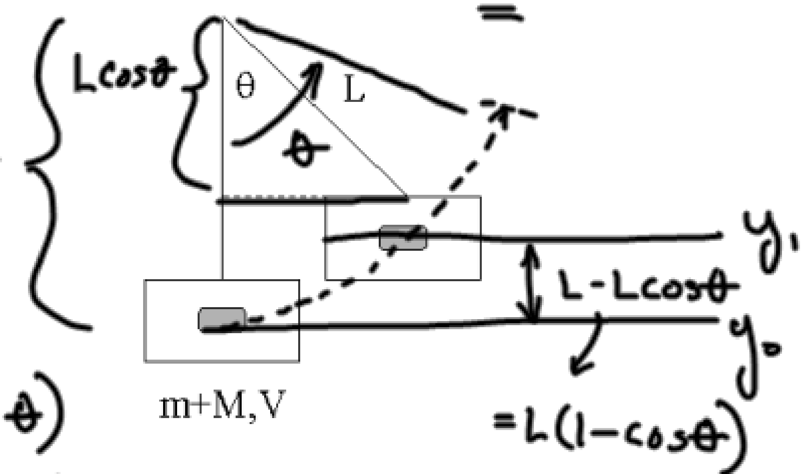
$$\frac{40{,}000\,N\cdot m\, m}{10{,}000\,N} = 4m^2 = x_2^2 \qquad x = 2m$$

A bullet of mass m strikes a stationary pendulum of mass M. Find the bullet velocity v in terms of the pendulum length L and the maximum angle θ.

K.E ~~before~~ at impact

K.E. at apogee = $\emptyset$

P.E. at apogee

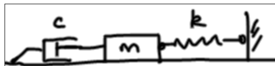


$$\frac{1}{2}mv^2 = (m+M)gL(1-\cos\theta)$$

$$v^2 = \frac{2(m+M)gL(1-\cos\theta)}{m}$$

$$v = \sqrt{2gL\left(\frac{m+M}{m}\right)(1-\cos\theta)}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)V^2$$



damping – friction
explicit damping

free vibrations

Forced vibration = f(t)

damping ratio ($0 \le \zeta < 1$)

$$\underbrace{m\ddot{x} + c\dot{x} + kx}_{\text{damping}} = 0 \qquad \ddot{x} + 2\zeta\omega_n\dot{x} + \boxed{\omega_n^2 x} = 0$$

mass spring natural frequency

Fourier analysis & undet. coeff

$$x(t) = \exp(-\zeta\omega_n t)\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$\boxed{\omega_n = \sqrt{k/m}}\ ; \boxed{\zeta = \frac{c}{2\sqrt{mk}}}\ ; \boxed{\omega = \omega_n\sqrt{1-\zeta^2}}$$

ζ=1 critically damped, ζ>1 over-damped

periodic motion stops rapidly

periodic motion impossible

No damping $c = 0$

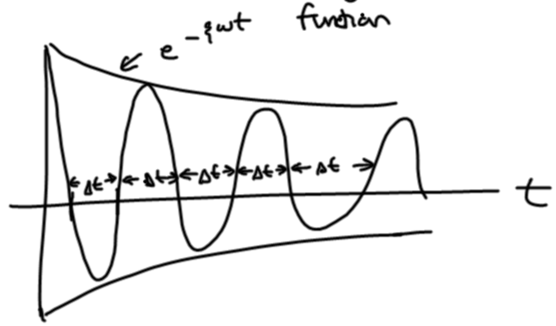
$\zeta = 0$

$x(t) = A\cos\omega t + B\sin\omega t$

If No spring, then no vibration

$$m\ddot{x} + c\dot{x} + kx = \underbrace{F_0 \sin\omega t}_{\text{Forcing function}}$$

at forcing frequency



8. Find the damping ratio and natural frequency of a system described by the following equation.

$$\ddot{y} + 8\dot{y} + 25y = \underline{\underline{16\sin(\Omega t)}}$$

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = 16\sin\Omega t$$

$$2\xi\omega_n = 8 \qquad \xi = \frac{8}{2\omega_n} = \frac{4}{\omega_n} = \frac{4}{5}$$

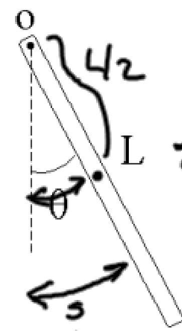
$$\cancel{\omega_n = \sqrt{\frac{k}{m}} =}$$

$$\omega_n^2 = 25\ ; \quad \omega_n = 5 = \sqrt{\frac{k}{m}}$$

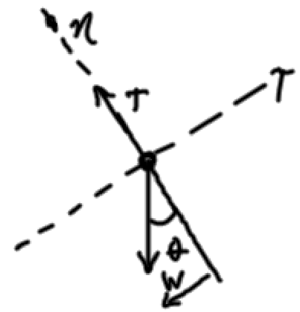
$$\sqrt{k} = 5$$

$$\therefore \quad k = 25$$

What is the natural frequency of a slender rod of length L and mass m that is pinned at one end?



≈ pendulum



$$\Sigma F_t = -mg\sin\theta = m a_t$$

$$a_t = \frac{d^2 s}{dt^2}$$

$$s = \frac{L}{2}\theta$$

$$\dot{s} = \frac{L}{2}\dot{\theta}$$

$$\ddot{s} = \frac{L}{2}\ddot{\theta}$$

$$-\cancel{m}g\sin\theta = \cancel{m}\frac{L}{2}\ddot{\theta}$$

$$\frac{L}{2}\ddot{\theta} + g\sin\theta = 0$$

$$\ddot{\theta} + \frac{2g}{L}\sin\theta = 0$$

for small displacements

$$\sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{2g}{L}\theta = 0$$

$$m=1,\ c=0,\ k=\frac{2g}{L} \qquad \omega_n = \sqrt{\frac{2g}{L}}$$

planar rigid body motion

$\Sigma \underline{F} = m\underline{a}$

$\Sigma \underline{M} = I\,\underline{\alpha}$

mass moment of inertia

8.2 p 344 review manual

mass

moment of inertia

$$M_c = I_c \alpha \;;\; \alpha = \dot{\omega}$$

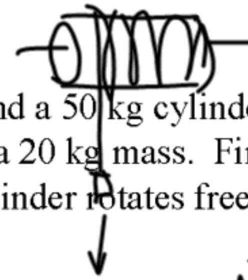
moment angular acceleration

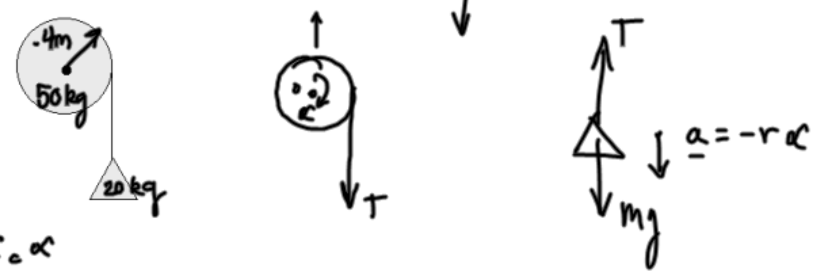
$$M_c = I_c \alpha$$

This equation applies at the center of mass ~~or a fixed pivot.~~

$$I = I_c + md^2$$

96. A string is wrapped around a 50 kg cylinder of radius 0.4 m and attached to a 20 kg mass. Find the tension in the string if the cylinder rotates freely?





$+\circlearrowleft \; \Sigma M_o = I_o \alpha$

$-T(0.4) = -\frac{mr^2}{2}\alpha$

$= -\frac{50(0.4)^2}{2}\alpha$

$T = \frac{50(.4)}{2}\alpha$

$\Sigma \underline{F} = m\underline{a}$

$T - mg = -r\alpha \cdot m$

$T - (20)(10) = -(0.4)\alpha(20)$

9.8

$\frac{50(.4)}{2}\alpha - 200 + 8\alpha = 0$

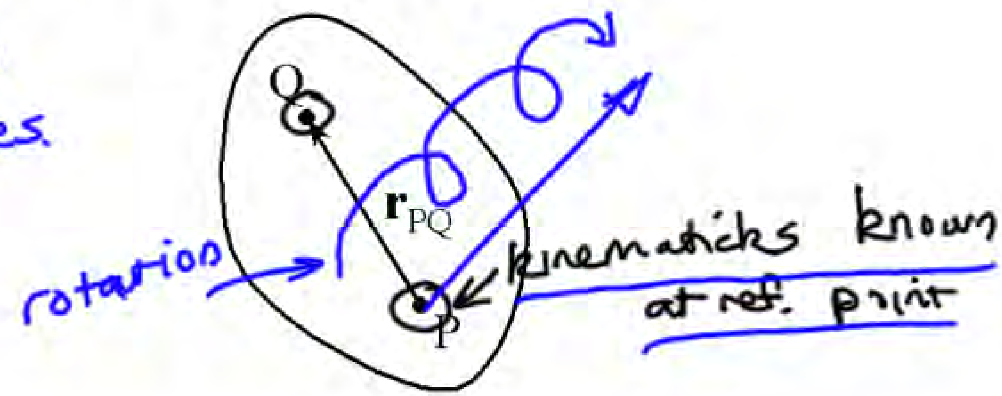
$18\alpha = 200$

$\underline{\underline{\alpha}} = 11.11\ \text{rad}/s^2$

$T = \frac{50(.4)}{2}(11.11\ \text{rad}/s^2) = \underline{\underline{111.1N}}$

velocity of a rigid body

know how "p" moves.



position vector

$$\mathbf{v}_Q = \mathbf{v}_P + \omega \times \mathbf{r}_{PQ}$$

velocity of Q = displacement of entre body + angular displacement of Q relative to P.

angular velocity

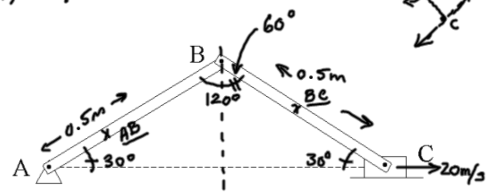
→ kinematics & trigonometry

94. Find the angular velocity of link AB if the angles are both 30°. Both links are 50 cm long, and the slider moves at velocity 20 m/s to the right.

$V_{B/A} = \perp AB$

$V_{C/B} = \perp BC$

V_C = horizontal, 20 m/s

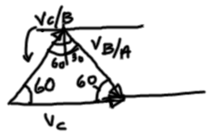


$V_B = \underbrace{V_A}_{=0 \text{ (fixed)}} + V_{B/A}$

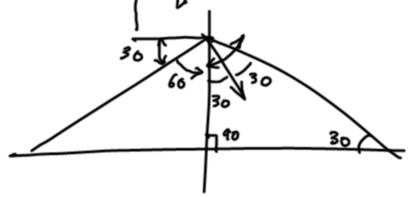
$V_{B/A} = r\omega_{AB}$

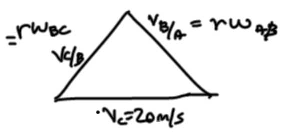
$V_C = V_B + V_{C/B}$

$V_{C/B} = r\omega_{BC}$



equilateral triangle





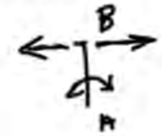
$\therefore\ 20\text{ m/s} = r\omega_{BC}$

$\omega_{BC} = \dfrac{20}{(0.5\text{m})} = 40\text{ rad/sec}$

$r\omega_{AB} = 20\text{ m/s}$

$\omega_{ab} = \dfrac{20\text{ m/s}}{(0.5\text{m})} = 40\text{ rad/sec}$

28. Find the slider velocity and the angular velocity of the 50 cm link BC. The 30 cm drive link is vertical with an angular velocity of 10 rad/s.



$V_B = V_A + V_{B/A}$

$V_A = \phi$ (A is fixed)

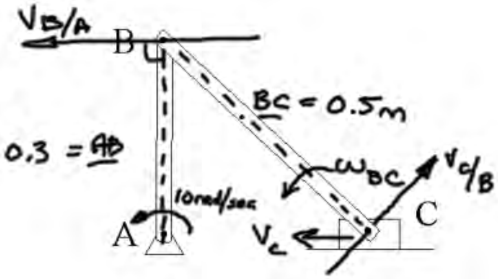
$V_C = V_B + V_{C/B}$

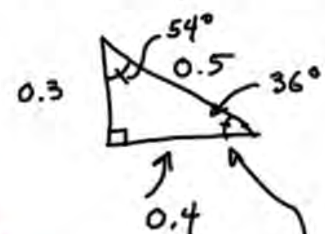
$= -(0.3)(10\,rad/s) + V_{C/B}$

$-r\omega_{AB}$

$V_{C/B} = r\omega_{BC}$

$= (0.5)\,\omega_{BC}$





$\tan^{-1}\left(\frac{3}{4}\right) = \phi$

$V_{B/A}$ & V_C must be horizontal

$\alpha = \phi$

$V_{C/B} = \emptyset \therefore \omega_{BC} = 0$