FE Civil

Practice

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1 Units

PRACTICE PROBLEMS

1. What SI unit is equal to the combination of base units $kg\cdot m^2/s^2$?

(A) joule

(B) pascal

(C) tesla

(D) watt

2. What is a kip?

(A) 1000 in-lbf (torque)

(B) 1000 lbm (mass)

(C) 1000 lbf (force)

(D) 1000 psi (pressure)

3. What is a metric ton?

(A) 200 kg

(B) 1000 kg

(C) 2000 kg

(D) 2000 N

SOLUTIONS

1. Kinetic energy is calculated in the SI system as $\frac{1}{2}m\mathrm{v}^2$, with units of $kg\cdot m^2/s^2$, which are equal to joules (J).

The answer is (A).

2. The abbreviation *kip* is used for *kilopound*, which is 1000 lbf (pounds of force).

The answer is (C).

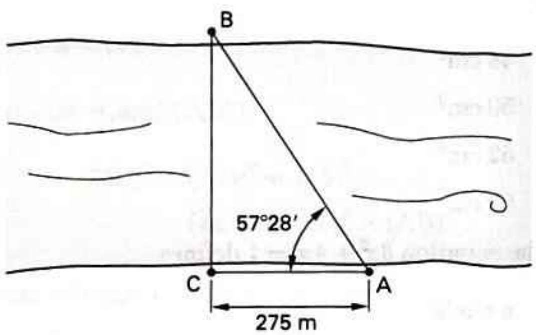
3. A metric ton, also known as a *tonne*, is 1000 kg.

The answer is (B).

2 Analytic Geometry and Trigonometry

PRACTICE PROBLEMS

1. To find the width of a river, a surveyor sets up a transit at point C on one river bank and sights directly across to point B on the other bank. The surveyor then walks along the bank for a distance of 275 m to point A. The angle CAB is 57° 28′.



What is the approximate width of the river?

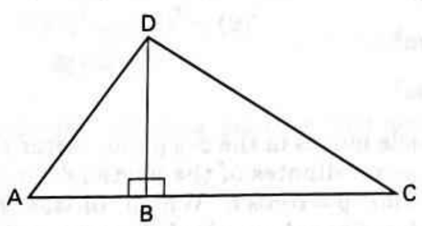
(A) 150 m

(B) 230 m

(C) 330 m

(D) 430 m

2. In the triangle shown, angles ABD and DBC are 90°, $AD = 15$, $DC = 20$, and $AC = 25$.



What are the lengths BC and BD, respectively?

(A) 12 and 16

(B) 13 and 17

(C) 16 and 12

(D) 18 and 13

3. A line with slope 4/3 passes through the point $(6,4)$. How long is the segment of the line that begins at the y-intercept and ends at the point $(6,4)$?

(A) 10

(B) 25

(C) 50

(D) 75

4. Which of the following expressions is equivalent to $\sin 2\theta$?

(A) $2\sin\theta\cos\theta$

(B) $\cos^2\theta - \sin^2\theta$

(C) $\sin\theta\cos\theta$

(D) $\dfrac{1-\cos 2\theta}{2}$

5. Which of the following equations describes a circle with center at $(2,3)$ and passing through the point $(-3,-4)$?

(A) $(x+3)^2+(y+4)^2=85$

(B) $(x+3)^2+(y+2)^2=\sqrt{74}$

(C) $(x-3)^2+(y-2)^2=74$

(D) $(x-2)^2+(y-3)^2=74$

6. The equation for a circle is $x^2+4x+y^2+8y=0$. What are the coordinates of the circle's center?

(A) $(-4,-8)$

(B) $(-4,-2)$

(C) $(-2,-4)$

(D) $(2,-4)$

Mathematics

7. Which of the following statements is FALSE for all noncircular ellipses?

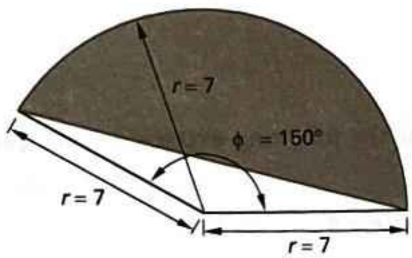
(A) The eccentricity, e, is less than one.

(B) The ellipse has two foci.

(C) The sum of the two distances from the two foci to any point on the ellipse is $2a$ (i.e., twice the semimajor distance).

(D) The coefficients A and C preceding the x^2 and y^2 terms in the general form of the equation are equal.

8. What is the area of the shaded portion of the circle shown?



(A) $\frac{5\pi}{6} - 1$

(B) $\left(\frac{49}{12}\right)(5\pi - 3)$

(C) $\frac{50\pi}{3}$

(D) $49\pi - \sqrt{3}$

9. A pipe with a 20 cm inner diameter is laying horizontally and is filled to a depth equal to one-third of its diameter with a substance that will prevent flow. What is the approximate area in flow in the remaining open cross section of the pipe?

(A) 33 cm^2

(B) 92 cm^2

(C) 223 cm^2

(D) 314 cm^2

10. The equation $y = a_1 + a_2x$ is an algebraic expression for which of the following?

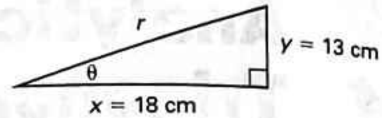
(A) a cosine expansion series

(B) projectile motion

(C) a circle in polar form

(D) a straight line

11. For the right triangle shown, $x = 18$ cm and $y = 13$ cm.



Most nearly, what is csc θ?

(A) 0.98

(B) 1.2

(C) 1.7

(D) 15

12. A circular sector has a radius of 8 cm and an arc length of 13 cm. The area of the circular sector is most nearly

(A) 48 cm^2

(B) 50 cm^2

(C) 52 cm^2

(D) 60 cm^2

13. The equation $3x^2 + 4y^2 = 1$ defines

(A) a circle

(B) an ellipse

(C) a hyperbola

(D) a parabola

14. What is the approximate surface area (including both side and base) of a 4 m high right circular cone with a base 3 m in diameter?

(A) 24 m^2

(B) 27 m^2

(C) 32 m^2

(D) 36 m^2

15. A particle moves in the x-y plane. After t seconds, the x- and y-coordinates of the particle's location are $x = 8 \sin t$ and $y = 6 \cos t$. Which of the following equations describes the path of the particle?

(A) $36x^2 + 64y^2 = 2304$

(B) $36x^2 - 64y^2 = 2304$

(C) $64x^2 + 36y^2 = 2304$

(D) $64x^2 - 36y^2 = 2304$

Mathematics

SOLUTIONS

1. Use the formula for the tangent of an angle in a right triangle. The angle may be entered in a calculator in degrees and minutes if the calculator is in that mode or the minutes can be converted to decimal form with 60 minutes per degree.

$$\begin{aligned} \tan\theta &= \mathrm{BC}/\mathrm{AC} \\ \mathrm{BC} &= \mathrm{AC}\tan\theta \\ &= (275\ \mathrm{m})\tan(57^\circ 28') \\ &= (275\ \mathrm{m})\tan\left(57^\circ + (28')\left(\frac{1^\circ}{60'}\right)\right) \\ &= (275\ \mathrm{m})\tan(57.467^\circ) \\ &= 431.1\ \mathrm{m} \quad (430\ \mathrm{m}) \end{aligned}$$

The answer is (D).

2. For right triangle ABD,

$$\begin{aligned} (\mathrm{BD})^2 + (\mathrm{AB})^2 &= (15)^2 \\ (\mathrm{BD})^2 &= (15)^2 - (\mathrm{AB})^2 \end{aligned}$$

For right triangle DBC, BC = (25-AB)

$$\begin{aligned} (\mathrm{BD})^2 + (25-\mathrm{AB})^2 &= (20)^2 \\ (\mathrm{BD})^2 &= (20)^2 - (25-\mathrm{AB})^2 \end{aligned}$$

Equate the two expressions for $(\mathrm{BD})^2$.

$$\begin{aligned} (15)^2 - (\mathrm{AB})^2 &= (20)^2 - (25)^2 + 50(\mathrm{AB}) - (\mathrm{AB})^2 \\ \mathrm{AB} &= \frac{(15)^2 - (20)^2 + (25)^2}{50} = 9 \\ \mathrm{BC} &= 25 - \mathrm{AB} = 25 - 9 = 16 \\ (\mathrm{BD})^2 &= (15)^2 - (9)^2 \\ \mathrm{BD} &= 12 \end{aligned}$$

Alternatively, this problem can be solved using the law of cosines.

The answer is (C).

3. The equation of the line is of the form

$$y = mx + b$$

The slope is $m = 4/3$, and a known point is $(x, y) = (6, 4)$. Find the y-intercept, b.

$$4 = \left(\frac{4}{3}\right)(6) + b$$

$$b = 4 - \left(\frac{4}{3}\right)(6) = -4$$

The complete equation is

$$y = \tfrac{4}{3}x - 4$$

b is the y-intercept, so the intersection with the y-axis is at point $(0, -4)$. The distance between these two points is

$$\begin{aligned} d &= \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \\ &= \sqrt{(4 - (-4))^2 + (6 - 0)^2} \\ &= 10 \end{aligned}$$

The answer is (A).

4. The double angle identity is

$$\sin 2\theta = 2\sin\theta\cos\theta$$

The answer is (A).

5. Substitute the known points into the center-radius form of the equation of a circle.

$$\begin{aligned} r^2 &= (x-h)^2 + (y-k)^2 \\ &= (-3-2)^2 + (-4-3)^2 \\ &= 74 \end{aligned}$$

The equation of the circle is

$$(x-2)^2 + (y-3)^2 = 74$$

$r^2 = 74$, so the radius is $\sqrt{74}$.

The answer is (D).

6. To find the circle's center, put the equation of the circle into standard form. Complete the square.

$$\begin{aligned} x^2 + 4x + y^2 + 8y &= 0 \\ x^2 + 4x + 4 + y^2 + 8y + 16 &= 4 + 16 \\ (x+2)^2 + (y+4)^2 &= 20 \end{aligned}$$

The center is at $(-2, -4)$. The opposite of the offsets of x and y are the origin of the circle.

The answer is (C).

7. The general form of the equation for an ellipse is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

The coefficients preceding the squared terms in the general equation are equal only for a straight line or circle, not for a noncircular ellipse.

The answer is (D).

8. The angle ϕ expressed in radians is

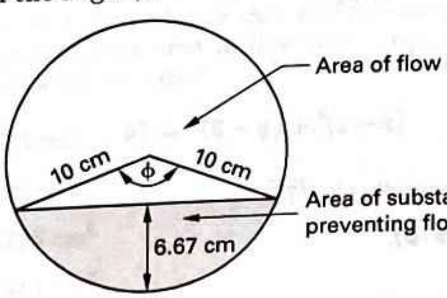
$$\phi = (150°)\left(\frac{2\pi \text{ rad}}{360°}\right) = \frac{5\pi}{6} \text{ rad}$$

The area of the shaded portion of the circular segment is

$$\begin{aligned} A &= \frac{r^2(\phi - \sin\phi)}{2} \\ &= \frac{(7)^2\left(\frac{5\pi}{6} - \sin\frac{5\pi}{6}\right)}{2} \\ &= \left(\frac{49}{2}\right)\left(\frac{5\pi}{6} - \frac{1}{2}\right) \\ &= \left(\frac{49}{12}\right)(5\pi - 3) \end{aligned}$$

The answer is (B).

9. Find the angle ϕ.



$$\begin{aligned} \phi &= 2\{\arccos[(r - d)/r]\} \\ &= 2\arccos\left(\frac{10 \text{ cm} - 6.67 \text{ cm}}{10 \text{ cm}}\right) \\ &= 141.1°\left(\frac{\pi \text{ rad}}{180°}\right) \\ &= 2.46 \text{ rad} \end{aligned}$$

Find the area of substance preventing flow.

$$\begin{aligned} A &= [r^2(\phi - \sin\phi)]/2 \\ &= \frac{(10 \text{ cm})^2(2.46 - \sin 2.46)}{2} \\ &= \frac{(10 \text{ cm})^2(2.46 - 0.63)}{2} \\ &= 91.5 \text{ cm}^2 \end{aligned}$$

Find area of flow

$$\begin{aligned} A_{flow} &= A_{total} - A \\ &= \pi(10 \text{ cm})^2 - 91.5 \text{ cm}^2 \\ &= 314.2 \text{ cm}^2 - 91.5 \text{ cm}^2 = 222.7 \text{ cm}^2 \quad (223 \text{ cm}^2) \end{aligned}$$

The answer is (C).

10. $y = mx + b$ is the slope-intercept form of the equation of a straight line. a_1 and a_2 are both constants, so $y = a_1 + a_2 x$ describes a straight line.

The answer is (D).

11. Find the length of the hypotenuse, r.

$$r = \sqrt{x^2 + y^2} = \sqrt{(18 \text{ cm})^2 + (13 \text{ cm})^2} = 22.2 \text{ cm}$$

Find $\csc\theta$.

$$\csc\theta = r/y = \frac{22.2 \text{ cm}}{13 \text{ cm}} = 1.7$$

The answer is (C).

12. Find the area of the circular sector.

$$A = \frac{sr}{2} = \left(\frac{(13 \text{ cm})(8 \text{ cm})}{2}\right) = 52 \text{ cm}^2$$

The answer is (C).

13. The general form of the conic section equation is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$A = 3$, $C = 4$, $F = -1$, and $B = D = E = 0$. A and C are different, so the equation does not define a circle. Calculate the discriminant.

$$B^2 - 4AC = (0)^2 - (4)(3)(4) = -48$$

This is less than zero, so the equation defines an ellipse.

The answer is (B).

14. Find the total surface area of a right circular cone. The radius is $r = d/2 = 3\text{ m}/2 = 1.5\text{ m}$.

$$\begin{aligned} A &= \text{side area} + \text{base area} = \pi r\left(r + \sqrt{r^2 + h^2}\right) \\ &= \pi(1.5\text{ m})\left(1.5\text{ m} + \sqrt{(1.5\text{ m})^2 + (4\text{ m})^2}\right) \\ &= 27.2\text{ m}^2 \quad (27\text{ m}^2) \end{aligned}$$

The answer is (B).

15. Rearrange the two coordinate equations.

$$\sin t = \frac{x}{8}$$
$$\cos t = \frac{y}{6}$$

Use the following trigonometric identity.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{x}{8}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$

To clear the fractions, multiply both sides by $(8)^2 \times (6)^2 = 2304$.

$$36x^2 + 64y^2 = 2304$$

The answer is (A).

3 Algebra and Linear Algebra

PRACTICE PROBLEMS

1. What is the name for a vector that represents the sum of two vectors?

(A) scalar

(B) resultant

(C) tensor

(D) moment

2. The second and sixth terms of a geometric progression are 3/10 and 243/160, respectively. What is the first term of this sequence?

(A) 1/10

(B) 1/5

(C) 3/5

(D) 3/2

3. Using logarithmic identities, what is most nearly the numerical value for the following expression?

$$\log_3 \frac{3}{2} + \log_3 12 - \log_3 2$$

(A) 0.95

(B) 1.33

(C) 2.00

(D) 2.20

4. Which of the following statements is true for a power series with the general term $a_i x^i$?

I. An infinite power series converges for $x < 1$.

II. Power series can be added together or subtracted within their interval of convergence.

III. Power series can be integrated within their interval of convergence.

(A) I only

(B) II only

(C) I and III

(D) II and III

5. What is most nearly the length (magnitude) of the resultant vector given the following vectors?

$$\begin{aligned} 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} \\ 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \\ -16\mathbf{i} - 14\mathbf{j} + 2\mathbf{k} \end{aligned}$$

(A) 3

(B) 4

(C) 10

(D) 14

6. What is the solution to the following system of simultaneous linear equations?

$$\begin{aligned} 10x + 3y + 10z &= 5 \\ 8x - 2y + 9z &= 3 \\ 8x + y - 10z &= 7 \end{aligned}$$

(A) $x = 0.326$; $y = -0.192$; $z = 0.586$

(B) $x = 0.148$; $y = 1.203$; $z = 0.099$

(C) $x = 0.625$; $y = 0.186$; $z = -0.181$

(D) $x = 0.282$; $y = -1.337$; $z = -0.131$

7. What is the inverse of matrix **A**?

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$$

(A) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

8. If the determinant of matrix **A** is -40, what is the determinant of matrix **B**?

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 1.5 & 1 & 0.5 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(A) -80

(B) -40

(C) -20

(D) 0.5

9. Given the origin-based vector $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, what is most nearly the angle between **A** and the x-axis?

(A) 22°

(B) 24°

(C) 66°

(D) 80°

10. Which is a true statement about these two vectors?

$$\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{B} = \mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$$

(A) Both vectors pass through the point $(0, -1, 6)$.

(B) The vectors are parallel.

(C) The vectors are orthogonal.

(D) The angle between the vectors is 17.4°.

11. What is most nearly the acute angle between vectors $\mathbf{A} = (3, 2, 1)$ and $\mathbf{B} = (2, 3, 2)$, both based at the origin?

(A) 25°

(B) 33°

(C) 35°

(D) 59°

12. Force vectors **A**, **B**, and **C** are applied at a single point.

$$\mathbf{A} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$$
$$\mathbf{B} = 2\mathbf{i} + 7\mathbf{j} - \mathbf{k}$$
$$\mathbf{C} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

What is most nearly the magnitude of the resultant force vector, **R**?

(A) 13

(B) 14

(C) 15

(D) 16

13. What is the sum of $12 + 13j$ and $7 - 9j$?

(A) $19 - 22j$

(B) $19 + 4j$

(C) $25 - 22j$

(D) $25 + 4j$

14. What is the product of the complex numbers $3 + 4j$ and $7 - 2j$?

(A) $10 + 2j$

(B) $13 + 22j$

(C) $13 + 34j$

(D) $29 + 22j$

Mathematics

SOLUTIONS

1. By definition, the sum of two vectors is known as the resultant.

The answer is (B).

2. Use the formula for geometric progression to find the common ratio.

$$l = ar^{n-1}$$
$$\frac{l_6}{l_2} = \frac{ar^{6-1}}{ar^{2-1}} = r^4$$
$$r = \sqrt[4]{\frac{l_6}{l_2}}$$
$$= \sqrt[4]{\frac{\frac{243}{160}}{\frac{3}{10}}}$$
$$= 3/2$$

The term before 3/10 is

$$a_1 = \frac{\frac{3}{10}}{\frac{3}{2}} = 1/5$$

The answer is (B).

3. Use the logarithmic identities.

$$\log xy = \log x + \log y$$
$$\log x/y = \log x - \log y$$

$$\log_3 \frac{3}{2} + \log_3 12 - \log_3 2 = \log_3 \frac{\left(\frac{3}{2}\right)(12)}{2}$$
$$= \log_3 9$$

Since $(3)^2 = 9$, use the logarithmic identities.

$$\log_b(b^n) = n\log_b(b) = n$$

$$\log_3(9) = \log_3(3^2)$$
$$= 2\log_3(3)$$
$$= 2.00$$

Alternately, the problem may be solved with the logarithmic identity for changing the base.

$$\log_b x = \frac{\log_a x}{\log_a b}$$
$$= \frac{\log_{10}(9)}{\log_{10}(3)}$$
$$= 2.00$$

The answer is (C).

4. Power series can be added together, subtracted from each other, differentiated, and integrated within their interval of convergence. The interval of convergence is $-1 < x < 1$.

The answer is (D).

5. The resultant vector is produced by adding the vectors.

$$\begin{array}{r} 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} \\ 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \\ -16\mathbf{i} - 14\mathbf{j} + 2\mathbf{k} \\ \hline -6\mathbf{i} - 8\mathbf{j} + 0\mathbf{k} \end{array}$$

The length (magnitude) of the resultant vector is

$$|\mathbf{R}| = \sqrt{(-6)^2 + (-8)^2 + (0)^2}$$
$$= 10$$

The answer is (C).

6. There are several ways of solving this problem. One is to write the equations in matrix form and solve for the variable matrix, **X**.

$$\mathbf{AX} = \mathbf{B}$$
$$\begin{bmatrix} 10 & 3 & 10 \\ 8 & -2 & 9 \\ 8 & 1 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix}$$
$$\mathbf{AA^{-1}X} = \mathbf{A^{-1}B}$$
$$\mathbf{IX} = \mathbf{A^{-1}B}$$
$$\mathbf{X} = \mathbf{A^{-1}B}$$

$$\mathbf{X} = \begin{bmatrix} \frac{11}{806} & \frac{20}{403} & \frac{47}{806} \\ \frac{76}{403} & \frac{-90}{403} & \frac{-5}{403} \\ \frac{12}{403} & \frac{7}{403} & \frac{-22}{403} \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} (5)\left(\frac{11}{806}\right) + (3)\left(\frac{20}{403}\right) + (7)\left(\frac{47}{806}\right) \\ (5)\left(\frac{76}{403}\right) + (3)\left(\frac{-90}{403}\right) + (7)\left(\frac{-5}{403}\right) \\ (5)\left(\frac{12}{403}\right) + (3)\left(\frac{7}{403}\right) + (7)\left(\frac{-22}{403}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0.625 \\ 0.186 \\ -0.181 \end{bmatrix}$$

Another method is to manipulate the equations to more easily solve for two equations with two unknowns, and then substitute into the third equation to solve for the third unknown.

Starting with the three original equations:

$$10x + 3y + 10z = 5 \text{ (Eq. 1)}$$

$$8x - 2y + 9z = 3 \text{ (Eq. 2)}$$

$$8x + y - 10z = 7 \text{ (Eq. 3)}$$

Inspecting the three given equations, it appears that the y coefficients are easy to work with for elimination.

In the second equation, the y coefficient is -2. If the third equation is multiplied by 2 and added to Eq. 2, the y coefficient can be eliminated. The three equations now become:

$$10x + 3y + 10z = 5 \text{ (Eq. 1)}$$

$$24x - 11z = 17 \text{ (Eq. 2a)(Multiplied Eq. 3 by 2 and added to Eq. 2.)}$$

$$8x + y - 10z = 7 \text{ (Eq. 3)}$$

Multiply Eq. 3 by 3 and subtract Eq. 1 from that and the y coefficient will be eliminated:

$$10x + 3y + 10z = 5 \text{ (Eq. 1)}$$

$$24x - 11z = 17 \text{ (Eq. 2a)}$$

$$14x - 40z = 16 \text{ (Eq. 3a)(Multiplied Eq. 3 by 3 and subtracted Eq. 1.)}$$

Equation 2a and 3a represent two equations with two unknowns:

$$24x - 11z = 17 \text{ (Eq. 2a)}$$

$$14x - 40z = 16 \text{ (Eq. 3a)}$$

From Eq. 2a, solving for x in terms of z:

$$24x = 17 + 11z$$

$$x = (17/24) + (11/24)z$$

By substituting x into Eq. 3a, the value of z can be solved:

$$14[(17/24) + (11/24)z] - 40z = 16$$

$$14[(0.708) + (0.458)z] - 40z = 16$$

$$9.912 + 6.412z - 40z = 16$$

$$-33.588z = 6.088$$

$$z = -0.181$$

By inspection, the only answer amongst the four answers that have $z = -0.181$ is answer (C).

Alternatively, substituting the four answer options directly into the original equations is probably the fastest way to solve the problem.

The answer is (C).

7. Find the determinant.

$$|\mathbf{A}| = 2 \times 1 - 1 \times 3 = -1$$

The inverse of a 2×2 matrix is

$$\mathbf{A}^{-1} = \frac{\text{adj}(\mathbf{A})}{|\mathbf{A}|} = \frac{\begin{bmatrix} b_2 & -a_2 \\ -b_1 & a_1 \end{bmatrix}}{|\mathbf{A}|}$$

$$= \frac{\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}}{-1}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

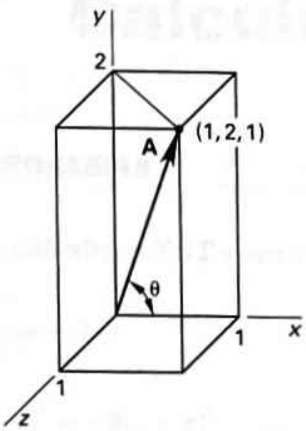
The answer is (D).

8. The first row of matrix **B** is half that of **A**, and the other rows are the same in **A** and **B**, so the determinant of **B** is half the determinant of **A**.

The answer is (C).

9. The magnitude of vector **A** is

$$|\mathbf{A}| = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6}$$



The x-component of the vector is 1, so the direction cosine is

$$\cos\theta_x = \frac{1}{\sqrt{6}}$$

The angle is

$$\theta = \arccos\frac{1}{\sqrt{6}} = 65.9^\circ \quad (66^\circ)$$

The answer is (C).

10. The magnitudes of the two vectors are

$$|\mathbf{A}| = \sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6}$$
$$|\mathbf{B}| = \sqrt{(1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

The angle between them is

$$\begin{aligned}\phi &= \arccos\left(\frac{a_xb_x + a_yb_y + a_zb_z}{|\mathbf{A}|\,|\mathbf{B}|}\right)\\ &= \arccos\left(\frac{(1)(1) + (2)(3) + (1)(-7)}{\sqrt{6}\sqrt{59}}\right)\\ &= 90^\circ\end{aligned}$$

The vectors are orthogonal.

The answer is (C).

11. The angle between the two vectors is

$$\begin{aligned}\theta &= \arccos\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|\,|\mathbf{B}|}\right)\\ &= \arccos\left(\frac{a_xb_x + a_yb_y + a_zb_z}{|\mathbf{A}|\,|\mathbf{B}|}\right)\\ &= \arccos\left(\frac{(3)(2) + (2)(3) + (1)(2)}{\sqrt{(3)^2 + (2)^2 + (1)^2}\,\sqrt{(2)^2 + (3)^2 + (2)^2}}\right)\\ \theta &= \arccos\left(\frac{14}{(3.74)(4.12)}\right)\\ &= \arccos\left(\frac{14}{15.41}\right)\\ &= \arccos(0.91)\\ &= 24.8^\circ \quad (25^\circ)\end{aligned}$$

The answer is (A).

12. The magnitude of **R** is

$$\begin{aligned}|\mathbf{R}| &= \sqrt{(1+2-1)^2 + (3+7+4)^2 + (4-1+2)^2}\\ &= \sqrt{4+196+25}\\ &= \sqrt{225}\\ &= 15\end{aligned}$$

The answer is (C).

13. Add the real parts and the imaginary parts of each complex number.

$$\begin{aligned}(a+jb) + (c+jd) &= (a+c) + j(b+d)\\ (12+13j) + (7-9j) &= (12+7) + j(13+(-9))\\ &= 19+4j\end{aligned}$$

The answer is (B).

14. Use the algebraic distributive law and the equivalency $j^2 = -1$.

$$\begin{aligned}(a+jb)(c+jd) &= (ac-bd) + j(ad+bc)\\ (3+4j)(7-2j) &= 21 - 8j^2 + 28j - 6j\\ &= 21 + 8 + 28j - 6j\\ &= 29 + 22j\end{aligned}$$

The answer is (D).

4 Calculus

PRACTICE PROBLEMS

1. Which of the following is NOT a correct derivative?

(A) $\frac{d}{dx}\cos x = -\sin x$

(B) $\frac{d}{dx}(1-x)^3 = -3(1-x)^2$

(C) $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$

(D) $\frac{d}{dx}\csc x = -\cot x$

2. What is the the partial derivative $\partial f(x, y)/\partial x$, of the equation
$x^2y - e^{2x} = \sin y$?

(A) $\dfrac{2e^{2x}}{x^2 - \cos y}$

(B) $\dfrac{2e^{2x} - 2xy}{x^2 - \cos y}$

(C) $2xy - 2e^{2x}$

(D) $x^2 - \cos y$

3. What is the approximate area bounded by the curves $y = 8 - x^2$ and $y = -2 + x^2$?

(A) 22

(B) 27

(C) 30

(D) 45

4. What are the absolute minimum and maximum values, respectively, of the equation $f(x) = 5x^3 - 2x^2 + 1$ on the interval $[-2, 2]$?

(A) $-47, 33$

(B) $-4, 4$

(C) $0.95, 1$

(D) $0, 0.27$

5. In vector calculus, a gradient is a

I. vector that points in the direction of a general rate of change of a scalar field

II. vector that points in the direction of the maximum rate of change of a scalar field

III. scalar that indicates the magnitude of the rate of change of a vector field in a general direction

IV. scalar that indicates the maximum magnitude of the rate of change of a vector field in any particular direction

(A) I only

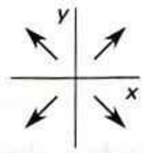
(B) II only

(C) I and III

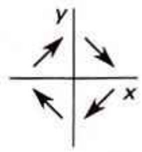
(D) II and IV

6. Which of the illustrations shown represents the vector field, $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$, for nonzero values of x and y?

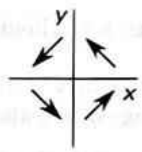
(A)



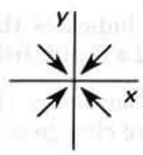
(B)



(C)



(D)



7. If a crop of peaches is picked now, 1000 lugs of peaches will be obtained, which can be sold at $1.00 per lug. For each week that picking is delayed, the crop will increase by 60 lugs, but the price will drop by $0.025 per lug. In addition, 10 lugs will spoil for each week of delay. In order to maximize revenue, after how many weeks should the peaches be picked?

(A) 2 weeks

(B) 5 weeks

(C) 7 weeks

(D) 10 weeks

8. Determine the following indefinite integral.

$$\int \frac{x^3 + x + 4}{x^2} dx$$

(A) $\frac{x}{4} + \ln|x| - \frac{4}{x} + C$

(B) $\frac{-x}{2} + \log x - 8x + C$

(C) $\frac{x^2}{2} + \ln|x| - \frac{2}{x^2} + C$

(D) $\frac{x^2}{2} + \ln|x| - \frac{4}{x} + C$

9. Find dy/dx for the parametric equations given.

$$x = 2t^2 - t$$
$$y = t^3 - 2t + 1$$

(A) $3t^2$

(B) $3t^2/2$

(C) $4t - 1$

(D) $(3t^2 - 2)/(4t - 1)$

10. A two-dimensional function, $f(x, y)$, is defined as

$$f(x, y) = 2x^2 - y^2 + 3x - y$$

What is the direction of the line passing through the point $(1, -2)$ that has the maximum slope?

(A) $4\mathbf{i} + 2\mathbf{j}$

(B) $7\mathbf{i} + 3\mathbf{j}$

(C) $7\mathbf{i} + 4\mathbf{j}$

(D) $9\mathbf{i} - 7\mathbf{j}$

11. Evaluate the following limit.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 2

(C) 4

(D) ∞

12. If $f(x,y) = x^2y^3 + xy^4 + \sin x + \cos^2 x + \sin^3 y$, what is $\partial f/\partial x$?

(A) $(2x+y)y^3 + 3\sin^2 y \cos y$

(B) $(4x-3y^2)xy^2 + 3\sin^2 y \cos y$

(C) $(3x+4y^2)xy + 3\sin^2 y \cos y$

(D) $(2x+y)y^3 + (1-2\sin x)\cos x$

13. What is dy/dx if $y = (2x)^x$?

(A) $(2x)^x(2 + \ln 2x)$

(B) $2x(1+\ln 2x)^x$

(C) $(2x)^x(\ln 2x^2)$

(D) $(2x)^x(1+\ln 2x)$

SOLUTIONS

1. Determine each of the derivatives.

$$\frac{d}{dx}\cos x = -\sin x \quad \text{[OK]}$$

$$\frac{d}{dx}(1-x)^3 = (3)(1-x)^2(-1) = (-3)(1-x)^2 \quad \text{[OK]}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = (-1)(x^{-2}) = \frac{-1}{x^2} \quad \text{[OK]}$$

$$\frac{d}{dx}\csc x = -\cot x \quad \text{[Not OK]}$$

Option D is not a correct derivative. The correct derivative trigonometric function is

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

The answer is (D).

2. Since neither x nor y can be extracted from the equation, rearrange to obtain a homogeneous equation in x and y.

$$x^2y - e^{2x} = \sin y$$
$$f(x,y) = x^2y - e^{2x} - \sin y = 0$$

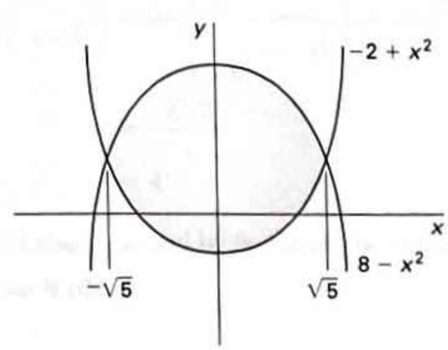
Take the partial derivative with respect to x.

$$\frac{\partial f(x,y)}{\partial x} = 2xy - 2e^{2x}$$

The answer is (C).

3. Find the intersection points by setting the two functions equal.

$$-2 + x^2 = 8 - x^2$$
$$2x^2 = 10$$
$$x = \pm\sqrt{5}$$



The integral of $f_1(x) - f_2(x)$ represents the area between the two curves between the limits of integration.

$$\begin{aligned} A &= \int_{x_1}^{x_2} (f_1(x) - f_2(x))\,dx \\ &= \int_{-\sqrt{5}}^{\sqrt{5}} ((8 - x^2) - (-2 + x^2))\,dx \\ &= \int_{-\sqrt{5}}^{\sqrt{5}} (10 - 2x^2)\,dx \\ &= \left(10x - \tfrac{2}{3}x^3\right)\Big|_{-\sqrt{5}}^{\sqrt{5}} \\ &= 29.8 \quad (30) \end{aligned}$$

The answer is (C).

4. The critical points are located where the first derivative is zero.

$$\begin{aligned} f(x) &= 5x^3 - 2x^2 + 1 \\ f'(x) &= 15x^2 - 4x \\ 15x^2 - 4x &= 0 \\ x(15x - 4) &= 0 \\ x = 0 \quad \text{or} \quad x &= 4/15 \end{aligned}$$

Test each critical point to determine whether it is a maximum, minimum, or inflection point.

$$\begin{aligned} f''(x) &= 30x - 4 \\ f''(0) &= (30)(0) - 4 \\ &= -4 \end{aligned}$$

This is less than zero, so the critical point at $x = 0$ is a maximum.

$$\begin{aligned} f''\left(\frac{4}{15}\right) &= (30)\left(\frac{4}{15}\right) - 4 \\ &= 4 \end{aligned}$$

This is greater than zero, so the critical point at $x = 4/15$ is a minimum.

These two critical points could be a local maximum and minimum. Compare the values of the function at the critical points with the values of the function at the endpoints.

$$\begin{aligned} f(-2) &= (5)(-2)^3 - (2)(-2)^2 + 1 = -47 \\ f(2) &= (5)(2)^3 - (2)(2)^2 + 1 = 33 \\ f(0) &= (5)(0)^3 - (2)(0)^2 + 1 = 1 \\ f\left(\frac{4}{15}\right) &= (5)\left(\frac{4}{15}\right)^3 - (2)\left(\frac{4}{15}\right)^2 + 1 \\ &= 0.95 \end{aligned}$$

The minimum and maximum values of the equation over the entire interval, -47 and 33, respectively, are at the endpoints.

The answer is (A).

5. A gradient (gradient vector) at some point P is described by use of the gradient (del, grad, nabla, etc.) function, $\nabla f_P \cdot \mathbf{a}$, where $\mathbf{a}$ is a unit vector. In three-dimensional rectangular coordinates, the gradient is equivalent to the partial derivative vector.

$$\nabla f \cdot \mathbf{a} = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

This is a vector that points in the direction of the maximum rate of change (i.e., maximum slope).

The answer is (B).

6. From the term $-y\mathbf{i}$, it can be concluded that

(a) for positive values of y, the vector field points to the left

(b) for negative values of y, the vector field points to the right

From the term $+x\mathbf{j}$, it can be concluded that

(a) for positive values of x, the vector field points upward

(b) for negative values of x, the vector field points downward

The answer is (C).

7. Let x represent the number of weeks. The equation describing the price as a function of time is

$$\frac{\text{price}}{\text{lug}} = \$1 - \$0.025x$$

The equation describing the yield is

$$\begin{aligned}\text{lugs sold} &= 1000 + (60-10)x \\ &= 1000 + 50x\end{aligned}$$

The revenue function is

$$\begin{aligned}R &= \left(\frac{\text{price}}{\text{lug}}\right)(\text{lugs sold}) \\ &= (1-0.025x)(1000+50x) \\ &= 1000 + 50x - 25x - 1.25x^2 \\ &= 1000 + 25x - 1.25x^2\end{aligned}$$

To find the maximum of the revenue function, set its derivative equal to zero.

$$\begin{aligned}\frac{dR}{dx} &= 25 - 2.5x = 0 \\ x &= 10 \text{ weeks}\end{aligned}$$

The answer is (D).

8. Separate the fraction into parts and integrate each one.

$$\begin{aligned}\int \frac{x^3+x+4}{x^2}\,dx &= \int \frac{x^3}{x^2}\,dx + \int \frac{x}{x^2}\,dx + \int \frac{4}{x^2}\,dx \\ &= \int x\,dx + \int \frac{1}{x}\,dx + 4\int \frac{1}{x^2}\,dx \\ &= \frac{x^2}{2} + \ln|x| + 4\left(\frac{x^{-1}}{-1}\right) + C \\ &= \frac{x^2}{2} + \ln|x| - \frac{4}{x} + C\end{aligned}$$

The answer is (D).

9. Calculate the derivatives of x and y with respect to t.

$$\begin{aligned}\frac{dy}{dt} &= 3t^2 - 2 \\ \frac{dx}{dt} &= 4t - 1\end{aligned}$$

The derivative of y with respect to x is

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \\ &= \frac{3t^2-2}{4t-1}\end{aligned}$$

The answer is (D).

10. The direction of the line passing through $(1,-2)$ with maximum slope is found by inserting $x=1$ and $y=-2$ into the gradient vector function.

The gradient of the function is

$$\begin{aligned}\nabla f(x,y,z) &= \frac{\partial f(x,y,z)}{\partial x}\mathbf{i} + \frac{\partial f(x,y,z)}{\partial y}\mathbf{j} + \frac{\partial f(x,y,z)}{\partial z}\mathbf{k} \\ &= \frac{\partial(2x^2-y^2+3x-y)}{\partial x}\mathbf{i} \\ &\quad + \frac{\partial(2x^2-y^2+3x-y)}{\partial y}\mathbf{j} \\ &= (4x+3)\mathbf{i} - (2y+1)\mathbf{j}\end{aligned}$$

At $(1,-2)$,

$$\begin{aligned}\nabla f(1,-2) &= ((4)(1)+3)\mathbf{i} - ((2)(-2)+1)\mathbf{j} \\ &= 7\mathbf{i} + 3\mathbf{j}\end{aligned}$$

The answer is (B).

11. The expression approaches $0/0$ at the limit.

$$\frac{(2)^2-4}{2-2} = \frac{0}{0}$$

Use L'Hôpital's rule.

$$\begin{aligned}\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) &= \lim_{x\to 2}\left(\frac{\frac{d}{dx}(x^2-4)}{\frac{d}{dx}(x-2)}\right) = \lim_{x\to 2}\left(\frac{2x}{1}\right) \\ &= \frac{(2)(2)}{1} \\ &= 4\end{aligned}$$

This could also be solved by factoring the numerator.

The answer is (C).

12. The partial derivative with respect to x is found by treating all other variables as constants. Therefore, all terms that do not contain x have zero derivatives.

$$\begin{aligned}\frac{\partial f}{\partial x} &= 2xy^3 + y^4 + \cos x + 2\cos x(-\sin x)\\ &= (2x + y)y^3 + (1 - 2\sin x)\cos x\end{aligned}$$

The answer is (D).

13. From the table of derivatives,

$$\begin{aligned}\frac{d(u^v)}{dx} &= vu^{v-1}\,\frac{du}{dx} + (\ln\, u)u^v\,\frac{dv}{dx}\\ u(x) &= 2x\\ v(x) &= x\\ \frac{d(2x)^x}{dx} &= x(2x)^{x-1}(2) + (\ln 2x)(2x)^x(1)\\ &= (2x)(2x)^{x-1} + (2x)^x \ln\, 2x\\ &= (2x)^x + (2x)^x \ln 2x\\ &= (2x)^x(1 + \ln 2x)\end{aligned}$$

The answer is (D).

5 Differential Equations and Transforms

PRACTICE PROBLEMS

1. What is the solution to the following differential equation?

$$y' + 5y = 0$$

(A) $y = 5x + C$

(B) $y = Ce^{-5x}$

(C) $y = Ce^{5x}$

(D) either (A) or (B)

2. What is the solution to the following linear difference equation?

$$(k+1)(y(k+1)) - ky(k) = 1$$

(A) $y(k) = 12 - \frac{1}{k}$

(B) $y(k) = 1 - \frac{12}{k}$

(C) $y(k) = 12 + 3k$

(D) $y(k) = 3 + \frac{1}{k}$

3. What is the general solution to the following differential equation?

$$2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

(A) $y = C_1 \cos x + C_2 \sin x$

(B) $y = C_1 e^{x} + C_2 e^{-x}$

(C) $y = e^{-x}(C_1 \cos x - C_2 \sin x)$

(D) $y = e^{x}(C_1 \cos x + C_2 \sin x)$

4. What is the general solution to the following differential equation?

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

(A) $y = C_1 \sin x - C_2 \cos x$

(B) $y = C_1 \cos x - C_2 \sin x$

(C) $y = C_1 \cos x + C_2 \sin x$

(D) $y = e^{-x}(C_1 \cos x + C_2 \sin x)$

5. What is the complementary solution to the following differential equation?

$$y'' - 4y' + \tfrac{25}{4}y = 10\cos 8x$$

(A) $y = 2C_1x + C_2x - C_3x$

(B) $y = C_1e^{2x} + C_2e^{1.5x}$

(C) $y = C_1e^{2x}\cos 1.5x + C_2e^{2x}\sin 1.5x$

(D) $y = C_1e^{x}\tan x + C_2e^{x}\cot x$

6. What is the general solution to the following differential equation?

$$y'' + y' + y = 0$$

(A) $y = e^{-(1/2)x}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right)$

(B) $y = e^{-(1/2)x}\left(C_1 \cos \frac{3}{2}x + C_2 \sin \frac{3}{2}x\right)$

(C) $y = e^{-2x}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right)$

(D) $y = e^{-2x}\left(C_1 \cos \frac{3}{2}x + C_2 \sin \frac{3}{2}x\right)$

7. What is the solution to the following differential equation if $x=1$ at $t=0$, and $dx/dt=0$ at $t=0$?

$$\frac{1}{2}\frac{d^2x}{dt^2}+4\frac{dx}{dt}+8x=5$$

(A) $x=e^{-4t}+4te^{-4t}$

(B) $x=\frac{3}{8}e^{-2t}(\cos 2t+\sin 2t)+\frac{5}{8}$

(C) $x=e^{-4t}+4te^{-4t}+\frac{5}{8}$

(D) $x=\frac{3}{8}e^{-4t}+\frac{3}{2}te^{-4t}+\frac{5}{8}$

8. In the following differential equation with the initial condition $x(0)=12$, what is the value of $x(2)$?

$$\frac{dx}{dt}+4x=0$$

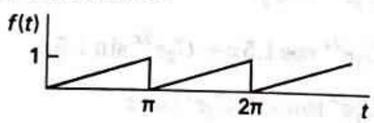
(A) 3.4×10^{-3}

(B) 4.0×10^{-3}

(C) 5.1×10^{-3}

(D) 6.2×10^{-3}

9. What are the three general Fourier coefficients for the sawtooth wave shown?



(A) $a_0=0,\ a_n=0,\ b_n=\frac{-1}{\pi n}$

(B) $a_0=\frac{1}{2},\ a_n=0,\ b_n=\frac{-1}{\pi n}$

(C) $a_0=1,\ a_n=1,\ b_n=\frac{1}{\pi n}$

(D) $a_0=\frac{1}{2},\ a_n=\frac{1}{2},\ b_n=\frac{1}{\pi n}$

10. The values of an unknown function follow a Fibonacci number sequence. It is known that $f(1)=4$ and $f(2)=1.3$. What is $f(4)$?

(A) −4.1

(B) 0.33

(C) 2.7

(D) 6.6

SOLUTIONS

1. This is a first-order linear equation with characteristic equation $r+5=0$. The form of the solution is

$$y=Ce^{-5x}$$

In the preceding equation, the constant, C, could be determined from additional information.

Alternate Solution

An alternate solution for this is to substitute the options into the differential equation and test if the equation balances. For option B, the derivative of y is

$$y'=\frac{dy}{dx}=-5Ce^{-5x}$$

Substitute into the differential equation.

$$-5Ce^{-5x}+5Ce^{-5x}=0$$

This equation balances for any value of constant C and any variable x. The equation does not balance for options (A), (C), and (D).

The answer is (B).

2. Since nothing is known about the general form of $y(k)$, the only way to solve this problem is by trial and error, substituting each answer option into the equation in turn. Option B is

$$y(k)=1-\frac{12}{k}$$

Substitute this into the difference equation.

$$\begin{aligned}(k+1)(y(k+1))-k(y(k))&=1\\(k+1)\left(1-\frac{12}{k+1}\right)-k\left(1-\frac{12}{k}\right)&=1\\(k+1)\left(\frac{k+1-12}{k+1}\right)-k\left(\frac{k-12}{k}\right)&=1\\k+1-12-k+12&=1\\1&=1\end{aligned}$$

$y(k)=1-12/k$ solves the difference equation.

The answer is (B).

3. This is a second-order, homogeneous, linear differential equation. Start by putting it in general form.

$$\begin{aligned} y'' + ay' + by &= 0 \\ 2y'' - 4y' + 4y &= 0 \\ y'' - 2y' + 2y &= 0 \\ a &= -2 \\ b &= 2 \end{aligned}$$

Since $a^2 < 4b$, the roots will be complex, so the form of the equation is

$$\begin{aligned} y &= e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta_x) \\ \alpha &= \frac{-a}{2} = \frac{-(-2)}{2} = 1 \\ \beta &= \frac{\sqrt{4b - a^2}}{2} \\ &= \frac{\sqrt{4 \times 2 - (-2)^2}}{2} \\ &= 1 \\ y &= e^{x}(C_1 \cos x + C_2 \sin x) \end{aligned}$$

The answer is (D).

4. The characteristic equation is

$$\begin{aligned} r^2 + 2r + 2 &= 0 \\ a &= 2 \\ b &= 2 \end{aligned}$$

The roots are

$$\begin{aligned} r_{1,2} &= \frac{-a \pm \sqrt{a^2 - 4b}}{2} \\ &= \frac{-2 \pm \sqrt{(2)^2 - (4)(2)}}{2} \\ &= (-1 + i), (-1 - i) \end{aligned}$$

Since $a^2 < 4b$, the solution is

$$\begin{aligned} y &= e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x) \\ \alpha &= \frac{-a}{2} = \frac{-2}{2} = -1 \\ \beta &= \frac{\sqrt{4b - a^2}}{2} = \frac{\sqrt{(4)(2) - (2)^2}}{2} \\ &= 1 \\ y &= e^{-x}(C_1 \cos x + C_2 \sin x) \end{aligned}$$

The answer is (D).

5. The complementary solution to a nonhomogeneous differential equation is the solution of the homogeneous differential equation.

The characteristic equation is

$$\begin{aligned} r^2 + ar + b &= 0 \\ r^2 - 4r + \frac{25}{4} &= 0 \end{aligned}$$

So, $a = -4$, and $b = 25/4$.

The roots are

$$\begin{aligned} r_{1,2} &= \frac{-a \pm \sqrt{a^2 - 4b}}{2} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - (4)\left(\frac{25}{4}\right)}}{2} \\ &= 2 \pm 1.5i \end{aligned}$$

Since the roots are imaginary, the homogeneous solution has the form of

$$\begin{aligned} y &= e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x) \\ \alpha &= \frac{-a}{2} \\ &= \frac{-(-4)}{2} \\ &= 2 \\ \beta &= \frac{\sqrt{4b - a^2}}{2} \\ &= \frac{\sqrt{(4)(\frac{25}{4}) - (-4)^2}}{2} \\ &= 1.5 \end{aligned}$$

The complementary solution is

$$\begin{aligned} y &= e^{2x}(C_1 \cos 1.5x + C_2 \sin 1.5x) \\ &= C_1 e^{2x} \cos 1.5x + C_2 e^{2x} \sin 1.5x \end{aligned}$$

The answer is (C).

6. This is a second-order, homogeneous, linear differential equation with $a = b = 1$. This differential equation can be solved by the method of undetermined coefficients with a solution in the form $y = Ce^{rx}$. The substitution of the solution gives

$$(r^2 + ar + b)\,Ce^{rx} = 0$$

Because Ce^{rx} can never be zero, the characteristic equation is

$$r^2 + ar + b = 0$$

Because $a^2 = 1 < 4b = 4$, the general solution is in the form

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$$

Then,

$$\alpha = -\frac{a}{2} = -1/2$$

$$\beta = \frac{\sqrt{4b - a^2}}{2} = \frac{\sqrt{(4)(1) - (1)^2}}{2} = \frac{\sqrt{3}}{2}$$

Therefore, the general solution is

$$y = e^{-(1/2)x}\left(C_1 \cos \tfrac{\sqrt{3}}{2}x + C_2 \sin \tfrac{\sqrt{3}}{2}x\right)$$

The answer is (A).

7. Multiplying the equation by 2 gives

$$x'' + 8x' + 16x = 10$$

The characteristic equation is

$$r^2 + 8r + 16 = 0$$

The roots of the characteristic equation are

$$r_1 = r_2 = -4$$

The homogeneous (natural) response is

$$x_{\text{natural}} = Ae^{-4t} + Bte^{-4t}$$

To find a particular solution, assume $x_p(t) = C$. Differentiate twice and substitute accordingly in the differential equation.

$$x_p = C$$

$$\frac{dx_p}{dt} = 0$$

$$\frac{d^2x_p}{dt^2} = 0$$

$$\frac{1}{2}(0) + 4(0) + 8(C) = 5$$

$$C = \frac{5}{8}$$

Hence, 5/8 is a particular solution that solves the nonhomogeneous equation.

$$x = Ae^{-4t} + Bte^{-4t} + \frac{5}{8}$$

Since $x = 1$ at $t = 0$,

$$1 = Ae^0 + \frac{5}{8}$$

$$A = \frac{3}{8}$$

Differentiating x,

$$x' = \left(\frac{3}{8}\right)(-4)e^{-4t} + B(-4te^{-4t} + e^{-4t}) + 0$$

Since $x' = 0$ at $t = 0$,

$$0 = -\frac{3}{2} + B(0 + 1)$$

$$B = \frac{3}{2}$$

$$x = \tfrac{3}{8}e^{-4t} + \tfrac{3}{2}te^{-4t} + \frac{5}{8}$$

The answer is (D).

8. This is a first-order, linear, homogeneous differential equation with characteristic equation $r+4=0$.

$$\begin{aligned} x'+4x &= 0 \\ x &= x_0e^{-4t} \\ x(0) &= x_0e^{(-4)(0)} \\ &= 12 \\ x_0 &= 12 \\ x &= 12e^{-4t} \\ x(2) &= 12e^{(-4)(2)} \\ &= 12e^{-8} \\ &= 4.03\times 10^{-3} \quad (4.0\times 10^{-3}) \end{aligned}$$

The answer is (B).

9. By inspection, $f(t)=t/\pi$, with the period $T=\pi$. The angular frequency is

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

The average is

$$\begin{aligned} a_0 &= (1/T)\int_0^T f(t)\,dt = (1/T)\int_0^T \frac{t}{\pi}\,dt = \left(\frac{1}{\pi^2}\right)\left(\frac{1}{2}\right)t^2\Big|_0^\pi \\ &= \frac{1}{2}-0 \\ &= \frac{1}{2} \end{aligned}$$

The general a term is

$$\begin{aligned} a_n &= (2/T)\int_0^T f(t)\cos(n\omega_0t)\,dt \\ &= \frac{2}{\pi}\int_0^\pi \frac{t}{\pi}\cos(2nt)\,dt \\ &= \frac{2}{\pi^2}\int_0^t t\cos(2nt)\,dt \\ &= \frac{2}{\pi^2}\left[\frac{\cos(2nt)+t\sin(2nt)}{\frac{d(2nt)}{dt}}\right]_0^\pi \\ &= \frac{2}{\pi^2}\left[\frac{\cos(2nt)+t\sin(2nt)}{2n}\right]_0^\pi \\ &= \frac{1}{\pi^2n}[\cos(2n\pi)+\pi\sin(2n\pi)-\cos(0)-(0)\sin(0)] \\ &= \frac{1}{\pi^2n}[1+\pi(0)-1-(0)(0)] \\ &= 0 \end{aligned}$$

The general b term is

$$\begin{aligned} b_n &= (2/T)\int_0^T f(t)\sin(n\omega_0t)\,dt \\ &= \frac{2}{\pi}\int_0^\pi \frac{t}{\pi}\sin(2nt)\,dt \\ &= \frac{2}{\pi^2}\int_0^\pi t\sin(2nt)\,dt \\ &= \frac{2}{\pi^2}\left[\frac{\sin(2nt)-t\cos(2nt)}{\frac{d(2nt)}{dt}}\right]_0^\pi \\ &= \frac{2}{\pi^2}\left[\frac{\sin(2nt)-t\cos(2nt)}{2n}\right]_0^\pi \\ &= \frac{1}{\pi^2n}[\sin(2n\pi)-\pi\cos(2n\pi)-\sin(0)-(0)\cos(0)] \\ &= \frac{1}{\pi^2n}[0-\pi(1)-0+(0)(1)] \\ &= \frac{-1}{\pi n} \end{aligned}$$

The answer is (B).

10. The value of a number in a Fibonacci sequence is the sum of the previous two numbers in the sequence.

Use the second-order difference equation.

$$\begin{aligned} f(k) &= f(k-1)+f(k-2) \\ f(3) &= f(2)+f(1) = 1.3+4 \\ &= 5.3 \\ f(4) &= f(3)+f(2) = 5.3+1.3 \\ &= 6.6 \end{aligned}$$

The answer is (D).

6 Probability and Statistics

PRACTICE PROBLEMS

1. What is the approximate probability that no people in a group of seven have the same birthday (ignoring leap year birthdays)?

(A) 0.056

(B) 0.43

(C) 0.92

(D) 0.94

2. A study gives the following results for a total sample size of 12.

$$3, 4, 4, 5, 8, 8, 8, 10, 11, 15, 18, 20$$

What is most nearly the mean?

(A) 8.9

(B) 9.5

(C) 11

(D) 12

3. A study gives the following results for a sample size of 8 taken from the population.

$$2, 3, 5, 8, 8, 10, 10, 12$$

The mean of the samples is 7.25. What is most nearly the standard deviation of the sample?

(A) 2.5

(B) 2.9

(C) 3.3

(D) 3.6

4. A study gives the following results for a total sample size of 6.

$$10, 12, 13, 14, 14, 15$$

The mean of the sample is 13. What is most nearly the sample standard deviation?

(A) 0.85

(B) 0.90

(C) 1.6

(D) 1.8

5. A population has a population standard deviation of 10.4, and a sample standard deviation of 11.6. What is most nearly the population variance?

(A) 46

(B) 52

(C) 110

(D) 130

6. A study has a sample size of 9, a standard deviation of 4.0, and a sample standard deviation of 4.2. What is most nearly the sample variance?

(A) 16

(B) 18

(C) 34

(D) 36

7. A bag contains 100 balls numbered 1 to 100. One ball is drawn from the bag. What is the probability that the number on the ball selected will be odd or greater than 80?

(A) 0.1

(B) 0.5

(C) 0.6

(D) 0.7

8. Measurements of the water content of soil from a borrow site are normally distributed with a mean of 14.2% and a standard deviation of 2.3%. What is the probability that a sample taken from the site will have a water content above 16% or below 12%?

(A) 0.13

(B) 0.25

(C) 0.37

(D) 0.42

9. What is the probability that either exactly two heads or exactly three heads will be thrown if six fair coins are tossed at once?

(A) 0.35

(B) 0.55

(C) 0.59

(D) 0.63

10. Which of the following statements about probability is NOT valid?

(A) The probability of an event is always positive and within the range of zero and one.

(B) The probability of an event which cannot occur in the population being examined is zero.

(C) If events A and B are mutually exclusive, then the probability of either event occurring in the same population is zero.

(D) The probability of either of two events, A or B, occurring is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

11. One fair die is used in a dice game. A player wins \$10 if he rolls either a 1 or a 6. He loses \$5 if he rolls any other number. What is the expected value for this game?

(A) \$0.00

(B) \$3.30

(C) \$5.00

(D) \$6.70

12. A simulation model for a transportation system is run for 30 replications, and the mean percentage utilization of the transporter used by the system is recorded for each replication. Those 30 data points are then used to form a confidence interval on mean transporter utilization for the system. At a 95% confidence level, the confidence interval is found to be 37.2% ± 3.4%.

Given this information, which of the following facts can be definitively stated about the system?

(A) At 95% confidence, the sample mean of transporter utilization lies in the range 37.2% ± 3.4%.

(B) At 95% confidence, the population mean of transporter utilization lies in the range 37.2% ± 3.4%.

(C) At 95% confidence, the population mean of transporter utilization lies outside of the range 37.2% ± 3.4%.

(D) At 5% confidence, the population mean of transporter utilization lies inside of the range 37.2% ± 3.4%.

13. What is the approximate probability of exactly two people in a group of seven having a birthday on April 15?

(A) 1.2×10^{-18}

(B) 2.4×10^{-17}

(C) 7.4×10^{-6}

(D) 1.6×10^{-4}

14. What are the arithmetic mean and sample standard deviation of the following numbers?

$$71.3, 74.0, 74.25, 78.54, 80.6$$

(A) 74.3, 2.7

(B) 74.3, 3.8

(C) 75.7, 2.7

(D) 75.7, 3.8

15. Four fair coins are tossed at once. What is the probability of obtaining three heads and one tail?

(A) 1/4

(B) 3/8

(C) 1/2

(D) 3/4

16. Set A and set B are subsets of the universal set U. The values within each set are shown.

$$A = \{4, 7, 9\}$$
$$B = \{4, 5, 9, 10\}$$
$$U = \{4, 5, 6, 7, 8, 9, 10\}$$

What is the union of the complement of set A with set B, $\overline{A} \cup B$?

(A) $\{4,5,6,7,8,9,10\}$

(B) $\{4,5,7,9,10\}$

(C) $\{4,5,6,8,9,10\}$

(D) $\{5,10\}$

17. Set A consists of elements $\{1,3,6\}$, and set B consists of elements $\{1,2,6,7\}$. Both sets come from the universal set of $\{1,2,3,4,5,6,7,8\}$. What is the intersection of the complement of set A with set B, $\overline{A} \cap B$?

(A) $\{2,7\}$

(B) $\{2,3,7\}$

(C) $\{2,4,5,7,8\}$

(D) $\{4,5,8\}$

18. A given thermometer always measures temperature as being 2.5°C higher than the actual temperature. The thermometer is used to measure a water sample and finds the temperature is 25°C. The random error of the measurement is known to be zero. Most nearly, what is the true temperature of the sample?

(A) 23°C

(B) 25°C

(C) 28°C

(D) 30°C

19. A 1 L water sample has a known sulfate concentration of 225 mg/L. The sample is separated into ten aliquots, and each is tested separately for sulfate concentration, producing the test results shown.

aliquot	concentration (mg/L)
1	221
2	229
3	221
4	227
5	226
6	223
7	228
8	223
9	222
10	229

Most nearly, what is the random error associated with the laboratory measurements?

(A) 3.0 mg/L

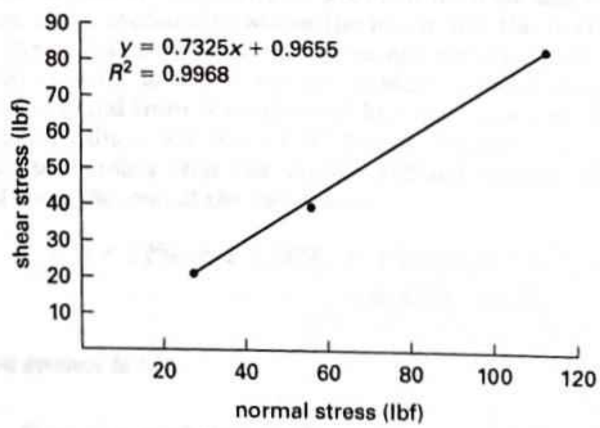
(B) 5.0 mg/L

(C) 10 mg/L

(D) 25 mg/L

20. Strength tests of aggregate limestone are carried out on three field samples, producing the results shown.

x (lbf/ft^2)	y (lbf/ft^2)	$\hat{y}$ (lbf/ft^2)
27.77	22.65	21.31
56.35	40.22	42.24
112.74	84.23	83.55



The linear model of the relationship is found to be $\hat{y} = 0.7325x + 0.9655$. Most nearly, what is the mean of the residuals?

(A) −2.02

(B) 0.00

(C) 0.01

(D) 0.68

Probability/ Statistics

21. A linear model of the form $\widehat{y} = (\widehat{a} + \widehat{b}\,x_i)$ has parameters $\widehat{a} = 1.24$ and $\widehat{b} = 15.35$. The observed dependent variable, y, has a value of 156.21 at $x = 10$. Most nearly, what is the model residual at $x = 10$?

(A) −1.5

(B) −1.0

(C) 1.0

(D) 1.5

Probability/Statistics

SOLUTIONS

1. This is the classic "birthday problem." The problem is to find the probability that all seven people have distinctly different birthdays. The solution can be found from simple counting.

The first person considered can be born on any day, which means the probability that the first person will not be born on one of the 365 days of the year is 0.

$$P(1) = 1 - P(\text{not } 1) = 1 - 0 = 1 \quad (365/365)$$

The probability the second person will be born on the same day as the first person is 1 in 365. (The second person can be born on any other of the 364 days.) The probability that the second person is born on any other day is

$$P(2) = 1 - P(\text{not } 2) = 1 - \frac{1}{365} = \frac{364}{365}$$

The third person cannot have been born on either of the same days as the first and second people, which has a 2 in 365 probability of happening. The probability that the third person is born on any other day is

$$P(3) = 1 - P(\text{not } 3) = 1 - \frac{2}{365} = \frac{363}{365}$$

This logic continues to the seventh person. The probability that all seven conditions are simultaneously satisfied is

$$\begin{aligned} &P(7 \text{ distinct birthdays}) \\ &\quad = P(1) \times P(2) \times P(3) \times P(4) \times P(5) \\ &\qquad \times P(6) \times P(7) \\ &\quad = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)\left(\frac{362}{365}\right)\left(\frac{361}{365}\right) \\ &\qquad \times \left(\frac{360}{365}\right)\left(\frac{359}{365}\right) \\ &\quad = 0.9438 \quad (0.94) \end{aligned}$$

The answer is (D).

2. The mean is

$$\begin{aligned} \overline{X} &= (1/n)\sum_{i=1}^{n} X_i \\ &= \left(\frac{1}{12}\right)\begin{pmatrix} 3+4+4+5 \\ +8+8+8+10 \\ +11+15+18+20 \end{pmatrix} \\ &= 9.5 \end{aligned}$$

The answer is (B).

3. The standard deviation of the sample is calculated as follows.

$$s = \sqrt{\left(1/_{n-1}\right)\sum\left(X_i - \overline{X}\right)^2}$$

$$= \sqrt{\left(\frac{1}{8-1}\right)\left(\begin{array}{l}(2-7.25)^2 + (3-7.25)^2 \\ +(5-7.25)^2 + (8-7.25)^2 \\ +(8-7.25)^2 + (10-7.25)^2 \\ +(10-7.25)^2 + (12-7.25)^2\end{array}\right)}$$

$$= 3.5757 \quad (3.6)$$

The answer is (D).

4. The sample standard deviation is

$$s = \sqrt{[1/(n-1)]\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}$$

$$= \sqrt{\left(\frac{1}{6-1}\right)\left(\begin{array}{l}(10-13)^2 + (12-13)^2 \\ +(13-13)^2 + (14-13)^2 \\ +(14-13)^2 + (15-13)^2\end{array}\right)}$$

$$= 1.79 \quad (1.8)$$

The answer is (D).

5. The population variance is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2$$

The population standard deviation is

$$\sigma_{\text{population}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2}$$

The population variance is the square of the population standard deviation.

$$\sigma^2 = \sigma^2_{\text{population}} = (10.4)^2 = 108 \quad (110)$$

The answer is (C).

6. The sample variance is the square of the sample standard deviation.

$$s^2 = (4.2)^2 = 17.64 \quad (18)$$

The answer is (B).

7. There are 50 odd-numbered balls. Including ball 100, there are 20 balls with numbers greater than 80.

$$P(A) = P(\text{ball is odd}) = \frac{50}{100} = 0.5$$

$$P(B) = P(\text{ball} > 80) = \frac{19}{100} = 0.19$$

It is possible for the number on the selected ball to be both odd and greater than 80. Use the law of total probability.

$$\begin{aligned}P(A+B) &= P(A) + P(B) - P(A,B) \\ &= P(A) + P(B) - P(A)P(B)\end{aligned}$$

$$\begin{aligned}P(\text{odd or} > 80) &= 0.5 + 0.19 - (0.5)(0.19) \\ &= 0.595 \quad (0.6)\end{aligned}$$

The answer is (C).

8. Find the standard normal values for the two points of interest.

$$\begin{aligned}Z_{16\%} &= \frac{x-\mu}{\sigma} = \frac{16\% - 14.2\%}{2.3\%} \\ &= 0.78 \quad [\text{use } 0.80]\end{aligned}$$

$$\begin{aligned}Z_{12\%} &= \frac{x-\mu}{\sigma} = \frac{12\% - 14.2\%}{2.3\%} \\ &= -0.96 \quad [\text{use } -1.00]\end{aligned}$$

Use the unit normal distribution table. The $2R(x)$ column cannot be used for this problem because the portion of the probability above the mean and the portion of the probability below the mean are not equal, so the $R(x)$ column is used. The probabilities being sought can be found from the values of $R(x)$ for both standard normal values. $R(0.80) = 0.2119$ and $R(1.00) = 0.1587$. The probability that the sample will fall outside these values is the sum of the two values.

$$\begin{aligned}P(x < 12\% \text{ or } x > 16\%) &= 0.2119 + 0.1587 \\ &= 0.3706 \quad (0.37)\end{aligned}$$

The answer is (C).

9. Find the probability of exactly two heads being thrown. The probability will be the quotient of the total number of possible combinations of six objects taken two at a time and the total number of possible outcomes from tossing six fair coins. The total number of possible outcomes is $(2)^6 = 64$. The total number of possible combinations in which exactly two heads are thrown is

$$C(n,r) = \frac{n!}{r!(n-r)!} = \frac{6!}{2!(6-2)!} = 15$$

The probability of exactly two heads out of six fair coins is

$$P(A) = P(2 \text{ heads}) = \frac{15}{64} = 0.234$$

The probability of exactly three heads being thrown is found similarly. The total number of possible combinations in which exactly three heads are thrown is

$$\begin{aligned} C(n,r) &= \frac{n!}{r!(n-r)!} = \frac{6!}{3!(6-3)!} \\ &= 20 \end{aligned}$$

The probability of exactly three heads out of six fair coins is

$$P(B) = P(3 \text{ heads}) = \frac{20}{64} = 0.313$$

From the law of total probability, the probability that either of these outcomes will occur is the sum of the individual probabilities that the outcomes will occur, minus the probability that both will occur. These two outcomes are mutually exclusive (i.e., both cannot occur), so the probability of both happening is zero.

The total probability is

$$\begin{aligned} P(2 \text{ heads or } 3 \text{ heads}) &= P(A) + P(B) - P(A,B) \\ &= 0.234 + 0.313 - 0 \\ &= 0.547 \quad (0.55) \end{aligned}$$

The answer is (B).

10. If events A and B are mutually exclusive, the probability of both occurring is zero. However, either event could occur by itself, and the probability of that is nonzero.

The answer is (C).

11. For a fair die, the probability of any face turning up is $1/6$. There are two ways to win, and there are four ways to lose. The expected value is

$$\begin{aligned} E[X] &= \sum_{k=1}^{n} x_k f(x_k) = (\$10)\left((2)\left(\frac{1}{6}\right)\right) + (-\$5)\left((4)\left(\frac{1}{6}\right)\right) \\ &= \$0.00 \end{aligned}$$

The answer is (A).

12. A 95% confidence interval on mean transporter utilization means there is a 95% chance the population (or true) mean transporter utilization lies within the given interval based on the sample of 30 replications.

The answer is (B).

13. Use the binomial probability function to calculate the probability that two of the seven samples will have been born on April 15. $x = 2$, and the sample size, n, is 7.

The probability that a person will have been born on April 15 is $1/365$. Therefore, the probability of "success," p, is $1/365$, and the probability of "failure," $q = 1 - p$, is $364/365$.

$$\begin{aligned} P_n(x) &= \frac{n!}{x!(n-x)!} p^x q^{n-x} \\ P_7(2) &= \left(\frac{7!}{2!(7-2)!}\right)\left(\frac{1}{365}\right)^2\left(\frac{364}{365}\right)^{7-2} \\ &= (21)\left(\frac{1}{365}\right)^2\left(\frac{364}{365}\right)^5 \\ &= 1.555 \times 10^{-4} \quad (1.6 \times 10^{-4}) \end{aligned}$$

The answer is (D).

14. The arithmetic mean is

$$\begin{aligned} \overline{X} &= (1/n)\sum_{i=1}^{n} X_i \\ &= \left(\frac{1}{5}\right)(71.3 + 74.0 + 74.25 + 78.54 + 80.6) \\ &= 75.738 \quad (75.7) \end{aligned}$$

The sample standard deviation is

$$\begin{aligned} s &= \sqrt{[1/(n-1)]\sum_{i=1}^{n}(X_i - \overline{X})^2} \\ &= \sqrt{\left(\frac{1}{5-1}\right)\left(\begin{array}{l} (71.3 - 75.738)^2 + (74.0 - 75.738)^2 \\ \quad + (74.25 - 75.738)^2 \\ \quad + (78.54 - 75.738)^2 \\ \quad + (80.6 - 75.738)^2 \end{array}\right)} \\ &= 3.756 \quad (3.8) \end{aligned}$$

The answer is (D).

15. The binomial probability function can be used to determine the probability of three heads in four trials. The probability of heads is independent for the coins, so they can be treated as four separate trials; that is, the probability of tossing a single fair coin four times and getting three heads and one tail is the same as the probability of tossing four fair coins at the same time and getting three heads and one tail.

$$\begin{aligned} p &= P(\text{heads}) = 0.5 \\ q &= P(\text{not heads}) = 1 - 0.5 = 0.5 \\ n &= \text{number of trials} = 4 \\ x &= \text{number of successes} = 3 \end{aligned}$$

From the binomial function,

$$\begin{aligned} P_n(x) &= \frac{n!}{x!(n-x)!} p^x q^{n-x} \\ &= \left(\frac{4!}{3!(4-3)!}\right)(0.5)^3(0.5)^{4-3} \\ &= 0.25 \quad (1/4) \end{aligned}$$

This problem may be solved directly because each of the possible outcomes has the same probability. There are 16 possible outcomes flipping four fair coins at the same time. There are four outcomes with three heads and one tail. The probability of three heads and one tail is four divided by 16 or 0.25.

The answer is (A).

16. The complement of set A contains all of the members of set U that are not members of set A.

$$\overline{A} = \{5, 6, 8, 10\}$$

The union of the complement of set A with set B is the set of all members appearing in either.

$$\begin{aligned} \overline{A} \cup B &= \{5, 6, 8, 10\} \cup \{4, 5, 9, 10\} \\ &= \{4, 5, 6, 8, 9, 10\} \end{aligned}$$

The answer is (C).

17. The complement of set A consists of all elements in the universal set that are not in set A: $\{2, 4, 5, 7, 8\}$.

The intersection of the complement of set A with set B is the set of all elements appearing in both.

$$\begin{aligned} \overline{A} \cap B &= \{2, 4, 5, 7, 8\} \cap \{1, 2, 6, 7\} \\ &= \{2, 7\} \end{aligned}$$

The answer is (A).

18. Use the equation for measurement error, and solve for the true measurement.

$$\begin{aligned} x &= x_{\text{true}} + x_{\text{bias}} + x_{\text{re}} \\ x_{\text{true}} &= x - x_{\text{bias}} - x_{\text{re}} \\ &= 25^\circ\text{C} - 2.5^\circ\text{C} - 0^\circ\text{C} \\ &= 22.5^\circ\text{C} \quad (23^\circ\text{C}) \end{aligned}$$

The answer is (A).

19. Find the mean value of the aliquot testing results.

$$\begin{aligned} \mu &= \frac{\begin{array}{r} 221\ \frac{\text{mg}}{\text{L}} + 229\ \frac{\text{mg}}{\text{L}} + 221\ \frac{\text{mg}}{\text{L}} + 227\ \frac{\text{mg}}{\text{L}} \\ +226\ \frac{\text{mg}}{\text{L}} + 223\ \frac{\text{mg}}{\text{L}} + 228\ \frac{\text{mg}}{\text{L}} \\ +223\ \frac{\text{mg}}{\text{L}} + 222\ \frac{\text{mg}}{\text{L}} + 229\ \frac{\text{mg}}{\text{L}} \end{array}}{10} \\ &= 224.9\ \text{mg/L} \end{aligned}$$

The random error is

$$\begin{aligned} \sigma_{\text{population}} = \sigma_{\text{re}} &= \sqrt{\frac{1}{N}\sum(x_i - \mu)^2} \\ &= \sqrt{\frac{1}{10}\left(\begin{array}{l} \left(221\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(229\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(221\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(227\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(226\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(223\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(228\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(223\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(221\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \\ +\left(229\ \frac{\text{mg}}{\text{L}} - 224.9\ \frac{\text{mg}}{\text{L}}\right)^2 \end{array}\right)} \\ &= 3.1\ \text{mg/L} \quad (3.0\ \text{mg/L}) \end{aligned}$$

The answer is (A).

Probability/ Statistics

20. From the equation for the residual,

$$e_i = y_i - \hat{y}$$

$$e_1 = 22.65\ \frac{\text{lbf}}{\text{ft}^2} - 21.31\ \frac{\text{lbf}}{\text{ft}^2} = 1.34\ \text{lbf/ft}^2$$

$$e_2 = 40.22\ \frac{\text{lbf}}{\text{ft}^2} - 42.24\ \frac{\text{lbf}}{\text{ft}^2} = -2.02\ \text{lbf/ft}^2$$

$$e_3 = 84.23\ \frac{\text{lbf}}{\text{ft}^2} - 83.55\ \frac{\text{lbf}}{\text{ft}^2} = 0.68\ \text{lbf/ft}^2$$

$$\frac{1}{N}\sum_{i=1}^{N} e_i = \frac{1.34 - 2.02 + 0.68}{3} = 0.00$$

The answer is (B).

21. From the equation for the residual,

$$\begin{aligned} e_i &= y_i - \hat{y} = y_i - (\hat{a} + \hat{b}x_i) \\ &= 156.21 - \big(1.24 + (15.35)(10)\big) \\ &= 1.47 \quad (1.5) \end{aligned}$$

The answer is (D).

7 Fluid Properties

PRACTICE PROBLEMS

1. A leak from a faucet comes out in separate drops instead of a stream. What is the main cause of this phenomenon?

(A) gravity

(B) air resistance

(C) viscosity

(D) surface tension

2. A solid cylinder is concentric with a straight pipe. The cylinder is 0.5 m long and has an outside diameter of 8 cm. The pipe has an inside diameter of 8.5 cm. The annulus between the cylinder and the pipe contains stationary oil. The oil has a specific gravity of 0.92 and a kinematic viscosity of 5.57×10^{-4} m^2/s. Most nearly, what is the force needed to move the cylinder along the pipe at a constant velocity of 1 m/s?

(A) 5.9 N

(B) 12 N

(C) 26 N

(D) 55 N

3. Kinematic viscosity can be expressed in

(A) m^2/s

(B) s^2/m

(C) $kg{\cdot}s^2/m$

(D) kg/s

4. Which three of the following must be satisfied by the flow of any fluid, whether real or ideal?

I. Newton's second law of motion

II. the continuity equation

III. uniform velocity distribution

IV. Newton's law of viscosity

V. conservation of energy

(A) I, II, and III

(B) I, II, and V

(C) I, III, and V

(D) II, IV, and V

5. 15 kg of a fluid with a density of 790 kg/m^3 is mixed with 10 kg of water. The volumes are additive, and the resulting mixture is homogeneous. Most nearly, what is the specific volume of the resulting mixture?

(A) 0.0012 m^3/kg

(B) 0.0027 m^3/kg

(C) 0.0047 m^3/kg

(D) 0.0061 m^3/kg

6. The rise or fall of liquid in a small-diameter capillary tube is NOT affected by

(A) adhesive forces

(B) cohesive forces

(C) surface tension

(D) fluid viscosity

7. A capillary tube 3.8 mm in diameter is placed in a beaker of 40°C distilled water. The surface tension is 0.0696 N/m, and the angle made by the water with the wetted tube wall is negligible. The specific weight of water at this temperature is 9.730 kN/m^3. Most nearly, what is the height to which the water will rise in the tube?

(A) 1.2 mm

(B) 3.6 mm

(C) 7.5 mm

(D) 9.2 mm

SOLUTIONS

1. Surface tension is caused by the molecular cohesive forces in a fluid. It is the main cause of the formation of drops of water.

The answer is (D).

2. Treat the cylinder as a moving plate, and use Newton's law of viscosity. Find the absolute viscosity of the oil using the kinematic viscosity.

$$\nu = \frac{\mu}{\rho}$$

$$\begin{aligned}\mu = \nu\rho &= \left(5.57 \times 10^{-4}\ \frac{\text{m}^2}{\text{s}}\right)(0.92)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) \\ &= 0.512\ \text{Pa·s}\end{aligned}$$

From geometry, the width of the separation between the cylinder and the pipe is

$$\begin{aligned}\delta &= \frac{d_{\text{pipe}} - d_{\text{cylinder}}}{2} \\ &= \frac{8.5\ \text{cm} - 8\ \text{cm}}{2} \\ &= 0.25\ \text{cm}\end{aligned}$$

The surface area of the cylinder is

$$\begin{aligned}A &= \pi d L \\ &= \pi\left(\frac{8\ \text{cm}}{100\ \frac{\text{cm}}{\text{m}}}\right)(0.5\ \text{m}) \\ &= 0.126\ \text{m}^2\end{aligned}$$

Find the force needed by using the stress equation.

$$\frac{F}{A} = \mu\left(\frac{d\text{v}}{dy}\right)$$

$$\begin{aligned}F &= A\mu\left(\frac{d\text{v}}{dy}\right) \approx A\mu\left(\frac{\Delta\text{v}}{\Delta y}\right) \\ &= (0.126\ \text{m}^2)(0.512\ \text{Pa·s})\left(\frac{1\ \frac{\text{m}}{\text{s}}}{0.25\ \text{cm}}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right) \\ &= 25.8\ \text{N} \quad (26\ \text{N})\end{aligned}$$

The answer is (C).

3. Typical units of kinematic viscosity are m^2/s.

The answer is (A).

4. Newton's second law, the continuity equation, and the principle of conservation of energy always apply for any fluid.

The answer is (B).

5. Calculate the volumes. Use a standard water density of 1000 kg/m^3.

$$\rho = \frac{m}{V}$$
$$V = \frac{m}{\rho}$$
$$V_{\text{water}} = \frac{m}{\rho} = \frac{10 \text{ kg}}{1000 \ \frac{\text{kg}}{\text{m}^3}} = 0.010 \text{ m}^3$$
$$V_{\text{fluid}} = \frac{m}{\rho} = \frac{15 \text{ kg}}{790 \ \frac{\text{kg}}{\text{m}^3}} = 0.019 \text{ m}^3$$

The total volume is

$$V_{\text{total}} = V_{\text{water}} + V_{\text{fluid}} = 0.010 \text{ m}^3 + 0.019 \text{ m}^3 = 0.029 \text{ m}^3$$

The density of the mixture is the total mass divided by the total volume.

$$\rho = \frac{m_{\text{water}} + m_{\text{fluid}}}{V_{\text{total}}} = \frac{10 \text{ kg} + 15 \text{ kg}}{0.029 \text{ m}^3} = 862 \text{ kg/m}^3$$

The specific volume of the mixture is the reciprocal of its density.

$$\upsilon_{\text{mixture}} = \frac{1}{\rho_{\text{mixture}}} = \frac{1}{862 \ \frac{\text{kg}}{\text{m}^3}} = 0.00116 \text{ m}^3/\text{kg} \quad (0.0012 \text{ m}^3/\text{kg})$$

The answer is (A).

6. The height of capillary rise is

$$h = 4\sigma \cos\beta/\gamma d$$

σ is the surface tension of the fluid, β is the angle of contact, γ is the specific weight of the liquid, and d is the diameter of the tube.

The viscosity of the fluid is not directly relevant to the height of capillary rise.

The answer is (D).

7. Since the contact angle is negligible, use 0° for β. The capillary rise in liquid is

$$h = 4\sigma \cos\beta/\gamma d$$
$$= \frac{(4)\left(0.0696 \ \frac{\text{N}}{\text{m}}\right)\cos 0°\left(1000 \ \frac{\text{mm}}{\text{m}}\right)}{\left(9.730 \ \frac{\text{kN}}{\text{m}^3}\right)\left(1000 \ \frac{\text{N}}{\text{kN}}\right)(3.8 \text{ mm})}$$
$$= 7.53 \times 10^{-3} \text{ m} \quad (7.5 \text{ mm})$$

The answer is (C).

Fluid Mechanics

8 Fluid Statics

PRACTICE PROBLEMS

1. A barometer contains mercury with a density of 13 600 kg/m^3. Atmospheric conditions are 95.8 kPa and 20°C. At 20°C, the vapor pressure of the mercury is 0.000173 kPa. The column of mercury will rise to a height of most nearly

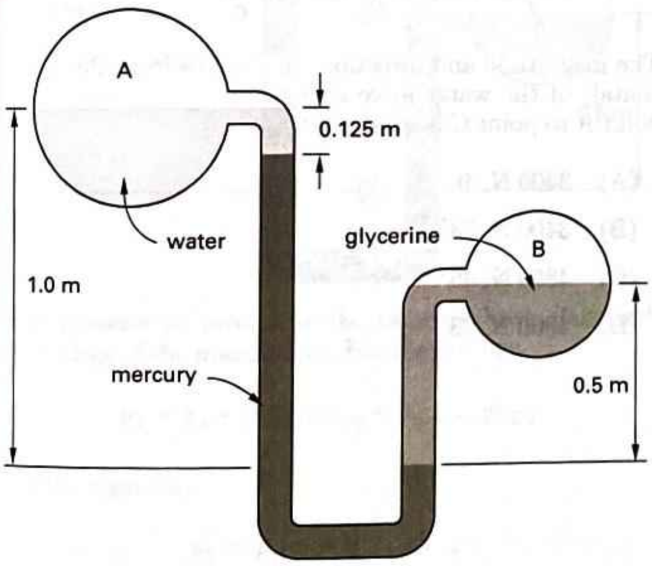
(A) 0.38 m

(B) 0.48 m

(C) 0.72 m

(D) 0.82 m

2. The manometer shown contains water, mercury, and glycerine. The specific gravity of mercury is 13.6, and the specific gravity of glycerine is 1.26.



What is most nearly the difference in pressure between points A and B?

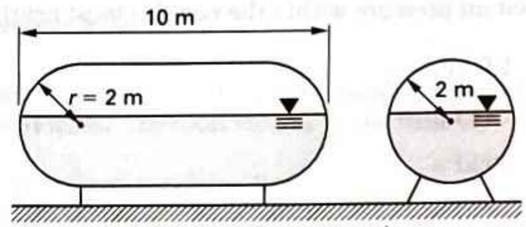
(A) 35 kPa

(B) 42 kPa

(C) 55 kPa

(D) 110 kPa

3. An open water manometer is used to measure the pressure in a tank. The tank is cylindrical with hemispherical ends. The tank is half-filled with 50 000 kg of a liquid chemical that is not miscible in water. The manometer tube is filled with liquid chemical up to the water.



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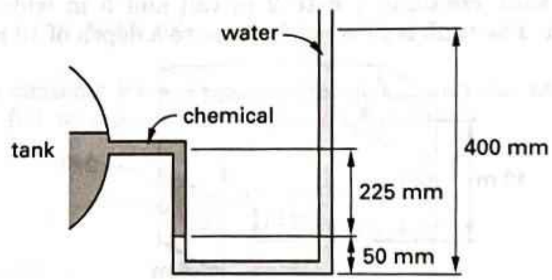


What is most nearly the pressure in the tank relative to the atmospheric pressure?

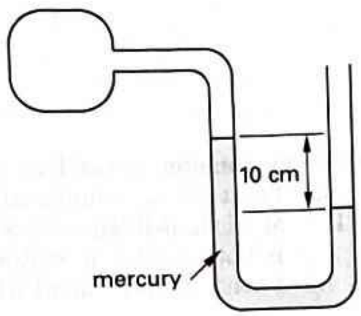
(A) 1.4 kPa

(B) 1.9 kPa

(C) 2.4 kPa

(D) 3.4 kPa

4. A pressure vessel is connected to a simple U-tube open to the atmosphere as shown. A 10 cm deflection of mercury is observed. The density of mercury is 13 600 kg/m^3.



The vacuum pressure within the vessel is most nearly

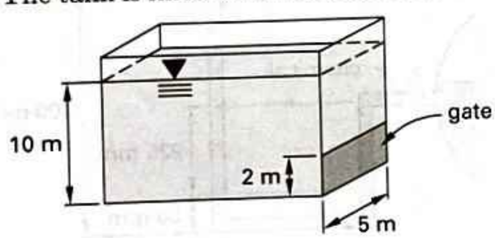
(A) 1.0 kPa

(B) 13 kPa

(C) 39 kPa

(D) 78 kPa

5. A tank contains a gate 2 m tall and 5 m wide as shown. The tank is filled with water to a depth of 10 m.



The total force from hydrostatic pressure on the gate is most nearly

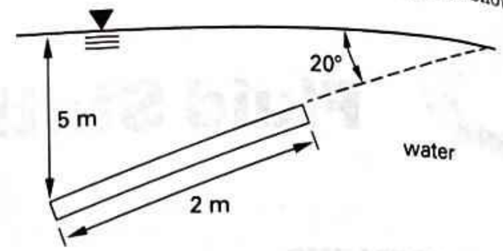
(A) 90 kN

(B) 440 kN

(C) 880 kN

(D) 980 kN

6. A 1 m × 2 m inclined plate is submerged as shown.



The normal force acting on the upper surface of the plate due to the water (when taking atmospheric pressure into account) is most nearly

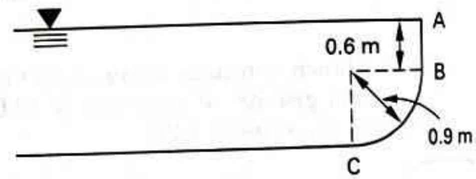
(A) 68 kN

(B) 91 kN

(C) 203 kN

(D) 294 kN

7. The water tank shown has a width of 0.3 m. The rounded corner has a radius of 0.9 m.



The magnitude and direction (in degrees from the horizontal) of the water force over the length of wall from point A to point C is most nearly

(A) 3400 N, 46°

(B) 3400 N, 73°

(C) 4800 N, 46°

(D) 4800 N, 73°

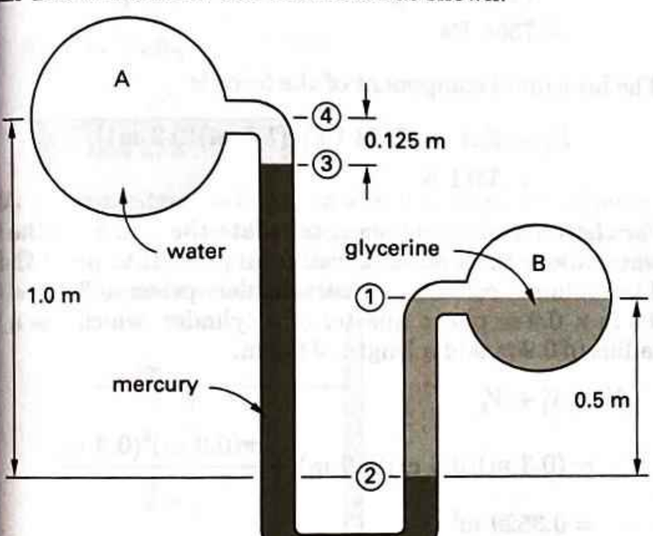
SOLUTIONS

1. Find the height of the mercury in the column.

$$p_A = p_B + \rho g h$$
$$h = \frac{p_A - p_B}{\rho g}$$
$$= \frac{(95.8\ \text{kPa} - 0.000173\ \text{kPa})\left(1000\ \frac{\text{Pa}}{\text{kPa}}\right)}{\left(13\,600\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 0.7181\ \text{m} \quad (0.72\ \text{m})$$

The answer is (C).

2. The manometer can be labeled as shown.



The pressure at level 2 is the same in both (left and right) legs of the manometer. For the left leg,

$$p_2 = p_A + \rho_{\text{water}} g h_{3\text{-}4} + \rho_{\text{mercury}} g h_{2\text{-}3}$$

For the right leg,

$$p_2 = p_B + \rho_{\text{glycerine}} g h_{1\text{-}2}$$

Where $h_{1\text{-}2}$ is the distance between levels 1 and 2, $h_{2\text{-}3}$ is the distance between levels 2 and 3, and $h_{3\text{-}4}$ is the distance between levels 3 and 4.

Equating these two equations for p_2 and solving for the pressure difference $p_A - p_B$ gives

$$p_A - p_B = g(\rho_{\text{glycerine}} h_{1\text{-}2} - \rho_{\text{water}} h_{3\text{-}4} - \rho_{\text{Hg}} h_{2\text{-}3})$$
$$= g\rho_{\text{water}}\begin{pmatrix} \text{SG}_{\text{glycerine}} h_{1\text{-}2} - \text{SG}_{\text{water}} h_{3\text{-}4} \\ -\text{SG}_{\text{Hg}} h_{2\text{-}3} \end{pmatrix}$$
$$= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) \times \begin{pmatrix} (1.26)(0.5\ \text{m}) - (1.00)(0.125\ \text{m}) \\ -(13.6)(1.0\ \text{m} - 0.125\ \text{m}) \end{pmatrix}$$
$$= -111\,785\ \text{Pa} \quad (110\ \text{kPa})$$

The answer is (D).

3. Calculate the density of the chemical from the volume and mass. The total volume of the tank is

$$V = \tfrac{4}{3}\pi r^3 + \pi r^2 (L - 2r)$$
$$= \tfrac{4}{3}\pi(2\ \text{m})^3 + \pi(2\ \text{m})^2\big(10\ \text{m} - (2)(2\ \text{m})\big)$$
$$= 108.9\ \text{m}^3$$

The contents have a mass of 50 000 kg, and the tank is half full, so the density of the chemical is

$$\rho_{\text{chemical}} = \frac{m}{V} = \frac{50\,000\ \text{kg}}{\left(\frac{1}{2}\right)(108.9\ \text{m}^3)}$$
$$= 918.2\ \text{kg/m}^3$$

The relative pressure is

$$p_0 - p_2 = \rho_{\text{water}} g h_2 - \rho_{\text{chemical}} g h_1$$
$$= \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{400\ \text{mm} - 50\ \text{mm}}{1000\ \frac{\text{mm}}{\text{m}}}\right) - \left(918.2\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{225\ \text{mm}}{1000\ \frac{\text{mm}}{\text{m}}}\right)$$
$$= 1407\ \text{Pa} \quad (1.4\ \text{kPa})$$

The answer is (A).

4. The mercury is higher at the vessel end of the U-tube, so the pressure in the vessel is less than atmospheric pressure, and the U-tube is measuring the vacuum pressure in the vessel. (If the mercury were higher at the end open to the atmosphere, the U-tube would be measuring gage pressure.) The vacuum pressure in the vessel is

$$\begin{aligned} p_{vac} &= \rho g h \\ &= \left(13\,600\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{10\ \text{cm}}{100\ \frac{\text{cm}}{\text{m}}}\right) \\ &= 13\,342\ \text{Pa} \quad (13\ \text{kPa}) \end{aligned}$$

The answer is (B).

5. $h_1 = 10\ \text{m} - 2\ \text{m} = 8\ \text{m}$. Hydrostatic pressure is determined based on the height of fluid above any one point. Since the top of the gate is 2 meters higher than the bottom of the gate, the pressure at the bottom of the gate will be higher than that at the top by the equivalent pressure of 2 meters of water. To calculate the average pressure exerted on the gate, an average can be taken of the height of water over the gate. The average pressure is

$$\begin{aligned} \bar{p} &= \tfrac{1}{2}\rho g(h_1 + h_2) \\ &= \left(\frac{1}{2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(8\ \text{m} + 10\ \text{m}) \\ &= 88\,290\ \text{Pa} \end{aligned}$$

The total force acting on the gate is

$$\begin{aligned} R &= \bar{p}A \\ &= (88\,290\ \text{Pa})\big((2\ \text{m})(5\ \text{m})\big) \\ &= 882\,900\ \text{N} \quad (880\ \text{kN}) \end{aligned}$$

The answer is (C).

6. The upper edge of the plate is at a depth of

$$\begin{aligned} h_1 &= 5\ \text{m} - (2\ \text{m})\sin 20^\circ \\ &= 4.32\ \text{m} \end{aligned}$$

The value of standard atmospheric pressure, P_{atm}, is found in the Units and Conversions table of the *NCEES Handbook*:

$$P_{atm} = 1.013 \times 10^5\ \text{Pa}.$$

The average pressure is calculated as

$$\begin{aligned} \bar{p} &= \tfrac{1}{2}(p_1 + p_2) \\ &= \tfrac{1}{2}\big((p_{atm} + \rho g h_1) + (p_{atm} + \rho g h_2)\big) \\ &= \tfrac{1}{2}\rho g(h_1 + h_2) + p_{atm} \\ &= \left(\frac{1}{2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(4.32\ \text{m} + 5\ \text{m}) + 101\,300\ \text{Pa} \\ &= 146\,995\ \text{Pa} \end{aligned}$$

The normal force acting on the plate is

$$\begin{aligned} R &= \bar{p}A \\ &= (146\,995\ \text{Pa})\big((1\ \text{m})(2\ \text{m})\big) \\ &= 293\,990\ \text{N} \quad (294\ \text{kN}) \end{aligned}$$

The answer is (D).

7. Find separately the horizontal and vertical components of the force acting on the wall from point A to point C. For the horizontal component, $h_1 = 0$ m, and $h_2 = 0.6\ \text{m} + 0.9\ \text{m} = 1.5\ \text{m}$.

$$\begin{aligned} \bar{p}_x &= \tfrac{1}{2}\rho g(h_1 + h_2) \\ &= \left(\frac{1}{2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0\ \text{m} + 1.5\ \text{m}) \\ &= 7358\ \text{Pa} \end{aligned}$$

The horizontal component of the force is

$$\begin{aligned} R_x &= \bar{p}_x A = (7358\ \text{Pa})\big((1.5\ \text{m})(0.3\ \text{m})\big) \\ &= 3311\ \text{N} \end{aligned}$$

For the vertical component, calculate the weight of the water above the section of wall from point B to point C. The volume consists of rectangular prism 0.3 m × 0.6 m × 0.9 m plus a quarter of a cylinder, which has a radius of 0.9 m and a length of 0.3 m.

$$\begin{aligned} V &= V_1 + V_2 \\ &= (0.3\ \text{m})(0.6\ \text{m})(0.9\ \text{m}) + \frac{\pi(0.9\ \text{m})^2(0.3\ \text{m})}{4} \\ &= 0.3529\ \text{m}^3 \end{aligned}$$

The vertical component of the force equals the weight of this volume of water.

$$\begin{aligned} R_y &= \rho g V \\ &= \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.3529\ \text{m}^3) \\ &= 3461\ \text{N} \end{aligned}$$

The resultant force acting on this section of wall is

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} = \sqrt{(3311\ \text{N})^2 + (3461\ \text{N})^2} \\ &= 4790\ \text{N} \quad (4800\ \text{N}) \end{aligned}$$

The direction of the resultant force from the horizontal is

$$\begin{aligned} \theta &= \arctan\frac{R_y}{R_x} = \arctan\frac{3461\ \text{N}}{3311\ \text{N}} \\ &= 46.27^\circ \quad (46^\circ) \end{aligned}$$

The answer is (C).

Fluid Mechanics

9 Fluid Measurement and Similitude

PRACTICE PROBLEMS

1. A pitot tube is used to measure the flow of an incompressible fluid with a density of 926 kg/m^3. The velocity is measured as 2 m/s, and the stagnation pressure is 14.1 kPa. Most nearly, what is the static pressure of the fluid where the measurement is taken?

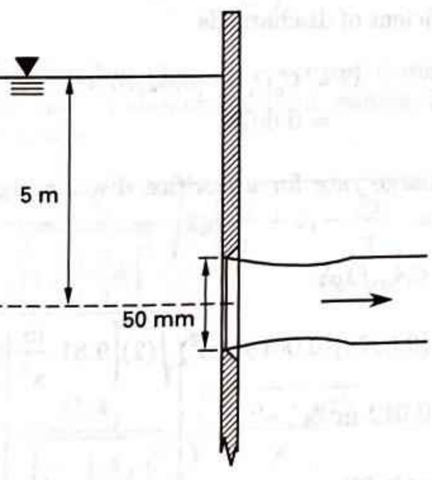
(A) 10.4 kPa

(B) 11.7 kPa

(C) 12.2 kPa

(D) 13.5 kPa

2. A sharp-edged orifice with a 50 mm diameter opening in the vertical side of a large tank discharges under a head of 5 m. The coefficient of contraction is 0.62, and the coefficient of velocity is 0.98.



Most nearly, what is the rate of discharge?

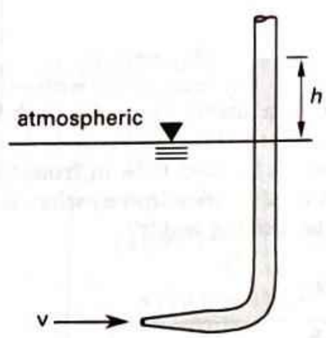
(A) 0.00031 m^3/s

(B) 0.0040 m^3/s

(C) 0.010 m^3/s

(D) 0.012 m^3/s

3. The velocity of the water in the stream shown is 1.2 m/s.



Most nearly, what is the height of water in the pitot tube?

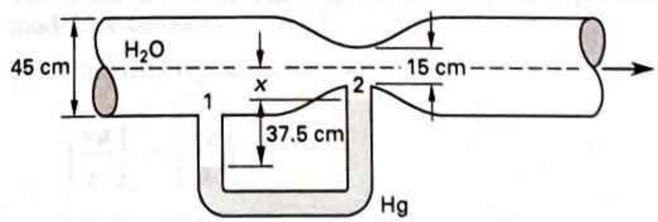
(A) 3.7 cm

(B) 4.6 cm

(C) 7.3 cm

(D) 9.2 cm

4. A horizontal venturi meter with a diameter of 15 cm at the throat is installed in a 45 cm water main. A differential manometer gauge is partly filled with mercury (the remainder of the tube is filled with water) and connected to the meter at the throat and inlet. The mercury column stands 37.5 cm higher in one leg than in the other. The specific gravity of mercury is 13.6.



Neglecting friction, what is most nearly the flow through the meter?

(A) 0.10 m^3/s

(B) 0.17 m^3/s

(C) 0.23 m^3/s

(D) 0.28 m^3/s

Fluid Mechanics

Fluid Mechanics

5. A 1:1 model of a torpedo is tested in a wind tunnel according to the Reynolds number similarity. At the testing temperature, the kinematic viscosity of air is 1.41×10^{-5} m²/s, and the kinematic viscosity of water is 1.31×10^{-6} m²/s. The velocity of the torpedo in water is 7 m/s. Most nearly, what should be the air velocity in the wind tunnel?

(A) 0.62 m/s

(B) 7.0 m/s

(C) 18 m/s

(D) 75 m/s

6. A 2 m tall, 0.5 m inside diameter tank is filled with water. A 10 cm diameter circular hole is opened. The center of the hole is located 0.75 m from the bottom of the tank. Ignoring all orifice losses, what is most nearly the velocity of the exiting water?

(A) 4.75 m/s

(B) 4.80 m/s

(C) 4.85 m/s

(D) 4.95 m/s

7. Water flows from one reservoir to another through a perfectly insulated pipe. Between the two reservoirs, 100 m of head is lost due to friction. Water has a specific heat of 4180 J/kg·K. Most nearly, what is the increase in water temperature between the reservoirs?

(A) 0.23°C

(B) 0.52°C

(C) 0.70°C

(D) 1.0°C

SOLUTIONS

1. Solve the equation for velocity in a pitot tube for the static pressure.

$$\mathrm{v} = \sqrt{\left(\frac{2}{\rho}\right)(P_0 - P_s)}$$

$$\begin{aligned} P_s &= P_0 - \frac{\rho \mathrm{v}^2}{2} \\ &= 14.1\ \mathrm{kPa} - \frac{\left(926\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(2\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)\left(1000\ \frac{\mathrm{Pa}}{\mathrm{kPa}}\right)} \\ &= 12.2\ \mathrm{kPa} \end{aligned}$$

The answer is (C).

2. The cross-sectional area of flow is

$$\begin{aligned} A_0 &= \frac{\pi D^2}{4} = \frac{\pi (50\ \mathrm{mm})^2}{(4)\left(1000\ \frac{\mathrm{mm}}{\mathrm{m}}\right)^2} \\ &= 0.00196\ \mathrm{m}^2 \end{aligned}$$

The coefficient of discharge is

$$\begin{aligned} C &= C_c C_\mathrm{v} = (0.62)(0.98) \\ &= 0.6076 \end{aligned}$$

The discharge rate for an orifice discharging in atmosphere is

$$\begin{aligned} Q &= C A_0 \sqrt{2gh} \\ &= (0.6076)(0.00196\ \mathrm{m}^2)\sqrt{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(5\ \mathrm{m})} \\ &= 0.012\ \mathrm{m}^3/\mathrm{s} \end{aligned}$$

The answer is (D).

3. The difference in height between the pitot tube and the free-water surface is a measure of the difference in static and stagnation pressures. Solve for the height of the water.

$$\begin{aligned} \mathrm{v} &= \sqrt{\left(\frac{2}{\rho}\right)(p_0 - p_s)} = \sqrt{\left(\frac{2}{\rho}\right)\rho g h} \\ &= \sqrt{2gh} \\ h &= \frac{\mathrm{v}^2}{2g} = \frac{\left(1.2\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} \\ &= 0.073\ \mathrm{m} \quad (7.3\ \mathrm{cm}) \end{aligned}$$

The answer is (C).

4. The areas of the pipes are

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi(45\ \mathrm{cm})^2}{(4)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)^2} = 0.159\ \mathrm{m}^2$$

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi(15\ \mathrm{cm})^2}{(4)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)^2} = 0.0177\ \mathrm{m}^2$$

The equation for flow through a venturi meter can be written in terms of a manometer fluid reading. For horizontal flow, $z_1 = z_2$.

$$\begin{aligned} Q &= \frac{C_\mathrm{v} A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \\ &= \left(\frac{C_\mathrm{v} A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\right) \sqrt{\frac{2g(\rho_m - \rho)h}{\rho}} \end{aligned}$$

Because friction is to be neglected, $C_\mathrm{v} = 1$. (For venturi meters, C_v is usually very close to one because the diameter changes are gradual and there is little friction loss.)

$$\begin{aligned} Q &= \left(\frac{C_\mathrm{v} A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\right) \sqrt{\frac{2g(\rho_m - \rho)h}{\rho}} \\ &= \left(\frac{(1)(0.0177\ \mathrm{m}^2)}{\sqrt{1 - \left(\frac{0.0177\ \mathrm{m}^2}{0.159\ \mathrm{m}^2}\right)^2}}\right) \\ &\quad \times \sqrt{\frac{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right) \times (13.6 - 1)(37.5\ \mathrm{cm})}{\left(1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)}} \\ &= 0.171\ \mathrm{m}^3/\mathrm{s} \quad (0.17\ \mathrm{m}^3/\mathrm{s}) \end{aligned}$$

The answer is (B).

5. From the Reynolds number similarity,

$$\begin{aligned} \left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m &= \left[\frac{\mathrm{v} l \rho}{\mu}\right]_p = \left[\frac{\mathrm{v} l \rho}{\mu}\right]_m \\ &= [\mathrm{Re}]_p \\ &= [\mathrm{Re}]_m \end{aligned}$$

The scale is 1:1, so the lengths of the prototype and model are the same ($l_m = l_p$).

The similarity equation reduces to

$$\left(\frac{\mathrm{v}\rho}{\mu}\right)_p = \left(\frac{\mathrm{v}\rho}{\mu}\right)_m$$

$$\left(\frac{\mathrm{v}}{\nu}\right)_p = \left(\frac{\mathrm{v}}{\nu}\right)_m$$

$$\begin{aligned} \mathrm{v}_m = \mathrm{v}_p\left(\frac{\nu_m}{\nu_p}\right) &= \left(7\ \frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{1.41 \times 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}}{1.31 \times 10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}}\right) \\ &= 75.3\ \mathrm{m/s} \quad (75\ \mathrm{m/s}) \end{aligned}$$

The answer is (D).

6. The hydraulic head at the hole is the height from the top of the tank to the center of the hole.

$$h = 2\text{ m} - 0.75\text{ m} = 1.25\text{ m}$$

For an orifice discharging freely into the atmosphere,

$$Q = CA_0\sqrt{2gh}$$

As orifice losses are neglected, $C = 1$. Dividing both sides by A_0 gives

$$\begin{aligned}\text{v} &= C\sqrt{2gh} = 1\sqrt{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.25\text{ m})}\\ &= 4.95\text{ m/s}\end{aligned}$$

The answer is (D).

7. Convert the frictional head loss to specific energy loss. Note that the conversion between m^2/s^2 and J/kg is 1.

$$\begin{aligned}\Delta E &= h_f g = (100\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\\ &= 981\text{ m}^2/\text{s}^2 \quad (981\text{ J/kg})\end{aligned}$$

The temperature increase is

$$\Delta T = \frac{\Delta E}{c_p} = \frac{981\ \frac{\text{J}}{\text{kg}}}{4180\ \frac{\text{J}}{\text{kg·K}}} = 0.2347\text{K} \quad (0.23°\text{C})$$

Note that the unit difference between Celsius and Kelvin is the same though both scales have different starting points. Thus when temperature difference is being measured, the value is the same in Kelvins and Celsius.

The answer is (A).

10 Hydrology

PRACTICE PROBLEMS

1. Two adjacent fields contribute runoff to a collector. Field 1 is 2 ac in size and has a runoff coefficient of 0.35. Field 2 is 4 ac in size and has a runoff coefficient of 0.65. The rainfall intensity of the storm after the time to concentration is 3.9 in/hr. The peak runoff is most nearly

(A) 8.7 ft^3/sec

(B) 10 ft^3/sec

(C) 13 ft^3/sec

(D) 16 ft^3/sec

2. The rational formula runoff coefficient of a 950 ft × 600 ft property is 0.35. A storm occurs with a rainfall intensity of 4.5 in/hr. The peak runoff from this property is most nearly

(A) 21 ft^3/sec

(B) 30 ft^3/sec

(C) 62 ft^3/sec

(D) 90 ft^3/sec

3. The table shown contains curve numbers based on land use and soil type. A watershed contains 10 ac of residential land of soil type B and 5 ac of grassland of soil type A.

	soil type	
land use	type A	type B
residential	57	72
grassland	30	58

If the total precipitation is 11 in, what will be the approximate runoff from the watershed?

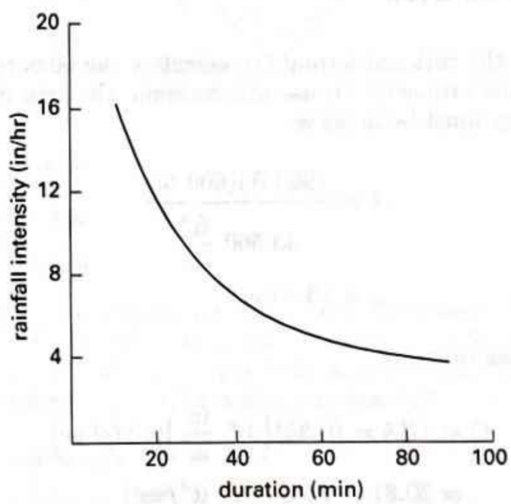
(A) 5.4 in

(B) 6.0 in

(C) 6.7 in

(D) 7.2 in

4. A watershed occupies a 70 ac site. 45 ac of the site have been cleared and are used for pasture land with a runoff coefficient of 0.13; 3 ac are occupied by farm buildings, a house, and paved surfaces and have a runoff coefficient of 0.75; the remaining 22 ac are woodland with a runoff coefficient of 0.20. The total time to concentration for the watershed is 30 min. The 20 yr storm is characterized by the intensity duration curve shown.



The peak runoff for the 20 yr storm is most nearly

(A) 50 ft^3/sec

(B) 110 ft^3/sec

(C) 240 ft^3/sec

(D) 530 ft^3/sec

5. A drainage basin covers an area of 2.4 ac. During a storm with a sustained rainfall intensity of 0.6 in/hr, the peak runoff from the basin is 320 gal/min. What is most nearly the runoff coefficient for the basin?

(A) 0.38

(B) 0.50

(C) 0.65

(D) 0.85

Hydraulics/ Hydrologic Sys.

SOLUTIONS

1. The runoff coefficients are given for each area. Combine the coefficients, weighting them by their respective contributing areas.

$$C = \frac{(2\ \text{ac})(0.35) + (4\ \text{ac})(0.65)}{2\ \text{ac} + 4\ \text{ac}} = 0.55$$

Find the peak flow from the rational formula.

$$Q = CIA = (0.55)\left(3.9\ \frac{\text{in}}{\text{hr}}\right)(6\ \text{ac}) = 12.87\ \text{ft}^3/\text{sec} \quad (13\ \text{ft}^3/\text{sec})$$

The answer is (C).

2. Use the rational formula to calculate the peak runoff from this property. To use this formula, the area of the property must be in acres.

$$A = \frac{(950\ \text{ft})(600\ \text{ft})}{43{,}560\ \frac{\text{ft}^2}{\text{ac}}} = 13.09\ \text{ac}$$

The peak runoff is

$$Q = CIA = (0.35)\left(4.5\ \frac{\text{in}}{\text{hr}}\right)(13.09\ \text{ac}) = 20.61\ \text{ft}^3/\text{sec} \quad (21\ \text{ft}^3/\text{sec})$$

The answer is (A).

3. Find the weighted curve number for the watershed. From the table, the watershed contains 10 ac of land with a curve number of 72 and 5 ac of land with a curve number of 30.

$$CN = \frac{(10\ \text{ac})(72) + (5\ \text{ac})(30)}{10\ \text{ac} + 5\ \text{ac}} = 58$$

Find the storage capacity of the watershed.

$$S = \frac{1000}{CN} - 10 = \frac{1000}{58} - 10 = 7.241\ \text{in}$$

Find the runoff from 11 in of precipitation.

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} = \frac{\big(11\ \text{in} - (0.2)(7.241\ \text{in})\big)^2}{11\ \text{in} + (0.8)(7.241\ \text{in})} = 5.433\ \text{in} \quad (5.4\ \text{in})$$

The answer is (A).

4. The weighted average runoff coefficient for the watershed is

$$C = \frac{(0.13)(45\ \text{ac}) + (0.75)(3\ \text{ac}) + (0.20)(22\ \text{ac})}{70\ \text{ac}} = 0.179$$

From the graph, at $t_c = 30$ min, $I = 9$ in/hr. Use the rational formula to calculate the peak runoff.

$$Q = CIA = (0.179)\left(9\ \frac{\text{in}}{\text{hr}}\right)(70\ \text{ac}) = 112.5\ \text{ft}^3/\text{sec} \quad (110\ \text{ft}^3/\text{sec})$$

The answer is (B).

5. Convert the runoff to cubic feet per second.

$$Q = \frac{320\ \frac{\text{gal}}{\text{min}}}{\left(7.48\ \frac{\text{gal}}{\text{ft}^3}\right)\left(60\ \frac{\text{sec}}{\text{min}}\right)} = 0.7130\ \text{ft}^3/\text{sec}$$

Use the rational formula to determine the runoff coefficient.

$$Q = CIA$$

$$C = \frac{Q}{IA} = \frac{0.7130\ \frac{\text{ft}^3}{\text{sec}}}{\left(0.6\ \frac{\text{in}}{\text{hr}}\right)(2.4\ \text{ac})} = 0.4951 \quad (0.50)$$

The answer is (B).

11 Hydraulics

PRACTICE PROBLEMS

1. A model of a dam has been constructed so that the scale of dam to model is 15:1. The similarity is based on Froude numbers. At a certain point on the spillway of the model, the velocity is 5 m/s. At the corresponding point on the spillway of the actual dam, the velocity would most nearly be

(A) 6.7 m/s

(B) 7.5 m/s

(C) 15 m/s

(D) 19 m/s

2. An open channel has a cross-sectional area of flow of 0.5 m^2, a hydraulic radius of 0.15 m, and a roughness coefficient of 0.15. Most nearly, what is the slope of the hydraulic gradient needed to achieve a flow rate of 10 L/s?

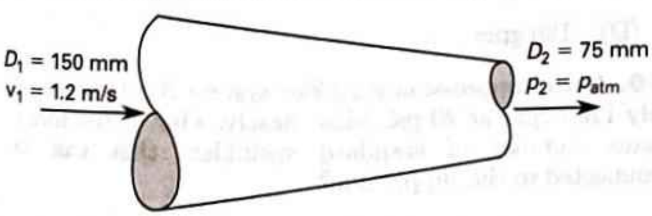
(A) 1.1×10^{-4}

(B) 6.7×10^{-4}

(C) 1.1×10^{-3}

(D) 6.7×10^{-3}

3. Water flows through a converging fitting as shown and discharges freely to the atmosphere at the exit. Flow is incompressible, and friction is negligible.



Most nearly, what is the gage pressure at the inlet?

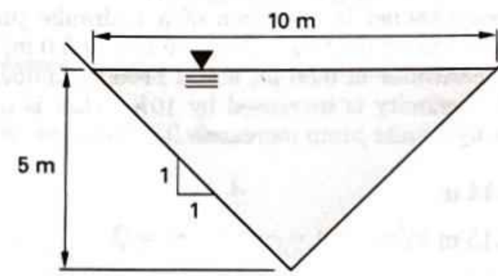
(A) 10.2 kPa

(B) 10.8 kPa

(C) 11.3 kPa

(D) 12.7 kPa

4. The hydraulic radius of a 5 m deep triangular channel with a 1:1 side slope is most nearly



(A) 1.0 m

(B) 1.8 m

(C) 2.0 m

(D) 2.8 m

5. A pipe carrying an incompressible fluid has a diameter of 100 mm at point 1 and a diameter of 50 mm at point 2. The velocity of the fluid at point 1 is 0.3 m/s. Most nearly, what is the velocity at point 2?

(A) 0.95 m/s

(B) 1.2 m/s

(C) 2.1 m/s

(D) 3.5 m/s

6. A rectangular channel 3 m wide and 1.25 m deep is flowing at steady state. The channel momentum is 4.5 m^3. The discharge of the channel is most nearly

(A) 5.3 m^3/s

(B) 5.5 m^3/s

(C) 6.6 m^3/s

(D) 8.9 m^3/s

7. A hydraulic jump occurs in an open channel where the upstream Froude number is 3.96 and the upstream depth is 0.65 m. Most nearly, what is the height of the hydraulic jump?

(A) 0.9 m

(B) 2.7 m

(C) 3.0 m

(D) 6.0 m

8. An open channel is upstream of a hydraulic jump. The channel has an upstream flow velocity of 5.0 m/s, a constant headwater of 0.50 m, and a Froude number of 2.26. If the velocity is increased by 10%, what is most nearly the hydraulic jump increase?

(A) 0.14 m

(B) 0.15 m

(C) 0.20 m

(D) 0.32 m

9. A fire hydrant has a rated capacity of 1235 gpm at 20 psi. A field test shows that the static pressure of the hydrant's water distribution system is 90 psi, and the residual pressure is 45 psi. What is most nearly the test flow rate?

(A) 250 gpm

(B) 620 gpm

(C) 970 gpm

(D) 1000 gpm

10. A fire hydrant flow test is conducted. The ratio of the target pressure drop to the observed pressure drop is 1.67, and the observed test flow is 789 gpm. Most nearly, what is the rated discharge capacity of the hydrant?

(A) 600 gpm

(B) 1000 gpm

(C) 1200 gpm

(D) 1300 gpm

11. A fire hydrant is discharging to the atmosphere at a rate of 1125 gpm from a 2.5 in, smooth, well-rounded outlet. Most nearly, what is the pitot pressure of the discharge?

(A) 7.0 psi

(B) 36 psi

(C) 45 psi

(D) 74 psi

12. A fire hydrant discharging to the atmosphere has a 3 in diameter square, projecting outlet. Another fire hydrant has a 3 in diameter smooth, well-rounded outlet. The pitot pressure at both hydrants is 62 psi. Most nearly, what is the difference in the discharge rates of the two hydrants?

(A) 210 gpm

(B) 320 gpm

(C) 420 gpm

(D) 640 gpm

13. A fire hydrant being rated for capacity has a square, sharp outlet with a 2.5 in diameter. The pitot pressure of the discharge is 53 psi, and the residual pressure during discharge is 36 psi. The static pressure of the system is 93 psi. Most nearly, what is the rated capacity of the hydrant?

(A) 1240 gpm

(B) 1390 gpm

(C) 1400 gpm

(D) 1720 gpm

14. A fire hydrant with a sharp, square 2 in outlet is rated for capacity twice. At the time of the first test, the pitot pressure reading is 83 psi. At the time of the second test, the pitot pressure reading is 63 psi. The residual pressure during discharge is 50 psi during both tests. The static pressure of the system is 106 psi. Most nearly, how much lower is the discharge of the fire hydrant after the second test?

(A) 20 gpm

(B) 85 gpm

(C) 110 gpm

(D) 120 gpm

15. A fire suppression sprinkler system is rated to supply 1209 gpm at 20 psi. Most nearly, what is the maximum number of standard sprinklers that can be connected to the supply line?

(A) 33

(B) 36

(C) 48

(D) 50

Hydraulics/ Hydrologic Sys.

16. A fire sprinkler system can discharge water at a rate of 71 gpm at each head. The sprinkler orifices are extra large. Most nearly, what is the design pressure at which the sprinkler system should operate?

(A) 40 psi

(B) 80 psi

(C) 120 psi

(D) 160 psi

SOLUTIONS

1. The Froude numbers must be equal.

$$\mathrm{Fr}_{\mathrm{dam}} = \mathrm{Fr}_{\mathrm{model}}$$

$$\frac{\mathrm{v}_{\mathrm{dam}}}{\sqrt{g y_{h,\mathrm{dam}}}} = \frac{\mathrm{v}_{\mathrm{model}}}{\sqrt{g y_{h,\mathrm{model}}}}$$

$$\begin{aligned}\mathrm{v}_{\mathrm{dam}} &= \mathrm{v}_{\mathrm{model}}\sqrt{\frac{y_{h,\mathrm{dam}}}{y_{h,\mathrm{model}}}} = \left(5\ \frac{\mathrm{m}}{\mathrm{s}}\right)\sqrt{\frac{15}{1}} \\ &= 19.36\ \mathrm{m/s} \quad (19\ \mathrm{m/s})\end{aligned}$$

The answer is (D).

2. The volumetric flow rate is

$$Q = \frac{10\ \frac{\mathrm{L}}{\mathrm{s}}}{1000\ \frac{\mathrm{L}}{\mathrm{m}^3}} = 0.01\ \mathrm{m}^3/\mathrm{s}$$

The velocity needed is

$$\mathrm{v} = \frac{Q}{A} = \frac{0.01\ \frac{\mathrm{m}^3}{\mathrm{s}}}{0.5\ \mathrm{m}^2} = 0.02\ \mathrm{m/s}$$

Use Manning's equation to find the slope needed to achieve this velocity.

$$\mathrm{v} = (K/n)R_H^{2/3}S^{1/2}$$

$$\begin{aligned}S &= \left(\frac{\mathrm{v}n}{KR_H^{2/3}}\right)^2 \\ &= \left(\frac{\left(0.02\ \frac{\mathrm{m}}{\mathrm{s}}\right)(0.15)}{(1.0)(0.15\ \mathrm{m})^{2/3}}\right)^2 \\ &= 0.0001129 \quad (1.1 \times 10^{-4})\end{aligned}$$

The answer is (A).

3. From the continuity equation for incompressible flow,

$$A_1\mathrm{v}_1 = A_2\mathrm{v}_2$$

$$\begin{aligned}\mathrm{v}_2 &= \frac{A_1\mathrm{v}_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2\mathrm{v}_1 \\ &= \left(\frac{150\ \mathrm{mm}}{75\ \mathrm{mm}}\right)^2\left(1.2\ \frac{\mathrm{m}}{\mathrm{s}}\right) \\ &= 4.8\ \mathrm{m/s}\end{aligned}$$

Hydraulics/ Hydrologic Sys.

Use the Bernoulli equation.

$$\frac{p_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{p_1}{\rho} + \frac{v_1^2}{2} + z_1 g$$

$$z_1 = z_2$$

$$p_{\text{gage}} = p_1 - p_2$$

$$\begin{aligned} p_{\text{gage}} &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\ &= \left(\frac{1000\ \frac{\text{kg}}{\text{m}^3}}{2}\right)\left(\left(4.8\ \frac{\text{m}}{\text{s}}\right)^2 - \left(1.2\ \frac{\text{m}}{\text{s}}\right)^2\right) \\ &= 10\,800\ \text{Pa} \quad (10.8\ \text{kPa}) \end{aligned}$$

The answer is (B).

4. The hydraulic radius is found by dividing the cross-sectional area of the channel by the wetted perimeter. Because the sides have a 1:1 slope, the cross section of the channel is a 90°-45°-45° triangle with the vertex down. The width of the channel surface (the base of the inverted triangle) is 10 m, and the area of the triangular cross section is

$$A = \frac{bh}{2} = \frac{(10\ \text{m})(5\ \text{m})}{2} = 25\ \text{m}^2$$

The wetted length of each side of the channel is equal to the hypotenuse of a 90°-45°-45° triangle with sides of 5 m, and the wetted perimeter is the sum of these two wetted lengths.

$$\begin{aligned} \text{hypotenuse} &= \sqrt{a^2 + b^2} = \sqrt{(5\ \text{m})^2 + (5\ \text{m})^2} \\ &= 7.071\ \text{m} \end{aligned}$$

$$\text{wetted perimeter} = (2)(7.071\ \text{m}) = 14.14\ \text{m}$$

The hydraulic radius is

$$\begin{aligned} R_H &= \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{25\ \text{m}^2}{14.14\ \text{m}} \\ &= 1.768\ \text{m} \quad (1.8\ \text{m}) \end{aligned}$$

The answer is (B).

5. Use the continuity equation for an incompressible fluid and solve for the velocity at point 2.

$$A_1 v_1 = A_2 v_2$$

$$\left(\frac{\pi D_1^2}{4}\right)v_1 = \left(\frac{\pi D_2^2}{4}\right)v_2$$

$$\begin{aligned} v_2 &= \left(\frac{D_1}{D_2}\right)^2 v_1 = \left(\frac{100\ \text{mm}}{50\ \text{mm}}\right)^2\left(0.3\ \frac{\text{m}}{\text{s}}\right) \\ &= 1.2\ \text{m/s} \end{aligned}$$

The answer is (B).

6. The area of the channel is

$$A = (3\ \text{m})(1.25\ \text{m}) = 3.75\ \text{m}^2$$

The flow depth to the centroid is half the total depth because the cross section is rectangular.

$$h_c = (0.5)(1.25\ \text{m}) = 0.625\ \text{m}$$

From the equation for momentum for steady flow rate,

$$M = \frac{Q^2}{gA} + Ah_c$$

$$\begin{aligned} Q &= \sqrt{gA(M - Ah_c)} \\ &= \sqrt{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3.75\ \text{m}^2)\left(4.5\ \text{m}^3 - (3.75\ \text{m}^2)(0.625\ \text{m})\right)} \\ &= 8.9\ \text{m}^3/\text{s} \end{aligned}$$

The answer is (D).

7. The height of the hydraulic jump, h_f, is the downstream depth minus the upstream depth, $y_2 - y_1$. From the hydraulic jump equation,

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_1^2}\right)$$

$$\begin{aligned} h_f &= y_2 - y_1 \\ &= \frac{0.65\ \text{m}}{2}\left(-1 + \sqrt{1 + (8)(3.96)^2}\right) - 0.65\ \text{m} \\ &= 2.7\ \text{m} \end{aligned}$$

The answer is (B).

Hydraulics/ Hydrologic Sys.

8. From the hydraulic jump equation, the initial downstream flow depth is

$$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$$
$$= \left(\frac{0.50\ \mathrm{m}}{2}\right)\left(-1+\sqrt{1+(8)(2.26)^2}\right)$$
$$= 1.37\ \mathrm{m}$$

Find the Froude number after the upstream velocity is increased.

$$\mathrm{Fr} = \frac{\mathrm{v}}{\sqrt{gy_h}}$$
$$= \frac{(1.10)\left(5.0\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{\sqrt{\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.50\ \mathrm{m})}}$$
$$= 2.48$$

After the upstream velocity is increased, the downstream flow depth becomes

$$y_2 = \frac{y_1}{2}\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$$
$$= \left(\frac{0.50\ \mathrm{m}}{2}\right)\left(-1+\sqrt{1+(8)(2.48)^2}\right)$$
$$= 1.52\ \mathrm{m}$$

The increase in the hydraulic jump is

$$1.52\ \mathrm{m} - 1.37\ \mathrm{m} = 0.15\ \mathrm{m}$$

The answer is (B).

9. The observed residual pressure drop parameter is

$$H_R = P_S - 20\ \mathrm{psi}$$
$$= 90\ \frac{\mathrm{lbf}}{\mathrm{in}^2} - 20\ \frac{\mathrm{lbf}}{\mathrm{in}^2}$$
$$= 70\ \mathrm{lbf/in}^2$$

The target residual pressure drop parameter is

$$H_F = P_S - P_R$$
$$= 90\ \frac{\mathrm{lbf}}{\mathrm{in}^2} - 45\ \frac{\mathrm{lbf}}{\mathrm{in}^2}$$
$$= 45\ \mathrm{lbf/in}^2$$

Rearrange the equation for the hydrant's rated capacity flow, and solve for the observed discharge.

$$Q_R = Q_F \times (H_R/H_F)^{0.54}$$
$$Q_F = Q_R\left(\frac{H_F}{H_R}\right)^{0.54}$$
$$= \left(1235\ \frac{\mathrm{gal}}{\mathrm{min}}\right)\left(\frac{45\ \frac{\mathrm{lbf}}{\mathrm{in}^2}}{70\ \frac{\mathrm{lbf}}{\mathrm{in}^2}}\right)^{0.54}$$
$$= 972.9\ \mathrm{gpm}\quad (970\ \mathrm{gpm})$$

The answer is (C).

10. The rated capacity flow of the fire hydrant is

$$Q_R = Q_F \times (H_R/H_F)^{0.54}$$
$$= \left(789\ \frac{\mathrm{gal}}{\mathrm{min}}\right)(1.67)^{0.54}$$
$$= 1040.7\ \mathrm{gpm}\quad (1000\ \mathrm{gpm})$$

The answer is (B).

11. Rearrange the equation for fire hydrants discharging to the atmosphere to solve for the pitot pressure. For a smooth, well-rounded outlet, the hydraulic outlet coefficient is 0.90.

$$Q = 29.8D^2C_dP^{1/2}$$
$$P = \left(\frac{Q}{29.8D^2C_d}\right)^2$$
$$= \left(\frac{1125\ \frac{\mathrm{gal}}{\mathrm{min}}}{(29.8)(2.5\ \mathrm{in})^2(0.90)}\right)^2$$
$$= 45\ \mathrm{psi}$$

The answer is (C).

12. From the equation for fire hydrants discharging to the atmosphere, the discharge rate for the hydrant with the square, projecting outlet is

$$Q = 29.8D^2C_dP^{1/2}$$
$$= (29.8)(3\ \mathrm{in})^2(0.70)\left(62\ \frac{\mathrm{lbf}}{\mathrm{in}^2}\right)^{1/2}$$
$$= 1478.3\ \mathrm{gpm}$$

The discharge rate for the hydrant with the smooth, well-rounded outlet is

$$\begin{aligned} Q &= 29.8D^2C_dP^{1/2} \\ &= (29.8)(3\text{ in})^2(0.90)\left(62\ \frac{\text{lbf}}{\text{in}^2}\right)^{1/2} \\ &= 1900.6\text{ gpm} \end{aligned}$$

The difference between the discharge rates is

$$1900.6\ \frac{\text{gal}}{\text{min}} - 1478.3\ \frac{\text{gal}}{\text{min}} = 422.4\text{ gpm} \quad (420\text{ gpm})$$

The answer is (C).

13. Use the equation for fire hydrants discharging to the atmosphere to find the discharge. The hydraulic outlet coefficient for a square, sharp outlet is 0.80.

$$\begin{aligned} Q_F &= 29.8D^2C_dP^{1/2} \\ &= (29.8)(2.5\text{ in})^2(0.80)\left(53\ \frac{\text{lbf}}{\text{in}^2}\right)^{1/2} \\ &= 1084.7\text{ gpm} \end{aligned}$$

Find the observed and target residual pressure drop parameters.

$$\begin{aligned} H_R &= P_S - 20\text{ psi} \\ &= 93\ \frac{\text{lbf}}{\text{in}^2} - 20\ \frac{\text{lbf}}{\text{in}^2} \\ &= 73\text{ psi} \end{aligned}$$

$$\begin{aligned} H_F &= P_S - P_R \\ &= 93\ \frac{\text{lbf}}{\text{in}^2} - 36\ \frac{\text{lbf}}{\text{in}^2} \\ &= 57\text{ psi} \end{aligned}$$

The rated capacity flow is

$$\begin{aligned} Q_R &= Q_F \times (H_R/H_F)^{0.54} \\ &= \left(1084.7\ \frac{\text{gal}}{\text{min}}\right)\left(\frac{73\ \frac{\text{lbf}}{\text{in}^2}}{57\ \frac{\text{lbf}}{\text{in}^2}}\right)^{0.54} \\ &= 1239.8\text{ gpm} \quad (1240\text{ gpm}) \end{aligned}$$

The answer is (A).

14. Use the equation for fire hydrants discharging to the atmosphere to find the discharge during the first test. For a square, sharp outlet, the hydraulic outlet coefficient is 0.80.

$$\begin{aligned} Q &= 29.8D^2C_dP^{1/2} \\ &= (29.8)(2\text{ in})^2(0.80)\left(83\ \frac{\text{lbf}}{\text{in}^2}\right)^{1/2} \\ &= 868.8\text{ gpm} \end{aligned}$$

Find the discharge during the second test.

$$\begin{aligned} Q &= 29.8D^2C_dP^{1/2} \\ &= (29.8)(2\text{ in})^2(0.80)\left(63\ \frac{\text{lbf}}{\text{in}^2}\right)^{1/2} \\ &= 756.9\text{ gpm} \end{aligned}$$

The difference between the two cases is

$$868.8\ \frac{\text{gal}}{\text{min}} - 756.9\ \frac{\text{gal}}{\text{min}} = 111.9\text{ gpm} \quad (110\text{ gpm})$$

The answer is (C).

15. A standard sprinkler has an orifice of ½ in diameter. From a table of K-factor values, the K factor for an orifice this size is 5.6 gpm/(psi$^{1/2}$). The capacity of a single orifice at a pressure of 20 psi is

$$\begin{aligned} Q &= KP^{1/2} \\ &= \left(5.6\ \frac{\frac{\text{gal}}{\text{min}}}{\sqrt{\frac{\text{lbf}}{\text{in}^2}}}\right)\left(\sqrt{20\ \frac{\text{lbf}}{\text{in}^2}}\right) \\ &= 25\text{ gpm} \end{aligned}$$

The maximum number of sprinklers the system can support is

$$\frac{1209\ \frac{\text{gal}}{\text{min}}}{25\ \frac{\frac{\text{gal}}{\text{min}}}{\text{sprinkler}}} = 48.4\text{ sprinklers} \quad (48\text{ sprinklers})$$

The answer is (C).

16. Rearrange the equation for fire sprinkler discharge to solve for the design pressure. The K factor for extra-large sprinkler orifices is 11.2 gpm/(psi$^{1/2}$).

$$Q = KP^{1/2}$$

$$P = \left(\frac{Q}{K}\right)^2$$

$$= \left(\frac{71\ \dfrac{\text{gal}}{\text{min}}}{11.2\ \dfrac{\dfrac{\text{gal}}{\text{min}}}{\sqrt{\dfrac{\text{lbf}}{\text{in}^2}}}}\right)^2$$

$$= 40\ \text{psi}$$

The answer is (A).

Hydraulics/ Hydrologic Sys.

12 Groundwater

PRACTICE PROBLEMS

1. Water drains at a constant rate through a saturated soil column with a diameter of 1 ft and a height of 2 ft. The hydraulic head is maintained at 5 ft at the top of the column and 0.5 ft at the bottom. After a period of 1 hr, 100 in^3 of water has drained through the column. What is most nearly the hydraulic conductivity of the soil?

(A) 3.5×10^{-6} ft/sec

(B) 4.6×10^{-6} ft/sec

(C) 7.1×10^{-6} ft/sec

(D) 9.1×10^{-6} ft/sec

2. Darcy's law is primarily associated with flow through

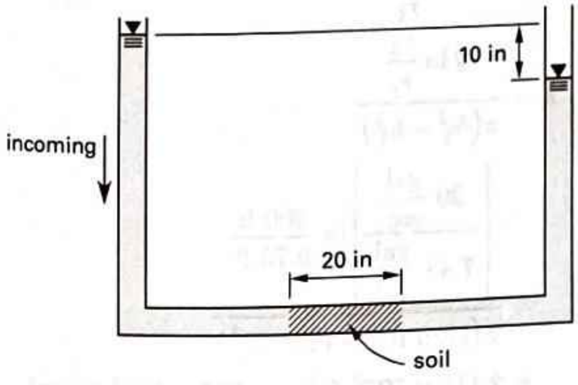
(A) open channels

(B) pipes

(C) pitot tubes and venturi meters

(D) porous media

3. A soil sample with a permeability of 5×10^{-6} in/sec will be tested using the pipe setup shown. The pipe's diameter is 2 in. The 10 in head differential will be maintained.



The volume of flow is most nearly

(A) 2.5×10^{-6} in^3/sec

(B) 4.9×10^{-6} in^3/sec

(C) 7.9×10^{-6} in^3/sec

(D) 3.0×10^{-5} in^3/sec

4. An aquifer has a thickness of 52 ft and a transmissivity of 650 ft^2/day. What is most nearly the hydraulic conductivity of the aquifer?

(A) 2.5 ft/day

(B) 6.3 ft/day

(C) 13 ft/day

(D) 33 ft/day

5. The results of well pumping tests from a homogeneous, unconfined aquifer are shown. At the time of the tests, the pumping had continued long enough for the well discharge to become steady.

parameter	value
pumping rate	20 gal/sec
well diameter	1.5 ft
radius of influence	900 ft
depth of aquifer at radius of influence	135 ft
drawdown in well	11 ft

The hydraulic conductivity of the aquifer is most nearly

(A) 1.2×10^{-3} ft/sec

(B) 2.1×10^{-3} ft/sec

(C) 5.0×10^{-3} ft/sec

(D) 8.7×10^{-3} ft/sec

SOLUTIONS

1. The cross-sectional area of the column is

$$A = \frac{\pi D^2}{4} = \frac{\pi(1 \text{ ft})^2}{4} = 0.785 \text{ ft}^2$$

The change in hydraulic head over the length of the soil sample is

$$\frac{dh}{dx} = \frac{0.5 \text{ ft} - 5 \text{ ft}}{2 \text{ ft}} = -2.25 \text{ ft/ft}$$

Use Darcy's law.

$$\begin{aligned} Q &= -KA\left(\frac{dh}{dx}\right) \\ K &= -\frac{Q}{A\frac{dh}{dx}} \\ &= -\frac{100 \frac{\text{in}^3}{\text{hr}}}{(0.785 \text{ ft}^2)\left(-2.25 \frac{\text{ft}}{\text{ft}}\right)} \\ &\quad \times \left(12 \frac{\text{in}}{\text{ft}}\right)^3\left(3600 \frac{\text{sec}}{\text{hr}}\right) \\ &= 9.097 \times 10^{-6} \text{ ft/sec} \quad (9.1 \times 10^{-6} \text{ ft/sec}) \end{aligned}$$

The answer is (D).

2. Darcy's law is primarily associated with flow though porous media.

The answer is (D).

3. The cross-sectional area of the pipe is

$$A = \frac{\pi D^2}{4} = \frac{\pi(2 \text{ in})^2}{4} = 3.142 \text{ in}^2$$

The change in hydraulic head in the direction of flow over the length of the soil sample is

$$\frac{dh}{dx} = \frac{-10 \text{ in}}{20 \text{ in}} = -0.5 \text{ in/in}$$

Use Darcy's law.

$$\begin{aligned} Q &= -KA\left(\frac{dh}{dx}\right) \\ &= -\left(5 \times 10^{-6} \frac{\text{in}}{\text{sec}}\right)(3.142 \text{ in}^2)\left(-0.5 \frac{\text{in}}{\text{in}}\right) \\ &= 7.854 \times 10^{-6} \text{ in}^3\text{/sec} \quad (7.9 \times 10^{-6} \text{ in}^3\text{/sec}) \end{aligned}$$

The answer is (C).

4. From the formula for transmissivity, the permeability is

$$\begin{aligned} T &= KD \\ K &= \frac{T}{D} = \frac{650 \frac{\text{ft}^2}{\text{day}}}{52 \text{ ft}} \\ &= 12.5 \text{ ft/day} \quad (13 \text{ ft/day}) \end{aligned}$$

The answer is (C).

5. The radius of the well is

$$r_1 = \frac{D}{2} = \frac{1.5 \text{ ft}}{2} = 0.75 \text{ ft}$$

The thickness of the saturated aquifer at the well is

$$h_1 = D - D_{w,2} = 135 \text{ ft} - 11 \text{ ft} = 124 \text{ ft}$$

Use Dupuit's equation to find the hydraulic conductivity of the aquifer.

$$\begin{aligned} Q &= \frac{\pi k(h_2^2 - h_1^2)}{\ln\frac{r_2}{r_1}} \\ k &= \frac{Q \ln\frac{r_2}{r_1}}{\pi(h_2^2 - h_1^2)} \\ &= \frac{\left(\frac{20 \frac{\text{gal}}{\text{sec}}}{7.48 \frac{\text{gal}}{\text{ft}^3}}\right)\ln\frac{900 \text{ ft}}{0.75 \text{ ft}}}{\pi\left((135 \text{ ft})^2 - (124 \text{ ft})^2\right)} \\ &= 2.118 \times 10^{-3} \text{ ft/sec} \quad (2.1 \times 10^{-3} \text{ ft/sec}) \end{aligned}$$

The answer is (B).

13 Water Quality

PRACTICE PROBLEMS

1. What is most nearly the equivalent weight of aluminum sulfate, $Al_2(SO_4)_3$?

(A) 57 g/mol

(B) 110 g/mol

(C) 170 g/mol

(D) 340 g/mol

2. A water sample contains 57 mg/L of phosphate, PO_4^{---}, measured as substance. What is most nearly the concentration as $CaCO_3$?

(A) 32 mg/L as $CaCO_3$

(B) 36 mg/L as $CaCO_3$

(C) 60 mg/L as $CaCO_3$

(D) 90 mg/L as $CaCO_3$

3. The alkalinity of water was determined to be 200 mg/L as $CaCO_3$. If a nitric acid (HNO_3; molecular weight of 63 g/mol) solution with an acidity of 35 mg/L as $CaCO_3$ is available, most nearly how much acid solution would be required to completely neutralize 700 L of the water?

(A) 440 L

(B) 3200 L

(C) 4000 L

(D) 5000 L

4. A water analysis of lake water has the results shown, with all values reported as $CaCO_3$.

alkalinity	151.5 mg/L
sodium	120.0 mg/L
calcium	127.5 mg/L
iron (III)	0.107 mg/L
magnesium	43.5 mg/L
potassium	8.24 mg/L
chloride	39.5 mg/L
fluoride	1.05 mg/L
nitrate	1.06 mg/L
sulfate	106 mg/L

The water's hardness is most nearly

(A) 150 mg/L

(B) 170 mg/L

(C) 290 mg/L

(D) 300 mg/L

5. Water has a carbonate hardness of 92 mg/L as $CaCO_3$. Each mol of free electrons provided by CaO will reduce hardness by eliminating the equivalent mol of hardness as carbonate. Most nearly, what dose of lime (CaO; oxidation number 2) as substance should be added to the water to reduce the hardness to 30 mg/L as $CaCO_3$?

(A) 35 mg/L

(B) 60 mg/L

(C) 90 mg/L

(D) 180 mg/L

6. Which of the following is generally NOT attributable to hard water?

(A) scum rings in bathtubs

(B) stains on porcelain bath fixtures

(C) foam in clothes-washing equipment

(D) scale buildup in boiler tubes

7. A fate and transport study was performed on phosphorus in a small pond. Biological processes in the pond biota convert phosphorus to a nonbioavailable form at the rate of 22% per year. Recycling of sediment phosphorus by rooted plants and by anaerobic conditions in the hypolimnion converts 12% per year of the nonbioavailable phosphorus back to bioavailable forms. Runoff into the pond evaporates during the year, so no change in pond volume

Environmental Engineering

occurs. Which of the following actions would best reduce the phosphorus accumulation in the pond?

(A) adding chemicals to precipitate phosphorus in the pond

(B) adding chemicals to combine with phosphorus in the pond

(C) reducing use of phosphorus-based fertilizers in the surrounding fields

(D) using natural-based soaps and detergents in surrounding homes

8. Hardness in natural water is caused by the presence of which of the following?

(A) weakly acidic ions

(B) nitrites and nitrates

(C) polyvalent metallic cations

(D) colloidal solids

9. In aquatic systems, phosphorus recycling is significantly aided by which of the following?

I. algae

II. fungi

III. macrophytes

IV. phytoplankton

(A) I and II only

(B) I, II, and III only

(C) I, III, and IV only

(D) II, III, and IV only

10. Which of the following are sources of color in water?

I. copper ions

II. iron ions

III. manganese ions

IV. industrial colloidal solids

(A) IV only

(B) I and II only

(C) I, II, and IV only

(D) I, II, III, and IV

11. Which of the following processes is used to measure the amount of acid in water?

(A) titration

(B) filtration

(C) spectrophotometry

(D) digestion

12. Acidity in water is typically specified in terms of which of the following?

(A) H_2CO_3

(B) H^+

(C) OH^-

(D) $CaCO_3$

13. The National Primary Drinking Water Regulations apply to every public water supply serving at least how many service connections?

(A) 15

(B) 50

(C) 100

(D) 250

14. The molar nitrogen-to-phosphorus (N:P) ratio for ideal algae growth is which of the following?

(A) 10:1

(B) 12:1

(C) 16:1

(D) 18:1

15. In water, excess amounts of which of the following can contribute to methemoglobinemia, or "blue baby" syndrome?

(A) phosphorus

(B) manganese

(C) carbonates

(D) nitrates

Environmental Engineering

SOLUTIONS

1. For aluminum sulfate, $Al_2(SO_4)_3$, the molecular weight is

$$\begin{aligned}MW_{Al_2(SO_4)_3} &= 2(MW_{Al}) + 3(MW_S) + 12(MW_O)\\ &= (2)\left(26.981\ \frac{g}{mol}\right) + (3)\left(32.066\ \frac{g}{mol}\right)\\ &\quad + (12)\left(15.999\ \frac{g}{mol}\right)\\ &= 342.148\ g/mol\end{aligned}$$

The aluminum ion is triply charged, and there are two aluminum ions, so the number of charges involved is $(3)(2) = 6$. The equivalent weight is

$$\begin{aligned}EW_{Al_2(SO_4)_3} &= \frac{MW_{Al_2(SO_4)_3}}{\text{oxidation number}}\\ &= \frac{342.148\ \frac{g}{mol}}{6}\\ &= 57.025\ g/mol\quad (57\ g/mol)\end{aligned}$$

The answer is (A).

2. The molecular weight of phosphate, PO_4^{---}, is

$$\begin{aligned}MW_{PO_4^{---}} &= MW_P + 4(MW_O)\\ &= 30.974\ \frac{g}{mol} + (4)\left(15.999\ \frac{g}{mol}\right)\\ &= 94.97\ g/mol\end{aligned}$$

The ion is triply charged, so the equivalent weight is

$$\begin{aligned}EW_{PO_4^{---}} &= \frac{MW_{PO_4^{---}}}{\text{oxidation number}}\\ &= \frac{94.97\ \frac{g}{mol}}{3}\\ &= 31.657\ g/mol\end{aligned}$$

The concentration as $CaCO_3$ is

$$\begin{aligned}C_{\text{as }CaCO_3} &= C_{\text{as substance}}\left(\frac{EW_{CaCO_3}}{EW_{PO_4^{---}}}\right)\\ &= \left(57\ \frac{mg}{L}\right)\left(\frac{50.1\ \frac{g}{mol}}{31.657\ \frac{g}{mol}}\right)\\ &= 90.21\ mg/L\quad (90\ mg/L\text{ as }CaCO_3)\end{aligned}$$

The answer is (D).

3. Since the alkalinity and the acidity concentrations were both given as $CaCO_3$ equivalents, the volumes can be determined without converting the concentrations. Balance the total masses of alkalinity and acidity.

$$\begin{aligned}\Delta m_{\text{alkalinity}} &= \Delta m_{\text{acidity}}\\ V_{\text{water}}\Delta C_{\text{water,as }CaCO_3} &= V_{\text{acid}}\Delta C_{\text{acid,as }CaCO_3}\\ V_{\text{acid}} &= \frac{V_{\text{water}}\Delta C_{\text{water,as }CaCO_3}}{\Delta C_{\text{acid,as }CaCO_3}}\\ &= \frac{(700\ L)\left(200\ \frac{mg}{L} - 0\ \frac{mg}{L}\right)}{35\ \frac{mg}{L} - 0\ \frac{mg}{L}}\\ &= 4000\ L\end{aligned}$$

The answer is (C).

4. Water hardness is determined from the polyvalent metallic cations, which are calcium (Ca^{++}), iron (Fe^{+++}), and magnesium (Mg^{++}). Sodium and potassium are singly charged and do not contribute to hardness.

$$\begin{aligned}\text{hardness} &= C_{Ca^{++}} + C_{Fe^{+++}} + C_{Mg^{++}}\\ &= 127.5\ \frac{mg}{L} + 0.107\ \frac{mg}{L} + 43.5\ \frac{mg}{L}\\ &= 171.107\ mg/L\quad (170\ mg/L)\end{aligned}$$

The answer is (B).

5. The lime is being added to the water, so the volume of solution is the same as the volume of treated water. Determine the $CaCO_3$ dose.

$$V_{water}\Delta C_{water,as\,CaCO_3} = V_{lime}D_{lime,as\,CaCO_3}$$

$$\begin{aligned} D_{lime,as\,CaCO_3} &= \frac{V_{water}\Delta C_{water,as\,CaCO_3}}{V_{lime}} \\ &= \frac{(1\text{ L})\left(92\ \frac{\text{mg}}{\text{L}} - 30\ \frac{\text{mg}}{\text{L}}\right)}{1\text{ L}} \\ &= 62\text{ mg/L} \end{aligned}$$

Determine the $CaCO_3$ equivalent of CaO. The molecular weight of CaO is

$$\begin{aligned} MW_{lime} &= MW_{Ca} + MW_{O} \\ &= 40.078\ \frac{\text{g}}{\text{mol}} + 15.999\ \frac{\text{g}}{\text{mol}} \\ &= 56.077\text{ g/mol} \end{aligned}$$

The calcium ion is doubly charged, so the equivalent weight of lime is

$$\begin{aligned} EW_{lime} &= \frac{MW_{lime}}{\text{oxidation number}} = \frac{56.077\ \frac{\text{g}}{\text{mol}}}{2} \\ &= 28.039\text{ g/mol} \end{aligned}$$

The concentration as substance is

$$\begin{aligned} D_{lime,as\,substance} &= D_{lime,as\,CaCO_3}\left(\frac{EW_{lime}}{EW_{CaCO_3}}\right) \\ &= \left(62\ \frac{\text{mg}}{\text{L}}\right)\left(\frac{28.039\ \frac{\text{g}}{\text{mol}}}{50.1\ \frac{\text{g}}{\text{mol}}}\right) \\ &= 34.7\text{ mg/L}\quad (35\text{ mg/L}) \end{aligned}$$

The answer is (A).

6. Hard water interferes with foam/bubble formation, turning soap into scum.

The answer is (C).

7. To reduce the phosphorus accumulation in the pond, the arrival of additional bioavailable phosphorus must be reduced. This entails watershed management to reduce the amount of phosphorus applied as fertilizer and released through other sources. As eutrophication is a natural process accelerated by the availability of plant nutrients (nitrogen and phosphorus especially), reducing phosphorus can slow the process.

While there are chemical means to alter the forms of phosphorus to nonbioavailable states, this usually is not practical on a large scale and is potentially harmful in itself.

It is unlikely that the pond receives untreated discharge from local homes. Use of phosphate-rich detergents in the home will not affect the pond.

The answer is (C).

8. Hardness in natural water is caused by the presence of polyvalent metallic cations.

The answer is (C).

9. In aquatic systems, phosphorus recycling is significantly aided by algae, macrophytes, and phytoplankton.

The answer is (C).

10. Copper, iron, and manganese ions, and industrial colloidal solids are all sources of color in water.

The answer is (D).

11. Titration is used to measure the amount of acid in water.

The answer is (A).

12. The measurement of acidity in water is typically given in terms of the $CaCO_3$ equivalent that would neutralize the acid.

The answer is (D).

13. Every public water supply serving 15 or more service connections must meet the National Primary Drinking Water Regulations.

The answer is (A).

14. The molar N:P ratio for ideal algae growth is 16:1.

The answer is (C).

15. Excess amounts of nitrate in water can contribute to methemoglobinemia.

The answer is (D).

14 Water Supply Treatment and Distribution

PRACTICE PROBLEMS

1. Primary treatment of wastewater consists of

(A) chlorination

(B) removal of toxic industrial wastes

(C) regulating the flow of the wastewater through tanks of varied sizes so that the flow is a constant in secondary and tertiary treatment

(D) removing solids and particles of various sizes

2. In a packed bed operating at 25°C, a vapor contained in an air stream is dissolved in water. The Henry's law constant of the vapor is 0.10 atm·L/mol. The volumetric flow rate of air is 1 m^3/s. Most nearly, what volumetric flow rate of water is required?

(A) 2 L/s

(B) 4 L/s

(C) 6 L/s

(D) 10 L/s

3. At 15°C, the Henry's law constant for ammonia is 0.62 atm. The concentration of ammonia in a water solution at this temperature is 8.1×10^{-3} mol/L. The partial pressure of ammonia vapor above the liquid phase is most nearly

(A) 9.0×10^{-5} atm

(B) 4.4×10^{-5} atm

(C) 3.8×10^{-5} atm

(D) 5.0×10^{-3} atm

4. The equilibrium equation for the dissolution of sulfur dioxide in water is

$$SO_2(g) + 2H_2O(l) \rightleftharpoons HSO_3^-(aq) + H_3O^+(aq)$$

The equilibrium constant for this equation is

$$K_{eq} = \frac{[HSO_3^-][H_3O^+]}{p_{SO_2}} = 2.1 \times 10^{-2}\ (\text{mol/L})^2/\text{atm}$$

The variable p_{SO_2} represents the partial pressure of sulfur dioxide in the gaseous phase. The molar concentration of hydronium ions in water in equilibrium with air that contains 0.1 parts per million volume (ppmv) of sulfur dioxide is most nearly

(A) 2.1×10^{-9} mol/L

(B) 4.6×10^{-5} mol/L

(C) 5.3×10^{-5} mol/L

(D) 2.1×10^{-3} mol/L

5. Which of the following diseases should be of concern to the environmental engineer when designing, operating, or managing water supply projects?

I. acquired immunodeficiency syndrome (AIDS)

II. Rocky Mountain spotted fever

III. botulism

IV. tuberculosis

V. Legionnaires' disease

VI. hepatitis A

(A) I, III, and VI

(B) II, IV, and VI

(C) V only

(D) V and VI

6. Which of the following substances are used to remove calcium and magnesium from hard water?

I. lime

II. soda ash

III. ozone

IV. activated carbon

(A) I and II

(B) I and III

(C) II and III

(D) III and IV

Environmental Engineering

7. Water is recarbonated after it has been softened to

I. remove unwanted odors

II. lower its pH

III. improve its taste

IV. reduce its scale-forming potential

(A) I and II

(B) II and III

(C) II and IV

(D) III and IV

8. In addition to lime-soda treatment, what method can be used to reduce the hardness of water?

(A) neutralizing alkalinity with hydrochloric acid

(B) replacing calcium ions with sodium ions

(C) adsorbing sodium and iron with granular activated carbon

(D) flocculating the water with magnesium sulfate ($MgSO_4$)

9. Noncarbonate hardness can be removed by which of the following?

I. heating

II. distillation

III. precipitation softening processes

IV. ion exchange processes

(A) I and II

(B) I and III

(C) II and IV

(D) III and IV

10. Which of the following are generally true for water treatment relative to the adsorption of a contaminate by activated carbon?

I. The adsorption is a chemical reaction and typically irreversible.

II. The adsorption is a physical reaction (van der Waals forces) and generally reversible.

III. Water soluble, inorganic contaminants with low molecular weights are best adsorbed by activated carbon.

IV. The contaminant sticks to the surface of the activated carbon particles.

(A) II and IV

(B) III and IV

(C) I, II, and III

(D) I, II, III, and IV

11. 1 L of water having a contaminant concentration of 1 mg/L is to be treated with activated carbon. A residual or equilibrium contaminant concentration of 0.1 mg/L is desired. The activated carbon can adsorb 5% of its weight at the desired equilibrium concentration. Most nearly, what mass of activated carbon is required?

(A) 0.9 mg

(B) 18 mg

(C) 20 mg

(D) 29 mg

12. The electrodialysis of a 0.1 N NaCl solution is carried out in 100 cells. The flow rate of the solution is 1 L/s, the removal efficiency is 60%, and the electrical efficiency is 98%. The current required for operation is most nearly

(A) 60 A

(B) 200 A

(C) 500 A

(D) 1000 A

Environmental Engineering

SOLUTIONS

1. The primary treatment of wastewater is the removal of solids and particles.

The answer is (D).

2. The volumetric flow rate is

$$Q_W \cdot C_{in} = Q_A H' C_{in}$$

$$\begin{aligned} Q_W &= Q_A H' = \frac{Q_A H}{RT} \\ &= \frac{\left(1\ \frac{\text{m}^3}{\text{s}}\right)\left(1000\ \frac{\text{L}}{\text{m}^3}\right)\left(0.10\ \frac{\text{atm·L}}{\text{mol}}\right)}{\left(0.08206\ \frac{\text{atm·L}}{\text{mol·K}}\right)(25°\text{C} + 273°)} \\ &= 4.09\ \text{L/s} \quad (4\ \text{L/s}) \end{aligned}$$

The answer is (B).

3. The concentration of ammonia is 8.1×10^{-3} moles per liter of solution. The solution is essentially 100% water. The molecular weight of water (H_2O) is

$$\begin{aligned} \text{MW}_{\text{H}_2\text{O}} &= 2(\text{MW}_\text{H}) + \text{MW}_\text{O} \\ &= (2)\left(1.0079\ \frac{\text{g}}{\text{mol}}\right) + 15.999\ \frac{\text{g}}{\text{mol}} \\ &= 18.0148\ \text{g/mol} \end{aligned}$$

The number of moles of water in a liter of water is

$$\begin{aligned} n_W &= \frac{m}{\text{MW}_{\text{H}_2\text{O}}} = \frac{\rho V}{\text{MW}_{\text{H}_2\text{O}}} \\ &= \frac{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)(1\ \text{L})\left(1000\ \frac{\text{g}}{\text{kg}}\right)}{\left(18.0148\ \frac{\text{g}}{\text{mol}}\right)\left(1000\ \frac{\text{L}}{\text{m}^3}\right)} \\ &= 55.51\ \text{mol} \end{aligned}$$

The mole fraction of ammonia in the solution is

$$x = \frac{n_{\text{ammonia}}}{n_W} = \frac{8.1 \times 10^{-3}\ \text{mol}}{55.51\ \text{mol}} = 1.459 \times 10^{-4}$$

Use Henry's law.

$$\begin{aligned} P_i &= h x_i \\ &= (0.62\ \text{atm})(1.459 \times 10^{-4}) \\ &= 9.047 \times 10^{-5}\ \text{atm} \quad (9.0 \times 10^{-5}\ \text{atm}) \end{aligned}$$

The answer is (A).

4. For mixtures of ideal gases, the volumetric fraction and the mole fraction are the same.

$$y_{\text{SO}_2} = 0.1 \times 10^{-6}$$

Use Henry's law. The partial pressure of SO_2 is

$$\begin{aligned} P_{\text{SO}_2} &= P_{\text{total}} y_{\text{SO}_2} \\ &= (1\ \text{atm})(0.1 \times 10^{-6}) \\ &= 1.0 \times 10^{-7}\ \text{atm} \end{aligned}$$

When SO_2 dissociates, an equal number of HSO_3^- and H_3O^+ ions are created, so

$$[\text{HSO}_3^-][\text{H}_3\text{O}^+] = [\text{H}_3\text{O}^+]^2$$

Solve the equilibrium constant equation for the $[H_3O^+]$ concentration.

$$\begin{aligned} [\text{H}_3\text{O}^+] &= \sqrt{p_{\text{SO}_2} K_{\text{eq}}} \\ &= \sqrt{(1.0 \times 10^{-7}\ \text{atm})\left(2.1 \times 10^{-2}\ \frac{\left(\frac{\text{mol}}{\text{L}}\right)^2}{\text{atm}}\right)} \\ &= 4.6 \times 10^{-5}\ \text{mol/L} \end{aligned}$$

The answer is (B).

5. Items I through IV are not concerns: AIDS is not a waterborne disease; Rocky Mountain spotted fever is passed to humans by ticks; botulism is foodborne or can enter the body through contaminated soil or needles; and tuberculosis is spread by inhalation of infectious droplets.

Legionnaires' disease, on the other hand, is associated with heat transfer systems, warm temperature water, and stagnant water. Hepatitis A can also be spread through contaminated water.

The answer is (D).

6. Lime and soda ash are used to remove calcium and magnesium from hard water.

The answer is (A).

7. Water is recarbonated after it has been softened to lower its pH and reduce its scale-forming potential.

The answer is (C).

8. Various ion exchange methods are used to replace multivalent ions such as calcium (Ca^{++}) and magnesium (Mg^{++}) with monovalent ions such as sodium (Na^{+}).

The answer is (B).

9. Noncarbonate hardness cannot be removed by heating. It can be removed by precipitation softening processes (typically, the lime-soda ash process) or by ion exchange processes using resins selective for ions causing hardness.

The answer is (D).

10. Adsorption is a physical process wherein the contaminant adheres to the surface of an adsorbent such as granular activated charcoal. Following saturation, the adsorbent can be reactivated in a number of ways and reused. Adsorbents are particularly effective at removing large organic molecules from the processing stream.

The answer is (A).

11. Since 1 L of water is to be treated, and since the concentrations are given per liter, work on a per-liter basis. The mass of contaminant removed is

$$\begin{aligned} x_{\text{removed}} &= V(C_{\text{in}} - C_{\text{out}}) \\ &= (1\ \text{L})\left(1\ \frac{\text{mg}}{\text{L}} - 0.1\ \frac{\text{mg}}{\text{L}}\right) \\ &= 0.9\ \text{mg} \end{aligned}$$

Solve the equation for mass ratio of the solid phase for the mass of adsorbent.

$$m_{\text{adsorbent}} = \frac{x_{\text{removed}}}{X} = \frac{0.9\ \text{mg}}{0.05\ \frac{\text{g}}{\text{g}}} = 18\ \text{mg}$$

The answer is (B).

12. The required current is

$$I = (FQN/n) \times E_1/E_2$$

$$= \left(\frac{\left(96\,485\ \frac{\text{C}}{\text{g·equivalent}}\right)\left(1\ \frac{\text{L}}{\text{s}}\right) \times \left(0.1\ \frac{\text{g·equivalent}}{\text{L}}\right)}{100}\right)\left(\frac{0.60}{0.98}\right)$$

$$= 59.07\ \text{A} \quad (60\ \text{A})$$

The answer is (A).

15 Wastewater Collection and Treatment

PRACTICE PROBLEMS

1. The solids loading rate for a 30.5 m diameter clarifier with a flow rate of 5 MGD and an influent total suspended solids content of 150 mg/L is most nearly

(A) 1.1 kg/d·m^2

(B) 2.2 kg/d·m^2

(C) 3.9 kg/d·m^2

(D) 4.2 kg/d·m^2

2. What can treated wastewater be used for?

I. irrigation

II. fire fighting

III. road maintenance

IV. bathing

(A) I and II

(B) II and III

(C) I, II, and III

(D) I, II, III, and IV

3. How is most wastewater phosphorus removed?

(A) primary sedimentation

(B) flocculation and sedimentation

(C) biological processes

(D) adsorption in granular activated carbon towers

4. What is the primary factor that determines whether a stabilization pond will be aerobic or anaerobic?

(A) detention time

(B) depth

(C) surface (plan) area

(D) temperature

5. Which type of stabilization ponds are the most common for small communities?

(A) facultative ponds

(B) aerobic ponds

(C) anaerobic ponds

(D) aerated lagoons

6. Which of the following are used in the secondary treatment of wastewater?

I. biological beds

II. electrodialysis

III. rotating contactors

IV. air stripping

(A) III only

(B) I and II

(C) I and III

(D) I, II, III, and IV

Environmental Engineering

SOLUTIONS

1. The solids loading rate is

$$\text{solids loading rate} = \frac{QX}{A} = \frac{QX}{\frac{\pi D^2}{4}}$$

$$= \frac{\left(5\times 10^6 \ \frac{\text{gal}}{\text{d}}\right)\left(150 \ \frac{\text{mg}}{\text{L}}\right) \times \left(3.7854 \ \frac{\text{L}}{\text{gal}}\right)}{\left(\pi \frac{(30.5 \text{ m})^2}{4}\right)\left(1000 \ \frac{\text{mg}}{\text{g}}\right) \times \left(1000 \ \frac{\text{g}}{\text{kg}}\right)}$$

$$= 3.886 \text{ kg/d·m}^2 \quad (3.9 \text{ kg/d·m}^2)$$

The answer is (C).

2. Treated wastewater is generally not considered potable. Treated wastewater can be used for irrigation, firefighting, and road maintenance.

The answer is (C).

3. Most phosphorus is soluble. It must be removed through flocculation and sedimentation.

The answer is (B).

4. The average depth of an aerobic stabilization pond is about 1.3 m, so aerobic conditions can be maintained throughout through photosynthesis and mixing due to wind and convection. Anaerobic ponds are much deeper, and the biological organisms rapidly deplete the oxygen at the deeper levels.

The answer is (B).

5. Facultative ponds are the most common pond type selected for small communities.

The answer is (A).

6. Rotating contactors and biological beds are used in the secondary treatment of wastewater. Electrodialysis and air stripping are tertiary treatments.

The answer is (C).

Environmental Engineering

16 Activated Sludge and Sludge Processing

PRACTICE PROBLEMS

1. A 770,000 gal conventional activated sludge process has a recycle ratio of 0.25, an influent flow rate of 1 MGD, a mixed liquor volatile suspended solids (MLVSS) concentration of 700 mg/L, and an influent BOD_5 of 160 mg/L. What is most nearly the F:M ratio of the process?

(A) 0.1 lbm/day-lbm

(B) 0.2 lbm/day-lbm

(C) 0.3 lbm/day-lbm

(D) 0.4 lbm/day-lbm

2. Sludge is digested after being removed from a secondary treatment tank. During the digestion process, the gas that is produced is mostly

(A) carbon dioxide

(B) methane

(C) nitrogen oxide

(D) hydrogen sulfide

3. A 7400 ft^3 conventional activated sludge system has a recycle ratio of 0.26, an influent flow rate of 9100 ft^3/day, a mixed liquor volatile suspended solids (MLVSS) concentration of 490 mg/L, and an influent BOD_5 of 110 mg/L. What is most nearly the F:M ratio of the system?

(A) 0.028 day^{-1}

(B) 0.28 day^{-1}

(C) 0.72 day^{-1}

(D) 1.1 day^{-1}

4. Activated sludge is treated in a 1400 ft^3 aeration basin. The biomass concentration is 1800 mg/L. The waste sludge flows at a rate of 140 ft^3/day, and the waste sludge suspended solids concentration is 600 mg/L. The effluent flows at a rate of 240 ft^3/day, and the effluent suspended solids concentration is 1900 mg/L. What is most nearly the solids residence time?

(A) 1 day

(B) 3 days

(C) 5 days

(D) 8 days

5. A sample of influent water has a sludge volume index of 65 mL/g. After 30 min of settling, the sludge volume is 260 mL/L. What is most nearly the concentration of mixed liquor suspended solids (MLSS) in the sample?

(A) 2.0×10^3 mg/L

(B) 3.0×10^3 mg/L

(C) 4.0×10^3 mg/L

(D) 5.0×10^3 mg/L

6. An aerobic digester has an influent average flow rate of 110 ft^3/day and a suspended solids concentration of 12 mg/L in the reactor. The influent 5-day BOD is 310 mg/L, and the influent suspended solids concentration is 6.0 mg/L. The reaction rate constant is 0.75 day^{-1}, and the volatile fraction is 0.3. The solids resistance time (sludge age) is 3 days. The fraction of influent 5-day BOD consisting of raw sewage is 0.5. What is most nearly the volume of the digester?

(A) 1700 ft^3

(B) 2100 ft^3

(C) 2400 ft^3

(D) 2600 ft^3

7. A high-rate, two-stage anaerobic digester treats raw sludge with an input rate of 420 ft^3/day. The digester sludge accumulates at a rate of 110 ft^3/day. The sludge reacts after 10 hr, thickens after 7 hr, and is stored for 15 hr. What is most nearly the reactor volume of the first stage?

(A) 140 ft^3

(B) 180 ft^3

(C) 280 ft^3

(D) 460 ft^3

8. Activated sludge is treated in an aeration tank. The hydraulic residence time is 5 hr, and the solids residence time is 6 days. The influent BOD concentration is 170 mg/L, and the effluent BOD concentration is 5.2 mg/L. With a yield coefficient of 0.4 and a microbial death ratio of 0.1 day^{-1}, what is most nearly the biomass concentration in the tank?

(A) 1200 mg/L

(B) 2400 mg/L

(C) 3100 mg/L

(D) 4300 mg/L

9. A high-rate, two-stage anaerobic digester treats raw sludge with an input rate of 1200 $\text{ft}^3\text{/day}$. The digester sludge accumulates at a rate of 550 $\text{ft}^3\text{/day}$. The sludge reacts after 12 hr, thickens after 10 hr, and is stored for 20 hr. What is most nearly the reactor volume at the second stage?

(A) 460 ft^3

(B) 660 ft^3

(C) 820 ft^3

(D) 980 ft^3

10. A 15,000 ft^3 conventional activated sludge system has a recycle ratio of 0.38, an influent flow rate of 17,000 $\text{ft}^3\text{/day}$, a mixed liquor volatile suspended solids (MLVSS) concentration of 280 mg/L, and an influent BOD_5 of 60 mg/L. What is most nearly the recycle rate of the system?

(A) 2800 $\text{ft}^3\text{/day}$

(B) 3600 $\text{ft}^3\text{/day}$

(C) 5300 $\text{ft}^3\text{/day}$

(D) 6500 $\text{ft}^3\text{/day}$

Environmental Engineering

SOLUTIONS

1. The food-to-microorganism ratio is

$$\begin{aligned}\text{F:M} &= Q_0S_0/(\text{Vol}\,X_A)\\ &= \frac{\left(1\times10^6\ \frac{\text{gal}}{\text{day}}\right)\left(160\ \frac{\text{mg}}{\text{L}}\right)}{(770{,}000\ \text{gal})\left(700\ \frac{\text{mg}}{\text{L}}\right)}\\ &= 0.297\ \text{lbm/day-lbm}\quad (0.3\ \text{lbm/day-lbm})\end{aligned}$$

The answer is (C).

2. The gas produced is primarily methane.

The answer is (B).

3. The food-to-microorganism ratio is

$$\begin{aligned}\text{F:M} &= Q_0S_0/(\text{Vol}\,X_A)\\ &= \frac{\left(9100\ \frac{\text{ft}^3}{\text{day}}\right)\left(110\ \frac{\text{mg}}{\text{L}}\right)}{(7400\ \text{ft}^3)\left(490\ \frac{\text{mg}}{\text{L}}\right)}\\ &= 0.28\ \text{day}^{-1}\end{aligned}$$

The answer is (B).

4. The solids residence time is

$$\begin{aligned}\theta_c &= \frac{V(X_A)}{Q_wX_w + Q_eX_e}\\ &= \frac{(1400\ \text{ft}^3)\left(1800\ \frac{\text{mg}}{\text{L}}\right)}{\left(140\ \frac{\text{ft}^3}{\text{day}}\right)\left(600\ \frac{\text{mg}}{\text{L}}\right) + \left(240\ \frac{\text{ft}^3}{\text{day}}\right)\left(1900\ \frac{\text{mg}}{\text{L}}\right)}\\ &= 4.667\ \text{days}\quad (5\ \text{days})\end{aligned}$$

The answer is (C).

5. Calculate the amount of MLSS.

$$\text{SVI} = \frac{\text{sludge volume after settling (mL/L)} * 1000}{\text{MLSS (mg/L)}}$$

$$\begin{aligned}\text{MLSS} &= \frac{\text{sludge volume after settling (mL/L)} * 1000}{\text{SVI}} \\ &= \frac{\left(260\ \frac{\text{mL}}{\text{L}}\right)\left(1000\ \frac{\text{mg}}{\text{g}}\right)}{65\ \frac{\text{mL}}{\text{g}}} \\ &= 4.0 \times 10^3\ \text{mg/L}\end{aligned}$$

The answer is (C).

6. The volume of the aerobic digester is

$$\begin{aligned}V &= \frac{Q_i(X_i + FS_i)}{X_d(k_d P_v + 1/\theta_c)} \\ &= \frac{\left(110\ \frac{\text{ft}^3}{\text{day}}\right)\left(\left(6.0\ \frac{\text{mg}}{\text{L}}\right) + (0.5)\left(310\ \frac{\text{mg}}{\text{L}}\right)\right)}{\left(12\ \frac{\text{mg}}{\text{L}}\right)\left((0.75\ \text{day}^{-1})(0.3) + \left(\frac{1}{3\ \text{days}}\right)\right)} \\ &= 2643\ \text{ft}^3 \quad (2600\ \text{ft}^3)\end{aligned}$$

The answer is (D).

7. The reactor volume of the first stage is

$$\begin{aligned}\text{reactor volume} &= V_1 t_r \\ &= \frac{\left(420\ \frac{\text{ft}^3}{\text{day}}\right)(10\ \text{hr})}{24\ \frac{\text{hr}}{\text{day}}} \\ &= 175\ \text{ft}^3 \quad (180\ \text{ft}^3)\end{aligned}$$

The answer is (B).

8. The tank concentration is

$$\begin{aligned}X_A &= \frac{\theta_c Y(S_0 - S_e)}{\theta(1 + k_d\theta_c)} \\ &= \frac{(6\ \text{days})(0.4)\left(170\ \frac{\text{mg}}{\text{L}} - 5.2\ \frac{\text{mg}}{\text{L}}\right)\left(24\ \frac{\text{hr}}{\text{day}}\right)}{(5\ \text{hr})\left(1 + (0.1\ \text{day}^{-1})(6\ \text{days})\right)} \\ &= 1187\ \text{mg/L} \quad (1200\ \text{mg/L})\end{aligned}$$

The answer is (A).

9. The reactor volume is

$$\begin{aligned}\text{reactor volume} &= \frac{V_1 + V_2}{2} t_t + V_2 t_s \\ &= \frac{\left(\frac{1200\ \frac{\text{ft}^3}{\text{day}} + 550\ \frac{\text{ft}^3}{\text{day}}}{2}\right)(10\ \text{hr}) + \left(550\ \frac{\text{ft}^3}{\text{day}}\right)(20\ \text{hr})}{24\ \frac{\text{hr}}{\text{day}}} \\ &= 822.9\ \text{ft}^3 \quad (820\ \text{ft}^3)\end{aligned}$$

The answer is (C).

10. The recycle flow rate is

$$\begin{aligned}Q_R &= Q_0 R \\ &= \left(17{,}000\ \frac{\text{ft}^3}{\text{day}}\right)(0.38) \\ &= 6460\ \text{ft}^3/\text{day} \quad (6500\ \text{ft}^3/\text{day})\end{aligned}$$

The answer is (D).

Environmental Engineering

17 Air Quality

PRACTICE PROBLEMS

1. Which of the following pollutants is NOT a criteria pollutant as defined by the National Ambient Air Quality Standards (NAAQS)?

(A) carbon dioxide

(B) lead

(C) nitrogen dioxide

(D) dust

2. Carbon monoxide has been measured in an automotive repair garage at 9 parts per million by volume (ppmv). The garage air has a pressure of 1 standard atmosphere and a temperature of 18°C. Most nearly, what is the observed concentration per cubic meter?

(A) 11 mg/m^3

(B) 24 mg/m^3

(C) 52 mg/m^3

(D) 94 mg/m^3

3. The abbreviation VOC stands for

(A) volatile organic carbon

(B) various ocean chemicals

(C) virtual objectionable chemicals

(D) volatile organic compound

4. Which of the following equations represents the formation of acid rain?

(A) $S + O_3 + H_2O \rightarrow H_2SO_4$

(B) $SO + O_2 + H_2O \rightarrow H_2SO_4$

(C) $SO_2 + H_2O \rightarrow H_2SO_3$

(D) $SO_3 + H_2O \rightarrow H_2SO_4$

5. Incomplete combustion of fossil fuels typically results in the selective production of

(A) ozone

(B) smog

(C) carbon monoxide

(D) fly ash

6. What is the maximum size of thoracic particles generally considered to be inhalable hazards?

(A) 2.5 μm

(B) 10 μm

(C) 50 μm

(D) 250 μm

7. Which of the following devices is the LEAST effective at removing fly ash particles with a diameter of 1 μm?

(A) electrostatic precipitator

(B) fabric baghouse

(C) venturi water scrubber

(D) air cyclone

SOLUTIONS

1. The criteria pollutants are carbon monoxide, lead, nitrogen dioxide, coarse and fine particles, ozone, and sulfur dioxide.

The answer is (A).

2. Since the carbon monoxide concentration as a mass per cubic meter is desired, calculate the mass of carbon monoxide in one cubic meter of air. The molecular weight of carbon monoxide is

$$\text{MW} = 12.011\ \frac{\text{g}}{\text{mol}} + 15.999\ \frac{\text{g}}{\text{mol}} = 28.01\ \text{g/mol}$$

The volume of carbon monoxide in a cubic meter is $9 \times 10^{-6}\ \text{m}^3/\text{m}^3$, and using the atmospheric conditions (1 atm and 18°C), the mass of this volume is

$$m = \frac{pV}{\left(\frac{\overline{R}}{\text{MW}}\right)T}$$

$$= \frac{(1\ \text{atm})(101.3\ \text{kPa})\left(9 \times 10^{-6}\ \frac{\text{m}^3}{\text{m}^3}\right) \times \left(1000\ \frac{\text{mg}}{\text{g}}\right)\left(1000\ \frac{\text{mol}}{\text{kmol}}\right)}{\left(\frac{8.314\ \frac{\text{kPa}\cdot\text{m}^3}{\text{kmol}\cdot\text{K}}}{28.01\ \frac{\text{g}}{\text{mol}}}\right)(18°\text{C} + 273°)}$$

$$= 10.55\ \text{mg/m}^3 \quad (11\ \text{mg/m}^3)$$

The answer is (A).

3. VOC is an abbreviation for volatile organic compound.

The answer is (D).

4. Acid rain occurs when SO_2 or NO_x gases react in the atmosphere with oxygen, water and other chemicals to form mild solutions of sulfuric or nitric acid.

$$2SO_2 + O_2 \rightarrow 2SO_3$$
$$SO_3 + H_2O \rightarrow H_2SO_4$$

The answer is (D).

5. Incomplete combustion is caused by a deficiency of oxygen during combustion. Carbon burns to CO because there is inadequate oxygen to produce CO_2.

The answer is (C).

6. Thoracic particles are generally less than 10 μm in size and are designated PM_{10}.

The answer is (B).

7. Cyclones rely on gravitational and centrifugal forces to collect particles. Small particles have small masses, rendering force-dependent collection methods less effective.

The answer is (D).

Environmental Engineering

18 Soil Properties and Testing

PRACTICE PROBLEMS

1. An undisturbed sample of clay has a wet mass of 100 kg, a dry mass of 93 kg, and a total volume of 0.0491 m^3. The solids have a specific gravity of 2.65. The void ratio is most nearly

(A) 0.31

(B) 0.40

(C) 0.61

(D) 1.0

2. A saturated sample of undisturbed clay has a wet mass of 318 kg and a dry mass of 204 kg. The total volume of the sample is 0.193 m^3. Most nearly, what is the specific gravity of the soil solids?

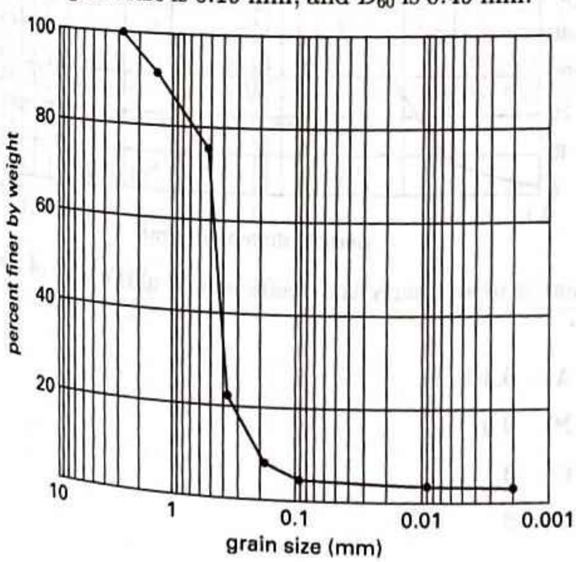
(A) 2.4

(B) 2.6

(C) 2.7

(D) 2.9

3. A soil's grain-size distribution curve is as shown. The effective grain size is 0.19 mm, and D_{60} is 0.49 mm.



The coefficient of concavity is most nearly

(A) 0.17

(B) 0.44

(C) 1.6

(D) 3.0

4. A soil has the following characteristics.

percentage fines, F	69%
liquid limit	72
plastic limit	48

Using the Unified Soil Classification System, what is the classification of the soil?

(A) GW

(B) ML

(C) MH

(D) CH

5. A sample of soil has the following characteristics.

% passing no. 40 screen	95
% passing no. 200 screen	57
liquid limit	37
plastic limit	18

What is the AASHTO group index number?

(A) 5

(B) 6

(C) 7

(D) 8

Geotechnical Engineering

6. The lines in a graphical seepage flow net intersect to form a pattern of

(A) triangles

(B) trapezoids

(C) squares

(D) rectangles

7. A flow net is constructed of

(A) isobars and aquicludes

(B) isobars and streamlines

(C) streamlines and isochrones

(D) isogons and isopleths

8. Granular soils tend to densify and consolidate if subjected to vibrations and seismic shaking. When saturated granular soils undergo seismic vibrations, they become subject to cyclic shear deformation, which causes an increase in pore water pressure. The result is usually a severe reduction in effective stress. This hazardous soil condition is called

(A) consolidation

(B) liquefaction

(C) heave

(D) boiling

9. The specific gravity of the soil solids in a given sample is 3.11. The porosity of the soil is 27%. What is most nearly the effective unit weight of the soil sample?

(A) 96 lbf/ft^3

(B) 110 lbf/ft^3

(C) 130 lbf/ft^3

(D) 160 lbf/ft^3

10. The total moist unit weight of a given soil is 120 lbf/ft^3. The soil has a porosity of 0.26 and is 100% saturated. The dry unit weight of the soil is most nearly

(A) 58 lbf/ft^3

(B) 86 lbf/ft^3

(C) 100 lbf/ft^3

(D) 120 lbf/ft^3

11. A 1 L field sample contains soil with an initial weight of 30 N. The soil is dried completely in an oven at 100°C. The weight of the soil after drying is 22 N. The maximum volume of the dry soil sample is 1.5 L, and the minimum volume is 0.60 L. The relative density of the soil is most nearly

(A) 37%

(B) 56%

(C) 62%

(D) 75%

12. An undisturbed 10 cm wide × 10 cm long × 5 cm high sample of clay has a dry weight of 32 kg. The dried soil sample is compacted into a minimum volume, a rectangular prism 8.5 cm wide × 9 cm long × 3.5 cm high. What is most nearly the relative compaction of the soil sample?

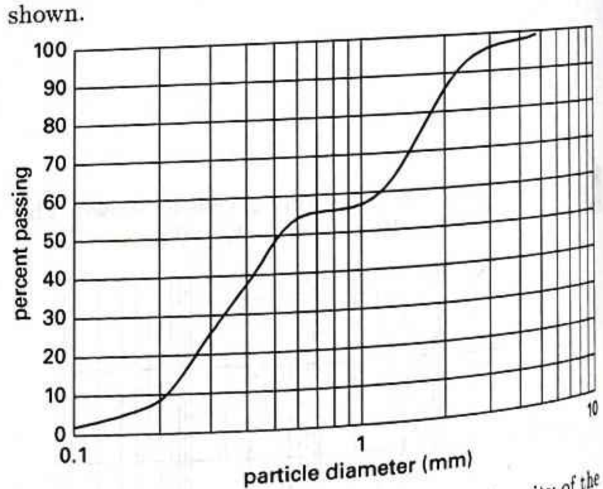
(A) 33%

(B) 45%

(C) 49%

(D) 54%

13. A soil sample has the particle size distribution shown.



What is most nearly the coefficient of uniformity of the soil?

(A) 0.4

(B) 0.8

(C) 3

(D) 6

Geotechnical Engineering

14. A soil has a void ratio of 0.41. The unit weight of the solids is 33 kN/m^3. What is most nearly the saturated unit weight of the soil?

(A) 26 kN/m^3

(B) 29 kN/m^3

(C) 33 kN/m^3

(D) 37 kN/m^3

15. The water content of a saturated soil sample is 26%, and the void ratio is 0.57. The unit weight of the solids is most nearly

(A) 4.5 kN/m^3

(B) 9.8 kN/m^3

(C) 22 kN/m^3

(D) 28 kN/m^3

16. An undisturbed soil sample with a volume of 0.30 ft^3 is weighed in a 0.2 lbf pan. The combined weight of the soil sample and the pan is 40.5 lbf. The soil is then completely dried in an oven at 100°C. After drying, the combined weight of the soil and the pan is 35.2 lbf. The unit weight of the soil solids is 194 lbf/ft^3. What is most nearly the volume of the air in the original soil sample?

(A) 0.010 ft^3

(B) 0.035 ft^3

(C) 0.10 ft^3

(D) 0.21 ft^3

17. A 0.10 m^3 saturated soil sample is weighed in a 34 kg pan. The combined weight of the soil and the pan is 450 kg. The soil sample is completely dried in an oven. After drying, the combined weight of the soil sample and the pan is 385 kg. What is most nearly the unit weight of the saturated soil sample?

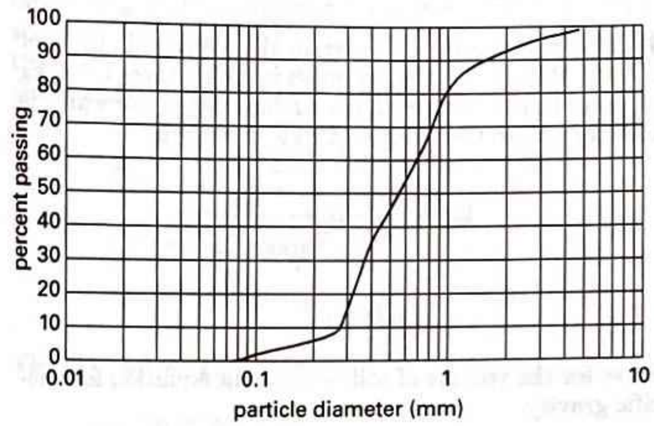
(A) 27 kN/m^3

(B) 34 kN/m^3

(C) 41 kN/m^3

(D) 44 kN/m^3

18. The results of a sieve analysis are shown.



Most nearly, what is the coefficient of curvature of the soil sample?

(A) 0.3

(B) 0.7

(C) 2

(D) 3

19. A 4 ft thick clay layer with sandy gravel above and impervious shale below is being surcharged in order to achieve 95% consolidation in 60 days. Most nearly, what must the coefficient of consolidation of the clay be in order to achieve the desired consolidation?

(A) 0.27 ft^2/day

(B) 0.30 ft^2/day

(C) 0.34 ft^2/day

(D) 0.40 ft^2/day

Geotechnical Engineering

SOLUTIONS

1. Find the volume of water in the sample. If the wet mass is 100 kg and the dry mass is 93 kg, there are 7 kg of water in the sample. The standard density of water is 1000 kg/m^3, so the volume of 7 kg of water is

$$V_W = \frac{m_W}{\rho_W} = \frac{7\ \text{kg}}{1000\ \frac{\text{kg}}{\text{m}^3}} = 0.007\ \text{m}^3$$

Solve for the volume of solids using the equation for specific gravity.

$$G_S = \frac{\frac{m_S}{V_S}}{\rho_W}$$

$$V_S = \frac{m_S}{G_S \rho_W} = \frac{93\ \text{kg}}{(2.65)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)} = 0.0351\ \text{m}^3$$

The volume of air in the sample is

$$\begin{aligned} V_A &= V_{\text{total}} - V_W - V_S \\ &= 0.0491\ \text{m}^3 - 0.007\ \text{m}^3 - 0.0351\ \text{m}^3 \\ &= 0.007\ \text{m}^3 \end{aligned}$$

The volume of voids is

$$\begin{aligned} V_V &= V_A + V_W = 0.007\ \text{m}^3 + 0.007\ \text{m}^3 \\ &= 0.014\ \text{m}^3 \end{aligned}$$

The void ratio is

$$\begin{aligned} e &= \frac{V_V}{V_S} = \frac{0.014\ \text{m}^3}{0.0351\ \text{m}^3} \\ &= 0.399 \quad (0.40) \end{aligned}$$

The answer is (B).

2. If the sample is saturated, there is no air in the sample. The mass of water in the sample is

$$m_W = 318\ \text{kg} - 204\ \text{kg} = 114\ \text{kg}$$

The density of water is 1000 kg/m^3, so the volume of water in the sample is

$$V_W = \frac{m_W}{\rho} = \frac{114\ \text{kg}}{1000\ \frac{\text{kg}}{\text{m}^3}} = 0.114\ \text{m}^3$$

The volume of solids in the sample is

$$V_S = V - V_W = 0.193\ \text{m}^3 - 0.114\ \text{m}^3 = 0.079\ \text{m}^3$$

The specific gravity of the solids is

$$\begin{aligned} G_S &= \frac{\frac{W_S}{V_S}}{\gamma_W} = \frac{\frac{204\ \text{kg}}{0.079\ \text{m}^3}}{1000\ \frac{\text{kg}}{\text{m}^3}} \\ &= 2.58 \quad (2.6) \end{aligned}$$

The answer is (B).

3. As read from the distribution curve, $D_{30} = 0.39$ mm. The coefficient of concavity is

$$\begin{aligned} C_C &= \frac{D_{30}^2}{D_{10}D_{60}} = \frac{(0.39\ \text{mm})^2}{(0.19\ \text{mm})(0.49\ \text{mm})} \\ &= 1.63 \quad (1.6) \end{aligned}$$

The answer is (C).

4. The plasticity index is PI = LL − PL = 72 − 48 = 24. When F is 69%, the soil is first classified as fine-grained. From the plasticity chart, for a liquid limit of 72 and a plasticity index of 24 (plots below the line), the soil is classified as MH.

The answer is (C).

5. The plasticity index is

$$\text{PI} = \text{LL} - \text{PL} = 37 - 18 = 19$$

The group index is

$$\begin{aligned} \text{GI} &= (F-35)\big(0.2 + 0.005(\text{LL}-40)\big) \\ &\quad + 0.01(F-15)(\text{PI}-10) \\ &= (57-35)\big(0.2 + (0.005)(37-40)\big) \\ &\quad + (0.01)(57-15)(19-10) \\ &= 7.85 \quad (8) \end{aligned}$$

The answer is (D).

Geotechnical Engineering

6. The intersections of streamlines and equipotential lines must be perpendicular and form a grid of four-sided shapes. These shapes can be referred to as curvilinear squares. The sides of the square may be curved, but perpendicular intersections are maintained.

The answer is (C).

7. A flow net uses streamlines (flow lines) and equipotential lines (isobars) to show pressure and flow path graphically. Equipotential lines represent levels of constant hydraulic head, and streamlines represent paths of flow.

The answer is (B).

8. The occurrence of liquefaction can result in loss of bearing capacity and immediate settlement of foundation elements. Excessive settlement and foundation movement can cause severe structural damage and possibly life-threatening destruction of buildings.

The answer is (B).

9. Rearrange the equation for porosity to calculate the void ratio of the soil sample.

$$\begin{aligned} n &= \frac{e}{1+e} \\ e &= \frac{n}{1-n} \\ &= \frac{0.27}{1-0.27} \\ &= 0.37 \end{aligned}$$

Calculate the saturated unit weight of the soil sample.

$$\begin{aligned} \gamma_{\text{sat}} &= \frac{(G_S+e)\gamma_W}{1+e} \\ &= \frac{(3.11+0.37)\left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)}{1+0.37} \\ &= 158.5\ \text{lbf/ft}^3 \end{aligned}$$

The effective unit weight of the soil sample is

$$\begin{aligned} \gamma' &= \gamma_{\text{sat}} - \gamma_W \\ &= 158.5\ \frac{\text{lbf}}{\text{ft}^3} - 62.4\ \frac{\text{lbf}}{\text{ft}^3} \\ &= 96\ \text{lbf/ft}^3 \end{aligned}$$

The answer is (A).

10. The total moist unit weight of the soil is the combined weight of the soil and the water. For a saturated soil, the water completely fills the voids. The weight of the water is the unit weight of water multiplied by the volume of the voids.

$$\begin{aligned} \gamma_{\text{sat}} &= \frac{W}{V} = \frac{W_S+W_W}{V} = \frac{W_S}{V} + \frac{W_W}{V} \\ &= \frac{W_S}{V} + \frac{\gamma_W V_V}{V} \\ &= \gamma_D + \gamma_W n \end{aligned}$$

The dry unit weight of the soil is

$$\begin{aligned} \gamma_D &= \gamma_{\text{sat}} - \gamma_W n \\ &= 120\ \frac{\text{lbf}}{\text{ft}^3} - \left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)(0.26) \\ &= 103.8\ \text{lbf/ft}^3 \quad (100\ \text{lbf/ft}^3) \end{aligned}$$

The answer is (C).

11. Calculate the dry unit weight, maximum dry unit weight, and minimum dry unit weight of the sample. The minimum density condition corresponds to the maximum volume condition.

$$\begin{aligned} \gamma_{d\text{-field}} &= \frac{W_S}{V} = \frac{22\ \text{N}}{1\ \text{L}} \\ &= 22\ \text{N/L} \\ \gamma_{d\text{-min}} &= \frac{W_S}{V} = \frac{22\ \text{N}}{1.5\ \text{L}} \\ &= 14.67\ \text{N/L} \\ \gamma_{d\text{-max}} &= \frac{W_S}{V} = \frac{22\ \text{N}}{0.60\ \text{L}} \\ &= 36.67\ \text{N/L} \end{aligned}$$

The relative density of the soil sample is

$$\begin{aligned} D_r &= \frac{(\gamma_{d\text{-field}} - \gamma_{d\text{-min}})(\gamma_{d\text{-max}})}{(\gamma_{d\text{-max}} - \gamma_{d\text{-min}})(\gamma_{d\text{-field}})} \times 100\% \\ &= \frac{\left(22\ \frac{\text{N}}{\text{L}} - 14.67\ \frac{\text{N}}{\text{L}}\right)\left(36.67\ \frac{\text{N}}{\text{L}}\right)}{\left(36.67\ \frac{\text{N}}{\text{L}} - 14.67\ \frac{\text{N}}{\text{L}}\right)\left(22\ \frac{\text{N}}{\text{L}}\right)} \times 100\% \\ &= 56\% \end{aligned}$$

The answer is (B).

12. Substitute the equation for dry unit weight into the equation for relative compaction of the soil sample.

$$\gamma_D = \frac{W_S}{V}$$

$$\text{RC} = \frac{\gamma_{d\text{-field}}}{\gamma_{d\text{-max}}} \times 100\% = \frac{\left(\frac{W_S}{V}\right)_{\text{field}}}{\left(\frac{W_S}{V}\right)_{\text{max}}} \times 100\%$$

The volume in the field is the initial volume of the soil sample, and the maximum density occurs at the minimum volume. Calculate the relative compaction.

$$\begin{aligned}\text{RC} &= \frac{\frac{W_S}{V_0}}{\frac{W_S}{V_{\min}}} \times 100\% \\ &= \frac{V_{\min}}{V_0} \times 100\% \\ &= \frac{(8.5 \text{ cm})(9 \text{ cm})(3.5 \text{ cm})}{(10 \text{ cm})(10 \text{ cm})(5 \text{ cm})} \times 100\% \\ &= 54\%\end{aligned}$$

The answer is (D).

13. The diameter at which 10% of the particles are finer is 0.2 mm, and the diameter at which 60% of the particles are finer is 1.2 mm. Calculate the coefficient of uniformity.

$$\begin{aligned}C_u &= \frac{D_{60}}{D_{10}} \\ &= \frac{1.2 \text{ mm}}{0.2 \text{ mm}} \\ &= 6\end{aligned}$$

The answer is (D).

14. Substitute the equation for unit weight of the solids into the equation for the saturated unit weight of the soil.

$$G_S = \frac{\frac{W_S}{V_S}}{\gamma_W}$$

$$\gamma_S = \frac{W_S}{V_S} = G_S\gamma_W$$

$$\begin{aligned}\gamma_{\text{sat}} &= \frac{(G_S + e)\gamma_W}{1+e} = \frac{G_S\gamma_W + e\gamma_W}{1+e} \\ &= \frac{\gamma_S + e\gamma_W}{1+e} \\ &= \frac{33 \ \frac{\text{kN}}{\text{m}^3} + (0.41)\left(9.81 \ \frac{\text{kN}}{\text{m}^3}\right)}{1+0.41} \\ &= 26 \text{ kN/m}^3\end{aligned}$$

The answer is (A).

15. For a saturated soil, the volume of voids is the volume of the water.

$$\omega = \frac{W_W}{W_S}$$

$$e = \frac{V_V}{V_S} = \frac{V_W}{V_S}$$

The unit weight of the solids is

$$\frac{\omega}{e} = \frac{\frac{W_W}{W_S}}{\frac{V_W}{V_S}} = \frac{\frac{W_W}{V_W}}{\frac{W_S}{V_S}} = \frac{\gamma_W}{\gamma_S}$$

$$\begin{aligned}\gamma_S &= \frac{\gamma_W e}{\omega} \\ &= \frac{\left(9.81 \ \frac{\text{kN}}{\text{m}^3}\right)(0.57)}{0.26} \\ &= 22 \text{ kN/m}^3\end{aligned}$$

The answer is (C).

16. The weight of the water in the sample is

$$\begin{aligned}W_W &= W_{\text{pan and soil,initial}} - W_{\text{pan and soil,final}} \\ &= 40.5 \text{ lbf} - 35.2 \text{ lbf} \\ &= 5.3 \text{ lbf}\end{aligned}$$

Geotechnical Engineering

The weight of the solids in the sample is

$$\begin{aligned} W_S &= W_{\text{pan and soil,final}} - W_{\text{pan}} \\ &= 35.2 \text{ lbf} - 0.2 \text{ lbf} \\ &= 35.0 \text{ lbf} \end{aligned}$$

Calculate the volume of water in the sample.

$$\begin{aligned} V_W &= \frac{W_W}{\gamma_W} = \frac{5.3 \text{ lbf}}{62.4 \ \frac{\text{lbf}}{\text{ft}^3}} \\ &= 0.085 \text{ ft}^3 \end{aligned}$$

Calculate the volume of solids in the sample.

$$\begin{aligned} V_S &= \frac{W_S}{\gamma_S} = \frac{35.0 \text{ lbf}}{194 \ \frac{\text{lbf}}{\text{ft}^3}} \\ &= 0.180 \text{ ft}^3 \end{aligned}$$

The volume of air is

$$\begin{aligned} V_{\text{total}} &= V_A + V_W + V_S \\ V_A &= V_{\text{total}} - V_W - V_S \\ &= 0.30 \text{ ft}^3 - 0.085 \text{ ft}^3 - 0.180 \text{ ft}^3 \\ &= 0.035 \text{ ft}^3 \end{aligned}$$

The answer is (B).

17. The unit weight of the saturated soil sample is the combined weight of the soil and water divided by the volume of the sample. Calculate the weight of the saturated soil sample.

$$\begin{aligned} W_{\text{sat}} &= W_{\text{sat,soil and pan}} - W_{\text{pan}} \\ &= \frac{(450 \text{ kg} - 34 \text{ kg})\left(9.81 \ \frac{\text{N}}{\text{kg}}\right)}{1000 \ \frac{\text{N}}{\text{kN}}} \\ &= 4.08 \text{ kN} \end{aligned}$$

The saturated unit weight is

$$\begin{aligned} \gamma_{\text{sat}} &= \frac{W_{\text{sat}}}{V} \\ &= \frac{4.08 \text{ kN}}{0.10 \text{ m}^3} \\ &= 41 \text{ kN/m}^3 \end{aligned}$$

The answer is (C).

18. The diameter at which 10% of the particles are finer is 0.28 mm. The diameter at which 30% of the particles are finer is 0.37 mm. The diameter at which 60% of the particles are finer is 0.7 mm. Calculate the coefficient of curvature.

$$\begin{aligned} C_C &= \frac{D_{30}^2}{D_{10}D_{60}} = \frac{(0.37 \text{ mm})^2}{(0.28 \text{ mm})(0.7 \text{ mm})} \\ &= 0.7 \end{aligned}$$

The answer is (B).

19. The clay layer experiences one-way drainage. From a table of time factor values, for a degree of consolidation of 95%, the time factor is 1.129.

$$\begin{aligned} T_v &= \frac{c_v t}{H_{dr}^2} \\ c_v &= \frac{H_{dr}^2 T_v}{t} \\ &= \frac{(4 \text{ ft})^2(1.129)}{60 \text{ days}} \\ &= 0.301 \text{ ft}^2\text{/day} \quad (0.30 \ \text{ ft}^2\text{/day}) \end{aligned}$$

The answer is (B).

Geotechnical Engineering

19 Foundations

PRACTICE PROBLEMS

1. The ultimate bearing capacity of a designed column is 25 kPa, whereas the toe bearing capacity of the column is 50 kPa. Most nearly, what is the value of the bearing capacity safety factor,

(A) 0.5

(B) 1.1

(C) 1.6

(D) 7.3

2. In cohesionless soils, the depth at which the point-bearing of a pile reaches a constant value is called the

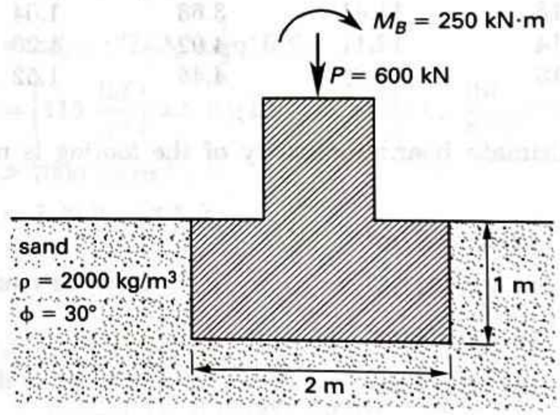
(A) ultimate depth

(B) effective depth

(C) critical depth

(D) maximum depth

3. A 2 m wide continuous wall footing is designed to support an axial column load of 600 kN (per meter of wall length) and a moment of 250 kN·m (per meter of wall length), as shown. The footing is placed 1 m into a sandy soil with a density of 2000 kg/m³, a cohesion of 0.5 Pa, and an angle of internal friction of 30°. The Terzaghi bearing capacity factors are $\phi = 30°$, $N_C = 37.2$, $N_q = 22.5$, and $N_\gamma = 19.7$.



Most nearly, what is the ultimate bearing capacity per meter of footing length?

(A) 84 kPa

(B) 210 kPa

(C) 430 kPa

(D) 830 kPa

4. A 3 m wide continuous footing is designed to support an axial column load of 2000 kN per meter of wall length. The footing is placed 1.8 m into a soil with a unit weight of 2500 kN/m³, a cohesion of 20 kPa, and an angle of internal friction of 10°. The table shown gives the Terzaghi bearing capacity factors applicable to footings.

Geotechnical Engineering

Terzaghi Bearing Capacity Table

ϕ (deg)	N_c	N_q	N_Υ
0	5.70	1.00	0.00
1	6.00	1.10	0.01
2	6.30	1.22	0.04
3	6.62	1.35	0.06
4	6.97	1.49	0.10
5	7.34	1.64	0.14
6	7.73	1.81	0.20
7	8.15	2.00	0.27
8	8.60	2.21	0.35
9	9.09	2.44	0.44
10	9.61	2.69	0.56
11	10.16	2.98	0.69
12	10.76	3.29	0.85
13	11.41	3.63	1.04
14	12.11	4.02	1.26
15	12.86	4.45	1.52

The ultimate bearing capacity of the footing is most nearly

(A) 12 000 kN/m^2

(B) 14 000 kN/m^2

(C) 16 000 kN/m^2

(D) 18 000 kN/m^2

5. A 5 ft wide strip footing is placed in a clayey sand 3 ft below the ground surface. The soil has a unit weight of 112 lbf/ft^3, a cohesion of 125 lbf/ft^2, and an angle of internal friction of 20°. The Terzaghi bearing capacity factors are $N_c = 17.7$, $N_q = 7.4$, and $N_\gamma = 5.0$, and the factor of safety is 1. Most nearly, what is the ultimate bearing capacity for the footing?

(A) 3000 psf

(B) 3500 psf

(C) 5500 psf

(D) 6000 psf

6. A strip footing is placed in a sandy soil with a unit weight of 115 lbf/ft^3 and an angle of internal friction of 25°. A load of 7 $kips/ft^2$ is applied to the footing. The footing is placed at a depth of 3.5 ft. The Terzaghi bearing capacity factors are $N_q = 12.7$ and $N_\gamma = 9.7$, and the factor of safety is 1. Most nearly, what is the minimum width of the footing?

(A) 3.0 ft

(B) 3.5 ft

(C) 4.0 ft

(D) 4.5 ft

7. A strip footing is designed to bear a maximum applied load of 15 $kips/ft^2$ in a sandy soil with a unit weight of 110 lbf/ft^3 and an angle of internal friction of 30°. Due to site constraints, the maximum footing width is 6 ft. The bearing capacity factors are $N_q = 22.5$ and $N_\gamma = 19.7$, and the factor of safety is 1. What is most nearly the minimum depth of the continuous footing?

(A) 3 ft

(B) 3.5 ft

(C) 4 ft

(D) 4.5 ft

Geotechnical Engineering

SOLUTIONS

1. Calculate the bearing capacity safety factor.

$$\begin{aligned}\text{FS}_{\text{bearing capacity}} &= \frac{q_{\text{ULT}}}{q_{\text{toe}}}\\ &= \frac{25\text{ kPa}}{50\text{ kPa}}\\ &= 0.5\end{aligned}$$

The answer is (A).

2. The critical depth for loose sands is taken as an installation depth of ten times the diameter of the pile. For dense sands, the critical depth is taken as 20 times the diameter of the pile.

The answer is (C).

3. The ultimate bearing capacity is

$$\begin{aligned}q_u &= cN_c + \gamma' D_f N_q + \tfrac{1}{2}\gamma' B N_\gamma\\ &= (0.5\text{ Pa})(37.2)\\ &\quad + \left(2000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1\text{ m})(22.5)\\ &\quad + \left(\frac{1}{2}\right)\left(2000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\text{ m})(19.7)\\ &= 827\,983\text{ Pa}\quad (830\text{ kPa})\end{aligned}$$

The answer is (D).

4. For an internal fraction angle of 10°, the Terzaghi bearing capacity factors are $N_c = 9.61$, $N_q = 2.69$, and $N_\gamma = 0.56$. Use the equation for ultimate bearing capacity.

$$\begin{aligned}q_{\text{ULT}} &= cN_c + \gamma' D_f N_q + \tfrac{1}{2}\gamma' B N_\gamma\\ &= (20\,000\text{ Pa})(9.61) + \left(2500\times 10^3\ \frac{\text{N}}{\text{m}^3}\right)(1.8\text{ m})(2.69)\\ &\quad + \left(\frac{1}{2}\right)\left(2500\times 10^3\ \frac{\text{N}}{\text{m}^3}\right)(3\text{ m})(0.56)\\ &= 14\,397\,200\text{ N/m}^2\quad (14\,000\text{ kN/m}^2)\end{aligned}$$

The answer is (B).

5. From the Terzaghi-Meyerhof equation, the ultimate bearing capacity is

$$\begin{aligned}q_u &= cN_c + \gamma' D_f N_q + \tfrac{1}{2}\gamma' B N_\gamma\\ &= \left(125\ \frac{\text{lbf}}{\text{ft}^2}\right)(17.7) + \left(112\ \frac{\text{lbf}}{\text{ft}^3}\right)(3\text{ ft})(7.4)\\ &\quad + \left(\frac{1}{2}\right)\left(112\ \frac{\text{lbf}}{\text{ft}^3}\right)(5\text{ ft})(5.0)\\ &= 6098.9\text{ lbf/ft}^2\quad (6000\text{ psf})\end{aligned}$$

The answer is (D).

6. Rearrange the Terzaghi-Meyerhof equation to solve for the footing width.

$$\begin{aligned}q_u &= cN_c + \gamma' D_f N_q + \tfrac{1}{2}\gamma' B N_\gamma\\ &= \left(115\ \frac{\text{lbf}}{\text{ft}^3}\right)(3.5\text{ ft})(12.7) + \left(\frac{1}{2}\right)\left(115\ \frac{\text{lbf}}{\text{ft}^3}\right)B(9.7)\\ &\geq 7000\text{ lbf/ft}^2\\ B &= 3.39\text{ ft}\quad (3.5\text{ ft})\end{aligned}$$

The answer is (B).

7. Use the Terzaghi-Meyerhof equation. The minimum depth is the depth of the footing when the maximum load is applied ($q_u = 15{,}000$ lbf/ft²). The cohesion is zero.

$$\begin{aligned}q_u &= cN_c + \gamma' D_f N_q + \tfrac{1}{2}\gamma' B N_\gamma\\ D_f &= \frac{q_u - cN_c - \frac{1}{2}\gamma' B N_\gamma}{\gamma' N_q}\\ &= \frac{15{,}000\ \frac{\text{lbf}}{\text{ft}^2} - 0\ \frac{\text{lbf}}{\text{ft}^3} - \left(\frac{1}{2}\right)\left(110\ \frac{\text{lbf}}{\text{ft}^3}\right)(6\text{ ft})(19.7)}{\left(110\ \frac{\text{lbf}}{\text{ft}^3}\right)(22.5)}\\ &= 3.43\text{ ft}\quad (3.5\text{ ft})\end{aligned}$$

The answer is (B).

Geotechnical Engineering

20 Rigid Retaining Walls

PRACTICE PROBLEMS

1. A soil has an angle of internal friction of 25°. What is most nearly the Rankine active earth pressure coefficient?

(A) 0.34

(B) 0.41

(C) 0.52

(D) 0.58

2. A soil has an angle of internal friction of 25°. What is most nearly the Rankine passive earth pressure coefficient?

(A) 0.59

(B) 1.6

(C) 2.5

(D) 4.1

3. A retaining wall supports soil with a vertical height of 2 m. The soil has an angle of internal friction of 32° and a specific (unit) weight of 25 kN/m^3. Most nearly, what is the active lateral soil resultant?

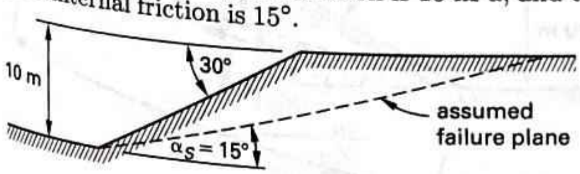
(A) 5.0 kN/m

(B) 15 kN/m

(C) 46 kN/m

(D) 92 kN/m

4. An excavated slope in uniform soil is shown. The specific weight is 14 kN/m^3, the cohesion is 15 kPa, and the angle of internal friction is 15°.



What is most nearly the available shearing resistance along the assumed failure plane?

(A) 550 kN/m

(B) 600 kN/m

(C) 680 kN/m

(D) 940 kN/m

5. A uniform soil slope has a planar slip surface length of 100 m. The soil's cohesion is 5 kPa, and the angle of internal friction is 40°. The angle that the slope makes with respect to the horizontal is 25°. The weight of the soil mound above the failure plane is 2000 kN/m. What is most nearly the mobilized shear force along the failure plane?

(A) 500 kN/m

(B) 850 kN/m

(C) 1500 kN/m

(D) 2000 kN/m

6. An excavation is made in uniform soil. The available shearing resistance along an assumed slip surface is 1350 kN/m. The mobilized shear force along the slip surface is 465 kN/m. The slip surface makes an angle of 45° with respect to the horizontal. What is most nearly the factor of safety against slope instability?

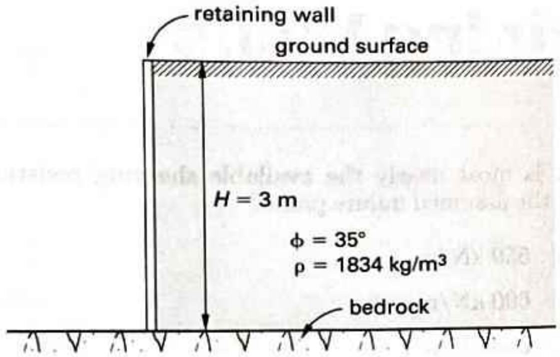
(A) 1.5

(B) 2.1

(C) 2.9

(D) 4.1

7. A retaining wall extends 3 m from the top of bedrock to the ground surface. The soil is cohesionless and has the properties shown.



Using Rankine theory, the total active earth pressure per unit width of retaining wall is most nearly

(A) 15 kN/m

(B) 22 kN/m

(C) 44 kN/m

(D) 82 kN/m

SOLUTIONS

1. The active lateral earth pressure coefficient is

$$\begin{aligned} K_A &= \tan^2\left(45° - \frac{\phi}{2}\right) \\ &= \tan^2\left(45° - \frac{25°}{2}\right) \\ &= 0.406 \quad (0.41) \end{aligned}$$

The answer is (B).

2. The passive earth pressure coefficient is

$$\begin{aligned} K_P &= \tan^2\left(45° + \frac{\phi}{2}\right) = \tan^2\left(45° + \frac{25°}{2}\right) \\ &= 2.46 \quad (2.5) \end{aligned}$$

The answer is (C).

3. The active lateral earth pressure coefficient is

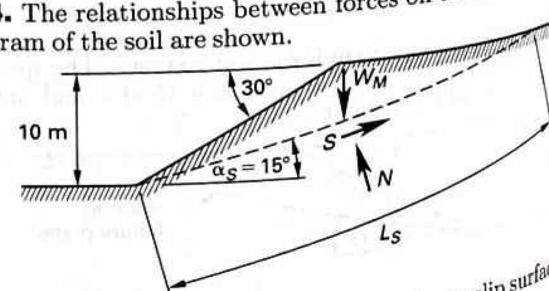
$$\begin{aligned} K_A &= \tan^2\left(45° - \frac{\phi}{2}\right) \\ &= \tan^2\left(45° - \frac{32°}{2}\right) \\ &= 0.307 \end{aligned}$$

Find the resultant active force.

$$\begin{aligned} P_A &= \tfrac{1}{2}K_AH^2\gamma = \left(\frac{1}{2}\right)(0.307)(2\ \text{m})^2\left(25\ \frac{\text{kN}}{\text{m}^3}\right) \\ &= 15.4\ \text{kN/m} \quad (15\ \text{kN/m}) \end{aligned}$$

The answer is (B).

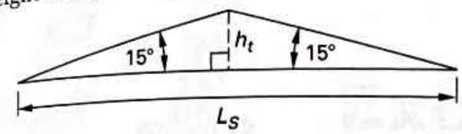
4. The relationships between forces on a free-body diagram of the soil are shown.



Calculate the length of the assumed planar slip surface.

$$L_S = \frac{h}{\sin\alpha_S} = \frac{10\ \text{m}}{\sin 15°} = 38.64\ \text{m}$$

The height of the triangle wedge, h_t, is



$$\begin{aligned} h_t &= \frac{L_S}{2}\tan\alpha \\ &= \left(\frac{38.64 \text{ m}}{2}\right)\tan 15° \\ &= 5.176 \text{ m} \end{aligned}$$

Calculate the weight of the soil above the failure plane.

$$\begin{aligned} W_M &= (\text{area of soil wedge})\gamma = \tfrac{1}{2}h_t L_S \gamma \\ &= \left(\frac{1}{2}\right)(5.176 \text{ m})(38.64 \text{ m})\left(14 \ \frac{\text{kN}}{\text{m}^3}\right) \\ &= 1400 \text{ kN/m} \end{aligned}$$

The available shearing resistance along the failure plane is

$$\begin{aligned} T_{\text{FF}} &= cL_S + W_M \cos\alpha_S \tan\phi \\ &= (15 \text{ kPa})(38.64 \text{ m}) + \left(1400 \ \frac{\text{kN}}{\text{m}}\right)\cos 15° \tan 15° \\ &= 942 \text{ kN/m} \quad (940 \text{ kN/m}) \end{aligned}$$

The answer is (D).

5. The mobilized shear force along the failure plane is

$$\begin{aligned} T_{\text{MOB}} &= W_M \sin\alpha_S \\ &= \left(2000 \ \frac{\text{kN}}{\text{m}}\right)\sin 25° \\ &= 845.2 \text{ kN/m} \quad (850 \text{ kN/m}) \end{aligned}$$

The answer is (B).

6. The factor of safety against slope instability is

$$\begin{aligned} FS &= \frac{T_{\text{FF}}}{T_{\text{MOB}}} \\ &= \frac{1350 \ \frac{\text{kN}}{\text{m}}}{465 \ \frac{\text{kN}}{\text{m}}} \\ &= 2.9 \end{aligned}$$

The answer is (C).

7. The coefficient of active lateral earth pressure is

$$\begin{aligned} K_A &= \tan^2\left(45° - \frac{\phi}{2}\right) \\ &= \tan^2\left(45° - \frac{35°}{2}\right) \\ &= 0.27 \end{aligned}$$

The total active force is

$$\begin{aligned} P_A &= \tfrac{1}{2}K_A H^2 \gamma \\ &= \left(\frac{1}{2}\right)(0.27)(3 \text{ m})^2\left(1834 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right) \\ &= 21\,940 \text{ N/m} \quad (22 \text{ kN/m}) \end{aligned}$$

The answer is (B).

Geotechnical Engineering

21 Excavations

PRACTICE PROBLEMS

1. An anchored bulkhead is typically supported by an embedded base with a tieback near the top. Sheeting can be used to span the distance between the embedded piles. The LEAST commonly observed failure with this type of construction is typically

(A) soil heave of the base layer

(B) failure of the sheeting

(C) pull-out of the anchorage

(D) embedment at the base such as toe kick-out

2. For the purposes of calculating the maximum stress in a braced cut, what are the primary factors in determining the shape of the pressure distribution?

(A) lateral earth pressure coefficient, specific weight, and cut depth

(B) specific weight, cut depth, and cohesion

(C) cut depth, soil shear strength, and specific weight

(D) angle of internal friction, lateral earth pressure coefficient, and cohesion

3. A 30 ft deep, 40 ft square excavation in sand is being designed. The sand will be dewatered before excavation. The angle of internal friction is 40°, and the specific weight is 121 lbf/ft^3. Bracing consists of horizontal shoring supported by 8 in soldier piles separated horizontally by 8 ft. Assuming a simple uniform pressure distribution behind the shoring, what is most nearly the maximum pressure on the piles?

(A) 500 lbf/ft^2

(B) 1100 lbf/ft^2

(C) 2400 lbf/ft^2

(D) 3600 lbf/ft^2

SOLUTIONS

1. For anchored bulkheads, the sheeting material spanning between the piles is typically the most reliable portion of the system. The highest failure occurrence is usually in the base heave of soils followed by anchorage pull-out. Failure of the sheeting is rare.

The answer is (B).

2. While the value of the maximum pressure in a braced cut depends on the lateral earth pressure coefficient, specific weight, and cut depth, the shape of the pressure distribution depends on the stability number, $\gamma H/c$.

The answer is (B).

3. The maximum pressure is

$$\begin{aligned} p_{\text{max}} &= 0.65K_A\gamma H \\ &= 0.65\gamma H\tan^2\left(45^\circ - \frac{\theta}{2}\right) \\ &= (0.65)\left(121\ \frac{\text{lbf}}{\text{ft}^3}\right)(30\ \text{ft})\tan^2\left(45^\circ - \frac{40^\circ}{2}\right) \\ &= 513\ \text{lbf/ft}^2 \quad (500\ \text{lbf/ft}^2) \end{aligned}$$

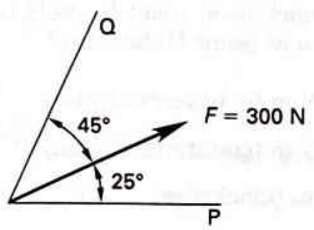
The answer is (A).

Geotechnical Engineering

22 Systems of Forces and Moments

PRACTICE PROBLEMS

1. In the system shown, force F, line P, and line Q are coplanar.



Resolve the 300 N force into two components, one along line P and the other along line Q.

(A) $F_Q = 272$ N; $F_P = 126$ N

(B) $F_Q = 232$ N; $F_P = 186$ N

(C) $F_Q = 135$ N; $F_P = 226$ N

(D) $F_Q = 212$ N; $F_P = 226$ N

2. Which type of load is NOT resisted by a pinned joint?

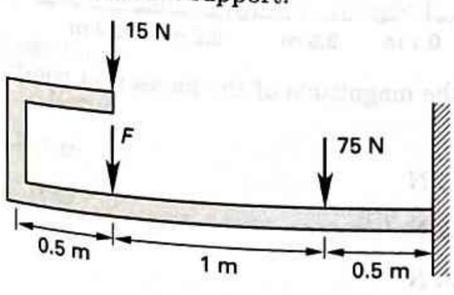
(A) moment

(B) shear

(C) axial

(D) distributed

3. The loading shown requires a clockwise resisting moment of 20 N·m at the support.



What is most nearly the value of force F?

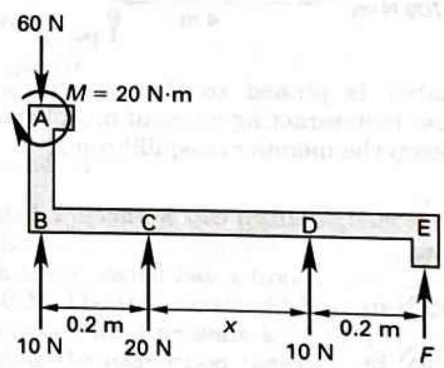
(A) 25 N (up)

(B) 27 N (up)

(C) 38 N (down)

(D) 43 N (down)

4. A bent beam is acted upon by a moment and several concentrated forces, as shown.



Approximate the unknown force, F, and distance, x, that will maintain equilibrium on the beam.

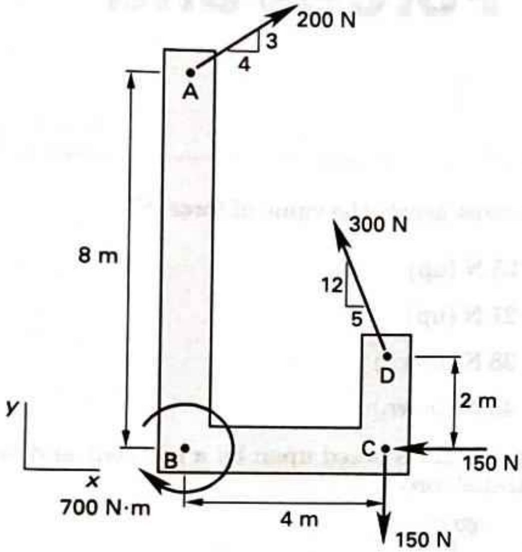
(A) $F = 5$ N; $x = 0.8$ m

(B) $F = 10$ N; $x = 0.6$ m

(C) $F = 20$ N; $x = 0.2$ m

(D) $F = 20$ N; $x = 0.4$ m

5. For the member shown, the 700 N·m moment is applied at point B.



If the member is pinned so that it rotates around point B, what counteracting moment must be applied at point A to keep the member in equilibrium?

(A) 650 N·m

(B) 890 N·m

(C) 1150 N·m

(D) 1240 N·m

6. A force is defined by the vector $\mathbf{A} = 3.5\mathbf{i} - 1.5\mathbf{j} + 2.0\mathbf{k}$. What is most nearly the angle between the force and the positive y-axis?

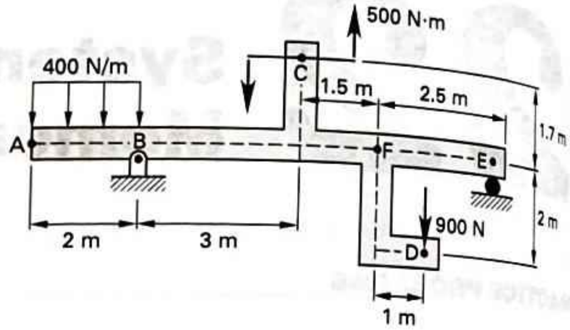
(A) 20°

(B) 66°

(C) 70°

(D) 110°

7. In the structure shown, the beam is pinned at point B. Point E is a roller support. The beam is loaded with a distributed load from point A to point B of 400 N/m, a 500 N·m couple at point C, and a vertical 900 N force at point D.



If the distributed load and the vertical load are removed and replaced with a vertically upward force of 1700 N at point F, what moment at point F would be necessary to keep the reaction at point E the same?

(A) −9000 N·m (counterclockwise)

(B) −6500 N·m (counterclockwise)

(C) 3500 N·m (clockwise)

(D) 12 000 N·m (clockwise)

8. Where can a couple be moved on a rigid body to have an equivalent effect?

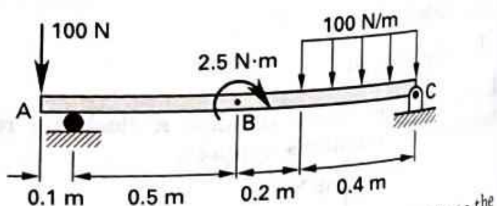
(A) along the line of action

(B) in a parallel plane

(C) along the perpendicular bisector joining the two original forces

(D) anywhere on the rigid body

9. The overhanging beam shown is supported by a roller and a pinned support. The moment is removed and replaced by a couple consisting of forces applied at points A and C.



What is the magnitude of the forces that constitute the couple?

(A) 2.1 N

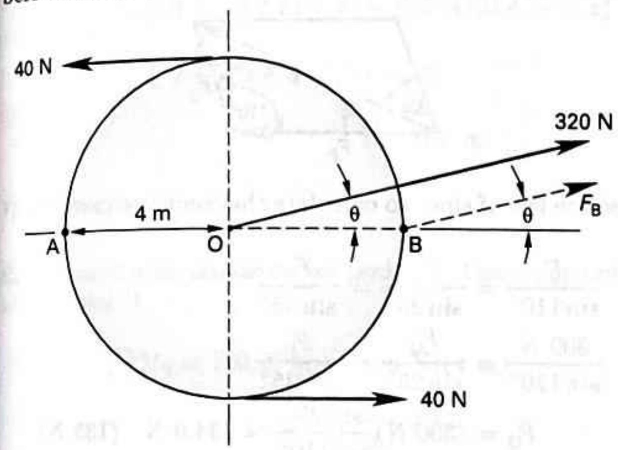
(B) 4.2 N

(C) 6.3 N

(D) 8.3 N

Statics

10. A disk-shaped body with a 4 m radius has a 320 N force acting through the center at an unknown angle θ, and two 40 N loads acting as a couple, as shown. All of these forces are removed and replaced by a single 320 N force at point B, parallel to the original 320 N force. The before and after moments at the center O are equal.



Most nearly, what is the angle θ?

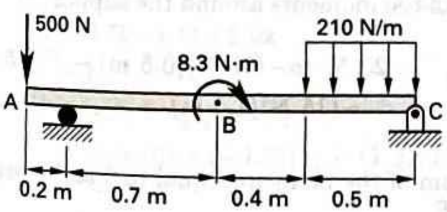
(A) 0°

(B) 7.6°

(C) 15°

(D) 29°

11. The overhanging beam shown is supported by a roller and a pinned support. The moment is removed and replaced by a couple consisting of forces applied at points A and C.



What is most nearly the magnitude of the couple that exactly replaces the moment that is removed?

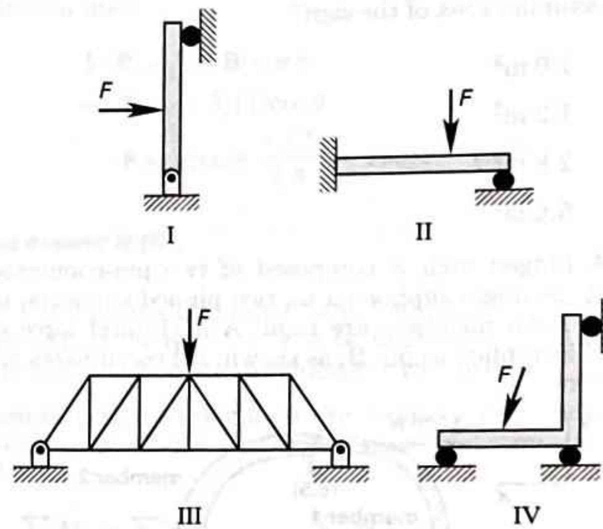
(A) 0.080 N·m

(B) 0.16 N·m

(C) 8.3 N·m

(D) 15 N·m

12. Which structure is statically determinate and stable with the loadings shown?



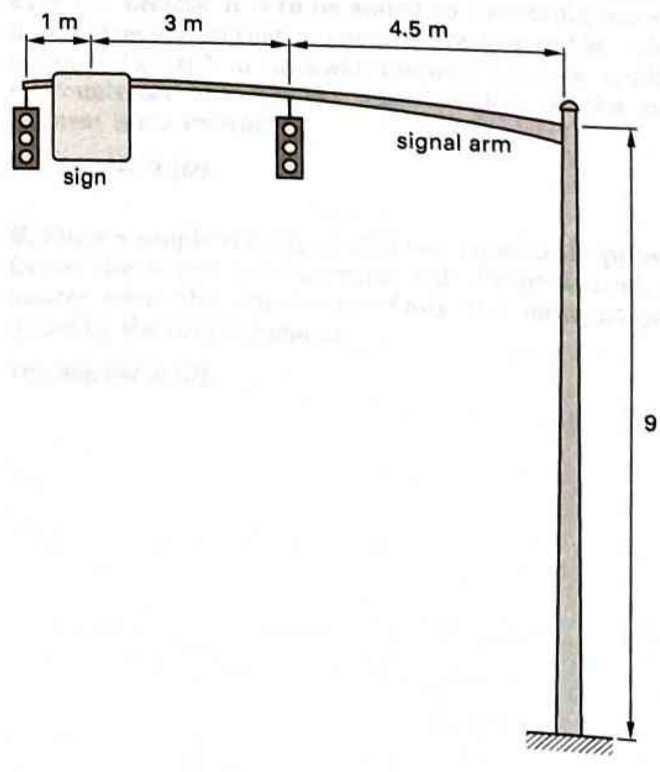
(A) I only

(B) I and III

(C) I and IV

(D) II and III

13. A signal arm carries two traffic signals and a sign, as shown. The signals and sign are rigidly attached to the arm. Each traffic signal has a frontal area of $0.2\ m^2$ and weighs 210 N. The sign weighs 60 N per square meter of area. The design wind pressure is 575 Pa. The maximum moment that the connection between the arm and pole can withstand due to wind is 6000 N·m, and due to gravity (weight) is 4000 N·m.



Neglecting the weight of the pole and limited by the moment capacity of the connection, what is most nearly the maximum area of the sign?

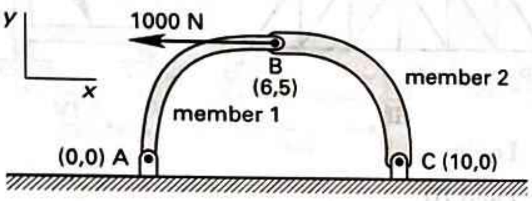
(A) $1.0\ \text{m}^2$

(B) $1.2\ \text{m}^2$

(C) $2.8\ \text{m}^2$

(D) $5.6\ \text{m}^2$

14. A hinged arch is composed of two pin-connected curved members supported on two pinned supports, as shown. Both members are rigid. A horizontal force of 1000 N is applied to pin B, as shown. All coordinates are in meters.



Most nearly, what are the reactions and moments at joint A?

(A) $F_x = 500$ N; $F_y = 600$ N; $M_A = 5000$ N·m

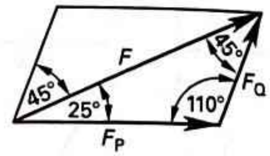
(B) $F_x = 600$ N; $F_y = 500$ N; $M_A = 0$ N·m

(C) $F_x = 680$ N; $F_y = 400$ N; $M_A = 5000$ N·m

(D) $F_x = 616$ N; $F_y = 480$ N; $M_A = 0$ N·m

SOLUTIONS

1. One of the characteristics of the components of a force is that they combine as vectors into the total force. Draw the vector addition triangle and determine all of the angles and sides.



Use the law of sines to calculate the components.

$$\frac{F}{\sin 110°} = \frac{F_Q}{\sin 25°} = \frac{F_P}{\sin 45°}$$

$$\frac{300\ \text{N}}{\sin 110°} = \frac{F_Q}{\sin 25°} = \frac{F_P}{\sin 45°}$$

$$F_Q = (300\ \text{N})\frac{\sin 25°}{\sin 110°} = 134.9\ \text{N} \quad (135\ \text{N})$$

$$F_P = (300\ \text{N})\frac{\sin 45°}{\sin 110°} = 225.7\ \text{N} \quad (226\ \text{N})$$

The answer is (C).

2. A pinned support will resist forces but not moments.

The answer is (A).

3. The choice of sign on the moments is arbitrary. Clockwise moments can be assumed to be positive.

The sum of the moments around the support is

$$\sum M = 20\ \text{N}\cdot\text{m} - (75\ \text{N})(0.5\ \text{m}) - F(1.5\ \text{m}) - (15\ \text{N})(1.5\ \text{m})$$

Set the sum of the moments equal to 0 and rearrange to solve for F.

$$0 = 20\ \text{N·m} - (75\ \text{N})(0.5\ \text{m}) - F(1.5\ \text{m}) - (15\ \text{N})(1.5\ \text{m})$$

$$\begin{aligned} F &= \frac{20\ \text{N·m} - (75\ \text{N})(0.5\ \text{m}) - (15\ \text{N})(1.5\ \text{m})}{1.5\ \text{m}} \\ &= -26.67\ \text{N} \quad (27\ \text{N}) \quad [\text{up}] \end{aligned}$$

The answer is (B).

4. The sum of the forces in the y-direction is

$$\begin{aligned} \sum F_y &= 0 \\ &= -60\ \text{N} + 10\ \text{N} + 20\ \text{N} + 10\ \text{N} + F \\ F &= 20\ \text{N} \end{aligned}$$

Statics

Clockwise moments are positive. The sum of the moments around point A is

$$\begin{aligned}\sum M_A &= 0\\ &= 20\ \text{N·m} - (20\ \text{N})(0.2\ \text{m})\\ &\quad - (10\ \text{N})(0.2\ \text{m} + x) - (20\ \text{N})(0.4\ \text{m} + x)\end{aligned}$$

$$\begin{aligned}4 + 2 + 10x + 8 + 20x &= 20\\ 30x &= 6\\ x &= 0.2\ \text{m}\end{aligned}$$

The answer is (C).

5. Let clockwise moments be positive. Take moments about point B.

$$\begin{aligned}\sum M_B &= 700\ \text{N·m} + (150\ \text{N})(4\ \text{m})\\ &\quad -(300\ \text{N})\left(\frac{5}{13}\right)(2\ \text{m})\\ &\quad -(300\ \text{N})\left(\frac{12}{13}\right)(4\ \text{m})\\ &\quad +(200\ \text{N})\left(\frac{4}{5}\right)(8\ \text{m})\\ &= 1242\ \text{N·m} \quad (1240\ \text{N·m})\end{aligned}$$

The application point of the moment is irrelevant.

The answer is (D).

6. Calculate the dot product of two vectors.

$$\begin{aligned}\mathbf{A} &= 3.5\mathbf{i} - 1.5\mathbf{j} + 2.0\mathbf{k}\\ \mathbf{B} &= \mathbf{j}\\ \mathbf{A}\cdot\mathbf{B} &= a_x b_x + a_y b_y + a_z b_z\\ &= (3.5)(0) + (-1.5)(1) + (2.0)(0)\\ &= -1.5\end{aligned}$$

The magnitude of B (a unit vector) is 1. The magnitude of A (same as F) is

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2}\\ &= \sqrt{(3.5)^2 + (-1.5)^2 + (2.0)^2}\\ &= 4.3\end{aligned}$$

The dot product is defined as the product of the vector magnitudes multiplied by the cosine of the angle between them.

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= |\mathbf{A}||\mathbf{B}|\cos\theta\\ -1.5 &= (4.3)(1)\cos\theta\\ \theta &= \arccos\frac{-1.5}{4.3} = 110.4^\circ \quad (110^\circ)\end{aligned}$$

The answer is (D).

7. The reaction at point E is unknown, but it is irrelevant. Since the reaction is to be unchanged, it is necessary only to calculate the change in the loading.

Assume clockwise moments are positive. Take moments about point B for the forces that are removed and added.

$$\begin{aligned}\sum M_B &= \sum M_{B,\text{removed}} - \sum M_{B,\text{added}}\\ &= -\left(400\ \frac{\text{N}}{\text{m}}\right)(2\ \text{m})\left(\frac{2\ \text{m}}{2}\right)\\ &\quad +(900\ \text{N})(1\ \text{m} + 1.5\ \text{m} + 3\ \text{m})\\ &\quad -(-1700\ \text{N})(1.5\ \text{m} + 3\ \text{m})\\ &= 11\,800\ \text{N·m} \quad (12\,000\ \text{N·m}) \quad [\text{clockwise}]\end{aligned}$$

This is the moment that is applied by the forces that are removed, reduced by the moment of the new force that is applied. The distributed force applies a counterclockwise moment, so it is negative, and the force at point D applies a clockwise moment, so it is positive. The force to be added at point F is upward, so it is counterclockwise, but because it is to be added to the configuration, it is subtracted, so that moment is positive in the calculation. A 12 000 N·m clockwise moment must be applied to counteract the change. The location of the new moment is not relevant.

The answer is (D).

8. Since a couple is composed of two equal but opposite forces, the x- and y-components will always cancel, no matter what the orientation. Only the moment produced by the couple remains.

The answer is (D).

Statics

9. The distance between the forces is

$$\begin{aligned} d &= 0.4\ \text{m} + 0.2\ \text{m} + 0.5\ \text{m} + 0.1\ \text{m} \\ &= 1.2\ \text{m} \end{aligned}$$

The forces are

$$\begin{aligned} F &= \frac{M}{d} = \frac{2.5\ \text{N·m}}{1.2\ \text{m}} \\ &= 2.08\ \text{N} \quad (2.1\ \text{N}) \end{aligned}$$

The answer is (A).

10. Assume clockwise moments are positive. Take moments about the center for the original forces. The 320 N force has no moment arm, so it does not contribute to the moment. The couple is

$$\begin{aligned} M &= 2Fr = -(2)(40\ \text{N})(4\ \text{m}) \\ &= -320\ \text{N m} \end{aligned}$$

The single replacement force must produce a moment of −320 N·m. The horizontal component of the replacement force acts through the center; only the vertical component of the force contributes to the moment.

$$\begin{aligned} M &= -320\ \text{N m} = Fr = -(320\ \text{N})(\sin\theta)(4\ \text{m}) \\ \theta &= 14.48^\circ \quad (15^\circ) \end{aligned}$$

The answer is (C).

11. A couple is a moment. When a moment of 8.3 N·m is removed, it must be replaced by the same moment.

The answer is (C).

12. Structure I is simply supported and determinate. Structure II is a propped cantilever beam, always indeterminate by one degree. Structure III is a truss that is pinned at both ends, also indeterminate by one degree. Structure IV is a beam with three rollers, two in the vertical direction and one in the horizontal direction. It is determinate, but not stable.

The answer is (A).

13. The length of the signal arm is

$$1\ \text{m} + 3\ \text{m} + 4.5\ \text{m} = 8.5\ \text{m}$$

Set the moment on the arm due to the wind equal to the maximum allowed.

$$\begin{aligned} \sum M_{\text{wind}} &= (0.2\ \text{m}^2)(575\ \text{Pa})(8.5\ \text{m} + 4.5\ \text{m}) \\ &\quad + A_{\text{sign}}(575\ \text{Pa})(7.5\ \text{m}) = 6000\ \text{N·m} \\ A_{\text{sign}} &= 1.04\ \text{m}^2 \end{aligned}$$

Set the moment on the arm due to vertical loading equal to the maximum allowed.

$$\begin{aligned} \sum M_{\text{loads}} &= (210\ \text{N})(8.5\ \text{m} + 4.5\ \text{m}) \\ &\quad + \left(60\ \frac{\text{N}}{\text{m}^2}\right)A_{\text{sign}}(7.5\ \text{m}) = 4000\ \text{N·m} \\ A_{\text{sign}} &= 2.82\ \text{m}^2 \end{aligned}$$

The maximum area of the sign is the smaller of these two values, 1.04 m² (1.0 m²).

The answer is (A).

14. Sum moments about point C. The x-component of the reaction at point A has a zero moment arm.

$$\begin{aligned} \sum M_{\text{C}} &= 10F_y - (1000\ \text{N})(5) = 0 \\ F_y &= 500\ \text{N} \end{aligned}$$

Point B is a frictionless pin, which transmits no moment, so the member that connects point B to point C and the forces on point C impart no moment on the member that connects point A to point B. The sum of the moments acting on the member connecting point A to point B about point B must equal 0.

$$\begin{aligned} \sum M_{\text{B}} &= F_y(6) - F_x(5) = 0 \\ &= (500\ \text{N})(6) - F_x(5) = 0 \\ F_x &= 600\ \text{N} \end{aligned}$$

Point A is a frictionless pin, which transmits no moment.

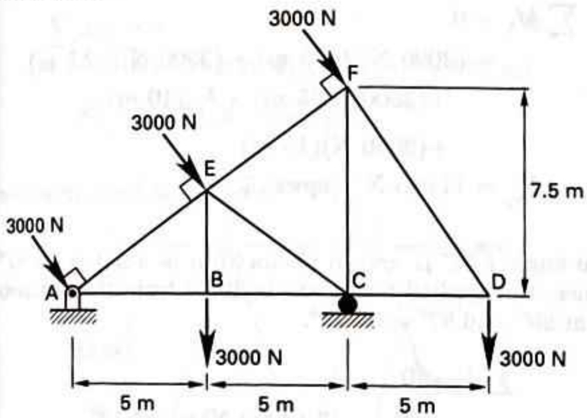
The answer is (B).

Statics

23 Trusses

PRACTICE PROBLEMS

1. Determine the approximate force in member BC for the truss shown.



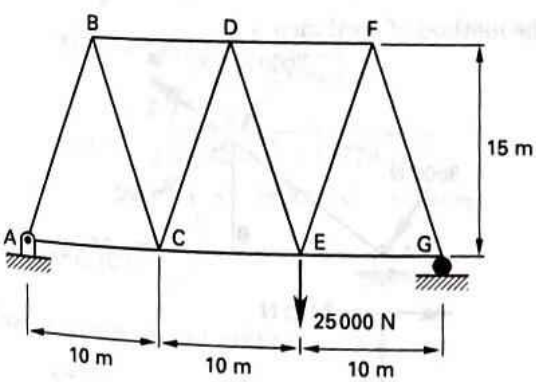
(A) 0 N

(B) 1000 N

(C) 1500 N

(D) 2500 N

2. Find the approximate force in member DE for the truss shown.



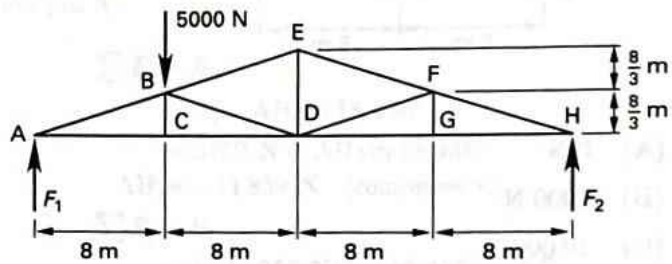
(A) 0 N

(B) 6300 N

(C) 8800 N

(D) 10 000 N

3. For the truss shown, what are most nearly the forces in members AC and BD?



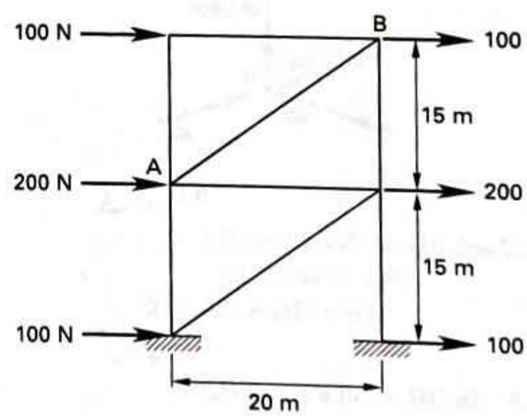
(A) AC = 11 000 N; BD = −7900 N

(B) AC = 0 N; BD = −2000 N

(C) AC = 1100 N; BD = 2500 N

(D) AC = 0 N; BD = −7900 N

4. The braced frame shown is constructed with pin-connected members and supports. All applied forces are horizontal.



Most nearly, what is the force in the diagonal member BA?

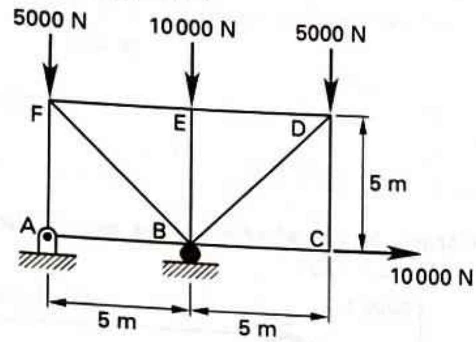
(A) 0 N

(B) 160 N

(C) 200 N

(D) 250 N

5. For the cantilever truss shown, what is most nearly the force in member AF?



(A) 0 N

(B) 5000 N

(C) 10 000 N

(D) 15 000 N

SOLUTIONS

1. Distance AF is

$$\text{AF} = \sqrt{(5\text{ m} + 5\text{ m})^2 + (7.5\text{ m})^2} = 12.5\text{ m}$$

Distance AE is

$$\text{AE} = \tfrac{1}{2}\text{AF} = \left(\frac{1}{2}\right)(12.5\text{ m}) = 6.25\text{ m}$$

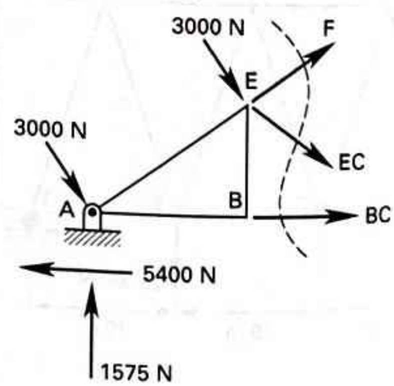
The sum of the moments around point A is

$$\begin{aligned}\sum M_{\text{A}} &= 0\\ &= (3000\text{ N})(6.25\text{ m}) + (3000\text{ N})(12.5\text{ m})\\ &\quad + (3000\text{ N})(5\text{ m}) - F_{\text{C}_y}(10\text{ m})\\ &\quad + (3000\text{ N})(15\text{ m})\\ F_{\text{C}_y} &= 11\,625\text{ N} \quad \text{[upward]}\end{aligned}$$

The angle FAC is arctan $(7.5\text{m}/(5\text{m} + 5\text{m})) = 36.87°$. Hence, the applied forces are inclined from the horizontal at $90° - 36.87° = 53.13°$.

$$\begin{aligned}\sum F_y &= 0\\ &= F_{\text{A}_y} - (3)(3000\text{ N})\sin 53.13°\\ &\quad - (2)(3000\text{ N}) + 11\,625\text{ N}\\ F_{\text{A}_y} &= 1575\text{ N} \quad \text{[upward]}\\ \sum F_x &= 0\\ &= F_{\text{A}_x} + (3)(3000\text{ N})\cos 53.13°\\ F_{\text{A}_x} &= -5400\text{ N} \quad \text{[to the left]}\end{aligned}$$

Use the method of sections.



Take moments about point E. The vertical downward force at point B passes through point E, and it does not generate a moment.

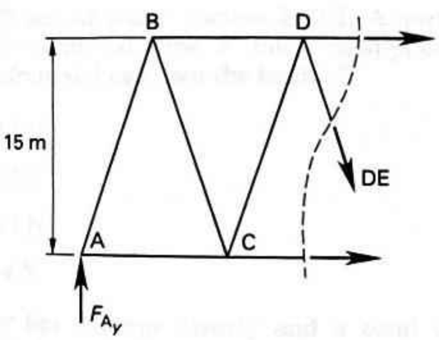
$$\begin{aligned}\sum M_{\mathrm{E}} &= 0\\ &= (5400\ \mathrm{N})(3.75\ \mathrm{m}) + (1575\ \mathrm{N})(5\ \mathrm{m})\\ &\quad -(3000\ \mathrm{N})(6.25\ \mathrm{m}) - \mathrm{BC}(3.75\ \mathrm{m})\\ \mathrm{BC} &= 2500\ \mathrm{N}\end{aligned}$$

The answer is (D).

2. Take moments about point G.

$$\begin{aligned}\sum M_{\mathrm{G}} &= 0\\ &= (-25\,000\ \mathrm{N})(10\ \mathrm{m}) + F_{\mathrm{A}_y}(30\ \mathrm{m})\\ F_{\mathrm{A}_y} &= 8333\ \mathrm{N}\quad [\text{upward}]\end{aligned}$$

Use the method of sections.



$$\begin{aligned}\sum F_y &= 0\\ &= 8333\ \mathrm{N} - \mathrm{DE}_y\\ \mathrm{DE}_y &= 8333\ \mathrm{N}\\ \mathrm{DE}_x &= (8333\ \mathrm{N})\left(\frac{5\ \mathrm{m}}{15\ \mathrm{m}}\right)\\ &= 2778\ \mathrm{N}\\ \mathrm{DE} &= \sqrt{(8333\ \mathrm{N})^2 + (2778\ \mathrm{N})^2}\\ &= 8784\ \mathrm{N}\quad (8800\ \mathrm{N})\quad [\text{tension}]\end{aligned}$$

The answer is (C).

3. Take moments about point H.

$$\begin{aligned}\sum M_{\mathrm{H}} &= 0\\ &= (5000\ \mathrm{N})(24\ \mathrm{m}) - F_1(32\ \mathrm{m})\\ F_1 &= 3750\ \mathrm{N}\end{aligned}$$

The angle made by the inclined members with the horizontal is

$$\arctan\frac{\frac{8}{3}\ \mathrm{m}}{8\ \mathrm{m}} = 18.435^\circ$$

(Alternatively, the force components could be determined from geometry.)

Use the method of joints.

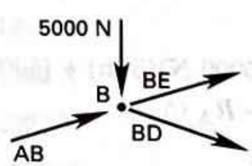
For pin A,

$$\begin{aligned}\sum F_y &= 0\\ &= F_1 + \mathrm{AB}\sin 18.435^\circ\\ &= 3750\ \mathrm{N} + \mathrm{AB}\sin 18.435^\circ\\ \mathrm{AB} &= -11\,859\ \mathrm{N}\quad [\text{compression}]\\ \sum F_x &= 0\\ &= (-11\,859\ \mathrm{N})\cos 18.435^\circ + \mathrm{AC}\\ \mathrm{AC} &= 11\,250\ \mathrm{N}\quad (11\,000\ \mathrm{N})\quad [\text{tension}]\end{aligned}$$

For pin C,

$$\begin{aligned}\sum F_y &= 0\\ \mathrm{BC} &= 0\quad [\text{zero-force member}]\end{aligned}$$

For pin B,



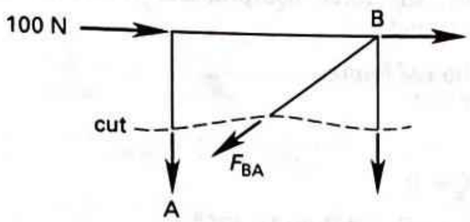
$$\begin{aligned}\sum F_x &= 0\\ &= \mathrm{AB}\cos 18.435^\circ + \mathrm{BE}\cos 18.435^\circ\\ &\quad +\mathrm{BD}\cos 18.435^\circ\\ 0 &= \mathrm{AB} + \mathrm{BE} + \mathrm{BD}\\ \sum F_y &= 0\\ &= \mathrm{AB}\sin 18.435^\circ + \mathrm{BE}\sin 18.435^\circ\\ &\quad -\mathrm{BD}\sin 18.435^\circ - 5000\ \mathrm{N}\\ \mathrm{BD}\sin 18.435^\circ &= \mathrm{AB}\ \sin 18.435^\circ + (-\mathrm{AB} - \mathrm{BD})\\ &\quad \times(\sin 18.435^\circ) - 5000\ \mathrm{N}\\ &= \sin 18.435^\circ(\mathrm{AB} - \mathrm{AB} - \mathrm{BD})\\ &\quad -5000\ \mathrm{N}\\ 2\mathrm{BD}\sin 18.435^\circ &= -5000\ \mathrm{N}\\ \mathrm{BD} &= -7906\ \mathrm{N}\quad (-7900\ \mathrm{N})\\ &\quad \left[\begin{array}{c}\text{compression in opposite}\\ \text{direction shown}\end{array}\right]\end{aligned}$$

The answer is (A).

4. Determine the length of member BA by recognizing this configuration to be a 3-4-5 triangle.

$$L_{BA} = 25 \text{ m}$$

Use the method of sections. Cut the frame horizontally through member BA.



By inspection, the horizontal component of F_{BA} balan ces the two applied horizontal loads.

$$BA_x = 100 \text{ N} + 100 \text{ N} = 200 \text{ N}$$

By similar triangles,

$$BA = \left(\frac{5}{4}\right)(200 \text{ N}) = 250 \text{ N}$$

The answer is (D).

5. Take moments about point B.

$$\begin{aligned}\sum M_B &= 0 \\ &= -(5000 \text{ N})(5 \text{ m}) + (5000 \text{ N})(5 \text{ m}) \\ &\quad + R_{A_y}(5 \text{ m}) \\ R_{A_y} &= 0 \text{ N}\end{aligned}$$

By inspection, the force in member AF is equal to the vertical component of the reaction at point A.

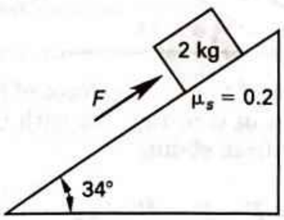
The answer is (A).

Statics

24 Pulleys, Cables, and Friction

PRACTICE PROBLEMS

1. A 2 kg block rests on a 34° incline.



The coefficient of static friction is 0.2. Approximately how much additional force, F, must be applied to keep the block from sliding down the incline?

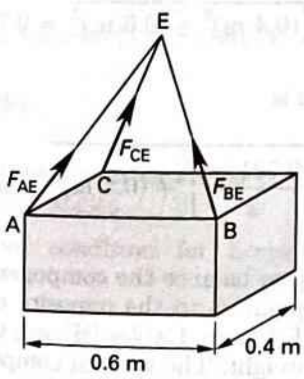
(A) 7.7 N

(B) 8.8 N

(C) 9.1 N

(D) 14 N

2. A box has uniform density and a total weight of 600 N. It is suspended by three cables, AE, BE, and CE, as shown. Point E is 0.5 m directly above the center of the box's top surface.



Most nearly, what is the tension in cable CE?

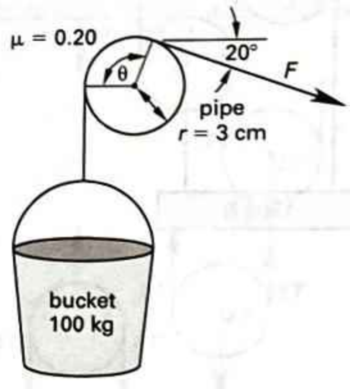
(A) 130 N

(B) 200 N

(C) 370 N

(D) 400 N

3. A rope is wrapped over a 6 cm diameter pipe to support a bucket of tools being lowered. The coefficient of static friction between the rope and the pipe is 0.20. The combined mass of the bucket and tools is 100 kg.



Most nearly, what is the range of force that can be applied to the free end of the rope such that the bucket remains stationary?

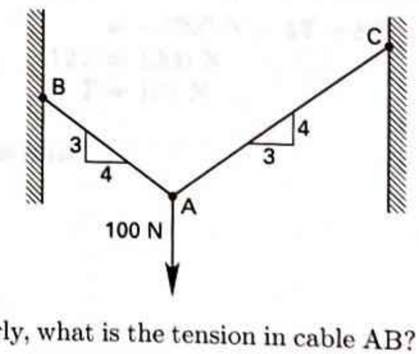
(A) 560 N to 1360 N

(B) 670 N to 1440 N

(C) 720 N to 1360 N

(D) 720 N to 1510 N

4. The two cables shown carry a 100 N vertical load.



Most nearly, what is the tension in cable AB?

(A) 40 N

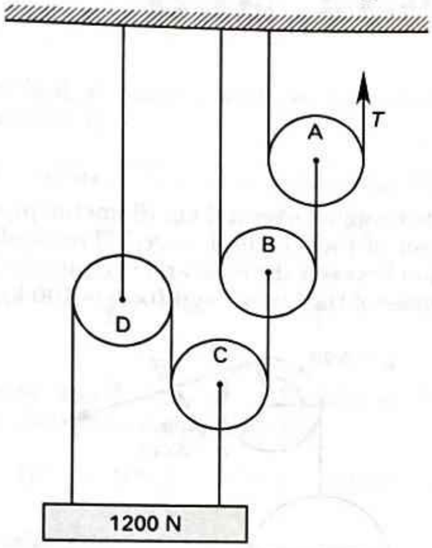
(B) 50 N

(C) 60 N

(D) 80 N

Statics

5. Most nearly, what is the tension, T, that must be applied to pulley A to lift the 1200 N weight?



(A) 75 N

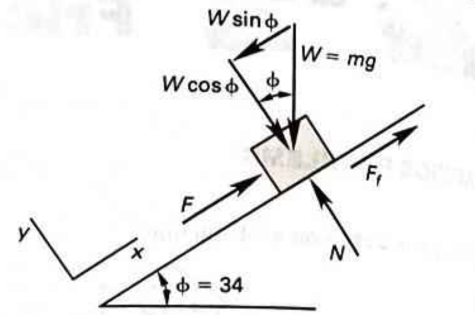
(B) 100 N

(C) 300 N

(D) 400 N

SOLUTIONS

1. Choose coordinate axes parallel and perpendicular to the incline.



Calculate the sum of forces. The force of friction opposes motion, so it acts in coordination with the force added to keep the block from sliding.

$$\begin{aligned}
\sum F_x &= 0 = F + F_f - W\sin\phi \\
F &= W\sin\phi - F_f \\
&= mg\sin\phi - \mu_s N \\
&= mg\sin\phi - \mu_s mg\cos\phi \\
&= mg(\sin\phi - \mu\cos\phi) \\
&= (2\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\sin 34° - 0.2\cos 34°) \\
&= 7.7\text{ N}
\end{aligned}$$

The answer is (A).

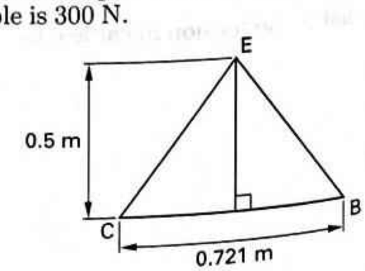
2. The length of the diagonal is

$$\text{BC} = \sqrt{(0.4\text{ m})^2 + (0.6\text{ m})^2} = 0.721\text{ m}$$

The cable length is

$$\text{BE} = \sqrt{\left(\frac{0.721\text{ m}}{2}\right)^2 + (0.5\text{ m})^2} = 0.616\text{ m}$$

There is nothing to balance the component force in the direction from point A to the opposite corner, so the force in cable AE is zero. Cables BE and CE each carry half of the box weight. The vertical component of force in each cable is 300 N.



Statics

By similar triangles, the tensile force in each cable is

$$T = \frac{(300\ \text{N})(0.616\ \text{m})}{0.5\ \text{m}} = 370\ \text{N}$$

The answer is (C).

3. The angle of wrap is

$$\theta = (90° + 20°)\left(\frac{2\pi\ \text{rad}}{360°}\right) = 1.92\ \text{rad}$$

(This must be expressed in radians.)

The tensile force in the rope due to the bucket's mass is

$$\begin{aligned} F &= mg = (100\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 981\ \text{N} \end{aligned}$$

The free end of the rope can be on either the tight or loose side. These two options define the range of force that will keep the bucket stationary.

The ratio of tight-side to loose-side forces is

$$\begin{aligned} \frac{F_1}{F_2} &= e^{\mu\theta} = e^{(0.20)(1.92\ \text{rad})} \\ &= 1.468 \end{aligned}$$

Multiply and divide the bucket-end tension by this ratio.

$$\text{minimum tension: } \frac{981\ \text{N}}{1.468} = 668\ \text{N} \quad (670\ \text{N})$$

$$\text{maximum tension: } (1.468)(981\ \text{N}) = 1440\ \text{N}$$

The answer is (B).

4. Recognize that the orientations of both cables are defined by 3-4-5 triangles.

The equilibrium condition for horizontal forces at point A is

$$\begin{aligned} F_x &= T_{\text{AC}} - T_{\text{AB}} = 0 \\ \tfrac{3}{5}T_{\text{AC}} &- \tfrac{4}{5}T_{\text{AB}} = 0 \\ T_{\text{AC}} &= \tfrac{4}{3}T_{\text{AB}} \end{aligned}$$

The equilibrium condition for vertical forces at point A is

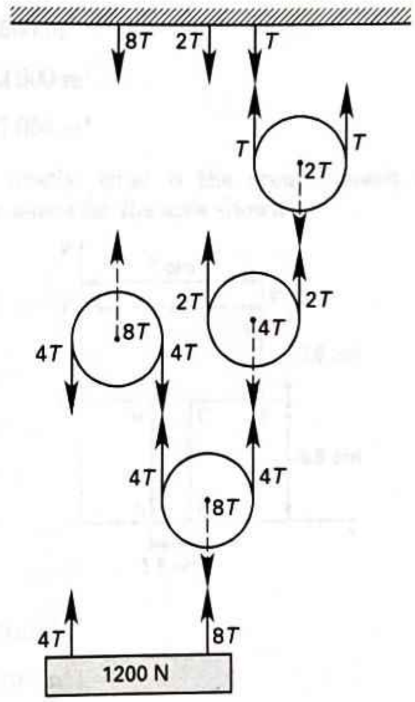
$$\begin{aligned} F_y &= T_{\text{AB}} + T_{\text{AC}} - 100\ \text{N} = 0 \\ \tfrac{3}{5}T_{\text{AB}} &+ \tfrac{4}{5}T_{\text{AC}} - 100\ \text{N} = 0 \end{aligned}$$

Substitute $(4/3)T_{\text{AB}}$ for T_{AC}.

$$\begin{aligned} \tfrac{3}{5}T_{\text{AB}} + \left(\frac{4}{5}\right)\left(\frac{4}{3}\right)T_{\text{AB}} &= 100\ \text{N} \\ \left(\frac{3}{5} + \frac{16}{15}\right)T_{\text{AB}} &= 100\ \text{N} \\ T_{\text{AB}} &= 60\ \text{N} \end{aligned}$$

The answer is (C).

5. The free bodies of the system are shown.



$$\begin{aligned} \sum F_y &= 0 \\ &= -1200\ \text{N} + 4T + 8T \\ 12T &= 1200\ \text{N} \\ T &= 100\ \text{N} \end{aligned}$$

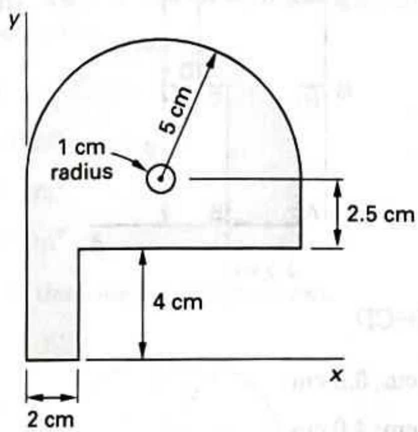
The answer is (B).

Statics

25 Centroids and Moments of Inertia

PRACTICE PROBLEMS

1. Refer to the complex shape shown.



What is most nearly the moment of inertia about the x-axis?

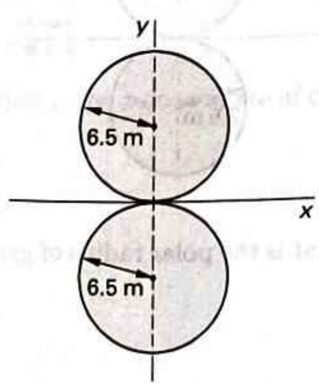
(A) 1500 cm^4

(B) 3400 cm^4

(C) 3600 cm^4

(D) 5200 cm^4

2. Refer to the composite plane areas shown.



What is the approximate polar moment of inertia about the composite centroid?

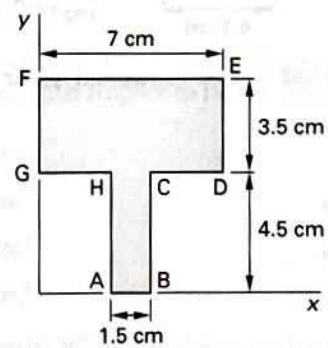
(A) 2400 m^4

(B) 5500 m^4

(C) 12 000 m^4

(D) 17 000 m^4

3. Most nearly, what is the area moment of inertia about the x-axis for the area shown?



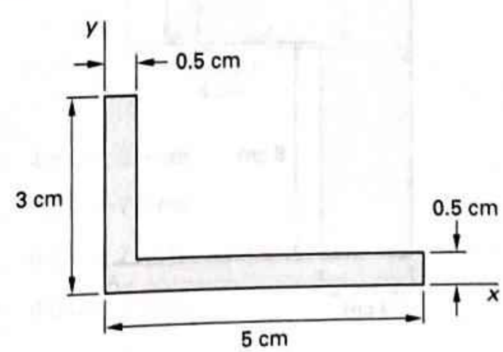
(A) 89 cm^4

(B) 170 cm^4

(C) 510 cm^4

(D) 1000 cm^4

4. Refer to the cross section of the angle shown.



Statics

Most nearly, what is the x-coordinate of the centroid of the area?

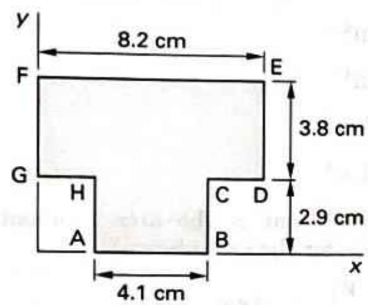
(A) 1.56 cm

(B) 1.66 cm

(C) 1.75 cm

(D) 1.80 cm

5. The centroidal moment of inertia about the x-axis for the area shown is 142.41 cm^4.



Most nearly, what is the centroidal polar moment of inertia?

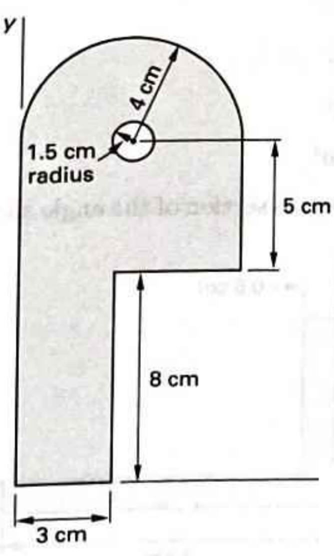
(A) 79 cm^4

(B) 110 cm^4

(C) 330 cm^4

(D) 450 cm^4

6. Refer to the complex shape shown.



Most nearly, what is the y-coordinate of the centroid?

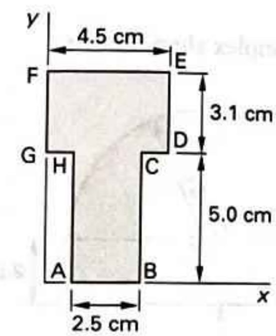
(A) 5.5 cm

(B) 7.3 cm

(C) 9.7 cm

(D) 11 cm

7. Most nearly, what are the x- and y-coordinates of the centroid of the perimeter line for the area shown?



NOTE: GH=CD

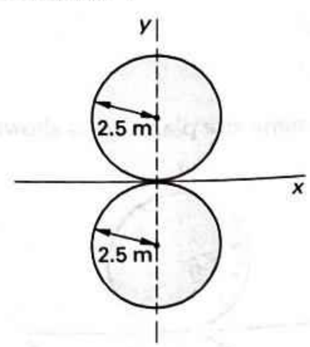
(A) 1.0 cm; 3.8 cm

(B) 1.0 cm; 4.0 cm

(C) 2.3 cm; 4.4 cm

(D) 2.3 cm; 4.8 cm

8. Refer to the composite plane areas shown. The polar moment of inertia is $3\pi r^4$.



Most nearly, what is the polar radius of gyration?

(A) 2.5 m

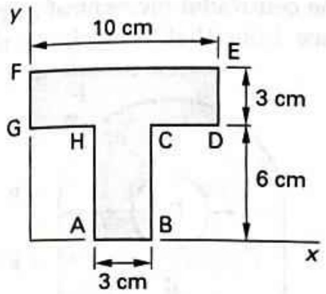
(B) 2.7 m

(C) 2.9 m

(D) 3.1 m

Statics

9. The y-coordinate of the centroid of the area shown is 5.8125 cm.



Most nearly, what is the centroidal moment of inertia with respect to the x-axis?

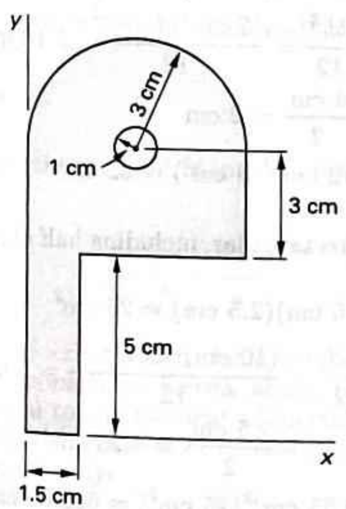
(A) 82 cm^4

(B) 100 cm^4

(C) 220 cm^4

(D) 300 cm^4

10. Refer to the complex shape shown.



Most nearly, what is the x-coordinate of the centroid?

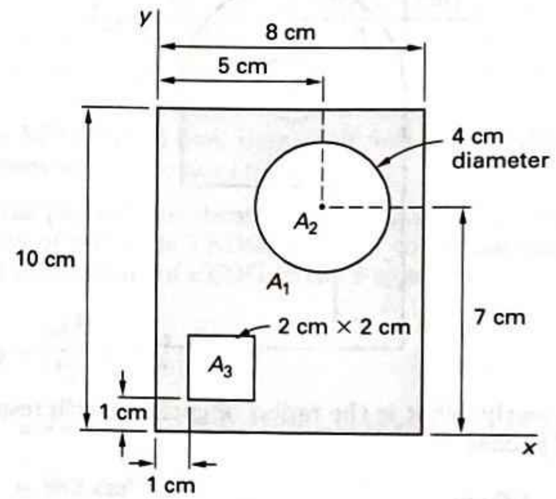
(A) 2.4 cm

(B) 2.5 cm

(C) 2.8 cm

(D) 3.2 cm

11. The centroidal moment of inertia of area A_2 with respect to the composite centroidal x-axis is 73.94 cm^4. The y-coordinate of the composite centroid is 4.79 cm. The moment of inertia of area A_3 with respect to the composite centroidal x-axis is 32.47 cm^4.



Most nearly, what is the moment of inertia of the entire composite area with respect to its centroidal x-axis?

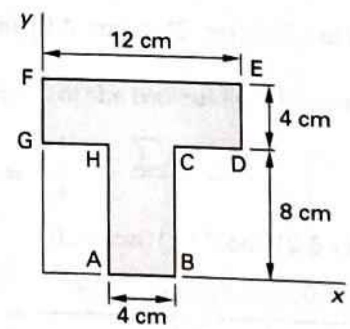
(A) 350 cm^4

(B) 460 cm^4

(C) 480 cm^4

(D) 560 cm^4

12. Most nearly, what are the x- and y-coordinates of the centroid of the area shown?



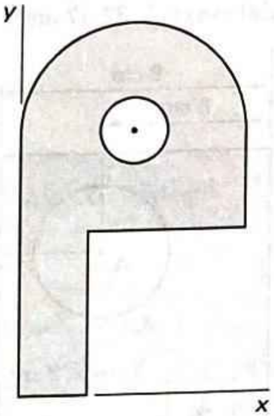
(A) 4.8 cm; 6.8 cm

(B) 6.0 cm; 7.2 cm

(C) 6.0 cm; 7.6 cm

(D) 6.0 cm; 8.0 cm

13. Refer to the complex shape shown. The moment of inertia about the y-axis is 352 cm^4, and the total area of the shape is 36.5 cm^2.



Most nearly, what is the radius of gyration with respect to the y-axis?

(A) 1.9 cm

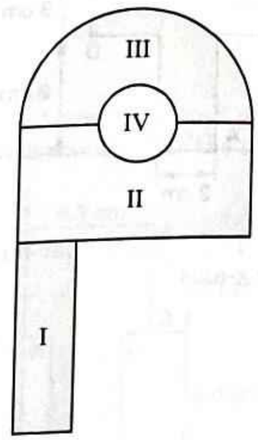
(B) 3.1 cm

(C) 3.3 cm

(D) 4.0 cm

SOLUTIONS

1. Calculate the centroidal moment of inertia, the area, and the distance from that centroid to the x-axis for each region.



For region I (rectangular),

$$A = (2\ \text{cm})(4\ \text{cm}) = 8\ \text{cm}^2$$

$$I_c = \frac{bh^3}{12} = \frac{(2\ \text{cm})(4\ \text{cm})^3}{12} = 10.67\ \text{cm}^4$$

$$d = \frac{4\ \text{cm}}{2} = 2\ \text{cm}$$

$$d^2A = (2\ \text{cm})^2(8\ \text{cm}^2) = 32\ \text{cm}^4$$

For region II (rectangular, including half of region IV),

$$A = (10\ \text{cm})(2.5\ \text{cm}) = 25\ \text{cm}^2$$

$$I_c = \frac{bh^3}{12} = \frac{(10\ \text{cm})(2.5\ \text{cm})^3}{12} = 13.02\ \text{cm}^4$$

$$d = 4\ \text{cm} + \frac{2.5\ \text{cm}}{2} = 5.25\ \text{cm}$$

$$d^2A = (5.25\ \text{cm})^2(25\ \text{cm}^2) = 689.06\ \text{cm}^4$$

For region III (semicircular, including half of region IV),

$$A = \frac{\pi r^2}{2} = \frac{\pi(5\ \text{cm})^2}{2} = \frac{25\pi}{2}$$

$$I_c = 0.1098r^4 = (0.1098)(5\ \text{cm})^4 = 68.63\ \text{cm}^4$$

$$\begin{aligned} d &= 4\ \text{cm} + 2.5\ \text{cm} + \frac{4r}{3\pi} \\ &= 6.5\ \text{cm} + \frac{(4)(5\ \text{cm})}{3\pi} \\ &= 6.5\ \text{cm} + \frac{20}{3\pi}\ \text{cm} \end{aligned}$$

$$d^2A = \left(6.5\ \text{cm} + \frac{20}{3\pi}\ \text{cm}\right)^2\left(\frac{25\pi}{2}\ \text{cm}^2\right) = 2919.33\ \text{cm}^4$$

For region IV (circular),

$$A = \pi r^2 = \pi(1\text{ cm})^2 = \pi\text{ cm}^2$$

$$\begin{aligned} I_c &= \frac{\pi r^4}{4} = \frac{\pi(1\text{ cm})^4}{4} \\ &= \frac{\pi}{4}\text{ cm}^4 \end{aligned}$$

$$\begin{aligned} d &= 4\text{ cm} + 2.5\text{ cm} \\ &= 6.5\text{ cm} \end{aligned}$$

$$\begin{aligned} d^2A &= (6.5\text{ cm})^2(\pi\text{ cm}^2) \\ &= 132.73\text{ cm}^4 \end{aligned}$$

Use the parallel axis theorem for each shape.

$$\begin{aligned} I_x &= \sum I_{x_c} + \sum d^2 A \\ &= 10.67\text{ cm}^4 + 32\text{ cm}^4 \\ &\quad + 13.02\text{ cm}^4 + 689.06\text{ cm}^4 \\ &\quad + 68.63\text{ cm}^4 + 2919.33\text{ cm}^4 \\ &\quad - \frac{\pi}{4}\text{ cm}^4 - 132.73\text{ cm}^4 \\ &= 3599.2\text{ cm}^4 \quad (3600\text{ cm}^4) \end{aligned}$$

The answer is (C).

2. For a circle, the moment of inertia about the circle's centroid,

$$I_{x_c} = I_{y_c} = \frac{\pi r^4}{4}$$

Use the parallel axis theorem for a composite area. In this case the moment of inertia about the composite centroid is equal to the moment of inertia about the x axis, Ix. While the moment of inertia about the y-axis remains the same as Iyc.

$$I_x = (2)\left(\frac{\pi r^4}{4} + (\pi r^2)r^2\right) = \frac{5\pi r^4}{2}$$

$$I_{y_c} = (2)\left(\frac{\pi r^4}{4}\right) = \frac{\pi r^4}{2}$$

$$\begin{aligned} J_c &= I_x + I_{y_c} = \frac{5\pi r^4}{2} + \frac{\pi r^4}{2} \\ &= 3\pi r^4 = 3\pi(6.5\text{ m})^4 \\ &= 16\,824\text{ m}^4 \quad (17\,000\text{ m}^4) \end{aligned}$$

The answer is (D).

3. Use the formula for the moment of inertia about an edge for a rectangular shape.

For rectangle HCBA,

$$\begin{aligned} I_{x,1} &= \frac{bh^3}{3} = \frac{(1.5\text{ cm})(4.5\text{ cm})^3}{3} \\ &= 45.56\text{ cm}^4 \end{aligned}$$

($I_c = bh^3/12$ could have been used, but the parallel axis theorem would also have to be used.)

Use the parallel axis theorem to calculate the moment of inertia of rectangle FEDG. $d = 6.25$ cm is the distance from the centroid of FEDG to the x-axis.

$$\begin{aligned} I_{x,2} &= \frac{bh^3}{12} + Ad^2 \\ &= \frac{(7\text{ cm})(3.5\text{ cm})^3}{12} + (7\text{ cm})(3.5\text{ cm})(6.25\text{ cm})^2 \\ &= 982\text{ cm}^4 \end{aligned}$$

The moment of inertia for the total area is

$$\begin{aligned} I_x &= I_{x,1} + I_{x,2} \\ &= 45.56\text{ cm}^4 + 982\text{ cm}^4 \\ &= 1028\text{ cm}^4 \quad (1000\text{ cm}^4) \end{aligned}$$

The answer is (D).

4. The area of the shape is

$$A = (0.5\text{ cm})(2.5\text{ cm}) + (5\text{ cm})(0.5\text{ cm}) = 3.75\text{ cm}^2$$

The x-coordinate of the centroid of the area is

$$\begin{aligned} x_{ac} &= \frac{M_{ay}}{A} = \frac{\sum x_n a_n}{A} \\ &= \frac{(0.25\text{ cm})\big((0.5\text{ cm})(2.5\text{ cm})\big) + (2.5\text{ cm})\big((5\text{ cm})(0.5\text{cm})\big)}{3.75\text{ cm}^2} \\ &= 1.75\text{ cm} \end{aligned}$$

The answer is (C).

Statics

5. The shape is symmetrical about a vertical axis, so determine the location of the centroid by inspection. Determine the centroidal moment of inertia about the y-axis. Move the y-axis 4.1 cm to the right, so that the y-axis passes through the centroid. The centroidal y-axis passes through the centroid of the area, so the parallel axis theorem is not needed. Since the centroids of the individual rectangles coincide with the centroid of the composite area about the centroidal y-axis, I_{yc} is simply the sum of the moments of inertia of the individual areas.

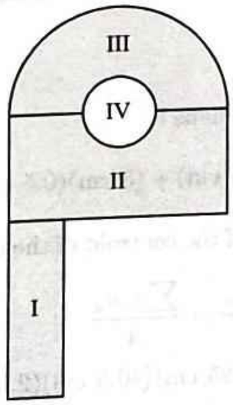
$$\begin{aligned} I_{y_c} &= \frac{(4.1 \text{ cm})^3(2.9 \text{ cm})}{12} + \frac{(8.2 \text{ cm})^3(3.8 \text{ cm})}{12} \\ &= 191.26 \text{ cm}^4 \end{aligned}$$

The centroidal polar moment of inertia is

$$\begin{aligned} J_c &= I_{x_c} + I_{y_c} \\ &= 142.41 \text{ cm}^4 + 191.26 \text{ cm}^4 \\ &= 333.7 \text{ cm}^4 \quad (330 \text{ cm}^4) \end{aligned}$$

The answer is (C).

6. Separate the shape into regions. Calculate the area and locate the centroid for each region.



For region I (rectangular),

$$\begin{aligned} A &= (3 \text{ cm})(8 \text{ cm}) \\ &= 24 \text{ cm}^2 \\ y_c &= \frac{8 \text{ cm}}{2} \\ &= 4 \text{ cm} \end{aligned}$$

For region II (rectangular, including half of region IV),

$$\begin{aligned} A &= (2)(4 \text{ cm})(5 \text{ cm}) \\ &= 40 \text{ cm}^2 \\ y &= 8 \text{ cm} + \frac{5 \text{ cm}}{2} \\ &= 10.5 \text{ cm} \end{aligned}$$

For region III (semicircular, including half of region IV),

$$\begin{aligned} A &= \frac{\pi r^2}{2} = \frac{\pi(4 \text{ cm})^2}{2} \\ &= 8\pi \text{ cm}^2 \\ y &= 8 \text{ cm} + 5 \text{ cm} + \frac{4r}{3\pi} \\ &= 13 \text{ cm} + \frac{(4)(4 \text{ cm})}{3\pi} \\ &= 13 \text{ cm} + \frac{16}{3\pi} \text{ cm} \end{aligned}$$

For region IV (circular),

$$\begin{aligned} A &= \pi r^2 = \pi(1.5 \text{ cm})^2 \\ &= 2.25\pi \text{ cm}^2 \\ y_c &= 8 \text{ cm} + 5 \text{ cm} = 13 \text{ cm} \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\sum y_{c,n} A_n}{\sum A_n} \\ &= \frac{\begin{array}{c} (4 \text{ cm})(24 \text{ cm}^2) + (10.5 \text{ cm})(40 \text{ cm}^2) \\ + \left(13 \text{ cm} + \frac{16}{3\pi} \text{ cm}\right)(8\pi \text{ cm}^2) \\ - (13 \text{ cm})(2.25\pi \text{ cm}^2) \end{array}}{24 \text{ cm}^2 + 40 \text{ cm}^2 + 8\pi \text{ cm}^2 - 2.25\pi \text{ cm}^2} \\ &= \frac{793.5 \text{ cm}^3}{82.06 \text{ cm}^2} \\ &= 9.67 \text{ cm} \quad (9.7 \text{ cm}) \end{aligned}$$

The answer is (C).

7. Calculate the centroid of the perimeter using the same concept of calculating the centroid of an area, but use lengths in lieu of areas. The total length of the perimeter is

$$\begin{aligned} L &= 2.5\text{ cm} + 5.0\text{ cm} + 1.0\text{ cm} + 3.1\text{ cm} \\ &\quad + 4.5\text{ cm} + 3.1\text{ cm} + 1.0\text{ cm} + 5.0\text{ cm} \\ &= 25.2\text{ cm} \end{aligned}$$

Starting with line segment FG and working counterclockwise, the x-coordinate of the centroid of the perimeter is

$$x_c = \frac{\sum x_i L_i}{L}$$

$$= \frac{\begin{array}{l}(0\text{ cm})(3.1\text{ cm}) + (0.5\text{ cm})(1.0\text{ cm}) \\ \quad + (1.0\text{ cm})(5.0\text{ cm}) \\ \quad + (2.25\text{ cm})(2.5\text{ cm}) \\ \quad + (3.5\text{ cm})(5.0\text{ cm}) \\ \quad + (4\text{ cm})(1\text{ cm}) \\ \quad + (4.5\text{ cm})(3.1\text{ cm}) + (2.25\text{ cm})(4.5\text{ cm})\end{array}}{25.2\text{ cm}}$$

$$= 2.25\text{ cm} \quad (2.3\text{ cm})$$

The y-coordinate is

$$y_c = \frac{\sum y_i L_i}{L}$$

$$= \frac{\begin{array}{l}(0\text{ cm})(2.5\text{ cm}) + (2)(2.5\text{ cm})(5.0\text{ cm}) \\ \quad + (2)(5.0\text{ cm})(1\text{ cm}) + (2)(6.55\text{ cm})(3.1\text{ cm}) \\ \quad + (8.1\text{ cm})(4.5\text{ cm})\end{array}}{25.2\text{ cm}}$$

$$= 4.447\text{ cm} \quad (4.4\text{ cm})$$

The answer is (C).

8. The polar radius of gyration is

$$\begin{aligned} r_p &= \sqrt{\frac{J}{A}} = \sqrt{\frac{3\pi r^4}{2\pi r^2}} = r\sqrt{\frac{3}{2}} \\ &= (2.5\text{ m})\sqrt{\frac{3}{2}} \\ &= 3.1\text{ m} \end{aligned}$$

The answer is (D).

9. Use the parallel axis theorem to find the centroidal moment of inertia of each rectangular area.

$$\text{HCBA} = \text{area } 1 = (3\text{ cm})(6\text{ cm}) = 18\text{ cm}^2$$

$$\text{FEDG} = \text{area } 2 = (10\text{ cm})(3\text{ cm}) = 30\text{ cm}^2$$

$$I_{X_c} = (I_{C,1} + A_1 d_1^2) + (I_{C,2} + A_2 d_2^2)$$

$$I_C = \text{centroidal moment} = \frac{bh^3}{12}\ (\text{units: cm}^4)$$

$$A = \text{area of rectangle}\ (\text{units: cm}^2)$$

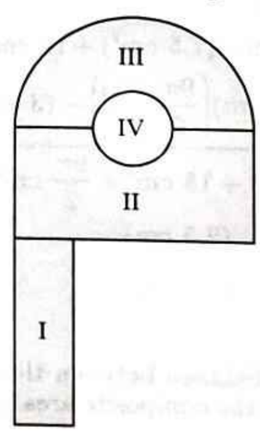
$$d = \begin{array}{l}\text{is the distance between the centroidal axis} \\ \text{and the second, parallel axis (units: cm)}\end{array}$$

$$= \left(\begin{array}{l}\dfrac{(3\text{ cm})(6\text{ cm})^3}{12} \\ \quad + (18\text{ cm}^2)(5.8125\text{ cm} - 3.0\text{ cm})^2\end{array}\right) + \left(\begin{array}{l}\dfrac{(10\text{ cm})(3\text{ cm})^3}{12} \\ \quad + (30\text{ cm}^2)(7.5\text{ cm} - 5.8125\text{ cm})^2\end{array}\right)$$

$$= 304.3\text{ cm}^4 \quad (300\text{ cm}^4)$$

The answer is (D).

10. Separate the shape into regions. Calculate the area and locate the centroid for each region.



For region I (rectangular),

$$\begin{aligned} A &= (1.5\text{ cm})(5\text{ cm}) \\ &= 7.5\text{ cm}^2 \\ x_c &= \frac{1.5\text{ cm}}{2} \\ &= 0.75\text{ cm} \end{aligned}$$

Statics

For region II (rectangular, including half of region IV),

$$A = (3 \text{ cm})(3 \text{ cm} + 3 \text{ cm}) = 18 \text{ cm}^2$$

$$x_c = \frac{(2)(3 \text{ cm})}{2} = 3 \text{ cm}$$

For region III (semicircular, including half of region IV),

$$A = \frac{\pi r^2}{2} = \frac{\pi(3 \text{ cm})^2}{2} = \frac{9\pi}{2} \text{ cm}^2$$

$$x_c = 3 \text{ cm} \quad \text{[by inspection]}$$

For region IV (circular),

$$A = \pi r^2 = \pi(1 \text{ cm})^2 = \pi \text{ cm}^2$$

$$x_c = 3 \text{ cm} \quad \text{[by inspection]}$$

Calculate the centroid for the whole shape. Note that region IV is to be subtracted from regions II and III.

$$x_c = \frac{\sum x_{c,n}A_n}{\sum A_n} = \frac{\begin{array}{c}(0.75 \text{ cm})(7.5 \text{ cm}^2) + (3 \text{ cm})(18 \text{ cm}^2) \\ + (3 \text{ cm})\left(\frac{9\pi}{2} \text{ cm}^2\right) - (3 \text{ cm})(\pi \text{ cm}^2)\end{array}}{7.5 \text{ cm}^2 + 18 \text{ cm}^2 + \frac{9\pi}{2} \text{ cm}^2 - \pi \text{ cm}^2} = 2.54 \text{ cm} \quad (2.5 \text{ cm})$$

The answer is (B).

11. The vertical distance between the centroidal location of area A_1 and the composite area's centroid is

$$d_1 = \frac{h}{2} - y_c = \frac{10 \text{ cm}}{2} - 4.79 \text{ cm}$$

The moment of inertia is

$$I_{x_c} = \left(I_{x_c,1} + A_1 d_1^2\right) - I_{x_c',2} - I_{x_c',3}$$

$$= \left(\begin{array}{c}\frac{(8 \text{ cm})(10 \text{ cm})^3}{12} \\ + (8 \text{ cm})(10 \text{ cm})\left(\frac{10 \text{ cm}}{2} - 4.79 \text{ cm}\right)^2\end{array}\right) - 73.94 \text{ cm}^4 - 32.47 \text{ cm}^4$$

$$= 563.8 \text{ cm}^4 \quad (560 \text{ cm}^4)$$

The answer is (D).

12. Divide the area into two rectangles, HCBA and FEDG. Their areas are

$$A_1 = (4 \text{ cm})(8 \text{ cm}) = 32 \text{ cm}^2$$

$$A_2 = (12 \text{ cm})(4 \text{ cm}) = 48 \text{ cm}^2$$

The total area is

$$A = A_1 + A_2 = 32 \text{ cm}^2 + 48 \text{ cm}^2 = 80 \text{ cm}^2$$

By inspection, the x-coordinates of the centroids of the rectangles are $x_{c,1} = x_{c,2} = 6$ cm. The x-coordinate of the centroid of the total area is

$$x_c = \frac{\sum x_{c,n}A_n}{A} = \frac{(6 \text{ cm})(32 \text{ cm}^2) + (6 \text{ cm})(48 \text{ cm}^2)}{80 \text{ cm}^2} = 6.0 \text{ cm}$$

By inspection, the y-coordinates of the rectangles are $y_{c,1} = 4$ cm and $y_{c,2} = 10$ cm. The y-coordinate of the centroid of the total area is

$$y_c = \frac{\sum y_{c,n}A_n}{A} = \frac{(4 \text{ cm})(32 \text{ cm}^2) + (10 \text{ cm})(48 \text{ cm}^2)}{80 \text{ cm}^2} = 7.6 \text{ cm}$$

The answer is (C).

13. The radius of gyration with respect to the y-axis is

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{352\ \text{cm}^4}{36.5\ \text{cm}^2}}$$
$$= 3.1\ \text{cm}$$

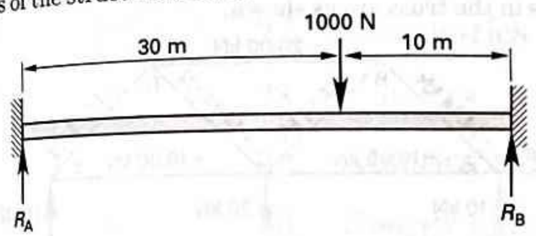
The answer is (B).

Statics

26 Indeterminate Statics

PRACTICE PROBLEMS

1. What are the approximate vertical reactions at the ends of the structure shown?



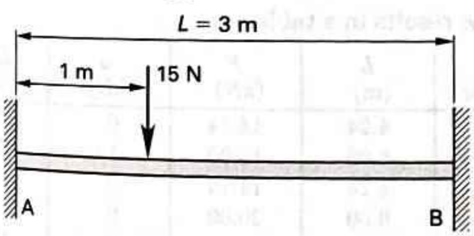
(A) $R_A = 120$ N; $R_B = 630$ N

(B) $R_A = 160$ N; $R_B = 840$ N

(C) $R_A = 630$ N; $R_B = 120$ N

(D) $R_A = 840$ N; $R_B = 160$ N

2. What is most nearly the fixed-end moment at point A when a 15 N load is applied?



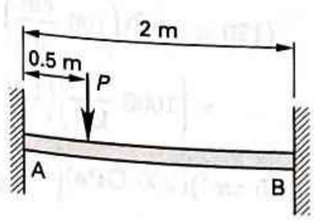
(A) 3.3 N·m

(B) 6.7 N·m

(C) 33 N·m

(D) 45 N·m

3. For the beam shown, the fixed-end moment at point A cannot exceed 2 N·m.



What is most nearly the maximum load that can be applied to the beam?

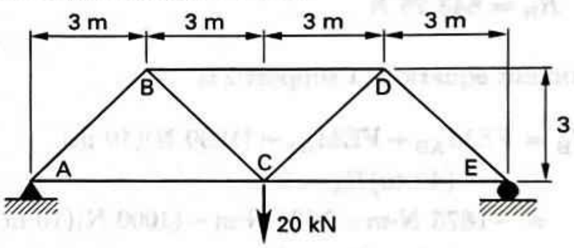
(A) 1.8 N

(B) 2.0 N

(C) 3.6 N

(D) 7.1 N

4. The truss shown is constructed of members having a modulus of elasticity of 200 GPa. The cross-sectional area of each member is 5 cm^2.



What is most nearly the horizontal displacement of the roller at location E due to the applied load?

(A) 0.1 cm

(B) 4.2 cm

(C) 6.0 cm

(D) 8.5 cm

SOLUTIONS

1. The total length is $L = a + b = 30\text{ m} + 10\text{ m} = 40\text{ m}$. Take clockwise moments to be positive.

$$\begin{aligned}\text{FEM}_{\text{AB}} &= \frac{-Pab^2}{L^2} = \frac{-(1000\text{ N})(30\text{ m})(10\text{ m})^2}{(40\text{ m})^2}\\ &= -1875\text{ N}\cdot\text{m}\\ \text{FEM}_{\text{BA}} &= \frac{Pa^2b}{L^2} = \frac{(1000\text{ N})(30\text{ m})^2(10\text{ m})}{(40\text{ m})^2}\\ &= 5625\text{ N}\cdot\text{m}\end{aligned}$$

The moment equation at support A is

$$\begin{aligned}\sum M_{\text{A}} &= \text{FEM}_{\text{AB}} + \text{FEM}_{\text{BA}}\\ &\quad + (1000\text{ N})(30\text{ m}) - (40\text{ m})R_{\text{B}} = 0\\ &= -1875\text{ N}\cdot\text{m} + 5625\text{ N}\cdot\text{m}\\ &\quad + (1000\text{ N})(30\text{ m}) - (40\text{ m})R_{\text{B}} = 0\\ R_{\text{B}} &= 843.75\text{ N}\end{aligned}$$

The moment equation at support 2 is

$$\begin{aligned}\sum M_{\text{B}} &= \text{FEM}_{\text{AB}} + \text{FEM}_{\text{BA}} - (1000\text{ N})(10\text{ m})\\ &\quad + (40\text{ m})R_{\text{A}} = 0\\ &= -1875\text{ N}\cdot\text{m} + 5625\text{ N}\cdot\text{m} - (1000\text{ N})(10\text{ m})\\ &\quad + (40\text{ m})R_{\text{A}} = 0\\ R_{\text{A}} &= 156.25\text{ N}\end{aligned}$$

The force equilibrium requirement is

$$\begin{aligned}R_{\text{A}} + R_{\text{B}} &= 1000\text{ N}\\ 156.25\text{ N} + 843.75\text{ N} &= 1000\text{ N} \quad \text{[check]}\end{aligned}$$

$$\begin{aligned}R_{\text{A}} &= 156.25\text{ N};\ R_{\text{B}} = 843.75\text{ N}\\ &(R_{\text{A}} = 160\text{ N};\ R_{\text{B}} = 840\text{ N})\end{aligned}$$

The answer is (B).

2.

$$\begin{aligned}a &= 1\text{ m}\\ b &= L - a = 3\text{ m} - 1\text{ m} = 2\text{ m}\end{aligned}$$

The fixed-end moment at point A is

$$\begin{aligned}\text{FEM}_{\text{AB}} &= \frac{Pab^2}{L^2} = \frac{(15\text{ N})(1\text{ m})(2\text{ m})^2}{(3\text{ m})^2}\\ &= 6.667\text{ N}\cdot\text{m} \quad (6.7\text{ N}\cdot\text{m})\end{aligned}$$

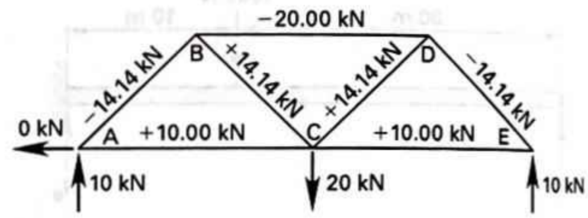
The answer is (B).

3. The maximum load that can be applied to the beam is

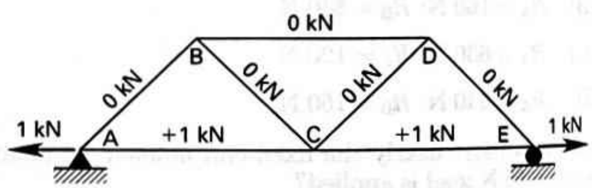
$$\begin{aligned}\text{FEM}_{\text{AB}} &= \frac{Pab^2}{L^2}\\ P &= \frac{\text{FEM}_{\text{AB}}L^2}{ab^2} = \frac{(2\text{ N}\cdot\text{m})(2\text{ m})^2}{(0.5\text{ m})(1.5\text{ m})^2}\\ &= 7.11\text{ N} \quad (7.1\text{ N})\end{aligned}$$

The answer is (D).

4. Use the unit load method. The reaction and member forces in the truss are as shown.



Remove the applied loads and place a unit load at joint E.



Place the results in a table.

member	L (m)	F (kN)	u (kN)	LFu (m·kN²)
AB	4.24	14.14	0	0
AC	6.00	10.00	1.00	60.00
BC	4.24	14.14	0	0
BD	6.00	20.00	0	0
CD	4.24	14.14	0	0
CE	6.00	10.00	1.00	60.00
DE	4.24	14.14	0	0
			total	120.00

The displacement at joint E is

$$f\delta = \sum \frac{SuL}{AE}$$

$$(1\text{ kN})\delta = \frac{(120\text{ m}\cdot\text{kN}^2)\left(100\ \frac{\text{cm}}{\text{m}}\right)^2 \times \left(1000\ \frac{\text{N}}{\text{kN}}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)}{(5\text{ cm}^2)(200\text{ GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)}$$

$$\delta = 0.12\text{ cm} \quad (0.1\text{ cm})$$

The answer is (A).

27 Kinematics

PRACTICE PROBLEMS

1. A particle's curvilinear motion is represented by the equation $s(t) = 20t + 4t^2 - 3t^3$. What is most nearly the particle's initial velocity?

(A) 20 m/s

(B) 25 m/s

(C) 30 m/s

(D) 32 m/s

2. A vehicle is traveling at 72 km/h when the driver sees a traffic light in an intersection 500 m ahead turn red. The light's red cycle duration is 40 s. The driver wants to enter the intersection without stopping the vehicle, just as the light turns green. If the vehicle decelerates at a constant rate, what will be its approximate speed when the light turns green?

(A) 11 km/h

(B) 18 km/h

(C) 25 km/h

(D) 31 km/h

3. A projectile has an initial velocity of 110 m/s and a launch angle of 20° from the horizontal. The surrounding terrain is level, and air friction is to be disregarded. What is most nearly the flight time of the projectile?

(A) 3.8 s

(B) 7.7 s

(C) 8.9 s

(D) 12 s

4. A particle's position is defined by

$$\mathbf{s}(t) = 2\sin t\mathbf{i} + 4\cos t\mathbf{j} \quad [t \text{ in radians}]$$

What is most nearly the magnitude of the particle's velocity when $t = 4$ rad?

(A) 2.6

(B) 2.7

(C) 3.3

(D) 4.1

5. A roller coaster train climbs a hill with a constant gradient. During a 10 s period, the acceleration is constant at 0.4 m/s^2, and the average velocity of the train is 40 km/h. What is most nearly the velocity of the train after 10 s?

(A) 9.1 m/s

(B) 11 m/s

(C) 13 m/s

(D) 15 m/s

6. Choose the equation that best represents a rigid body or particle under constant acceleration.

(A) $a = 9.81 \text{ m/s}^2 + \mathrm{v}_0/t$

(B) $\mathrm{v} = a_0(t - t_0) + \mathrm{v}_0$

(C) $\mathrm{v} = \mathrm{v}_0 + \int_0^t a(t)\,dt$

(D) $a = \mathrm{v}_t^2/r$

7. A particle's curvilinear motion is represented by the equation $s(t) = 40t + 5t^2 - 8t^3$. What is most nearly the initial acceleration of the particle?

(A) 2 m/s^2

(B) 3 m/s^2

(C) 8 m/s^2

(D) 10 m/s^2

8. The rotor of a steam turbine is rotating at 7200 rpm when the steam supply is suddenly cut off. The rotor decelerates at a constant rate and comes to rest after 5 min. What is most nearly the angular deceleration of the rotor?

(A) $0.40\ \text{rad/s}^2$

(B) $2.5\ \text{rad/s}^2$

(C) $5.8\ \text{rad/s}^2$

(D) $16\ \text{rad/s}^2$

9. The angular position of a car traveling around a curve is described by the following function of time (in seconds).

$$\theta(t) = t^3 - 2t^2 - 4t + 10$$

What is most nearly the angular acceleration of the car at a time of 5 s?

(A) $4.0\ \text{rad/s}^2$

(B) $6.0\ \text{rad/s}^2$

(C) $26\ \text{rad/s}^2$

(D) $30\ \text{rad/s}^2$

10. A vehicle is traveling at 70 km/h when the driver sees a traffic light in the next intersection turn red. The intersection is 250 m away, and the light's red cycle duration is 15 s. What is most nearly the uniform deceleration that will put the vehicle in the intersection the moment the light turns green?

(A) $0.18\ \text{m/s}^2$

(B) $0.25\ \text{m/s}^2$

(C) $0.37\ \text{m/s}^2$

(D) $1.3\ \text{m/s}^2$

11. A projectile has an initial velocity of 85 m/s and a launch angle of 60° from the horizontal. The surrounding terrain is level, and air friction is to be disregarded. What is most nearly the horizontal distance traveled by the projectile?

(A) 80 m

(B) 400 m

(C) 640 m

(D) 1200 m

12. A particle's position is defined by

$$\mathbf{s}(t) = 15\sin t\,\mathbf{i} + 8.5\cos t\,\mathbf{j} \quad [t \text{ in radians}]$$

What is most nearly the magnitude of the particle's acceleration when $t = \pi$?

(A) 6.5

(B) 8.5

(C) 15

(D) 17

13. A particle's curvilinear motion is represented by the equation $s(t) = 30t - 8t^2 + 6t^3$. What is most nearly the minimum speed reached by the particle?

(A) 26 m/s

(B) 30 m/s

(C) 35 m/s

(D) 48 m/s

14. A projectile has an initial velocity of 80 m/s and a launch angle of 42° from the horizontal. The surrounding terrain is level, and air friction is to be disregarded. What is most nearly the maximum elevation achieved by the projectile?

(A) 72 m

(B) 150 m

(C) 350 m

(D) 620 m

Dynamics

SOLUTIONS

1. The initial velocity at $t=0$ is

$$\begin{aligned} v &= \frac{dr}{dt} = \frac{ds}{dt} = 20 + 8t - 9t^2 \\ &= 20 + (8)(0\text{ s}) - (9)(0\text{ s})^2 \\ &= 20\text{ m/s} \end{aligned}$$

The answer is (A).

2. Start with the time when the driver sees the signal turn red, $t_0 = 0$ s

Convert the initial speed of the car to m / s.

$$v_0 = \frac{\left(72\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{\text{km}}\right)}{\frac{3600\text{ s}}{\text{h}}} = 20\text{ m / s}$$

Determine the deceleration required by using the initial speed, the distance covered and the traffic light duration.

$$s = v_0 t + \frac{at^2}{2}$$

Solve for a and substitute the known values.

$$\begin{aligned} a = \frac{2(s - v_0 t)}{t^2} &= \frac{2\left(500\text{ m} - 20\frac{\text{m}}{\text{s}} \times 40\text{ s}\right)}{(40\text{ s})^2} \\ &= -0.375\text{ m / s}^2 \end{aligned}$$

Calculate the speed at the signal.

$$\begin{aligned} v_t = v_0 + at &= 20\frac{\text{m}}{\text{s}} + \left(-0.375\frac{\text{m}}{\text{s}^2}\right)(40\text{ s}) \\ &= 5\text{ m / s} \end{aligned}$$

Convert the speed to km / hr.

$$v_t = \left(5\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{\text{h}}\right) = 18\text{ km / h}$$

The answer is (B).

3. The vertical component of velocity is zero at the apex. Calculate the time to reach the apex.

$$\begin{aligned} v_y &= -gt + v_0 \sin(\theta) \\ 0 &= -\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)t + \left(110\,\frac{\text{m}}{\text{s}}\right)\sin 20^\circ \\ t &= 3.84\text{ s} \end{aligned}$$

The projectile takes the same amount of time to return to the ground from the apex as it took to reach the apex after launch. The total flight time is

$$t_{\text{total}} = (2)(3.84\text{ s}) = 7.67\text{ s} \quad (7.7\text{ s})$$

The answer is (B).

4. The velocity is

$$\begin{aligned} \mathbf{v}(t) &= \frac{ds(t)}{dt} = \frac{d}{dt}(2\sin t\mathbf{i} + 4\cos t\mathbf{j}) \\ &= 2\cos t\mathbf{i} - 4\sin t\mathbf{j} \end{aligned}$$

At $t = 4$ rad,

$$\begin{aligned} \mathbf{v}(4) &= 2\cos(4\text{ rad})\,\mathbf{i} - 4\sin(4\text{ rad})\mathbf{j} \\ &= -1.31\mathbf{i} - (-3.03)\mathbf{j} \\ |\mathbf{v}(4)| &= \sqrt{(-1.31)^2 + (3.03)^2} \\ &= 3.3 \end{aligned}$$

The answer is (C).

5. If the train travels for 10 s at an average velocity of 40 km/h, then the distance traveled in 10 s is

$$\begin{aligned} s(t) = v_{\text{ave}}t &= \frac{\left(40\,\frac{\text{km}}{\text{h}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right)(10\text{ s})}{3600\,\frac{\text{s}}{\text{h}}} \\ &= 111.1\text{ m} \end{aligned}$$

Dynamics

Rearrange the equation for distance as a function of initial velocity and acceleration, and solve for the initial velocity.

$$\begin{aligned} s(t) &= a_0(t-t_0)^2/2 + v_0(t-t_0) + s_0 \\ v_0 &= \frac{s(t) - s_0 - \frac{a_0(t-t_0)^2}{2}}{t-t_0} \\ &= \frac{111.1\ \text{m} - 0\ \text{m} - \frac{\left(0.4\ \frac{\text{m}}{\text{s}^2}\right)(10\ \text{s} - 0\ \text{s})^2}{2}}{10\ \text{s} - 0\ \text{s}} \\ &= 9.11\ \text{m/s} \end{aligned}$$

For an initial velocity of 9.11 m/s and an acceleration of 0.4 m/s^2 over 10 s, the final velocity after 10 s is

$$\begin{aligned} v_f &= v_0 + a_0 t \\ &= 9.11\ \frac{\text{m}}{\text{s}} + \left(0.4\ \frac{\text{m}}{\text{s}^2}\right)(10\ \text{s}) \\ &= 13.11\ \text{m/s} \quad (13\ \text{m/s}) \end{aligned}$$

The answer is (C).

6. Option A is an expression for acceleration that varies with time. Option C is an expression for velocity with a generalized time-varying acceleration. The expression in option D relates tangential and normal accelerations, respectively, along a curved path, to the tangential velocity. For a generalized curved path, these accelerations are not constant.

Option B is the expression for the velocity of a linear system under constant acceleration.

$$v(t) = a_0 \int dt = a_0(t - t_0) + v_0$$

The answer is (B).

7. The acceleration at $t=0$ is

$$\begin{aligned} a &= \frac{d^2r}{dt^2} = \frac{d^2s}{dt^2} = 10 - 48t \\ &= 10\ \text{m/s}^2 \end{aligned}$$

The answer is (D).

8. The angular deceleration (velocity) is

$$\begin{aligned} \omega_f &= \omega_0 - \alpha t \\ \alpha &= \frac{\omega_0 - \omega_f}{t} \\ &= \frac{\left(7200\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right) - 0\ \frac{\text{rad}}{\text{s}}}{(5\ \text{min})\left(60\ \frac{\text{s}}{\text{min}}\right)^2} \\ &= 2.51\ \text{rad/s}^2 \quad (2.5\ \text{rad/s}^2) \end{aligned}$$

The answer is (B).

9. The angular acceleration is

$$\begin{aligned} \alpha(t) &= \frac{d^2\theta}{dt^2} = 6t - 4 \\ \alpha(5) &= (6)(5\ \text{s}) - 4 = 26\ \text{rad/s}^2 \end{aligned}$$

The answer is (C).

10. Rearrange the equation for the distance traveled under a constant acceleration. Let the initial distance traveled equal 0 m, and the initial time equal 0 s.

$$\begin{aligned} s(t) &= a_0(t-t_0)^2/2 + v_0(t-t_0) + s_0 \\ a_0 &= \frac{(2)\big(-v_0(t-t_0) + s(t) - s_0\big)}{(t-t_0)^2} \\ &= \frac{(2)\left(\frac{\left(-70\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)(15\ \text{s} - 0\ \text{s})}{3600\ \frac{\text{s}}{\text{h}}} + 250\ \text{m} - 0\ \text{m}\right)}{(15\ \text{s} - 0\ \text{s})^2} \\ &= -0.37\ \text{m/s}^2 \quad (0.37\ \text{m/s}^2\ \text{deceleration}) \end{aligned}$$

Note: It is important that −70km/h be used in the equation. If the sign is not maintained a large deceleration value is calculated incorrectly.

The answer is (C).

11. Calculate the total flight time. The vertical component of velocity is zero at the apex.

$$\begin{aligned} v_y &= -gt + v_0 \sin(\theta) \\ 0 &= -\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t + \left(85\ \frac{\text{m}}{\text{s}}\right)\sin 60° \\ t &= 7.50\ \text{s} \end{aligned}$$

Dynamics

The projectile takes the same amount of time to return to the ground from the apex as it took to reach the apex after launch. The total flight time is

$$t = (2)(7.50\text{ s}) = 15.0\text{ s}$$

The horizontal distance traveled is

$$\begin{aligned} x &= \mathrm{v}_0 \cos(\theta) t + x_0 \\ &= \left(85\ \frac{\text{m}}{\text{s}}\right)\cos 60^\circ (15.0\text{ s}) + 0\text{ m} \\ &= 638\text{ m} \quad (640\text{ m}) \end{aligned}$$

The answer is (C).

12. The velocity is

$$\begin{aligned} \mathbf{v}(t) &= \frac{d\mathbf{s}(t)}{dt} = \frac{d}{dt}(15\sin t\mathbf{i} + 8.5\cos t\mathbf{j}) \\ &= 15\cos t\mathbf{i} - 8.5\sin t\mathbf{j} \\ \mathbf{a}(t) &= \frac{d\mathbf{v}(t)}{dt} = -15\sin t\mathbf{i} - 8.5\cos t\mathbf{j} \\ \mathbf{a}(\pi) &= -15\sin \pi\mathbf{i} - 8.5\cos \pi\mathbf{j} \\ &= 0\mathbf{i} + 8.5\mathbf{j} \\ |\mathbf{a}(\pi)| &= \sqrt{(0\mathbf{i})^2 + (8.5\mathbf{j})^2} \\ &= 8.5 \end{aligned}$$

The answer is (B).

13. The minimum of the velocity function is found by equating the derivative of the velocity function to zero and solving for t.

$$\begin{aligned} \mathrm{v}(t) &= \frac{ds}{dt} = \frac{d}{dt}(30t - 8t^2 + 6t^3) \\ &= 30 - 16t + 18t^2 \end{aligned}$$

$$\begin{aligned} \frac{d\mathrm{v}}{dt} &= \frac{d}{dt}(30 - 16t + 18t^2) = -16 + 36t = 0 \\ t &= 0.444\text{ s} \end{aligned}$$

$$\begin{aligned} \mathrm{v}_{\min} &= 30 - 16t + 18t^2 \\ &= 30 - (16)(0.444\text{ s}) + (18)(0.444\text{ s})^2 \\ &= 26.4\text{ m/s} \quad (26\text{ m/s}) \end{aligned}$$

The answer is (A).

14. The maximum elevation is achieved when the projectile is at the apex. The vertical component of velocity is zero at the apex. Calculate the time to reach the apex.

$$\begin{aligned} \mathrm{v}_y &= -gt + \mathrm{v}_0 \sin(\theta) \\ 0 &= -\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t + \left(80\ \frac{\text{m}}{\text{s}}\right)\sin 42^\circ \\ t &= 5.46\text{ s} \end{aligned}$$

The elevation at time t is

$$\begin{aligned} y &= -gt^2/2 + \mathrm{v}_0 \sin(\theta)t + y_0 \\ &= \frac{-\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5.46\text{ s})^2}{2} \\ &\quad + \left(80\ \frac{\text{m}}{\text{s}}\right)\sin 42^\circ (5.46\text{ s}) + 0\text{ m} \\ &= 146\text{ m} \quad (150\text{ m}) \end{aligned}$$

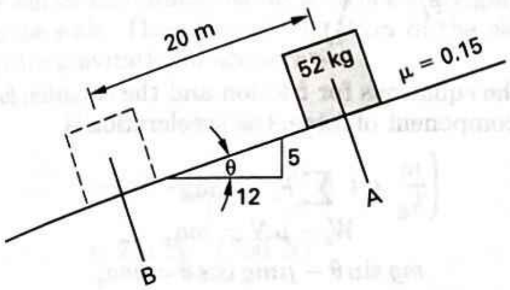
The answer is (B).

Dynamics

28 Kinetics

PRACTICE PROBLEMS

1. The 52 kg block shown starts from rest at position A and slides down the inclined plane to position B. The coefficient of friction between the block and the plane is $\mu = 0.15$.



What is most nearly the velocity of the block at position B?

(A) 2.4 m/s

(B) 4.1 m/s

(C) 7.0 m/s

(D) 9.8 m/s

2. A 5 kg block begins from rest and slides down an inclined plane. After 4 s, the block has a velocity of 6 m/s. The angle of inclination of the plane is 45°. Approximately how far has the block traveled after 4 s?

(A) 1.5 m

(B) 3.0 m

(C) 6.0 m

(D) 12 m

3. The elevator in a 20-story apartment building has a mass of 1800 kg. Its maximum velocity and maximum acceleration are 2.5 m/s and 1.4 m/s^2, respectively. A passenger weighing 67 kg stands on a scale in the elevator as the elevator ascends at its maximum acceleration. When the elevator reaches its maximum acceleration, the scale most nearly reads

(A) 67 N

(B) 560 N

(C) 660 N

(D) 750 N

4. A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is most nearly the greatest acceleration that the car can reach without breaking the rope?

(A) 1.2 m/s^2

(B) 2.5 m/s^2

(C) 3.8 m/s^2

(D) 4.5 m/s^2

5. An 8 kg block begins from rest and slides down an inclined plane. After 10 s, the block has a velocity of 15 m/s. The plane's angle of inclination is 30°. What is most nearly the coefficient of friction between the plane and the block?

(A) 0.15

(B) 0.22

(C) 0.40

(D) 0.85

6. If the sum of the forces on a particle is not equal to zero, the particle is

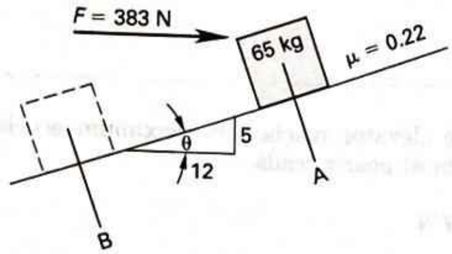
(A) moving with constant velocity in the direction of the resultant force

(B) accelerating in a direction opposite to the resultant force

(C) accelerating in the same direction as the resultant force

(D) moving with a constant velocity opposite to the direction of the resultant force

7. A 383 N horizontal force is applied to the 65 kg block shown. Beginning at position A, the block is moving down the slope at an initial velocity of 12.5 m/s and comes to a complete stop at position B. The coefficient of friction between the block and the plane is $\mu = 0.22$.



What is most nearly the distance between positions A and B?

(A) 6.1 m

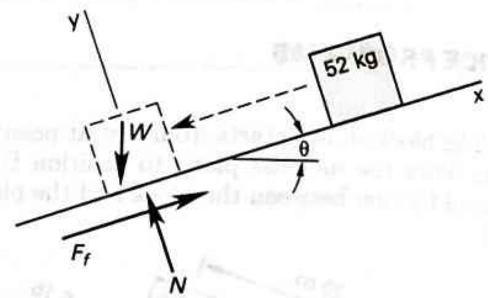
(B) 9.1 m

(C) 15 m

(D) 19 m

SOLUTIONS

1. Choose a coordinate system parallel and perpendicular to the plane, as shown. Let the x-axis be positive in the direction of motion (to the left). Recognize that this is a 5-12-13 triangle. Alternatively, calculate $\sqrt{(5)^2 + (12)^2} = 13$.



From the equations for friction and the equation for the radial component of force, the acceleration is

$$\sum F_x = ma_x$$
$$W_x - \mu N = ma_x$$
$$mg \sin\theta - \mu mg \cos\theta = ma_x$$

$$\begin{aligned} a_x &= g\sin\theta - \mu g\cos\theta = g(\sin\theta - \mu\cos\theta) \\ &= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{5}{13} - (0.15)\left(\frac{12}{13}\right)\right) \\ &= 2.415\ \text{m/s}^2 \end{aligned}$$

The velocity at position B is

$$\begin{aligned} \text{v}^2 &= \text{v}_0^2 + 2a_x(x - x_0) \\ \text{v}_0 &= x_0 = 0 \\ \text{v}^2 &= 2a_x x \\ &= (2)\left(2.415\ \frac{\text{m}}{\text{s}^2}\right)(20\ \text{m}) \\ &= 96.6\ \text{m}^2/\text{s}^2 \end{aligned}$$

$$\text{v} = \sqrt{96.6\ \frac{\text{m}^2}{\text{s}^2}} = 9.83\ \text{m/s} \quad (9.8\ \text{m/s})$$

The answer is (D).

2. Calculate the initial acceleration.

$$\text{v}(t) = a_x t + \text{v}_0$$

$$a_x = \frac{\text{v}(t) - \text{v}_0}{t} = \frac{6\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{4\ \text{s}}$$
$$= 1.5\ \text{m/s}^2$$

Dynamics

After 4 s the block will have moved

$$x = \frac{a_x t^2}{2} + v_0 t + x_0$$
$$= \frac{\left(1.5\ \frac{m}{s^2}\right)(4\ s)^2}{2} + \left(0\ \frac{m}{s}\right)(4\ s) + 0\ m$$
$$= 12\ m$$

The answer is (D).

3. Use Newton's second law. As the elevator ascends, it exerts a force on the passenger whose inertia resists the movement of the elevator. The inertia force of the passenger will be manifested as an additional weight reading on the scale. Thus, the acceleration of the elevator adds to the gravitational acceleration.

$$F = ma = m(a_1 + a_2)$$
$$= (67\ kg)\left(9.81\ \frac{m}{s^2} + 1.4\ \frac{m}{s^2}\right)$$
$$= 751\ N \quad (750\ N)$$

The answer is (D).

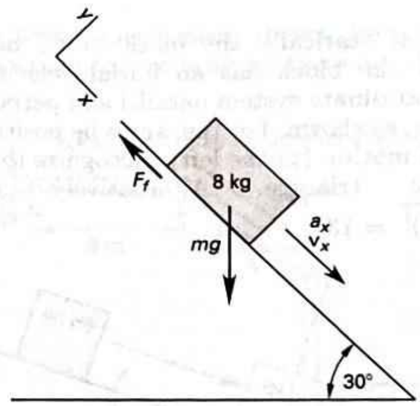
4. From Newton's second law, the maximum acceleration is

$$F = ma$$
$$a = \frac{F}{m} = \frac{2000\ N}{800\ kg}$$
$$= 2.5\ m/s^2$$

The answer is (B).

5. Calculate the initial acceleration.

$$v_x = a_x t + v_0$$
$$a_x = \frac{v_x - v_0}{t} = \frac{15\ \frac{m}{s} - 0\ \frac{m}{s}}{10\ s}$$
$$= 1.5\ m/s^2$$



Choose a coordinate system so that the x-direction is parallel to the inclined plane. From the equations for friction, normal force, and the parallel component of force, the coefficient of friction is

$$\sum F_x = ma_x = mg_x - F_f$$
$$ma_x = mg \sin\phi - \mu mg \cos\phi$$
$$\mu = \frac{mg \sin\phi - ma_x}{mg \cos\phi}$$
$$= \frac{g \sin\phi - a_x}{g \cos\phi}$$
$$= \frac{\left(9.81\ \frac{m}{s^2}\right)\sin 30^\circ - 1.5\ \frac{m}{s^2}}{\left(9.81\ \frac{m}{s^2}\right)\cos 30^\circ}$$
$$= 0.40$$

The answer is (C).

6. Newton's second law can be applied separately to any direction in which forces are resolved into components, including the resultant direction.

$$F_R = ma_R$$

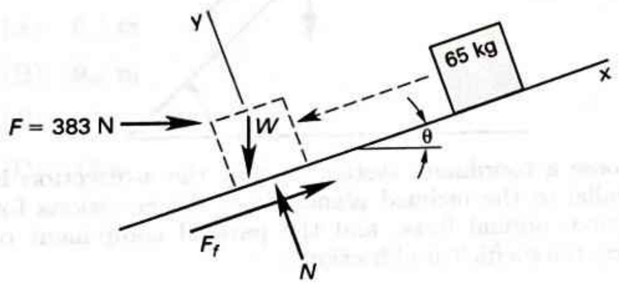
Dynamics

Since force and acceleration are both vectors, and mass is a scalar, the direction of acceleration is the same as the resultant force.

$$\mathbf{a}_R = \frac{\mathbf{F}_R}{m}$$

The answer is (C).

7. Note that statically the block would not move. Recall that the block has an initial velocity given. Choose a coordinate system parallel and perpendicular to the plane, as shown. Let the x-axis be positive in the direction of motion (to the left). Recognize that this is a 5-12-13 triangle. Alternatively, calculate $\sqrt{(5)^2+(12)^2} = 13$.



Using the sum of forces and the equations for friction and normal force, solve for the acceleration.

$$\sum F_t = ma_t$$
$$W_x - F_x - \mu N = ma_t$$
$$mg\sin\theta - F\cos\theta - \mu(mg\cos\theta + F\sin\theta) = ma_t$$

$$\begin{aligned} a_t &= \left(\frac{1}{m}\right)\big(mg\sin\theta - F\cos\theta - \mu(mg\cos\theta + F\sin\theta)\big) \\ &= g(\sin\theta - \mu\cos\theta) - \frac{F}{m}(\cos\theta + \mu\sin\theta) \\ &= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{5}{13} - (0.22)\left(\frac{12}{13}\right)\right) \\ &\quad - \left(\frac{383\ \text{N}}{65\ \text{kg}}\right)\left(\frac{12}{13} + (0.22)\left(\frac{5}{13}\right)\right) \\ &= -4.157\ \text{m/s}^2 \end{aligned}$$

The velocity is

$$\text{v}^2 = \text{v}_0^2 + 2a_t(x - x_0)$$
$$\text{v}_0 = 12.5\ \text{m/s}$$
$$\text{v} = x_0 = 0$$

The distance between positions A and B is

$$\begin{aligned} x &= \frac{-\text{v}_0^2}{2a_t} = \frac{-\left(12.5\ \frac{\text{m}}{\text{s}}\right)^2}{(2)\left(-4.157\ \frac{\text{m}}{\text{s}^2}\right)} \\ &= 18.79\ \text{m} \quad (19\ \text{m}) \end{aligned}$$

The answer is (D).

29 Kinetics of Rotational Motion

PRACTICE PROBLEMS

1. A 1530 kg car is towing a 300 kg trailer. The coefficient of friction between all tires and the road is 0.80. The car and trailer are traveling at 100 km/h around a banked curve of radius 200 m. Most nearly, what is the necessary banking angle such that tire friction will NOT be necessary to prevent skidding?

(A) 8.0°

(B) 21°

(C) 36°

(D) 78°

2. Why does a spinning ice skater's angular velocity increase as she brings her arms in toward her body?

(A) Her mass moment of inertia is reduced.

(B) Her angular momentum is constant.

(C) Her radius of gyration is reduced.

(D) all of the above

3. A 1 m long uniform rod has a mass of 10 kg. It is pinned at one end to a frictionless pivot. Most nearly, what is the mass moment of inertia of the rod taken about the pivot point?

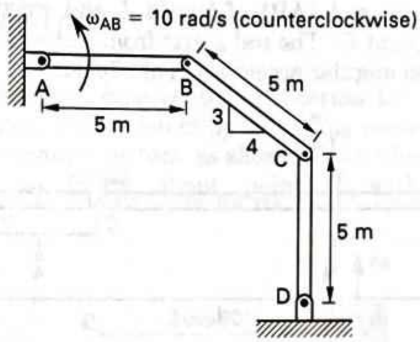
(A) 0.83 kg·m^2

(B) 2.5 kg·m^2

(C) 3.3 kg·m^2

(D) 10 kg·m^2

4. In the linkage mechanism shown, link AB rotates with an instantaneous counterclockwise angular velocity of 10 rad/s.



Most nearly, what is the instantaneous angular velocity of link BC when link AB is horizontal and link CD is vertical?

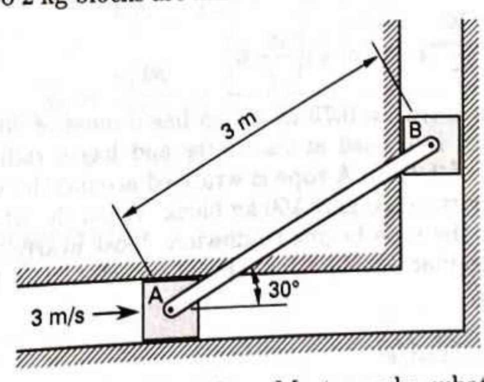
(A) 2.3 rad/s (clockwise)

(B) 3.3 rad/s (counterclockwise)

(C) 5.5 rad/s (clockwise)

(D) 13 rad/s (clockwise)

5. Two 2 kg blocks are linked as shown.



The surfaces are frictionless. Most nearly, what is the velocity of block B if block A is moving at a speed of 3 m/s?

(A) 0 m/s

(B) 1.3 m/s

(C) 1.7 m/s

(D) 5.2 m/s

Dynamics

6. A car travels on a perfectly horizontal, unbanked circular track of radius r. The coefficient of friction between the tires and the track is 0.3. The car's velocity is 10 m/s. Most nearly, what is the smallest radius the car can travel without skidding?

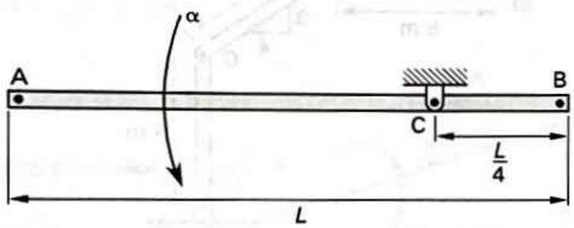
(A) 10 m

(B) 34 m

(C) 50 m

(D) 68 m

7. A uniform rod (AB) of length L and weight W is pinned at point C. The rod starts from rest and accelerates with an angular acceleration of $12g/7L$.



What is the instantaneous reaction at point C at the moment rotation begins?

(A) $\dfrac{W}{4}$

(B) $\dfrac{W}{3}$

(C) $\dfrac{4W}{7}$

(D) $\dfrac{7W}{12}$

8. A wheel with a 0.75 m radius has a mass of 200 kg. The wheel is pinned at its center and has a radius of gyration of 0.25 m. A rope is wrapped around the wheel and supports a hanging 100 kg block. When the wheel is released, the rope begins to unwind. Most nearly, what is the angular acceleration of the wheel as the block descends?

(A) 5.9 rad/s^2

(B) 6.5 rad/s^2

(C) 11 rad/s^2

(D) 14 rad/s^2

9. A car travels around an unbanked 50 m radius curve without skidding. The coefficient of friction between the tires and road is 0.3. Most nearly, what is the car's maximum velocity?

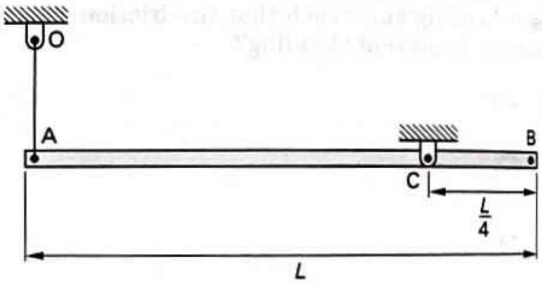
(A) 14 km/h

(B) 25 km/h

(C) 44 km/h

(D) 54 km/h

10. A uniform rod (AB) of length L and weight W is pinned at point C and restrained by cable OA. The cable is suddenly cut. The rod starts to rotate about point C, with point A moving down and point B moving up.



What is the instantaneous linear acceleration of point B?

(A) $\dfrac{3g}{16}$

(B) $\dfrac{g}{4}$

(C) $\dfrac{3g}{7}$

(D) $\dfrac{3g}{4}$

Dynamics

SOLUTIONS

1. The necessary superelevation angle without relying on friction is

$$\theta = \arctan \frac{v^2}{gr}$$

$$= \arctan \frac{\left(\left(100\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)\right)^2}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(200\ \text{m})\left(3600\ \frac{\text{s}}{\text{h}}\right)^2}$$

$$= 21.47^\circ \quad (21^\circ)$$

The answer is (B).

2. As the skater brings her arms in, her radius of gyration and mass moment of inertia decrease. However, in the absence of friction, her angular momentum, H, is constant.

$$\omega = \frac{H}{I}$$

Since angular velocity, ω, is inversely proportional to the mass moment of inertia, the angular velocity increases when the mass moment of inertia decreases.

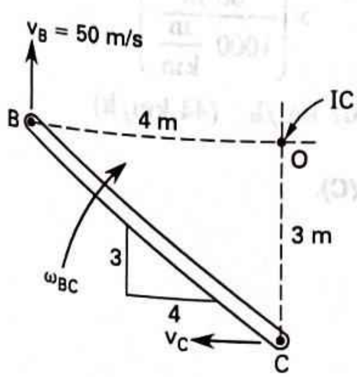
The answer is (D).

3. The mass moment of inertia of the rod taken about one end is

$$I_{\text{rod}} = \frac{ML^2}{3} = \frac{(10\ \text{kg})(1\ \text{m})^2}{3}$$
$$= 3.33\ \text{kg·m}^2 \quad (3.3\ \text{kg·m}^2)$$

The answer is (C).

4. Find the instantaneous center of rotation. The absolute velocity directions at points B and C are known. The instantaneous center is located by drawing perpendiculars to these velocities, as shown. The angular velocity of any point on rigid body link BC is the same at this instant.



The velocity of point B is

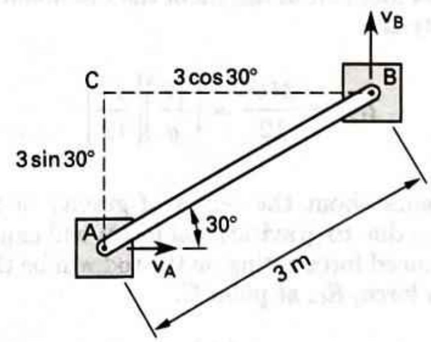
$$v_B = AB\omega_{AB} = (5\ \text{m})\left(10\ \frac{\text{rad}}{\text{s}}\right) = 50\ \text{m/s}$$

The angular velocity of link BC is

$$\omega_{BC} = \frac{v_B}{OB} = \frac{50\ \frac{\text{m}}{\text{s}}}{4\ \text{m}}$$
$$= 12.5\ \text{rad/s} \quad (13\ \text{rad/s}) \quad \text{[clockwise]}$$

The answer is (D).

5. The instantaneous center of rotation for the slider rod assembly can be found by extending perpendiculars from the velocity vectors, as shown. Both blocks can be assumed to rotate about point C with angular velocity ω.



The velocity of block B is

$$\omega = \frac{v_A}{CA} = \frac{v_B}{BC}$$

$$v_B = \frac{v_A BC}{CA} = \frac{\left(3\ \frac{\text{m}}{\text{s}}\right)(3\ \text{m})\cos 30^\circ}{(3\ \text{m})\sin 30^\circ}$$
$$= 5.2\ \text{m/s}$$

The answer is (D).

Dynamics

6. If there is no skidding, the frictional force, F_f, will equal the centrifugal force, F_c. From the equations for centrifugal force and frictional force, the smallest possible radius is

$$F_c = \frac{m\mathrm{v}^2}{r}$$

$$F_f = \mu N = \mu m g$$

$$\frac{m\mathrm{v}^2}{r} = \mu m g$$

$$r = \frac{\mathrm{v}^2}{\mu g} = \frac{\left(10\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(0.3)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 34\ \mathrm{m}$$

The answer is (B).

7. The mass moment of inertia of the rod about its center of gravity is

$$I_{\mathrm{CG}} = \frac{ML^2}{12} = \left(\frac{W}{g}\right)\left(\frac{L^2}{12}\right)$$

Take moments about the center of gravity of the rod. All moments due to gravitational forces will cancel. The only unbalanced force acting on the rod will be the vertical reaction force, R_{C}, at point C.

$$\sum M_{\mathrm{CG}} = R_{\mathrm{C}}\left(\frac{L}{4}\right) = I_{\mathrm{CG}}\alpha_{\mathrm{CG}}$$

$$R_{\mathrm{C}}\left(\frac{L}{4}\right) = \left(\left(\frac{W}{g}\right)\left(\frac{L^2}{12}\right)\right)\left(\frac{12g}{7L}\right)$$

$$R_{\mathrm{C}} = \frac{4W}{7}$$

The angular velocity is zero, so the center of the mass does not have a component of acceleration in the horizontal direction. There is no horizontal force component at point C.

The answer is (C).

8. From the equation for the radius of gyration, the mass moment of inertia of the wheel is

$$r = \sqrt{\frac{I}{m_{\mathrm{wheel}}}}$$

$$I = m_{\mathrm{wheel}}r^2$$

The unbalanced moment on the wheel is

$$M = FR = (mg - ma)R = mR(g - a)$$

Substitute the acceleration, a, which is the radius, R, multiplied by the angular acceleration, α.

$$M = m_{\mathrm{block}}R(g - R\alpha)$$

The unbalanced moment on the wheel is also the moment of inertia, I, multiplied by the angular acceleration, α.

$$M = I\alpha$$

$$m_{\mathrm{block}}R(g - R\alpha) = m_{\mathrm{wheel}}r^2\alpha$$

Solve for the angular acceleration.

$$\alpha = \frac{m_{\mathrm{block}}Rg}{m_{\mathrm{wheel}}r^2 + m_{\mathrm{block}}R^2}$$

$$= \frac{(100\ \mathrm{kg})(0.75\ \mathrm{m})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)}{(200\ \mathrm{kg})(0.25\ \mathrm{m})^2 + (100\ \mathrm{kg})(0.75\ \mathrm{m})^2}$$

$$= 10.7\ \mathrm{rad/s^2} \quad (11\ \mathrm{rad/s^2})$$

The answer is (C).

9. If the car does not skid, the frictional force and the centrifugal force must be equal. From the equations for centrifugal force and frictional force, the car's maximum velocity is

$$F_c = \frac{m\mathrm{v}^2}{r}$$

$$F_f = \mu N = \mu m g$$

$$\frac{m\mathrm{v}^2}{r} = \mu m g$$

$$\mathrm{v} = \sqrt{\mu g r}$$

$$= \sqrt{(0.3)\left(\frac{\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(60\ \frac{\mathrm{s}}{\mathrm{min}}\right)^2\left(60\ \frac{\mathrm{min}}{\mathrm{h}}\right)^2}{1000\ \frac{\mathrm{m}}{\mathrm{km}}}\right) \times \left(\frac{50\ \mathrm{m}}{1000\ \frac{\mathrm{m}}{\mathrm{km}}}\right)}$$

$$= 43.67\ \mathrm{km/h} \quad (44\ \mathrm{km/h})$$

The answer is (C).

Dynamics

10. Point C is $L/4$ from the center of gravity of the rod. The mass moment of inertia about point C is

$$I_C = I_{CG} + Md^2 = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \left(\frac{7}{48}\right)ML^2$$

The sum of moments on the rod is

$$\begin{aligned}\sum M_C = \sum Fr &= \left(\frac{3W}{4}\right)\left(\frac{\frac{3L}{4}}{2}\right) - \left(\frac{W}{4}\right)\left(\frac{\frac{L}{4}}{2}\right)\\ &= \frac{WL}{4}\\ &= \frac{MgL}{4}\end{aligned}$$

The angular acceleration is

$$\alpha = \frac{\sum M_C}{I_C} = \frac{\frac{MgL}{4}}{\left(\frac{7}{48}\right)ML^2} = \frac{12g}{7L}$$

The tangential acceleration of point B is

$$a_{t,B} = r\alpha = \left(\frac{L}{4}\right)\left(\frac{12g}{7L}\right) = \frac{3g}{7}$$

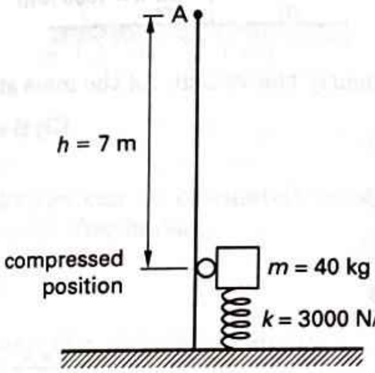
The answer is (C).

Dynamics

30 Energy and Work

PRACTICE PROBLEMS

1. The 40 kg mass, m, shown is guided by a frictionless rail. The spring constant, k, is 3000 N/m. The spring is compressed sufficiently and released, such that the mass barely reaches point A.



What is most nearly the initial spring compression?

(A) 0.96 m

(B) 1.2 m

(C) 1.4 m

(D) 1.8 m

2. Two balls, both of mass 2 kg, collide head on. The velocity of each ball at the time of the collision is 2 m/s. The coefficient of restitution is 0.5. Most nearly, what are the final velocities of the balls?

(A) 1 m/s and −1 m/s

(B) 2 m/s and −2 m/s

(C) 3 m/s and −3 m/s

(D) 4 m/s and −4 m/s

3. A 1500 kg car traveling at 100 km/h is towing a 250 kg trailer. The coefficient of friction between the tires and the road is 0.8 for both the car and trailer. Approximately what energy is dissipated by the brakes if the car and trailer are braked to a complete stop?

(A) 96 kJ

(B) 390 kJ

(C) 580 kJ

(D) 680 kJ

4. A 3500 kg car traveling at 65 km/h skids and hits a wall 3 s later. The coefficient of friction between the tires and the road is 0.60. What is most nearly the speed of the car when it hits the wall?

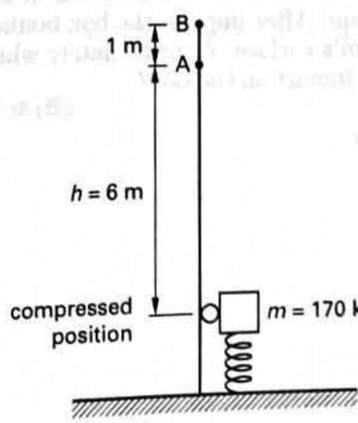
(A) 0.14 m/s

(B) 0.40 m/s

(C) 5.1 m/s

(D) 6.2 m/s

5. The 170 kg mass, m, shown is guided by a frictionless rail. The spring is compressed sufficiently and released, such that the mass barely reaches point B.



What is most nearly the kinetic energy of the mass at point A?

(A) 20 J

(B) 220 J

(C) 390 J

(D) 1700 J

Dynamics

6. A pickup truck is traveling forward at 25 m/s. The bed is loaded with boxes whose coefficient of friction with the bed is 0.40. What is most nearly the shortest time that the truck can be brought to a stop such that the boxes do not shift?

(A) 2.3 s

(B) 4.7 s

(C) 5.9 s

(D) 6.4 s

7. Two balls both have a mass of 8 kg and collide head on. The velocity of each ball at the time of collision is 18 m/s. The velocity of each ball decreases to 10 m/s in opposite directions after the collision. Approximately how much energy is lost in the collision?

(A) 0.57 kJ

(B) 0.91 kJ

(C) 1.8 kJ

(D) 2.3 kJ

8. The impulse-momentum principle is mostly useful for solving problems involving

(A) force, velocity, and time

(B) force, acceleration, and time

(C) velocity, acceleration, and time

(D) force, velocity, and acceleration

9. A 12 kg aluminum box is dropped from rest onto a large wooden beam. The box travels 0.2 m before contacting the beam. After impact, the box bounces 0.05 m above the beam's surface. Approximately what impulse does the beam impart on the box?

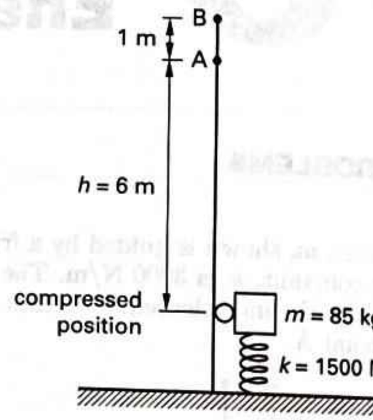
(A) 8.6 N·s

(B) 12 N·s

(C) 36 N·s

(D) 42 N·s

10. The 85 kg mass, m, shown is guided by a frictionless rail. The spring is compressed sufficiently and released, such that the mass barely reaches point B. The spring constant, k, is 1500 N/m.



What is most nearly the velocity of the mass at point A?

(A) 3.1 m/s

(B) 4.4 m/s

(C) 9.8 m/s

(D) 20 m/s

SOLUTIONS

1. At the point just before the spring is released, all of the energy in the system is elastic potential energy; while at point A, all of the energy is potential energy due to gravity.

$$\frac{kx^2}{2} = mgh$$

$$x = \sqrt{\frac{2mgh}{k}}$$

$$= \sqrt{\frac{(2)(40\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(7\ \text{m})}{3000\ \frac{\text{N}}{\text{m}}}}$$

$$= 1.35\ \text{m} \quad (1.4\ \text{m})$$

The answer is (C).

2. The velocities can be calculated using the relationships of velocity after impact,

Impact

$$(v_1')_n = \frac{m_2(v_2)_n(1+e) + (m_1 - em_2)(v_1)_n}{(m_1 + m_2)}$$

$$= \frac{2\ \text{kg}(-2\text{m/s})(1+0.5) + (2\ \text{kg} - 0.5(2\text{kg}))(2\text{m/s})}{2\ \text{kg} + 2\ \text{kg}}$$

$$= -1\text{m/s}$$

$$(v_2')_n = \frac{m_1(v_1)_n(1+e) - (em_1 - m_2)(v_2)_n}{(m_1 + m_2)}$$

$$= \frac{2\ \text{kg}(2\ \text{m/s})(1+0.5) - (0.5(2\ \text{kg}) - 2\ \text{kg})(-2\ \text{m/s})}{2\ \text{kg} + 2\ \text{kg}}$$

$$= 1\ \text{m/s}$$

The answer is (A).

3. The original velocity of the car and trailer is

$$v = \frac{\left(100\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)}{\left(60\ \frac{\text{s}}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{h}}\right)}$$

$$= 27.78\ \text{m/s}$$

Since the final velocity is zero, the energy dissipated is the original kinetic energy.

$$T = \frac{mv^2}{2}$$

$$= \frac{(1500\ \text{kg} + 250\ \text{kg})\left(27.78\ \frac{\text{m}}{\text{s}}\right)^2}{2}$$

$$= 675\,154\ \text{J} \quad (680\ \text{kJ})$$

The answer is (D).

4. The frictional force (negative because it opposes motion) decelerating the car is

$$F_f = -\mu N = -\mu mg$$

$$= -(0.60)(3500\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= -20\,601\ \text{N}$$

Use the impulse-momentum principle.

$$F_f(t_1 - t_2) = m(v_1 - v_2)$$

$$v_2 = v_1 - \frac{F_f(t_1 - t_2)}{m}$$

$$= \frac{\left(65\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)}{\left(60\ \frac{\text{s}}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{h}}\right)} - \frac{(-20\,601\ \text{N})(0\ \text{s} - 3\ \text{s})}{3500\ \text{kg}}$$

$$= 0.3976\ \text{m/s} \quad (0.40\ \text{m/s})$$

The answer is (B).

Dynamics

5. At point A, the energy of the mass is a combination of kinetic and gravitational potential energies. The total energy of the system is constant, and the kinetic energy at B is 0.

$$
\begin{aligned}
E_A &= E_B \\
U_A + T_A &= U_B \\
mgh + \frac{m\mathrm{v}^2}{2} &= mg(h + 1\ \mathrm{m}) \\
T_A &= mg(h + 1\ \mathrm{m}) - mgh \\
&= mg(1\ \mathrm{m}) \\
&= (170\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(1\ \mathrm{m}) \\
&= 1670\ \mathrm{J} \quad (1700\ \mathrm{J})
\end{aligned}
$$

The answer is (D).

6. The frictional force is the only force preventing the boxes from shifting. The forces on each box are its weight, the normal force, and the frictional force. The normal force on each box is equal to the box weight.

$$N = W = mg$$

The frictional force is

$$F_f = \mu N = \mu mg$$

Use the impulse-momentum principle. $\mathrm{v}_2 = 0$. The frictional force is opposite of the direction of motion, so it is negative.

$$
\begin{aligned}
\mathrm{Imp} &= \Delta p \\
F_f \Delta t &= m\Delta \mathrm{v} \\
\Delta t &= \frac{m(\mathrm{v}_2 - \mathrm{v}_1)}{F_f} = \frac{-m\mathrm{v}_1}{-\mu mg} = \frac{\mathrm{v}_1}{\mu g} \\
&= \frac{25\ \frac{\mathrm{m}}{\mathrm{s}}}{(0.40)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} \\
&= 6.37\ \mathrm{s} \quad (6.4\ \mathrm{s})
\end{aligned}
$$

The answer is (D).

7. Each ball possesses kinetic energy before and after the collision. The velocity of each ball is reduced from $|18\ \mathrm{m/s}|$ to $|10\ \mathrm{m/s}|$.

$$
\begin{aligned}
\Delta T &= T_2 - T_1 \\
&= (2)\left(\frac{m(\mathrm{v}_2^2 - \mathrm{v}_1^2)}{2}\right) \\
&= (2)\left(\frac{(8\ \mathrm{kg})\left(\left(18\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 - \left(10\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2\right)}{2}\right) \\
&= 1792\ \mathrm{J} \quad (1.8\ \mathrm{kJ})
\end{aligned}
$$

The answer is (C).

8. Impulse is calculated from force and time. Momentum is calculated from mass and velocity. The impulse-momentum principle is useful in solving problems involving force, time, velocity, and mass.

The answer is (A).

9. Initially, the box has potential energy only. (This takes the beam's upper surface as the reference plane.) When the box reaches the beam, all of the potential energy will have been converted to kinetic energy.

$$
\begin{aligned}
mgh_1 &= \frac{m\mathrm{v}_1^2}{2} \\
\mathrm{v}_1 &= \sqrt{2gh_1} \\
&= \sqrt{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.2\ \mathrm{m})} \\
&= 1.98\ \mathrm{m/s} \quad \text{[downward]}
\end{aligned}
$$

When the box rebounds to its highest point, all of its remaining energy will be potential energy once again.

$$
\begin{aligned}
mgh_2 &= \frac{m\mathrm{v}_2^2}{2} \\
\mathrm{v}_2 &= \sqrt{2gh_2} \\
&= \sqrt{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.05\ \mathrm{m})} \\
&= 0.99\ \mathrm{m/s} \quad \text{[upward]}
\end{aligned}
$$

Use the impulse-momentum principle. (Downward is taken as the positive velocity direction.)

$$
\begin{aligned}
\text{Imp} = \Delta p &= m(\mathrm{v}_1 - \mathrm{v}_2) \\
&= (12\ \text{kg})\left(1.98\ \frac{\text{m}}{\text{s}} - \left(-0.99\ \frac{\text{m}}{\text{s}}\right)\right) \\
&= 35.66\ \text{N·s} \quad (36\ \text{N·s})
\end{aligned}
$$

The answer is (C).

10. At point A, the energy of the mass is a combination of kinetic and gravitational potential energies. The total energy of the system is constant, and the kinetic energy at B is 0.

$$
\begin{aligned}
E_\text{A} &= E_\text{B} \\
U_\text{A} + T_\text{A} &= U_\text{B} \\
mgh + \frac{m\mathrm{v}^2}{2} &= mg(h + 1\ \text{m}) \\
T_\text{A} &= mg(h + 1\ \text{m}) - mgh \\
&= mg(1\ \text{m}) \\
&= (85\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1\ \text{m}) \\
&= 833.9\ \text{J}
\end{aligned}
$$

Therefore, the velocity of the mass at point A is

$$
\begin{aligned}
T_\text{A} &= \frac{m\mathrm{v}^2}{2} = 833.9\ \text{J} \\
\mathrm{v} &= \sqrt{\frac{2T_\text{A}}{m}} \\
&= \sqrt{\frac{(2)(833.9\ \text{J})}{85\ \text{kg}}} \\
&= 4.43\ \text{m/s} \quad (4.4\ \text{m/s})
\end{aligned}
$$

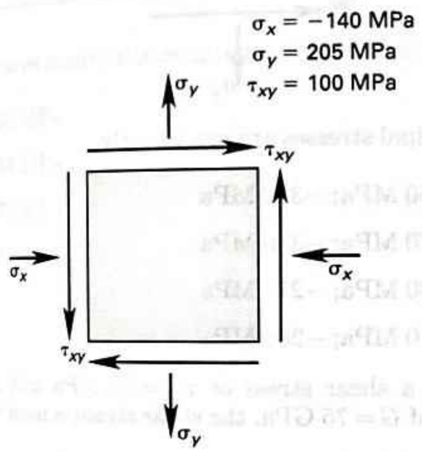
The answer is (B).

Dynamics

31 Stresses and Strains

PRACTICE PROBLEMS

1. The element is subjected to the plane stress condition shown.



What is the maximum shear stress?

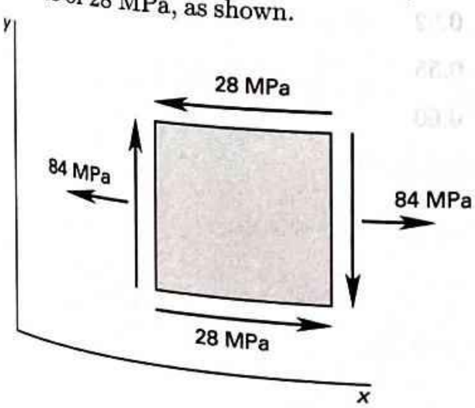
(A) 100 MPa

(B) 160 MPa

(C) 200 MPa

(D) 210 MPa

2. A plane element in a body is subjected to a normal tensile stress in the x-direction of 84 MPa, as well as shear stresses of 28 MPa, as shown.



Most nearly, what are the principal stresses?

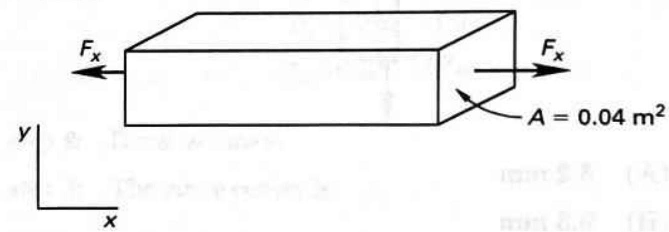
(A) 70 MPa; 14 MPa

(B) 84 MPa; 28 MPa

(C) 92 MPa; −8.5 MPa

(D) 112 MPa; −28 MPa

3. What is most nearly the lateral strain, ε_y, of the steel specimen shown if $F_x = 3000$ kN, $E = 193$ GPa, and $\nu = 0.29$?



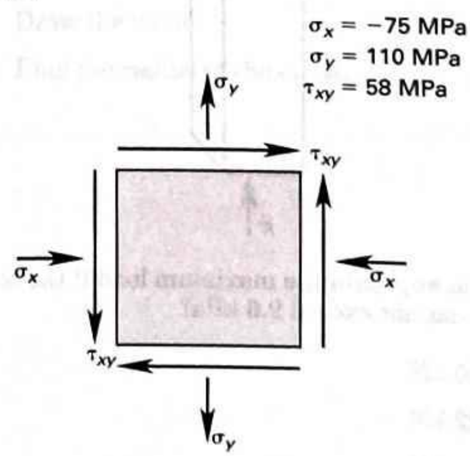
(A) -4.0×10^{-4}

(B) -1.1×10^{-4}

(C) 1.0×10^{-4}

(D) 4.0×10^{-4}

4. The elements are subjected to the plane stress condition shown.



What is the maximum shear stress?

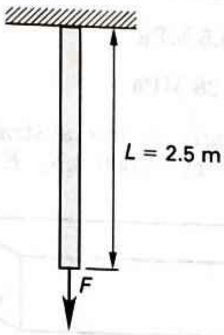
(A) 58.1 MPa

(B) 60.6 MPa

(C) 75.8 MPa

(D) 109.2 MPa

5. What is most nearly the elongation of the aluminum bar (cross section of 3 cm × 3 cm) shown when loaded to its yield point? The modulus of elasticity is 69 GPa, and the yield strength in tension is 255 MPa. Neglect the weight of the bar.



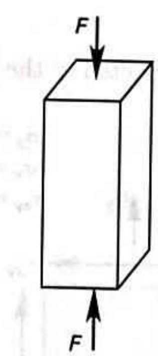
(A) 3.3 mm

(B) 9.3 mm

(C) 12 mm

(D) 15 mm

6. The column shown has a cross-sectional area of 13 m^2.



What is the approximate maximum load if the compressive stress cannot exceed 9.6 kPa?

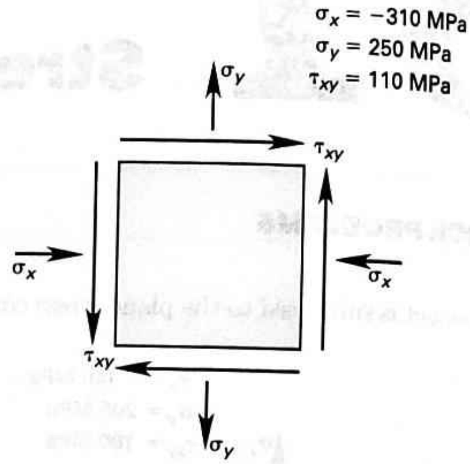
(A) 120 kN

(B) 122 kN

(C) 124 kN

(D) 130 kN

7. The element is subjected to the plane stress condition shown. The maximum shear stress is 300 MPa.



The principal stresses are most nearly

(A) 250 MPa; −310 MPa

(B) 270 MPa; −330 MPa

(C) 330 MPa; −270 MPa

(D) 310 MPa; −250 MPa

8. Given a shear stress of $\tau_{xy} = 35$ MPa and a shear modulus of $G = 75$ GPa, the shear strain is most nearly

(A) 2.5×10^{-5} rad

(B) 4.7×10^{-4} rad

(C) 5.5×10^{-4} rad

(D) 8.3×10^{-4} rad

9. Which of the following could be the Poisson ratio of a material?

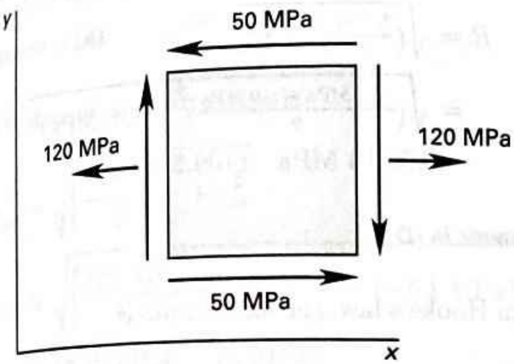
(A) 0.35

(B) 0.52

(C) 0.55

(D) 0.60

10. A plane element in a body is subjected to the stresses shown.



What is most nearly the maximum shear stress?

(A) 50 MPa

(B) 64 MPa

(C) 72 MPa

(D) 78 MPa

SOLUTIONS

1. There are two methods for solving the problem. The first method is to use the equation for $\tau_{\max}$; the second method is to draw Mohr's circle.

Solving by the equation for $\tau_{\max}$,

$$\begin{aligned}\tau_{\max} &= \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\\ &= \sqrt{\left(\frac{-140\text{ MPa} - 205\text{ MPa}}{2}\right)^2 + (100\text{ MPa})^2}\\ &= 199.4\text{ MPa}\quad(200\text{ MPa})\end{aligned}$$

Solving by Mohr's circle,

step 1:

$$\begin{aligned}\sigma_x &= -140\text{ MPa}\\ \sigma_y &= 205\text{ MPa}\\ \tau_{xy} &= 100\text{ MPa}\end{aligned}$$

step 2: Draw σ-τ axes.

step 3: The circle center is

$$\begin{aligned}C &= \tfrac{1}{2}(\sigma_x + \sigma_y)\\ &= \left(\tfrac{1}{2}\right)(-140\text{ MPa} + 205\text{ MPa})\\ &= 32.5\text{ MPa}\end{aligned}$$

step 4: Plot the points (−140 MPa, −100 MPa) and (205 MPa, 100 MPa).

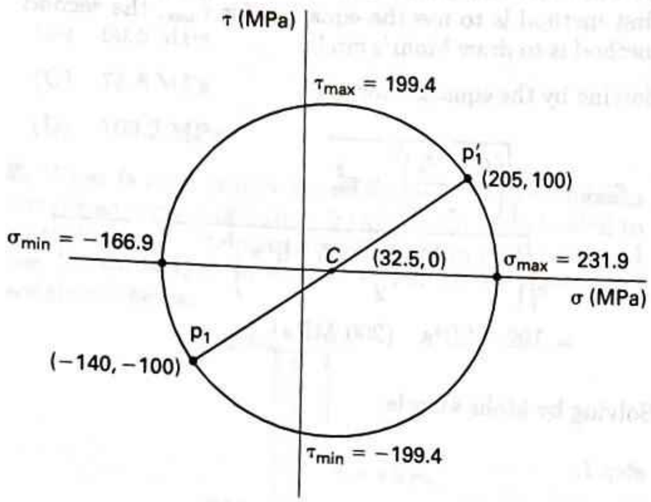
step 5: Draw the diameter of the circle.

step 6: Draw the circle.

step 7: Find the radius of the circle.

Mechanics of Materials

step 8: Maximum shear stress is at the top of the circle, $\tau_{max} = 199.4$ MPa (200 MPa).



The answer is (C).

2. τ_{xy} is negative according to the standard sign convention.

$$\begin{aligned}\sigma_{\text{max,min}} &= \tfrac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \left(\tfrac{1}{2}\right)(84 \text{ MPa} + 0 \text{ MPa}) \\ &\quad \pm \sqrt{\left(\frac{84 \text{ MPa} - 0 \text{ MPa}}{2}\right)^2 + (-28 \text{ MPa})^2} \\ &= 42 \text{ MPa} \pm 50.478 \text{ MPa} \\ &= 92.478 \text{ MPa};\ -8.478 \text{ MPa} \\ &\quad (92 \text{ MPa};\ -8.5 \text{ MPa})\end{aligned}$$

The answer is (C).

3. From Hooke's law and the equation for axial stress,

$$\begin{aligned}\varepsilon_x = \frac{\sigma_x}{E} = \frac{F_x}{EA} &= \frac{(3000 \text{ kN})\left(1000 \ \frac{\text{N}}{\text{kN}}\right)}{(193 \text{ GPa})\left(10^9 \ \frac{\text{Pa}}{\text{GPa}}\right)(0.04 \text{ m}^2)} \\ &= 3.89 \times 10^{-4}\end{aligned}$$

Use Poisson's ratio.

$$\begin{aligned}\varepsilon_y = -\nu\varepsilon_x &= (-0.29)(3.89 \times 10^{-4}) \\ &= -1.13 \times 10^{-4} \quad (-1.1 \times 10^{-4})\end{aligned}$$

The answer is (B).

4. The maximum shear stress, $\tau_{\text{in}} = R$. R is given by the following equation

$$\begin{aligned}R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{-75 \text{ MPa} - 110 \text{ MPa}}{2}\right)^2 + (58 \text{ MPa})^2} \\ &= 109.18 \text{ MPa} \quad (109.2 \text{ MPa})\end{aligned}$$

The answer is (D).

5. From Hooke's law, the axial strain is

$$\varepsilon = \frac{\sigma}{E} = \frac{(255 \text{ MPa})\left(10^6 \ \frac{\text{Pa}}{\text{MPa}}\right)}{(69 \text{ GPa})\left(10^9 \ \frac{\text{Pa}}{\text{GPa}}\right)} = 0.0037$$

The elongation is

$$\delta = \varepsilon L = (0.0037)(2.5 \text{ m}) = 0.00925 \text{ m} \quad (9.3 \text{ mm})$$

The answer is (B).

6. The maximum force is

$$\begin{aligned}F_{\text{max}} = S_a A &= (9.6 \text{ kPa})(13 \text{ m}^2) \\ &= 124.8 \text{ kN} \quad (124 \text{ kN})\end{aligned}$$

The answer is (C).

7. The principal stresses are

$$\begin{aligned}\sigma_{\text{max}}, \sigma_{\text{min}} &= \tfrac{1}{2}(\sigma_x + \sigma_y) \pm \tau_{\text{max}} \\ &= \left(\tfrac{1}{2}\right)(-310 \text{ MPa} + 250 \text{ MPa}) \pm 300 \text{ MPa} \\ &= -30 \text{ MPa} \pm 300 \text{ MPa} \\ \sigma_{\text{max}} &= 270 \text{ MPa} \\ \sigma_{\text{min}} &= -330 \text{ MPa}\end{aligned}$$

The answer is (B).

8. Use Hooke's law for shear.

$$\begin{aligned}\gamma = \frac{\tau_{xy}}{G} &= \frac{(35 \text{ MPa})\left(10^6 \ \frac{\text{Pa}}{\text{MPa}}\right)}{(75 \text{ GPa})\left(10^9 \ \frac{\text{Pa}}{\text{GPa}}\right)} \\ &= 4.67 \times 10^{-4} \text{ rad} \quad (4.7 \times 10^{-4} \text{ rad})\end{aligned}$$

The answer is (B).

9. The Poisson ratio is almost always in the range $0 < \nu < 0.5$. Option (A) (0.35) is the only answer that satisfies this condition.

The answer is (A).

10. The maximum shear stress is

$$\begin{aligned}\tau_{\max} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\\ &= \sqrt{\left(\frac{120\ \text{MPa} - 0\ \text{MPa}}{2}\right)^2 + (-50\ \text{MPa})^2}\\ &= 78.10\ \text{MPa} \quad (78\ \text{MPa})\end{aligned}$$

The answer is (D).

32 Thermal, Hoop, and Torsional Stress

PRACTICE PROBLEMS

1. The maximum torque on a 0.15 m diameter solid shaft is 13 500 N·m. What is most nearly the maximum shear stress in the shaft?

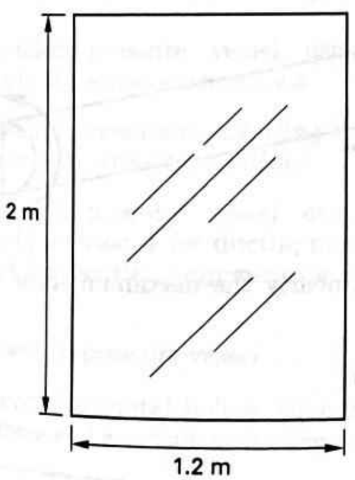
(A) 20 MPa

(B) 23 MPa

(C) 28 MPa

(D) 34 MPa

2. The unrestrained glass window shown is subjected to a temperature change from 0°C to 50°C. The coefficient of thermal expansion for the glass is 8.8×10^{-6} 1/°C.



What is most nearly the change in area of the glass?

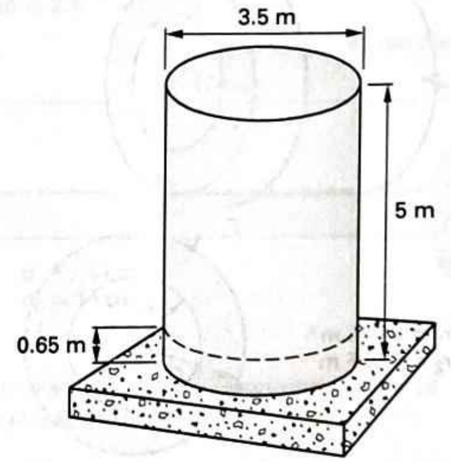
(A) 0.00040 m^2

(B) 0.0013 m^2

(C) 0.0021 m^2

(D) 0.0028 m^2

3. The cylindrical steel tank shown is 3.5 m in diameter, 5 m high, and filled to the top with a brine solution. Brine has a density of 1198 kg/m^3. The thickness of the steel shell is 12.5 mm. Neglect the weight of the tank.



What is the approximate hoop stress in the steel 0.65 m above the rigid concrete pad?

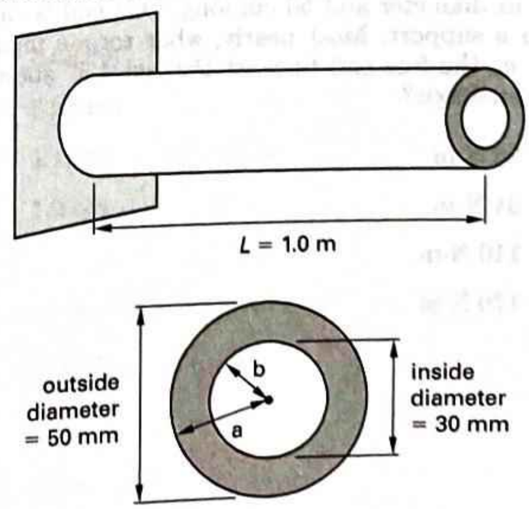
(A) 1.2 MPa

(B) 1.4 MPa

(C) 7.2 MPa

(D) 11 MPa

4. A steel shaft is shown. The shear modulus is 80 GPa.



Most nearly, what torque should be applied to the end of the shaft in order to produce a twist of 1.5°?

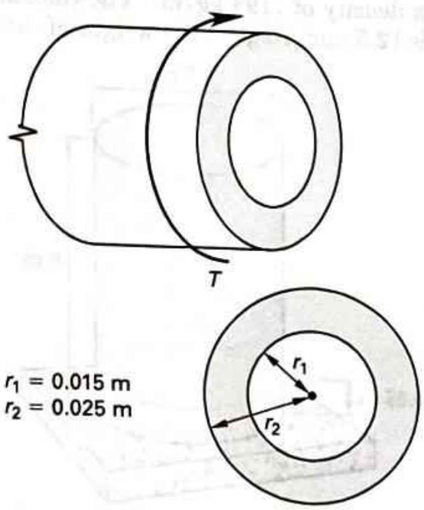
(A) 420 N·m

(B) 560 N·m

(C) 830 N·m

(D) 1100 N·m

5. For the shaft shown, the shear stress is not to exceed 110 MPa.



What is most nearly the largest torque that can be applied?

(A) 1700 N·m

(B) 1900 N·m

(C) 2300 N·m

(D) 3400 N·m

6. An aluminum (shear modulus = 2.8×10^{10} Pa) rod is 25 mm in diameter and 50 cm long. One end is rigidly fixed to a support. Most nearly, what torque must be applied at the free end to twist the rod 4.5° about its longitudinal axis?

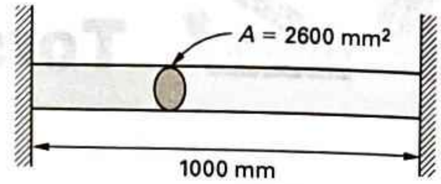
(A) 26 N·m

(B) 84 N·m

(C) 110 N·m

(D) 170 N·m

7. A circular bar at 10°C is constrained by rigid concrete walls at both ends. The bar is 1000 mm long and has a cross-sectional area of 2600 mm².



E = modulus of elasticity
= 200 GPa
α = coefficient of thermal expansion
= 9.4×10^{-6} 1/°C

What is most nearly the axial force in the bar if the temperature is raised to 40°C?

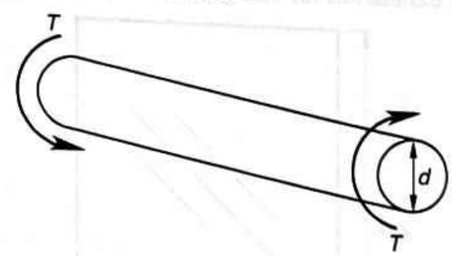
(A) 92 kN

(B) 110 kN

(C) 130 kN

(D) 150 kN

8. A 3 m diameter solid bar experiences opposing torques of 280 N·m at each end.



What is most nearly the maximum shear stress in the bar?

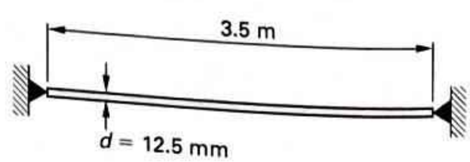
(A) 2.2 Pa

(B) 31 Pa

(C) 42 Pa

(D) 53 Pa

9. A 12.5 mm diameter steel rod is pinned between two rigid walls. The rod is initially unstressed. The rod's temperature subsequently increases 50°C. The rod is adequately stiffened and supported such that buckling does not occur. The coefficient of linear thermal expansion for steel is 11.7×10^{-6} 1/°C. The modulus of elasticity for steel is 210 GPa.



What is the approximate axial force in the rod?

(A) 2.8 kN

(B) 15 kN

(C) 19 kN

(D) 58 kN

10. 10 km of steel railroad track are placed when the temperature is 20°C. The linear coefficient of thermal expansion for the rails is 11×10^{-6} 1/°C. The track is free to slide forward. Most nearly, how far apart will the ends of the track be when the temperature reaches 50°C?

(A) 10.0009 km

(B) 10.0027 km

(C) 10.0033 km

(D) 10.0118 km

11. A deep-submersible diving bell has a cylindrical pressure hull with an outside diameter of 3.5 m and a wall thickness of 15 cm constructed from a ductile material. The hull is expected to experience an external pressure of 50 MPa. The hull should be designed as a

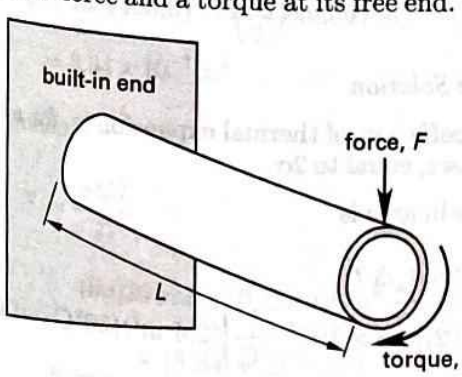
(A) thin-walled pressure vessel using the outer radius in the stress calculations

(B) thin-walled pressure vessel using the logarithmic mean area in stress calculations

(C) thin-walled pressure vessel using factors of safety of at least 4 for ductile materials and at least 8 for brittle components such as viewing ports

(D) thick-walled pressure vessel

12. A cantilever horizontal hollow tube is acted upon by a vertical force and a torque at its free end.



Where is the maximum stress in the cylinder?

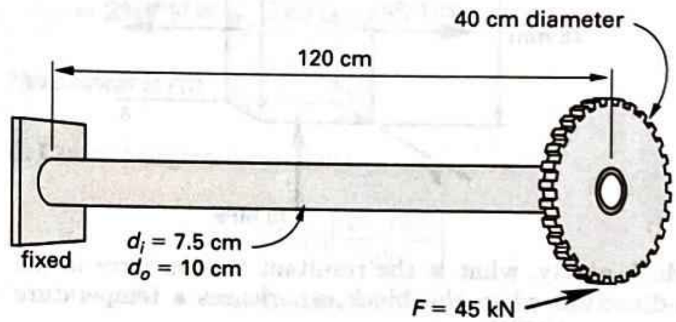
(A) at the upper surface at midlength ($L/2$)

(B) at the lower surface at the built-in end

(C) at the upper surface at the built-in end

(D) at both the upper and lower surfaces at the built-in end

13. One end of the hollow aluminum shaft is fixed, and the other end is connected to a gear with an outside diameter of 40 cm as shown. The gear is subjected to a tangential gear force of 45 kN. The shear modulus of the aluminum is 2.8×10^{10} Pa.



What are most nearly the maximum angle of twist and the shear stress in the shaft?

(A) 0.016 rad, 14 MPa

(B) 0.025 rad, 220 MPa

(C) 0.057 rad, 67 MPa

(D) 0.25 rad, 200 MPa

14. A compressed gas cylinder for use in a laboratory has an internal gage pressure of 8 MPa at the time of delivery. The outside diameter of the cylinder is 25 cm. If the steel has an allowable stress of 90 MPa, what is the required thickness of the wall?

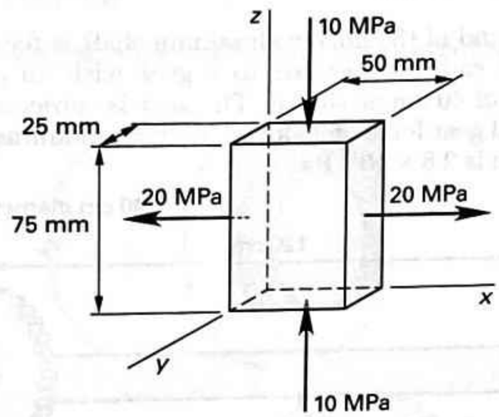
(A) 0.69 cm

(B) 0.95 cm

(C) 1.1 cm

(D) 1.9 cm

15. The steel block shown is subjected to a uniformly distributed stress of 20 MPa (tension) in the x-direction, a strain of 4.23×10^{-4} cm/cm (tension) in the y-direction, and a uniformly distributed stress of 10 MPa (compression) in the z-direction. The steel has a modulus of elasticity of 200,000 MPa, a Poisson's ratio of 0.30, and a coefficient of linear thermal expansion of 6.5×10^{-6} cm/cm/°C.



Most nearly, what is the resultant tension force in the y-direction when the block experiences a temperature increase of 30°C?

(A) 153 kN

(B) 182 kN

(C) 201 kN

(D) 225 kN

SOLUTIONS

1. The polar moment of inertia is

$$J = \frac{\pi r^4}{2} = \left(\frac{\pi}{2}\right)\left(\frac{0.15 \text{ m}}{2}\right)^4 = 4.97 \times 10^{-5} \text{ m}^4$$

The shear stress is

$$\tau = \frac{Tr}{J} = \frac{(13\,500 \text{ N}\cdot\text{m})\left(\frac{0.15 \text{ m}}{2}\right)}{4.97 \times 10^{-5} \text{ m}^4} = 20.37 \times 10^6 \text{ Pa} \quad (20 \text{ MPa})$$

The answer is (A).

2. Changes in temperature affect each linear dimension.

$$\delta_{\text{width}} = \alpha L(T - T_o) = \left(8.8 \times 10^{-6} \frac{1}{°\text{C}}\right)(1.2 \text{ m})(50°\text{C} - 0°\text{C}) = 0.000528 \text{ m}$$

$$\delta_{\text{height}} = \left(8.8 \times 10^{-6} \frac{1}{°\text{C}}\right)(2 \text{ m})(50°\text{C} - 0°\text{C}) = 0.00088 \text{ m}$$

$$A_{\text{initial}} = (2 \text{ m})(1.2 \text{ m}) = 2.4 \text{ m}^2$$

$$A_{\text{final}} = (2 \text{ m} + 0.00088 \text{ m}) \times (1.2 \text{ m} + 0.000528 \text{ m}) = 2.40211 \text{ m}^2$$

$$\Delta A = A_{\text{final}} - A_{\text{initial}} = 2.40211 \text{ m}^2 - 2.4 \text{ m}^2 = 0.00211 \text{ m}^2 \quad (0.0021 \text{ m}^2)$$

Alternative Solution

The area coefficient of thermal expansion is, for all practical purposes, equal to 2α.

The change in area is

$$\Delta A = 2\alpha A_o \Delta T = (2)\left(8.8 \times 10^{-6} \frac{1}{°\text{C}}\right)(2.4 \text{ m}^2)(50°\text{C} - 0°\text{C}) = 0.00211 \text{ m}^2 \quad (0.0021 \text{ m}^2)$$

The answer is (C).

3. Determine whether the tank is thin-walled or thick-walled.

$$\frac{t}{r} = \frac{12.5 \text{ mm}}{\left(\frac{3.5 \text{ m}}{2}\right)\left(1000 \ \frac{\text{mm}}{\text{m}}\right)} = 0.007 < 0.1$$

Use formulas for thin-walled cylindrical tanks. The pressure is

$$\begin{aligned} p &= \rho g h \\ &= \left(1198 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(5 \text{ m} - 0.65 \text{ m}) \\ &= 51\,123 \text{ Pa} \end{aligned}$$

The hoop stress is

$$\begin{aligned} \sigma_t &= \frac{pr}{t} = \frac{pd}{2t} = \frac{(51\,123 \text{ Pa})(3.5 \text{ m})}{(2)\left(\frac{12.5 \text{ mm}}{1000 \ \frac{\text{mm}}{\text{m}}}\right)} \\ &= 7.157 \times 10^6 \text{ Pa} \quad (7.2 \text{ MPa}) \end{aligned}$$

The answer is (C).

4. Convert the twist angle to radians.

$$\phi = (1.5°)\left(\frac{2\pi \text{ rad}}{360°}\right) = 0.026 \text{ rad}$$

Calculate the polar moment of inertia, J.

$$\begin{aligned} b_{(\text{inner radius})} &= 15 \text{ mm} \quad (0.015 \text{ m}) \\ a_{(\text{outer radius})} &= 25 \text{ mm} \quad (0.025 \text{ m}) \\ J &= \frac{\pi}{2}(a_o^4 - b_i^4) = \left(\frac{\pi}{2}\right)\left((0.025 \text{ m})^4 - (0.015 \text{ m})^4\right) \\ &= 5.34 \times 10^{-7} \text{ m}^4 \end{aligned}$$

The torque is

$$\begin{aligned} T &= \frac{\phi G J}{L} \\ &= \frac{(0.026 \text{ rad})(80 \text{ GPa})\left(10^9 \ \frac{\text{Pa}}{\text{GPa}}\right) \times (5.34 \times 10^{-7} \text{ m}^4)}{1.0 \text{ m}} \\ &= 1110.72 \text{ N·m} \quad (1100 \text{ N·m}) \end{aligned}$$

The answer is (D).

5. Since the shear stress is largest at the outer diameter, the maximum torque is found using this radius. For an annular region,

$$\begin{aligned} J &= \frac{\pi}{2}(r_o^4 - r_i^4) = \left(\frac{\pi}{2}\right)\left((0.025 \text{ m})^4 - (0.015 \text{ m})^4\right) \\ &= 5.34 \times 10^{-7} \text{ m}^4 \end{aligned}$$

The torque is

$$\begin{aligned} T_{\max} &= \frac{\tau J}{r_2} = \frac{(110 \text{ MPa})\left(10^6 \ \frac{\text{Pa}}{\text{MPa}}\right)(5.34 \times 10^{-7} \text{ m}^4)}{0.025 \text{ m}} \\ &= 2349.9 \text{ N·m} \quad (2300 \text{ N·m}) \end{aligned}$$

The answer is (C).

6. Convert degrees to radians.

$$\begin{aligned} \phi &= (4.5°)\left(\frac{2\pi \text{ rad}}{360°}\right) \\ &= 7.854 \times 10^{-2} \text{ rad} \end{aligned}$$

The polar moment of inertia is

$$\begin{aligned} J &= \frac{\pi r^4}{2} = \left(\frac{\pi}{2}\right)\left(\frac{25 \text{ mm}}{(2)\left(1000 \ \frac{\text{mm}}{\text{m}}\right)}\right)^4 \\ &= 3.83 \times 10^{-8} \text{ m}^4 \end{aligned}$$

Rearrange the twist angle equation to solve for torque.

$$\begin{aligned} T &= \frac{\phi G J}{L} \\ &= \frac{(7.854 \times 10^{-2} \text{ rad})(2.8 \times 10^{10} \text{ Pa})(3.83 \times 10^{-8} \text{ m}^4)}{\frac{50 \text{ cm}}{100 \ \frac{\text{cm}}{\text{m}}}} \\ &= 168.7 \text{ N·m} \quad (170 \text{ N·m}) \end{aligned}$$

The answer is (D).

7. The elongation due to temperature change is

$$\begin{aligned} \delta &= \alpha L(T_2 - T_1) \\ &= \left(9.4 \times 10^{-6} \ \frac{1}{°\text{C}}\right)(1000 \text{ mm})(40°\text{C} - 10°\text{C}) \\ &= 0.282 \text{ mm} \end{aligned}$$

Rearrange the elongation equation to solve for force.

$$F = \frac{\delta EA}{L}$$
$$= \frac{(0.282\ \text{mm})(200\ \text{GPa})\left(10^6\ \frac{\text{kPa}}{\text{GPa}}\right)(2600\ \text{mm}^2)}{(1\ \text{m})\left(1000\ \frac{\text{mm}}{\text{m}}\right)^3}$$
$$= 146.6\ \text{kN} \quad (150\ \text{kN})$$

The answer is (D).

8. Maximum shear stress occurs at the outer surface. The shear is

$$\tau = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{(280\ \text{N·m})\left(\frac{3\ \text{m}}{2}\right)}{\left(\frac{\pi}{32}\right)(3\ \text{m})^4}$$
$$= 52.8\ \text{Pa} \quad (53\ \text{Pa})$$

The answer is (D).

9. The thermal strain is

$$\varepsilon_t = \alpha\Delta T = \left(11.7\times 10^{-6}\ \frac{1}{°\text{C}}\right)(50°\text{C})$$
$$= 0.000585\ \text{m/m}$$

The thermal stress is

$$\sigma_t = E\varepsilon_t = (210\ \text{GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)\left(0.000585\ \frac{\text{m}}{\text{m}}\right)$$
$$= 1.2285\times 10^8\ \text{Pa}$$

(This is less than the yield strength of steel.)

The compressive force in the rod is

$$F = \sigma A$$
$$= (1.2285\times 10^8\ \text{Pa})\pi\left(\frac{12.5\ \text{mm}}{(2)\left(1000\ \frac{\text{mm}}{\text{m}}\right)}\right)^2$$
$$= 15\,076\ \text{N} \quad (15\ \text{kN})$$

The answer is (B).

10. The total change in length is

$$\delta_t = \alpha L_{\text{initial}}(T - T_o)$$
$$= \left(11\times 10^{-6}\ \frac{1}{°\text{C}}\right)(10\ \text{km})(50°\text{C} - 20°\text{C})$$
$$= 0.0033\ \text{km}$$

Add the change in length to the initial length.

$$L = L_{\text{initial}} + \delta_t$$
$$= 10\ \text{km} + 0.0033\ \text{km}$$
$$= 10.0033\ \text{km}$$

The answer is (C).

11. Tanks under external pressure fail by buckling (i.e., collapse), not by yielding. They should not be designed using the simplistic formulas commonly used for thin-walled tanks under internal pressure.

The answer is (D).

12. The torsional shear stress is maximum at the outer surface and is the same everywhere on the tube. The maximum moment occurs at the built-in end, tensile at the upper surface and compressive at the lower surface. The absolute value of the combined stress at the upper and lower surfaces at the built-in end will be the same.

The answer is (D).

13. Calculate the torque.

$$T = rF = \left(\frac{40\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right)(45\ \text{kN})\left(1000\ \frac{\text{N}}{\text{kN}}\right)$$
$$= 9000\ \text{N·m}$$

The polar moment of inertia is

$$J = \frac{\pi}{2}(r_o^4 - r_i^4)$$
$$= \left(\frac{\pi}{2}\right)\left(\left(\frac{10\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right)^4 - \left(\frac{7.5\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right)^4\right)$$
$$= 6.71\times 10^{-6}\ \text{m}^4$$

Find the angle of twist.

$$\phi = \frac{TL}{GJ} = \frac{(9000\ \text{N·m})(120\ \text{cm})}{(2.8\times 10^{10}\ \text{Pa})(6.71\times 10^{-6}\ \text{m}^4)\left(100\ \frac{\text{cm}}{\text{m}}\right)}$$
$$= 0.057\ \text{rad}$$

Find the shear stress in the shaft.

$$\tau = \frac{Tr}{J} = \frac{(9000\ \text{N·m})\left(\frac{10\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right)}{6.71\times 10^{-6}\ \text{m}^4}$$
$$= 67.05\times 10^{6}\ \text{Pa}\quad (67\ \text{MPa})$$

The answer is (C).

14. Assume a thin-walled tank. Solve the equation for tangential (hoop) stress for the wall thickness. Although the inner radius is used by convention, the outer radius can be used.

$$t = \frac{pd}{2\sigma_t} = \frac{p(d_o - 2t)}{2\sigma_t} \approx \frac{pd_o}{2\sigma_t}$$
$$= \frac{(8\ \text{MPa})(25\ \text{cm})}{(2)(90\ \text{MPa})}$$
$$= 1.11\ \text{cm}\quad (1.1\ \text{cm})$$

Check the thin-wall assumption.

$$\frac{t}{r_i} = \frac{t}{\frac{d_o - 2t}{2}} = \frac{1.11\ \text{cm}}{\frac{25\ \text{cm} - (2)(1.11\ \text{cm})}{2}}$$
$$= 0.098 < 0.1\quad [\text{thin wall}]$$

The answer is (C).

15. For triaxial deformation, tension is considered positive and compression is considered negative. From the generalized equation of Hooke's law,

$$\varepsilon_y = \frac{1}{E}\big(\sigma_y - \nu(\sigma_x + \sigma_z)\big) + \alpha(T_f - T_i)$$
$$\sigma_y = E\big(\varepsilon_y - \alpha(T_f - T_i)\big) + \nu(\sigma_x + \sigma_z)$$
$$= (200\,000\ \text{MPa})\left(4.23\times 10^{-4}\ \frac{\text{cm}}{\text{cm}} - \left(6.5\times 10^{-6}\ \frac{\frac{\text{cm}}{\text{cm}}}{^\circ\text{C}}\right)(30^\circ\text{C})\right)$$
$$+ (0.30)\big(20\ \text{MPa} + (-10\ \text{MPa})\big)$$
$$= 48.6\ \text{MPa}$$

The resultant tension force in the y-direction is

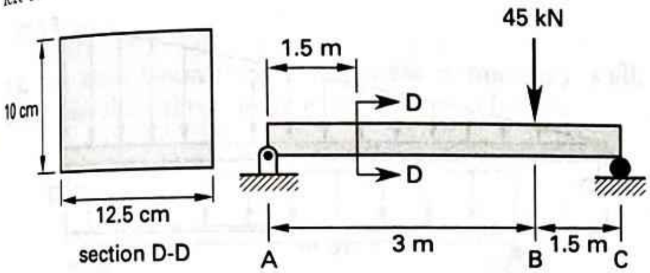
$$P_y = \sigma_y A_y$$
$$= (48.6\ \text{MPa})\big((75\ \text{mm})(50\ \text{mm})\big)\left(\frac{\left(10^6\ \frac{\text{Pa}}{\text{MPa}}\right)}{\left(1000\ \frac{\text{mm}}{\text{m}}\right)^2}\right)$$
$$= 182\,250\ \text{N}\quad (182\ \text{kN})$$

The answer is (B).

33 Beams

PRACTICE PROBLEMS

1. For the beam shown, what is most nearly the maximum compressive stress at section D-D, 1.5 m from the left end?



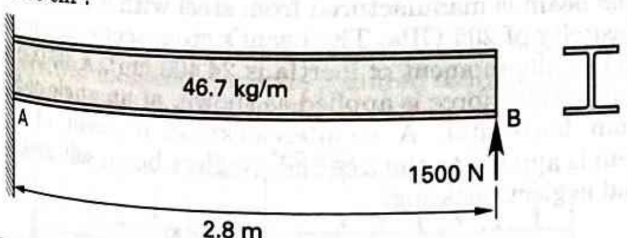
(A) 63 MPa

(B) 110 MPa

(C) 230 MPa

(D) 330 MPa

2. Refer to the beam shown. The beam is fixed at one end. The beam has a mass of 46.7 kg/m. The modulus of elasticity of the beam is 200 GPa; the moment of inertia is 4680 cm^4.



The upward force at B is 1500 N. What is most nearly the net deflection of the beam at a point 1.2 m from the fixed end?

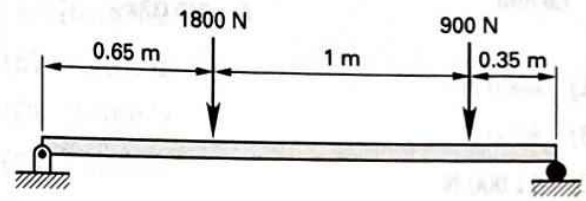
(A) −0.32 mm (downward)

(B) −0.29 mm (downward)

(C) 0.12 mm (upward)

(D) 0.17 mm (upward)

3. Refer to the simply supported beam shown.



What is most nearly the maximum bending moment?

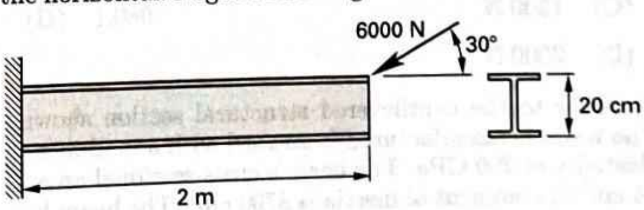
(A) 340 N·m

(B) 460 N·m

(C) 660 N·m

(D) 890 N·m

4. Refer to the cantilevered structural section shown. The beam is manufactured from steel with a modulus of elasticity of 210 GPa. The beam's cross-sectional area is 37.9 cm^2; its moment of inertia is 2880 cm^4. The beam has a mass of 45.9 kg/m. A 6000 N compressive force is applied at the top of the beam, at an angle of 30° from the horizontal. Neglect buckling.



What is most nearly the maximum vertical shear force in the beam?

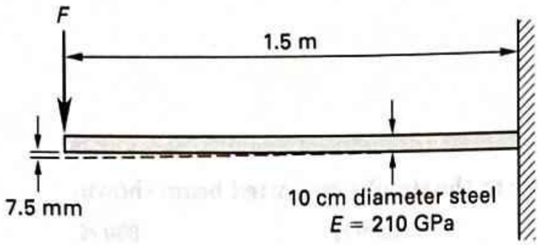
(A) 3000 N

(B) 3900 N

(C) 5200 N

(D) 6100 N

5. For the cantilever steel rod shown, what is most nearly the force, F, necessary to deflect the rod a vertical distance of 7.5 mm?



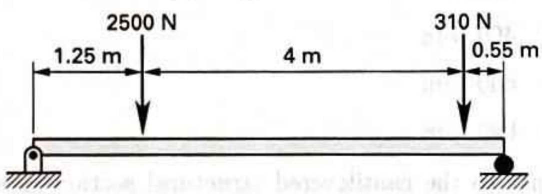
(A) 6900 N

(B) 8800 N

(C) 11 000 N

(D) 17 000 N

6. Refer to the simply supported beam shown.



What is most nearly the maximum shear?

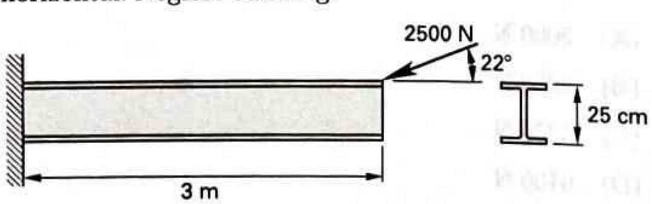
(A) 500 N

(B) 1000 N

(C) 1500 N

(D) 2000 N

7. Refer to the cantilevered structural section shown. The beam is manufactured from steel with a modulus of elasticity of 200 GPa. The beam's cross-sectional area is 74 cm²; its moment of inertia is 8700 cm⁴. The beam has a mass of 60 kg/m. A 2500 N compressive force is applied at the top of the beam, at an angle of 22° from horizontal. Neglect buckling.



What is most nearly the approximate absolute value of the maximum bending moment in the beam?

(A) 5000 N·m

(B) 5200 N·m

(C) 5900 N·m

(D) 6100 N·m

8. A rectangular beam has a cross section of 5 cm wide × 10 cm deep and experiences a maximum shear of 2250 N. What is most nearly the maximum shear stress in the beam?

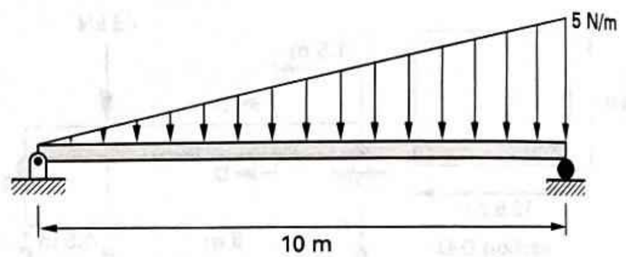
(A) 450 kPa

(B) 570 kPa

(C) 680 kPa

(D) 790 kPa

9. A simply supported beam supports a triangular distributed load as shown. The peak load at the right end of the beam is 5 N/m.



What is the approximate bending moment at a point 7 m from the left end of the beam?

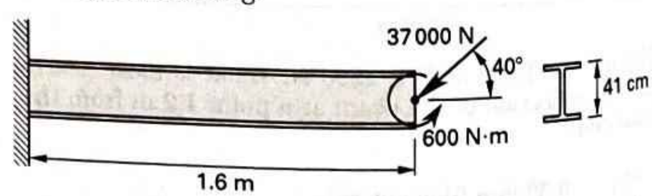
(A) 15 N·m

(B) 17 N·m

(C) 28 N·m

(D) 30 N·m

10. Refer to the cantilevered structural section shown. The beam is manufactured from steel with a modulus of elasticity of 205 GPa. The beam's cross-sectional area is 86 cm²; its moment of inertia is 24 400 cm⁴. A 37 000 N compressive force is applied as shown, at an angle of 40° from horizontal. A counterclockwise moment of 600 N·m is applied to the free end. Neglect beam self-weight, and neglect buckling.



What is most nearly the deflection at the tip of the beam due to the external force alone (i.e., neglecting the beam's own mass)?

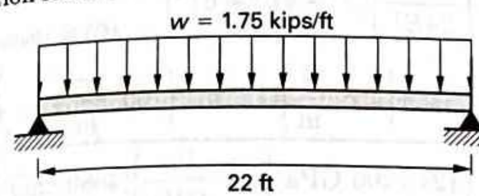
(A) 0.63 mm

(B) 0.82 mm

(C) 1.2 mm

(D) 2.5 mm

11. For the structural steel beam shown, the moment of inertia is 250 in4. Most nearly, what is the maximum deflection for the beam?



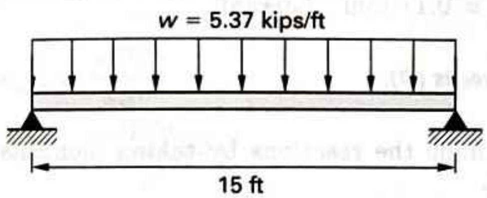
(A) 0.11 in

(B) 0.25 in

(C) 1.27 in

(D) 3.6 in

12. The steel beam shown supports a masonry wall. Local codes limit the masonry deflection to L/360.



What is most nearly the minimum moment of inertia for the beam?

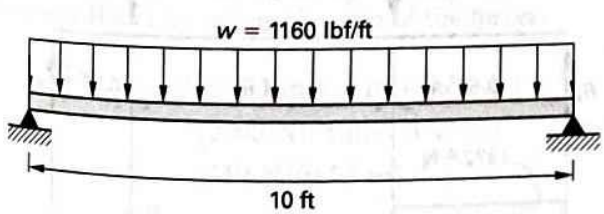
(A) 285 in^4

(B) 422 in^4

(C) 660 in^4

(D) 880 in^4

13. A steel beam is shown. What is most nearly the MINIMUM moment of inertia required to limit the elastic slope to less than 5×10^{-4}?



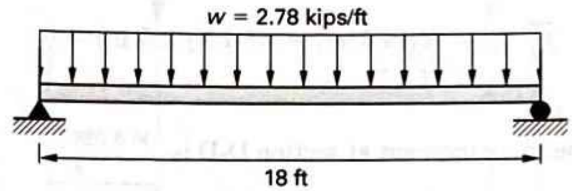
(A) 425 in^4

(B) 480 in^4

(C) 507 in^4

(D) 525 in^4

14. For the steel beam shown, the modulus of elasticity is 29,000 ksi, and the moment of inertia is 468 in^4. What is most nearly the deflection 5 ft from the left support?



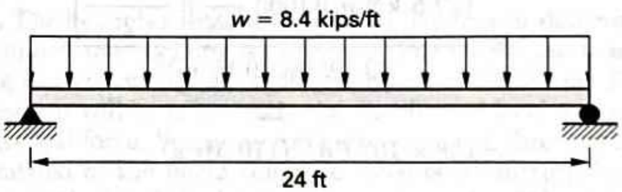
(A) 0.0300 in

(B) 0.3000 in

(C) 0.4000 in

(D) 0.6700 in

15. For the steel beam shown, the moment of inertia is 267 in^4. Most nearly, what is the maximum deflected slope for the beam?



(A) 0.0060

(B) 0.0700

(C) 0.0900

(D) 1.080

SOLUTIONS

1. Find the reaction at A.

$$\sum M_C = R_A(4.5\text{ m}) - (45\text{ kN})(1.5\text{ m}) = 0$$
$$R_A = 15\text{ kN}$$

The bending moment at section D-D is

$$M = (15\text{ kN})(1.5\text{ m}) = 22.5\text{ kN·m}$$

The maximum compressive stress is at the top fiber of the beam section.

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\frac{h}{2}}{\frac{bh^3}{12}}$$
$$= \frac{(22.5\text{ kN·m})\left(1000\ \frac{\text{N}}{\text{kN}}\right)\left(\frac{0.10\text{ m}}{2}\right)}{\frac{(0.125\text{ m})(0.10\text{ m})^3}{12}}$$
$$= 108\times 10^6\text{ Pa}\quad (110\text{ MPa})$$

The answer is (B).

2. Use the principle of superposition to determine the deflection. The total deflection is the upward deflection due to the concentrated force less the downward deflection due to the weight of the beam.

The distance to the point of interest, x, is 1.2 m, and the distance to the point where the force is applied, a, is 2.8 m, so $a > x$. The force is upward, and the force in the equation is defined as positive in the downward direction, so the force is negative. The upward deflection of the beam due to the concentrated force is

$$v_{x,1} = \frac{-Px^2}{6EI}(-x+3a),\ \text{for } x \leq a$$
$$= \frac{-(-1500\text{ N})(1.2\text{ m})^2\left(100\ \frac{\text{cm}}{\text{m}}\right)^4}{(6)(200\text{ GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)(4680\text{ cm}^4)}\times(-1.2\text{ m}+(3)(2.8\text{ m}))$$

The downward deflection is due to the beams own mass. Distance x is measured from the fixed end. The load per unit length is

$$w = mg = \left(46.7\ \frac{\text{kg}}{\text{m}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 458\text{ N/m}$$

The downward deflection is

$$v_{x,2} = \left(\frac{-wx^2}{24EI}\right)(x^2 - 4Lx + 6L^2)$$
$$= \left(\frac{-\left(458\ \frac{\text{N}}{\text{m}}\right)(1.2\text{ m})^2\left(100\ \frac{\text{cm}}{\text{m}}\right)^4}{(24)(200\text{ GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)(4680\text{ cm}^4)}\right)\times\left(\begin{array}{c}(1.2\text{ m})^2 - (4)(2.8\text{ m})(1.2\text{ m})\\ +(6)(2.8\text{ m})^2\end{array}\right)$$
$$= -0.000103\text{ m}\quad(-0.10\text{ mm})\quad[\text{downward}]$$

The net deflection is

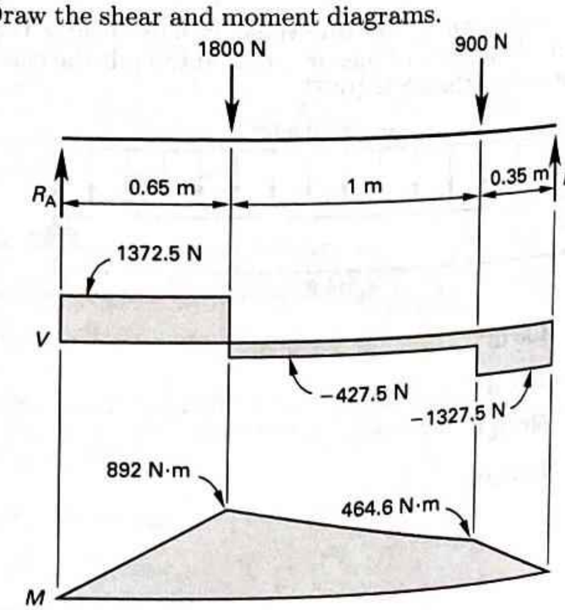
$$v = v_{x,1} + v_{x,2} = 0.28\text{ mm} + (-0.10\text{ mm})$$
$$= 0.17\text{ mm}\quad[\text{upward}]$$

The answer is (D).

3. Determine the reactions by taking moments about each end.

$$\sum M_B = -R_A(0.65\text{ m} + 1\text{ m} + 0.35\text{ m}) + (1800\text{ N})(1\text{ m} + 0.35\text{ m}) + (900\text{ N})(0.35\text{ m}) = 0$$
$$R_A = 1372.5\text{ N}$$
$$\sum F_y = R_B + 1372.5\text{ N} - 1800\text{ N} - 900\text{ N} = 0$$
$$R_B = 1327.5\text{ N}$$

Draw the shear and moment diagrams.



$$M = VR_A = (1372.5\text{ N})(0.65\text{ m}) = 892\text{ N·m}$$

The maximum moment occurs 0.65 m from the left end (where V goes through zero) of the beam and is equal to 892 N·m (890 N·m).

The answer is (D).

4. The maximum vertical shear in the beam will occur at the fixed end.

$$\begin{aligned} V &= wL + F_y \\ &= mgL + F_y \\ &= \left(45.9\ \frac{\text{kg}}{\text{m}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\ \text{m}) + (6000\ \text{N})(\sin 30°) \\ &= 3900\ \text{N} \end{aligned}$$

The answer is (B).

5. For a cantilever beam loaded at its tip, with $x = L$,

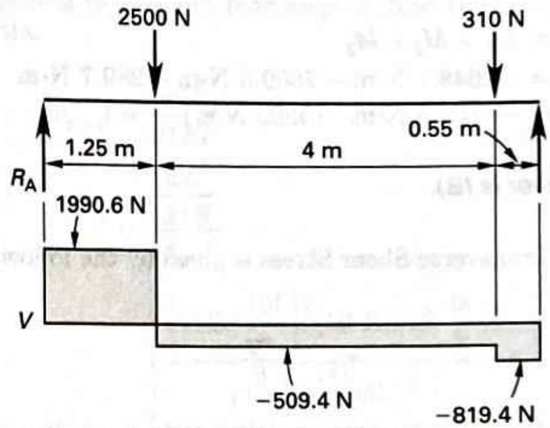
$$\begin{aligned} v_{\max} &= \frac{-PL^3}{3EI} \\ P &= \frac{-3EIv_{\max}}{L^3} \\ &= \frac{-(3)(210\ \text{GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right) \times\left(\frac{\pi}{4}\right)(0.05\ \text{m})^4(7.5\ \text{mm})}{(1.5\ \text{m})^3\left(1000\ \frac{\text{mm}}{\text{m}}\right)} \\ &= -6872\ \text{N} \quad (6900\ \text{N}) \quad [\text{downward}] \end{aligned}$$

The answer is (A).

6. Determine the reactions by taking the moments about end B and by taking the sum of the forces.

$$\begin{aligned} \sum M_{\text{B}} &= -R_{\text{A}}(1.25\ \text{m} + 4\ \text{m} + 0.55\ \text{m}) \\ &\quad + (2500\ \text{N})(4\ \text{m} + 0.55\ \text{m}) \\ &\quad + (310\ \text{N})(0.55\ \text{m}) \\ &= 0 \\ R_{\text{A}} &= 1990.6\ \text{N} \\ \sum F_y &= R_{\text{B}} + 1990.6\ \text{N} - 2500\ \text{N} - 310\ \text{N} \\ &= 0 \\ R_{\text{B}} &= 819.4\ \text{N} \end{aligned}$$

Draw the shear diagram.



From the shear diagram, the maximum shear is 1990.6 N (2000 N).

The answer is (D).

7. The beam is assumed to have no bending or deformation because buckling is neglected. The maximum bending moment will occur at the fixed end of the beam. The moment will be affected by the distributed load and the external force. Since the force does not act through the centroid of the beam (i.e., the force is eccentric), both the vertical and the horizontal components of the external force must be included.

The moment due to the beams own mass is

$$\begin{aligned} M_1 &= -\tfrac{1}{2}wL^2 = -\tfrac{1}{2}mgL^2 \\ &= -\left(\frac{1}{2}\right)\left(60\ \frac{\text{kg}}{\text{m}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3\ \text{m})^2 \\ &= -2648.7\ \text{N·m} \end{aligned}$$

The moment due to the vertical component of the external force is

$$\begin{aligned} M_2 &= -F_yL = -(2500\ \text{N})(\sin 22°)(3\ \text{m}) \\ &= -2809.5\ \text{N·m} \end{aligned}$$

The force is not applied through the beams centroid. The horizontal component of the force causes the beam to bend upward, while the other forces bend the beam downward. The moment due to the eccentricity is

$$\begin{aligned} M_3 &= F_xe = (2500\ \text{N})(\cos 22°)\left(\frac{25\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right) \\ &= 289.7\ \text{N·m} \end{aligned}$$

The total moment is

$$\begin{aligned} M &= M_1 + M_2 + M_3 \\ &= -2648.7\ \text{N·m} - 2809.5\ \text{N·m} + 289.7\ \text{N·m} \\ &= -5168.5\ \text{N·m} \quad (5200\ \text{N·m}) \end{aligned}$$

The answer is (B).

8. The Transverse Shear Stress is given by the following equation

$$\tau_{xy} = \frac{VQ}{Ib}$$

The maximum shear stress occurs at the centerline of a rectangular section, so the previous equation resolves as follows, where Q, in this case, is the first moment of area above the beam section centerline, I, the moment of inertia of the shape, b, the width of the beam, and h, the depth of the beam.

$$\begin{aligned} \tau_{xy} &= \frac{VQ}{Ib} \\ &= \frac{VA'\bar{y}}{\frac{bh^3}{12} \times b} = \frac{VA'\bar{y}}{\frac{b^2h^3}{12}} \\ &= \frac{Vb \times \frac{h^2}{8}}{\frac{b^2h^3}{12}} = \frac{3V}{2bh} \\ &= \frac{(3)(2250)}{(2)(5\ \text{cm})(10\ \text{cm})\left(100\ \frac{\text{cm}}{\text{m}}\right)^2} \\ &= 675 \times 10^3\ \text{Pa} \quad (680\ \text{kPa}) \end{aligned}$$

The answer is (C).

9. The total force from the distributed load is

$$\left(\frac{1}{2}\right)(10\ \text{m})\left(5\ \frac{\text{N}}{\text{m}}\right) = 25\ \text{N}$$

This force can be assumed to act at two-thirds of the beam length from the left end, or one-third of the beam length from the right end.

Sum the moments around the right end to find the left reaction.

$$\begin{aligned} \sum M_{\text{right end}} &= (25\ \text{N})\left(\frac{10\ \text{m}}{3}\right) - R_{\text{left}}(10\ \text{m}) = 0 \\ R_{\text{left}} &= 8.33\ \text{N} \end{aligned}$$

The load increases linearly to 5 N/m at 10 m. At 7 m, the loading is (0.7)(5 N/m). The total distributed force over the first 7 m of the beam is

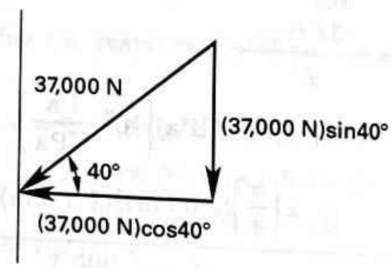
$$\left(\frac{1}{2}\right)(7\ \text{m})\left((0.7)\left(5\ \frac{\text{N}}{\text{m}}\right)\right) = 12.25\ \text{N}$$

Take the sum of the moments about the point of interest (7 m from the left end) due to the distributed force to the left of the point of interest and the reaction at the left end. The calculation is easier to the left.

$$\begin{aligned} \sum M &= (12.25\ \text{N})\left(\frac{7\ \text{m}}{3}\right) - (8.33\ \text{N})(7\ \text{m}) \\ &= -29.73\ \text{N·m} \quad (30\ \text{N·m}) \end{aligned}$$

The answer is (D).

10. The vertical component of the linear force causes deflection. The vertical component is shown in the following figure.



With $x = L$, the deflection due to the vertical component of the force is

$$\begin{aligned} v_1 &= \frac{-PL^3}{3EI} = \frac{-(37\,000\ \text{N})(\sin 40°)(1.6\ \text{m})^3\left(100\ \frac{\text{cm}}{\text{m}}\right)^4}{(3)(205\ \text{GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)(24\,400\ \text{cm}^4)} \\ &= -0.000649\ \text{m} \quad (-0.649\ \text{mm}) \quad \text{[downward]} \end{aligned}$$

The eccentric application of the force causes an upward deflection. The deflection due to the end moment is

$$\begin{aligned} v_2 &= \frac{M_0L^2}{2EI} = \frac{(600\ \text{N·m})(1.6\ \text{m})^2\left(100\ \frac{\text{cm}}{\text{m}}\right)^4}{(2)(205\ \text{GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)(24\,400\ \text{cm}^4)} \\ &= 0.0000154\ \text{m} \quad (0.0154\ \text{mm}) \quad \text{[upward]} \end{aligned}$$

The total deflection due to the external force alone is

$$\begin{aligned} v &= v_1 + v_2 = -0.649\ \text{mm} + 0.0154\ \text{mm} \\ &= -0.634\ \text{mm} \quad (0.63\ \text{mm}) \quad \text{[downward]} \end{aligned}$$

The answer is (A).

11. Maximum deflection occurs halfway down the beam from the support. The modulus of elasticity for structural steel is 29,000 ksi. From the equation for maximum deflection for a simply supported beam,

$$|v_{\text{max}}| = \frac{5wL^4}{384EI}$$

$$= \frac{5\left(\dfrac{1.75\ \frac{\text{kips}}{\text{ft}}}{12\ \frac{\text{in}}{\text{ft}}}\right)\left((22\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^4}{(384)\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)(250\text{ in}^4)}$$

$$= 1.27\text{ in}$$

The answer is (C).

12. From the local code, the maximum deflection is

$$v_{\text{max}} = \frac{L}{360} = \frac{(15\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{360} = 0.5\text{ in}$$

Use the equation for maximum deflection for a simply supported beam, and rearrange to find the minimum moment of inertia. Maximum deflection occurs halfway down the beam from the support. The modulus of elasticity for steel is 29,000 ksi.

$$|v_{\text{max}}| = \frac{5wL^4}{384EI}$$

$$I = \frac{5wL^4}{384E|v_{\text{max}}|}$$

$$= \frac{5\left(\dfrac{5.37\ \frac{\text{kips}}{\text{ft}}}{12\ \frac{\text{in}}{\text{ft}}}\right)\left((15\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^4}{(384)\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)(0.5\text{ in})}$$

$$= 421.85\text{ in}^4 \quad (422\text{ in}^4)$$

The answer is (B).

13. Use the equation for maximum slope for a simply supported beam, and rearrange to find the moment of inertia.

$$|\theta_{\text{max}}| = \frac{wL^3}{24EI}$$

$$I = \frac{\dfrac{wL^3}{24E}}{|\theta_{\text{max}}|}$$

$$= \frac{\dfrac{\left(1160\ \frac{\text{lbf}}{\text{ft}}\right)\left((10\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3}{\left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}}{(24)\left(29{,}000\ \frac{\text{kips}}{\text{ft}}\right)(0.0005)}$$

$$= 480\text{ in}^4$$

The answer is (B).

14. Use the equation for elastic curve for a simply supported beam.

$$v_{\text{max}}(x) = \frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$= \left(\frac{\left(\dfrac{2.78\ \frac{\text{kips}}{\text{ft}}}{12\ \frac{\text{in}}{\text{ft}}}\right)(5\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(24)\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)(468\text{ in}^4)}\right) \times \left(\begin{array}{l}\left((18\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3 - (2)(18\text{ ft})(5\text{ ft})^2\left(12\ \frac{\text{in}}{\text{ft}}\right)^3 \\ \quad + \left((5\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3\end{array}\right)$$

$$= 0.3729\text{ in} \quad (0.4000\text{ in})$$

The answer is (C).

15. Use the equation for maximum slope for a simply supported beam. The modulus of elasticity for steel is 29,000 ksi.

$$|\theta_{\max}| = \frac{wL^3}{24EI}$$

$$= \left| \frac{\left(\dfrac{8.4\ \frac{\text{kips}}{\text{ft}}}{12\ \frac{\text{in}}{\text{ft}}}\right)\left((24\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^3}{(24)\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)(267\ \text{in}^4)} \right|$$

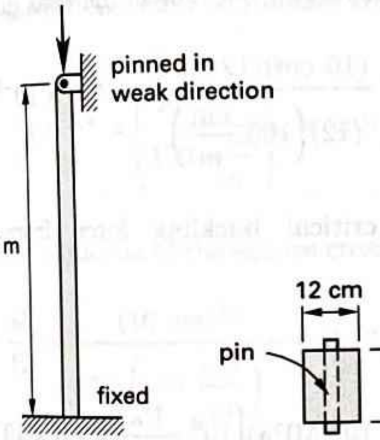
$$= 0.0899 \quad (0.0900)$$

The answer is (C).

34 Columns

PRACTICE PROBLEMS

1. A steel column with a cross section of 12 cm × 16 cm is 4 m in height and fixed at its base. The column is pinned against translation in its weak direction at the top but is unbraced in its strong direction. The column's modulus of elasticity is 2.1×10^5 MPa.



What is most nearly the maximum theoretical vertical load the column can support without buckling?

(A) 1.3 MN

(B) 5.2 MN

(C) 6.1 MN

(D) 11 MN

2. A 10 cm × 10 cm square column supports a compressive force of 9000 N. The load is concentrically applied. What is most nearly the maximum compressive stress in the column?

(A) 450 kPa

(B) 900 kPa

(C) 1400 kPa

(D) 2300 kPa

3. A square column with a solid cross section is placed in a building to support a load of 5 MN. The maximum allowable stress in the column is 350 MPa. The column reacts linearly to all loads. If the contractor is permitted to load the column anywhere in the central one-fifth of the column's cross section, what are most nearly the smallest possible dimensions of the column?

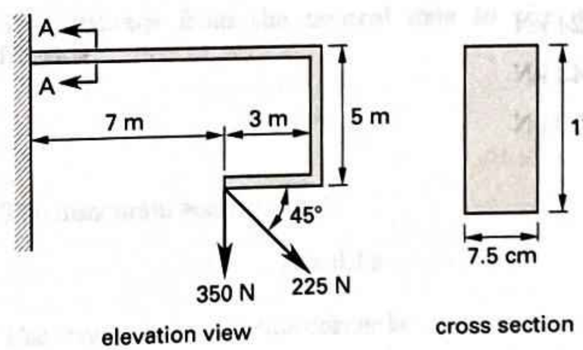
(A) 12 cm × 12 cm

(B) 14 cm × 14 cm

(C) 16 cm × 16 cm

(D) 18 cm × 18 cm

4. What is most nearly the maximum resultant normal stress at A for the cantilever beam shown?



(A) 7.2 MPa

(B) 9.4 MPa

(C) 9.8 MPa

(D) 9.9 MPa

5. A rectangular steel bar 37.5 mm wide and 50 mm thick is pinned at each end and subjected to axial compression. The bar has a length of 1.75 m. The modulus of elasticity is 200 GPa. What is most nearly the critical buckling load?

(A) 60 kN

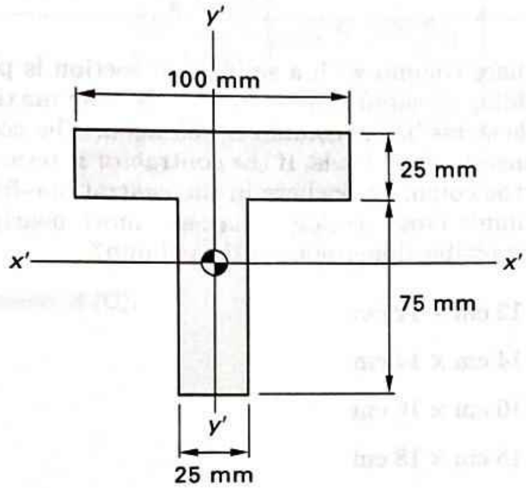
(B) 93 kN

(C) 110 kN

(D) 140 kN

6. What is most nearly the Euler buckling load for a 10 m long steel column with cup-and-ball ends and with the given properties and cross section?

$$I_{x'x'} = 3.70 \times 10^6 \text{ mm}^4$$
$$E = 200 \text{ GPa}$$



(A) 15 kN

(B) 24 kN

(C) 43 kN

(D) 73 kN

SOLUTIONS

1. The column can buckle in either the right-left direction or in the forward-back direction, so both must be checked to determine the maximum load the column can support. The column is 12 cm wide in the right-left direction and 16 cm wide in the forward-back direction, so right-left is the "weak" direction and forward-back is the "strong" direction.

Check the buckling force in the weak direction. Since the column is fixed at one end and pinned at the other, the theoretical end-restraint coefficient, K, is 0.7. The effective length for buckling in the weak direction is

$$K\ell = (0.7)(4 \text{ m}) = 2.8 \text{ m}$$

The moment of inertia for buckling in the weak direction is

$$I = \frac{bh^3}{12} = \frac{(16 \text{ cm})(12 \text{ cm})^3}{(12)\left(100 \ \frac{\text{cm}}{\text{m}}\right)^4} = 2.3 \times 10^{-5} \text{ m}^4$$

Calculate the critical buckling force from Euler's formula.

$$P_{cr} = \frac{\pi^2 EI}{(K\ell)^2}$$
$$= \frac{\pi^2(2.1 \times 10^5 \text{ MPa})\left(10^6 \ \frac{\text{Pa}}{\text{MPa}}\right)(2.3 \times 10^{-5} \text{ m}^4)}{(2.8 \text{ m})^2}$$
$$= 6.09 \times 10^6 \text{ N} \quad (6.1 \text{ MN})$$

Check the buckling force in the strong direction. The column is not braced in that direction, so for a column fixed at one end and free at the other, $K = 2$.

$$K\ell = (2)(4 \text{ m}) = 8 \text{ m}$$

The moment of inertia for buckling in the strong direction is

$$I = \frac{bh^3}{12} = \frac{(12 \text{ cm})(16 \text{ cm})^3}{(12)\left(100 \ \frac{\text{cm}}{\text{m}}\right)^4} = 4.1 \times 10^{-5} \text{ m}^4$$

Calculate the critical buckling force from Euler's formula.

$$P_{cr} = \frac{\pi^2 EI}{(K\ell)^2}$$
$$= \frac{\pi^2(2.1\times 10^5 \text{ MPa})\left(10^6 \frac{\text{Pa}}{\text{MPa}}\right)(4.1\times 10^{-5} \text{ m}^4)}{(8 \text{ m})^2}$$
$$= 1.3\times 10^6 \text{ N} \quad (1.3 \text{ MN})$$

This is less than for buckling in the weak direction. This force controls.

The answer is (A).

2. The cross-sectional area of a square column is

$$A = b^2 = \left(\frac{10 \text{ cm}}{100 \frac{\text{cm}}{\text{m}}}\right)^2 = 0.01 \text{ m}^2$$

The moment of inertia of the square cross section is

$$I = \frac{b^4}{12} = \frac{(10 \text{ cm})^4}{(12)\left(100 \frac{\text{cm}}{\text{m}}\right)^4} = 8.33\times 10^{-6} \text{ m}^4$$

The distance from the neutral axis to the extreme fibers is

$$c = \frac{b}{2} = \frac{10 \text{ cm}}{(2)\left(100 \frac{\text{cm}}{\text{m}}\right)} = 0.05 \text{ m}$$

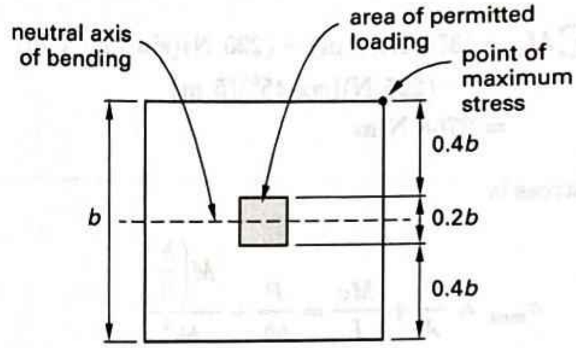
The stress is

$$\sigma = \frac{F}{A}$$
$$= \frac{-9000 \text{ N}}{0.01 \text{ m}^2}$$
$$= -9\times 10^5 \text{ Pa}$$
$$(-900 \text{ kPa})$$

Compressive forces and stresses are usually given a negative sign.

The answer is (B).

3. The middle one-fifth of the column is a square with dimensions of $b/5 \times b/5$ ($0.2b \times 0.2b$).



The maximum stress will be induced when the middle one-fifth square is loaded at one of its corners.

The cross-sectional area is

$$A = b^2$$

The moment of inertia of the square cross section is

$$I = \frac{b^4}{12}$$

The distance from the neutral axis to the extreme fibers is

$$c = \frac{b}{2}$$

The maximum eccentricity is

$$e = 0.1b$$

The stress at the extreme corner is

$$\sigma = \frac{F}{A} \pm \left(\frac{Fe_x c_x}{I_x} + \frac{Fe_y c_y}{I_y}\right) = F\left(\frac{1}{b^2} \pm \frac{(2)(0.1b)\left(\frac{b}{2}\right)}{\frac{b^4}{12}}\right)$$
$$= F\left(\frac{1}{b^2} \pm \frac{1.2}{b^2}\right)$$
$$= \frac{2.2F}{b^2}$$
$$b = \sqrt{\frac{2.2F}{\sigma}} = \sqrt{\frac{(2.2)(5 \text{ MN})\left(10^6 \frac{\text{N}}{\text{MN}}\right)}{(350 \text{ MPa})\left(10^6 \frac{\text{Pa}}{\text{MPa}}\right)}}$$
$$= 0.177 \text{ m} \quad (18 \text{ cm})$$

The answer is (D).

4. The beam experiences both axial tension and bending stresses, so it should be analyzed as a beam-column.

$$\begin{aligned}\sum M_A &= (350\ \text{N})(7\ \text{m}) + (225\ \text{N})(\sin 45°)(7\ \text{m}) \\ &\quad -(225\ \text{N})(\cos 45°)(5\ \text{m}) \\ &= 2768\ \text{N·m}\end{aligned}$$

The stress is

$$\begin{aligned}\sigma_{max} &= \frac{P}{A} + \frac{Mc}{I} = \frac{P}{bh} + \frac{M\left(\frac{h}{2}\right)}{\frac{bh^3}{12}} \\ &= \frac{(225\ \text{N})(\cos 45°)\left(100\ \frac{\text{cm}}{\text{m}}\right)^2}{(7.5\ \text{cm})(17.5\ \text{cm})} \\ &\quad + \frac{(2768\ \text{N·m})\left(\frac{17.5\ \text{cm}}{(2)\left(100\ \frac{\text{cm}}{\text{m}}\right)}\right)}{\frac{(7.5\ \text{cm})(17.5\ \text{cm})^3}{(12)\left(100\ \frac{\text{cm}}{\text{m}}\right)^4}} \\ &= 7.24 \times 10^6\ \text{Pa} \quad (7.2\ \text{MPa})\end{aligned}$$

The answer is (A).

5. Use Euler's formula. $K = 1$ since both ends are pinned. Though the problem states the bar width is 37.5 mm and its depth (thickness) is 50 mm, the critical buckling load is governed by the least moment of inertia of the cross-section. In this case, use b = 50 mm and h = 37.5 mm.

$$\begin{aligned}P_{cr} &= \frac{\pi^2 EI}{(K\ell)^2} \\ &= \frac{\pi^2 E\left(\frac{bh^3}{12}\right)}{(K\ell)^2} \\ &= \frac{\pi^2(200\ \text{GPa})\left(10^9\ \frac{\text{Pa}}{\text{GPa}}\right)\left(\frac{(50\ \text{mm})(37.5\ \text{mm})^3}{(12)\left(1000\ \frac{\text{mm}}{\text{m}}\right)^4}\right)}{\left((1)(1.75\ \text{m})\right)^2} \\ &= 141\,624\ \text{N} \quad (140\ \text{kN})\end{aligned}$$

The answer is (D).

6. $x'x'$ and $y'y'$ are centroidal axes. $I_{y'y'}$ is computed from the equation $I = bh^3/12$ about the centroidal axis of a rectangle. For this cross section, $b_1 = 25$ mm, $h_1 = 100$ mm, $b_2 = 75$ mm, and $h_2 = 25$ mm.

$$\begin{aligned}I_{y'y'} &= \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12} \\ &= \frac{(25\ \text{mm})(100\ \text{mm})^3}{(12)\left(1000\ \frac{\text{mm}}{\text{m}}\right)^4} + \frac{(75\ \text{mm})(25\ \text{mm})^3}{(12)\left(1000\ \frac{\text{mm}}{\text{m}}\right)^4} \\ &= 2.18 \times 10^{-6}\ \text{m}^4\end{aligned}$$

Find the Euler buckling load, P_{cr}. The smallest moment of inertia (corresponding to the least radius of gyration) should be used. $I_{y'y'}$ is less than $I_{x'x'}$.

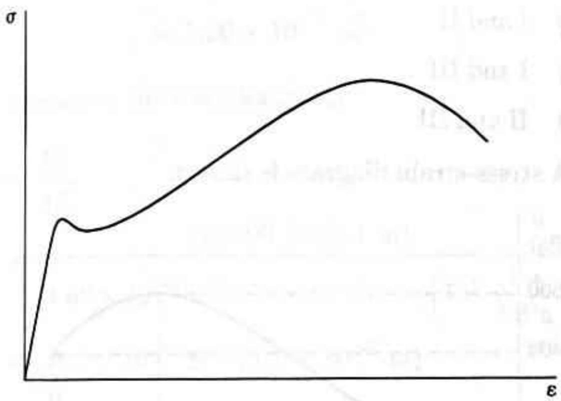
$$\begin{aligned}P_{cr} &= \frac{\pi^2 EI}{(K\ell)^2} \\ &= \frac{\pi^2(200\ \text{GPa})\left(10^6\ \frac{\text{kPa}}{\text{GPa}}\right)(2.18 \times 10^{-6}\ \text{m}^4)}{\left((1)(10\ \text{m})\right)^2} \\ &= 43\ \text{kN}\end{aligned}$$

The answer is (C).

35 Material Properties and Testing

PRACTICE PROBLEMS

1. A stress-strain diagram is shown.



What kind of test might result from this diagram?

(A) resilience test

(B) rotating beam test

(C) ductility test

(D) tensile test

2. A 0.4 m long steel rod has a diameter of 0.05 m and a modulus of elasticity of 20×10^4 MPa. The rod supports a 10 000 N compressive load. Most nearly, what is the decrease in the steel rod's length?

(A) 1.3×10^{-6} m

(B) 2.5×10^{-6} m

(C) 5.1×10^{-6} m

(D) 1.0×10^{-5} m

3. What term is used for the ratio of stress to strain below the proportional limit?

(A) modulus of rigidity

(B) Hooke's constant

(C) Poisson's ratio

(D) Young's modulus

4. What does an impact test measure?

(A) hardness

(B) yield strength

(C) toughness

(D) creep strength

5. The density of a particular metal is 2750 kg/m^3. The modulus of elasticity for this metal is 210 GPa. A circular bar of this metal 3.5 m long and 160 cm^2 in cross-sectional area is suspended vertically from one end. What is most nearly the elongation of the bar due to its own mass?

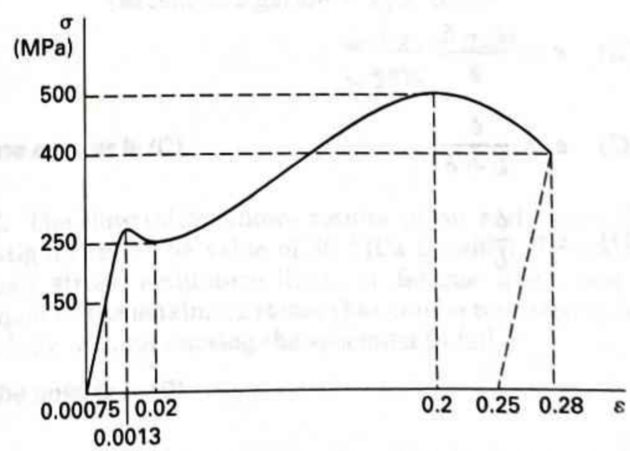
(A) 0.00055 mm

(B) 0.00079 mm

(C) 0.0016 mm

(D) 0.0024 mm

6. A stress-strain diagram is shown.



Most nearly, what is the percent elongation at failure?

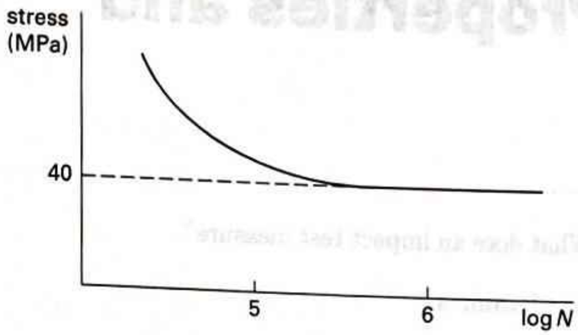
(A) 14%

(B) 19%

(C) 25%

(D) 28%

7. In this illustration, what does the value of 40 MPa represent?



I. fatigue limit
II. endurance limit
III. proportional limit
IV. yield stress

(A) I only
(B) I and II
(C) II and IV
(D) I, II, and IV

8. If δ is deformation, and L is the original length of the specimen, what is the definition of normal strain, ε?

(A) $\varepsilon = \dfrac{L+\delta}{L}$

(B) $\varepsilon = \dfrac{L+\delta}{\delta}$

(C) $\varepsilon = \dfrac{\delta}{L+\delta}$

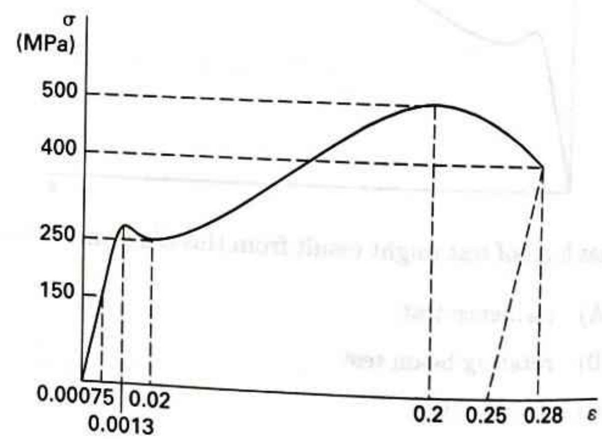
(D) $\varepsilon = \dfrac{\delta}{L}$

9. Which of the following statements regarding the ductile-to-brittle transition temperature is true?

I. It is important for structures used in cold environments.
II. It is the point at which the size of the shear lip or tearing rim goes to zero.
III. It is the temperature at which 20 J of energy causes failure in a Charpy V-notch specimen of standard dimensions.

(A) I only
(B) I and II
(C) I and III
(D) II and III

10. A stress-strain diagram is shown.



Most nearly, what is the modulus of elasticity of the material?

(A) 20 GPa
(B) 80 GPa
(C) 100 GPa
(D) 200 GPa

SOLUTIONS

1. The illustration shows the results from a tensile test. Both resilience and ductility may be calculated from the results, but the test is not known by those names. The rotating beam is a cyclic test and does not yield a monotonic stress-strain curve.

The answer is (D).

2. The area of the steel rod is

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.05\ \text{m})^2}{4} = 1.96 \times 10^{-3}\ \text{m}^2$$

The decrease in the rod's length is

$$\begin{aligned}\delta &= \frac{PL}{AE} \\ &= \frac{(10\,000\ \text{N})(0.4\ \text{m})}{(1.96 \times 10^{-3}\ \text{m}^2)(20 \times 10^4\ \text{MPa})\left(10^6\ \frac{\text{Pa}}{\text{MPa}}\right)} \\ &= 1.019 \times 10^{-5}\ \text{m} \quad (1.0 \times 10^{-5}\ \text{m})\end{aligned}$$

The answer is (D).

3. Young's modulus is defined by Hooke's law.

$$\sigma = E\varepsilon$$

E is Young's modulus, also called the modulus of elasticity, and is equal to stress divided by strain within the proportional region of the stress-strain curve.

The answer is (D).

4. An impact test measures the energy needed to fracture a test sample. This is a measure of toughness.

The answer is (C).

5. The mass of the bar is

$$\begin{aligned}m &= \rho V = \rho A L \\ &= \left(2750\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{160\ \text{cm}^2}{\left(100\ \frac{\text{cm}}{\text{m}}\right)^2}\right)(3.5\ \text{m}) \\ &= 154\ \text{kg}\end{aligned}$$

The total gravitational force is experienced by the metal at the suspension point. Farther down the rod, however, there is less volume contributing to the force, and the stress is reduced. The average force on the metal in the bar is half of the maximum value.

$$\begin{aligned}F_{\text{ave}} &= \tfrac{1}{2}F_{\text{max}} = \tfrac{1}{2}mg \\ &= \left(\frac{1}{2}\right)(154\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 755\ \text{N}\end{aligned}$$

The elongation is

$$\varepsilon = \frac{\Delta L}{L_0}$$

$$\begin{aligned}\Delta L &= \varepsilon L_0 = \frac{\sigma}{E}L_0 = \frac{F}{AE}L_0 \\ &= \left(\frac{755\ \text{N}}{(160\ \text{cm}^2)(210 \times 10^9\ \text{Pa})}\right)(3.5\ \text{m})\left(100\ \frac{\text{cm}}{\text{m}}\right)^2 \\ &= 7.868 \times 10^{-7}\ \text{m} \quad (0.00079\ \text{mm})\end{aligned}$$

The answer is (B).

6. The strain at failure used in the equation is found by extending a line from the failure point to the strain axis, parallel to the linear portion of the curve. The percent elongation is an indicator of the ductility of a material, but it is not the same as the ductility. The percent elongation is

$$\begin{aligned}\text{percent elongation} &= \varepsilon_f \times 100\% \\ &= 0.25 \times 100\% \\ &= 25\%\end{aligned}$$

The answer is (C).

7. The illustration shows results of an endurance (or fatigue) test. The value of 40 MPa is called the endurance stress, endurance limit, or fatigue limit, and is equal to the maximum stress that can be repeated indefinitely without causing the specimen to fail.

The answer is (B).

8. Strain is defined as elongation, δ, per unit length, L.

The answer is (D).

9. Statement II is the only one that is false. A test piece that breaks at 20 J of energy usually has a small shear lip.

The answer is (C).

10. The modulus of elasticity (Young's modulus) is the slope of the stress-strain line in the proportional region.

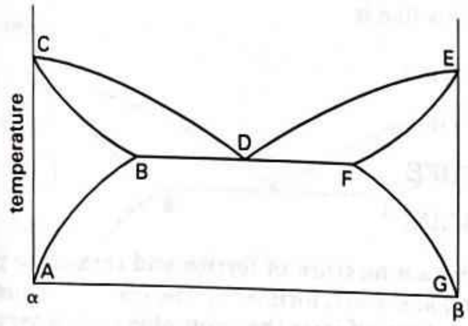
$$\begin{aligned}\sigma &= E\varepsilon \\ E &= \frac{\sigma}{\varepsilon} = \frac{150\ \text{MPa}}{0.00075} \\ &= 200\,000\ \text{MPa} \quad (200\ \text{GPa})\end{aligned}$$

The answer is (D).

36 Engineering Materials

PRACTICE PROBLEMS

1. Refer to the phase diagram shown.



The region enclosed by points DEF can be described as a

(A) mixture of solid β component and liquid α component

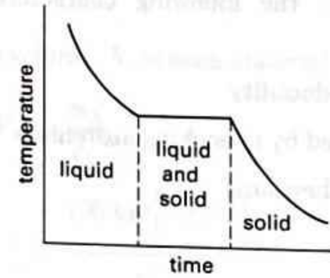
(B) mixture of solid β component and liquid β component

(C) peritectic composition

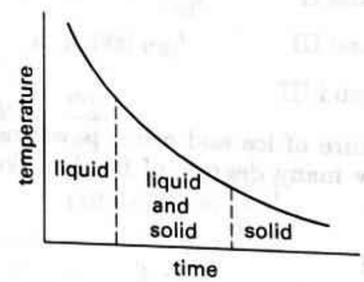
(D) mixture of solid β component and a molten mixture of α and β components

2. Which of the following figures is a cooling curve of a pure metal?

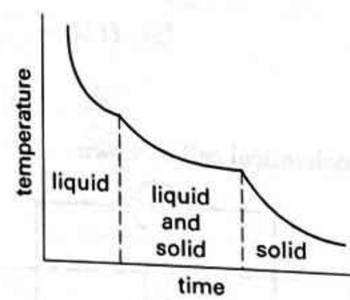
(A)



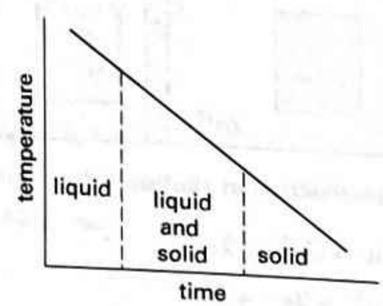
(B)



(C)



(D)



3. A composite material consists of 20 kg of material A, 10 kg of material B, and 5 kg of material C. The densities of materials A, B, and C are 2 g/cm^3, 3 g/cm^3, and 4 g/cm^3, respectively. Most nearly, what is the density of the composite material?

(A) 2.1 g/cm^3
(B) 2.4 g/cm^3
(C) 2.7 g/cm^3
(D) 3.3 g/cm^3

4. Which of the following characteristics describes martensite?

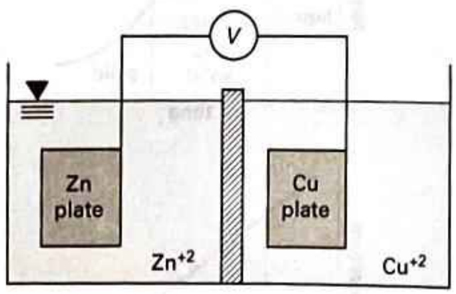
I. high ductility
II. formed by quenching austenite
III. high hardness

(A) I only
(B) I and II
(C) I and III
(D) II and III

5. A mixture of ice and water is warmed from 0°C to 10°C. How many degrees of freedom does the mixture have?

(A) −1
(B) 0
(C) 1
(D) 2

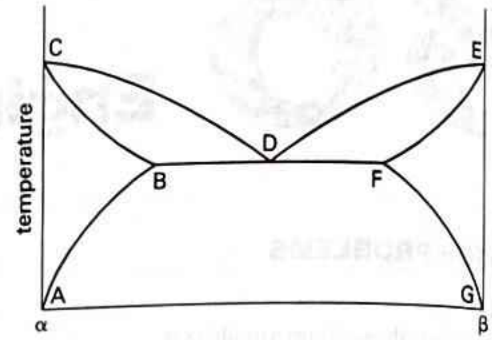
6. An electrochemical cell is shown.



What is the reaction at the anode?

(A) $Cu \rightarrow Cu^{2+} + 2e^-$
(B) $Cu^{2+} + 2e^- \rightarrow Cu$
(C) $Zn \rightarrow Zn^{2+} + 2e^-$
(D) $Zn^{2+} + 2e^- \rightarrow Zn$

7. Refer to the phase diagram shown.



The liquidus line is

(A) CDFG
(B) CDE
(C) CBFE
(D) ABDE

8. Pearlite is a mixture of ferrite and cementite present in cast iron and steel, formed by the cooling of austenite. Ferrite is a form of iron that contains only a very small amount of carbon. Bainite is an aggregate of iron carbide and ferrite, formed from austenite below the temperature at which pearlite forms and above the temperature at which martensite forms. Rapid quenching of austenite can result in martensite. What is the hardest form of steel?

(A) pearlite
(B) ferrite
(C) bainite
(D) martensite

9. Which of the following processes can increase the deformation resistance of steel?

I. tempering
II. hot working
III. adding alloying elements
IV. hardening

(A) I and II
(B) I and IV
(C) II and III
(D) III and IV

Materials

10. Corrosion of iron can be inhibited with a more electropositive coating, while a less electropositive coating tends to accelerate corrosion. Which of these coatings will contribute to corrosion of iron products?

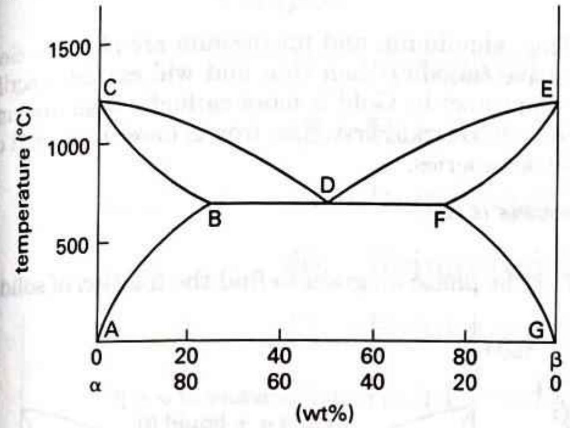
(A) zinc

(B) gold

(C) aluminum

(D) magnesium

11. Refer to the phase diagram shown.



Approximately how much solid (as a percentage by weight) exists when the mixture is 30% α and 70% β and the temperature is 800°C?

(A) 0%

(B) 19%

(C) 30%

(D) 50%

SOLUTIONS

1. The region describes a mixture of solid β component and a liquid of components α and β.

The answer is (D).

2. The solidification of a molten metal is no different than the solidification of water into ice. During the phase change, the temperature remains constant as the heat of fusion is removed. The temperature remains constant during the phase change.

The answer is (A).

3. Calculate the volume, V, of each material.

$$\begin{aligned} V_A &= \frac{m_A}{\rho_A} \\ &= \frac{(20\ \text{kg})\left(1000\ \frac{\text{g}}{\text{kg}}\right)}{2\ \frac{\text{g}}{\text{cm}^3}} \\ &= 10\,000\ \text{cm}^3 \end{aligned}$$

$$\begin{aligned} V_B &= \frac{m_B}{\rho_B} \\ &= \frac{(10\ \text{kg})\left(1000\ \frac{\text{g}}{\text{kg}}\right)}{3\ \frac{\text{g}}{\text{cm}^3}} \\ &= 3333\ \text{cm}^3 \end{aligned}$$

$$\begin{aligned} V_C &= \frac{m_C}{\rho_C} \\ &= \frac{(5\ \text{kg})\left(1000\ \frac{\text{g}}{\text{kg}}\right)}{4\ \frac{\text{g}}{\text{cm}^3}} \\ &= 1250\ \text{cm}^3 \end{aligned}$$

The total volume is

$$\begin{aligned} V_{tot} &= V_A + V_B + V_C \\ &= 10\,000\ \text{cm}^3 + 3333\ \text{cm}^3 + 1250\ \text{cm}^3 \\ &= 14\,583\ \text{cm}^3 \end{aligned}$$

The density of the composite material is

$$\begin{aligned}\rho_c &= \sum f_i\rho_i = f_A\rho_A + f_B\rho_B + f_C\rho_C \\ &= \frac{V_A\rho_A + V_B\rho_B + V_C\rho_C}{V_{tot}} \\ &= \frac{(10\,000\ \text{cm}^3)\left(2\ \frac{\text{g}}{\text{cm}^3}\right) + (3333\ \text{cm}^3)\left(3\ \frac{\text{g}}{\text{cm}^3}\right) + (1250\ \text{cm}^3)\left(4\ \frac{\text{g}}{\text{cm}^3}\right)}{14\,583\ \text{cm}^3} \\ &= 2.4\ \text{g/cm}^3\end{aligned}$$

The answer is (B).

4. Martensite is a hard, strong, and brittle material formed by rapid cooling of austenite.

Ductility is the property of a material to be able to change its shape easily. Because quenching makes ferrous metals hard, and martensite is a very hard metal, it has very low ductility.

The answer is (D).

5. Solid and liquid phases are present simultaneously, so the number of phases, P, is 2. Only water is involved, so the number of compounds, C, is 1.

Gibbs phase rule is applicable when both temperature and pressure can be varied. When the temperature is varied, Gibbs phase rule becomes

$$\begin{aligned}P + F &= C + 2 \\ F &= C + 2 - P \\ &= 1 + 2 - 2 \\ &= 1\end{aligned}$$

The answer is (C).

6. Zinc has a higher potential and will act as the anode. By definition, the anode is where electrons are lost. The reaction at the anode of the electrochemical cell is $Zn \rightarrow Zn^{2+} + 2e^-$.

The answer is (C).

7. The liquidus line divides the diagram into two regions. Above the liquidus line, the alloy is purely liquid, while below the liquidus line, the alloy may exist as solid phase or as a mixture of solid and liquid phases. The liquidus line is CDE.

The answer is (B).

8. Hardness in steel is obtained by rapid quenching. Martensite is quenched rapidly, so it has a high hardness (and low ductility).

The answer is (D).

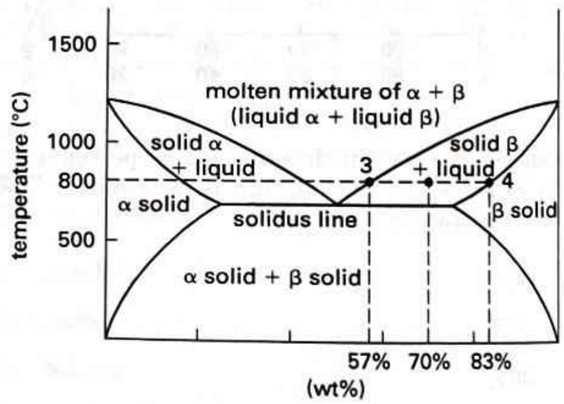
9. Steel's hardness, or resistance to deformation, can be increased by surface hardening processes and by some alloying metals. However, tempering and hot working increase the ductility (deformation capability) of steel, not its hardness.

The answer is (D).

10. Zinc, aluminum, and magnesium are all more electropositive (anodic) than iron and will corrode sacrificially to protect it. Gold is more cathodic than iron and the iron will corrode first. The iron is lower than gold on the galvanic series.

The answer is (B).

11. Use the phase diagram to find the fraction of solid.



$$\begin{aligned}\text{wt\% fraction solid} &= \frac{x - x_3}{x_4 - x_3} \times 100\% \\ &= \frac{70\% - 57\%}{83\% - 57\%} \times 100\% \\ &= 50\%\end{aligned}$$

The answer is (D).

37 Structural Design: Materials and Basic Concepts

PRACTICE PROBLEMS

1. Which statement about the modulus of elasticity, E, is true?

(A) It is the same as the rupture modulus.

(B) It is the slope of the stress-strain curve in the linearly elastic region.

(C) It is the ratio of stress to volumetric strain.

(D) Its value depends only on the temperature of the material.

2. What modulus of elasticity is predicted by ACI 318 for normal weight concrete with a compressive strength of 3000 lbf/in^2?

(A) 0.26×10^6 psi

(B) 1.9×10^6 psi

(C) 2.8×10^6 psi

(D) 3.2×10^6 psi

3. Which of the following criteria must be met in order for the compressive strength of concrete to be satisfied?

I. No single strength test falls below the specified compressive strength, f_c', by more than $0.10f_c'$.

II. No single strength test falls below the specified compressive strength, f_c', by more than $0.20f_c'$.

III. The average of every three consecutive strength tests equals or exceeds the specified compressive strength.

IV. The average of every three consecutive strength tests must not equal or exceed the specified compressive strength.

(A) I and III

(B) I and IV

(C) II and III

(D) II and IV

4. A waste product of coal-burning power-generation stations, fly ash, is the most common pozzolamic additive. Which of the following statements are true about fly ash?

I. Fly ash reacts with calcium hydroxide to increase binding.

II. Fly ash reacts with calcium silicate to form a binder.

III. Fly ash acts as a microfiller between cement particles, increasing strength and durability while reducing permeability.

IV. When used as a replacement for less than 45% of the portland cement, fly ash meeting ASTM C618 enhances resistance to scaling from road-deicing chemicals.

(A) I and II

(B) III and IV

(C) I, III, and IV

(D) II, III, and IV

5. Which category of steel contains 0.15–0.29% carbon?

(A) low-carbon

(B) mild-carbon

(C) medium-carbon

(D) high-carbon

6. Which property of steel allows it to undergo large inelastic deformations without fracture?

(A) yield

(B) elasticity

(C) ductility

(D) toughness

Structural Design

7. A simply supported reinforced concrete beam 12 in wide and 28 in deep spans 20 ft. The beam is subjected to a uniform service dead load equal to 2.0 kips/ft (exclusive of beam weight) and to a uniform service live load of 2.4 kips/ft. The factored uniform load is most nearly

(A) 6.7 kips/ft

(B) 7.4 kips/ft

(C) 8.0 kips/ft

(D) 9.2 kips/ft

8. A simply supported reinforced concrete beam 12 in wide, 24 in deep, and 30 ft long is subjected to a dead load of 1.2 kips/ft and a live load of 0.8 kip/ft in addition to its own dead weight. Most nearly, what moment should be used to determine the steel reinforcement at the center of the beam?

(A) 12 ft-kips

(B) 350 ft-kips

(C) 380 ft-kips

(D) 420 ft-kips

SOLUTIONS

1. The modulus of elasticity is the slope of the stress-strain diagram in the linearly elastic region.

The answer is (B).

2. The modulus of elasticity is

$$\begin{aligned} E_c &= 33 w_c^{1.5}\sqrt{f_c'} \\ &= (33)\left(145\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5}\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}} \\ &= 3.16\times 10^6\ \text{psi}\quad (3.2\times 10^6\ \text{psi}) \end{aligned}$$

Note that this equation is not dimensionally consistent.

The answer is (D).

3. The compressive strength of concrete is considered satisfactory if two criteria are met: (a) no single strength test falls below the specified compressive strength, f_c', by more than $0.10f_c'$, and (b) the average of every three consecutive strength tests equals or exceeds the specified compressive strength.

The answer is (A).

4. As the cement sets, calcium silicate hydrate and calcium hydroxide are formed. Calcium silicate hydrate is a binder that holds concrete together, while calcium hydroxide does not contribute to binding. However, fly ash reacts with some of the calcium hydroxide to increase binding.

Fly ash acts as a microfiller between cement particles to increase strength and durability while decreasing permeability, and enhances resistance to scaling from road-deicing chemicals when it meets ASTM C618 and is used as a replacement for less than 45% of the portland cement.

The answer is (C).

5. Carbon steels are divided into four categories based on the percentages of carbon: low-carbon (less than 0.15%), mild-carbon (0.15–0.29%), medium-carbon (0.30–0.59%), and high-carbon (0.60–1.70%).

The answer is (B).

6. Ductility is the ability of steel to undergo large inelastic deformations without fracture. Yield stress is the unit tensile stress at which the stress-strain curve exhibits a well-defined increase in strain without an increase in stress. The modulus of elasticity is the slope of the

initial straight-line portion of the stress-strain diagram. Toughness is the ability of a specimen to absorb energy.

The answer is (C).

7. Although the unit weight of unreinforced normal weight concrete is taken as 145 lbf/ft^3, the unit weight of reinforced normal weight concrete is usually assumed to be 150 lbf/ft^3. The weight of the beam per unit length is

$$W = bhw = \frac{(12 \text{ in})(28 \text{ in})\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)}{\left(12 \ \frac{\text{in}}{\text{ft}}\right)^2\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)} = 0.35 \text{ kip/ft}$$

The factored uniform load is the maximum of

$$\begin{aligned} U &= 1.4D = (1.4)\left(2.0 \ \frac{\text{kips}}{\text{ft}} + 0.35 \ \frac{\text{kip}}{\text{ft}}\right) \\ &= 3.29 \text{ kips/ft} \quad \text{[does not control]} \\ U &= 1.2D + 1.6L \\ &= (1.2)\left(2.0 \ \frac{\text{kips}}{\text{ft}} + 0.35 \ \frac{\text{kip}}{\text{ft}}\right) + (1.6)\left(2.4 \ \frac{\text{kips}}{\text{ft}}\right) \\ &= 6.66 \text{ kips/ft} \quad (6.7 \text{ kips/ft}) \end{aligned}$$

The larger value controls, so use 6.7 kips/ft.

The answer is (A).

8. The specific weight of steel-reinforced concrete is assumed to be 150 lbf/ft^3. The weight of the beam is

$$W = bhw = \frac{(12 \text{ in})(24 \text{ in})\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)}{\left(12 \ \frac{\text{in}}{\text{ft}}\right)^2\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)} = 0.3 \text{ kip/ft}$$

The factored uniform load is

$$\begin{aligned} U &= 1.2D + 1.6L \\ &= (1.2)\left(1.2 \ \frac{\text{kips}}{\text{ft}} + 0.3 \ \frac{\text{kip}}{\text{ft}}\right) + (1.6)\left(0.8 \ \frac{\text{kip}}{\text{ft}}\right) \\ &= 3.08 \text{ kips/ft} \end{aligned}$$

For a uniformly loaded beam, the factored moment is maximum at the center of the beam and is

$$\begin{aligned} M_u &= \frac{UL^2}{8} = \frac{\left(3.08 \ \frac{\text{kips}}{\text{ft}}\right)(30 \text{ ft})^2}{8} \\ &= 347 \text{ ft-kips} \quad (350 \text{ ft-kips}) \end{aligned}$$

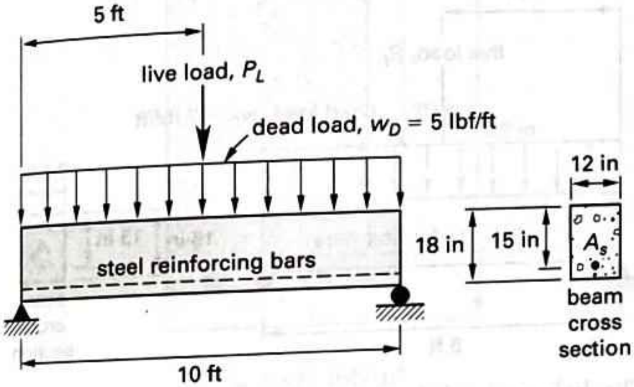
The answer is (B).

Structural Design

38 Reinforced Concrete: Beams

PRACTICE PROBLEMS

1. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f_c' = 3000$ lbf/in^2, $f_y = 40{,}000$ lbf/in^2, and $A_s = 3$ in^2.



Neglecting beam self-weight and based only on the allowable moment capacity of the beam as determined using American Concrete Institute (ACI) strength design specifications, the maximum allowable live load is most nearly

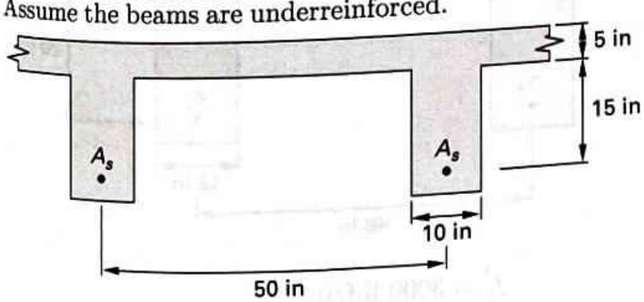
(A) 23,000 lbf

(B) 29,000 lbf

(C) 35,000 lbf

(D) 50,000 lbf

2. A floor system consists of ten 30 ft long reinforced concrete beams and a continuous 5 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.



For each beam in the floor system, the ACI-specified effective top flange width is most nearly

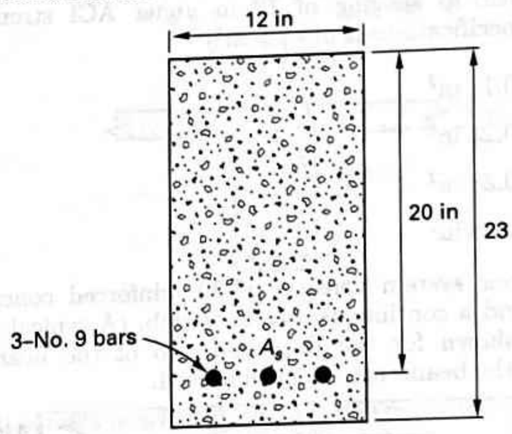
(A) 36 in

(B) 50 in

(C) 60 in

(D) 90 in

3. The cross section of a reinforced concrete beam with tension reinforcement is shown. Assume that the beam is underreinforced.



$f_c' = 3000$ lbf/in^2

$f_y = 40{,}000$ lbf/in^2

$A_s = 3$ in^2 [three no. 9 bars]

$A_s' = 1$ in^2

If the dead load shear force in the beam is 5 kips and the live load shear force in the beam is 15 kips, then the minimum amount of shear reinforcement needed for a center-to-center stirrup spacing of 8 in based on ACI strength design is most nearly

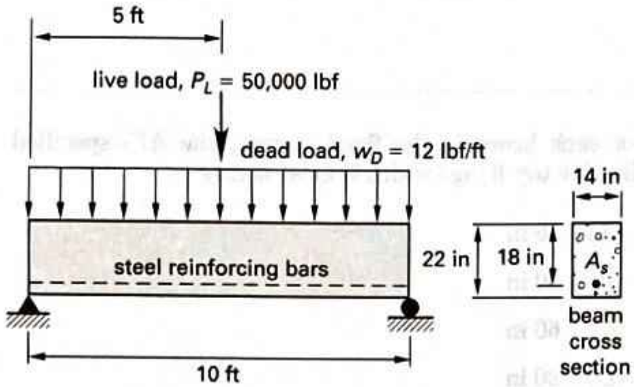
(A) 0.10 in^2

(B) 0.12 in^2

(C) 0.14 in^2

(D) 0.18 in^2

4. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f'_c = 2500\ \text{lbf/in}^2$, $f_y = 50{,}000\ \text{lbf/in}^2$, and $A_s = 3.8\ \text{in}^2$.



The beam supports a concentrated live load of 50,000 lbf. Neglect beam self-weight. The minimum amount of shear reinforcement required for a center-to-center stirrup spacing of 12 in under ACI strength design specifications is most nearly

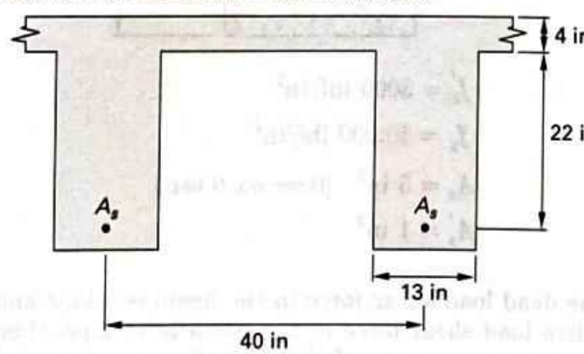
(A) 0.17 in²

(B) 0.23 in²

(C) 0.38 in²

(D) 0.78 in²

5. A floor system consists of 30 reinforced concrete beams and a continuous 4 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.



$$f'_c = 2800\ \text{lbf/in}^2$$
$$f_y = 42{,}000\ \text{lbf/in}^2$$
$$L = 30\ \text{ft} \quad [\text{simple span length}]$$

Assume the effective flange width for this beam is 40 in. If the area of reinforcing steel per beam is 6.00 in², the nominal moment capacity of each beam based on ACI strength design is most nearly

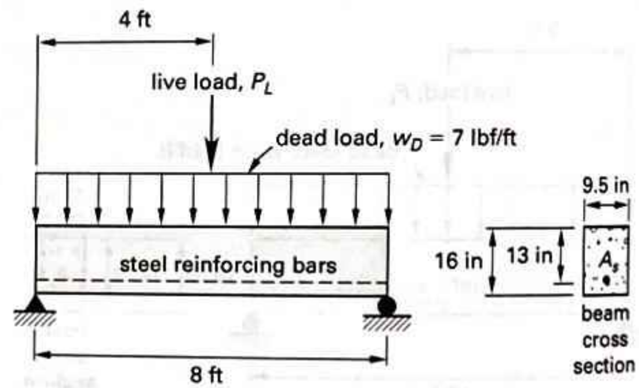
(A) 150 ft-kips

(B) 160 ft-kips

(C) 520 ft-kips

(D) 650 ft-kips

6. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f'_c = 3100\ \text{lbf/in}^2$, $f_y = 35{,}000\ \text{lbf/in}^2$, and $A_s = 2.5\ \text{in}^2$.



The balanced reinforcing steel ratio for this beam in accordance with ACI specifications is most nearly

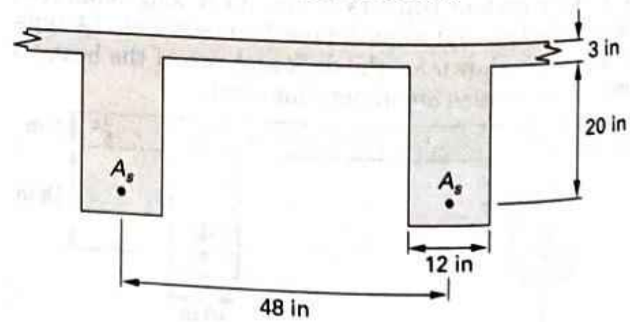
(A) 0.037

(B) 0.046

(C) 0.051

(D) 0.058

7. A floor system consists of 20 reinforced concrete beams and a continuous 3 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.



$$f'_c = 3000\ \text{lbf/in}^2$$
$$f_y = 60{,}000\ \text{lbf/in}^2$$
$$L = 30\ \text{ft} \quad [\text{simple span length}]$$

Assume the effective flange width for this beam is 48 in. If the area of reinforcing steel per beam is 7.25 in^2, the nominal moment capacity of each beam based on ACI strength design is most nearly

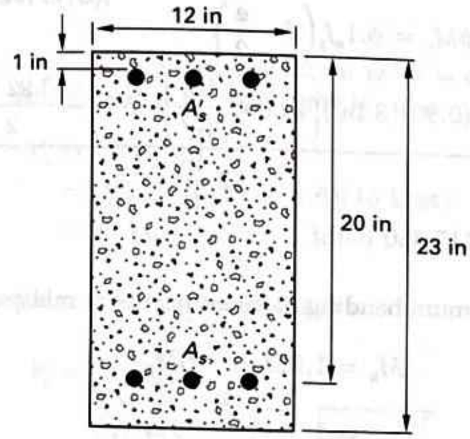
(A) 680 ft-kips

(B) 770 ft-kips

(C) 800 ft-kips

(D) 880 ft-kips

8. The cross section of a reinforced concrete beam with compression reinforcement is shown.



$$f_c' = 3000 \text{ lbf/in}^2$$
$$f_y = 40{,}000 \text{ lbf/in}^2$$
$$A_s = 3 \text{ in}^2$$
$$A_s' = 1 \text{ in}^2$$

The nominal moment capacity of the beam is most nearly

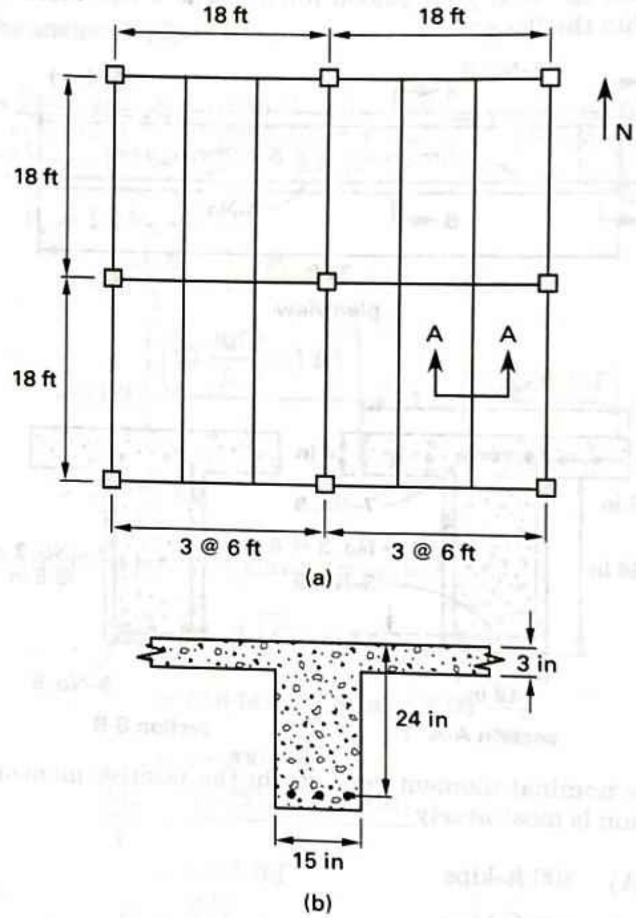
(A) 130 ft-kips

(B) 150 ft-kips

(C) 170 ft-kips

(D) 190 ft-kips

9. A monolithic slab-beam floor system is supported on a column grid of 18 ft on centers as shown. The dimensions of the cross section for the beams running in the north-south direction have been determined.



What is most nearly the effective flange width

(A) 45 in

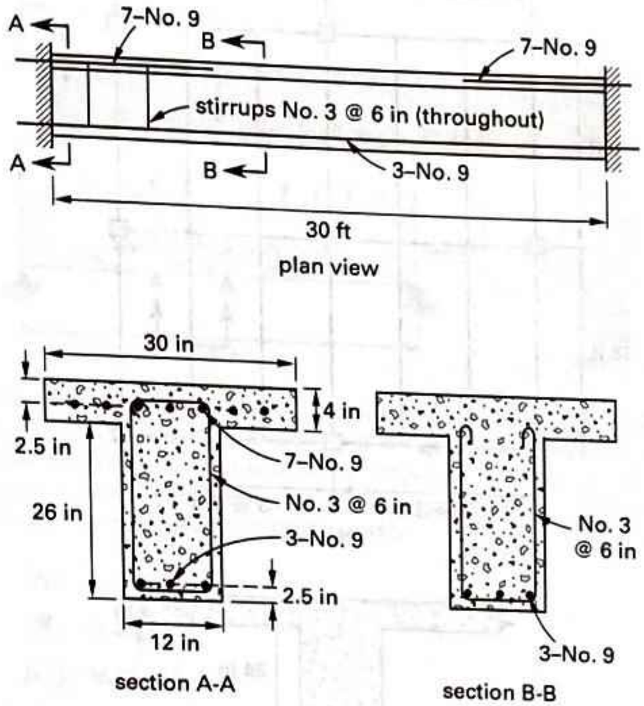
(B) 54 in

(C) 63 in

(D) 72 in

Structural Design

10. A reinforced concrete T-beam with a 30 ft span and a 30 in effective width in a floor slab system is fixed at both ends and is reinforced as shown. $f'_c = 3000$ lbf/in^2, $A_s = 3$ in^2, and $f_y = 60{,}000$ lbf/in^2. The stress block is within the flange.



The nominal moment capacity in the positive moment region is most nearly

(A) 300 ft-kips

(B) 360 ft-kips

(C) 390 ft-kips

(D) 410 ft-kips

SOLUTIONS

1. The height of the stress block is

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3\ \text{in}^2)\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(12\ \text{in})}$$
$$= 3.92\ \text{in}$$

For flexure, the strength reduction factor, ϕ, is 0.90.

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$
$$= \frac{(0.90)(3\ \text{in}^2)\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)\left(15\ \text{in} - \frac{3.92\ \text{in}}{2}\right)}{12\ \frac{\text{in}}{\text{ft}}}$$
$$= 117{,}350\ \text{ft-lbf}$$

The maximum bending moment occurs at midspan.

$$M_u = 1.2M_D + 1.6M_L$$
$$= 1.2\frac{w_D L^2}{8} + 1.6\frac{P_L L}{4}$$

This can be solved for the maximum allowable live load.

$$P_L = \frac{4\left(M_u - 1.2\frac{w_D L^2}{8}\right)}{1.6L}$$
$$= \frac{(4)\left((117{,}350\ \text{ft-lbf}) - (1.2)\left(\frac{\left(5\ \frac{\text{lbf}}{\text{ft}}\right)(10\ \text{ft})^2}{8}\right)\right)}{(1.6)(10\ \text{ft})}$$
$$= 29{,}319\ \text{lbf}\quad (29{,}000\ \text{lbf})$$

The answer is (B).

2. The effective flange width is

$$b_e = \text{smallest}\begin{cases} \left(\frac{1}{4}\right)(\text{span length}) = \left(\frac{1}{4}\right)(30\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ \qquad = 90\text{ in} \\ b_w + 16h_f = 10\text{ in} + (16)(5\text{ in}) \\ \qquad = 90\text{ in} \\ \text{beam centerline spacing} = 50\text{ in} \end{cases}$$

Therefore, the effective flange width is 50 in.

The answer is (B).

3. For shear, the capacity reduction ratio is $\phi = 0.75$. The ultimate shear force in the beam is

$$\begin{aligned} V_u &= 1.2V_D + 1.6V_L \\ &= (1.2)(5\text{ kips}) + (1.6)(15\text{ kips}) \\ &= 30\text{ kips} \end{aligned}$$

The nominal concrete shear strength is

$$\begin{aligned} V_c &= 2b_wd\sqrt{f_c'} \\ &= \frac{(2)(12\text{ in})(20\text{ in})\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}}{1000\ \frac{\text{kip}}{\text{lbf}}} \\ &= 26.29\text{ kips} \\ \frac{\phi V_c}{2} &= \frac{(0.75)(26.29\text{ kips})}{2} \\ &= 9.9\text{ kips} \end{aligned}$$

Since $V_u > \phi V_c/2$, shear reinforcement is required.

$$\phi V_c = (0.75)(26.29\text{ kips}) = 19.72\text{ kips}$$

Since $V_u = 30\text{ kips} > \phi V_c = 19.72\text{ kips}$, the required shear strength provided by the steel is

$$V_s = \frac{V_u}{\phi} - V_c = \frac{30\text{ kips}}{0.75} - 26.29\text{ kips} = 13.71\text{ kips}$$

The required steel area is

$$\begin{aligned} A_v &= \frac{sV_s}{f_yd} = \frac{(8\text{ in})(13.71\text{ kips})\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}{\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)(20\text{ in})} \\ &= 0.1371\text{ in}^2 \quad (0.14\text{ in}^2) \end{aligned}$$

Although not required for this problem, in an actual design and analysis situation, a check should be made to ensure that V_s does not exceed the ACI-allowed maximum shear reinforcement given by $V_{s,\max} = 8\sqrt{f_c'}\,bd$.

The answer is (C).

4. For shear, the strength reduction factor, ϕ, is 0.75. The maximum factored shear force in the beam is at either one of the supports and is

$$\begin{aligned} V_u &= 1.2V_D + 1.6V_L \\ &= 1.2\frac{w_DL}{2} + 1.6\frac{P_L}{2} \\ &= (1.2)\left(\frac{\left(12\ \frac{\text{lbf}}{\text{ft}}\right)(10\text{ ft})}{2}\right) + (1.6)\left(\frac{50{,}000\text{ lbf}}{2}\right) \\ &= 40{,}072\text{ lbf} \end{aligned}$$

The nominal concrete shear strength is

$$\begin{aligned} V_c &= 2b_wd\sqrt{f_c'} \\ &= (2)(14\text{ in})(18\text{ in})\sqrt{2500\ \frac{\text{lbf}}{\text{in}^2}} \\ &= 25{,}200\text{ lbf} \\ \frac{\phi V_c}{2} &= \frac{(0.75)(25{,}200\text{ lbf})}{2} \\ &= 9450\text{ lbf} \\ V_u &> \frac{\phi V_c}{2} \end{aligned}$$

Therefore, shear reinforcement is required.

In accordance with ACI specifications, the minimum required amount of shear reinforcement for a stirrup spacing of 12 in is

$$\begin{aligned} A_v &= \frac{50b_ws}{f_y} \\ &= \frac{(50)(14\text{ in})(12\text{ in})}{50{,}000\ \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.168\text{ in}^2 \end{aligned}$$

Structural Design

The amount of shear reinforcement based on factored loading can be determined as follows.

$$V_s = A_v f_y \frac{d}{s}$$

$$\phi(V_c + V_s) \geq V_u$$

$$\begin{aligned} A_v &= \frac{V_u - \phi V_c}{\phi f_y \frac{d}{s}} \\ &= \frac{40{,}072 \text{ lbf} - (0.75)(25{,}200 \text{ lbf})}{(0.75)\left(50{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{18 \text{ in}}{12 \text{ in}}\right)} \\ &= 0.376 \text{ in}^2 \quad (0.38 \text{ in}^2) \end{aligned}$$

The larger value for A_v controls. Use $A_v = 0.38 \text{ in}^2$.

The answer is (C).

5. This problem asks for the nominal moment capacity, M_n, not the allowable moment capacity, ϕM_n. Therefore, the reduction factor, ϕ, is not needed.

First, assume that each beam is rectangular with a width $b = b_e = 40$ in. The depth of the concrete compressive stress block is

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b} \\ &= \frac{(6.00 \text{ in}^2)\left(42{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(2800 \ \frac{\text{lbf}}{\text{in}^2}\right)(40 \text{ in})} \\ &= 2.65 \text{ in} \end{aligned}$$

Since $a <$ slab depth of 4 in, the nominal moment capacity of the beam is the same as for a rectangular singly reinforced concrete beam. Using $b = b_e = 40$ in, the nominal moment capacity is

$$\begin{aligned} M_n &= 0.85 f'_c ab\left(d - \frac{a}{2}\right) \\ &= \frac{(0.85)\left(2800 \ \frac{\text{lbf}}{\text{in}^2}\right)(2.65 \text{ in}) \times (40 \text{ in})\left(26 \text{ in} - \frac{2.65 \text{ in}}{2}\right)}{\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 520 \text{ ft-kips} \end{aligned}$$

This can also be calculated by using the following equation in accordance with rectangular singly reinforced concrete beam theory.

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

Even though the problem statement assumes that the beam is underreinforced, the actual reinforcing steel ratio and its limits should always be checked in real design and analysis problems.

The answer is (C).

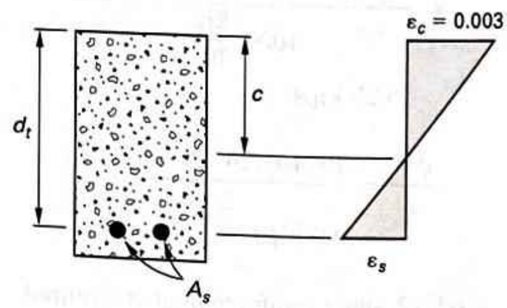
6. The ratio of the rectangular stress block depth to the neutral axis depth is

$$\beta_1 = 0.85 \geq \left(0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)\right) \geq 0.65$$

Since $f'_c < 4000 \text{ lbf/in}^2$, $\beta_1 = 0.85$. The reinforcement ratio is

$$\rho = \frac{A_s}{bd_t}$$

Expressions for A_s and d_t are needed.



From similar triangles,

$$\frac{c}{d_t} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s}$$

For a balanced condition, the concrete strain is 0.003, and the steel is at yield.

$$\frac{c}{d_t} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} = \frac{\varepsilon_c}{\varepsilon_c + \frac{f_y}{E_s}}$$

The modulus of elasticity for steel is $E_s = 29 \times 10^6 \text{ lbf/in}^2$. Since $\varepsilon_c = 0.003$, for the balanced condition,

$$\frac{1}{d_t} = \frac{87{,}000}{c(87{,}000 + f_y)}$$

The required steel area can be found from the nominal moment strength.

$$M_n = 0.85f_c'ab\left(d - \frac{a}{2}\right)$$
$$= A_s f_y\left(d - \frac{a}{2}\right)$$

$$A_s = \frac{0.85f_c'ab}{f_y}$$

And, since $a = \beta_1 c$,

$$A_s = \frac{0.85f_c'\beta_1 cb}{f_y}$$

Combining the expressions for ρ, $1/d_t$, and A_s, the balanced reinforcement ratio is

$$\rho_b = \left(\frac{0.85\beta_1 f_c'}{f_y}\right)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

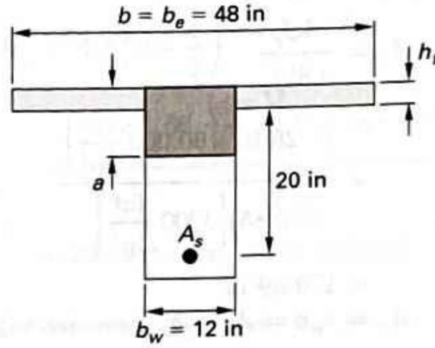
$$= \left(\frac{0.85\beta_1 f_c'}{f_y}\right)\left(\frac{87{,}000\ \frac{\text{lbf}}{\text{in}^2}}{87{,}000\ \frac{\text{lbf}}{\text{in}^2} + f_y}\right)$$

$$= \left(\frac{(0.85)(0.85)\left(3100\ \frac{\text{lbf}}{\text{in}^2}\right)}{35{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)$$
$$\times \left(\frac{87{,}000\ \frac{\text{lbf}}{\text{in}^2}}{87{,}000\ \frac{\text{lbf}}{\text{in}^2} + 35{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)$$
$$= 0.0456 \quad (0.046)$$

The answer is (B).

7. This problem asks for the nominal moment capacity, M_n, not the allowable moment capacity, ϕM_n. Therefore, the reduction factor, ϕ, is not needed.

The depth of the concrete compressive stress block must be checked to see whether or not it exceeds the 3 in deck thickness. If the depth of this compressive stress block exceeds the deck thickness, then each beam is a T-beam and T-beam formulas apply for determination of the nominal moment capacity. If, however, the depth of the concrete compressive stress block does not exceed the deck thickness, then each beam is a rectangular beam and rectangular beam formulas apply for determination of the nominal moment capacity.



First, assume that each beam is rectangular with a width of $b = b_e = 48$ in. The depth of the concrete compressive stress block is

$$a = \frac{A_s f_y}{0.85f_c'b} = \frac{(7.25\ \text{in}^2)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(48\ \text{in})}$$
$$= 3.55\ \text{in}$$

Since $a >$ slab depth of 3 in, the beam is a T-beam.

Since $a = 3.55$ in was found by assuming that the beam was a rectangular beam with width $b = 48$ in, this depth only indicates whether or not the beam is a T-beam and is not the correct depth for determining the nominal moment capacity. The correct depth is now found by applying the concepts of static equilibrium to the beam.

To find the correct depth, a, sum horizontal forces in the T-beam to show that the upper (above the neutral axis) compressive concrete stress block force is equal to the lower (below the neutral axis) maximum tensile force sustained by the reinforcing bars. By dividing the entire concrete compressive stress block section into three parts (a rectangular part and two overhanging flanges), depth a can be found.

A_f is the area of overhanging flanges. A_c is the total area of concrete compressive stress block. A_r is the area of concrete compressive stress block in the rectangular part of the T-beam between the overhanging flanges.

$$A_f = (b_e - b_w)h_f$$
$$= (48\ \text{in} - 12\ \text{in})(3\ \text{in})$$
$$= 108\ \text{in}^2$$

From equilibrium of horizontal forces,

$$0.85f_c'A_c = A_sf_y$$

$$\begin{aligned} A_c &= \frac{A_sf_y}{0.85f_c'} \\ &= \frac{(7.25\ \text{in}^2)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)} \\ &= 170.59\ \text{in}^2 \end{aligned}$$

$$A_r = b_wa = A_c - A_f$$

$$\begin{aligned} a &= \frac{A_c - A_f}{b_w} \\ &= \frac{170.59\ \text{in}^2 - 108\ \text{in}^2}{12\ \text{in}} \\ &= 5.22\ \text{in} \end{aligned}$$

Alternatively, a redefined stress block depth could be used.

$$\begin{aligned} a &= \frac{A_sf_y}{0.85f_c'b_w} - \frac{h_f(b_e - b_w)}{b_w} \\ &= \frac{(7.25\ \text{in}^2)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(12\ \text{in})} - \frac{(3\ \text{in})(48\ \text{in} - 12\ \text{in})}{12\ \text{in}} \\ &= 5.22\ \text{in} \end{aligned}$$

The nominal moment capacity of the T-beam is

$$\begin{aligned} M_n &= 0.85f_c'h_f(b_e - b_w)\left(d - \frac{h_f}{2}\right) \\ &\quad + 0.85f_c'ab_w\left(d - \frac{a}{2}\right) \\ &= \frac{\begin{array}{c} (0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(3\ \text{in}) \\ \times(48\ \text{in} - 12\ \text{in})\left(23\ \text{in} - \frac{3\ \text{in}}{2}\right) \\ +(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(5.22\ \text{in}) \\ \times(12\ \text{in})\left(23\ \text{in} - \frac{5.22\ \text{in}}{2}\right) \end{array}}{\left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 765\ \text{ft-kips} \quad (770\ \text{ft-kips}) \end{aligned}$$

Even though the problem statement assumes that the beam is underreinforced, the actual reinforcing steel ratio and its limits should always be checked in real design and analysis problems.

The answer is (B).

8. Determine whether compression steel yields.

$$\begin{aligned} A_s - A_s' &= 3\ \text{in}^2 - 1\ \text{in}^2 \\ &= 2\ \text{in}^2 \end{aligned}$$

$$\begin{aligned} &\frac{0.85\beta_1f_c'd'b}{f_y}\left(\frac{87{,}000}{87{,}000 - f_y}\right) \\ &= \frac{(0.85)(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(1\ \text{in})(12\ \text{in})}{40{,}000\ \frac{\text{lbf}}{\text{in}^2}} \\ &\quad \times\left(\frac{87{,}000\ \frac{\text{lbf}}{\text{in}^2}}{87{,}000\ \frac{\text{lbf}}{\text{in}^2} - 40{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right) \\ &= 1.2\ \text{in}^2 \end{aligned}$$

Compare both sides of the equation.

$$A_s - A_s' \geq \frac{0.85\beta_1f_c'd'b}{f_y}\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

$$2\ \text{in}^2 \geq 1.2\ \text{in}^2$$

Therefore, compression steel yields.

Calculate the depth of the concrete compressive stress block.

$$\begin{aligned} a &= \frac{(A_s - A_s')f_y}{0.85f_c'b} \\ &= \frac{(3\ \text{in}^2 - 1\ \text{in}^2)\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)(12\ \text{in})} \\ &= 2.61\ \text{in} \end{aligned}$$

The nominal moment strength is

$$M_n = f_y\left((A_s - A_s')\left(d - \frac{a}{2}\right) + A_s'(d - d')\right)$$

$$= \frac{\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\begin{array}{l}(3\ \text{in}^2 - 1\ \text{in}^2)\left(20\ \text{in} - \frac{2.61\ \text{in}}{2}\right) \\ +(1\ \text{in}^2)(20\ \text{in} - 1\ \text{in})\end{array}\right)}{\left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}$$

$$= 187.95\ \text{ft-kips} \quad (190\ \text{ft-kips})$$

The answer is (D).

9. The effective width of the flange is determined as the smallest of

$$b_e = \text{smallest}\begin{cases} \left(\frac{1}{4}\right)(\text{span length}) = \left(\frac{1}{4}\right)(18\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ \quad = 54\ \text{in} \\ b_w + 16h_f = 15\ \text{in} + (16)(3\ \text{in}) \\ \quad = 63\ \text{in} \\ \text{beam centerline spacing} = (6\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ \quad = 72\ \text{in} \end{cases}$$

Therefore, the effective flange width is 54 in.

The answer is (B).

10. The positive moment region for slabs is the central region, so cross-section B-B is applicable. Since the stress block is within the flange, $a < h_f$, and the T-beam can be analyzed like a rectangular beam. The distance from the extreme compression fibers to the extreme tensile steel is

$$d = 4\ \text{in} + 26\ \text{in} - 2.5\ \text{in}$$
$$= 27.5\ \text{in}$$

Determine the height of the stress block.

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{(3\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(30\ \text{in})} = 2.35\ \text{in}$$

The moment capacity can be calculated either from the steel properties (with $b = b_e$) or from the concrete properties. Using the steel properties,

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$= \frac{(3\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)\left(27.5\ \text{in} - \frac{2.35\ \text{in}}{2}\right)}{12\ \frac{\text{in}}{\text{ft}}}$$

$$= 394.9\ \text{ft-kips} \quad (390\ \text{ft-kips})$$

Using the concrete properties,

$$M_n = 0.85 f_c' a b_e\left(d - \frac{a}{2}\right)$$

$$= \frac{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(2.35\ \text{in})(30\ \text{in}) \times \left(27.5\ \text{in} - \frac{2.35\ \text{in}}{2}\right)}{12\ \frac{\text{in}}{\text{ft}}}$$

$$= 394.9\ \text{ft-kips} \quad (390\ \text{ft-kips})$$

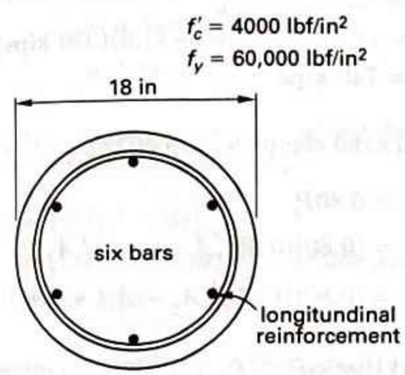
The answer is (C).

Structural Design

39 Reinforced Concrete: Columns

PRACTICE PROBLEMS

1. For the short, concentrically loaded round tied column shown, the applied axial dead load is 150 kips, and the applied axial live load is 350 kips.



round tied column (cross section)

Assuming that the longitudinal reinforcing bars are all the same size, the minimum required size of each longitudinal reinforcing bar is

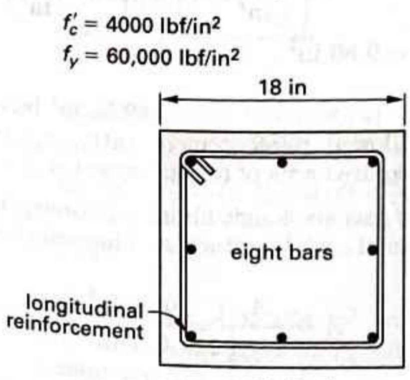
(A) no. 10

(B) no. 11

(C) no. 12

(D) no. 14

2. For the short, concentrically loaded square tied column shown, the applied axial dead load is 150 kips, and the applied axial live load is 250 kips.



square tied column (cross section)

Assuming that the longitudinal reinforcing bars are all the same size, the minimum required size of each longitudinal reinforcing bar is

(A) no. 3

(B) no. 4

(C) no. 5

(D) no. 6

3. A reinforced concrete tied column is subjected to a design axial compression force of 1090 kips that is concentrically applied. Slenderness effects are negligible, and the column is to be designed using ACI 318. Given a specified compressive strength of 5000 psi, grade 60 rebars, and a specified longitudinal steel ratio of 0.02, what is most nearly the width of the sides of the smallest square column that will support the load?

(A) 12 in

(B) 16 in

(C) 20 in

(D) 24 in

4. A 16 in (gross dimension) square tied column must carry 220 kip dead and 250 kip live loads. The dead load includes the column self-weight. The column is not exposed to any moments. Sidesway is prevented at the top, and slenderness effects are to be disregarded. The concrete compressive strength is 4000 lbf/in^2, and the steel tensile yield strength is 60,000 lbf/in^2. The longitudinal reinforcement ratio of this column is most nearly

(A) 0.028

(B) 0.061

(C) 0.092

(D) 0.11

5. An 18 in square tied column is reinforced with 12 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in². The column, which is braced against sidesway, has an unsupported height of 9 ft and supports axial load only without end moments. What is most nearly the design axial load capacity?

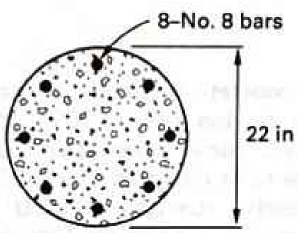
(A) 930 kips

(B) 970 kips

(C) 1800 kips

(D) 1900 kips

6. The short tied column shown uses eight no. 8 bars. Assume the loading has a low eccentricity. $f_c' = 3500$ lbf/in², and $f_y = 40{,}000$ lbf/in².



The design strength is most nearly

(A) 710 kips

(B) 890 kips

(C) 1100 kips

(D) 7100 kips

SOLUTIONS

1. Determine the amount of reinforcing steel required by the minimum required reinforcement ratio, ρ_g, of 0.01. The minimum area of reinforcing steel required is

$$\begin{aligned} A_s = \rho_g A_g &= (0.01)\left(\frac{\pi(18\text{ in})^2}{4}\right) \\ &= 2.54\text{ in}^2 \end{aligned}$$

Determine the required amount of reinforcing steel based on the factored axial load, P_u.

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ &= (1.2)(150\text{ kips}) + (1.6)(350\text{ kips}) \\ &= 740\text{ kips} \end{aligned}$$

The nominal axial compressive load capacity is

$$\begin{aligned} P_n &= 0.80P_o \\ &= (0.80)(0.85f_c' A_{\text{concrete}} + f_y A_s) \\ &= (0.80)\big(0.85f_c'(A_g - A_s) + f_y A_s\big) \end{aligned}$$

It is required that $\phi P_n \geq P_u$. For axial compression with tied reinforcement, $\phi = 0.65$. Setting $\phi P_n = P_u$ and solving for the area of longitudinal reinforcing steel gives

$$A_s = \frac{\dfrac{P_u}{0.80\phi} - 0.85f_c' A_g}{f_y - 0.85f_c'}$$

$$= \frac{\dfrac{(740\text{ kips})\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.80)(0.65)} - (0.85)\left(4000\ \dfrac{\text{lbf}}{\text{in}^2}\right)\left(\dfrac{\pi(18\text{ in})^2}{4}\right)}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2} - (0.85)\left(4000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}$$

$$= 9.86\text{ in}^2$$

$A_s = 9.86$ in² is greater than $A_s = 2.54$ in² based on the minimum allowed reinforcement ratio, $\rho_g = 0.01$. The minimum required area of reinforcement is $A_s = 9.86$ in².

The column has six longitudinal reinforcing bars. The required area of each longitudinal reinforcing bar is

$$\begin{aligned} A = \frac{A_s}{n_{\text{bars}}} &= \frac{9.86\text{ in}^2}{6\text{ bars}} \\ &= 1.64\text{ in}^2/\text{bar} \end{aligned}$$

A bar area of 1.64 in² is satisfied by a no. 14 bar, which has a nominal area of 2.25 in². The no. 12 bar would also satisfy the requirements with a nominal area of 1.76 in² however this rebar size is rarely used in conventional applications.

In an actual design and analysis situation, a check should also be made to see that the actual longitudinal reinforcement ratio does not exceed the maximum allowable ratio of 0.08.

The answer is (D).

2. The minimum required reinforcement ratio, ρ_g, is 0.01. The minimum area of reinforcing steel required is

$$\begin{aligned} A_s &= \rho_g A_g = (0.01)(18 \text{ in})^2 \\ &= 3.24 \text{ in}^2 \end{aligned}$$

Determine the required amount of reinforcing steel based on the factored axial load, P_u.

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ &= (1.2)(150 \text{ kips}) + (1.6)(250 \text{ kips}) \\ &= 580 \text{ kips} \end{aligned}$$

The nominal axial compressive load capacity is

$$\begin{aligned} P_n &= 0.80P_o \\ &= (0.80)(0.85f_c'A_{\text{concrete}} + f_yA_s) \\ &= (0.80)\big(0.85f_c'(A_g - A_s) + f_yA_s\big) \end{aligned}$$

It is required that $\phi P_n \geq P_u$. For axial compression with tied reinforcement, $\phi = 0.65$. Setting $\phi P_n = P_u$ and solving for the area of longitudinal reinforcing steel gives

$$\begin{aligned} A_s &= \frac{\dfrac{P_u}{0.80\phi} - 0.85f_c'A_g}{f_y - 0.85f_c'} \\ &= \frac{\dfrac{(580 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.80)(0.65)} - (0.85)\left(4000 \dfrac{\text{lbf}}{\text{in}^2}\right)(18 \text{ in})^2}{60{,}000 \dfrac{\text{lbf}}{\text{in}^2} - (0.85)\left(4000 \dfrac{\text{lbf}}{\text{in}^2}\right)} \\ &= 0.244 \text{ in}^2 \end{aligned}$$

$A_s = 0.244$ in² is less than $A_s = 3.24$ in² based on the minimum allowed reinforcement ratio, $\rho_g = 0.01$. Therefore, the minimum required area of reinforcement is $A_s = 3.24$ in².

The square tied column has eight uniformly sized longitudinal reinforcing bars. The required area of each longitudinal reinforcing bar is

$$\begin{aligned} A &= \frac{A_s}{n_{\text{bars}}} = \frac{3.24 \text{ in}^2}{8 \text{ bars}} \\ &= 0.405 \text{ in}^2/\text{bar} \end{aligned}$$

A bar area of 0.405 in² corresponds to a no. 6 bar, which has a nominal area of 0.44 in².

In an actual design and analysis situation, a check should also be made to see that the actual longitudinal reinforcement ratio does not exceed the maximum allowable ratio of 0.08.

The answer is (D).

3. For a tied column, $\phi = 0.65$. For a concentrically loaded tied column, the design strength is given by

$$\phi P_{n,\max} = 0.80\phi\big(0.85f_c'(A_g - A_{st}) + f_yA_{st}\big)$$

For a specified longitudinal steel ratio of 0.02,

$$A_{st} = \rho_g A_g = 0.02A_g$$

Substituting gives

$$\begin{aligned} 1090 \text{ kips} &= \phi P_{n,\max} \\ &= (0.80)(0.65) \\ &\quad \times \left(\begin{array}{l} (0.85)\left(5 \dfrac{\text{kips}}{\text{in}^2}\right)(A_g - 0.02A_g) \\ \quad + \left(60 \dfrac{\text{kips}}{\text{in}^2}\right)(0.02A_g) \end{array} \right) \end{aligned}$$

$$\begin{aligned} A_g &= 391 \text{ in}^2 \\ b &= \sqrt{A_g} = \sqrt{391 \text{ in}^2} \\ &= 19.8 \text{ in} \quad (20 \text{ in}) \end{aligned}$$

The answer is (C).

4. Determine the design load, P_u.

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ &= (1.2)(220 \text{ kips}) + (1.6)(250 \text{ kips}) \\ &= 664 \text{ kips} \end{aligned}$$

The gross column area is

$$A_g = (16 \text{ in})(16 \text{ in}) = 256 \text{ in}^2$$

Structural Design

For a tied column, $\phi = 0.65$. The capacity is

$$\begin{aligned}\phi P_{n,\max} &= 0.80\phi(0.85f'_c(A_g - A_{st}) + f_y A_{st})\\ &= (0.80)(0.65)\\ &\quad\times\left(\begin{array}{l}(0.85)\left(4\ \frac{\text{kips}}{\text{in}^2}\right)(256\ \text{in}^2 - A_{st})\\ \quad+\left(60\ \frac{\text{kips}}{\text{in}^2}\right)A_{st}\end{array}\right)\\ &= (0.80)(0.65)\times\left[\left(870.4\ \text{kips} - 3.4\ \frac{\text{kips}}{\text{in}^2}A_{st}\right) + \left(60\ \frac{\text{kips}}{\text{in}^2}\right)A_{st}\right]\\ &= (0.80)(0.65)\times\left[\left(870.4\ \text{kips} + 56.6\ \frac{\text{kips}}{\text{in}^2}A_{st}\right)\right]\\ &= 452.6\ \text{kips} + \left(29.4\ \frac{\text{kips}}{\text{in}^2}\right)A_{st}\end{aligned}$$

The design criterion is

$$\begin{aligned}\phi P_{n,\max} &\geq P_u\\ 664\ \text{kips} &= 452.6\ \text{kips} + \left(29.4\ \frac{\text{kips}}{\text{in}^2}\right)A_{st}\\ A_{st} &= 7.19\ \text{in}^2\end{aligned}$$

$$\rho = \frac{A_{st}}{A_g} = \frac{7.19\ \text{in}^2}{256\ \text{in}^2} = 0.028$$

Check the limits.

$$\begin{aligned}&\rho_{\min} < \rho < \rho_{\max}\\ &0.01 < 0.028 < 0.08 \quad \text{[OK]}\end{aligned}$$

The answer is (A).

5. The effective length factor for a column braced against sidesway is

$$K = 1.0$$

The radius of gyration is

$$\begin{aligned}r &= 0.288h = (0.288)(18\ \text{in})\\ &= 5.2\ \text{in}\end{aligned}$$

The slenderness ratio is

$$\begin{aligned}\frac{KL}{r} &= \frac{(1.0)(9\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{5.2\ \text{in}}\\ &= 20.8\end{aligned}$$

The gross area of the column is

$$A_g = b^2 = (18\ \text{in})^2 = 324\ \text{in}^2$$

The area of longitudinal steel reinforcement is

$$A_{st} = NA_b = (12)(1\ \text{in}^2) = 12\ \text{in}^2$$

Since $M_1 = 0$, the column is a short column. $\phi = 0.65$ for a tied column. The design axial load capacity is

$$\begin{aligned}\phi P_n &= 0.80\phi(0.85f'_c(A_g - A_{st}) + A_{st}f_y)\\ &= (0.80)(0.65)\left(\begin{array}{l}(0.85)\left(4\ \frac{\text{kips}}{\text{in}^2}\right)\\ \quad\times(324\ \text{in}^2 - 12\ \text{in}^2)\\ \quad+(12\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)\end{array}\right)\\ &= 926\ \text{kips} \quad (930\ \text{kips})\end{aligned}$$

The answer is (A).

6. The gross area of the column is

$$A_g = \frac{\pi D_g^2}{4} = \frac{\pi(22\ \text{in})^2}{4} = 380.1\ \text{in}^2$$

Since there are eight no. 8 bars, the area of steel is

$$A_{st} = (8)(0.79\ \text{in}^2) = 6.32\ \text{in}^2$$

$\phi = 0.65$ for tied columns. The design strength is then

$$\begin{aligned}\phi P_n &= 0.80\phi(0.85f'_c(A_g - A_{st}) + A_{st}f_y)\\ &= (0.80)(0.65)\\ &\quad\times\left(\frac{\begin{array}{l}(0.85)\left(3500\ \frac{\text{lbf}}{\text{in}^2}\right)(380.1\ \text{in}^2 - 6.32\ \text{in}^2)\\ \quad+(6.32\ \text{in}^2)\left(40{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)\end{array}}{1000\ \frac{\text{lbf}}{\text{kip}}}\right)\\ &= 709.7\ \text{kips} \quad (710\ \text{kips})\end{aligned}$$

The answer is (A).

Structural Design

40 Reinforced Concrete: Slabs

PRACTICE PROBLEMS

1. A two-way slab supported on a column grid without the use of beams is known as a

(A) flat slab

(B) flat plate

(C) drop panel

(D) waffle slab

2. All of the following are conditions that must be satisfied in order to use the simplified method for computing shear and moments in one-way slabs, given in ACI 318 Sec. 8.3.3, EXCEPT

(A) There are two or more spans.

(B) Spans are approximately equal, with the longer of two adjacent spans not longer than the shorter by more than 20%.

(C) The loads are uniformly distributed.

(D) The ratio of live to dead loads is no more than 5.

SOLUTIONS

1. A two-way slab supported on a column grid without the use of beams is known as a flat plate.

The answer is (B).

2. Shear and moments in one-way slabs can be computed using the simplified method specified in ACI 318 Sec. 8.3.3 when the following conditions are satisfied: (a) There are two or more spans. (b) Spans are approximately equal, with the longer of two adjacent spans not longer than the shorter by more than 20%. (c) The loads are uniformly distributed. (d) The ratio of live to dead loads is no more than 3. (e) The slab has a uniform thickness.

The answer is (D).

41 Reinforced Concrete: Walls

PRACTICE PROBLEMS

1. According to ACI 318 Sec. 14.3.3, the minimum horizontal reinforcement for nonbearing walls is

(A) 0.0012 times the gross concrete area for deformed no. 5 bars or smaller and $f_y \geq 60{,}000$ psi

(B) 0.0015 times the gross concrete area for other deformed bars

(C) 0.0020 times the gross concrete area for smooth or deformed welded wire reinforcement not larger than W31 or D31

(D) 0.01 times the gross concrete area

SOLUTIONS

1. ACI 318 Sec. 14.3.3 gives the minimum horizontal reinforcement for nonbearing walls as (a) 0.0020 times the gross concrete area for deformed no. 5 bars or smaller and $f_y \geq 60{,}000$ psi; (b) 0.0025 times the gross concrete area for other deformed bars; and (c) 0.0020 times the gross concrete area for smooth or deformed welded wire reinforcement not larger than W31 or D31.

The answer is (C).

42 Reinforced Concrete: Footings

PRACTICE PROBLEMS

1. The critical sections are a distance, d, from the face of the column in

(A) double-action shear

(B) one-way shear

(C) punching shear

(D) two-way shear

SOLUTIONS

1. For one-way shear (also known as single-action shear and wide-beam shear), the critical sections are a distance, d, from the face of the column.

The answer is (B).

Structural Design

43 Structural Steel: Beams

PRACTICE PROBLEMS

1. A 25 ft long steel beam is loaded uniformly (live) at 4 kips/ft. Loading due to self-weight is negligible, and there is adequate lateral support provided to the beam. The required plastic section modulus for a W12 shape using grade-50 steel is most nearly

(A) 60 in^3

(B) 95 in^3

(C) 120 in^3

(D) 130 in^3

2. A W-shaped beam has a warping constant of 3450 in^6. The moment of inertia about the weak axis is 45.1 in^4, and its elastic section modulus about the strong axis is 99.1 in^3. The effective radius of gyration of the compression flange is most nearly

(A) 2.0 in

(B) 3.0 in

(C) 4.0 in

(D) 5.0 in

3. An 18 ft long custom manufactured and rolled continuous W14 × 68 shape beam is supported at each end and at the midpoint. The compression flange of the beam is restrained from moving laterally, and all forms of local buckling are prevented. The nominal shear strength as determined from LRFD principles is 146 kips. Determine the most likely steel variety used in the manufacture of the beam and whether or not this beam is compact.

(A) probably noncompact; yield strength = 25 $kips/in^2$

(B) probably noncompact; yield strength = 36 $kips/in^2$

(C) probably compact; yield strength = 42 $kips/in^2$

(D) probably compact; yield strength = 50 $kips/in^2$

SOLUTIONS

1. Determine the required moment strength.

$$\begin{aligned} M_u &= 1.6M_L = 1.6\frac{w_u L^2}{8} \\ &= (1.6)\left(\frac{\left(4\ \frac{\text{kips}}{\text{ft}}\right)(25\text{ ft})^2}{8}\right) \\ &= 499\text{ ft-kips} \end{aligned}$$

The required plastic moment is equal to the nominal moment.

$$M_p = M_n = \frac{M_u}{\phi}$$

The required plastic section modulus is

$$\begin{aligned} M_p &= F_y Z_x \\ \frac{M_u}{\phi} &= F_y Z_x \\ Z_x &= \frac{M_p}{\phi F_y} \\ &= \frac{(499\text{ ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(0.90)\left(50\ \frac{\text{kips}}{\text{in}^2}\right)} \\ &= 133.1\text{ in}^3 \quad (130\text{ in}^3) \end{aligned}$$

The answer is (D).

2. The effective radius of gyration is

$$\begin{aligned} r_{ts} &= \sqrt[4]{\frac{I_y C_w}{S_x^2}} \\ &= \sqrt[4]{\frac{(45.1\text{ in}^4)(3450\text{ in}^6)}{(99.1\text{ in}^3)^2}} \\ &= 1.995\text{ in} \quad (2.0\text{ in}) \end{aligned}$$

Structural Design

(The equation needed to solve this problem is no longer present in the *NCEES Handbook*.)

The answer is (A).

3. The *NCEES Handbook* does not provide sufficient information to calculate the quantities involved in determining compactness. However, virtually all standard rolled W-shapes are compact. (The few exceptions are W40 × 174, W14 × 99, W14 × 90, W12 × 65, W10 × 12, W8 × 10, and W6 × 15 made from A992 steel.) Based on this fact, the beam is probably compact.

From the W-shapes dimensions and properties table, for a W14 × 68 shape, the beam depth is 14 in, and the web thickness is 0.415 in. The nominal shear strength is

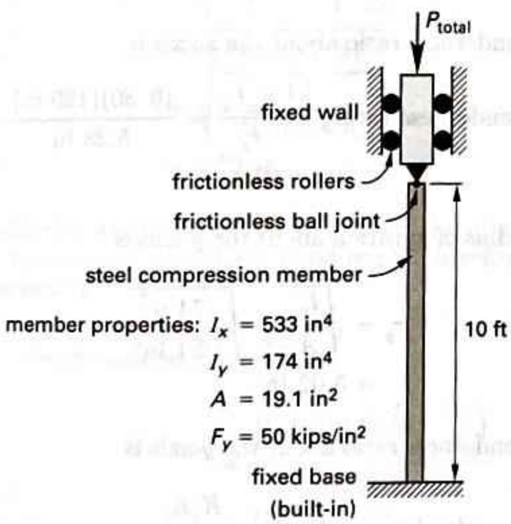
$$\begin{aligned} V_n &= 0.6F_y d t_w \\ F_y &= \frac{V_n}{0.6 d t_w} = \frac{146 \text{ kips}}{(0.6)(14 \text{ in})(0.415 \text{ in})} \\ &= 41.88 \text{ kips/in}^2 \quad (42 \text{ kips/in}^2) \end{aligned}$$

The answer is (C).

44 Structural Steel: Columns

PRACTICE PROBLEMS

1. A steel compression member has a fixed support at one end and a frictionless ball joint support at the other as shown. The total applied design load consists of a dead load of 7 kips (which includes the weight of the member) and an unspecified live load. Design (not theoretical) effective lengths are to be used.



This compression member is controlled by which type of buckling?

(A) local

(B) torsional

(C) inelastic

(D) elastic

2. A long column member has one end built-in and the other end pinned. The column is loaded in compression evenly until buckling occurs. Which statement about the column after buckling is true?

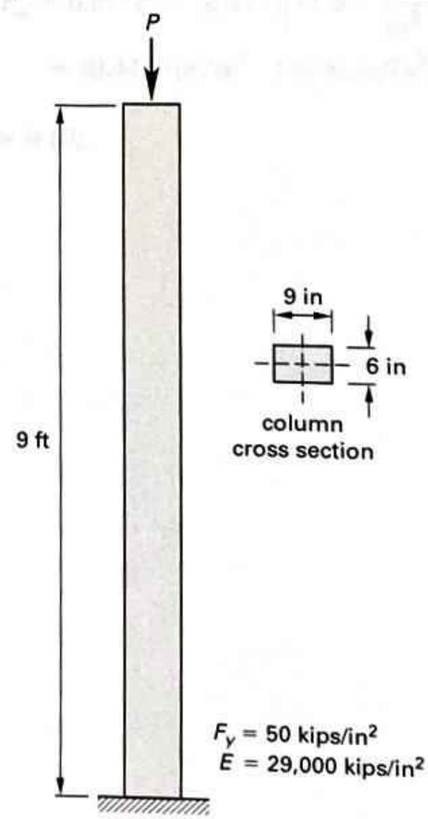
(A) The column experiences maximum deflection on its midpoint.

(B) The maximum deflection point is closer to the pinned end than the built-in end.

(C) The deflection curve is S-shaped.

(D) The column experiences no deflection under buckling.

3. A solid steel column with a fixed bottom support and free upper end is concentrically loaded. Material and geometric properties are shown.



The available axial compressive design stress is most nearly

(A) 13 $kips/in^2$

(B) 18 $kips/in^2$

(C) 29 $kips/in^2$

(D) 39 $kips/in^2$

4. A steel column is built-in at one end and free to translate and rotate at the other end. The column uses a 12 ft long W12 × 45 beam. If the yield strength of the steel is 50 $kips/in^2$, the critical stress in the column is most nearly

(A) 7.3 $kips/in^2$

(B) 9.0 $kips/in^2$

(C) 9.7 $kips/in^2$

(D) 10 $kips/in^2$

Structural Design

SOLUTIONS

1. The effective column length factor for design use about both the x-axis and y-axis is 0.80.

The unbraced length of the compression member is the same about the x-axis and the y-axis.

$$\begin{aligned} L_x = L_y &= (10\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ &= 120\ \text{in} \end{aligned}$$

The radius of gyration about the x-axis is

$$\begin{aligned} r_x &= \sqrt{\frac{I_x}{A}} = \sqrt{\frac{533\ \text{in}^4}{19.1\ \text{in}^2}} \\ &= 5.28\ \text{in} \end{aligned}$$

The slenderness ratio about the x-axis is

$$\begin{aligned} \text{slenderness ratio}_x &= \frac{K_xL_x}{r_x} = \frac{(0.80)(120\ \text{in})}{5.28\ \text{in}} \\ &= 18.2 \end{aligned}$$

The radius of gyration about the y-axis is

$$\begin{aligned} r_y &= \sqrt{\frac{I_y}{A}} = \sqrt{\frac{174\ \text{in}^4}{19.1\ \text{in}^2}} \\ &= 3.02\ \text{in} \end{aligned}$$

The slenderness ratio about the y-axis is

$$\begin{aligned} \text{slenderness ratio}_y &= \frac{K_yL_y}{r_y} \\ &= \frac{(0.80)(120\ \text{in})}{3.02\ \text{in}} \\ &= 31.8 \end{aligned}$$

The larger slenderness ratio controls, so use 31.8.

The modulus of elasticity, E, is 29,000 $kips/in^2$. The limiting slenderness ratio is

$$\begin{aligned} 4.71\sqrt{\frac{E}{F_y}} &= 4.71\sqrt{\frac{29{,}000\ \frac{\text{kips}}{\text{in}^2}}{50\ \frac{\text{kips}}{\text{in}^2}}} \\ &= 113.4 \end{aligned}$$

Since 31.8 is less than 113.4, the column fails by inelastic buckling.

The answer is (C).

2. The built-in end of the column is completely fixed against rotation and translation. The pinned end of the column is free to rotate but is fixed against translation. Therefore, the pinned end of the column experiences more rotational deflection, and the maximum deflection point is closer to the pinned end of the column.

The answer is (B).

3. Since the unsupported length of the column is the same about both the strong and weak axes, the largest slenderness ratio results from bending about the weak axis. Therefore, the least radius of gyration applies.

$$I_{weak} = r^2 A$$

$$\begin{aligned} r &= \sqrt{\frac{I_{weak}}{A}} = \sqrt{\frac{\frac{bh^3}{12}}{bh}} \\ &= \sqrt{\frac{\frac{(9\text{ in})(6\text{ in})^3}{12}}{(9\text{ in})(6\text{ in})}} \\ &= 1.73\text{ in} \end{aligned}$$

The effective length factor, K, is 2.10 for the given column end support conditions (fixed-free). Therefore, the slenderness ratio is

$$\begin{aligned} \text{slenderness ratio} &= \frac{KL}{r} \\ &= \frac{(2.10)(9\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{1.73\text{ in}} \\ &= 130.94\quad(131) \end{aligned}$$

From AISC Table 4–14, the available column strength for a compression member with a slenderness ratio of 131 is 13.2 kips/in^2 (13 kips/in^2).

The answer is (A).

4. The y-axis has the smallest radius of gyration. The radius of gyration about the y-axis is 1.95 in for a W12 × 45 beam. The effective length factor is 2.10 when one end is fixed and one end is free. Use these values to determine the slenderness ratio.

$$\begin{aligned} \text{slenderness ratio} &= \frac{KL}{r} \\ &= \frac{(2.10)(12\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{1.95\text{ in}} \\ &= 155.1 \end{aligned}$$

Determine the available design stress. The modulus of elasticity, E, is 29,000 kips/in^2.

$$\begin{aligned} 4.71\sqrt{\frac{E}{F_y}} &= 4.71\sqrt{\frac{29{,}000\ \frac{\text{kips}}{\text{in}^2}}{50\ \frac{\text{kips}}{\text{in}^2}}} \\ &= 113.4 \end{aligned}$$

The slenderness ratio is greater than 113.4. Therefore, the elastic buckling stress is

$$\begin{aligned} F_e &= \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \\ &= \frac{\pi^2\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)}{155.1^2} \\ &= 11.90\text{ kips/in}^2 \end{aligned}$$

The critical stress is

$$\begin{aligned} F_{cr} = 0.877F_e &= (0.877)\left(11.90\ \frac{\text{kips}}{\text{in}^2}\right) \\ &= 10.44\text{ kips/in}^2\quad(10\text{ kips/in}^2) \end{aligned}$$

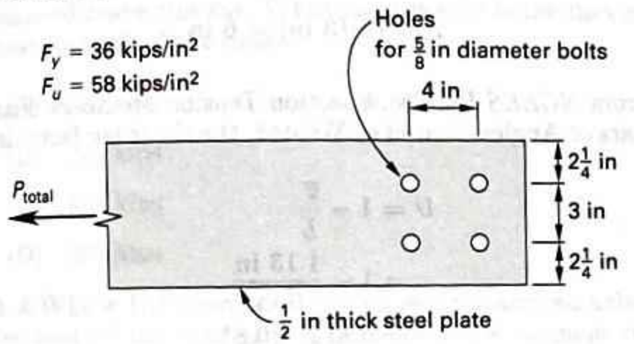
The answer is (D).

Structural Design

45 Structural Steel: Tension Members

PRACTICE PROBLEMS

1. A bolted steel tension member is shown.



What is most nearly the effective net area in tension for this plate?

(A) 2.3 in^2

(B) 3.1 in^2

(C) 3.4 in^2

(D) 3.8 in^2

2. A steel tension member is 5 in long and ½ in thick. There are two holes in the bar. The holes are in parallel and are made for a ¼-in bolt each. The net area is most nearly

(A) 2.0 in^2

(B) 2.2 in^2

(C) 2.5 in^2

(D) 2.7 in^2

3. A W-shape member (yield strength of 36 ksi; ultimate strength of 58 ksi) carries an axial live tensile load of 420 kips. The member's flanges are bolted to a connection bracket. The shear lag factor for the connection is 0.90. The required net area based on the fracture (rupture) criterion is most nearly

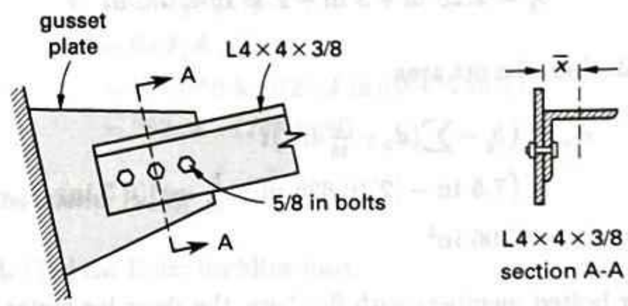
(A) 12 in^2

(B) 17 in^2

(C) 22 in^2

(D) 27 in^2

4. An L4 × 4 × ⅜ angle made from A36 steel is used as a tension member as shown. The angle is connected to a gusset plate with ⅝ in diameter bolts. The center-to-center bolt spacing is 3 in. The distance from the centroid of the area connected to the plane of connection (the edge), $\bar{x}$, is 1.13 in.



What is most nearly the shear lag factor?

(A) 0.72

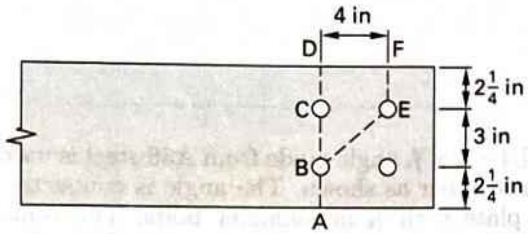
(B) 0.78

(C) 0.81

(D) 0.86

SOLUTIONS

1. For fracture, choose the shortest path, which is along line ABCD, as shown.



The gross member width is

$$b_g = 2.25 \text{ in} + 3 \text{ in} + 2.25 \text{ in} = 7.5 \text{ in}$$

Calculate the net area.

$$\begin{aligned} A_n &= \left(b_g - \sum(d_b + \tfrac{1}{16} \text{ in})\right)t \\ &= \left(7.5 \text{ in} - (2)(0.625 \text{ in} + \tfrac{1}{16} \text{ in})\right)(0.5 \text{ in}) \\ &= 3.06 \text{ in}^2 \end{aligned}$$

For bolted members with flat bars, the shear lag factor, U, is 1.0. The effective net area is

$$\begin{aligned} A_e &= UA_n = (1.0)(3.06 \text{ in}^2) \\ &= 3.06 \text{ in}^2 \quad (3.1 \text{ in}^2) \end{aligned}$$

The answer is (B).

2. The net area excludes area taken up by the holes.

$$\begin{aligned} A_n &= \left(b_g - \sum(d_b + \tfrac{1}{16} \text{ in})\right)t \\ &= \left(5 \text{ in} - (2 \text{ bolts})\left(0.25 \ \frac{\text{in}}{\text{bolt}} + \tfrac{1}{16} \text{ in}\right)\right) \\ &\quad \times (0.5 \text{ in}) \\ &= 2.19 \text{ in}^2 \quad (2.2 \text{ in}^2) \end{aligned}$$

The answer is (B).

3. Find the effective net area based on the fracture (rupture) criterion.

$$\begin{aligned} \phi_t &= 0.75 \\ T_u &= \phi_t T_n = \phi_t F_u A_e \\ (1.6)(420 \text{ kips}) &= (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)A_e \\ A_e &= 15.45 \text{ in}^2 \end{aligned}$$

The net area is

$$\begin{aligned} A_e &= UA_n \\ A_n &= \frac{A_e}{U} = \frac{15.45 \text{ in}^2}{0.90} \\ &= 17.2 \text{ in}^2 \quad (17 \text{ in}^2) \end{aligned}$$

The answer is (B).

4. The length of the connection for bolted connections is the distance between the outer holes.

$$L = (2)(3 \text{ in}) = 6 \text{ in}$$

From *NCEES Handbook* section Tension Members: Flat Bars or Angles, Bolted or Welded, the shear lag factor is

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{L} \\ &= 1 - \frac{1.13 \text{ in}}{6 \text{ in}} \\ &= 0.812 \quad (0.81) \end{aligned}$$

The answer is (C).

Structural Design

46 Structural Steel: Beam-Columns

PRACTICE PROBLEMS

1. A W21x73 beam (Gr. 50) is used to support a 5-ton overhead crane runway. What most nearly is the design shear strength of the beam?

(A) 260 kips

(B) 269 kips

(C) 288 kips

(D) 289 kips

2. A W14 × 132 beam has been chosen to carry an axial live load of 160 kips and a maximum live moment of 320 ft-kips about the strong axis. The unsupported length is 32 ft. $K=1$, $C_m=1$, and $I_x=1530\ \text{in}^4$. Taking into account the second-order effects, what is most nearly the required flexural strength?

(A) 340 ft-kips

(B) 380 ft-kips

(C) 420 ft-kips

(D) 460 ft-kips

SOLUTIONS

1. The design shear capacity of the beam is only dependent on the area of the web and its yield strength as shown

$$\begin{aligned} V_n &= 0.6F_yA_wC_{vl} \\ &= 0.6F_ydt_wC_{vl} \\ &= (0.6)(50\ \text{ksi})(21.2\ \text{in})(0.455\ \text{in})(1.0) \\ &= 289.38 \quad (289\ \text{kips}) \end{aligned}$$

The answer is (D).

2. Find the Euler buckling load.

$$\begin{aligned} P_{e1} &= \frac{\pi^2EI}{(KL)_x^2} = \frac{\pi^2\left(29{,}000\ \frac{\text{kips}}{\text{in}^2}\right)(1530\ \text{in}^4)}{\left((1)(32\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^2} \\ &= 2970\ \text{kips} \end{aligned}$$

Find the x-x axis flexural magnifier.

$$\begin{aligned} B_1 &= \frac{C_m}{1-\frac{P_r}{P_{e1}}} = \frac{1}{1-\frac{160\ \text{kips}}{2970\ \text{kips}}} \\ &= 1.057 \end{aligned}$$

The required flexural strength is

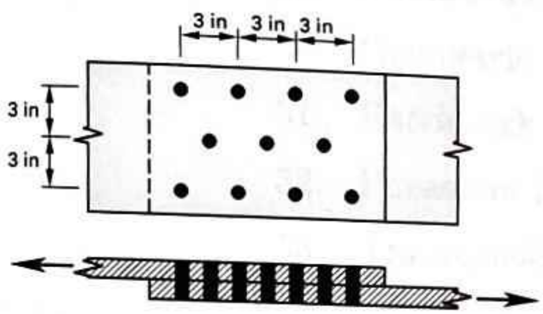
$$\begin{aligned} M_r &= B_1M_{nt} = (1.057)(320\ \text{ft-kips}) \\ &= 338.2\ \text{ft-kips} \quad (340\ \text{ft-kips}) \end{aligned}$$

The answer is (A).

47 Structural Steel: Connectors

PRACTICE PROBLEMS

1. The connection shown consists of 11 grade A307 ⅞ in diameter bolts. Bolt hole sizes are standard. The ultimate strength of the connected member is 58 ksi. The connected member is 0.5 in thick. The edge clear distance is 2.5 in, and the center-to-center spacing of the holes is 3 in.



The available bearing strength per bolt per inch of thickness in the connection is most nearly

(A) 76 kips/in

(B) 83 kips/in

(C) 91 kips/in

(D) 110 kips/in

2. A connection is made from two ¾ in bolts in parallel, placed on a 8.5 in wide steel bar. The bar is 1 in thick that has an ultimate strength of 65 ksi. The holes are 3 in from their centers to the side of the bar and 2.25 in from center to center. Bolt hole sizes are standard. The bearing resistance of the entire connection is most nearly

(A) 84 kips

(B) 95 kips

(C) 170 kips

(D) 210 kips

SOLUTIONS

1. Determine the bolt hole diameter.

$$\begin{aligned} d_h &= d_b + \tfrac{1}{16} \text{ in} \\ &= \tfrac{7}{8} \text{ in} + \tfrac{1}{16} \text{ in} \\ &= \tfrac{15}{16} \text{ in} \end{aligned}$$

Determine which clear distance, L_c, value controls for the bolts. Obtain the clear distance in between the interior holes.

$$\begin{aligned} L_c &= s - d_h \\ &= 3 \text{ in} - \tfrac{15}{16} \text{ in} \\ &= 2\tfrac{1}{16} \text{ in} < 2\tfrac{1}{2} \text{ in provided} \end{aligned}$$

The smaller value for the clear distance controls. Determine the available bearing strength.

$$\begin{aligned} \phi r_n &= \phi 1.2 L_c F_u \le \phi 2.4 d_b F_u \\ &= (0.75)(1.2)\left(2\tfrac{1}{16} \text{ in}\right)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right) \\ &\quad \le (0.75)(2.4)\left(\tfrac{7}{8} \text{ in}\right)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right) \\ &= 107.7 \ \frac{\text{kips}}{\text{in}} \le 91.35 \ \frac{\text{kips}}{\text{in}} \end{aligned}$$

Since the available bearing strength determined by the clear distance is larger than the available bearing strength determined by the bolt size, the smaller value controls, and the available bearing strength is 91 kips/in.

This value may also be determined using AISC tables for a center-to-center spacing of 3 in, a connected member strength of 58 ksi, and a bolt size of ⅞ in.

The answer is (C).

2. Calculate the hole diameter using the bolt diameter and standard clearance.

$$\begin{aligned} d_h &= d_b + \tfrac{1}{16}\ \text{in} \\ &= \tfrac{3}{4}\ \text{in} + \tfrac{1}{16}\ \text{in} \\ &= \tfrac{13}{16}\ \text{in} \quad (0.8125\ \text{in}) \end{aligned}$$

The center-to-center spacing of the interior holes is 3 in. Obtain the clear distance between the interior holes.

$$\begin{aligned} L_c &= s - d_h \\ &= 2.25\ \text{in} - 0.8125\ \text{in} \\ &= 1.4375\ \text{in} \end{aligned}$$

Determine the clear distance between the hole and the edge of the steel bar.

$$\begin{aligned} L_c &= L_e - \frac{d_h}{2} \\ &= 3\ \text{in} - \frac{0.8125\ \text{in}}{2} \\ &= 2.594\ \text{in} \end{aligned}$$

Use the smaller clear distance. Since each bolt has the same properties, calculate the available bearing strength for one bolt.

$$\begin{aligned} r_n &= \min\begin{cases} \phi 1.2 L_c F_u \\ \phi 2.4 d_b F_u \end{cases} \\ &= \min\begin{cases} (0.75)(1.2)(1.435\ \text{in})\left(65\ \dfrac{\text{kips}}{\text{in}^2}\right) \\ (0.75)(2.4)(0.75\ \text{in})\left(65\ \dfrac{\text{kips}}{\text{in}^2}\right) \end{cases} \\ &= \min\begin{cases} 84.09\ \dfrac{\text{kips}}{\text{in}} \\ 87.75\ \dfrac{\text{kips}}{\text{in}} \end{cases} \\ &= 84.09\ \text{kips/in} \end{aligned}$$

The available bearing strength is

$$\begin{aligned} \phi R_n &= \sum \phi r_n t \\ &= (2\ \text{bolts})\left(84.09\ \frac{\text{kips}}{\text{bolt-in}}\right)(1\ \text{in}) \\ &= 168.2\ \text{kips} \quad (170\ \text{kips}) \end{aligned}$$

The answer is (C).

48 Transportation Planning and Capacity

PRACTICE PROBLEMS

1. An interstate weigh station can weigh an average of 20 trucks per hour. Trucks arrive at the average rate of 12 trucks per hour. Performance is described by an $M/M/1$ model. What is most nearly the steady-state value of the trucks' time spent waiting to be weighed?

(A) 0.13 hr/truck

(B) 0.25 hr/truck

(C) 4.0 hr/truck

(D) 8.0 hr/truck

2. A traffic flow relationship is given by $q = kv$, where q is the traffic volume in veh/hr, k is the traffic density in veh/mi, and v is the mean speed in mi/hr. The mean speed on a road in mi/hr is given by the relationship

$$v = 60\ \frac{\text{mi}}{\text{hr}} - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k$$

The maximum capacity of overall traffic volume for this road is most nearly

(A) 3400 veh/hr

(B) 4300 veh/hr

(C) 4500 veh/hr

(D) 5000 veh/hr

3. A highway weigh station can inspect an average of 16 trucks per hour. Trucks arrive at the average rate of 10 trucks per hour. Performance is described by an $M/M/1$ model. What is most nearly the steady-state value of the number of trucks in the station?

(A) 0.16

(B) 0.60

(C) 1.7

(D) 2.5

4. Vehicles arrive at a parking garage at an average rate of 45 vehicles per hour. Vehicles are parked by either of two attendants at an average rate of one vehicle per minute for each attendant. The queue discipline is first come, first served. The arrival of vehicles is modeled as a Poisson's distribution and attendant parking time is modeled as an exponential distribution. What is most nearly the probability that at least one attendant will be idle?

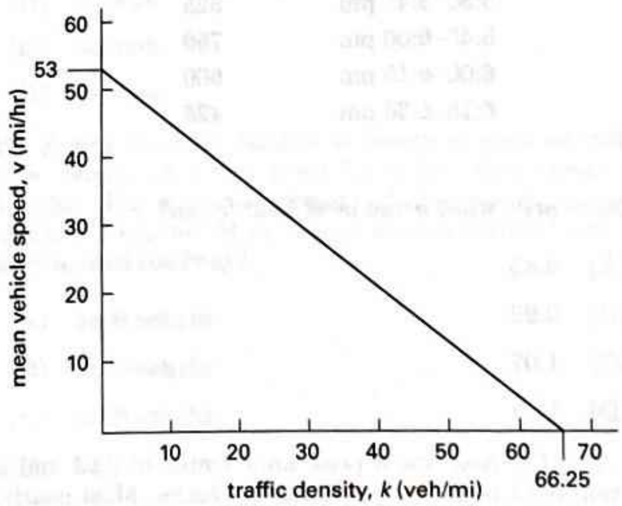
(A) 0.34

(B) 0.45

(C) 0.52

(D) 0.80

5. The relationship between traffic density and mean vehicle speed is shown for a particular road.



What is most nearly the maximum traffic volume for this road?

(A) 760 veh/hr

(B) 880 veh/hr

(C) 900 veh/hr

(D) 960 veh/hr

Transportation/ Surveying

6. A border's point of entry can weigh an average of 24 trucks per hour. Trucks arrive at the average rate of 15 trucks per hour. Performance is described by an $M/M/1$ model. What is most nearly the steady-state value of the probability that there will be five trucks being weighed or waiting to be weighed at any time?

(A) 0.010

(B) 0.036

(C) 0.38

(D) 0.63

7. 15 min interval traffic counts for a traffic flow are shown.

time	vehicles
3:30–3:45 pm	520
3:45–4:00 pm	475
4:00–4:15 pm	600
4:15–4:30 pm	625
4:30–4:45 pm	700
4:45–5:00 pm	500
5:00–5:15 pm	675
5:15–5:30 pm	800
5:30–5:45 pm	825
5:45–6:00 pm	750
6:00–6:15 pm	600
6:15–6:30 pm	425

Most nearly, what is the peak hour factor?

(A) 0.83

(B) 0.92

(C) 1.07

(D) 1.12

8. A traffic flow has a peak hour factor of 0.63 and a 15 min peak hour flow rate of 200 vehicles. Most nearly, what is the peak hour volume?

(A) 370 vph

(B) 500 vph

(C) 620 vph

(D) 700 vph

9. A traffic flow has a peak hour factor of 0.54 and a peak hour volume of 800 veh/hr. Most nearly, what is the hourly flow rate?

(A) 430 vph

(B) 800 vph

(C) 1200 vph

(D) 1500 vph

10. A freeway has two 10 ft lanes in each direction, with a 2 ft right side lateral clearance and a total ramp density of 4 ramps per mile. The hourly volume on the freeway is 1800 veh/hr. Most nearly, what is the free flow speed on the freeway?

(A) 55 mph

(B) 60 mph

(C) 65 mph

(D) 75 mph

11. A freeway has four 12 ft wide lanes in each direction, no right side lateral clearance, and a ramp density of 6 ramps per mile. Most nearly, what is the free flow speed?

(A) 60 mph

(B) 65 mph

(C) 70 mph

(D) 75 mph

12. A freeway has two 10 ft wide lanes in each direction, a 2 ft right side lateral clearance, and a ramp density of 3 ramps per mile. The demand flow rate is 2000 pc/hr/ln. Most nearly, what is the mean speed of traffic on the freeway?

(A) 55 mph

(B) 57 mph

(C) 60 mph

(D) 63 mph

13. A freeway segment has two 12 ft wide lanes in each direction through rolling terrain. The freeway traffic peak hour factor is 0.80, and the hourly flow rate is 1600 veh/hr one-way. The traffic flow is 15% trucks and buses. Most nearly, what is the demand flow rate?

(A) 900 pc/hr/ln

(B) 1000 pc/hr/ln

(C) 1300 pc/hr/ln

(D) 1500 pc/hr/ln

Transportation/ Surveying

14. A freeway has five lanes in each direction. The volume is 2300 veh/hr one way, the peak 15 min volume is 220 vehicles, and the peak hourly volume is 775 vehicles. The heavy vehicle factor of the traffic flow is 0.90. Most nearly, what is the demand flow rate?

(A) 580 pc/h/ln

(B) 650 pc/h/ln

(C) 730 pc/h/ln

(D) 1200 pc/h/ln

15. A freeway segment built through level terrain has five 11 ft lanes in each direction and a 6 ft right side lateral clearance. The traffic flow is 23% trucks and buses. Most nearly, what is the heavy vehicle factor for the traffic flow?

(A) 0.54

(B) 0.69

(C) 0.77

(D) 0.81

16. A 6 lane freeway has a vehicular density of 15.3 pc/mi/ln. The mean speed of travel on the freeway is 60 mph. Most nearly, what is the demand flow rate?

(A) 860 pc/hr/ln

(B) 870 pc/hr/ln

(C) 920 pc/hr/ln

(D) 1000 pc/hr/ln

17. A freeway segment built through rolling terrain has three lanes in each direction. The volume of the traffic flow is 3400 veh/hr one way, with a free flow speed of 55 mph and a peak hour factor of 0.75. The heavy vehicle factor is 0.82, and the factor for driver population is 1.0. What is the level of service, LOS, of the freeway segment?

(A) LOS A

(B) LOS B

(C) LOS C

(D) LOS D

18. A freeway has two lanes of traffic in each direction with a demand flow rate of 845 pc/hr/ln. The mean speed of travel is 70 mph. What is the level of service, LOS, of the freeway segment?

(A) LOS A

(B) LOS B

(C) LOS C

(D) LOS D

19. A 6 lane highway has a free flow speed of 65 mph, an observed density of 115 veh/mi/ln, and a jam density of 265 veh/mi/ln. Most nearly, what is the flow rate of the highway?

(A) 4200 veh/hr/ln

(B) 4500 veh/hr/ln

(C) 4800 veh/hr/ln

(D) 5000 veh/hr/ln

20. A roadway facility has a maximum flow rate of 9000 veh/hr for a free flow speed of 60 mph. The expected density during evening rush hour is 250 veh/mi. Most nearly, what is the speed of travel during rush hour?

(A) 25 mph

(B) 35 mph

(C) 50 mph

(D) 55 mph

21. A new roadway facility is designed with an optimum density of 150 veh/mi for a free flow speed of 50 mph. The expected density during morning rush hour is 175 veh/mi. Most nearly, what is the flow rate of the proposed roadway?

(A) 3300 veh/hr

(B) 3600 veh/hr

(C) 4000 veh/hr

(D) 4300 veh/hr

22. A new roadway facility has a maximum flow rate of 4500 veh/hr and a jam density of 275 veh/mi. Assume zero density and zero flow rate conditions. Most nearly, what is the theoretical speed of the first driver entering a facility?

(A) 50 mph

(B) 55 mph

(C) 60 mph

(D) 65 mph

23. A freeway facility has an optimum density of 125 veh/mi. Most nearly, what is the jam density?

(A) 62 veh/mi

(B) 120 veh/mi

(C) 250 veh/mi

(D) 500 veh/mi

SOLUTIONS

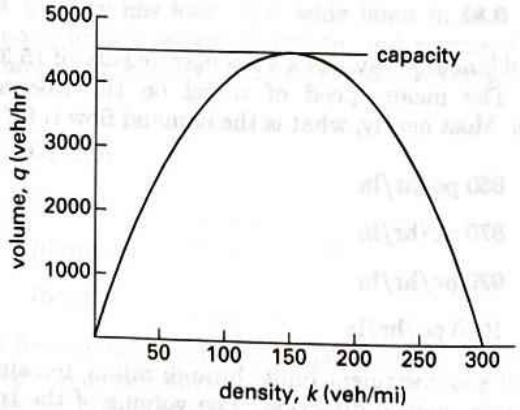
1. The average time spent waiting to be weighed is

$$W = \frac{1}{\mu - \lambda} = \frac{1}{20\ \frac{\text{trucks}}{\text{hr}} - 12\ \frac{\text{trucks}}{\text{hr}}}$$
$$= 0.125\ \text{hr/truck} \quad (0.13\ \text{hr/truck})$$

The answer is (A).

2. The mean speed relationship can be substituted into the traffic flow relationship, resulting in a quadratic relationship (i.e., a parabolic curve).

$$q = k\,\text{v} = k\left(60\ \frac{\text{mi}}{\text{hr}} - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k\right)$$
$$= \left(60\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2$$



To determine the traffic volume capacity, it is necessary to find the maximum point on the parabolic curve (i.e., the location where the slope of the curve equals 0).

$$\frac{dq}{dk} = 0$$

$$\frac{d\left[\left(60\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2\right]}{dk} = 60\ \frac{\text{mi}}{\text{hr}} - \left(0.4\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k$$
$$= 0$$

$$k = \frac{60\ \frac{\text{mi}}{\text{hr}}}{0.4\ \frac{\text{mi}^2}{\text{veh-hr}}}$$
$$= 150\ \text{veh/mi}$$

Substituting $k = 150$ veh/mi into the traffic flow relationship gives

$$\begin{aligned} q &= \left(60\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2 \\ &= \left(60\ \frac{\text{mi}}{\text{hr}}\right)\left(150\ \frac{\text{veh}}{\text{mi}}\right) \\ &\quad - \left(0.2\ \frac{\text{mi}^2}{\text{veh-hr}}\right)\left(150\ \frac{\text{veh}}{\text{mi}}\right)^2 \\ &= 4500\ \text{veh/hr} \end{aligned}$$

The answer is (C).

3. The average number of trucks in the station is

$$L = \frac{\lambda}{\mu - \lambda} = \frac{10\ \frac{\text{trucks}}{\text{hr}}}{16\ \frac{\text{trucks}}{\text{hr}} - 10\ \frac{\text{trucks}}{\text{hr}}} = 1.66 \quad (1.7)$$

The answer is (C).

4. The mean arrival rate of vehicles per minute is

$$\lambda = \frac{45}{(1\ \text{hr})\left(60\ \frac{\text{min}}{\text{hr}}\right)} = 0.75\ 1/\text{min}$$

The service rate per server is $\mu = 1\ 1/\text{min}$.

For two servers ($s = 2$),

$$\rho = \frac{\lambda}{s\mu} = \frac{0.75\ \frac{1}{\text{min}}}{(2)\left(1\ \frac{1}{\text{min}}\right)} = 0.375$$

Next, calculate the probability, P_0, of both attendants being idle (i.e., no units in the system).

$$P_0 = \frac{1-\rho}{1+\rho} = \frac{1-0.375}{1+0.375} = 0.4545$$

Calculate the probability, P_1, of one attendant being idle.

$$\begin{aligned} P_1 &= P_0\left(\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}\right) = (0.4545)\left(\frac{\left(\frac{0.75\ \frac{1}{\text{min}}}{1\ \frac{1}{\text{min}}}\right)^1}{1!}\right) \\ &= 0.3409 \end{aligned}$$

Finally, add P_0 and P_1 to calculate the probability of at least one idle attendant.

$$\begin{aligned} P &= P_0 + P_1 = 0.4545 + 0.3409 \\ &= 0.7954 \quad (0.80) \end{aligned}$$

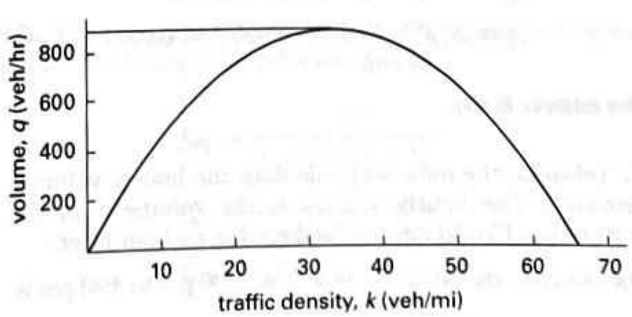
The answer is (D).

5. From the mean vehicle speed versus traffic density graph, the relationship is linear.

$$\begin{aligned} \text{v} &= 53\ \frac{\text{mi}}{\text{hr}} - \left(\frac{53\ \frac{\text{mi}}{\text{hr}}}{66.25\ \frac{\text{veh}}{\text{mi}}}\right)k \\ &= 53\ \frac{\text{mi}}{\text{hr}} - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k \quad \text{[velocity in mi/hr]} \end{aligned}$$

This expression for speed can be substituted into the standard relationship for traffic volume.

$$\begin{aligned} q &= k\text{v} \\ &= k\left(53\ \frac{\text{mi}}{\text{hr}} - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k\right) \\ &= \left(53\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2 \end{aligned}$$



Transportation/Surveying

The maximum traffic volume occurs where the slope of this curve equals 0.

$$\frac{dq}{dk} = \frac{d\left(\left(53\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2\right)}{dk}$$
$$= 53\ \frac{\text{mi}}{\text{hr}} - \left(1.6\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k$$
$$= 0$$
$$k_{\max q} = \frac{53\ \frac{\text{mi}}{\text{hr}}}{1.6\ \frac{\text{mi}^2}{\text{veh-hr}}}$$
$$= 33.125\ \text{veh/mi}$$

The density is 33.125 veh/mi at maximum traffic volume.

$$q = \left(53\ \frac{\text{mi}}{\text{hr}}\right)k - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)k^2$$
$$= \left(53\ \frac{\text{mi}}{\text{hr}}\right)\left(33.125\ \frac{\text{veh}}{\text{mi}}\right) - \left(0.8\ \frac{\text{mi}^2}{\text{veh-hr}}\right)\left(33.125\ \frac{\text{veh}}{\text{mi}}\right)^2$$
$$= 878\ \text{veh/hr} \quad (880\ \text{veh/hr})$$

The answer is (B).

6. The utilization factor is

$$\rho = \frac{\lambda}{s\mu} = \frac{15\ \frac{\text{trucks}}{\text{hr}}}{(1)\left(24\ \frac{\text{trucks}}{\text{hr}}\right)} = 0.625$$

The probability of five trucks being weighed or waiting to be weighed is

$$P_0 = 1 - \rho = 1 - 0.625 = 0.375$$
$$P_5 = P_0\rho^n = (0.375)(0.625)^5 = 0.036$$

The answer is (B).

7. Tabulate the data and calculate the hourly volumes provided. The hourly volume is the volume from the start of the 15 min interval and ending an hour later.

For example, the hourly volume from 3:30 pm to 4:30 pm is

$$520\ \text{vehicles} + 475\ \text{vehicles} + 600\ \text{vehicles} + 625\ \text{vehicles} = 2220\ \text{vehicles}$$

time	vehicles	hourly volume (vehicles)
3:30–3:45 pm	520	2220
3:45–4:00 pm	475	2400
4:00–4:15 pm	600	2425
4:15–4:30 pm	625	2500
4:30–4:45 pm	700	2675
4:45–5:00 pm	500	2800
5:00-5:15 pm	675	3050
5:15–5:30 pm	800	2975
5:30–5:45 pm	825	2600
5:45–6:00 pm	750	
6:00–6:15 pm	600	
6:15–6:30 pm	425	

The peak hour occurs between 5:00 pm and 6:00 pm. The peak hour volume is 3050 vehicles, and the peak 15 minute volume is 825 vehicles. The peak hour factor, PHF, is

$$\text{PHF} = \frac{\text{hourly volume}}{4 \times \text{peak 15 minute volume}}$$
$$= \frac{3050\ \text{vehicles}}{(4)(825\ \text{vehicles})}$$
$$= 0.924 \quad (0.92)$$

The answer is (B).

8. Rearrange the equation for the peak hour factor, PHF, to find the hourly volume.

$$\text{PHF} = \frac{\text{hourly volume}}{4 \times \text{peak 15 minute volume}}$$
$$\text{hourly volume} = (\text{PHF})(4 \times \text{peak 15 minute volume})$$
$$= (0.63)\big((4)(200\ \text{vehicles})\big)$$
$$= 504\ \text{veh/hr} \quad (500\ \text{vph})$$

The answer is (B).

9. Rearrange the equation for the peak hour factor, PHF, to find the hourly flow rate.

$$\text{PHF} = \frac{\text{hourly volume}}{\text{hourly flow rate}}$$
$$\text{hourly flow rate} = \frac{\text{hourly volume}}{\text{PHF}} = \frac{800\ \frac{\text{veh}}{\text{hr}}}{0.54}$$
$$= 1481\ \text{veh/hr} \quad (1500\ \text{vph})$$

The answer is (D).

10. From tables of adjustment factors, the lane width adjustment for 10 ft wide lanes is 6.6 mph.The adjustment for right-side lateral clearance for a 2 ft right side lateral clearance and two lanes in each direction is 2.4 mph. The free flow speed, FFS, is always rounded to the nearest multiple of 5 mph.

$$\begin{aligned}\text{FFS} &= 75.4\ \frac{\text{mi}}{\text{hr}} - f_{\text{LW}} - f_{\text{LC}} - 3.22\text{TRD}^{0.84}\\ &= 75.4\ \frac{\text{mi}}{\text{hr}} - 6.6\ \frac{\text{mi}}{\text{hr}} - 2.4\ \frac{\text{mi}}{\text{hr}} - (3.22)\left(4\ \frac{\text{ramps}}{\text{mi}}\right)^{0.84}\\ &= 56.1\ \text{mi/hr}\quad (55\ \text{mph})\end{aligned}$$

The answer is (A).

11. From tables of adjustment factors, the lane width adjustment, f_{LW}, for 12 ft lane widths is 0 mph, and the adjustment for right-side lateral clearance, f_{LC}, for no right-side clearance and four lanes in each direction is 1.2 mph. Round the free flow speed, FFS, to the nearest 5 mph.

$$\begin{aligned}\text{FFS} &= 75.4\ \frac{\text{mi}}{\text{hr}} - f_{\text{LW}} - f_{\text{LC}} - 3.22\text{TRD}^{0.84}\\ &= 75.4\ \frac{\text{mi}}{\text{hr}} - 0\ \frac{\text{mi}}{\text{hr}} - 1.2\ \frac{\text{mi}}{\text{hr}} - (3.22)\left(6\ \frac{\text{ramps}}{\text{mi}}\right)^{0.84}\\ &= 59.7\ \text{mi/hr}\quad (60\ \text{mph})\end{aligned}$$

The answer is (A).

12. Calculate the free flow speed, FFS. The lane width adjustment, f_{LW}, for 10 ft lane widths is 6.6 mph, and the adjustment for right-side lateral clearance, f_{LC}, for a 2 ft right-side lateral clearance and two lanes in each direction is 2.4 mph. Round to the nearest 5 mph.

$$\begin{aligned}\text{FFS} &= 75.4\ \frac{\text{mi}}{\text{hr}} - f_{\text{LW}} - f_{\text{LC}} - 3.22\text{TRD}^{0.84}\\ &= 75.4\ \frac{\text{mi}}{\text{hr}} - 6.6\ \frac{\text{mi}}{\text{hr}} - 2.4\ \frac{\text{mi}}{\text{hr}} - (3.22)\left(3\ \frac{\text{ramps}}{\text{mi}}\right)^{0.84}\\ &= 58.3\ \text{mi/hr}\quad (60\ \text{mph})\end{aligned}$$

From a table of breakpoint values, the breakpoint is 1600 pc/hr/ln. Since the demand flow rate is greater than the breakpoint, the mean speed of traffic is

$$\begin{aligned}S &= 60 - 0.00001814(v_p - 1600\ \text{pc/hr/ln})^2\\ &= 60 - (0.00001814)(2000\ \text{pc/hr/ln} - 1600\ \text{pc/hr/ln})^2\\ &= 57.1\ \text{mi/hr}\quad (57\ \text{mph})\end{aligned}$$

The answer is (B).

13. Calculate the heavy vehicle factor, f_{HV}. The passenger car equivalent for trucks/buses on rolling terrain is 3.0.

$$\begin{aligned}f_{\text{HV}} &= \frac{1}{1 + P_T(E_T - 1)}\\ &= \frac{1}{1 + (0.15)(3 - 1)}\\ &= 0.77\end{aligned}$$

The demand flow rate, v_p, is

$$\begin{aligned}v_p &= \frac{V}{\text{PHF} \times N \times f_{\text{HV}}}\\ &= \frac{1600\ \frac{\text{veh}}{\text{hr}}}{(0.80)(2\ \text{lanes})(0.77)}\\ &= 1298.7\ \text{pc/hr/ln}\quad (1300\ \text{pc/hr/ln})\end{aligned}$$

The answer is (C).

14. The peak hour factor, PHF, is

$$\begin{aligned}\text{PHF} &= \frac{\text{hourly volume}}{4 \times \text{peak 15 minute volume}}\\ &= \frac{775\ \frac{\text{veh}}{\text{hr}}}{(4)(220\ \text{vehicles})}\\ &= 0.881\end{aligned}$$

The demand flow rate, v_p, is

$$\begin{aligned}v_p &= \frac{V}{\text{PHF} \times N \times f_{\text{HV}}}\\ &= \frac{2300\ \frac{\text{vehicles}}{\text{hr}}}{(0.881)(5)(0.90)}\\ &= 580\ \text{pc/hr/ln}\end{aligned}$$

The answer is (A).

15. The passenger car equivalents for trucks and buses is 2.0. The heavy vehicle factor, f_{HV}, is

$$f_{HV} = \frac{1}{1 + P_T(E_T - 1)}$$

$$f_{HV} = \frac{1}{1 + 0.23(2.0 - 1)}$$

$$f_{HV} = 0.81$$

The answer is (D).

Transportation/ Surveying

16. Rearrange the equation for traffic density to find the demand flow rate, v_p.

$$\begin{aligned} D &= \frac{v_p}{S} \\ v_p &= DS \\ &= \left(15.3\ \frac{\frac{\text{pc}}{\text{mi}}}{\text{ln}}\right)\left(60\ \frac{\text{mi}}{\text{hr}}\right) \\ &= 918\ \text{pc/hr/ln} \quad (920\ \text{pc/hr/ln}) \end{aligned}$$

The answer is (C).

17. The demand flow rate, v_p, is

$$\begin{aligned} v_p &= \frac{V}{\text{PHF} \times N \times f_{\text{HV}} \times f_p} \\ &= \frac{3400\ \frac{\text{veh}}{\text{hr}}}{(0.75)(3\ \text{lanes})(0.82)(1.0)} \\ &= 1843\ \text{pc/hr/ln} \end{aligned}$$

The traffic density, D, is

$$\begin{aligned} D &= \frac{v_p}{S} \\ &= \frac{1843\ \frac{\frac{\text{pc}}{\text{hr}}}{\text{ln}}}{55\ \frac{\text{mi}}{\text{hr}}} \\ &= 33.5\ \text{pc/mi/ln} \end{aligned}$$

From a table of LOS ranges, a density of 33.5 pc/mi/ln falls within LOS D.

The answer is (D).

18. The vehicle density, D, in the freeway segment is

$$\begin{aligned} D &= \frac{v_p}{S} \\ &= \frac{845\ \frac{\frac{\text{pc}}{\text{hr}}}{\text{ln}}}{70\ \frac{\text{mi}}{\text{hr}}} \\ &= 12.07\ \text{pc/mi/ln} \end{aligned}$$

From a table of LOS ranges, a density of 12.07 pc/mi/ln falls within LOS B.

The answer is (B).

19. From the Greenshields' model for flow rate,

$$\begin{aligned} V &= S_f D - \frac{S_f}{D_j} D^2 \\ &= \left(65\ \frac{\text{mi}}{\text{hr}}\right)\left(115\ \frac{\text{veh}}{\text{mi-ln}}\right) - \left(\frac{65\ \frac{\text{mi}}{\text{hr}}}{265\ \frac{\text{veh}}{\text{mi-ln}}}\right)\left(115\ \frac{\text{veh}}{\text{mi-ln}}\right)^2 \\ &= 4231\ \text{veh/hr/ln} \quad (4200\ \text{veh/hr/ln}) \end{aligned}$$

The answer is (A).

20. Rearrange the Greenshields' model for maximum flow and speed to find the jam density, D_j.

$$\begin{aligned} V_m &= \frac{D_j S_f}{4} \\ D_j &= \frac{4 V_m}{S_f} \\ &= \frac{(4)\left(9000\ \frac{\text{veh}}{\text{hr}}\right)}{60\ \frac{\text{mi}}{\text{hr}}} \\ &= 600\ \text{veh/mi} \end{aligned}$$

The speed of travel, S, is

$$\begin{aligned} S &= S_f - \frac{S_f}{D_j} D \\ &= 60\ \frac{\text{mi}}{\text{hr}} - \left(\frac{60\ \frac{\text{mi}}{\text{hr}}}{600\ \frac{\text{veh}}{\text{mi}}}\right)\left(250\ \frac{\text{veh}}{\text{mi}}\right) \\ &= 35\ \text{mi/hr} \quad (35\ \text{mph}) \end{aligned}$$

The answer is (B).

21. Rearrange the equation for optimum density to find the jam density, D_j.

$$\begin{aligned} D_o &= \frac{D_j}{2} \\ D_j &= 2D_o \\ &= (2)\left(150\ \frac{\text{veh}}{\text{mi}}\right) \\ &= 300\ \text{veh/mi} \end{aligned}$$

Transportation/ Surveying

From the Greenshields' model equation for flow rate, the flow rate of the proposed roadway, V, is

$$V = S_f D - \frac{S_f}{D_j} D^2$$

$$= \left(50\ \frac{\text{mi}}{\text{hr}}\right)\left(175\ \frac{\text{veh}}{\text{mi}}\right) - \left(\frac{50\ \frac{\text{mi}}{\text{hr}}}{300\ \frac{\text{veh}}{\text{mi}}}\right)\left(175\ \frac{\text{veh}}{\text{mi}}\right)^2$$

$$= 3645\ \text{veh/hr} \quad (3600\ \text{veh/hr})$$

The answer is (B).

22. Rearrange the Greenshields' model for maximum flow equation for the theoretical speed for the first driver entering a facility, S_f.

$$V_m = \frac{D_j S_f}{4}$$

$$S_f = \frac{4V_m}{D_j}$$

$$= \frac{(4)\left(4500\ \frac{\text{veh}}{\text{hr}}\right)}{275\ \frac{\text{veh}}{\text{mi}}}$$

$$= 65.5\ \text{mph} \quad (65\ \text{mph})$$

The answer is (D).

23. From the Greenshields' model for optimum density,

$$D_o = \frac{D_j}{2}$$

$$D_j = 2D_o$$

$$= (2)\left(125\ \frac{\text{veh}}{\text{mi}}\right)$$

$$= 250\ \text{veh/mi}$$

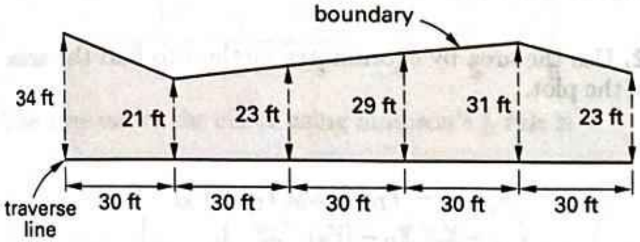
The answer is (C).

Transportation/ Surveying

49 Plane Surveying

PRACTICE PROBLEMS

1. Boundary and traverse lines bounding an irregular area are shown.



The total area between the irregular boundary and the traverse line is most nearly

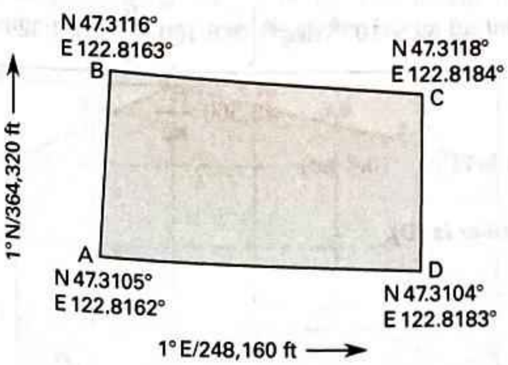
(A) 3600 ft^2

(B) 3800 ft^2

(C) 4000 ft^2

(D) 4200 ft^2

2. Global positioning system (GPS) latitudes and longitudes were taken of a plot of land. In the region where the plot is located, the length of a degree of latitude is 364,320 ft, and the length of a degree of longitude is 248,160 ft.



What is most nearly the area of the plot?

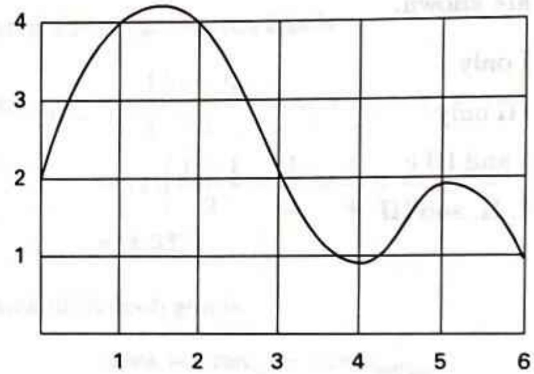
(A) 5.0 ac

(B) 5.1 ac

(C) 5.3 ac

(D) 5.4 ac

3. The illustration shows a curve from $x = 0$ to $x = 6$.



Using intervals of 1, what is most nearly the area under the curve predicted by Simpson's ⅓ rule?

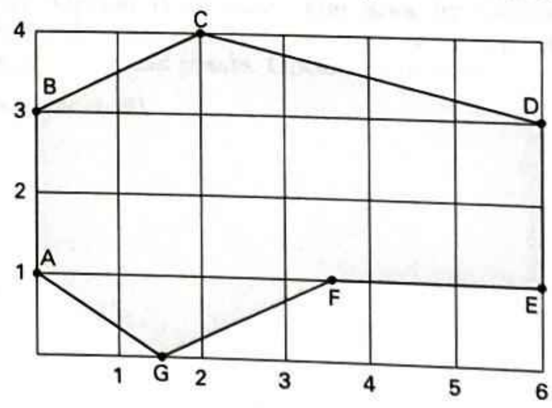
(A) 14

(B) 15

(C) 16

(D) 17

4. A polygon is created by enclosing lines as shown.



Using an interval of 1, what is most nearly the area of the polygon predicted by the trapezoidal rule?

(A) 15

(B) 16

(C) 17

(D) 18

Transportation/ Surveying

5. Which statement(s) concerning methods used to determine areas under a curve or line is/are true?

I. The trapezoidal rule applies to areas where the irregular sides are curved.

II. Simpson's ⅓ rule only applies to an odd number of segments.

III. The area by coordinates method can be used if the coordinates of the traverse leg end points are known.

(A) I only

(B) III only

(C) I and III

(D) I, II, and III

SOLUTIONS

1. The trapezoidal rule is

$$\begin{aligned}\text{area} &= w\left(\frac{h_1+h_6}{2}+h_2+h_3+h_4+h_5\right)\\ &= (30\text{ ft})\left(\begin{array}{l}\dfrac{34\text{ ft}+23\text{ ft}}{2}+21\text{ ft}\\ \quad +23\text{ ft}+29\text{ ft}+31\text{ ft}\end{array}\right)\\ &= 3975\text{ ft}^2\quad (4000\text{ ft}^2)\end{aligned}$$

The answer is (C).

2. Use the area by coordinates method to find the area of the plot.

$$\text{area} = \frac{\begin{array}{l}X_A(Y_B-Y_D)+X_B(Y_C-Y_A)\\ \quad +X_C(Y_D-Y_B)\\ \quad +X_D(Y_A-Y_C)\end{array}}{2}$$

$$= \frac{\begin{array}{l}(122.8162°)(47.3116°-47.3104°)\\ \quad +(122.8163°)(47.3118°-47.3105°)\\ \quad +(122.8184°)(47.3104°-47.3116°)\\ \quad +(122.8183°)(47.3105°-47.3118°)\end{array}}{2}$$

$$= -2.62\times 10^{-6}\text{ deg}^2$$

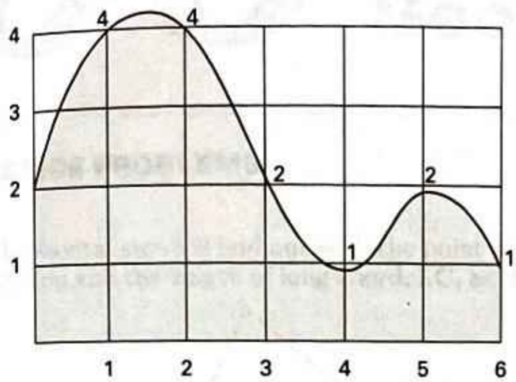
Use the lengths per degree of latitude and longitude to convert the angles to distance.

$$\begin{aligned}\text{area} &= \frac{(-2.62\times 10^{-6}\text{ deg}^2)\left(248{,}160\ \frac{\text{ft}}{°\text{E}}\right)\left(364{,}320\ \frac{\text{ft}}{°\text{N}}\right)}{43{,}560\ \frac{\text{ft}^2}{\text{ac}}}\\ &= 5.44\text{ ac}\quad (5.4\text{ ac})\end{aligned}$$

The answer is (D).

Transportation/ Surveying

3. There is an even number of intervals, so Simpson's ⅓ rule can be used. Use the graph to determine the measurement at each value of x. The starting measurement is at $x = 0$. That is, $h_1 = 2$.



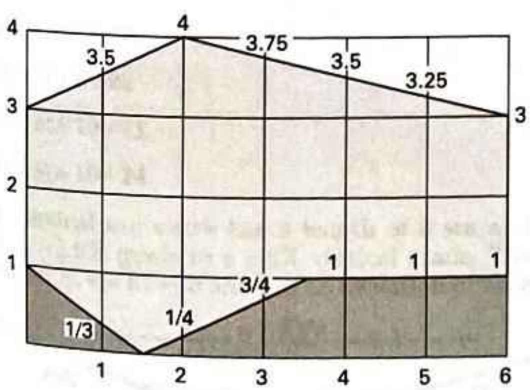
The area under the curve using Simpson's ⅓ rule is

$$\text{area} = \frac{w\left(h_1 + 2\left(\sum_{k=3,5,\ldots}^{n-2} h_k\right) + 4\left(\sum_{k=2,4,\ldots}^{n-1} h_k\right) + h_n\right)}{3}$$
$$= \frac{(1)\big(2 + (2)(4+1) + (4)(4+2+2) + 1\big)}{3}$$
$$= 15$$

The answer is (B).

4. The area of the polygon can be determined by subtracting the area under the bottom line from the area under the top line.

Since the common interval is 1, there will be seven height measurements (i.e., $n = 7$). The height of each trapezoid can be determined as shown in the figure.



The area under the top line is

$$\text{area}_{\text{top}} = w\left(\frac{h_1 + h_7}{2} + h_2 + h_3 + h_4 + h_5 + h_6\right)$$
$$= (1)\left(\frac{3+3}{2} + 3.5 + 4 + 3.75 + 3.5 + 3.25\right)$$
$$= 21$$

The area under the bottom line is

$$\text{area}_{\text{bottom}} = w\left(\frac{h_1 + h_7}{2} + h_2 + h_3 + h_4 + h_5 + h_6\right)$$
$$= (1)\left(\frac{1+1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{3}{4} + 1 + 1\right)$$
$$= 4.333$$

The area of the polygon is

$$\text{area} = \text{area}_{\text{top}} - \text{area}_{\text{bottom}}$$
$$= 21 - 4.333$$
$$= 16.667 \quad (17)$$

The answer is (C).

5. The trapezoidal rule is applicable to areas where the irregular sides are straight or nearly straight. Option I is false. Simpson's ⅓ rule is applicable to areas whose irregular sides are curved, but applies only to an even number of segments (i.e., an odd number of segment end points). Option II is false. The area by coordinates method calculates the traverse area using the coordinates of the leg end points. Option III is true.

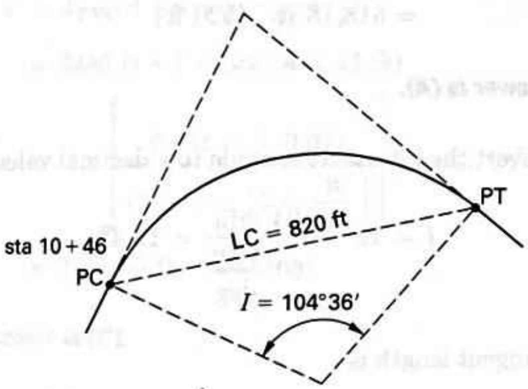
The answer is (B).

Transportation/ Surveying

50 Geometric Design

PRACTICE PROBLEMS

1. A horizontal curve is laid out with the point of curve, PC, station and the length of long chord, LC, as shown.



The radius of the curve is most nearly

(A) 520 ft

(B) 560 ft

(C) 620 ft

(D) 670 ft

2. A freeway route has a horizontal curve with a PI at sta 11+01.86, an intersection angle, I, of 12°24′00″ right, and a radius of 1760 ft. The PC is located at

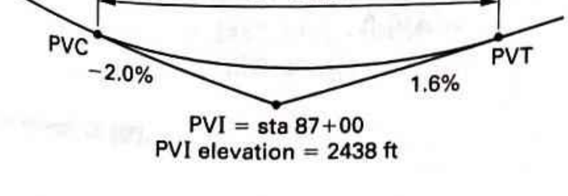
(A) sta 9+10

(B) sta 9+22

(C) sta 10+11

(D) sta 10+24

3. A vertical sag curve has a length of 8 sta and connects a −2.0% grade to a 1.6% vertical grade. The PVI is located at sta 87+00 and has an elevation of 2438 ft.



The elevation of the lowest point on the vertical curve is most nearly

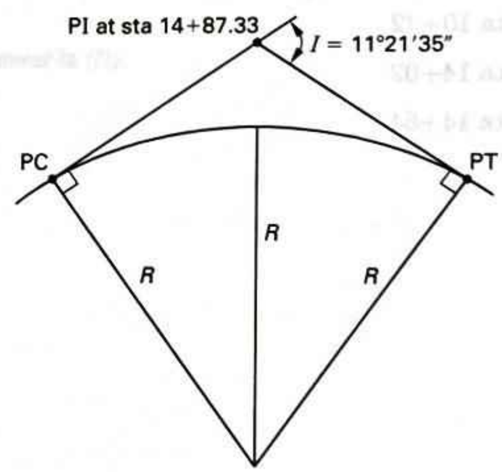
(A) 2420 ft

(B) 2430 ft

(C) 2440 ft

(D) 2450 ft

4. A 6° curve has forward and back tangents that intersect at sta 14+87.33.



The station of the point of curve, PC, is most nearly

(A) sta 5+32

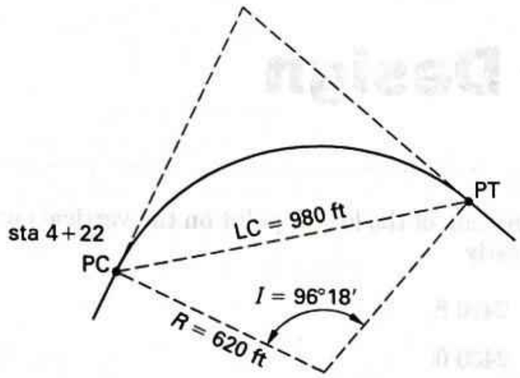
(B) sta 9+93

(C) sta 11+28

(D) sta 13+92

Transportation/ Surveying

5. A horizontal curve is laid out with the point of curve, PC, station and the length of long chord, LC, as shown.



The curve radius, R, is 620 ft. With stationing around the curve, the stationing of the point of tangent, PT, is most near to

(A) sta 6+20

(B) sta 10+42

(C) sta 14+02

(D) sta 14+64

SOLUTIONS

1. The intersection angle, I, in decimal degrees is

$$I = 104° + \frac{36 \text{ min}}{60 \ \frac{\text{min}}{\text{deg}}} = 104.6°$$

The radius of the curve is

$$\begin{aligned} R &= \frac{\text{LC}}{2\sin\frac{I}{2}} = \frac{820 \text{ ft}}{2\sin\frac{104.6°}{2}} \\ &= 518.18 \text{ ft} \quad (520 \text{ ft}) \end{aligned}$$

The answer is (A).

2. Convert the intersection angle to a decimal value.

$$I = 12° + \frac{24 \text{ min}}{60 \ \frac{\text{min}}{\text{deg}}} = 12.4°$$

The tangent length is

$$\begin{aligned} T &= R\tan\frac{I}{2} \\ &= (1760 \text{ ft})\tan\left(\frac{12.4°}{2}\right) \\ &= 191.20 \text{ ft} \end{aligned}$$

The PC is located at

$$\begin{aligned} \text{sta PC} &= \text{sta PI} - T \\ &= 1101.86 \text{ ft} - 191.20 \text{ ft} \\ &= 910 \text{ ft} \quad (\text{sta } 9{+}10) \end{aligned}$$

The answer is (A).

3. The PVI is located at the curve's midpoint. The elevation of the PVC is

$$\begin{aligned} Y_{\text{PVC}} &= Y_{\text{PVI}} + |g_1|\frac{L}{2} \\ &= 2438 \text{ ft} + (0.02)\left(\frac{8 \text{ sta}}{2}\right)\left(100 \ \frac{\text{ft}}{\text{sta}}\right) \\ &= 2446 \text{ ft} \end{aligned}$$

Transportation/ Surveying

The distance from the PVC to the lowest point on the curve is

$$\begin{aligned} x_m &= \frac{g_1 L}{g_1 - g_2} \\ &= \frac{(-0.02)(8\ \text{sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)}{-0.02 - 0.016} \\ &= 444.44\ \text{ft} \end{aligned}$$

Determine the elevation at the lowest point.

$$\begin{aligned} Y &= Y_{\text{PVC}} + g_1 x + \left(\frac{g_2 - g_1}{2L}\right)x^2 \\ &= 2446\ \text{ft} + (-0.02)(444.44\ \text{ft}) \\ &\quad + \left(\frac{0.016 - (-0.02)}{(2)(8\ \text{sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)}\right)(444.44\ \text{ft})^2 \\ &= 2441.55\ \text{ft} \quad (2440\ \text{ft}) \end{aligned}$$

The answer is (C).

4. The radius of the curve is

$$\begin{aligned} R &= \frac{5729.58}{D} = \frac{5729.58\ \text{ft-deg}}{6^\circ} \\ &= 954.93\ \text{ft} \end{aligned}$$

Convert the intersection angle to a decimal value.

$$\begin{aligned} I &= 11^\circ + \frac{21\ \text{min}}{60\ \frac{\text{min}}{\text{deg}}} + \frac{35\ \text{sec}}{\left(60\ \frac{\text{sec}}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{deg}}\right)} \\ &= 11.36^\circ \end{aligned}$$

The tangent length is

$$\begin{aligned} T &= R \tan\frac{I}{2} = (954.93\ \text{ft})\tan\left(\frac{11.36^\circ}{2}\right) \\ &= 94.98\ \text{ft} \end{aligned}$$

The station of the point of curve, PC, is

$$\begin{aligned} \text{sta PC} &= \text{sta PI} - T \\ &= 1487.33\ \text{ft} - 94.98\ \text{ft} \\ &= 1392\ \text{ft} \quad (\text{sta } 13{+}92) \end{aligned}$$

The answer is (D).

5. The intersection angle, I, in decimal degrees is

$$I = 96^\circ + \frac{18\ \text{min}}{60\ \frac{\text{min}}{\text{deg}}} = 96.3^\circ$$

The length of curve from PC to PT is

$$\begin{aligned} L &= RI\left(\frac{\pi}{180^\circ}\right) \\ &= (620\ \text{ft})(96.3^\circ)\left(\frac{\pi\ \text{rad}}{180^\circ}\right) \\ &= 1042.07\ \text{ft} \end{aligned}$$

The stationing of the point of tangent is

$$\begin{aligned} \text{sta PT} &= \text{sta PC} + L \\ &= 422\ \text{ft} + 1042.07\ \text{ft} \\ &= 1464\ \text{ft} \quad (\text{sta } 14{+}64) \end{aligned}$$

The answer is (D).

51 Earthwork

PRACTICE PROBLEMS

1. Earthwork quantities for a section of roadway indicate a transition from fill to cut. The following areas are scaled from the print cross sections. Where there are transitions between cut and fill, the cut and fill roadway cross sections are both triangular in shape.

station	cut (ft^2)	fill (ft^2)
20+00	–	1864.42
20+10.50	–	468.88
20+21.50	154.14	103.66
20+28.45	696.75	–
20+40	2017.37	–

The total volume of fill required for this section of road is most nearly

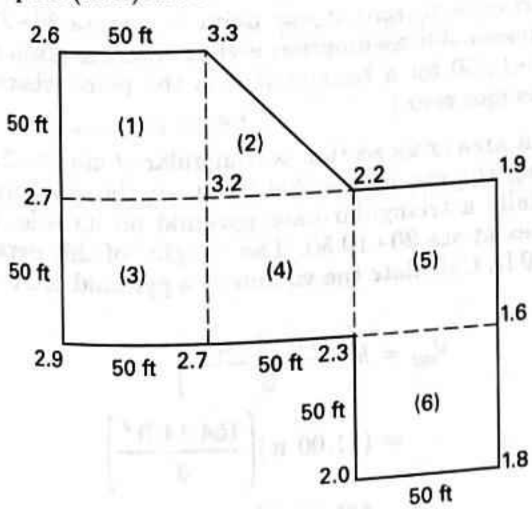
(A) 11,000 ft^3

(B) 16,000 ft^3

(C) 19,000 ft^3

(D) 21,000 ft^3

2. Consider the borrow pit grid shown. Existing excavation depths (in ft) are shown for each corner.



The total undercut volume of this borrow pit is most nearly

(A) 1300 yd^3

(B) 1600 yd^3

(C) 1900 yd^3

(D) 2100 yd^3

3. The table represents the areas of cut and fill at three consecutive roadway points (survey stations) along a rural road project.

point	cut (ft^2)	fill (ft^2)
10+00	1600	270
20+00	0	810
30+00	1100	270

The amount of borrow or waste between the points is most nearly

(A) 27,000 ft^3 borrow

(B) 240,000 ft^3 waste

(C) 250,000 ft^3 borrow

(D) 270,000 ft^3 waste

4. Earthwork quantities for a section of roadway indicate a transition from fill to cut. The following areas are scaled from the print cross sections. Where there are transitions between cut and fill, the cut and fill roadway cross sections are both triangular in shape.

station	cut (ft^2)	fill (ft^2)
10+30	–	126.5
10+60	160.7	50.6
10+82	505.0	–

The total volume of fill required for this section of road is most nearly

(A) 3000 ft^3

(B) 4900 ft^3

(C) 5900 ft^3

(D) 7200 ft^3

SOLUTIONS

1. Earthwork volumes for fill areas and cut areas can be calculated using the average end area formula. Since the cut and fill areas are triangular in shape, earthwork volumes in the transition region from fill to cut can be calculated from the formula that gives the volume of a pyramid.

For sta 20+00 to sta 20+10.50,

$$L = 10.50\ \text{ft} - 0\ \text{ft} = 10.50\ \text{ft}$$

The fill volume is

$$\begin{aligned} V_{\text{fill}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10.50\ \text{ft})\left(\frac{1864.42\ \text{ft}^2 + 468.88\ \text{ft}^2}{2}\right) \\ &= 12{,}249.83\ \text{ft}^3 \end{aligned}$$

For sta 20+10.50 to sta 20+21.50,

$$L = 21.50\ \text{ft} - 10.50\ \text{ft} = 11.00\ \text{ft}$$

The fill volume is

$$\begin{aligned} V_{\text{fill}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (11.00\ \text{ft})\left(\frac{468.88\ \text{ft}^2 + 103.66\ \text{ft}^2}{2}\right) \\ &= 3148.97\ \text{ft}^3 \end{aligned}$$

Cut is required at sta 20+21.50, but no cut is required at sta 20+10.50, so a cut-to-fill transition occurs between these two points. (The cut area could conceivably decrease to zero at any point before sta 20+21.50, but a reasonable assumption is that data was given at sta 20+10.50 for a reason: that is the point where the cut becomes zero.)

The cut area cross section is triangular at sta 20+21.50, tapering to zero at sta 20+10.50, so the soil mass is essentially a triangular-base pyramid on its side, with the apex at sta 20+10.50. The "height" of this pyramid is 11.00 ft. Calculate the volume of a pyramid of cut.

$$\begin{aligned} V_{\text{cut}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (11.00\ \text{ft})\left(\frac{154.14\ \text{ft}^2}{3}\right) \\ &= 565.18\ \text{ft}^3 \end{aligned}$$

For sta 20+21.50 to sta 20+28.45,

$$\begin{aligned} L &= 28.45 \text{ ft} - 21.50 \text{ ft} \\ &= 6.95 \text{ ft} \end{aligned}$$

Fill is required at sta 20+21.50, but no fill is required at sta 20+28.45, so a fill-to-cut transition occurs between these two points. The fill area cross section is triangular at sta 20+21.50, tapering to zero at sta 20+28.45. The "height" of the pyramid is 6.95 ft. Calculate the volume of a pyramid of fill.

$$\begin{aligned} V_{\text{fill}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (6.95 \text{ ft})\left(\frac{103.66 \text{ ft}^2}{3}\right) \\ &= 240.15 \text{ ft}^3 \end{aligned}$$

The cut volume is

$$\begin{aligned} V_{\text{cut}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (6.95 \text{ ft})\left(\frac{154.14 \text{ ft}^2 + 696.75 \text{ ft}^2}{2}\right) \\ &= 2956.84 \text{ ft}^3 \end{aligned}$$

For sta 20+28.45 to sta 20+40,

$$\begin{aligned} L &= 40 \text{ ft} - 28.45 \text{ ft} \\ &= 11.55 \text{ ft} \end{aligned}$$

The cut volume is

$$\begin{aligned} V_{\text{cut}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (11.55 \text{ ft})\left(\frac{696.75 \text{ ft}^2 + 2017.37 \text{ ft}^2}{2}\right) \\ &= 15{,}674.04 \text{ ft}^3 \end{aligned}$$

A table that summarizes earthwork volumes is now made.

station	cut area (ft^2)	fill area (ft^2)	cut volume (ft^3)	fill volume (ft^3)
20+00	–	1864.42		
			–	12,249.83
20+10.50	–	468.88		
			565.18	3148.97
20+21.50	154.14	103.66		
			2956.84	240.15
20+28.45	696.75	–		
			15,674.04	–
20+40	2017.37	–		
		total	19,196.06	15,638.95

Therefore, the total volume of fill required for this section of road is 15,638.95 ft^3 (16,000 ft^3).

The answer is (B).

2. Calculate the average depth of undercut by summing the undercut depths at each of the corners and dividing the total by the number of corners. Calculate the undercut volume by multiplying the area by the average depth of undercut.

The area of each full cell is

$$(50 \text{ ft})(50 \text{ ft}) = 2500 \text{ ft}^2$$

The area of each triangular half-cell is

$$\frac{2500 \text{ ft}^2}{2} = 1250 \text{ ft}^2$$

cell number	area (ft^2)	average depth of undercut (ft)	volume (ft^3)
1	2500	2.95	7375
2	1250	2.90	3625
3	2500	2.88	7200
4	2500	2.60	6500
5	2500	2.00	5000
6	2500	1.93	4825
		total	34,525

The total volume is

$$\begin{aligned} V_{\text{total}} &= \frac{34{,}525 \text{ ft}^3}{\left(3 \frac{\text{ft}}{\text{yd}}\right)^3} \\ &= 1279 \text{ yd}^3 \quad (1300 \text{ yd}^3) \end{aligned}$$

The answer is (A).

3. Use the average end area method. The points are 10 stations apart. 100 ft stations are used. The cut volumes are

$$V_{\text{cut,sta 1 to 2}} = L\left(\frac{A_1+A_2}{2}\right)$$
$$= (10 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{1600 \text{ ft}^2 + 0 \text{ ft}^2}{2}\right)$$
$$= 800{,}000 \text{ ft}^3$$
$$V_{\text{cut,sta 2 to 3}} = L\left(\frac{A_1+A_2}{2}\right)$$
$$= (10 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{0 \text{ ft}^2 + 1100 \text{ ft}^2}{2}\right)$$
$$= 550{,}000 \text{ ft}^3$$

The fill volumes are

$$V_{\text{fill,sta 1 to 2}} = L\left(\frac{A_1+A_2}{2}\right)$$
$$= (10 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{270 \text{ ft}^2 + 810 \text{ ft}^2}{2}\right)$$
$$= 540{,}000 \text{ ft}^3$$
$$V_{\text{fill,sta 2 to 3}} = L\left(\frac{A_1+A_2}{2}\right)$$
$$= (10 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{810 \text{ ft}^2 + 270 \text{ ft}^2}{2}\right)$$
$$= 540{,}000 \text{ ft}^3$$

Determine the total cut and fill volumes.

$$\text{cut: } V = 800{,}000 \text{ ft}^3 + 550{,}000 \text{ ft}^3 = 1{,}350{,}000 \text{ ft}^3$$
$$\text{fill: } V = 540{,}000 \text{ ft}^3 + 540{,}000 \text{ ft}^3 = 1{,}080{,}000 \text{ ft}^3$$

There is $1{,}350{,}000 \text{ ft}^3 - 1{,}080{,}000 \text{ ft}^3 = 270{,}000 \text{ ft}^3$ more waste than borrow.

The answer is (D).

4. Earthwork volumes for fill areas and cut areas can be calculated using the average end area formula. Since the cut and fill areas are triangular in shape, earthwork volumes in the transition region from fill to cut can be calculated from the formula that gives the volume of a pyramid.

For sta 10+30 to sta 10+60,

$$L = 60 \text{ ft} - 30 \text{ ft} = 30 \text{ ft}$$

The fill volume is

$$V_{\text{fill}} = L\left(\frac{A_1+A_2}{2}\right)$$
$$= (30 \text{ ft})\left(\frac{126.5 \text{ ft}^2 + 50.6 \text{ ft}^2}{2}\right)$$
$$= 2656.5 \text{ ft}^3$$

Cut is required at sta 10+60, but no cut is required at sta 10+30, so a cut-to-fill transition occurs between these two points. (The cut area could conceivably decrease to zero at a point before sta 10+60, but a reasonable assumption is that data was given at sta 10+30 for a reason: that is the point where the cut becomes zero.)

The cut area cross section is triangular at sta 10+60, tapering to zero at sta 10+30, so the soil mass is essentially a triangular-base pyramid on its side, with the apex at sta 10+30. The height of this pyramid is 30 ft. Calculate the volume of a pyramid of cut.

$$V_{\text{cut}} = h\left(\frac{\text{area of base}}{3}\right)$$
$$= (30 \text{ ft})\left(\frac{160.7 \text{ ft}^2}{3}\right)$$
$$= 1607.0 \text{ ft}^3$$

For sta 10+60 to sta 10+82,

$$L = 82 \text{ ft} - 60 \text{ ft} = 22 \text{ ft}$$

Fill is required at sta 10+60, but is not required at sta 10+82, so a fill-to-cut transition occurs between these two points. The fill area cross section is triangular at sta 10+60, tapering to zero at sta 10+82. The height of the pyramid is 22 ft. Calculate the volume of a pyramid of fill.

$$V_{\text{fill}} = h\left(\frac{\text{area of base}}{3}\right)$$
$$= (22 \text{ ft})\left(\frac{50.6 \text{ ft}^2}{3}\right)$$
$$= 371.1 \text{ ft}^3$$

The cut volume is

$$\begin{aligned} V_{cut} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (22 \text{ ft})\left(\frac{160.7 \text{ ft}^2 + 505.0 \text{ ft}^2}{2}\right) \\ &= 7322.7 \text{ ft}^3 \end{aligned}$$

A table that summarizes earthwork volumes is now made.

station	cut area (ft^2)	fill area (ft^2)	cut volume (ft^2)	fill volume (ft^3)
10+30	–	126.5	–	2656.5
10+60	160.7	50.6	1607.0	371.1
10+82	505.0	–	7322.7	–
		total	8929.7	3027.6

Therefore, the total volume of fill required for this section of road is 3027.6 ft^3 (3000 ft^3).

The answer is (A).

Transportation/ Surveying

52 Pavement Design

PRACTICE PROBLEMS

1. A Superpave design mixture for a highway with equivalent single-axle loads (ESALs) $< 10^7$ has a nominal maximum aggregate size of 19 mm. The mixture has been tested and has the following characteristics:

$$\begin{aligned}\text{air voids} &= 4.0\%\\ \text{VMA} &= 13.2\%\\ \text{VFA} &= 70\%\\ \text{dust-to-binder ratio} &= 0.97\\ \text{at } N_{\text{int}} = 8 \text{ gyrations},\ G_{\text{mm}} &= 87.1\%\\ \text{at } N_{\text{max}} = 174 \text{ gyrations},\ G_{\text{mm}} &= 97.5\%\end{aligned}$$

Do these characteristics satisfy their corresponding Superpave requirements?

(A) Yes, all the parameters are within an acceptable range.

(B) No, the VMA is excessive.

(C) No, the dust-to-binder ratio is too high.

(D) No, G_{mm} at N_{max} is too high.

2. What is the approximate load equivalency factor (LEF) for a 32,000 pound tandem-axle truck?

(A) 0.36

(B) 0.86

(C) 7.0

(D) 21

3. A road leading to a stone quarry is traveled by 40 trucks, with each truck making an average of 10 trips per day. When fully loaded, each truck consists of a front single axle transmitting a force of 10,000 lbf and two rear tandem axles, each tandem axle transmitting a force of 20,000 lbf. The load equivalency factor (LEF) for the front single axle is 0.0877. The load equivalency factor for each rear tandem axle is 0.1206.

The 18,000 lbf equivalent single-axle load (ESAL) for the truck traffic on this road for 5 years is most nearly

(A) 0.33 ESALs

(B) 130 ESALs

(C) 48,000 ESALs

(D) 240,000 ESALs

4. A highway pavement design has the material specifications shown.

layer	material	layer coefficient	layer thickness
subbase	sandy gravel	0.11 in^{-1}	12 in
base	crushed stone	0.14 in^{-1}	15 in
surface	asphalt concrete	0.44 in^{-1}	6 in

What is most nearly the structural number of the pavement?

(A) 2

(B) 4

(C) 6

(D) 9

Transportation/ Surveying

SOLUTIONS

1. This mixture would be designated as a 19 mm Superpave mixture. The limits for such a mixture are

$$\begin{aligned}\text{air voids} &= 4.0\% \\ \text{minimum VMA} &= 13\% \\ \text{VFA} &= 65-75\% \\ \text{dust-to-binder ratio} &= 0.6-1.2 \\ \text{at } N_{\text{int}} = 8 \text{ gyrations, maximum } G_{\text{mm}} &= 89\% \\ \text{at } N_{\text{max}} = 174 \text{ gyrations, maximum } G_{\text{mm}} &= 98\%\end{aligned}$$

All parameters in this mixture are within Superpave specifications.

The answer is (A).

2. From a load equivalency factor (LEF) table, the load equivalency factor for a 32,000 pound tandem-axle load is 0.857 (0.86).

The answer is (B).

3. The total ESALs per truck for each trip is

$$\begin{aligned}\text{ESALs}_{\text{truck}} &= (\text{no. of axles})(\text{LEF}) \\ &= (1 \text{ single axle})(0.0877) \\ &\quad +(2 \text{ tandem axles})(0.1206) \\ &= 0.3289 \text{ ESALs/truck-trip}\end{aligned}$$

The total daily ESALs for 40 trucks, each making 10 trips a day, is

$$\begin{aligned}\text{ESALs}_{\text{day}} &= (40 \text{ trucks})\left(10\ \frac{\text{trips}}{\text{day}}\right)\left(0.3289\ \frac{\text{ESALs}}{\text{truck-trip}}\right) \\ &= 131.56 \text{ ESALs/day}\end{aligned}$$

For 5 years, the total ESALs is

$$\begin{aligned}\text{ESALs}_{5\text{yr}} &= (5 \text{ yr})\left(365\ \frac{\text{days}}{\text{yr}}\right)\left(131.56\ \frac{\text{ESALs}}{\text{day}}\right) \\ &= 240{,}097 \text{ ESALs} \quad (240{,}000 \text{ ESALs})\end{aligned}$$

The answer is (D).

4. The structural number is

$$\begin{aligned}SN &= a_1D_1 + a_2D_2 + a_3D_3 \\ &= \left(0.44\ \frac{1}{\text{in}}\right)(6 \text{ in}) + \left(0.14\ \frac{1}{\text{in}}\right)(15 \text{ in}) \\ &\quad + \left(0.11\ \frac{1}{\text{in}}\right)(12 \text{ in}) \\ &= 6.06 \quad (6)\end{aligned}$$

The answer is (C).

Transportation/ Surveying

53 Traffic Safety

PRACTICE PROBLEMS

1. What is the design perception-reaction time recommended by the American Association of State Highway and Transportation Officials (AASHTO) for calculating stopping sight distance?

(A) 1.0 sec

(B) 1.5 sec

(C) 2.5 sec

(D) 4.0 sec

2. What is the accepted normal limit of peripheral vision?

(A) 45°

(B) 80°

(C) 160°

(D) 180°

3. What is the maximum light-intensity-contrast ratio perceptible to the human eye?

(A) 4:1

(B) 3:1

(C) 2:1

(D) 1:1

4. What percentage of roadway fatalities occur in a work zone annually?

(A) 1%

(B) 2%

(C) 5%

(D) 10%

SOLUTIONS

1. The typical range of reaction times is 1.5–3 sec. AASHTO recommends a design perception-reaction time of 2.5 sec.

The answer is (C).

2. The accepted normal limit of peripheral vision is 160°, although some individuals have peripheral vision up to 180°.

The answer is (C).

3. The maximum light-intensity-contrast ratio discernible to the human eye is 3:1.

The answer is (B).

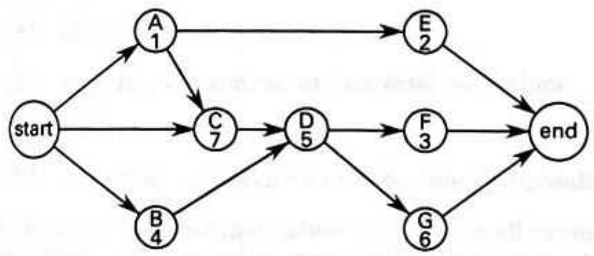
4. Work zones account for approximately 2% of all roadway fatalities each year.

The answer is (B).

54 Construction Management, Scheduling, and Estimating

PRACTICE PROBLEMS

1. An activity-on-node diagram for a construction project is given. (Activity letters and durations are shown in each node circle.)



What is most nearly the float time for activity G?

(A) 0 days

(B) 1 day

(C) 6 days

(D) 25 days

2. For which of the following is the Program Evaluation and Review Technique (PERT) NOT used?

(A) construction projects

(B) computer programming assignments

(C) preparation of bids and proposals

(D) queueing problems

SOLUTIONS

1. Solve this problem using the critical path method (CPM). There are several paths from the start to the end of this project. Identify the paths and calculate their durations.

start-A-E-end:

$$d = 1\text{ day} + 2\text{ days} = 3\text{ days}$$

start-A-C-D-F-end:

$$d = 1\text{ day} + 7\text{ days} + 5\text{ days} + 3\text{ days} = 16\text{ days}$$

start-A-C-D-G-end:

$$d = 1\text{ day} + 7\text{ days} + 5\text{ days} + 6\text{ days} = 19\text{ days}$$

start-B-D-F-end:

$$d = 4\text{ days} + 5\text{ days} + 3\text{ days} = 12\text{ days}$$

start-B-D-G-end:

$$d = 4\text{ days} + 5\text{ days} + 6\text{ days} = 15\text{ days}$$

The longest path is start-A-C-D-G-end, so this is the critical path. Because activity G is along the critical path, the float time for this activity is 0 days.

The answer is (A).

2. PERT is used to monitor the progress and predict the completion time for large projects. All of the choices given except option (D) are activities that have a finite completion time.

The answer is (D).

Construction

55 Procurement and Project Delivery Methods

PRACTICE PROBLEMS

1. Which of the following duties would normally NOT be a responsibility of the estimating department within a general contractor's organization?

(A) obtaining bid documents

(B) securing subcontractor/material quotations

(C) project cost accounting

(D) delivering competitive or negotiated proposals

2. In competitive bidding, when the bids are all opened, the owner will normally award the contract to the lowest

(A) available bidder

(B) qualified bidder

(C) responsible bidder

(D) bonded bidder

3. The term "design-build" means the

(A) design firm designs the project and the client builds it

(B) design firm both designs and builds the project

(C) client designs the project and the contractor builds it

(D) contractor designs the project and the subcontractors build it

SOLUTIONS

1. The estimation process does not include tracking costs or recording actual expenses.

The answer is (C).

2. In open competitive bidding by public and private owners, the bid will be awarded to the lowest responsible bidder.

The lowest responsible bidder is the lowest bidder whose offer best responds in quality, fitness, and capacity to fulfill the particular requirements of the proposed project, and who can fulfill these requirements with the qualifications needed to complete the job in accordance with the terms of the contract.

The answer is (C).

3. *Design-build* is a process where the client interacts only with a single entity, the "design-builder." While the design-builder is usually the general contractor, it can also be the design firm or a partnership consisting of the design firm and contractor.

The answer is (B).

Construction

56 Construction Documents

PRACTICE PROBLEMS

1. An activity is coded as 0793–31 62 13.16 using the Construction Specifications Institute's MasterFormat 2012. This coding represents construction specifications in the division of

(A) Thermal and Moisture Protection

(B) Special Construction

(C) Reserved for future expansion

(D) Earthwork

2. Which of the following organizations is NOT a contributor to the standard design and construction contract documents developed by the Engineers Joint Contract Documents Committee (EJCDC)?

(A) National Society of Professional Engineers

(B) Construction Specifications Institute

(C) Associated General Contractors of America

(D) American Institute of Architects

SOLUTIONS

1. The activity designation is translated as follows.

project 793
level 1 activity category 31
level 2 activity category 62
level 3 activity category 13

In the MasterFormat specifications, level 1 category 31 is earthwork, level 2 category 62 is driven piles, and level 3 category 13 is concrete piles. All of these subgroups fall under Division 31 - Earthwork.

The answer is (D).

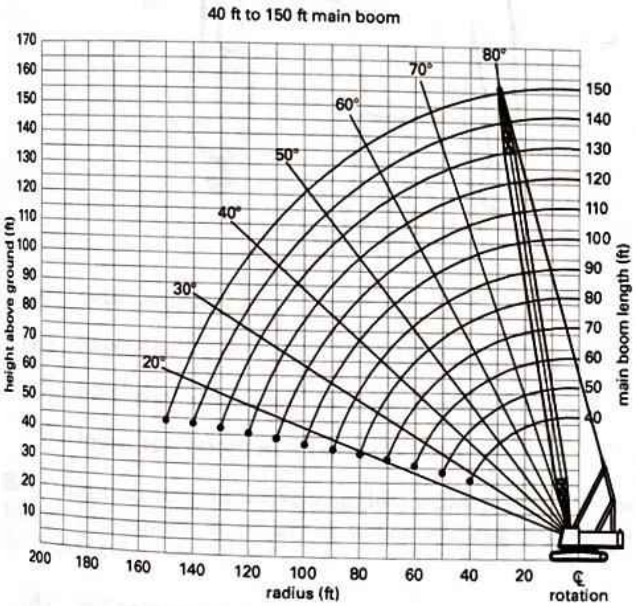
2. The Engineers Joint Contract Documents Committee (EJCDC) consists of the National Society of Professional Engineers, the American Council of Engineering Companies (formerly the American Consulting Engineers Council), the American Society of Civil Engineers, Construction Specifications Institute, and the Associated General Contractors of America. The American Institute of Architects is not a member, and it has its own standardized contract documents.

The answer is (D).

57 Construction Operations and Management

PRACTICE PROBLEMS

1. A crane chart for a crane set up to lift a rooftop onto a 60 ft tall building is shown. The crane's centerline of rotation is 50 ft away from the building, and a 140 ft boom is used at an angle of 60°. The crane is not allowed within 15 ft vertically of the top of the building. What is most nearly the maximum operating radius from the centerline of rotation?



(A) 60 ft

(B) 70 ft

(C) 85 ft

(D) 100 ft

2. What factor is responsible for an increase in production rates as a new task is repeated and practiced?

(A) the learning curve

(B) acceleration

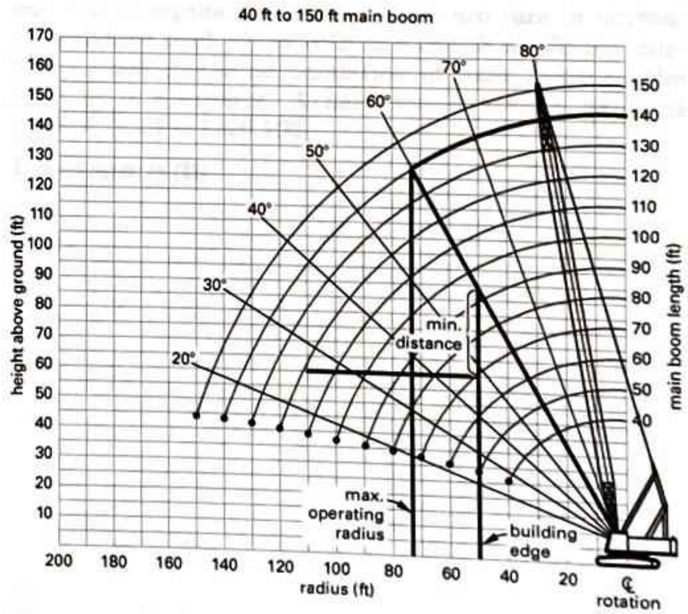
(C) phasing

(D) speed enhancement

SOLUTIONS

1. Draw the edges of the building on the chart, then follow the 140 ft boom curve to where it intersects with the 60° pitch line. Draw a vertical line from this intersection to the bottom of the chart. Check for vertical spacing requirements between the roof of the building and the boom of the crane.

The minimum distance from the roof of the building to the boom is 25 ft, which is greater than the 15 ft required. Therefore, the maximum operating radius is approximately 72 ft (70 ft).



The answer is (B).

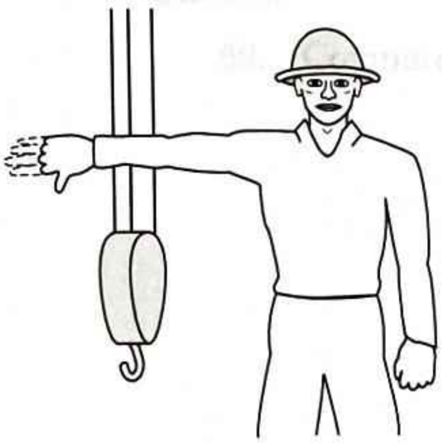
2. The learning curve is responsible for the usual increase in production rates as workers become skilled at a given task.

The answer is (A).

58 Construction Safety

PRACTICE PROBLEMS

1. What does the crane hand signal shown mean?



(A) lower the boom

(B) lower the load

(C) raise the boom and lower the load

(D) lower the boom and raise the load

2. In addition to protective goggles, what personal protective equipment is required when welding pipe in a deep trench?

(A) fall protection harness

(B) hard hat

(C) respirator

(D) hearing protection

SOLUTIONS

1. Hand signals in the crane industry are standardized. This signal means to lower the boom and raise the load. If the fingers were not shown (a fist was being made), the signal would mean to lower the boom only [OSHA 29 CFR 1926.550(a)(4)].

The answer is (D).

2. OSHA does not normally consider an open trench to be a confined space. Although there are some combinations of base metals and rods that produce toxic fumes and would require respirators, these are rare in normal construction work, so welding in a trench would not normally result in the accumulation of toxic gases or the need for a respirator. A hard hat is always required [OSHA 29 CFR 1926.100].

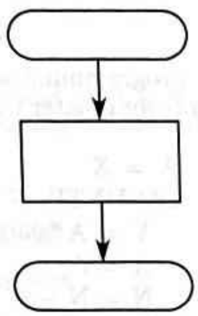
The answer is (B).

59 Computer Software

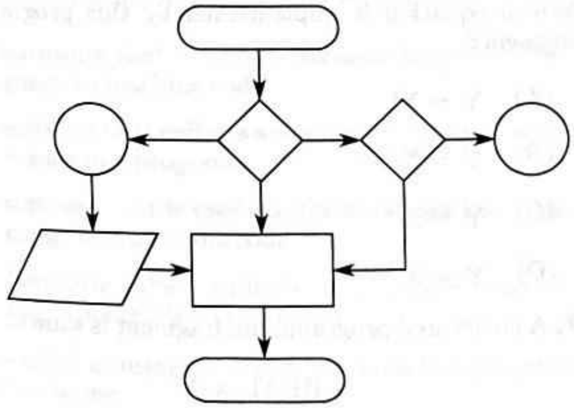
PRACTICE PROBLEMS

1. Which of the following flowcharts does NOT represent a complete program?

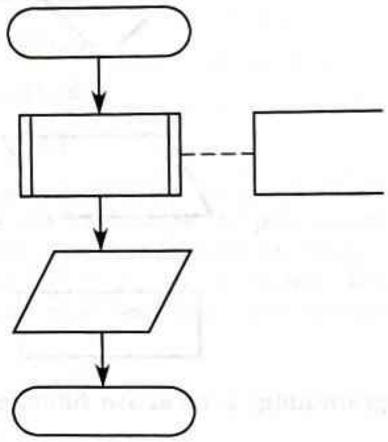
(A)



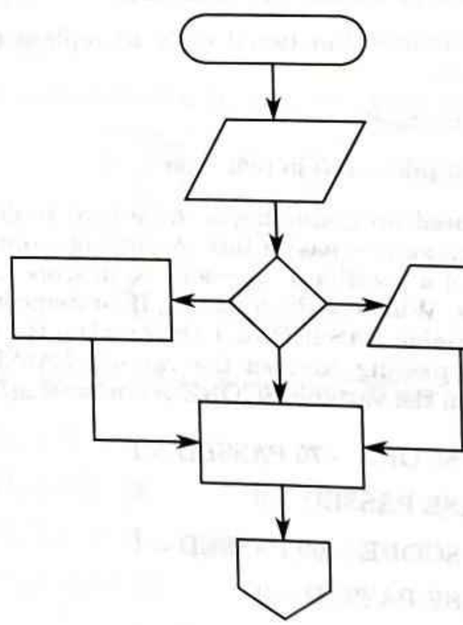
(B)



(C)



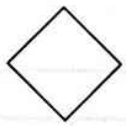
(D)



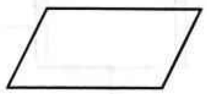
2. What flowchart element is used to represent an IF...THEN statement?

(A)

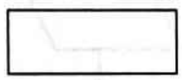
(B)



(C)



(D)



3. In programming, a recursive function is a function that

(A) calls previously used functions

(B) generates functional code to replace symbolic code

(C) calls itself

(D) compiles itself in real time

4. Structured programming is to be used to determine whether examinees pass a test. A passing score is 70 or more out of a possible 100, where exam scores can only be integers. Which of the following IF statements would set the variable PASSED to 1 (true) when the variable SCORE is passing, and set the variable PASSED to 0 (false) when the variable SCORE is not passing?

(A) IF SCORE > 70 PASSED = 1
ELSE PASSED = 0

(B) IF SCORE > 69 PASSED = 1
ELSE PASSED = 0

(C) IF SCORE < 69 PASSED = 1

(D) IF SCORE < 69 PASSED = 0
ELSE PASSED = 1

5. A structured programming segment is shown.

```
Y = 4
B = 4
Y = 3*B - 6
IF Y > B THEN Y = B - 2
IF Y < B THEN Y = Y + 2
IF Y = B THEN Y = B + 2
```

What is the value of Y after the segment is executed?

(A) 2

(B) 6

(C) 8

(D) 12

6. A structured programming segment is shown. The variable N is an integer greater than zero.

```
A = X
DO UNTIL N = 0
   Y = A*X
   A = Y
   N = N - 1
END UNTIL
```

Which equation is implemented by this programming segment?

(A) $Y = X!$

(B) $Y = X^{N-1}$

(C) $Y = X^{N}$

(D) $Y = X^{N+1}$

7. A structured programming fragment is shown.

```
1 REAL X,Y
2 X = 3
3 Y = COS(X)
4 PRINT Y
```

Line 2 is

(A) an assignment

(B) a command

(C) a declaration

(D) a function

Computational Tools

8. In a typical spreadsheet program, what cell is directly below cell AB4?

(A) AB5
(B) AC4
(C) AC5
(D) BC4

9. Which of the following terms is best defined as a formula or set of steps for solving a particular problem?

(A) program
(B) software
(C) firmware
(D) algorithm

10. The computer language that is executed within a computer's central processing unit is called

(A) MS-DOS
(B) a high-level language
(C) an assembly language
(D) a machine language

11. Which of the following best defines a compiler?

(A) hardware that is used to translate high-level language to machine code
(B) software that collects and stores executable commands in a program
(C) software that is used to translate high-level language into machine code
(D) hardware that collects and stores executable commands in a program

12. The effect of using recursive functions in a program is generally to use

(A) less code and less memory
(B) less code and more memory
(C) more code and less memory
(D) more code and more memory

13. In which of these situations can an 8-bit system correctly access more than 128 different integers?

(A) when the integers are in the range of [−255, 0]
(B) when the integers are in the range of [0, 256]
(C) when the integers are in the range of [−128, 128]
(D) when the integers are in the range of [0, 512]

14. In which of the following computer operating systems can a document in HTML (hypertext markup language) format be viewed?

I. Mac OS
II. Linux
III. Windows
IV. Unix

(A) I and III only
(B) I, II, and III only
(C) I, III, and IV only
(D) I, II, III, and IV

15. A typical spreadsheet for economic evaluation of alternatives uses cell F4 to store the percentage value of inflation rate. The percentage rate is assumed to be constant throughout the lifetime of the study. What variable should be used to access that value throughout the model?

(A) F4
(B) $F4
(C) %F4
(D) F4

16. Refer to the following portion of a spreadsheet.

	A	B	C
1	10	11	12
2	1	A2^2	
3	2	A3^2	
4	3	A4^2	
5	4	A5^2	

The top-to-bottom values in column B will be

(A) 11, 1, 2, 3, 4
(B) 11, 1, 3, 6, 10
(C) 11, 1, 4, 9, 16
(D) 11, 1, 5, 12, 22

Computational Tools

17. Refer to the following portion of a spreadsheet.

	A	B	C	D
1	10	11	12	13
2	5	4	B2*A$1	
3	6	5	B3*B$1	
4	7	6	B4*C$1	
5	8	7	B5*D$1	

The top-to-bottom values in column C will be

(A) 12, 20, 30, 42, 56

(B) 12, 40, 55, 72, 91

(C) 12, 50, 66, 84, 104

(D) 12, 100, 121, 144, 169

SOLUTIONS

1. A flowchart must begin and end with a terminal symbol. The symbol at the bottom of option D is the "off-page" symbol, which indicates that the flowchart continues on the next page. This is not a complete program.

The answer is (D).

2. At an IF...THEN statement, the flow of a program is decided based on a criterion that can be evaluated as true or false. The symbol used to represent a decision is the diamond.

The answer is (B).

3. A recursive function calls itself.

The answer is (C).

4. Option B sets PASSED to 1 for SCORE = 70 to 100 and sets PASSED to 0 for SCORE = 0 to 69.

Option A will not give the correct response when SCORE = 70. Option C will never set PASSED to 0. Option D will not give the correct response when SCORE = 69.

The answer is (B).

5. The first operation changes the value of Y.

$$Y = 3 \times 4 - 6 = 6$$

The first IF statement is satisfied, so the operation is performed.

$$Y = 4 - 2 = 2$$

However, the program execution does not end here.

The value of Y is then less than B, so the second IF statement is executed. This statement is satisfied, so the operation is performed.

$$Y = 2 + 2 = 4$$

The value of Y is then equal to B, so the third IF statement is executed. This statement is satisfied, so the operation is performed.

$$Y = 4 + 2 = 6$$

The answer is (B).

6. The DO/UNTIL loop will be executed N times. After one execution, $Y = X^2$. Each subsequent execution of the loop multiplies Y by X another time. Therefore, this segment calculates $Y = X^{N+1}$.

The answer is (D).

7. Line 2 is an assignment. A command, such as line 4, directs the computer to take some action such as PRINT. A declaration, such as line 1, states what type of data a variable will contain (like REAL) and reserves space for it in memory. A function, such as line 3, performs a specific operation (such as finding the cosine of a number) and returns a value.

The answer is (A).

8. Spreadsheets generally label a cell by giving its column and row, in that order. Cell AB4 is in column AB, row 4. The cell directly below AB4 is in column AB, row 5, designated as AB5.

The answer is (A).

9. An algorithm is a formula or set of steps for solving a particular problem.

A program is a sequence of instructions that implements a formula or set of steps, but the program is not itself the formula or set of steps. An algorithm is often implemented as a program. Software and firmware are programs stored on media.

The answer is (D).

10. The central processing unit executes a version of the program that has been compiled into the machine language. This version of the program is in the form of operations and operands specific to the machine's coding.

The answer is (D).

11. A compiler is a program (i.e., software) that converts programs written in higher-level languages to lower-level languages that the computer can understand.

The answer is (C).

12. A recursive function calls itself. Since the function does not need to be coded in multiple places, less code is used. Each subsequent call of the function must be carried out in a different location, so more memory is used.

The answer is (B).

13. An 8-bit system can represent $(2)^8 = 256$ different distinct integers. Normally, the eighth bit is used for the sign, and only seven bits are used for magnitude, resulting in a range of $[-127, 128]$ or $[-128, 127]$ (counting zero as one of the integers). If all the integers are known or assumed to have the same sign, a range of 256 integers is available. All the answer options except option A contain more than 256 distinct integers.

The answer is (A).

14. HTML may be viewed on any computer with a compatible browser.

The answer is (D).

15. The dollar sign symbol, "$", is used in spreadsheets to "fix" the column and/or row designator following it when other columns or rows are permitted to vary.

The answer is (D).

16. Except for the first entry (which is 11), column B calculates the square of the values in column A. The entries are 11, $(1)^2$, $(2)^2$, $(3)^2$, and $(4)^2$.

The answer is (C).

17. Except for the first entry (which is 12), column C is found by taking the numbers from column B and then multiplying by the entries in row 1. For example, B2*A$1 means to multiply the entry in B2, which is 4, by the number entered in cell A1, which is 10. This product is 40.

The entries are 12, 4×10, 5×11, 6×12, 7×13, or 12, 40, 55, 72, 91.

The answer is (B).

Computational Tools

60 Engineering Economics

PRACTICE PROBLEMS

1. Permanent mineral rights on a parcel of land are purchased for an initial lump-sum payment of $100,000. Profits from mining activities are $12,000 each year, and these profits are expected to continue indefinitely. Most nearly, what is the interest rate earned on the initial investment?

(A) 8.3%

(B) 9.0%

(C) 10%

(D) 12%

2. $1000 is deposited in a savings account that pays 6% annual interest, and no money is withdrawn for three years. Most nearly, what is the account balance after three years?

(A) $1120

(B) $1190

(C) $1210

(D) $1280

3. An oil company is planning to install a new 80 mm pipeline to connect storage tanks to a processing plant 1500 m away. The connection will be needed for the foreseeable future. An annual interest rate of 8% is assumed, and annual maintenance and pumping costs are considered to be paid in their entireties at the end of the years in which their costs are incurred.

initial cost	$1500
service life	12 yr
salvage value	$200
annual maintenance	$400
pump cost/hour	$2.50
pump operation	600 hr/yr

Most nearly, what is the capitalized cost of running and maintaining the 80 mm pipeline?

(A) $15,000

(B) $20,000

(C) $24,000

(D) $27,000

4. New 200 mm diameter pipeline is installed over a distance of 1000 m. Annual maintenance and pumping costs are considered to be paid in their entireties at the end of the years in which their costs are incurred. The pipe has the following costs and properties.

initial cost	$1350
annual interest rate	6%
service life	6 yr
salvage value	$120
annual maintenance	$500
pump cost/hour	$2.75
pump operation	2000 hr/yr

What is most nearly the equivalent uniform annual cost (EUAC) of the pipe?

(A) $5700

(B) $5900

(C) $6100

(D) $6300

5. New 120 mm diameter pipeline is installed over a distance of 5000 m. Annual maintenance and pumping costs are considered to be paid in their entireties at the end of the years in which their costs are incurred. The pipe has the following costs and properties.

Engineering Economics

initial cost	$2500
annual interest rate	10%
service life	12 yr
salvage value	$300
annual maintenance	$300
pump cost/hour	$1.40
pump operation	600 hr/yr

What is most nearly the equivalent uniform annual cost (EUAC) of the pipe?

(A) $1200

(B) $1300

(C) $1400

(D) $1500

6. A construction company purchases 100 m of 40 mm diameter steel cable with an initial cost of $4500. The annual interest rate is 4%, and annual maintenance costs are considered to be paid in their entireties at the end of the years in which their costs are incurred. The annual maintenance cost of the cable is $200/yr over a service life of nine years. Using Modified Accelerated Cost Recovery System (MACRS) depreciation and assuming a seven-year recovery period, what is most nearly the depreciation allowance for the cable in the first year of operation?

(A) $640

(B) $670

(C) $720

(D) $860

7. A piece of equipment has an initial cost of $5000. During years 1–3, the rate of inflation is 5%, and the effective annual rate of interest is 9%. The uninflated present worth of the equipment during year 3 is most nearly

(A) $3200

(B) $3300

(C) $3400

(D) $3500

8. A company is considering buying a computer with the following costs and interest rate.

initial cost	$3900
salvage value	$1800
useful life	10 years
annual maintenance	$390
interest rate	6%

Most nearly, what is the equivalent uniform annual cost (EUAC) of the computer?

(A) $740

(B) $780

(C) $820

(D) $850

9. A computer with a useful life of 13 years has the following costs and interest rate.

initial cost	$5500
salvage value	$3100
annual maintenance	
years 1–8	$275
years 9–13	$425
interest rate	6%

Most nearly, what is the equivalent uniform annual cost (EUAC) of the computer?

(A) $730

(B) $780

(C) $820

(D) $870

10. A computer with an initial cost of $1500 and an annual maintenance cost of $500/yr is purchased and kept indefinitely without any change in its annual maintenance costs. The interest rate is 4%. Most nearly, what is the present worth of all expenditures?

(A) $12,000

(B) $13,000

(C) $14,000

(D) $15,000

11. A computer with a useful life of 12 years has an initial cost of $2300 and a salvage value of $350. The interest rate is 6%. Using the straight-line method, what is most nearly the total depreciation of the computer for the first five years?

(A) $760

(B) $810

(C) $830

(D) $920

12. A computer with a useful life of 12 years has an initial cost of $3200 and a salvage value of $100. The interest rate is 10%. Using the Modified Accelerated Cost Recovery System (MACRS) method of depreciation and a 10-year recovery period, what is most nearly the book value of the computer after the second year?

(A) $1900

(B) $2100

(C) $2300

(D) $2400

13. A computer with a useful life of five years has an initial cost of $6000. The salvage value is $2300, and the annual maintenance is $210/yr. The interest rate is 8%. What is most nearly the present worth of the costs for the computer?

(A) $5200

(B) $5300

(C) $5600

(D) $5700

14. A company must purchase a machine that will be used over the next eight years. The purchase price is $10,000, and the salvage value after eight years is $1000. The annual insurance cost is 2% of the purchase price, the electricity cost is $300 per year, and maintenance and replacement parts cost $100 per year. The effective annual interest rate is 6%. Neglect taxes. Most nearly, what is the effective uniform annual cost (EUAC) of ownership?

(A) $1200

(B) $2100

(C) $2200

(D) $2300

15. A company purchases a piece of equipment for $15,000. After nine years, the salvage value is $900. The annual insurance cost is 5% of the purchase price, the electricity cost is $600/yr, and the maintenance and replacement parts cost is $120/yr. The effective annual interest rate is 10%. Neglecting taxes, what is most nearly the present worth of the equipment if it is expected to save the company $4500 per year?

(A) $2300

(B) $2800

(C) $3200

(D) $3500

Engineering Economics

SOLUTIONS

1. Use the capitalized cost equation to find the interest rate earned.

$$P = \frac{A}{i}$$

$$i = \frac{A_{\text{profit}}}{P_{\text{cost}}} = \frac{\$12{,}000}{\$100{,}000} = 0.12 \quad (12\%)$$

The answer is (D).

2. Find the future worth of \$1000.

$$F = P(1+i)^n = (\$1000)(1+0.06)^3 = \$1191 \quad (\$1190)$$

The answer is (B).

3. The problem statement asked for the capitalized cost of running and maintaining the pipeline, so the initial cost is not included in the calculation. The annual cost of running and maintaining the 80 mm pipeline is

$$A = \left(\frac{\$2.50}{\text{hr}}\right)(600\text{ hr}) + \$400 = \$1900$$

Capitalized costs are the present worth using an assumed perpetual period of time.

$$P = \frac{A}{i} = \frac{\$1900}{0.08} = \$23{,}750 \quad (\$24{,}000)$$

The answer is (C).

4. The equivalent uniform annual cost (EUAC) is the uniform annual amount equivalent of all cash flows. When calculating the EUAC, costs are positive and income is negative. All non-annual costs are converted to annual costs. The appropriate conversion factors are found in the Interest Rate Factor Table for 6% interest in the *NCEES Handbook*.

$$\begin{aligned}\text{EUAC}_{80} &= A_{\text{initial}} + A_{\text{maintenance}} + A_{\text{pump}} - A_{\text{salvage}} \\ &= (\$1350)(A/P, 6\%, 6) + \$500 + \left(\frac{\$2.75}{\text{hr}}\right)(2000\text{ hr}) - (\$120)(A/F, 6\%, 6) \\ &= (\$1350)(0.2034) + \$500 + \$5500 - (\$120)(0.1434) \\ &= \$6257 \quad (\$6300)\end{aligned}$$

The answer is (D).

5. The equivalent uniform annual cost (EUAC) is the uniform annual amount equivalent of all cash flows. When calculating the EUAC, costs are positive and income is negative.

$$\begin{aligned}\text{EUAC}_{12} &= A_{\text{initial}} + A_{\text{maintenance}} + A_{\text{pump}} - A_{\text{salvage}} \\ &= (\$2500)(A/P, 10\%, 12) + \$300 + \left(\frac{\$1.40}{\text{hr}}\right)(600\text{ hr}) - (\$300)(A/F, 10\%, 12) \\ &= (\$2500)(0.1468) + \$300 + \$840 - (\$300)(0.0468) \\ &= \$1493 \quad (\$1500)\end{aligned}$$

The answer is (D).

6. The MACRS factor for the first year, given a seven-year recovery period, is 14.29%. The annual interest rate and annual maintenance costs are distractors that should not be taken into account.

$$\begin{aligned}D_1 &= (\text{factor})C \\ &= (0.1429)(\$4500) \\ &= \$643 \quad (\$640)\end{aligned}$$

The answer is (A).

7. If the unadjusted interest rate is used to calculate the present worth, the answer will be in dollars affected by three years of inflation. To find the uninflated worth three years ago, the effect of inflation during those years must be eliminated from the calculation. To find the answer in uninflated dollars, determine the interest rate adjusted for inflation.

$$\begin{aligned} d &= i + f + (i \times f) \\ &= 0.09 + 0.05 + (0.09)(0.05) \\ &= 0.1445 \end{aligned}$$

Use this adjusted rate in the single payment present worth equation, substituting d for i.

$$\begin{aligned} P &= F(1+d)^{-n} \\ &= (\$5000)(1+0.1445)^{-3} \\ &= \$3335 \quad (\$3300) \end{aligned}$$

The answer is (B).

8. The equivalent uniform annual cost (EUAC) is the uniform annual amount equivalent to all cash flows. When calculating the EUAC, costs are positive and income is negative. Substitute the values from the economic factor table for 6% and the A/P and A/F columns. The annual maintenance cost does not need to be factored.

$$\begin{aligned} \text{EUAC} &= A_{\text{initial}} + A_{\text{maintenance}} - A_{\text{salvage}} \\ &= (\$3900)(A/P, 6\%, 10) + \$390 \\ &\quad -(\$1800)(A/F, 6\%, 10) \\ &= (\$3900)(0.1359) + \$390 \\ &\quad -(\$1800)(0.0759) \\ &= \$783 \quad (\$780) \end{aligned}$$

Alternately, the problem may be solved using the economic factor equations.

$$\begin{aligned} \text{EUAC} &= A_{\text{initial}} + A_{\text{maintenence}} - A_{\text{salvage}} \\ &= (\$3900)(A\,/\,P, 6\%, 10) + \$390 \\ &\quad -(\$3900)(A\,/\,F, 6\%, 10) \\ &= (\$3900)\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right) + \$390 \\ &\quad -(\$1800)\left(\frac{i}{(1+i)^n - 1}\right) \\ &= (\$3900)(0.1359) + \$390 \\ &\quad -(\$1800)(0.0759) \\ &= \$783 \qquad (\$780) \end{aligned}$$

The answer is (B).

9. An expedient way to find the annual worth of the maintenance for the computer is to divide the maintenance costs into two annual series, one of \$275 lasting from year 1 to year 13, and one of \$150 (the difference between \$425 and \$275) lasting for five years, from year 9 to year 13. Find the future value in year 13 for each series, add them, and then convert the result back into a single annual amount.

$$\begin{aligned} \frac{F}{A} &= (F/A, i\%, n) \\ F &= (F/A, i\%, n) \end{aligned}$$

The future annual compound amount factor for $(F/A, 6\%, 13)$ is 18.8821 and $(F/A, 6\%, 5)$ is 0.0530.

$$\begin{aligned} F_{\$275} &= A(F/A, 6\%, 13) = (\$275)(18.8821) \\ &= \$5192.58 \\ F_{\$150} &= A(F/A, 6\%, 5) = (\$150)(5.6371) \\ &= \$845.57 \\ F_{\text{maintenance}} &= F_{\$275} + F_{\$150} = \$5192.58 + \$845.57 \\ &= \$6038.15 \\ A_{\text{maintenance}} &= F_{\text{maintenance}}(A/F, 6\%, 13) \\ &= (\$6038.15)(0.0530) \\ &= \$320.02 \end{aligned}$$

Calculate the equivalent uniform annual cost (EUAC).

$$\begin{aligned} \text{EUAC} &= A_{\text{initial}} + A_{\text{maintenance}} - A_{\text{salvage}} \\ &= (\$5500)(A/P, 6\%, 13) + \$320.02 \\ &\quad -(\$3100)(A/F, 6\%, 13) \\ &= (\$5500)(0.1130) + \$320 \\ &\quad -(\$3100)(0.0530) \\ &= \$777.22 \quad (\$780) \end{aligned}$$

The answer is (B).

Engineering Economics

10. The expenditures for the computer are the initial cost of \$1500 and the annual maintenance cost of \$500. The annual costs continue indefinitely, so find the present worth of the maintenance cost, which is a capitalized cost.

$$\text{capitalized costs} = P = \frac{A}{i}$$

$$P_{\text{maintenance}} = \frac{A}{i} = \frac{\$500}{0.04} = \$12{,}500$$

The present worth of all expenditures is

$$\begin{aligned} P_{\text{total}} &= P_{\text{initial}} + P_{\text{annual}} = \$1500 + \$12{,}500 \\ &= \$14{,}000 \end{aligned}$$

The answer is (C).

11. With the straight-line method, the depreciation is the same every year. Find the annual depreciation.

$$D_j = \frac{C - S_n}{n} = \frac{\$2300 - \$350}{12} = \$162.50$$

The total depreciation for five years is

$$\sum D_{1-5} = (5)(\$162.50) = \$812.50 \quad (\$810)$$

The answer is (B).

12. Subtract the first two years' depreciation from the original cost.

year	factor (%)	D_j
1	10.00	$(0.10)(\$3200) = \$320$
2	18.00	$(0.18)(\$3200) = \underline{\$576}$
		$\sum D_j = \$896$

The book value is

$$\begin{aligned} \text{BV} &= \text{initial cost} - \sum D_j = \$3200 - \$896 \\ &= \$2304 \quad (\$2300) \end{aligned}$$

The answer is (C).

13. Bring all costs and benefits into the present.

$$\begin{aligned} P_{\text{total}} &= P_{\text{initial}} + P_{\text{maintenance}} - P_{\text{salvage}} \\ &= \$6000 + (\$210)(P/A, 8\%, 5) \\ &\quad -(\$2300)(P/F, 8\%, 5) \\ &= \$6000 + (\$210)(3.9927) \\ &\quad -(\$2300)(0.6806) \\ &= \$5273 \quad (\$5300) \end{aligned}$$

The answer is (B).

14. The effective uniform annual cost (EUAC) is the annual cost equivalent of all costs. When calculating the EUAC, costs are positive and income is negative. Find the annual equivalents of all costs and add them together to get the EUAC.

$$\begin{aligned} \text{EUAC} &= C_{\text{initial}}(A/P, 6\%, 8) \\ &\quad + A_{\text{electricity}} + A_{\text{maintenance}} \\ &\quad + A_{\text{insurance}} - S_8(A/F, 6\%, 8) \\ &= (\$10{,}000)(0.1610) + \$300 + \$100 \\ &\quad + (0.02)(\$10{,}000) \\ &\quad - (\$1000)(0.1010) \\ &= \$2109 \quad (\$2100) \end{aligned}$$

The answer is (B).

15. Add the present worths of all cash flows.

$$\begin{aligned} P_{\text{total}} &= -C_{\text{initial}} - A_{\text{electricity}}(P/A, 10\%, 9) \\ &\quad - A_{\text{maintenance}}(P/A, 10\%, 9) \\ &\quad - A_{\text{insurance}}(P/A, 10\%, 9) \\ &\quad + A_{\text{benefits}}(P/A, 10\%, 9) \\ &\quad + S_9(P/F, 10\%, 9) \\ &= -\$15{,}000 - (\$600)(5.7590) \\ &\quad -(\$120)(5.7590) \\ &\quad -(0.05)(\$15{,}000)(5.7590) + (\$4500)(5.7590) \\ &\quad +(\$900)(0.4241) \\ &= \$2831 \quad (\$2800) \end{aligned}$$

The answer is (B).

61 Professional Practice

PRACTICE PROBLEMS

1. What must be proven for damages to be collected from a strict liability in tort?

(A) that willful negligence caused an injury

(B) that willful or unwillful negligence caused an injury

(C) that the manufacturer knew about a product defect before the product was released

(D) none of the above

2. A material breach of a construction contract will always occur when the

(A) contractor uses material not approved by the contract for use

(B) contractor's material order arrives late

(C) customer becomes insolvent

(D) contractor installs a feature incorrectly

3. If a contract has a value engineering clause and a contractor suggests to the owner that a feature or method be used to reduce the annual maintenance cost of the finished project, what will be the most likely outcome?

(A) The contractor will be able to share one time in the owner's expected cost savings.

(B) The contractor will be paid a fixed amount (specified by the contract) for making a suggestion, but only if the suggestion is accepted.

(C) The contract amount will be increased by some amount specified in the contract.

(D) The contractor will receive an annuity payment over some time period specified in the contract.

4. A tort is

(A) a civil wrong committed against another person

(B) a section of a legal contract

(C) a legal procedure in which complaints are heard in front of an arbitrator rather than a judge or jury

(D) the breach of a contract

5. If a contract does not include the clause "Liquidated Damages" agreement that states how much must be paid in case of delay. Which of the following is true?

(A) It is difficult to recover losses for extra hours billed.

(B) Standard industry time guidelines apply.

(C) Damages for delay cannot be claimed.

(D) Workers need not be paid for downtime in the project.

6. Which statement is true regarding the legality and enforceability of contracts?

(A) For a contract to be enforceable, it must be in writing.

(B) A contract to perform illegal activity will still be enforced by a court.

(C) A contract must include a purchase order.

(D) Mutual agreement of all parties must be evident.

Ethics / Prof. Practice

7. Which option best describes the contractual lines of privity between parties in a general construction contract?

(A) The consulting engineer will have a contractual obligation to the owner, but will not have a contractual obligation with the general contractor or the subcontractors.

(B) The consulting engineer will have a contractual obligation to the owner and the general contractor.

(C) The consulting engineer will have a contractual obligation to the owner, general contractor, and subcontractors.

(D) The consulting engineer will have a contractual obligation to the general contractor, but will not have a contractual obligation to the owner or subcontractors.

8. A contract has a value engineering clause that allows the parties to share in improvements that reduce cost. The contractor had originally planned to transport concrete on-site for a small pour with motorized wheelbarrows. On the day of the pour, however, a concrete pump is available and is used, substantially reducing the contractor's labor cost for the day. This is an example of

(A) value engineering whose benefit will be shared by both contractor and owner

(B) efficient methodology whose benefit is to the contractor only

(C) value engineering whose benefit is to the owner only

(D) cost reduction whose benefit will be shared by both contractor and laborers

9. In which of the following fee structures is a specific sum paid to the engineer for each day spent on the project?

(A) salary plus

(B) per-diem fee

(C) lump-sum fee

(D) cost plus fixed fee

10. What damages are paid when responsibility is proven but the injury is slight or insignificant?

(A) nominal

(B) liquidated

(C) compensatory

(D) exemplary

SOLUTIONS

1. In order to prove strict liability in tort, it must be shown that a product defect caused an injury. Negligence need not be proven, nor must the manufacturer know about the defect before release.

The answer is (D).

2. A material breach of the contract is a significant event that is grounds for canceling the contract entirely.

If the customer becomes insolvent, the customer is not capable of paying for the construction, so the contract is in material breach of contract. Option (C) is correct.

There can be an agreement between the contractor and the customer to allow the use of material not approved by the contract for use. For example, the contractor could use a better material for the same price or a lesser material and make a price adjustment. A modification to the contract may be agreed to. So, the contract is not necessarily in material breach of contract if the contractor uses material not approved by the contract for use. This eliminates option (A).

The contractor may still be able to meet the contract schedule if the contractor's material order arrives late, or there could be some flexibility on schedule if materials are late in the terms of the contract. A modification to the contract may be agreed to. So, the contract is not necessarily in material breach of contract if the contractor's material order arrives late. This eliminates option (B).

The contractor could correct the problem at the contractor's expense if the contractor installs a feature incorrectly. So, the contract is not necessarily in material breach of contract if the contractor installs a feature incorrectly. This eliminates option (D).

The answer is (C).

3. Changes to a structure's performance, safety, appearance, or maintenance that benefit the owner in the long run will be covered by the value engineering clause of a contract. Normally, the contractor is able to share in cost savings in some manner by receiving a payment or credit to the contract.

The answer is (A).

4. A tort is a civil wrong committed against a person or his/her property which results in some form of damages. Torts are normally resolved through lawsuits.

The answer is (A).

5. This clause must be included in order to recover damages due to delay.

The answer is (C).

6. In order for a contract to be legally binding, it must

- be established for a legal purpose
- contain a mutual agreement by all parties
- have consideration, or an exchange of something of value (e.g., a service is provided in exchange for a fee)
- not obligate parties to perform illegal activity
- not be between parties that are mentally incompetent, minors, or do not otherwise have the power to enter into the contract

A contract does not need to use as its basis or include a purchase order to be enforceable. Oral contracts may be legally binding in some instances, depending on the circumstances and purpose of the contract. Oral contracts may be difficult to enforce, however, and should not be used for engineering and construction agreements.

The answer is (D).

7. With a general construction contract, a consulting engineer will be hired by the owner to develop the design and contract documents, as well as to assist in the preparation of the bid documents and provide contract administrative services during the construction phase. The contract documents produced by the engineer will form the basis of the owner's agreement with the contractor. Although the engineer will work closely with the contractor during the construction phase, and may work with subcontractors as well, the engineer will not have a contractual line of privity with either party.

The answer is (A).

8. The problem gives an example of efficient methodology, where the benefit is to the contractor only. It is not an example of value engineering, as the change affects the contractor, not the owner. Performance, safety, appearance, and maintenance are unaffected.

The answer is (B).

9. A specific fee is paid to the engineer for each day on the job in a per-diem fee structure.

The answer is (B).

10. Nominal damages are awarded for inconsequential injuries.

The answer is (A).

62 Ethics

PRACTICE PROBLEMS

1. An environmental engineer with five years of experience reads a story in the daily paper about a proposal being presented to the city council to construct a new sewage treatment plant near protected wetlands. Based on professional experience and the facts presented in the newspaper, the engineer suspects the plant would be extremely harmful to the local ecosystem. Which of the following would be an acceptable course of action?

(A) The engineer should contact appropriate agencies to get more data on the project before making a judgment.

(B) The engineer should write an article for the paper's editorial page urging the council not to pass the project.

(C) The engineer should circulate a petition through the community condemning the project, and present the petition to the council.

(D) The engineer should do nothing because he doesn't have enough experience in the industry to express a public opinion on the matter.

2. An engineer is consulting for a construction company that has been receiving bad publicity in the local papers about its waste-handling practices. Knowing that this criticism is based on public misperceptions and the paper's thirst for controversial stories, the engineer would like to write an article to be printed in the paper's editorial page. What statement best describes the engineer's ethical obligations?

(A) The engineer's relationship with the company makes it unethical for him to take any public action on its behalf.

(B) The engineer should request that a local representative of the engineering registration board review the data and write the article in order that an impartial point of view be presented.

(C) As long as the article is objective and truthful, and presents all relevant information including the engineer's professional credentials, ethical obligations have been satisfied.

(D) The article must be objective and truthful, present all relevant information including the engineer's professional credentials, and disclose all details of the engineer's affiliation with the company.

3. After making a presentation for an international project, an engineer is told by a foreign official that his company will be awarded the contract, but only if it hires the official's brother as an advisor to the project. The engineer sees this as a form of extortion and informs his boss. His boss tells him that, while it might be illegal in the United States, it is a customary and legal business practice in the foreign country. The boss impresses upon the engineer the importance of getting the project, but leaves the details up to the engineer. What should the engineer do?

(A) He should hire the official's brother, but insist that he perform some useful function for his salary.

(B) He should check with other companies doing business in the country in question, and if they routinely hire relatives of government officials to secure work, then he should do so too.

(C) He should withdraw his company from consideration for the project.

(D) He should inform the government official that his company will not hire the official's brother as a precondition for being awarded the contract, but invite the brother to submit an application for employment with the company.

4. If one is aware that a registered engineer willfully violates a state's rule of professional conduct, one should

(A) do nothing

(B) report the violation to the state's engineering registration board

(C) report the violation to the employer

(D) report the violation to the parties it affects

Ethics / Prof. Practice

5. Which of the following is an ethics violation specifically included in the National Council of Examiners for Engineering and Surveying (NCEES) *Model Rules*?

(A) an engineering professor "moonlighting" as a private contractor

(B) an engineer investing money in the stock of the company for which he/she works

(C) a civil engineer with little electrical experience signing the plans for an electric generator

(D) none of the above

6. A senior licensed professional engineer with 30 years of experience in geotechnical engineering is placed in charge of a multidisciplinary design team consisting of a structural group, a geotechnical group, and an environmental group. In this role, the engineer is responsible for supervising and coordinating the efforts of the groups when working on large interconnected projects. In order to facilitate coordination, designs are prepared by the groups under the direct supervision of the group leader, and then they are submitted to her for review and approval. This arrangement is ethical as long as

(A) the engineer signs and seals each design segment only after being fully briefed by the appropriate group leader

(B) the engineer signs and seals only those design segments pertaining to geotechnical engineering

(C) each design segment is signed and sealed by the licensed group leader responsible for its preparation

(D) the engineer signs and seals each design segment only after it has been reviewed by an independent consulting engineer who specializes in the field in which it pertains

7. The National Society of Professional Engineers' (NSPE) *Code of Ethics for Engineers* addresses competitive bidding. Which of the following is NOT stipulated?

(A) Engineers and their firms may refuse to bid competitively on engineering services.

(B) Clients are required to seek competitive bids for design services.

(C) Federal laws governing procedures for procuring engineering services (e.g., competitive bidding) remain in full force.

(D) Engineers and their societies may actively lobby for legislation that would prohibit competitive bidding for design services.

8. A city engineer is in charge of receiving bids on behalf of the city council. A contractor's bid arrives with two tickets to a professional football game. The bid is the lowest received and is considered realistic for the project. What should the engineer do?

(A) Return the tickets and accept the bid.

(B) Return the tickets and reject the bid.

(C) Discard the tickets and accept the bid.

(D) Discard the tickets and reject the bid.

9. A relatively new engineering firm is considering running an advertisement for their services in the local newspaper. An ad agency has supplied them with four concepts. Of the four ad concepts, which one(s) would be acceptable from the standpoint of professional ethics?

I. an advertisement contrasting their successes over the past year with their nearest competitors' failures

II. an advertisement offering a free television to anyone who hires them for work valued at over $10,000

III. an advertisement offering to beat the price of any other engineering firm for the same services

IV. an advertisement that tastefully depicts their logo against the backdrop of the Golden Gate Bridge

(A) I and III

(B) I, III, and IV

(C) II, III, and IV

(D) neither I, II, III, nor IV

10. "A professional engineer who took the licensing examination in mechanical engineering may

(A) not design in electrical engineering."

(B) design in electrical engineering if she feels competent."

(C) design in electrical engineering if she feels competent and the electrical portion of the design is insignificant and incidental to the overall job."

(D) design in electrical engineering if another engineer checks the electrical engineering work."

Ethics / Prof. Practice

11. An engineering firm is hired by a developer to prepare plans for a shopping mall. Prior to the final bid date, several contractors who have received bid documents and plans contact the engineering firm with requests for information relating to the project. What can the engineering firm do?

(A) The firm can supply requested information to the contractors as long as it does so fairly and evenly. It cannot favor or discriminate against any contractor.

(B) The firm should supply information to only those contractors that it feels could safely and economically perform the construction services.

(C) The firm cannot reveal facts, data, or information relating to the project that might prejudice a contractor against submitting a bid on the project.

(D) The firm cannot reveal facts, data, or information relating to the project without the consent of the client as authorized or required by law.

SOLUTIONS

1. The engineer certainly has more experience and knowledge in the field than the general public or even the council members who will have to vote on the issue. Therefore, the engineer is qualified to express his opinion if he wishes to do so. Before the engineer takes any public position, however, the engineer is obligated to make sure that all the available information has been collected.

The answer is (A).

2. It is ethical for the engineer to issue a public statement concerning a company he works for, provided he makes that relationship clear and provided the statement is truthful and objective.

The answer is (D).

3. Hiring the official's brother as a precondition for being awarded the contract is a form of extortion. Depending on the circumstances, however, it may be legal to do so according to U.S. law. (The Foreign Corrupt Practices Act of 1977 allows American companies to pay extortion in some cases.) This practice, however, is not approved by the National Council of Examiners for Engineering and Surveying (NCEES) *Model Rules*: B.8. Licensees shall not solicit or accept a professional contract from a governmental body on which a principal or officer of their organization serves as a member. Conversely, licensees serving as members, advisors, or employees of a government body or department, who are the principals or employees of a private concern, shall not participate in decisions with respect to professional services offered or provided by said concern to the governmental body that they serve.

The answer is (D).

4. A violation should be reported to the organization that has promulgated the rule.

The answer is (B).

5. The National Council of Examiners for Engineering and Surveying (NCEES) *Model Rules* specifically states that registrants may not perform work beyond their level of expertise. The other two examples may be unethical under some circumstances, but are not specifically forbidden by the NCEES code.

The answer is (C).

Ethics / Prof. Practice

6. The National Council of Examiners for Engineering and Surveying (NCEES) *Model Rules* states that

> *Licensees may accept assignments and assume responsibility for coordination of an entire project if each technical segment is signed and sealed by the licensee responsible for preparation of that technical segment.*

The answer is (C).

7. Clients are not required to seek competitive bids. In fact, many engineering societies discourage the use of bidding to procure design services because it is believed that competitive bidding results in lower-quality construction.

The answer is (B).

8. Registrants should not accept gifts from parties expecting special consideration, so the tickets cannot be kept. They also should not be merely discarded, for several reasons. Inasmuch as the motive of the contractor is not known with certainty, in the absence of other bidding rules, the bid may be accepted.

The answer is (A).

9. None of the ads is acceptable from the standpoint of professional ethics. Concepts I and II are explicitly prohibited by the National Council of Examiners for Engineering and Surveying (NCEES) *Model Rules*. Concept III demeans the profession of engineering by placing the emphasis on price as opposed to the quality of services. Concept IV is a misrepresentation; the picture of the Golden Gate Bridge in the background might lead some potential clients to believe that the engineering firm in question had some role in the design or construction of that project.

The answer is (D).

10. Although the laws vary from state to state, engineers are usually licensed generically. Engineers are licensed as "professional engineers." The scope of their work is limited only by their competence. In the states where the license is in a particular engineering discipline, an engineer may "touch upon" another discipline when the work is insignificant and/or incidental.

The answer is (C).

11. It is normal for engineers and architects to clarify the bid documents. However, some information may be proprietary to the developer. The engineering firm should only reveal information that has already been publicly disseminated or approved for release with the consent of the client.

The answer is (D).

63 Licensure

PRACTICE PROBLEMS

1. Which organization provides the authority for the private practice of engineering?

(A) National Council of Examiners for Engineering and Surveying (NCEES)

(B) Accreditation Board for Engineering and Technology (ABET)

(C) United States Department of Commerce

(D) state in which an engineer lives

2. Reciprocity is a term that describes the process by which one state may honor an engineer's professional engineering license from another state. Which word is commonly used as a synonym for "reciprocity"?

(A) normality

(B) comity

(C) suplurity

(D) anority

3. What term best defines the reason that engineers working for some companies do not need to be licensed as professional engineers?

(A) commercial exclusion

(B) corporate oversight

(C) industrial exemption

(D) caveat emptor

4. The purpose of professional licensure of engineers is to

(A) protect the engineering profession

(B) ensure public health

(C) limit competition to licensed engineers in a state

(D) protect engineering titles

SOLUTIONS

1. State laws provide the authority for professional practice. NCEES writes the examinations used by the states. ABET accredits four-year degree programs. The United States Department of Commerce is not involved.

The answer is (D).

2. The terms comity and reciprocity, though slightly different in meaning, are often used synonymously.

The answer is (B).

3. Professional licensure is intended to protect the public from engineers in private practice. States have various laws, commonly referred to as "industrial exemptions," that allow engineers to work in the industry (e.g., in companies that produce products) without being licensed.

The answer is (C).

4. Although engineering licensure laws do protect engineering titles, the purpose of professional licensure is to protect public health, safety, and welfare from unqualified practitioners.

The answer is (B).