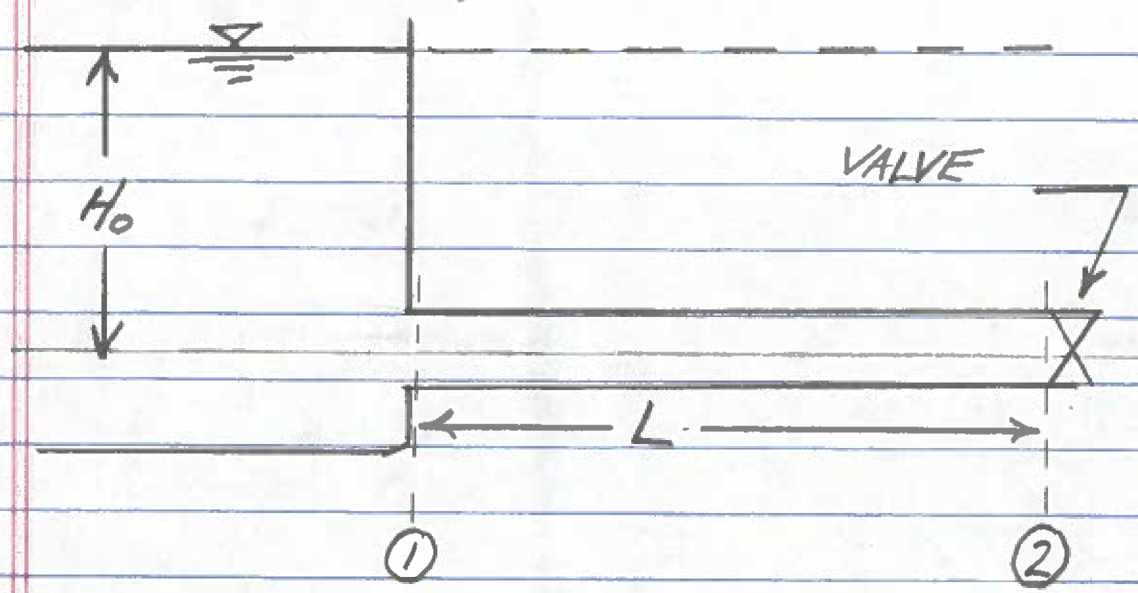
RIGID WATER COLUMN THEORY

VALVE OPENING SUDDENLY



PIPELINE AS SHOWN, VALVE CLOSED.
HEAD EVERYWHERE IS H_o

THEN VALVE IS SUDDENLY OPENED, AND LIQUID BEGINS TO ACCELERATE:

STARTING WITH EULER'S EQUATION

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{dz}{ds} - \frac{fV^2}{2gD} = \frac{1}{g}\frac{dV}{dt}$$

γ = SP. WEIGHT OF LIQUID
p = PRESSURE
s = PATHLINE (PIPE AXIAL DIRECTION)
z = ELEVATION OF CENTERLINE
V = VELOCITY

INTEGRATE FROM 1 TO 2

$$-\int_L \frac{1}{\gamma}\frac{\partial p}{\partial s}ds - \int_L \frac{dz}{ds}ds - \int_L \frac{fV^2}{2gD}ds = \int_L \frac{1}{g}\frac{dV}{dt}ds$$

TO OBTAIN

$$-\left(\frac{p_2}{\gamma} - \frac{p_1}{\gamma}\right) - \left(z_2 - z_1\right) - \frac{fLV^2}{2gD} = \frac{L}{g}\frac{dV}{dt}$$

$\frac{p_1}{\gamma} = H_o \qquad \frac{p_2}{\gamma} = 0$ (AT VALVE OPENING)

$z_1 = z_2 \quad \therefore \quad z_2 - z_1 = 0$ (HORIZONTAL)

SUBSTITUTE THE VARIOUS TERMS

$$H_o - \frac{fLV^2}{2gD} = \frac{L}{g}\frac{dV}{dt}$$

SEPERATE & INTEGRATE TO FIND
V vs t RELATIONSHIP

$$dt = \frac{L}{g}\left[\frac{dV}{H_o - \frac{fL}{2gD}V^2}\right]$$

COLLECT CONSTANTS (FOR THE INDEFINITE INTEGRAL)

$$\int dt = \frac{L}{g}\int \frac{dV}{H_0 - \frac{fL}{2gD}V^2}$$

$$\int dt = \frac{L}{g}\frac{1}{H_0}\int \frac{dV}{1 - \frac{fL}{2gDH_0}V^2}$$

$$\text{LET } U^2 = \frac{fLV^2}{2gDH_0}$$

$$U = \sqrt{\frac{fL}{2gDH_0}}\,V$$

$$dU = \sqrt{\frac{fL}{2gDH_0}}\,dV$$

MAKE SUBSTITUTION

$$\int dt = \frac{L}{g}\frac{1}{H_0}\frac{1}{\sqrt{\frac{fL}{2gDH_0}}}\int \frac{dU}{1-U^2}$$

INDEFINITE INTEGRAL

$$\int \frac{du}{1-u^2} = \frac{1}{2} \ln \frac{1+u}{1-u}$$

(GRADSHTEYN AND RYZHIK, 1980 p54 #16)

COMPLETE THE INTEGRATION

$$t + C = \frac{L}{g} \frac{1}{H_0} \frac{1}{\sqrt{\frac{fL}{2gDH_0}}} \cdot \frac{1}{2} \ln \frac{1 + \sqrt{\frac{fL}{2gDH_0}}\,V}{1 - \sqrt{\frac{fL}{2gDH_0}}\,V}$$

NOW COLLECT TERMS & SIMPLIFY

FIRST CONSIDER STEADY FLOW,

$$H_0 - \frac{fLV_0^2}{2gD} = 0$$

$$H_0 = \frac{fLV_0^2}{2gD}$$

$$\frac{2gDH_0}{fL} = V_0^2$$

THUS $\quad V_0 = \sqrt{\frac{2gDH_0}{fL}}$

NOW USE THIS TO SIMPLIFY

$$t + C = \frac{L}{g} \cdot \frac{1}{H_0} \cdot V_0 \cdot \frac{1}{2} \ln \frac{\left(1 + \frac{V}{V_0}\right)}{\left(1 - \frac{V}{V_0}\right)}$$

$$t + C = \frac{V_0 L}{g H_0} \cdot \frac{1}{2} \cdot \ln\left(\frac{\frac{V_0}{V_0}\left(1 + \frac{V}{V_0}\right)}{\frac{V_0}{V_0}\left(1 - \frac{V}{V_0}\right)}\right)$$

$$t + C = \frac{V_0 L}{g H_0} \cdot \frac{1}{2} \cdot \ln\left(\frac{\cancel{\frac{1}{V_0}}(V_0 + V)}{\cancel{\frac{1}{V_0}}(V_0 - V)}\right)$$

$$t + C = \frac{V_0 L}{g H_0} \cdot \frac{1}{2} \cdot \ln\left(\frac{V_0 + V}{V_0 - V}\right)$$

NOW EVALUATE C

AT $t = 0$, $V = 0$

$$\therefore \quad C = \frac{V_0 L}{2 g H_0} \underbrace{\ln\left(\frac{V_0}{V_0}\right)}_{=0} \qquad \therefore \; C = 0$$

SO THE MOTION EQUATION IS

$$t = \frac{V_0 L}{2gH_0} \ln\left(\frac{V_0 + V}{V_0 - V}\right)$$

AS STEADY FLOW (IN THIS MODEL) IS APPROACHED, $t \to \infty$ (BECAUSE OF $\ln(\to 0)$)

$\therefore$ STIPULATE THAT WHEN $V = 0.98 V_0$ WE HAVE ESSENTIALLY STEADY FLOW, CONSIDERING THIS STIPULATION:

$$t_{98} = \frac{V_0 L}{2gH_0} \ln\left(\frac{1.98 V_0}{0.02 V_0}\right)$$

$$t_{98} = 2.29 \frac{V_0 L}{gH_0}$$

NOW APPLY THE ANALYSIS TO AN EXAMPLE CASE

EXAMPLE:

A HORIZONTAL PIPE, 2 FEET IN DIAMETER, 10,000 FEET LONG LEAVES A RESERVOIR 100 FEET BELOW THE SURFACE AND TERMINATES AT A VALVE.

THE STEADY FLOW FRICTION FACTOR IS 0.018, ASSUMED CONSTANT DURING THE ACCELERATION PROCESS.

IF THE VALVE OPENS SUDDENLY, ESTIMATE HOW LONG IT WILL TAKE FOR THE VELOCITY TO REACH 98 PERCENT OF ITS FINAL (STEADY) VALUE.

NEGLECT MINOR LOSSES

$$t_{98} = 2.29\,\frac{V_o L}{g H_o} = \frac{2.29(10{,}000)\,V_o}{32.2(100)}$$

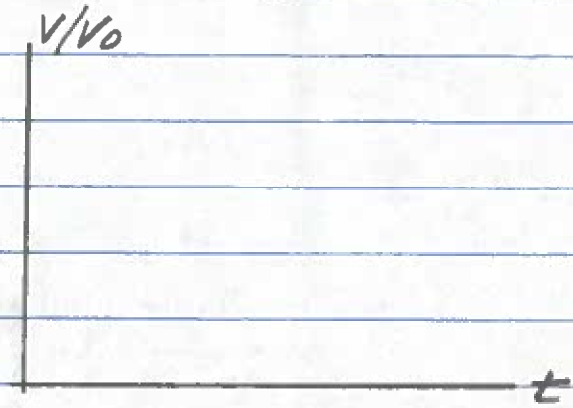
$$V_o = \sqrt{\frac{2g\;D\;H_o}{fL}} = \sqrt{\frac{2(32.2)(2)(100)}{0.018(10{,}000)}}$$

$$= 8.459 = 8.46 \text{ ft/sec}$$

$$\therefore\ t_{98} = \frac{2.29(10{,}000)(8.46)}{32.2(100)} = 60.159$$

$$= 60 \text{ seconds}$$

R SCRIPT TO PLOT



INPUTS: H_o, L, D, g, f

MODIFY LATER TO USE PIPE PROPERTIES & Re

OUTPUTS: $V_o, V(t)$

TABULAR & GRAPHICAL

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```r
# Script to Plot V(t)/Vo versue t
######## Prototype Functions #######
timeVelocity <- function(head,distance,diameter,gravity,friction,Vnow){
  Vo <- steadyVelocity(head,distance,diameter,gravity,friction);
  timeVelocity <- (Vo*distance)/(2.0*gravity*head) * log ((Vo+Vnow)/(Vo-Vnow))
  return(timeVelocity)
}
steadyVelocity <- function(head,distance,diameter,gravity,friction){
  steadyVelocity <- sqrt((2.0*gravity*diameter*head)/(friction*distance))
  return(steadyVelocity)
}
# Read Inputs
Ho <- as.numeric(readline(prompt = "Enter Reservoir Pool Elevation "))
L  <- as.numeric(readline(prompt = "         Enter Pipeline Length "))
D  <- as.numeric(readline(prompt = "           Enter Pipe Diameter "))
g  <- as.numeric(readline(prompt = "  Enter gravitational constant "))
f  <- as.numeric(readline(prompt = "   Enter darcy friction factor "))
# Echo Inputs
message("Reservoir Pool Elevation : ",Ho)
message("         Pipeline Length : ",L)
message("           Pipe Diameter : ",D)
message("  Gravitational Constant : ",g)
message("   Darcy Friction Factor : ",f)
# Compute Some Constants
Vzero <- steadyVelocity(Ho,L,D,g,f)
V999 <- 0.999 * Vzero # 99.9% of Vo
# Report Computed Constants
message("                      Vo : ",Vzero)
message("                  V_99.9 : ",V999)
VofT <- seq(0,V99,length.out=100) # Compute Time to 99.9% of Vo
TofV <- numeric(0)
for (i in 1:100) {
  TofV[i] <-  timeVelocity(Ho,L,D,g,f,VofT[i])
}
#print(cbind(VofT,TofV))
plot(TofV,VofT/Vzero,type="l",col="magenta",lwd=3,xlab="Time",ylab="V(t)/
 Vo",tck=1)
```

RStudio

Go to file/function | Addins | Project: (None)

ReservoirValveOpen.R | Source on Save | Run | Source

```r
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######## Prototype Functions #######
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D  <- as.numeric(readline(prompt = "          Enter Pipe Diameter "))
```

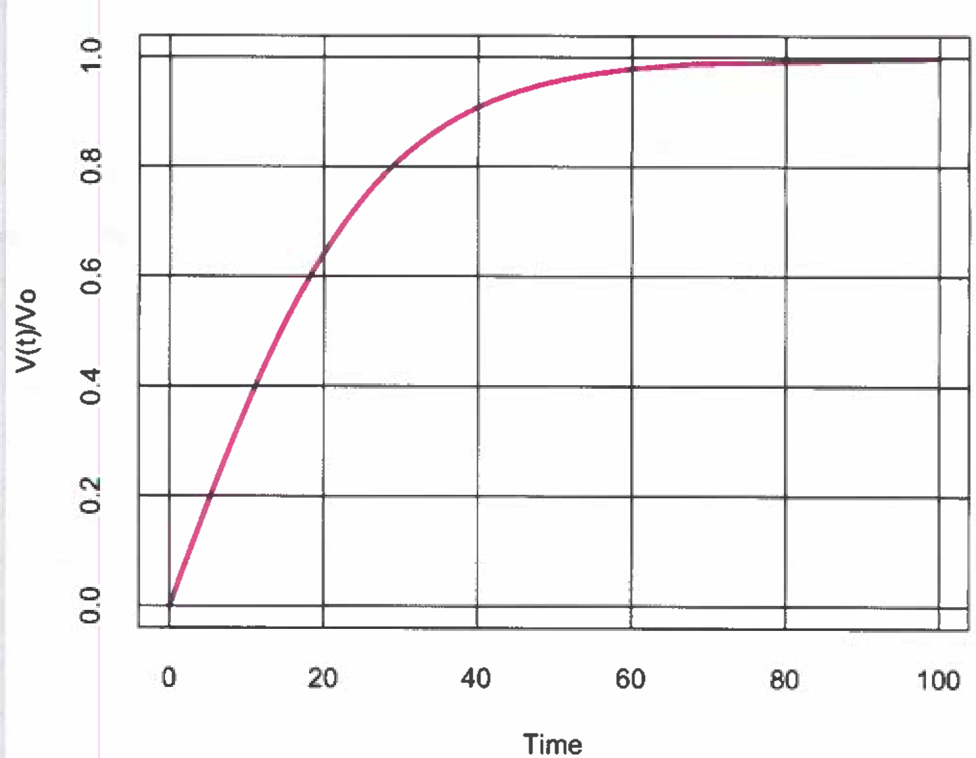
35:2 | Prototype Functions | R Script

Console ~/Dropbox/3-Research/NetworkSimulators/UnsteadyPipeNetwork/RigidWaterCol

```
> source('~/Dropbox/3-Research/NetworkSimulators/UnsteadyPipeNetwork/RigidWaterColumnTheory/ReservoirValveOpen.R')
Enter Reservoir Pool Elevation 100
         Enter Pipeline Length 10000
           Enter Pipe Diameter 2
  Enter gravitational constant 32.2
   Enter darcy friction factor 0.018
Reservoir Pool Elevation : 100
         Pipeline Length : 10000
           Pipe Diameter : 2
  Gravitational Constant : 32.2
    Darcy Friction Factor : 0.018
                       Vo : 8.45905169363301
                   V_99.9 : 8.45059264193938
>
```

Environment | History | Import Dataset | List | package:base | sapply

Files | Plots | Packages | Help | Viewer | Zoom | Export | Publish



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