4

Hydraulics of Pipelines and Pipe Networks

Hydraulics presented in this chapter is limited to turbulent flow of water in closed conduits (pipes) flowing full. Closed conduits flowing partially full, such as stormwater collection systems, are analyzed as open channel flow. Pipes are connected together in various configurations (called networks) to transport water from the supply to the user. When all pipes are connected in series, the system is called a pipeline. A pipe system can include many pipes of various lengths and diameters, along with valves to control the flow rate and pumps and turbines to convert between hydraulic energy and thermal, mechanical, and electrical energy.

Flow in a pipe can either be steady or unsteady. For unsteady flow, the velocity is a function of time and will change within a few seconds. For steady flow, the velocity is not considered a function of time, although it may change gradually. Steady flow equations can be used to analyze a water distribution system where the demands on the system change hourly, pumps turn on and off, and storage tanks fill and drain during the numerical simulation.

4.1 BASIC EQUATIONS FOR STEADY FLOW

The hydraulics of steady flow in pipe systems is described by the continuity and energy equations. Headloss is a key term in the energy equation.

4.1.1 Continuity Equation

For steady flow in pipelines and pipe networks, water is considered incompressible, and the conservation of mass equation (continuity equation) reduces to the volumetric flow rate (Q)

$$Q = AV \tag{4.1}$$

where A is the cross-section area of the pipe, and V is the average velocity. The flow rate (Q) is measured in cu m per sec (cms) or cu ft per sec (cfs). Discharge rate (Q) may also be specified in liters per second (lps), gallons per minute (gpm), or million gallons per day (mgd). The continuity equation between cross-sections 1 and 2 of a pipe is

$$A_1V_1 = A_2V_2 \tag{4.2}$$

Junction nodes are located where two or more pipes join together. A three-pipe junction node with a constant demand (C) is shown in Fig. 4.1. The continuity equation for the junction node is

$$Q_1 - Q_2 - Q_3 - C = 0 \tag{4.3}$$

In modeling pipe networks, all demands on the system are located at junction nodes, and the flow in pipes connecting nodes is assumed to be uniform. If a major demand is located between nodes, then an additional junction node is established at the location of the demand. Equation 4.2 serves as the continuity equation for a two-pipe junction node without a demand, where the subscripts refer to the pipe number.

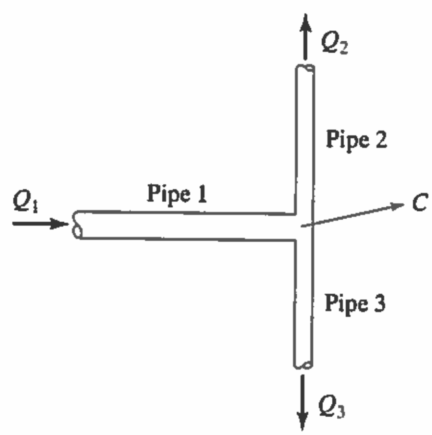


Figure 4.1 Three-pipe junction node with a constant demand.

4.1.2 Energy Equation

Figure 4.2 represents a pumped-storage hydroelectric plant where water is pumped to the upper reservoir during the off-peak power period and used to generate electricity during the peak power period. In Fig. 4.2a, water is being pumped from the lower supply reservoir through a pipeline to an upper storage reservoir. Water discharges from the upper reservoir in Fig. 4.2b through a pipeline and turbine into the lower reservoir.

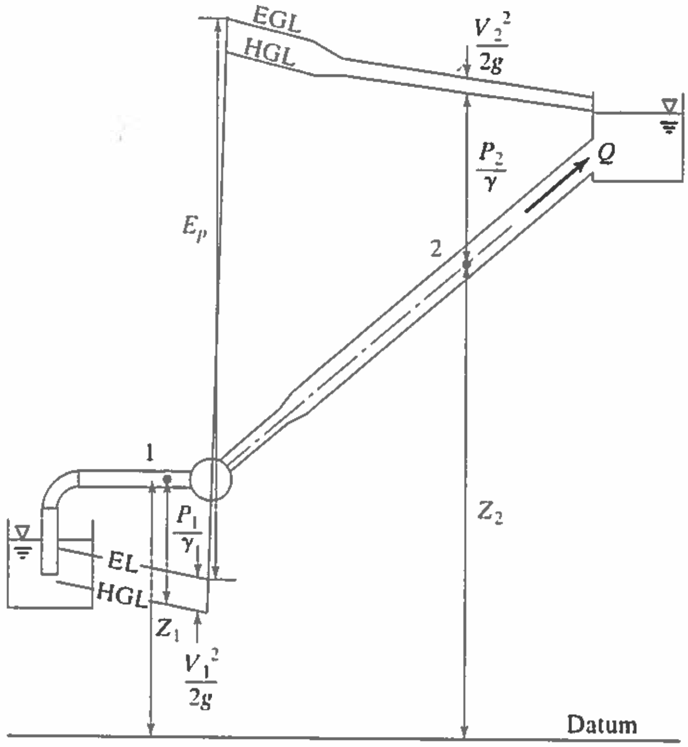


Figure 4.2a Pump system.

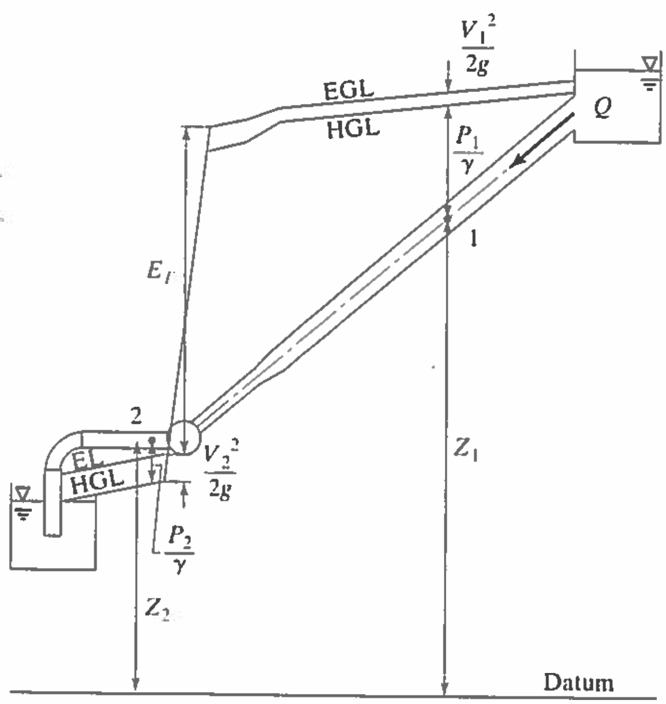


Figure 4.2b Turbine system.

The energy grade line (*EGL*) and hydraulic grade line (*HGL*) are shown in Fig. 4.2. The *HGL* is located one velocity head ($V^2/2g$) below the *EGL*. The *EGL* and *HGL* are parallel when the pipe size is uniform. The *EGL* slopes downward in the direction of flow because of energy loss. The vertical distance between the center of the pipe and the *HGL* is the pressure head (P/γ). If the *HGL* is above the pipe, the pressure is positive, and if the *HGL* is below the pipe, the pressure is negative. *Z* is the vertical distance above the datum (usually mean sea level—msl).

The energy equation written for flow in a pipeline (Fig. 4.3) is

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 + E_P = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + H_L \tag{4.4}$$

where z is the elevation of the pipe, P/γ is the pressure head, $V^2/2g$ is the velocity head, E_P is the energy head added by the pump, and H_L is the total headloss between points 1 and 2. Each term in the energy equation has units of length and represents energy per unit weight of fluid (Newton-meters per Newton of fluid flowing or ft-lbs per lb).

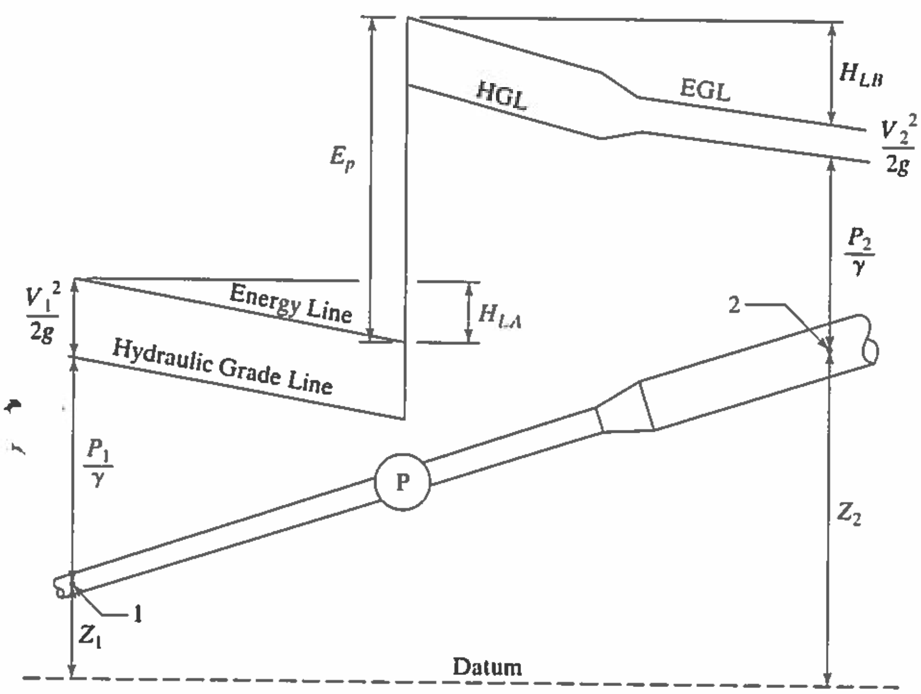
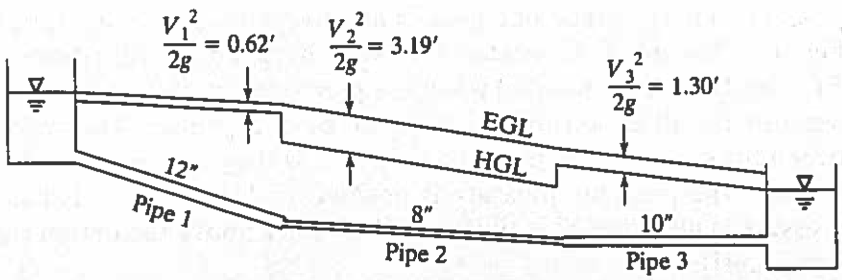


Figure 4.3 Energy equation for pipeline flow.

Example 4.1 Continuity Equation

If 5 cfs of water is flowing in the pipeline from the upper reservoir to the lower reservoir, determine the velocity in each line. Sketch the energy grade line (*EGL*) and hydraulic grade line (*HGL*) on the figure.



$$V_1 = \frac{Q}{A_1} = \frac{5}{0.785} = 6.34 \text{ fps}$$

$$\frac{V_1^2}{2g} = 0.62 \text{ ft}$$

$$V_2 = \frac{Q}{A_2} = \frac{5}{0.349} = 14.3 \text{ fps}$$

$$\frac{V_2^2}{2g} = 3.19 \text{ ft}$$

$$V_3 = \frac{Q}{A_3} = \frac{5}{0.545} = 9.16 \text{ fps}$$

$$\frac{V_3^2}{2g} = 1.30 \text{ ft}$$

4.1.3 Headloss

Headlosses in pipelines are caused by pipe friction, transitions, valves, bends, and fittings. For long pipelines, pipe friction is generally the major component of headloss and the other components are often neglected. Headlosses caused by transitions, valves, bends, and fittings are referred to as minor losses and in short pipelines such as highway culverts cannot be neglected.

4.1.3.1 Pipe friction headloss. Headloss caused by pipe friction can be estimated using the Darcy–Weisbach equation, Hazen–Williams equation, or the Manning equation. From Section 3.6, the Darcy–Weisbach equation is

$$H_L = f\frac{L}{D}\frac{V^2}{2g} \tag{4.5}$$

where f is the friction factor, L is the length of the pipe, and D is the diameter of the pipe. The friction factor can be estimated from the Moody diagram in Fig. 4.4 and is a function of the Reynolds number ($\text{Re} = DV/\nu$) of the flow and the relative roughness of the pipe (ε/D). In most pipelines, the flow will be fully turbulent.

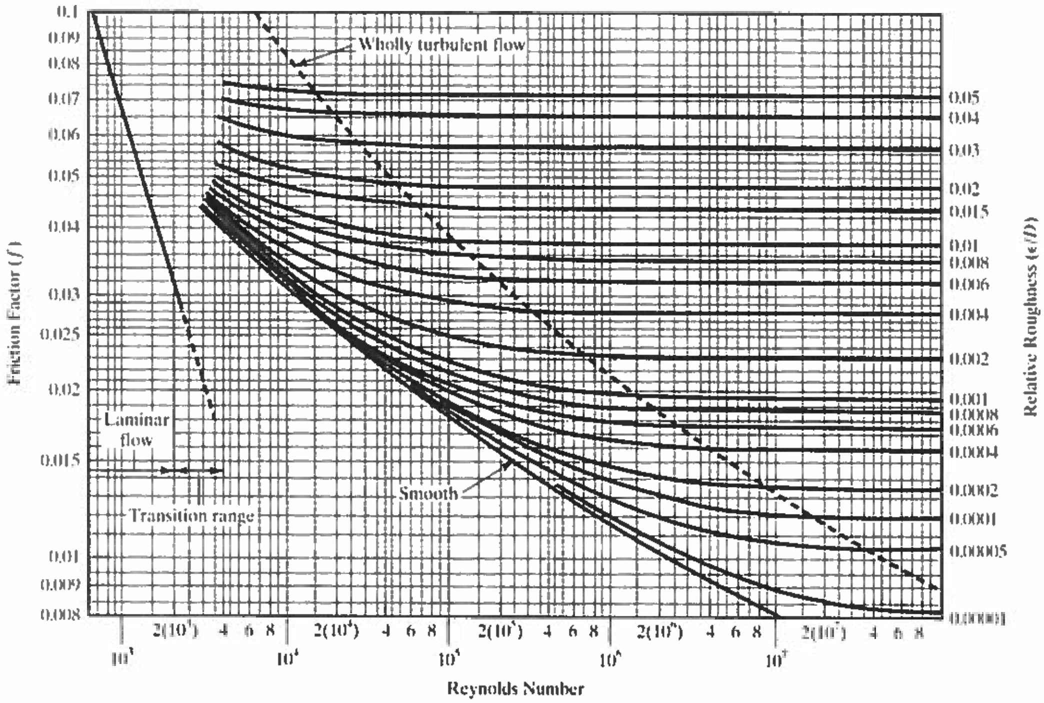


Figure 4.4 Moody diagram of Darcy-Weisbach friction factors.

The hydraulic radius (R) is defined as the area (A) of the flow cross-section divided by the wetted perimeter (P). For a circular pipe flowing full

$$R = \frac{A}{P} = \frac{\frac{1}{4}\pi D^2}{\pi D} = \frac{1}{4}D \tag{4.6}$$

For noncircular pipes, the headloss can be estimated using the equations for circular pipe with $4R$ substituted for D. Pipe roughness values are listed in Table 4.1 for common pipe materials.

After a pipe has been in service for some time, the diameter and roughness of the pipe may change, and it may be difficult to estimate the roughness of the pipe. The Hazen–Williams equation is often used in pipe network analysis. Tables are available relating the Hazen–Williams coefficient (C_H) to the age of the pipe.

The Hazen–Williams equation is

$$Q = C_u C_H A R^{0.63} S^{0.54} \tag{4.7}$$

where $C_u = 0.85$ for International System (SI) units [1.318 for British Gravitational (BG) units] and S is the slope of energy line. Writing the Hazen–Williams equation for headloss gives

$$H_L = S \times L = L\left(\frac{4}{D}\right)^{1.17}\left(\frac{V}{C_u C_H}\right)^{1.85} \tag{4.8}$$

TABLE 4.1 PIPE ROUGHNESS VALUES

Material	ε mm	C_H Hazen–Williams	n Manning
Plastic, PVC	0.001	150	0.009
Asbestos cement	—	140	0.011
Welded steel	0.045	120	0.012
Riveted steel	0.9–9	110	0.015
Concrete	0.3–3	130	0.012
Asphalted iron	0.12	—	0.013
Galvanized iron	0.15	—	0.016
Cast iron (new)	0.25	130	0.013
Cast iron (old)	—	100	0.025
Corrugated metal	—	—	0.025

The Manning equation is commonly used to estimate the friction headloss in culverts and storm sewers. The Manning equation is

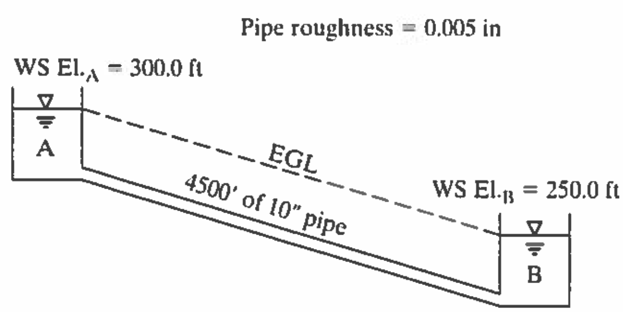
$$Q = \frac{C_m}{n} A R^{2/3} S^{1/2} \tag{4.9}$$

where $C_m = 1.00$ for SI units (1.49 for BG units) and n is the Manning roughness coefficient. Writing the Manning equation for headloss gives

$$H_L = S \times L = \frac{n^2 V^2 L}{C_m^2 R^{4/3}} \tag{4.10}$$

Example 4.2 Pipe Friction

Two reservoirs are connected with a 10-inch diameter pipeline 4500 ft long. If the pipe roughness is 0.005 inches, determine the discharge rate in the pipeline. Neglect minor losses.



$$\text{Relative roughness } \frac{\varepsilon}{D} = 0.0005$$

$$\text{from the Moody diagram } f = 0.017 \quad \text{for} \quad R_e > 10^6$$

$$\text{headloss } H_L = \frac{fL}{D}\frac{V^2}{2g}$$

$$300.0 - 250.0 = \frac{0.017 \times 4500}{10/12}\frac{V^2}{2g} = 91.8\frac{V^2}{2g}$$

$$\frac{V^2}{2g} = 0.545 \text{ ft}$$

$$V = (2g \times 0.545)^{1/2} = 5.90 \text{ fps}$$

$$R_e = \frac{DV}{\nu} = \frac{0.833 \times 5.9}{10^{-5}} = 4.9 \times 10^5$$

From the Moody diagram $f = 0.018$

$$H_L = 97.2\frac{V^2}{2g}$$

$$\frac{V^2}{2g} = 0.514 \text{ ft}$$

$$V = 5.75 \text{ fps}$$

$$R_e = 4.8 \times 10^5$$

$$Q = AV = 0.545 \times 5.75 = 3.14 \text{ cfs}$$

4.1.3.2 Minor losses. Minor losses are caused by excessive turbulence generated by a change in flow geometry. They represent the headloss that is in excess of the normal pipe friction at transitions, bends, valves, and other fittings. The coefficient (K) is used to give the minor headloss (H_M) as a function of the velocity head

$$H_M = K\frac{V^2}{2g} \tag{4.11}$$

At transitions V is the velocity in the smaller pipe. Minor loss coefficients are listed in Table 4.2.

TABLE 4.2 MINOR LOSS COEFFICIENTS (K)

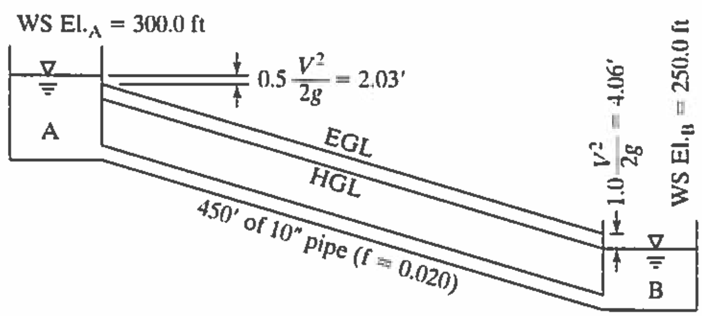
Transitions		
Diameter ratio	Expansion	Contraction
0	1.0	0.5
0.2	0.92	0.45
0.4	0.70	0.38
0.6	0.40	0.29
0.8	0.12	0.12
1.0	0.0	0.0
Entrance		
Pipe projection	0.8	
Square edge	0.5	
Rounded	0.1	
Exit	1.0	

TABLE 4.2 *(Continued)*

Bends		
Radius/diameter	90°	45°
1	0.5	0.37
2	0.3	0.22
4	0.25	0.19
6	0.15	0.11
Valves		
Globe (open)	10	
Swing check	2.0	
Gates (open)	0.2	
Gate (1/2 open)	5.6	
Butterfly (open)	1.2	
Ball (open)	0.05	

Example 4.3 Short Pipe Problem

The two reservoirs are connected with 450 ft of 10-in. diameter pipe ($f = 0.020$). The entrance loss coefficient is 0.5 at the upper reservoir, and the exit loss coefficient is 1.0 at the lower reservoir. Determine the discharge rate in the pipe. Draw the *EGL* and *HGL* on the sketch



$$H_L = 0.5\frac{V^2}{2g} + \frac{fL}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$300.0 - 250.0 = \left(1.5 + \frac{0.02 \times 450}{0.833}\right)\frac{V^2}{2g}$$

$$50.0 = 12.3\frac{V^2}{2g}$$

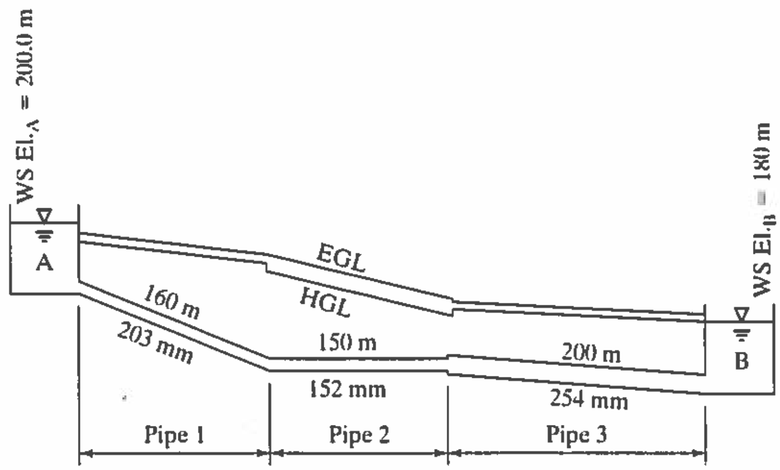
$$\frac{V^2}{2g} = 4.06 \text{ ft}$$

$$V = 16.2 \text{ fps}$$

$$Q = AV = 0.545 \times 16.2 = 8.83 \text{ cfs}$$

Example 4.4 Minor Losses

A pipeline consisting of three pipes in series ($f = 0.02$) extends from an upper reservoir (Elevation 200.0 m) to a lower reservoir (Elevation 180.0 m). Compute the discharge rate in the pipeline using minor loss coefficients of 0.5 for entrance, 0.15 for contraction at junction 1, 0.40 for expansion at junction 2, and 1.0 for exit. The minor loss coefficients at the two pipe junctions are based on the velocity in the smaller pipe (D = 152 mm). The headloss between A and B is the difference in water surface elevation ($WSEL$).



$$H_L = WSEL_A - WSEL_B$$

$$= 0.5\frac{V_1^2}{2g} + \frac{fL_1}{D_1}\frac{V_1^2}{2g} + 0.15\frac{V_2^2}{2g} + \frac{fL_2}{D_2}\frac{V_2^2}{2g} + 0.40\frac{V_2^2}{2g} + \frac{fL_3}{D_3}\frac{V_3^2}{2g} + 1.0\frac{V_3^2}{2g}$$

$$= (0.5 + 15.8)\frac{V_1^2}{2g} + (0.15 + 19.74 + 0.40)\frac{V_2^2}{2g} + (15.7 + 1.0)\frac{V_3^2}{2g}$$

$$= 16.3\frac{V_1^2}{2g} + 20.3\frac{V_2^2}{2g} + 16.7\frac{V_3^2}{2g}$$

Based on the continuity equation

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = 0.56V_2$$

$$V_3 = \left(\frac{D_2}{D_3}\right)^2 V_2 = 0.36V_2$$

$$H_L = 20 \text{ m} = \left[16.3(0.56)^2 + 20.3 + 16.7(0.36)^2\right]\frac{V_2^2}{2g} = 27.6\frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{20}{27.6} = 0.72 \text{ m}$$

$$V_2 = (2g \times 0.72)^{1/2} = (2.0 \times 9.81 \times 0.72)^{1/2} = 3.76 \text{ mps}$$

$$Q = A_2V_2 = 0.0181 \times 3.76 = 0.0682 \text{ cms}$$

4.2 PUMPS IN PIPELINES

Centrifugal pumps are used in pipelines to add energy to the water. The rotating element that transfers the energy from the motor to the water is called the impeller. In a radial flow centrifugal pump, the impeller is shaped to force water outward in a direction at a right angle to its axis. Radial flow pumps are high-head, low-capacity pumps. In an axial flow centrifugal pump, the impeller is shaped like a propeller, forcing the water in an axial direction. Axial flow pumps are low-head, high-capacity pumps. In a mixed-flow centrifugal pump, the impeller is shaped to give the water both axial and radial velocity components.

4.2.1 Pump Characteristics

A pump has operating characteristics that depend on the design and operating speed. Characteristic curves indicate the relation between head, discharge, and efficiency at a specific pump operating speed. Typical characteristic curves for a centrifugal pump are shown in Fig. 4.5. The pump in Fig. 4.5 has a cutoff head of 45 m (head at zero discharge) and a rated capacity of 0.25 m^3/s (discharge at maximum efficiency) at an operating speed of 1,750 rpm.

A pump can have more than one impeller. A multiple stage pump has the impellers arranged such that the discharge from one impeller flows into the next impeller. If a pump has three impellers in series, it is called a three-stage pump. The total energy transferred to the water by the pump (E_p) is equal to the head per stage (h_s) times the number of stages.

The water power (P) transferred from the impeller to the water in kw is

$$P_{water} = T\omega = \gamma Q E_p \tag{4.12}$$

where T is the torque exerted by the impeller on the water and ω is the rotation speed in radians/second (rad/s). The shaft power in kw is

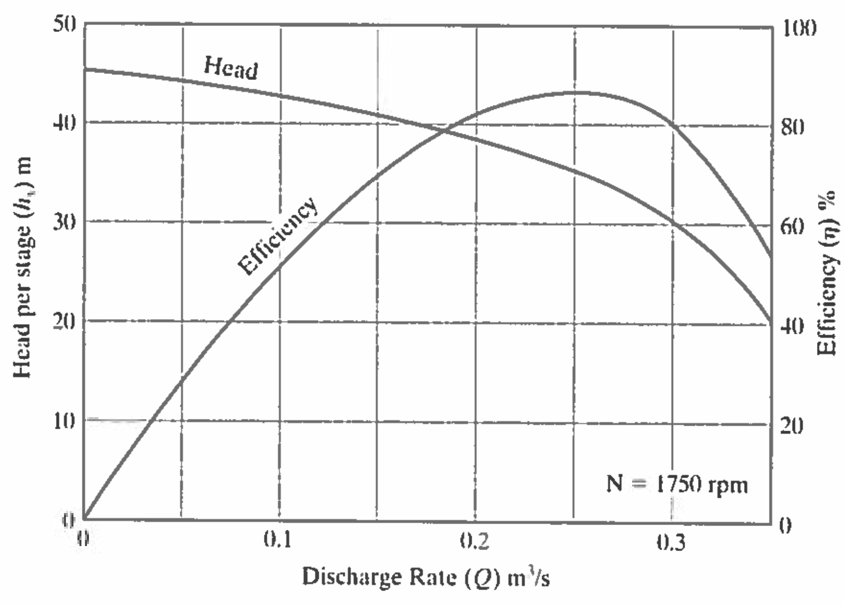


Figure 4.5 Pump characteristic curves.

$$P_{\text{shaft}} = \frac{\gamma Q E_p}{\eta} \tag{4.13}$$

where η is the efficiency of the pump.

Changing the pump speed or impeller diameter will change the pump characteristic curve. The head added per stage (h_s) is a function of impeller diameter (D), rotational speed (ω), discharge rate (Q), fluid density (ρ), and viscosity (μ). Written in functional form

$$gh_s = f(D, \omega, Q, \rho, \mu)$$

where gh_s is the head rise in terms of energy per unit mass. The pi one term (Section 3.6) is

$$\pi_1 = gh_s\omega^a D^b$$

where gh_s is the dependent variable and ω and D are repeating independent variables.

$$\begin{aligned}
F^0L^0T^0 &= L^2T^{-2}(T^{-1})^a(L)^b \\
L:\quad 0 &= 2 + b \quad b = -2 \\
T:\quad 0 &= -2 - a \quad a = -2 \\
\pi_1 &= C_H = \frac{gh_s}{\omega^2 D^2}
\end{aligned} \tag{4.14}$$

π_1 is called the head rise coefficient (C_H). Replacing the dependent variable with the nonrepeating variable Q gives

$$\begin{aligned}
\pi_2 &= Q\omega^a D^b \\
F^0L^0T^0 &= L^3T^{-1}(T^{-1})^a(L)^b \\
L:\quad 0 &= 3 + b \quad b = -3 \\
T:\quad 0 &= -1 - a \quad a = -1 \\
\pi_2 &= C_Q = \frac{Q}{\omega D^3}
\end{aligned} \tag{4.15}$$

π_2 is called the flow coefficient (C_Q).

For the last pi term, there are four variables

$$\begin{aligned}
\pi_3 &= \rho\mu^a\omega^b D^c \\
F^0L^0T^0 &= FT^2L^{-4}(FTL^{-2})^a(T^{-1})^b(L)^c \\
F:\quad 0 &= 1 + a \quad a = -1 \\
T:\quad 0 &= 2 - 1 - b \quad b = 1 \\
L:\quad 0 &= -4 + 2 + c \quad c = 2 \\
\pi_3 &= \rho\frac{\omega D^2}{\mu} = \frac{\omega D^2}{\nu}
\end{aligned} \tag{4.16}$$

The π_3 term represents the Reynolds number. For high Reynolds number flow, the effects of Reynolds number can be neglected. For geometrically similar pumps

$$\frac{gh_s}{\omega^2 D^2} = \phi\left(\frac{Q}{\omega D^3}\right) \tag{4.17}$$

For two geometrically similar pumps

$$\left(\frac{gh_s}{\omega^2 D^2}\right)_1 = \left(\frac{gh_s}{\omega^2 D^2}\right)_2$$

and

$$\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2$$

If the speed of a pump changes from N_o to N, then

$$Q = Q_o\left(\frac{N}{N_o}\right) \tag{4.18}$$

$$E_p = E_{po}\left(\frac{N}{N_o}\right)^2 \tag{4.19}$$

and

$$P = P_o\left(\frac{N}{N_o}\right)^3 \tag{4.20}$$

where subscript o indicates a point on the pump characteristic curve.

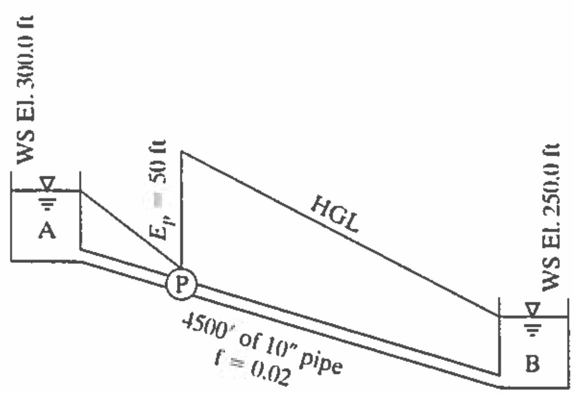
The specific speed (N_s) is a dimensionless parameter used to characterize the type of pump. N_s is obtained by eliminating the diameter (D) in the ratio of flow coefficient and head rise coefficient. Raising the flow coefficient to an exponent of 1/2 and dividing by the head coefficient raised to an exponent of 3/4 gives

$$N_s = \frac{\omega Q^{1/2}}{(gh_s)^{3/4}} \tag{4.21}$$

where ω is the pump rotation speed in rad/s, Q is the discharge rate in m^3/s, and h_s is the head per stage in m. The specific speed is sometimes called the shape number as it reflects the shape of the impeller. A specific speed between 0.2 and 1.3 indicates a radial flow pump, between 1.3 and 2.8 indicates a mixed flow pump, and between 2.8 and 4.4 indicates an axial flow pump. The specific speed of a pump is determined by the operating characteristics at the point of maximum efficiency.

Example 4.5 Booster Pump

A booster pump is installed in the pipeline between the two reservoirs shown below. If the energy added by the pump (E_p) is 50.0 ft, determine the flow rate in the pipeline. Neglect minor losses.



Write the energy equation from A to B in terms of water surface elevation ($WSEL$).

$$E_p + WSEL_A = H_L + WSEL_B$$

$$H_L = E_p + WSEL_A - WSEL_B$$

$$\frac{fL}{D}\frac{V^2}{2g} = 50.0 + 300.0 - 250.0$$

$$\frac{0.02 \times 4500}{0.833}\frac{V^2}{2g} = 100.0 \text{ ft}$$

$$\frac{V^2}{2g} = \frac{100.0}{108.0} = 0.926 \text{ ft}$$

$$V = 7.72 \text{ fps}$$

$$Q = AV = 0.545 \times 7.72 = 4.2 \text{ cfs}$$

Example 4.6 Pump Characteristic Curve

The pump characteristic curve shown below is for a pump speed of 1,700 rpm. Determine the pump characteristic curve for a pump speed of 1,500 rpm and 1,300 rpm, using Eqs. 4.18 and 4.19.

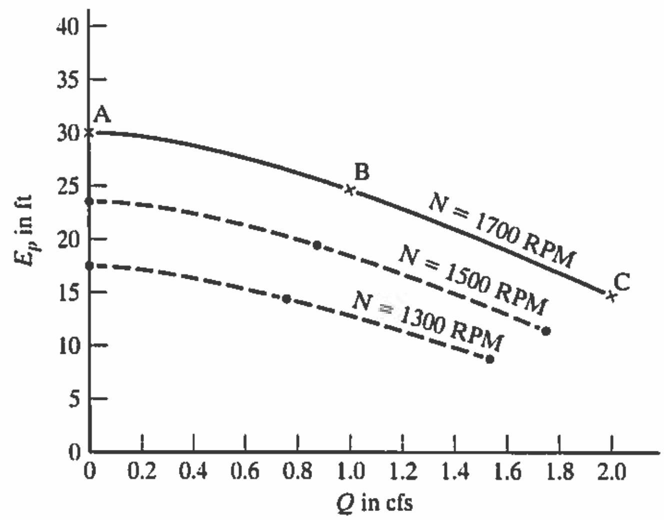
$$Q_2 = Q_1\left(\frac{N_2}{N_1}\right)$$

$$E_{p2} = E_{p1}\left(\frac{N_2}{N_1}\right)^2$$

Select 3 points (A, B, and C) on the pump characteristic curve.

	$N = 1700$		$N = 1500$		$N = 1300$	
Point	Q cfs	E_p ft	Q cfs	E_p ft	Q cfs	E_p ft
A	0	30	0	23.4	0	17.5
B	1.0	24.5	0.88	19.1	0.76	14.3
C	2.0	14.5	1.76	11.3	1.53	8.5

Curves are plotted on the sketch.



4.2.2 Pipeline with Pump

The pump and pipeline in Fig. 4.6 are used to transport water from the lower reservoir to the upper reservoir. Writing the energy equation between reservoirs A and B gives

$$El_A + E_p = El_B + H_L \tag{4.22}$$

or

$$E_p = El_B - El_A + K_{eq}Q^n \tag{4.23}$$

where El denotes elevation, $K_{eq}Q^n$ represents the headloss in the pipeline, and

$$K_{eq} = K_1 + K_2 + K_3$$

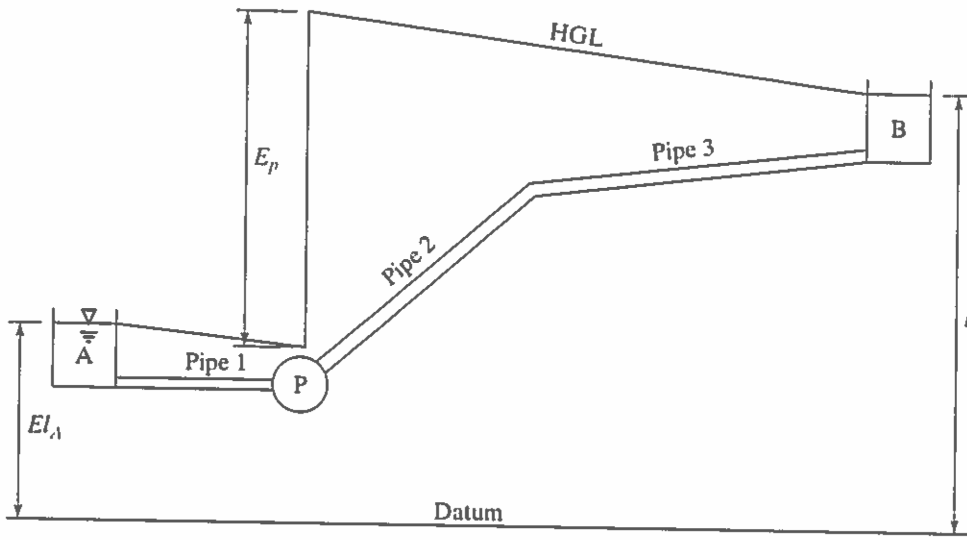
The energy added by the pump is equal to the difference in elevation between the two reservoirs plus the headloss in the pipeline. The discharge rate in the pipeline for a given pump can be determined graphically or numerically.

Graphic solution requires that the system curve (Eq. 4.23) be plotted on the same plot as the pump characteristic curve in Fig. 4.6. The pump operating point is at the intersection of the system curve and the pump characteristic curve. The discharge rate at the pump operating point can be read from the graph.

The pump characteristic curve shown in Fig. 4.6 can be approximated by the polynomial

$$E_p = AQ^2 + BQ + H_c \tag{4.24}$$

where H_c is the pump cutoff head (head when $Q = 0$). Coefficients A and B are evaluated from two points on the pump characteristic curve selected to represent the operating range of the pump. To ensure that the slope of pump characteristic



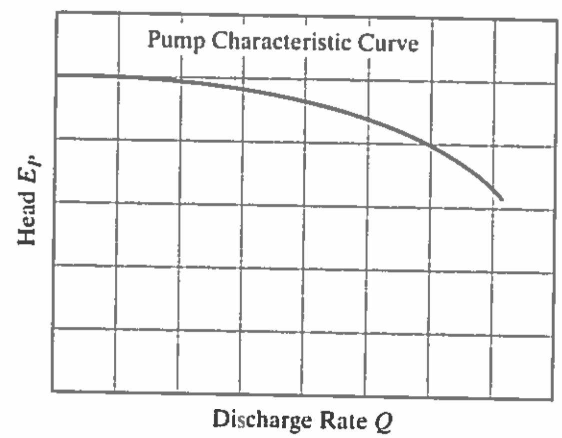


Figure 4.6 Pump in pipeline.

curve is always negative, both A and B should be negative. The energy equation becomes

$$AQ^2 + BQ + H_c = El_B - El_A + K_{eq}Q^n \tag{4.25}$$

Based on the Darcy–Weisbach headloss ($n = 2$), Eq. 4.25 can be solved using the quadratic equation

$$Q = \frac{-B - \sqrt{B^2 - 4ac}}{2a} \tag{4.26}$$

where

$$a = A - K_{eq}, \quad \text{and}$$
$$c = H_c + El_A - El_B$$

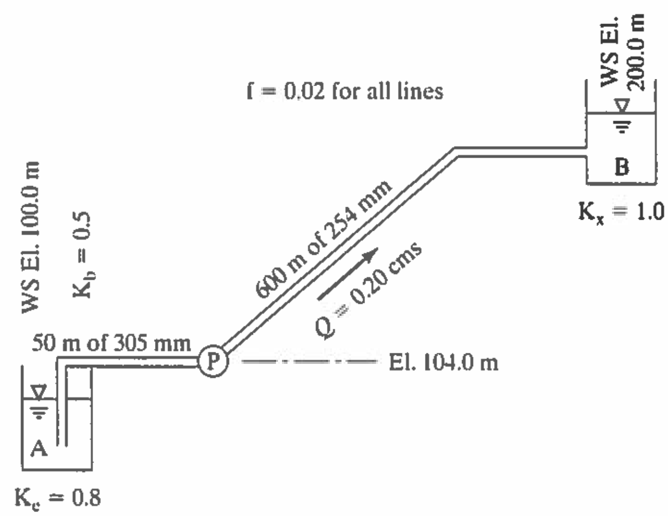
Example 4.7 Pump in Pipeline

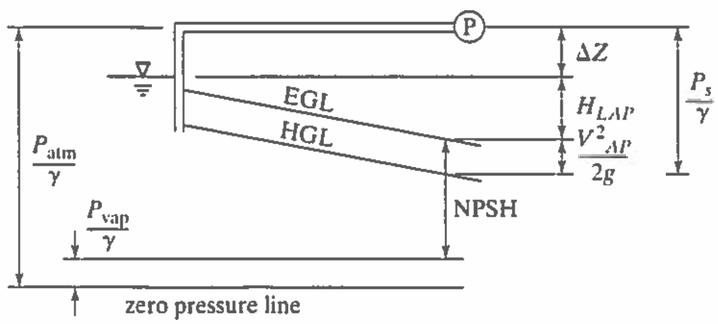
Determine the power required to lift 0.20 cms of water from a lower reservoir with a water surface elevation of 100.0 m to an upper reservoir with an elevation of 200.0 m. The pump has an efficiency of 0.88, and the motor has an efficiency of 0.83. The intake line to the pump is 50 m of 305-mm diameter pipe, and the discharge line is 600 m of a 254-mm diameter pipe. Both lines have a friction factor (f) of 0.02. If the pump is at an elevation of 104.0 m, determine the pressure at the inlet and discharge sides of the pump. Consider an entrance loss coefficient of 0.8, a bend loss coefficient of 0.5 in the inlet line, and an exit loss coefficient of 1.0 at the upper reservoir. If the atmospheric pressure is 95.5 kN/m^2 and the vapor pressure of the water is 2.3 kN/m^2, determine the net positive suction head (NPSH) for the pump. The NPSH is used to indicate the potential for cavitation.

$$\text{Headloss } K \text{ (Eq. 4.36) } K = \frac{fL}{DA^2 2g}$$

$$K_{AP} = \frac{0.02 \times 50}{(0.305 \times 0.073^2 \times 2 \times 9.81)} = 31.4$$

$$K_{PB} = \frac{0.02 \times 600}{(0.254 \times 0.051^2 \times 19.62)} = 926$$





Pumping head E_p (Eq. 4.23) $= El_B - El_A + \text{Headloss} = 100.0 + \text{Headloss}$

Pipe friction losses $= K_{AP}Q^2 + K_{PB}Q^2 = 957(0.2)^2 = 38.3$ m

$$\text{Minor losses} = 0.8\frac{V_{AP}^2}{2g} + 0.5\frac{V_{AP}^2}{2g} + 1.0\frac{V_{PB}^2}{2g} = 0.8 \times \frac{2.74^2}{19.62} + 0.5 \times \frac{2.74^2}{19.62} + 1.0 \times \frac{3.92^2}{19.62}$$
$$= 0.31 + 0.19 + 0.78 = 1.3 \text{ m}$$

$$\text{Pumping head } E_p = 100.0 + 38.3 + 1.3 = 139.6 \text{ m}$$

$$\text{Water power} = Q\gamma\frac{E_p}{1000} = 0.20 \times 9.79 \times 139.6 = 273 \text{ kw}$$

$$\text{Pump input power} = \frac{\text{water power}}{\text{efficiency pump}} = \frac{273}{0.88} = 310 \text{ kw}$$

$$\text{Motor input power} = \frac{\text{pump power}}{\text{efficiency motor}} = \frac{310}{0.83} = 373 \text{ kw}$$

Pressure at Pump Intake (P_s)

Energy equation between A and pump

$$Z_A = \frac{V_{AP}^2}{2g} + \frac{P_s}{\gamma} + Z_p + H_{LAP}$$

$$\frac{P_s}{\gamma} = (Z_A - Z_P) - \frac{V_{AP}^2}{2g} - H_{LAP}$$

$$H_{LAP} = K_{AP}Q^2 + (0.8 + 0.5)\frac{V_{AP}^2}{2g} = 31.4(0.2)^2 + 1.3 \times \frac{2.74^2}{19.62} = 1.25 + 0.50 = 1.75 \text{ m}$$

$$\frac{P_s}{\gamma} = (100.0 - 104.0) - 0.4 - 1.7 = -6.1 \text{ m}$$

$$P_s = -6.1 \times 9.79 = -59.7 \text{ kN/m}^2$$

Pressure at Pump Discharge (P_d)

Energy equation between pump and B

$$\frac{V_{PB}^2}{2g} + \frac{P_d}{\gamma} + Z_p = Z_B + H_{LPB}$$

$$\frac{P_d}{\gamma} = (Z_B - Z_p) - \frac{V_{PB}^2}{2g} + H_{LPB}$$

$$H_{LPB} = K_{PB}Q^2 + 1.0\frac{V_{PB}^2}{2g} = 926(0.2)^2 + 1.0\frac{3.92^2}{19.62} = 37.0 + 0.8 = 37.8 \text{ m}$$

$$\frac{P_d}{\gamma} = (200.0 - 104.0) - 0.8 + 37.8 = 133.0 \text{ m}$$

$$P_d = 133.0 \times 9.79 = 1{,}302 \text{ kN/m}^2$$

An enlargement of the intake side of the pump is included in the figure. By definition the NPSH is

$$\text{NPSH} = \frac{(P_{\text{atm}} - P_{\text{vap}})}{\gamma} - \Delta Z - H_{LAP} = \frac{(95.5 - 2.3)}{9.79} - 4.0 - 1.8 = 3.7\text{ m}$$

The cavitation parameter (σ) is defined as

$$\sigma = \frac{\text{NPSH}}{h_s}$$

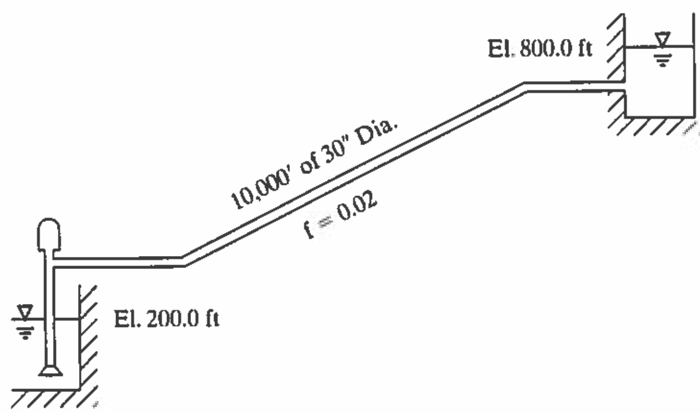
where h_s is the head per stage. For a 4 stage pump

$$\sigma = \frac{3.7(4)}{139.6} = 0.11$$

To prevent cavitation of the pump, the value of σ must be greater than a critical value which is normally provided by the pump manufacturer.

Example 4.8 Pump Characteristic Curve

(a) Determine the discharge rate for the pump and pipeline system shown below. The vertical turbine pump lifts water from the Columbia River (elevation 200 ft) and discharges through a 10,000-ft, 30-inch diameter pipeline ($f = 0.02$) to an upper reservoir at elevation 800 ft. The pump characteristic curve is also shown below. Determine the discharge rate using Eq. 4.26 and check the result using the graphic method.



Compute the headloss K value

$$K = \frac{fL}{DA^2 2g} = \frac{0.02 \times 10{,}000}{(2.5) \times 4.91^2 \times 64.4} = 0.0515$$

Pump curve

Three points

$$E_p = 800, \quad Q = 0$$
$$E_p = 777, \quad Q = 20$$
$$E_p = 664, \quad Q = 50$$

Pump curve

$$E_p = -0.0522Q^2 - 0.11Q + 800$$

Discharge rate, Eq. 4.26

$$Q = \frac{-B - \sqrt{B^2 - 4ac}}{2a}$$

where

$$B = -0.11$$

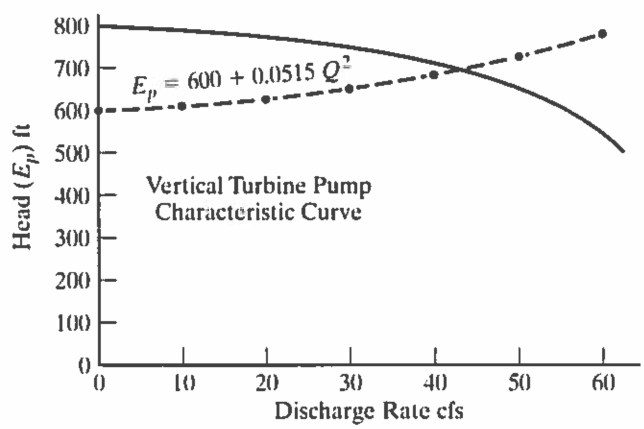
$$a = A - K = -0.0515 - 0.0522 = -0.1037$$

$$c = H_c - El_B + El_A = 800 - 800 + 200 = 200$$

$$Q = \frac{+0.11 - \sqrt{(0.11)^2 + 4 \times 0.1037 \times 200}}{-2 \times 0.1037} = 43.4 \text{ cfs}$$

Graphic solution is shown on characteristic curve plot for pump head Eq. 4.23

$$E_p = El_B - El_A + KQ^n = 600 + 0.0515Q^2$$



(b) Repeat example 4.8 for two pumps in parallel. For pumps in parallel, the points on the combined characteristic curve are

E_p ft	Q cfs
800	0
777	40
664	100

Pump curve

$$E_p = -0.01308Q^2 - 0.052Q + 800$$

Discharge rate, Eq. 4.26

$$Q = \frac{-B - \sqrt{B^2 - 4ac}}{2a}$$

$$= \frac{0.052 - \sqrt{0.052^2 + 4 \times 0.0646 \times 200}}{-2 \times 0.0646}$$

$$= 55.2 \text{ cfs}$$

(c) Repeat example 4.8 for two pumps in series: The points on the combined characteristic curve are

E_p ft	Q cfs
1,600	0
1,554	20
1,328	50

Pump curve

$$E_p = -0.1047Q^2 - 0.207Q + 1{,}600$$

Discharge rate, Eq. 4.26

$$Q = \frac{-B - \sqrt{B^2 - 4ac}}{2a}$$

$$= \frac{0.207 - \sqrt{0.207^2 + 4 \times 0.1562 \times 1{,}000}}{-2 \times 0.1562}$$

$$= 79.4 \text{ cfs}$$

(d) Repeat example 4.8 for pump speed increase of 10 percent. The points on the pump curve are

E_p ft	Q cfs
$800 \times 1.1^2 = 968$	$0 = 0$
$777 \times 1.1^2 = 940$	$20 \times 1.1 = 22$
$664 \times 1.1^2 = 803$	$50 \times 1.1 = 55$

Pump curve

$$E_p = -0.0523Q^2 - 0.121Q + 968$$

Discharge rate, Eq. 4.26

$$Q = \frac{-B - \sqrt{B^2 - 4ac}}{2a}$$

$$= \frac{0.12 - \sqrt{0.12^2 + 4 \times 0.1038 \times 368}}{-2 \times 0.1038}$$

$$= 59.5 \text{ cfs}$$

4.4 PIPELINES CONNECTING RESERVOIRS

Reservoirs in a pipeline or pipe network system are considered fixed-grade nodes, indicating a node with a constant EGL. A pipe is considered hydraulically long when pipe friction is the dominant headloss term and minor losses can be neglected. A pipe is considered hydraulically short when minor losses account for a significant part of the total headloss. A culvert (Section 4.3) is an example of short pipe flow.

4.4.1 Pipes in Series

The pipeline shown in Fig. 4.10 is designed to transport water from the upper reservoir to the lower reservoir. The pipeline consists of three pipes in series, two junction nodes and two fixed-grade nodes (A and B). For long pipe problems where the friction losses are much greater than the minor losses, minor losses are neglected and the friction headloss equation is written as

$$H_L = KQ^n \tag{4.35}$$

For the Darcy–Weisbach equation

$$K = \frac{fL}{DA^2 2g} \quad \text{and} \quad n = 2. \tag{4.36}$$

For the Hazen–Williams equation

$$K = \frac{L}{(C_u C_H A R^{0.63})^{1.85}} \quad \text{and} \quad n = 1.85. \tag{4.37}$$

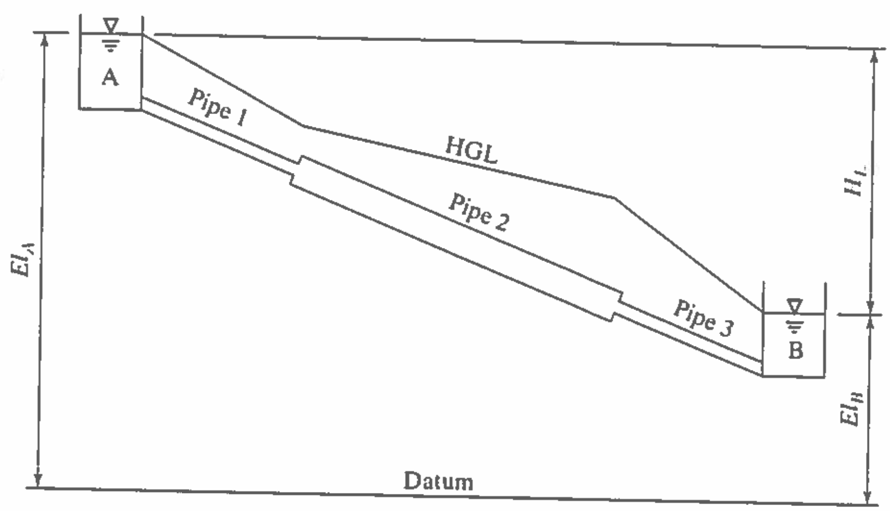


Figure 4.10 Pipes in series.

For the pipeline in Fig. 4.10, the flow rate in each pipe is unknown. Writing the continuity equation for the two junction nodes yields

$$Q_1 - Q_2 = 0 \tag{4.38}$$

$$Q_2 - Q_3 = 0 \tag{4.39}$$

and writing the energy equation between the two fixed-grade nodes yields

$$K_1Q_1^n + K_2Q_2^n + K_3Q_3^n = El_A - El_B \tag{4.40}$$

Combining Eqs. 4.38–4.40

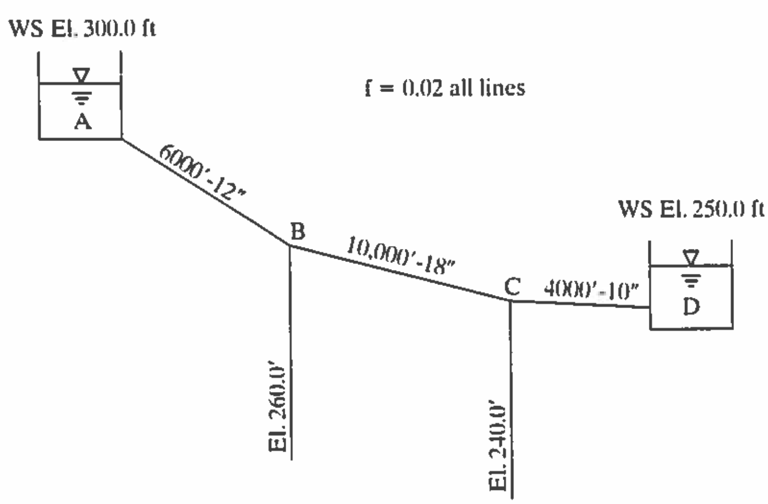
$$H_L = El_A - El_B = K_{eq}Q^n \tag{4.41}$$

where

$$K_{eq} = K_1 + K_2 + K_3 \tag{4.42}$$

Example 4.11 Pipes in Series

As shown in the sketch below, a pipeline ($f = 0.02$) consisting of three pipes in series (6,000 ft of 12-in. diameter pipe, 10,000 ft of 18-in. diameter pipe, and 4,000 ft of 10-in. diameter pipe) runs from an upper reservoir to a lower reservoir with water surface elevations of 300 ft and 250 ft, respectively. Determine discharge through the pipeline and the pressure at each of the junction nodes.



The headloss K value is computed for each line from Eq. 4.36

$$K = \frac{fL}{DA^2 2g}$$

Line A–B

$$K_{AB} = \frac{0.02 \times 6000}{(1.0 \times 0.78^2 \times 64.4)} = 3.02$$

Line B–C

$$K_{BC} = \frac{0.02 \times 10{,}000}{(1.5 \times 1.77^2 \times 64.4)} = 0.66$$

Line C–D

$$K_{CD} = \frac{0.02 \times 4000}{(0.83 \times 0.54^2 \times 64.4)} = 5.13$$

From Eq. 4.42, the equivalent K value

$$K_{eq} = K_{AB} + K_{BC} + K_{CD} = 3.02 + 0.66 + 5.13 = 8.81$$

The discharge rate computed from Eq. 4.41

$$Q = \left[\frac{El_A - El_D}{K_{eq}}\right]^{1/n} = \left[\frac{300.0 - 250.0}{8.81}\right]^{1/2} = 2.38 \text{ cfs}$$

Pressure at B

$El\ HGL$ at B

$$HGL_B = El_A - K_{AB}Q_{AB}^2 = 300.0 - 3.02(2.38)^2 = 282.9 \text{ ft}$$

$$\frac{P_B}{\gamma} = HGL_B - El_B = 282.9 - 260 = 22.9 \text{ ft}$$

$$P_B = 22.9 \times 62.4 = 1{,}429 \text{ lbs/ft}^2 = 9.9 \text{ psi}$$

Pressure at C

$$HGL_C = HGL_B - K_{BC}Q_{BC} = 282.9 - 0.66(2.38)^2 = 279.2 \text{ ft}$$

$$\frac{P_C}{\gamma} = HGL_c - EL_c = 279.2 - 240.0 = 39.2 \text{ ft}$$

$$P_C = 39.2 \times 62.4 = 2{,}446 \text{ lbs/ft}^2 = 17.0 \text{ psi}$$

4.4.2 Pipes in Parallel

The pipeline connecting the two reservoirs in Fig. 4.11 consists of pipes in series and pipes in parallel. Pipes 2 and 4 extend from the same junction nodes and are considered to be in parallel with each other. The flow rates in the four pipes shown in Fig. 4.11 are unknown. For the two pipes in parallel, the headloss in pipe 2 is equal to the headloss in pipe 4. Pipes 2 and 4 can be considered to form a closed pipe loop, and the summation of headloss (clockwise plus) around any closed pipe loop is equal to zero or

$$K_2Q_2^n - K_4Q_4^n = 0 \tag{4.43}$$

Writing the continuity equation for the two junction nodes

$$Q_1 - Q_2 - Q_4 = 0 \tag{4.44}$$

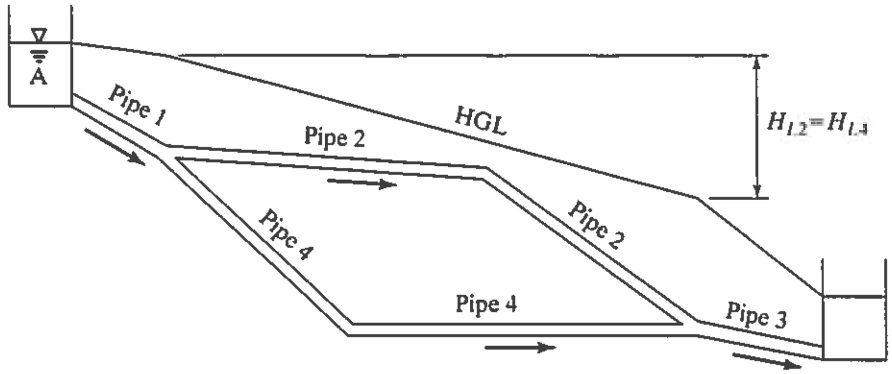


Figure 4.11 Pipes in parallel.

and

$$Q_2 + Q_4 - Q_3 = 0 \tag{4.45}$$

and writing the energy equation between the two fixed-grade nodes

$$K_1Q_1^n + K_2Q_2^n + K_3Q_3^n = El_A - El_B \tag{4.46}$$

or

$$K_1Q_1^n + K_4Q_4^n + K_3Q_3^n = El_A - El_B \tag{4.47}$$

yields four equations to solve for the four unknown flow rates. (Either Eq. 4.46 or Eq. 4.47 can be used in the computations.)

A different procedure is used to solve the equations manually than is used in computer analysis. For manual computation, the two parallel pipes are replaced with a single equivalent (imaginary) pipe that gives the same headloss based on total flow (Q) as pipe 2 or pipe 4 based on partial flow or

$$H_{Leq} = H_{L2} = H_{L4} = H_L \tag{4.48}$$

From Eqs. 4.44 and 4.45, the total flow is

$$Q = Q_1 = Q_3 \tag{4.49}$$

Solving the headloss Eq. 4.35 for Q and substituting into Eq. 4.44 yields

$$\left[\frac{H_L}{K_{eq}}\right]^{1/n} = \left[\frac{H_L}{K_2}\right]^{1/n} + \left(\frac{H_L}{K_4}\right)^{1/n} \tag{4.50}$$

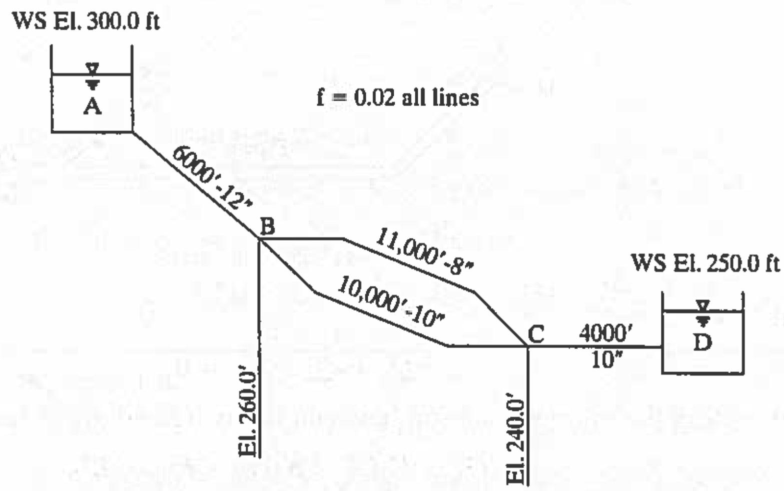
or

$$K_{eq} = \frac{1}{\left[\dfrac{1}{K_2^{1/n}} + \dfrac{1}{K_4^{1/n}}\right]^n} \tag{4.51}$$

The equivalent K value (Eq. 4.51) is used as K_2 in Eq. 4.42 to solve for Q in Eq. 4.41. Equations 4.35 and 4.48 are then used to solve for Q_2 and Q_4.

Example 4.12 Pipes in Parallel

Determine the flow in each line and the pressure at the junction nodes for the pipe network shown in the sketch below. The example is the same as Example 4.11, except the 18-inch diameter pipe has been replaced with parallel 8-inch and 10-inch pipes.



Compute the headloss K value

$$K = \frac{fL}{DA^2 2g}$$

$$K_{BC-8} = \frac{0.02 \times 11{,}000}{(0.67 \times 0.35^2 \times 64.4)} = 41.6$$

$$K_{BC-10} = \frac{0.02 \times 10{,}000}{(0.83 \times 0.54^2 \times 64.4)} = 12.8$$

The equivalent K value for parallel pipes (Eq. 4.51)

$$K_{eqBC} = \frac{1}{\left[\frac{1}{K_8^{1/n}} + \frac{1}{K_{10}^{1/n}}\right]^n} = \frac{1}{\left[\frac{1}{(41.6)^{1/2}} + \frac{1}{(12.8)^{1/2}}\right]^2} = 5.30$$

The equivalent K value for pipes in series (Eq. 4.42)

$$K_{eq} = K_{AB} + K_{eqBC} + K_{CD} = 3.02 + 5.30 + 5.13 = 13.45$$

The discharge rate from Eq. 4.41

$$Q = \left[\frac{El_A - El_D}{K_{eq}}\right]^{1/n} = \left[\frac{300.0 - 250.0}{13.45}\right]^{1/2} = 1.92 \text{ cfs}$$

Discharge rates in parallel pipes

Compute headloss for parallel pipes

$$H_{LBC} = K_{eqBC}Q^n = 5.30(1.92)^2 = 19.5 \text{ ft}$$

Discharge 8-inch line (Eq. 4.35)

$$Q_{8"} = \left[\frac{H_{LBC}}{K_{BC-8}}\right]^{1/n} = \left[\frac{19.5}{41.6}\right]^{1/2} = 0.68 \text{ cfs}$$

Discharge 10-inch line (Eq. 4.35)

$$Q_{10} = \left[\frac{H_{LBC}}{K_{BC-10}}\right]^{1/n} = \left[\frac{19.5}{12.8}\right]^{1/2} = 1.23 \text{ cfs}$$

Continuity check at B

$$Q_{AB} = Q_{BC-8} + Q_{BC-10}$$

$$1.92 = 0.68 + 1.23 = 1.91 \text{ cfs}$$

Pressure at B

$$HGL_B = El_A - H_{LAB} = 300.0 - 3.02 \times 1.92^2 = 288.9 \text{ ft}$$

$$\frac{P_B}{\gamma} = HGL_B - El_B = 28.9 \text{ ft}$$

$$P_B = 28.9 \times 62.4 = 1{,}803 \text{ lbs/ft}^2 = 12.5 \text{ psi}$$

Pressure at C

$$HGL_C = HGL_B - H_{LBC} = 288.9 - 19.5 = 269.4 \text{ ft}$$

$$\frac{P_C}{\gamma} = HGL_C - El_C = 269.4 - 240.0 = 29.4 \text{ ft}$$

$$P_C = 29.4 \times 62.4 = 1{,}835 \text{ lbs/ft}^2 = 12.7 \text{ psi}$$

4.4.3 Three-Reservoir System

Figure 4.12 shows a pipe network system connecting three reservoirs. The problem is included as an introduction to more complex pipe network systems. Since there are three unknowns in this problem (the discharge rate in the three pipes), three equations are required. The continuity equation can be written at the junction node, and two energy equations can be written between the reservoirs. If there are N

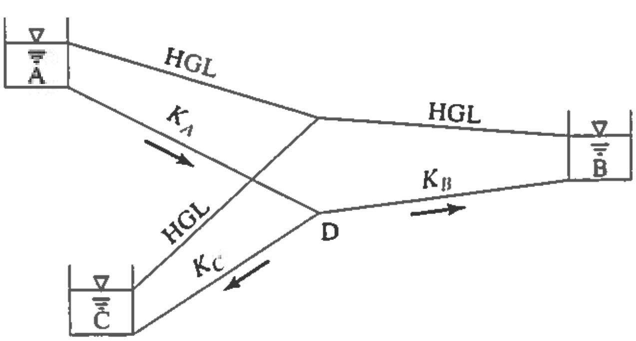


Figure 4.12 Pipe network connecting three reservoirs.

FGN (fixed-grade nodes or reservoirs) in a system, then there are $N - 1$ independent energy equations written between the reservoirs.

The continuity equation written at junction node D is

$$Q_A - Q_B - Q_C = 0 \tag{4.52}$$

and the two energy equations

$$El_A - K_A Q_A^n - K_C Q_C^n = El_C$$

or

$$K_A Q_A^n + K_C Q_C^n = El_A - El_C \tag{4.53}$$

and

$$El_C + K_C Q_C^n - K_B Q_B^n = El_B$$

or

$$K_C Q_C^n - K_B Q_B^n = El_B - El_C \tag{4.54}$$

For a computer solution, Eqs. 4.52–4.54 are solved simultaneously. For a manual solution, the problem is normally solved as follows:

1. The elevation of the HGL at junction node D is assumed HGL_D.
2. Based on the assumed HGL_D, the discharge rate in each line is computed

$$Q_A = \left[\frac{El_A - HGL_D}{K_A}\right]^{1/n} \tag{4.55}$$

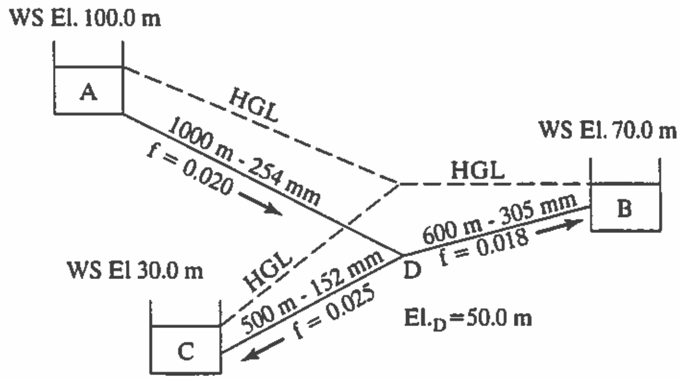
$$Q_B = \left[\frac{HGL_D - El_B}{K_B}\right]^{1/n} \tag{4.56}$$

$$Q_C = \left[\frac{HGL_D - El_C}{K_C}\right]^{1/n} \tag{4.57}$$

3. The continuity equation (Eq. 4.52) at the junction node is checked. If the continuity equation is satisfied, then the flow rates computed in step 2 are correct. If the continuity equation is not satisfied, then the elevation of the HGL at junction node D is adjusted, and the computations in steps 2 and 3 are repeated.

Example 4.13 Three Reservoirs

Determine the discharge rate in each pipeline for the following three-reservoir problems. This example is used to demonstrate a solution by trial and error and a numerical procedure. The example serves as an introduction to the section on pipe networks.



Compute the headloss K values

$$K = \frac{fL}{DA^2 2g}$$

$$K_A = 1{,}540$$

$$K_B = 340$$

$$K_C = 12{,}900$$

The assumed direction of flow is shown on the sketch.
Solve by trial and error

1. Assume $HGL_D = 70.0$ m
   A. Compute the flow in each line from Eq. 4.35

$$Q_A = \left[\frac{30.0}{1{,}540}\right]^{1/2} = 0.14 \text{ cms}$$

$$Q_B = 0$$

$$Q_C = \left[\frac{40}{12{,}900}\right]^{1/2} = 0.056 \text{ cms}$$

   B. Check continuity at D

$$0.14 \neq 0 + 0.056$$

2. Assume $HGL_D = 75.0$ m
   A. Compute the flow in each line

$$Q_A = \left[\frac{25}{1{,}540}\right]^{1/2} = 0.127 \text{ cms}$$

$$Q_B = \left[\frac{5}{340}\right]^{1/2} = 0.121 \text{ cms}$$

$$Q_C = \left[\frac{45}{12{,}900}\right]^{1/2} = 0.059 \text{ cms}$$

B. Check continuity at D

$$0.127 \neq 0.121 + 0.59$$

3. Assume $HGL_D = 72.0$ m

A. Compute the flow in each line

$$Q_A = \left[\frac{28}{1{,}540}\right]^{1/2} = 0.135 \text{ cms}$$

$$Q_B = \left[\frac{2.0}{340}\right]^{1/2} = 0.077 \text{ cms}$$

$$Q_C = \left[\frac{42.0}{12{,}900}\right]^{1/2} = 0.057 \text{ cms}$$

B. Check continuity at D

$$0.135 = 0.077 + 0.057$$

flows balance.

Trial and error solution

$$Q_A = 0.135 \text{ cms}, \quad Q_B = 0.077 \text{ cms}, \quad Q_C = 0.057 \text{ cms}$$

Simultaneous Equations Solution

The three equations include the linear continuity or node equation (Eq. 4.52) and the two nonlinear energy or loop equations. The nonlinear terms in the energy equations are linearized by using the first two terms in Taylor's series.

$$KQ^2 = Kq^2 + 2Kq(Q - q) = -Kq^2 + 2Kq(Q)$$

where Q is the new estimate of flow and q is the previous estimate of flow.

The three linear equations are

Continuity equation

$$(1) \quad Q_A - Q_B - Q_C = 0$$

Energy equations

$$(2) \quad (2K_Aq_A)Q_A + (2K_Cq_C)Q_C = K_Aq_A^2 + K_Cq_C^2 + El_A - El_C$$

$$(3) \quad -(2K_Bq_B)Q_B + (2K_Cq_C)Q_C = -K_Bq_B^2 + K_Cq_C^2 + El_B - E_C$$

Solution to the above equations is an iterative procedure. The initial estimate of flow is based on a velocity of 1.0 mps in each line giving

$$q_A = 0.05 \text{ cms}, \quad q_B = 0.07 \text{ cms}, \quad q_C = 0.02 \text{ cms}$$

Based on the estimated initial flow rates, the equations are

$$(1) \quad Q_A - Q_B - Q_C = 0$$

$$(2) \quad 154Q_A + 516Q_C = 79.0$$

$$(3) \quad -47.6Q_B + 516Q_C = 43.5$$

Solution yields

$$Q_A = 0.198, \quad Q_B = 0.104, \quad Q_C = 0.094$$

The linear equations based on the new flow rates are

$$(1) \qquad Q_A - Q_B - Q_C = 0$$

$$(2) \qquad 610Q_A + 2{,}425Q_C = 244$$

$$(3) \qquad -70.7Q_B + 2{,}425Q_C = 150$$

Solution yields

$$Q_A = 0.145, \quad Q_B = 0.081, \quad Q_C = 0.064$$

The new linear equations are

$$Q_A - Q_B - Q_C = 0$$

$$447Q_A + 1{,}651Q_C = 155$$

$$-55.1Q_B + 1{,}651Q_C = 90.6$$

Solution yields

$$Q_A = 0.135, \quad Q_B = 0.077, \quad Q_C = 0.058$$

The iterative procedure continues until the maximum change in flow is within some limit, for example, $|Q - q|_{max} \leq 0.002$. Based on the previously computed flow rates, the new linear equations are

$$Q_A - Q_B - Q_C = 0$$

$$416Q_A + 1{,}496Q_C = 141.5$$

$$-52.4Q_B + 1{,}496Q_C = 81.4$$

Final iteration yields

$$Q_A = 0.135 \text{ cms}$$

$$Q_B = 0.078 \text{ cms}$$

$$Q_C = 0.057 \text{ cms}$$

where $|Q - q|_{max} < 0.002$.

A set of computer programs developed in conjunction with this textbook is described by Section 1.4.5. A program for solving linear equations has been included under file SIMEQ.FOR. The input file for this example is SIMEQ.DAT. The FORTRAN program can be modified to compute K values and coefficients for the linearized energy equations.

4.5 PIPE NETWORK SYSTEMS

A municipal water distribution system is used to deliver water to the consumer. Although water is withdrawn from along the pipes in a pipe network system, for computational purposes all demands on the system are assumed to occur at the junction nodes. Pressure is the main concern in a water distribution system. At no time should the water pressure in the system be so low that contaminated groundwater could enter the system at points of leakage even with water hammer pressure waves.

The total water consumption for the water distribution service area is usually available from past records. The total water demand on the system is allocated to the junction nodes based on the estimated residential, industrial, and commercial water demands for each node.

A simple water distribution system is shown in Fig. 4.13. The system is connected to fixed-grade nodes (elevated water tanks labeled A and B in Fig. 4.13) to permit the computation of pressures throughout the system after the flow rates for all pipes have been determined. Pipes 1–9 and junction nodes 1–6 have been numbered on the sketch in Fig. 4.13. The demands (C) at the junction nodes are also shown on the sketch. The number of equations required for solution of the system is equal to the number of unknowns (number of pipes = 9) in the network. The number of independent equations (N_{eq}) available for solution is equal to the number of junction nodes (N_j) plus the number of loops (N_ℓ) plus the number of FGN (N_f) minus 1 or

$$N_{eq} = N_j + N_\ell + N_f - 1 \tag{4.58}$$

or for the example

$$N_{eq} = 6 + 2 + 2 - 1 = 9$$

The estimated direction of flow is also shown on the sketch. If the assumed direction of flow is wrong, the computed flow rate will be negative. Writing the six continuity equations for six junction nodes based on the assumed direction of flow gives

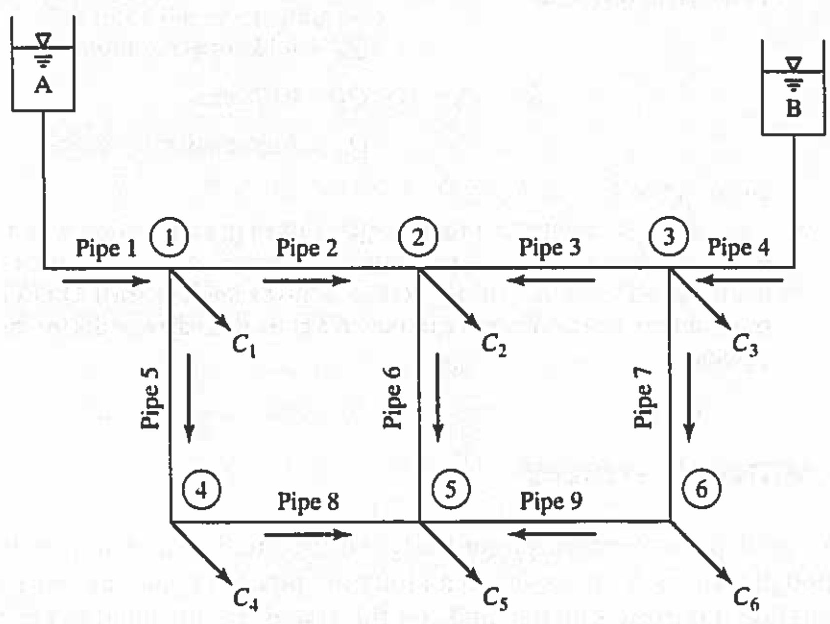


Figure 4.13 Pipe network system.

$$Q_1 - Q_2 - Q_5 - C_1 = 0 \tag{4.59.1}$$

$$Q_2 + Q_3 - Q_6 - C_2 = 0 \tag{4.59.2}$$

$$Q_4 - Q_3 - Q_7 - C_3 = 0 \tag{4.59.3}$$

$$Q_5 - Q_8 - C_4 = 0 \tag{4.59.4}$$

$$Q_6 + Q_8 + Q_9 - C_5 = 0 \tag{4.59.5}$$

$$Q_7 - Q_9 - C_6 = 0 \tag{4.59.6}$$

or in matrix notation

$$\begin{array}{lc} & \text{Pipe} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \\ \text{Equation number} & \begin{array}{c}1\\2\\3\\4\\5\\6\end{array} \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} \end{array} \tag{4.59.7}$$

The loop equations are written for the condition that the algebraic sum of headloss around any closed loop in the system is equal to zero. Nonoverlapping loops are generally used in the formulation of the loop equations to reduce the number of terms in the loop equations. The loop equations for the network in Fig. 4.13 (clockwise plus) are

$$K_2Q_2^n + K_6Q_6^n - K_8Q_8^n - K_5Q_5^n = 0 \tag{4.60.1}$$

$$-K_3Q_3^n + K_7Q_7^n + K_9Q_9^n - K_6Q_6^n = 0 \tag{4.60.2}$$

In addition, the algebraic sum of headloss around any loop containing two fixed-grade nodes (reservoirs) must equal the difference in water surface elevation between the reservoirs. These equations are often called pseudo-loop equations. The reader can visualize the loop by connecting the two reservoirs with a no flow pipe. If N_f equals the number of fixed-grade nodes, then there will be $N_f - 1$ pseudo-loop equations. There are numerous options for writing the pseudo-loop equations; however, they are generally written to include the smallest number of terms.

Starting at reservoir B and writing the pseudo-loop equation in terms of the elevation of the hydraulic grade line gives

$$El_B - K_4Q_4^n - K_3Q_3^n + K_2Q_2^n + K_1Q_1^n = El_A \tag{4.61}$$

or

$$K_4Q_4^n + K_3Q_3^n - K_2Q_2^n - K_1Q_1^n = El_B - El_A \tag{4.62}$$

To solve for the flow rates in each of the pipes in Fig. 4.13, the nine equations (six linear and three nonlinear) must be solved.

Many variations of methods are available for solving the system of equations representing conservation of mass and energy in a pipe network. The Hardy Cross and linear methods are presented here. The Hardy Cross method traditionally included in most texts since the 1940's is amiable to manual computations as well as computer programs. The alternative more computationally efficient linear method is incorporated in the widely applied computer models discussed in Section 4.7. The linear method is a very stable numerical procedure that is very effective in computer modeling of pipe systems, including complex networks with thousands of pipes. Both the linear and Hardy Cross methods compute the discharge in each pipe of a network.

After the flows have been computed for all pipes in the network, the elevation of the hydraulic grade line and the pressure are computed for each junction node. The hydraulic grade line elevation for any junction node is equal to the elevation of a fixed-grade node minus the algebraic sum of headlosses in the pipes connecting the fixed-grade node and the junction node. The headloss in a pipe is considered positive if the flow in the pipe is away from the fixed-grade node and toward the junction node. The pressure (P) at the junction node (at ground elevation) is computed as

$$P = (El_{HGL} - El_{GD})\gamma_\omega \tag{4.63}$$

where El_{HGL} is the elevation of the HGL at the junction node, El_{GD} is the ground elevation at the junction node, and γ_ω is the unit weight of water.

4.5.1 Hardy Cross Method

The Hardy Cross method of pipe network analyses requires an initial estimate of flow in each pipe so that the continuity equation for the junction nodes are satisfied. The loop (both closed and pseudo) equations are solved iteratively one at a time until the correction for each loop is within an acceptable magnitude. When the loop equations are solved simultaneously, it is generally referred to as the Newton–Raphson method of pipe network analysis.

If Q_i is the correct flow rate for pipe i and q_i is the assumed flow rate (or the flow rate from the previous iteration), then

$$Q_i = q_i + \Delta \tag{4.64}$$

where Δ is a correction term to be applied to all (N) pipes in the loop. The closed loop equation becomes

$$\sum_{i=1}^{N} K_i(q_i + \Delta)^n = 0 \tag{4.65}$$

Expanding

$$\sum_{i=1}^{N} K_i q_i^n + \sum_{i=1}^{N} nK_i\Delta q_i^{n-1} + \frac{n-1}{2}\sum_{i=1}^{N} nK_i\Delta^2 q_i^{n-2} + \cdots = 0 \tag{4.66}$$

Using only the first two terms in the binomial expansion and solving for Δ yields

$$\Delta = -\frac{\sum_{i=1}^{N} K_i q_i^n}{\sum_{i=1}^{N} |nK_i q_i^{n-1}|} \tag{4.67}$$

for the closed loops and

$$\Delta = -\frac{\sum_{i=1}^{N} K_i q_i^n - (El_B - El_A)}{\sum_{i=1}^{N} |nK_i q_i^{n-1}|} \tag{4.68}$$

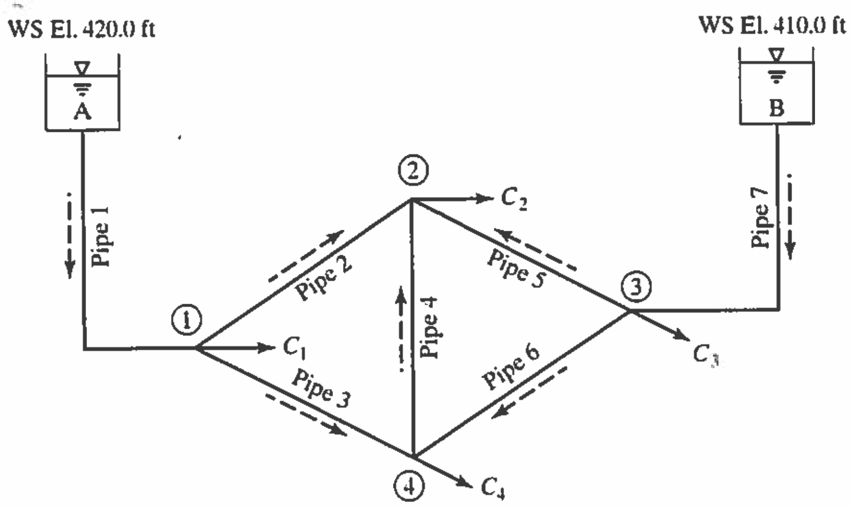
for the pseudo-loop.

The flow (q) and headloss term ($H_L = Kq^n$) for each pipe is considered positive if the flow is in the clockwise direction around the loop. Each term in the denominator can be considered as $n \times H_L/q$ and is always positive. The same correction term (Δ) is applied to all pipes in a loop. A positive correction term is added to the flow in all pipes that have flow in a clockwise direction around the loop and subtracted from the flow in all pipes that have flow in the counter clockwise direction around the loop. The continuity equations remain in balance after the flow in the pipes for a loop have been corrected.

Because only the first two terms were used in the binomial expansion of the headloss equation and because pipes that are in more than one loop have multiple corrections, the process is iterative. After applying one iterative correction to all loops, the process is repeated until convergence is achieved.

Example 4.14 Hardy Cross Pipe Network Problem

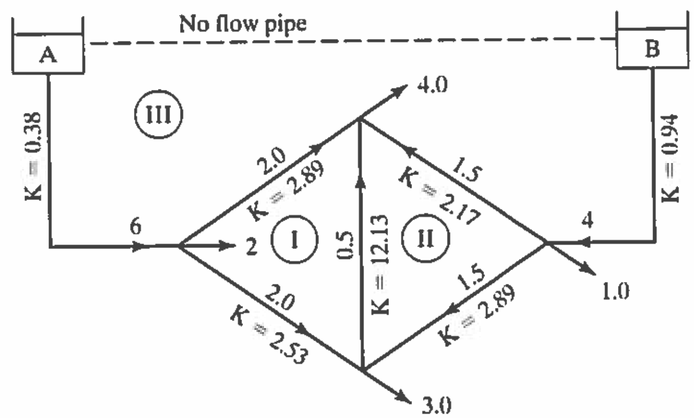
Determine the flow rate in each line and the pressure at each junction node for the pipe network in the sketch below using the Hardy Cross method of analysis. The pipe and junction data are listed below. The pipe area and headloss K values have also been computed and are listed in the table below.



Line	Nodes	Length ft	Diameter in.	f	A ft^2	K
1	A–1	1,000	12	0.015	0.78	0.38
2	1–2	800	8	0.019	0.35	2.89
3	1–4	700	8	0.019	0.35	2.53
4	4–2	750	6	0.020	0.196	12.13
5	3–2	600	8	0.019	0.35	2.17
6	3–4	800	8	0.019	0.35	2.89
7	B–3	900	10	0.017	0.55	0.94

Junction	Elevation ft	Demand cfs
1	320	2.0
2	330	4.0
3	310	1.0
4	300	3.0
	Total Demand	10.0

The Hardy Cross method requires an initial estimate of flow in each pipe such that the continuity equation is satisfied for each junction node. The estimated flows are shown below along with the K values for each pipe and the three loops.



For the two closed loops (Eq. 4.67)

$$\Delta_I = -\frac{\Sigma K q_i^n}{n\Sigma|Kq^{n-1}|} = -\frac{2.89 \times 2.0^2 - 12.13 \times 0.5^2 - 2.53 \times 2.0^2}{2(2.89 \times 2.0 + 12.13 \times 0.5 + 2.53 \times 2.0)} = +0.05$$

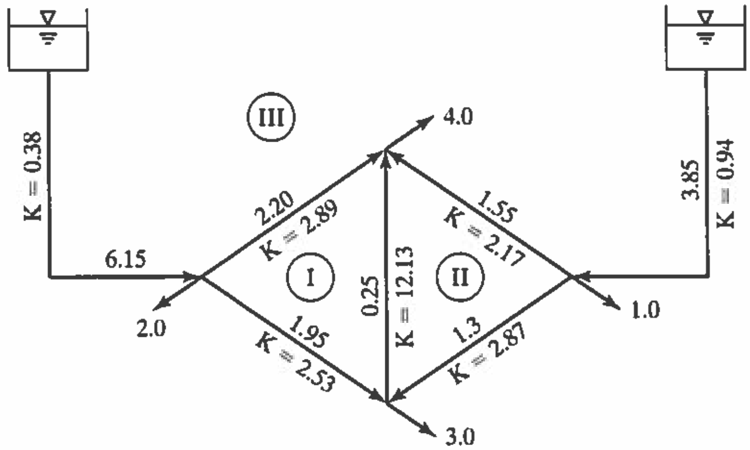
$$\Delta_{II} = -\frac{-2.17 \times 1.5^2 + 2.89 \times 1.5^2 + 12.13 \times 0.5^2}{2(2.17 \times 1.5 + 2.89 \times 1.5 + 12.13 \times 0.5)} = -0.2$$

For the pseudo-loop (Eq. 4.68)

$$\Delta_{III} = -\frac{\Sigma K q_i^n - El_B + El_A}{n\Sigma|Kq_i^{n-1}|}$$

$$= -\frac{0.94 \times 4^2 + 2.17 \times 1.5^2 - 2.89 \times 2.0^2 - 0.38 \times 6.0^2 + 10.0}{2(0.94 \times 4.0 + 2.17 \times 1.5 + 2.89 \times 2.0 + 0.38 \times 6.0)} = -0.15$$

The adjusted flows for each line are shown below. Lines 2, 4, and 5 are included in two loops and are adjusted twice. After the flows are adjusted, the continuity equation remains satisfied at each junction node.



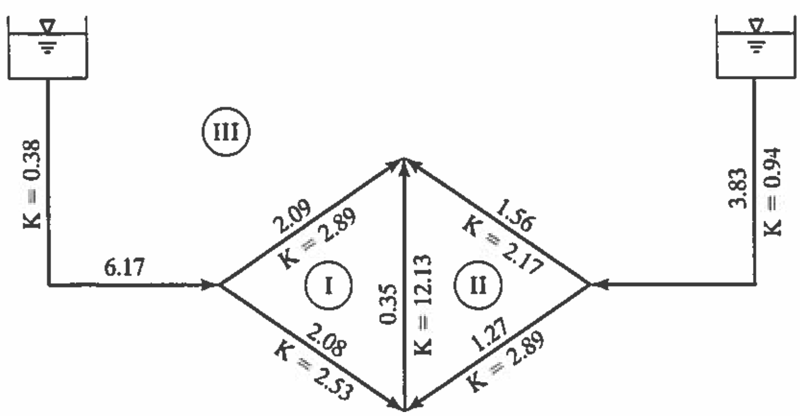
The correction terms for the second iteration are

$$\Delta_I = -\frac{2.89 \times 2.2^2 - 12.13 \times 0.25^2 - 2.53 \times 1.95^2}{2(2.89 \times 2.2 + 12.13 \times 0.25 + 2.53 \times 1.95)} = -0.13$$

$$\Delta_{II} = -\frac{-2.17 \times 1.55^2 + 2.89 \times 1.3^2 + 12.13 \times 0.25^2}{2(2.17 \times 1.55 + 2.89 \times 1.3 + 12.13 \times 0.25)} = -0.03$$

$$\Delta_{III} = -\frac{0.94 \times 3.85^2 + 2.17 \times 1.55^2 - 2.89 \times 2.20^2 - 0.38 \times 6.15^2 + 10}{2(0.94 \times 3.85 + 2.17 \times 1.55 + 2.89 \times 2.20 + 0.38 \times 6.15)} = -0.02$$

The adjusted flow rates for the second iteration are shown below.



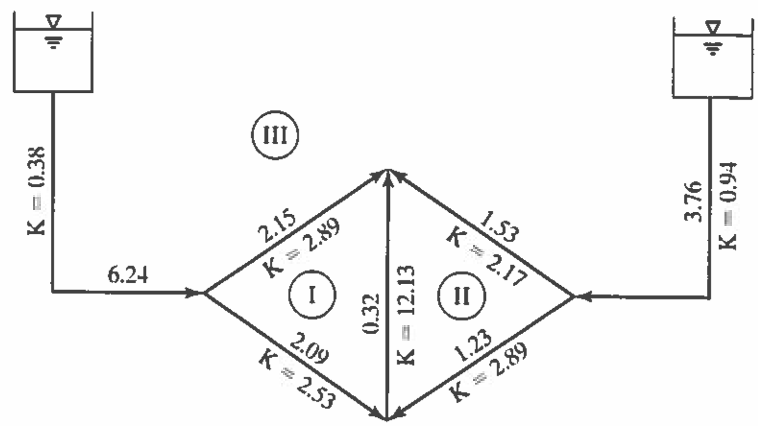
The correction terms for the third iteration are

$$\Delta_I = -\frac{2.89 \times 2.09^2 - 12.13 \times 0.35^2 - 2.53 \times 2.08^2}{2(2.89 \times 2.09 + 12.13 \times 0.35 + 2.53 \times 2.08)} = -0.01$$

$$\Delta_{II} = -\frac{-2.17 \times 1.56^2 + 2.89 \times 1.27^2 + 12.13 \times 0.35^2}{2(2.17 \times 1.56 + 2.89 \times 1.27 + 12.13 \times 0.35)} = -0.04$$

$$\Delta_{III} = -\frac{0.94 \times 3.83^2 + 2.17 \times 1.56^2 - 2.89 \times 2.09^2 - 0.38 \times 6.17^2 + 10}{2(0.94 \times 3.83 + 2.17 \times 1.56 + 2.89 \times 2.09 + 0.38 \times 6.17)} = -0.07$$

The adjusted flow rates for the third iteration are shown below



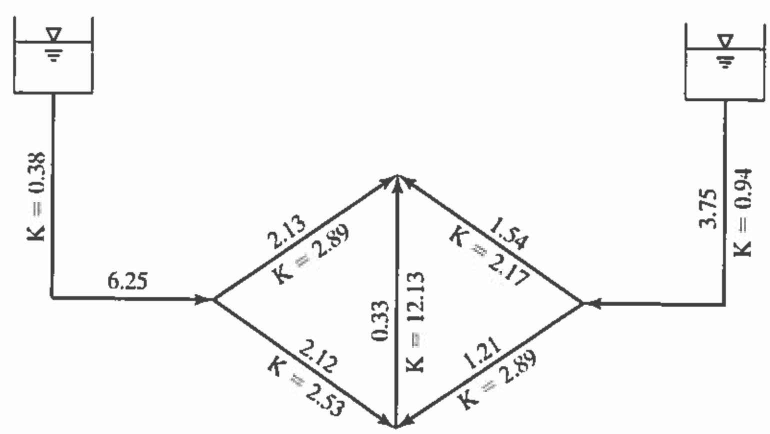
The correction terms for the fourth iteration are

$$\Delta_I = -\frac{2.89 \times 2.15^2 - 12.13 \times 0.32^2 - 2.53 \times 2.09^2}{2(2.89 \times 2.15 + 12.13 \times 0.32 + 2.53 \times 2.09)} = -0.03$$

$$\Delta_{II} = -\frac{-2.17 \times 1.53^2 + 2.89 \times 1.23^2 + 12.13 \times 0.32^2}{2(2.17 \times 1.53 + 2.89 \times 1.23 + 12.13 \times 0.32)} = -0.02$$

$$\Delta_{III} = -\frac{0.94 \times 3.76^2 + 2.17 \times 1.53^2 - 2.89 \times 2.15^2 - 0.38 \times 6.24^2 + 10}{2(0.94 \times 3.76 + 2.17 \times 1.53 + 2.89 \times 2.15 + 0.38 \times 6.24)} = -0.01$$

The adjusted flow rates for the fourth iteration are shown below.



The correction terms for the fifth iteration are

$$\Delta_I = -\frac{2.89 \times 2.13^2 - 12.13 \times 0.33^2 - 2.53 \times 2.12^2}{2(2.89 \times 2.13 + 12.13 \times 0.33 + 2.53 \times 2.12)} = -0.01$$

$$\Delta_{II} = -\frac{-2.17 \times 1.54^2 + 2.89 \times 1.21^2 + 12.13 \times 0.33^2}{2(2.17 \times 1.54 + 2.89 \times 1.21 + 12.13 \times 0.33)} = -0.02$$

$$\Delta_{III} = -\frac{0.94 \times 3.75^2 + 2.17 \times 1.54^2 - 2.89 \times 2.13^2 - 0.38 \times 6.25^2 + 10}{2(0.94 \times 3.75 + 2.17 \times 1.54 + 2.89 \times 2.13 + 0.38 \times 6.25)} = -0.01$$

The final flow rates are listed below along with the headloss for each line.

Line	Nodes	K	Q cfs	H_L ft
1	A–1	0.38	6.26	14.9
2	1–2	2.89	2.13	13.1
3	1–4	2.53	2.13	11.5
4	4–2	12.13	0.32	1.2
5	3–2	2.17	1.55	5.2
6	3–4	2.89	1.19	4.1
7	B–3	0.94	3.74	13.1

The pressure at each node is computed in the table below.

Node	Elevation ft	Line	H_L ft	HGL ft	P/γ ft	P psi
A	420			420.0		
		A–1	−14.9			
1	320			405.1	85.1	37
		1–2	−13.1			
2	330			392.0	62.0	27
		2–3	+5.2			
3	310			397.2	87.2	38
		3–4	−4.1			
4	300			393.1	93.1	40
		3–B	+13.1			
B	410			410.3		

4.5.2 Linear Method

A major difficulty with the Hardy Cross method of pipe network analysis is that the method requires an initial estimate of flow in each pipe. The initial estimates of flow must be reasonably accurate or the Hardy Cross method will not converge to a solution. With the linear method of analysis, all the equations (both continuity and loop equations) are solved simultaneously. This is a very stable numerical procedure that does not require the user to provide initial estimates of flow and will nearly always converge to a solution. The nonlinear loop equations are linearized, and all equations are solved iteratively. The user must specify the direction of flow in each line. If the computed flow rate is negative, the direction of flow in that line is reversed. For pipes with pumps, check valves, or pressure regulating valves, the flow direction cannot be reversed, and if the computed flow rate is negative the flow rate must be set equal to zero.

Each nonlinear headloss term $[f(Q)]$ in the loop equations is linearized using the first two terms in the Taylor series

$$f(Q) = f(q) + \frac{\partial f}{\partial q}(Q - q) \tag{4.69}$$

where q is the flow rate from the previous iteration and Q is the unknown flow rate. Each pipe friction term (KQ^n) in the loop equations is replaced with the linear term

$$Kq^n + nKq^{n-1}(Q - q) \tag{4.70}$$

or

$$DQ + D' \tag{4.71}$$

where

$$D = nKq^{n-1}$$

$$D' = (1 - n)Kq^n$$

and for loops with pumps the nonlinear pump characteristic equation ($AQ^2 + BQ + H_c$) is replaced with the linear equation

$$Aq^2 + Bq + H_c + (2Aq + B)(Q - q) \tag{4.72}$$

or

$$EQ + E' \tag{4.73}$$

where

$$E = 2Aq + B$$

$$E' = H_c - Aq^2$$

The nonlinear loop equations (Eqs. 4.60 and 4.62) in the example problem become

$$D_2Q_2 + D_6Q_6 - D_8Q_8 - D_5Q_5 = -D_2' - D_6' + D_8' + D_5' \tag{4.74}$$

$$-D_3Q_3 + D_7Q_7 + D_9Q_9 - D_6Q_6 = D'_3 - D'_7 - D'_9 + D'_6 \quad (4.75)$$

and

$$D_4Q_4 + D_3Q_3 - D_2Q_2 - D_1Q_1 = -D'_4 - D'_3 + D'_2 + D'_1 + El_B - El_A \quad (4.76)$$

In matrix notation, the set of linear equations are

$$\begin{array}{c|c} & \text{Pipe} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \\ \text{Equation number} & \begin{matrix}1\\2\\3\\4\\5\\6\\7\\8\\9\end{matrix} \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & D_2 & 0 & 0 & -D_5 & D_6 & 0 & -D_8 & 0 \\ 0 & 0 & -D_3 & 0 & 0 & -D_6 & D_7 & 0 & D_9 \\ -D_1 & -D_2 & D_3 & D_4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{bmatrix} \end{array} \quad (4.77)$$

where

$$C_1 - C_6 = \text{Demands at junctions}$$

$$D = nKq^{n-1}$$

$$C_7 = -D'_2 - D'_6 + D'_8 + D'_5$$

$$C_8 = D'_3 - D'_7 - D'_9 + D'_6$$

$$C_9 = -D'_4 - D'_3 + D'_2 + D'_1 + El_B - El_A$$

and

$$D' = (1 - n)Kq^n$$

Equation 4.77 can be written as

$$[A][Q] = [C] \quad (4.78)$$

where $[A]$ is the coefficient matrix and $[C]$ is a column vector of constants.

Solving for the new flow rates

$$[Q] = [A]^{-1}[C] \quad (4.79)$$

where $[A]^{-1}$ is the inverse of coefficient matrix.

For the initial iteration, the value of q for each pipe can be computed based on a velocity of 1 mps (3 fps). The flow rates (Q) computed from Eq. 4.79 became the estimated flow rates (q) for the next iteration. This iterative process is continued until the equations converge to a solution. Convergence is assumed to occur when the maximum change $|Q - q|$ is within a specified limit or the relative accuracy

$$\frac{\sum_{i=1}^{N} |Q_i - q_i|}{\sum_{i=1}^{N} |Q_i|} \tag{4.80}$$

is within a specified value.

Typically the demands specified at the junction nodes represent the average daily demand on the network systems and the model can be rerun using demand peaking factors to represent the peak daily demand on the system or the peak hourly demand on the system. Fire flows can be added at selected junction nodes to determine if the system can provide the fire demand at the required pressure.

Pressure-regulating valves (PRVs) are designed to maintain a specified discharge pressure (P_{RV}), which is lower than the upstream pressures. As shown in Fig. 4.14, the PRV can be modeled as a fixed-grade node located downstream of a junction node. The elevation of the fixed-grade node is equal to the ground elevation of the upstream junction node plus the PRV pressure head setting (P_{RV}/γ_w). The computed flow rate in the downstream pipe is added to the demand at the upstream junction node. If the computed flow in the downstream pipe is negative, the pipe is closed and the flow through the valve is zero. If the computed pressure at the upstream junction node is less than P_{RV}, the PRV is fully open and the system is analyzed without the PRV.

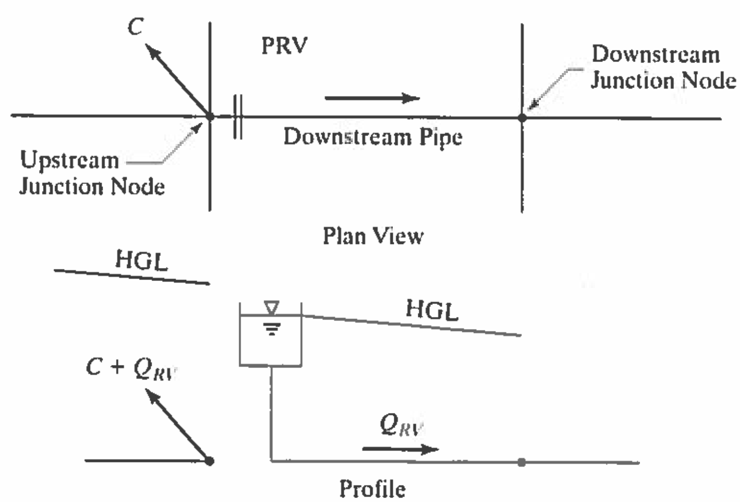
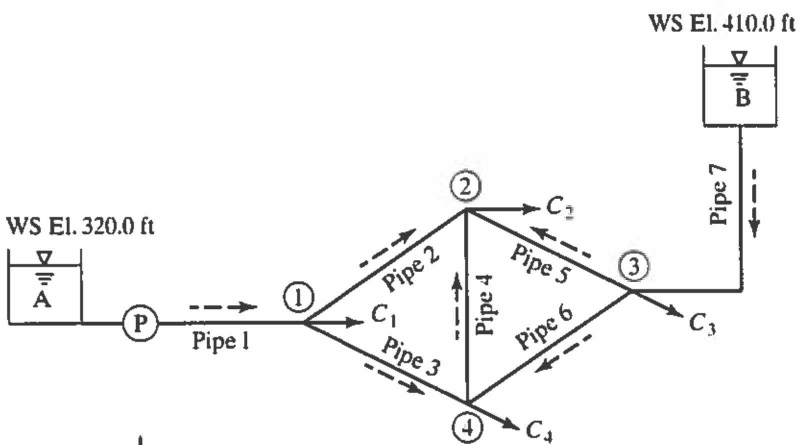
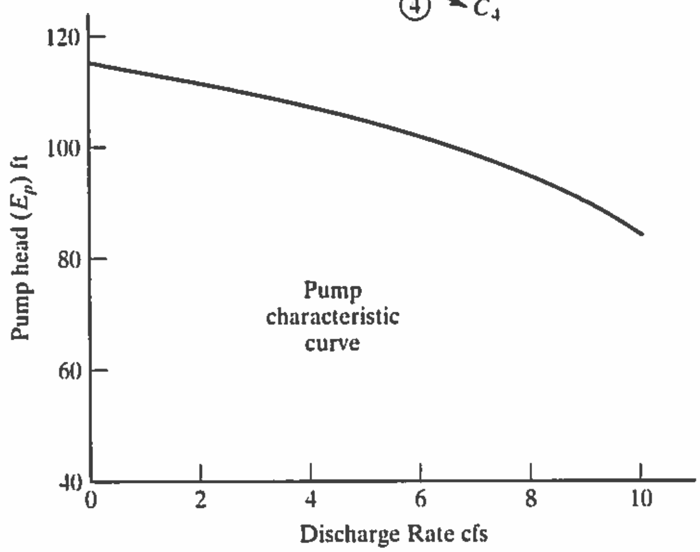


Figure 4.14 Modeling a pressure regulating valve.

Example 4.15 Linear Method of Pipe Network Analysis

For this problem, Example 4.14 was modified by changing the elevated storage tank at A to a ground-level storage tank and adding a pump in line A–1. The pump characteristic curve is shown below. The pipe and junction data for this problem are the same as Example 4.14. Determine the flow rate in each line and the pressure at each junction node using the linear method of pipe network analysis.





The number of pipes in the system ($N_P = 7$) must equal the number of junction nodes ($N_j = 4$) plus the number of loops ($N_l = 2$) plus the number of fixed-grade nodes ($N_f = 2$) minus 1. The four continuity (node) equations are

$$(1)\quad Q_1 - Q_2 - Q_3 = C_1$$

$$(2)\quad Q_2 + Q_4 + Q_5 = C_2$$

$$(3)\quad Q_7 - Q_5 - Q_6 = C_3$$

$$(4)\quad Q_3 - Q_4 + Q_6 = C_4$$

The two loop equations are

$$(5)\quad K_2Q_2^2 - K_4Q_4^2 - K_3Q_3^2 = 0$$

$$(6)\quad K_4Q_4^2 - K_5Q_5^2 + K_6Q_6^2 = 0$$

The one pseudo-loop equation is

$$(7)\quad K_7Q_7^2 + K_5Q_5^2 - K_2Q_2^2 - K_1Q_1^2 + AQ_1^2 + BQ_1 + H_C = 410 - 320$$

The three linearized energy equations are

$$(5)\quad D_2Q_2 - D_3Q_3 - D_4Q_4 = -D_2^1 + D_3^1 + D_4^1$$

$$(6)\quad D_4Q_4 - D_5Q_5 + D_6Q_6 = -D_4^1 + D_5^1 - D_6^1$$

$$(7)\quad D_7Q_7 + D_5Q_5 - D_2Q_2 - D_1Q_1 + E_1Q_1 = 410 - 320 - D_7^1 - D_5^1 + D_2^1 + D_1^1 - E_1^1$$

where

$$D = 2Kq$$

$$D^1 = -Kq^2$$

$$E = 2Aq + B$$

$$E^1 = H_c - Aq^2$$

$$q = \text{previous estimate of flow}$$

A computer program was written in FORTRAN to solve pipe networks using the linear method of analysis and is included in the HMP (Section 1.4.5) as NETWORK.FOR. The source code is available so students can modify the program as needed for other applications. The program was dimensioned for 50 pipes, 25 junction nodes, and 10 pumps. Comments are included in the source code explaining each section of the program. The program input file (NETWORK.DAT) and output file (NETWORK.OUT) are listed on the next page.

The first line of the input file is the project title (Ex. 4.15 pipe network). Line two includes the NAMELIST/NET/ and includes units (English), number of pipes (7), number of junction nodes (4), number of loops (2), number of fixed-grade nodes (2), number of pumps (1), and global demand factor (1.00). Pipe data follows the NAMELIST and is entered in the same order the pipes are numbered. Pipe data include pipe number upstream and downstream node numbers, pipe length (ft), pipe diameter (in.), and the Darcy–Weisbach friction factor and minor loss coefficient. If either node number is zero, indicating a fixed-grade node, the elevation of the fixed-grade node is entered on the following line. The junction data follow the pipe data and include the junction node number, the ground elevation (ft), and the demand (gpm). Pump data follow the junction data and include pipe number, cut-off head (ft), followed by the discharge (gpm) and head (ft) for two additional points on the pump characteristic curve. Loop data follow the pump data and include number of pipes in loop followed by pipe numbers in loop (clockwise plus). Pseudo-loops are listed last with pipe numbers starting with first pipe clockwise past the imaginary no-flow pipe.

```
EXAMPLE 4.15 PIPE NETWORK
&NET  IUN = 2,   NP = 7,    NJ = 4,   NL = 2,   NF = 2,   NPU = 1,   GDF = 1.0/
  1     0     1    1000.0       12.0      .015        .5
   320.0
  2     1     2     800.0        8.0      .019        .0
  3     4     1     700.0        8.0      .019        .0
  4     4     2     750.0        6.0      .020        .0
  5     3     2     600.0        8.0      .019        .0
  6     3     4     800.0        8.0      .019        .0
  7     3     0     900.0       10.0      .017        .5
   410.0
  1        320.0    896.0
  2        330.0   1792.0
  3        310.0    448.0
  4        300.0   1344.0
  1        115.0   2240.0      105.0    4480.0      85.0
  3
  2    -4     3
  3
 -5     6     4
  4
 -7     5    -2    -1
```

RESULTS

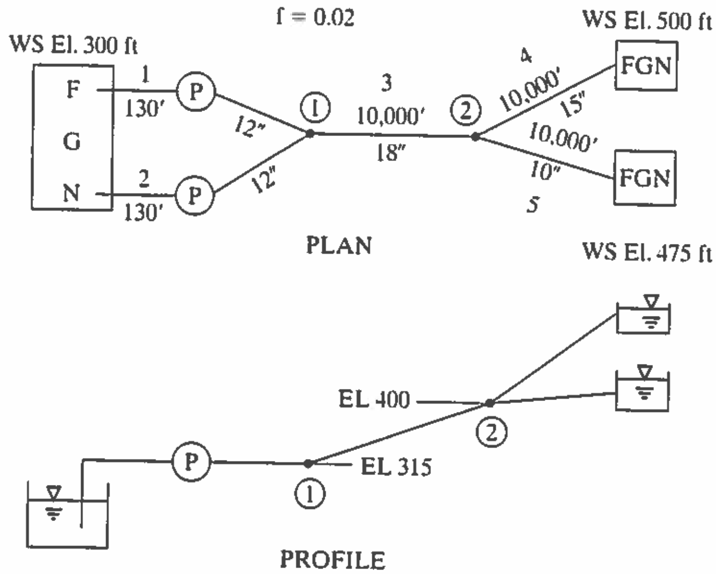
PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	0	1	1000.00	12.00	2835.22	8.06	15.63
2	1	2	800.00	8.00	961.66	6.15	13.39
3	4	1	700.00	8.00	-977.56	-6.25	-12.11
4	4	2	750.00	6.00	146.00	1.66	1.28
5	3	2	600.00	8.00	684.33	4.38	5.08
6	3	4	800.00	8.00	512.45	3.28	3.80
7	3	0	900.00	10.00	-1644.78	-6.73	-13.27

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	896.00	320.00	405.03	36.85
2	1792.00	330.00	391.65	26.71
3	448.00	310.00	396.73	37.58
4	1344.00	300.00	392.93	40.27

Example 4.16 Branching Pipeline

Compute the discharge in each line for the branching pipeline. There are 5 pipes, no loop, 2 pumps, and 4 fixed-grade nodes in the system. Use a friction factor of 0.02 for all lines.



Pump 1 E_p ft	Q cfs
400	0
350	6
250	12

Pump 2 E_p ft	Q cfs
400	0
350	6
250	12

The input file and a summary of the output file are listed below.

```
EXAMPLE 4.16  PIPELINE BRANCHING NETWORK
&NET  IUN = 2,    NP = 5,    NJ = 2,   NL = 0,   NF = 4,    NPU = 2,   GDF = 1.0/
   1    0     1     130.0      12.0      .020      1.0
      300.0
   2    0     1     130.0      12.0      .020      1.0
      300.0
   3    1     2   10000.0      18.0      .020       .0
   4    2     0   10000.0      15.0      .020      1.0
      500.0
   5    2     0   10000.0      10.0      .020      1.0
      475.0
   1        315.0     0.0
   2        400.0     0.0
   1        400.0  2688.0     350.0    5376.0     250.0
   2        400.0  2688.0     350.0    5376.0     250.0
   2
   1   -2
   3
  -4   -3    -1
   2
  -5    4
```

RESULTS

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	0	1	130.00	12.00	2301.20	6.54	2.39
2	0	1	130.00	12.00	2301.20	6.54	2.39
3	1	2	10000.00	18.00	4602.39	5.81	69.97
4	2	0	10000.00	15.00	3260.21	5.93	87.91
5	2	0	10000.00	10.00	1342.19	5.49	112.91

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	.00	315.00	657.88	148.58
2	.00	400.00	587.91	81.43

Example 4.17(a) Irrigation System

A 500-ac farm is to be irrigated with a sprinkler system. The crop requires 12.0 inches of water in July of which 2.0 inches is provided by rain and 10.0 inches is to be provided by the sprinkler system. Determine the water supply flow rate if the sprinkler system is 80 percent efficient in delivering water to the crop and the operation requires a 20 percent peaking factor.

$$\text{Discharge rate} = \text{Volume/time}$$

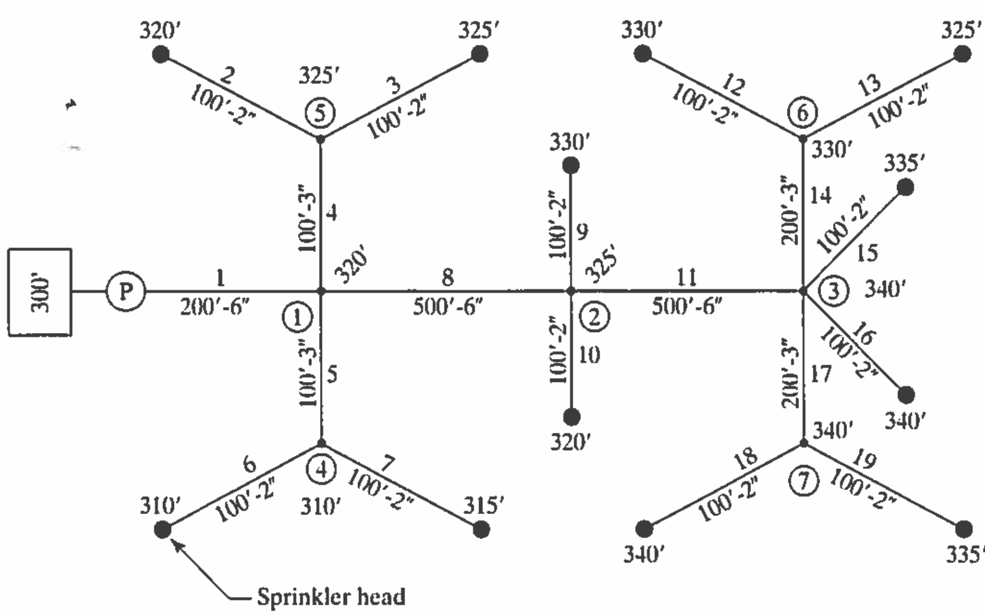
$$\text{Volume} = 500 \text{ ac} \times 43{,}560 \frac{\text{ft}^2}{\text{ac}} \times \frac{10}{12} \text{ ft} \times \frac{1}{0.80} \times 1.20$$

$$= 27.2 \times 10^6 \text{ ft}^3$$

$$Q = \frac{27.2 \times 10^6}{31 \times 86{,}400} = 10.2 \text{ cfs}$$

Example 4.17(b) Golf Course Sprinkler System

A golf course is to be watered by pumping from a lake (water surface elevation 300 ft) into a branching pipe network system shown below. The ground elevation varies from



310 to 340 ft. In this 19-pipe system, there are no demands at the junctions and all the flow is through the sprinkler heads. The sprinkler heads are modeled as a fixed-grade node at ground level plus a minor loss. For the example problem, the sprinkler heads were selected for a discharge of 75 gpm at a head of 50 ft. All lines connected to sprinkler heads are 2 inches in diameter. At the design discharge of 75 gpm, the velocity in the 2-inch pipe is 7.68 fps. The minor loss coefficient (M) for a head of 50 ft is

$$50 = M\frac{V^2}{2g} = M\frac{7.68^2}{64.4}$$

$$M = 55$$

This minor loss coefficient is included in all lines connected to a sprinkler head. The pump has the following characteristics

Head ft	Discharge cfs
200	0
175	1
125	2

Compute the flow at each sprinkler head. In this problem, there are 19 pipes, 7 junction nodes, 0 loops, and 13 fixed-grade nodes.

$$N_p = N_j + N_l + N_f - 1$$

$$19 = 7 + 0 + 13 - 1$$

A summary of the output file is listed below.

RESULTS

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	0	1	200.00	6.00	946.83	10.76	10.79
2	5	0	100.00	2.00	88.31	9.04	81.13
3	5	0	100.00	2.00	85.55	8.75	76.13
4	1	5	100.00	3.00	173.86	7.91	5.82
5	1	4	100.00	3.00	183.86	8.36	6.51
6	4	0	100.00	2.00	93.24	9.54	90.44
7	4	0	100.00	2.00	90.63	9.27	85.44
8	1	2	500.00	6.00	589.11	6.70	10.45
9	2	0	100.00	2.00	79.96	8.18	66.51
10	2	0	100.00	2.00	85.76	8.77	76.51
11	2	3	500.00	6.00	423.40	4.81	5.40
12	6	0	100.00	2.00	71.31	7.30	52.91
13	6	0	100.00	2.00	74.61	7.63	57.91
14	3	6	200.00	3.00	145.92	6.64	8.20
15	3	0	100.00	2.00	73.44	7.51	56.11

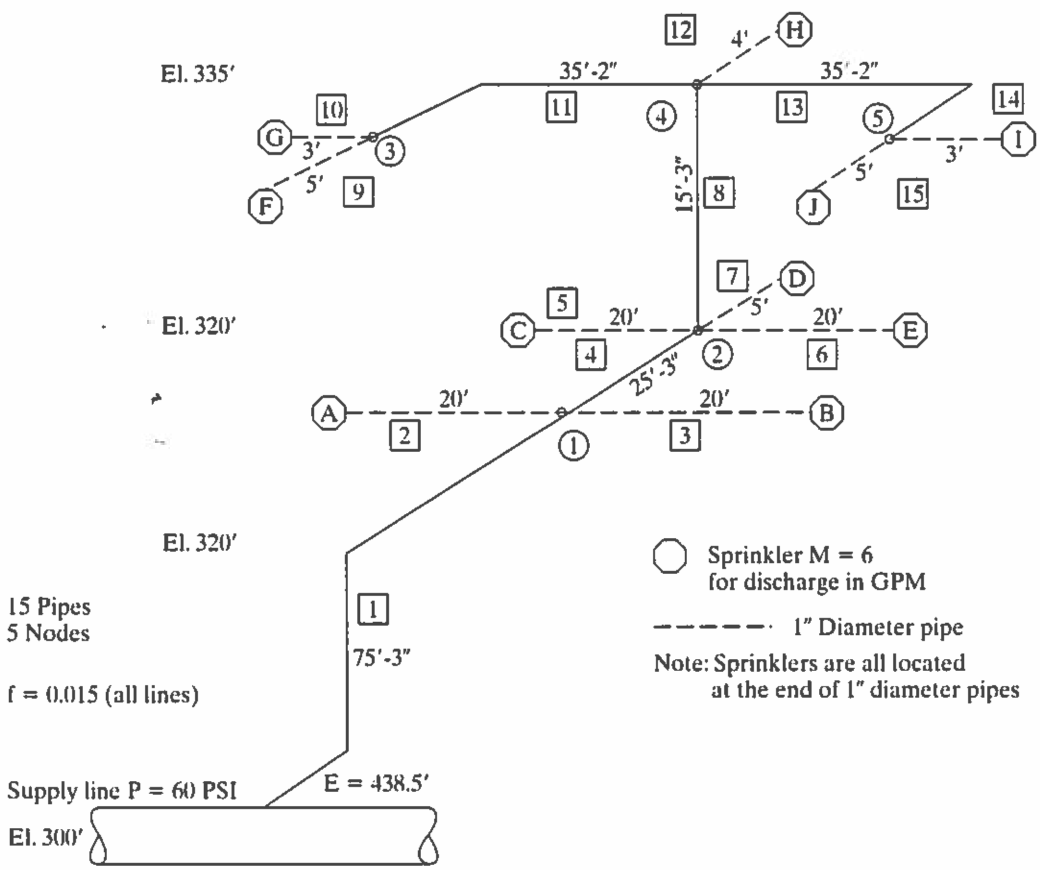
16	3	0	100.00	2.00	70.09	7.17	51.11
17	3	7	200.00	3.00	133.95	6.09	6.91
18	7	0	100.00	2.00	65.18	6.67	44.20
19	7	0	100.00	2.00	68.77	7.04	49.20

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	.00	320.00	406.95	37.68
2	.00	325.00	396.51	30.99
3	.00	340.00	391.11	22.15
4	.00	310.00	400.44	39.19
5	.00	325.00	401.13	32.99
6	.00	330.00	382.91	22.93
7	.00	340.00	384.20	19.15

Example 4.17(c) Building Sprinkler System

The 15-pipe branching system represents a 2-story building sprinkler system. Water supply is from a main pipeline with a total head of 438.5 ft. Sprinkler heads are all located at the end of a 1.0-inch diameter pipe and have a minor loss coefficient (M) equal to 6.0. Compute the discharge through each sprinkler head.



RESULTS

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	0	1	75.00	3.00	536.40	24.39	41.57
2	1	0	20.00	1.00	55.51	22.72	76.93
3	1	0	20.00	1.00	55.51	22.72	76.93
4	1	2	25.00	3.00	425.38	19.34	8.71
5	2	0	20.00	1.00	52.27	21.39	68.21
6	2	0	20.00	1.00	52.27	21.39	68.21
7	2	0	5.00	1.00	61.65	25.23	68.21
8	2	4	15.00	3.00	259.19	11.79	1.94
9	3	0	5.00	1.00	50.57	20.70	45.89
10	3	0	3.00	1.00	51.94	21.26	45.89
11	4	3	35.00	2.00	102.51	10.49	5.38
12	4	0	4.00	1.00	54.16	22.17	51.27
13	4	5	35.00	2.00	102.51	10.49	5.38
14	5	0	3.00	1.00	51.94	21.26	45.89
15	5	0	5.00	1.00	50.57	20.70	45.89

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	.00	320.00	396.93	33.34
2	.00	320.00	388.21	29.56
3	.00	335.00	380.89	19.89
4	.00	335.00	386.27	22.22
5	.00	335.00	380.89	19.89

Example 4.18(a) Municipal Water Supply Rate

Determine the water supply requirements for a city of 25,000 people if the average daily demand is 190 gpcd. The average daily demand (Q_a) is

$$Q_a = 25{,}000 \times 190/1440 = 3{,}300 \text{ gpm}$$

The water supply treatment plant is to be designed for the maximum daily demand. Determine the size of the supply line to the treatment plant, if the maximum daily demand is twice the average daily demand and the maximum velocity in the pipe is 5.0 fps. The area of the pipe is

$$A = \frac{Q}{V} = 6{,}600\,\frac{\text{gal}}{\text{min}}\,\frac{1\text{ ft}^3}{7.48\text{ gal}} \times \frac{1\text{ min}}{60\text{ sec}} \times \frac{1\text{ sec}}{5\text{ ft}} = 2.95\text{ ft}^2$$

$$D = \left(\frac{A \times 4}{\pi}\right)^{1/2} = 1.93\text{ ft}$$

Use a 24-inch diameter pipe.

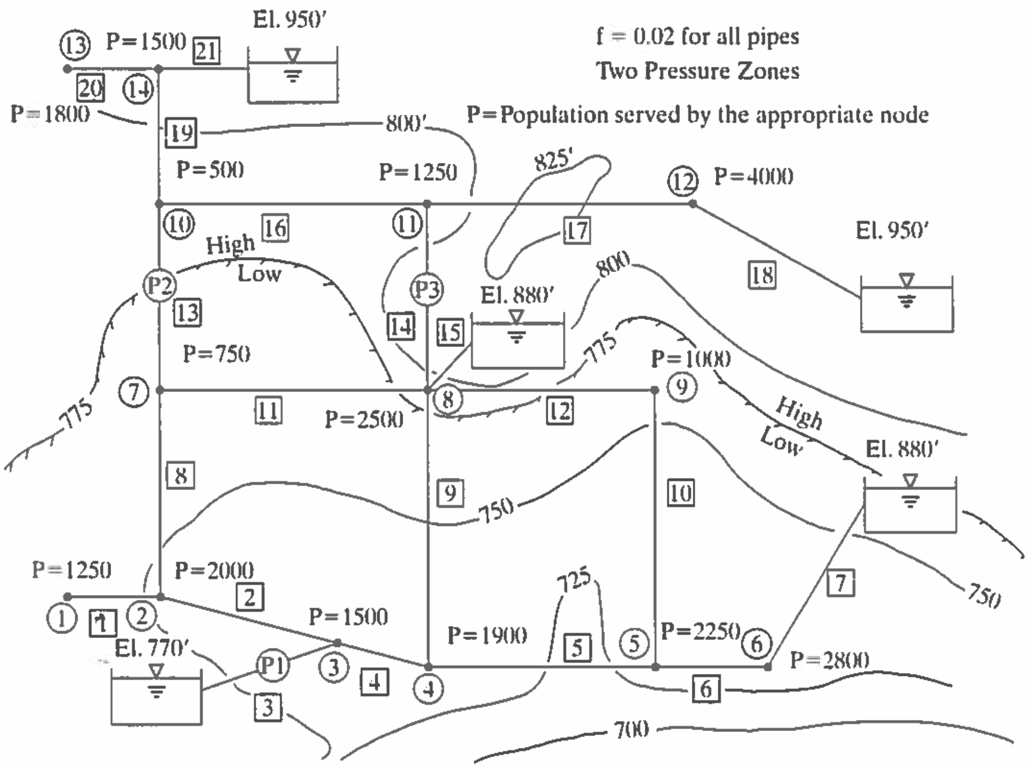
The water distribution system should be sized for the peak hourly demand with a minimum pressure of 35 psi. Determine the maximum hourly demand (Q_h) for the distribution system of the maximum hourly demand is three times the average daily demand

$$Q_h = Q_a \times 3.0 = 9{,}900 \text{ gpm}$$

Example 4.18(b) Municipal Water Distribution System

The water distribution system for Eagle Pass, Texas, is shown on the sketch. The low pressure zone has ground elevations ranging from 700 to 775 ft msl while the high pressure zone has ground elevations ranging from 800 to 825 ft msl. Contours showing the ground elevations are included in the sketch. Pump 1 pumps water from the ground-level storage into the lower pressure zone, whereas booster pumps 2 and 3 pump water from the low pressure zone to the high pressure zone. There are four elevated storage tanks (2 in each zone) in the system. Compute pressures in the system for

average daily demand (GDF = 1.0),
peak day demand (GDF = 2.0), and
peak hour demand (GDF = 3.0).



Pump Data

Pump 1		Pump 2		Pump 3	
E_p ft	Q cfs	E_p ft	Q cfs	E_p ft	Q cfs
200	0.0	150	0.0	160	0.0
130	20.0	90	1.1	100	3.3
80	25.0	60	1.6	60	4.5

Node	Population	Demand (gpm)	Elevation
1	1,250	166	760
2	2,000	264	749
3	1,500	197	735
4	1,900	250	730
5	2,250	296	726
6	2,800	367	726
7	750	98	765
8	2,500	332	787
9	1,000	130	760
10	500	67	785
11	1,250	166	790
12	4,000	529	815
13	1,800	237	805
14	1,500	197	807

Line	Diameter (in)	Length (ft)
1	12	1,400
2	14	1,300
3	14	700
4	14	1,500
5	12	2,000
6	12	1,000
7	10	2,500
8	12	2,000
9	12	3,000
10	12	3,000
11	12	2,600
12	10	2,000
13	12	1,100
14	10	1,100
15	10	400
16	10	2,600
17	10	3,400
18	10	1,800
19	10	1,400
20	10	900
21	10	550

Summaries of the output files are listed below. During the average demand, all elevated storage tanks are filling, and during the peak hourly demand, all elevated storage tanks are emptying. The minimum pressure during the peak hourly demand is 38.7 psi.

RESULTS (AVERAGE DAY)

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	2	1	1400.00	12.00	166.00	.47	.10
2	3	2	1300.00	14.00	2347.65	4.90	8.32
3	0	3	700.00	14.00	5897.95	12.32	28.26
4	3	4	1500.00	14.00	3353.29	7.00	19.58
5	4	5	2000.00	12.00	1633.35	4.64	13.38
6	5	6	1000.00	12.00	882.39	2.51	1.95
7	6	0	2500.00	10.00	515.39	2.11	4.15
8	2	7	2000.00	12.00	1917.65	5.45	18.45
9	4	8	3000.00	12.00	1469.94	4.18	16.26
10	5	9	3000.00	12.00	454.96	1.29	1.56
11	7	8	2600.00	12.00	1179.33	3.35	9.07
12	8	9	2000.00	10.00	-324.96	-1.33	-1.32
13	7	10	1100.00	12.00	640.32	1.82	1.13
14	8	11	1100.00	10.00	1506.21	6.16	15.58
15	8	0	400.00	10.00	1136.03	4.65	3.22
16	10	11	2600.00	10.00	-506.62	-2.07	-4.17
17	11	12	3400.00	10.00	833.59	3.41	14.75
18	12	0	1800.00	10.00	304.59	1.25	1.04
19	10	14	1400.00	10.00	1079.94	4.42	10.19
20	14	13	900.00	10.00	237.00	.97	.32
21	14	0	550.00	10.00	645.94	2.64	1.43

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	166.00	760.00	910.65	65.28
2	264.00	749.00	910.74	70.09
3	197.00	735.00	919.06	79.76
4	250.00	730.00	899.48	73.44
5	296.00	726.00	886.10	69.38
6	367.00	726.00	884.14	68.53
7	98.00	765.00	892.29	55.16
8	332.00	787.00	883.22	41.70
9	130.00	760.00	884.54	53.97
10	67.00	785.00	961.62	76.54
11	166.00	790.00	965.79	76.18
12	529.00	815.00	951.04	58.95
13	237.00	805.00	951.12	63.32
14	197.00	807.00	951.43	62.59

RESULTS (PEAK DAY)

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	2	1	1400.00	12.00	332.00	.94	.39
2	3	2	1300.00	14.00	2598.03	5.42	10.18
3	0	3	700.00	14.00	6272.02	13.10	31.96
4	3	4	1500.00	14.00	3279.99	6.85	18.73
5	4	5	2000.00	12.00	1509.48	4.29	11.43
6	5	6	1000.00	12.00	614.06	1.75	.95
7	6	0	2500.00	10.00	-119.94	-.49	-.22
8	2	7	2000.00	12.00	1738.03	4.94	15.15
9	4	8	3000.00	12.00	1270.51	3.61	12.15
10	5	9	3000.00	12.00	303.41	.86	.69
11	7	8	2600.00	12.00	921.46	2.62	5.54
12	8	9	2000.00	10.00	-43.41	-.18	-.02

13	7	10	1100.00	12.00	620.57	1.76	1.06
14	8	11	1100.00	10.00	1526.20	6.25	15.99
15	8	0	400.00	10.00	45.19	.18	.01
16	10	11	2600.00	10.00	−445.42	−1.82	−3.22
17	11	12	3400.00	10.00	748.78	3.06	11.90
18	12	0	1800.00	10.00	−309.22	−1.27	−1.07
19	10	14	1400.00	10.00	931.99	3.81	7.59
20	14	13	900.00	10.00	474.00	1.94	1.26
21	14	0	550.00	10.00	63.99	.26	.01

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	332.00	760.00	900.31	60.80
2	528.00	749.00	900.70	65.74
3	394.00	735.00	910.88	76.22
4	500.00	730.00	892.15	70.27
5	592.00	726.00	880.72	67.05
6	734.00	726.00	879.78	66.64
7	196.00	765.00	885.54	52.24
8	664.00	787.00	880.01	40.30
9	260.00	760.00	880.03	52.01
10	134.00	785.00	957.60	74.80
11	332.00	790.00	960.82	74.02
12	1058.00	815.00	948.93	58.03
13	474.00	805.00	948.75	62.29
14	394.00	807.00	950.01	61.97

RESULTS (PEAK HOUR)

PIPE	NODES		LENGTH	DIAMETER	DISCHARGE	VELOCITY	HEADLOSS
1	2	1	1400.00	12.00	498.00	1.42	.87
2	3	2	1300.00	14.00	2820.42	5.89	12.00
3	0	3	700.00	14.00	6624.81	13.83	35.65
4	3	4	1500.00	14.00	3213.40	6.71	17.98
5	4	5	2000.00	12.00	1408.38	4.00	9.95
6	5	6	1000.00	12.00	486.55	1.38	.59
7	6	0	2500.00	10.00	−614.45	−2.51	−5.89
8	2	7	2000.00	12.00	1530.42	4.35	11.75
9	4	8	3000.00	12.00	1055.02	3.00	8.38
10	5	9	3000.00	12.00	33.83	.10	.01
11	7	8	2600.00	12.00	631.38	1.79	2.60
12	8	9	2000.00	10.00	356.17	1.46	1.58
13	7	10	1100.00	12.00	605.04	1.72	1.01
14	8	11	1100.00	10.00	1555.27	6.37	16.61
15	8	0	400.00	10.00	−1221.05	−5.00	−3.72
16	10	11	2600.00	10.00	−298.32	−1.22	−1.44
17	11	12	3400.00	10.00	758.95	3.11	12.22
18	12	0	1800.00	10.00	−828.05	−3.39	−7.70
19	10	14	1400.00	10.00	702.35	2.87	4.31
20	14	13	900.00	10.00	711.00	2.91	2.84
21	14	0	550.00	10.00	−599.65	−2.45	−1.23

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	498.00	760.00	889.76	56.23
2	792.00	749.00	890.63	61.37
3	591.00	735.00	902.63	72.64

4	750.00	730.00	884.65	67.02
5	888.00	726.00	874.70	64.44
6	1101.00	726.00	874.11	64.18
7	294.00	765.00	878.88	49.35
8	996.00	787.00	876.28	38.69
9	390.00	760.00	874.69	49.70
10	201.00	785.00	953.08	72.83
11	498.00	790.00	954.52	71.29
12	1587.00	815.00	942.30	55.16
13	711.00	805.00	945.93	61.07
14	591.00	807.00	948.77	61.43

4.5.3 Design of Municipal Water Distribution Systems

As shown in the sketch of a water distribution system in Fig. 4.15, the major system components are the treatment unit, ground level storage, pumps, elevated storage, and pipe networks. The type of water treatment depends on the source of the water. Groundwater typically requires little treatment, whereas surface water

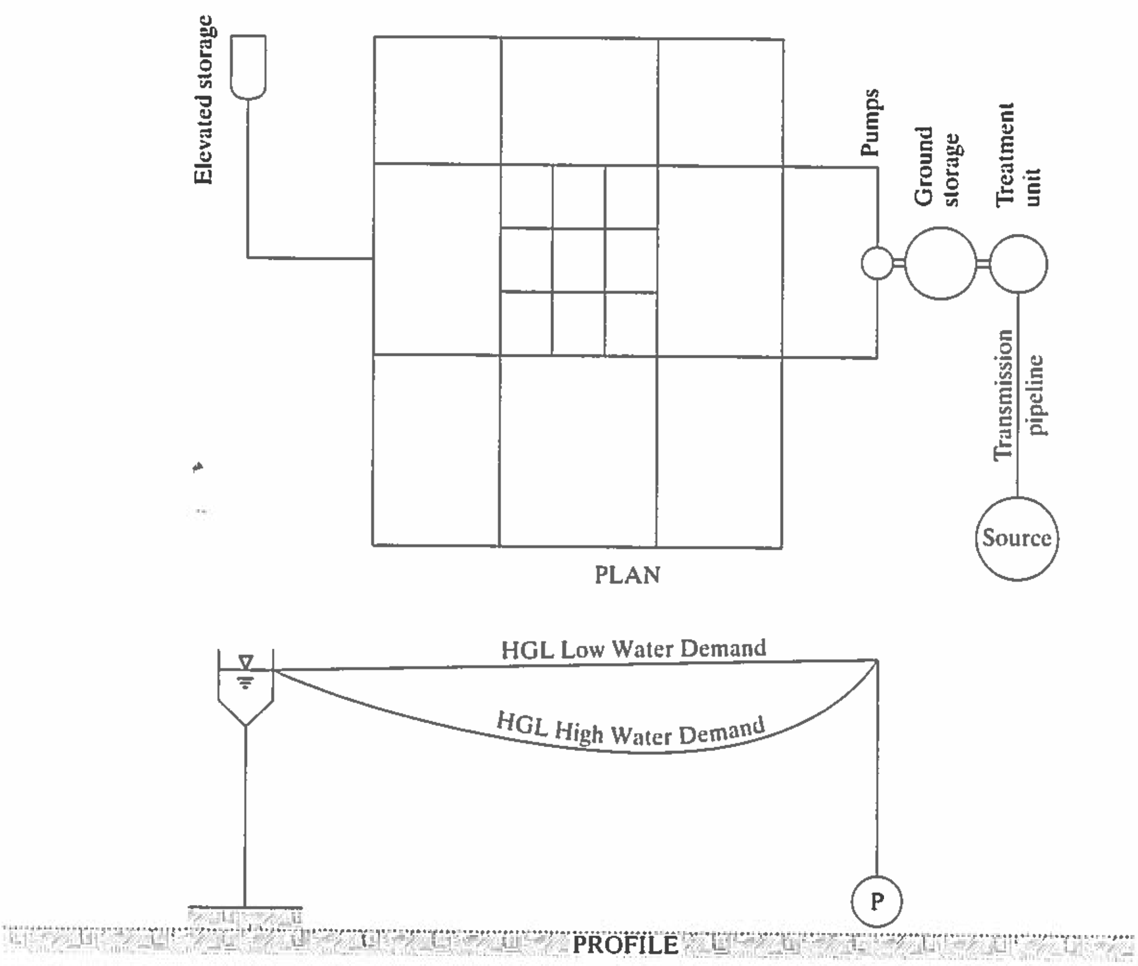


Figure 4.15 Sketch of water distribution system.

supply may require removal of suspended solids. Water is pumped from the ground level storage into the pipe network system. The elevated storage fills with water when the pumping rate is greater than the demand and empties when the demand is greater than the pumping rate. The height of the elevated storage is selected to provide the required system pressure.

Factors affecting the water demands for a system include water pressure, conservation programs, rainfall, water rates, and the economic status of residents. Average water requirements for a typical water distribution system are listed below.

Use	Average use per person	
	Gal/day	Liters/day
Domestic	70	260
Commercial	25	90
Industrial	45	170
Public	15	60
Loss	10	40
Total	165	620

Major industrial water users would be in addition to the values listed above.

Typical hourly variation in water demand is shown in Fig. 4.16. Unless historical records are available, the maximum daily water demand is often taken as twice the average daily water demand, and the peak hourly demand during the year is taken as three times the average daily demand. The future system demands are based on population projections. Water demand is assigned to each junction node in the system based on the estimated domestic, commercial, industrial, and public water use at the node.

The design flows for a municipal water distribution system are the maximum hourly demand and the maximum daily water demand plus the fire flow. Fire flows

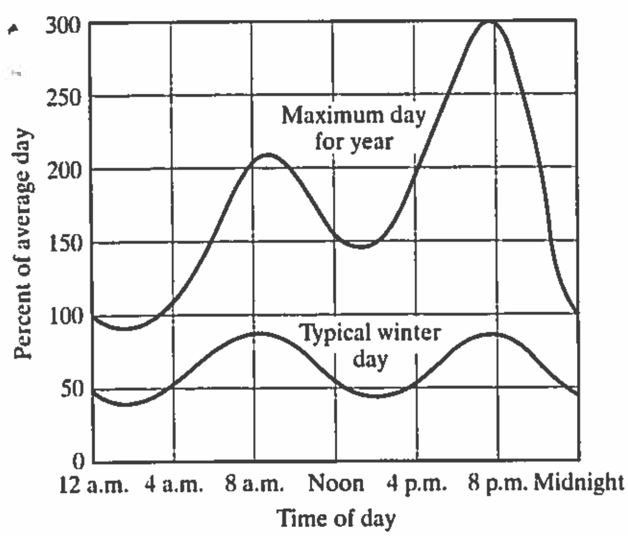


Figure 4.16 Typical hourly variation in water demand.

range from 500 gpm (1,900 lpm) for a residential fire to 12,000 gpm (45,000 lpm) for a fire in a major central business district. The duration of the fire ranges from 2 hr for a fire flow of less than 2,500 gpm (10,000 lpm) to 10 hr for a fire flow of 12,000 gpm (45,000 lpm). Fire hydrants are located throughout the service area generally at street intersections. Typical fire flows might be 500 gpm (1,900 lpm) for a residential fire, 2,000 gpm (7,500 lpm) for a small commercial or industrial area, and 3,000 gpm (11,400 lpm) in the main commercial area. For a large system, simultaneous fires may require consideration. The required maximum fire flow depends on the size of the city.

Elevated and ground level storage for a water distribution system is required to provide (a) equalization storage so the pumping does not have to match the demand, (b) fire demand, and (c) emergency storage. A schematic of system storage allocation is shown in Fig. 4.17. Emergency storage is typically a 1- to 2-day supply at the average daily demand. Emergencies that typically occur are line breaks on critical lines and system-wide power outages. Elevated equalization storage provides flexibility in the pumping schedule. With adequate elevated equalization storage, much of the pumping can be done at night during off-peak power demand. Typically, the amount of elevated storage may range from less than 10 percent for a large water distribution system to more than 50 percent of the total storage for a small water distribution system.

The height and location of the elevated storage in a water distribution system are selected to provide adequate pressure throughout the service area under various demand scenarios. During the peak hourly demand, the pressure should be greater than 35 psi (2.4×10^5 N/m^2) and greater than 20 psi (1.4×10^5 N/m^2) for the peak daily demand plus the fire flow. To reduce leakage and water use, the pressure in the water distribution system should be less than 90 psi (6.5×10^5 N/m^2).

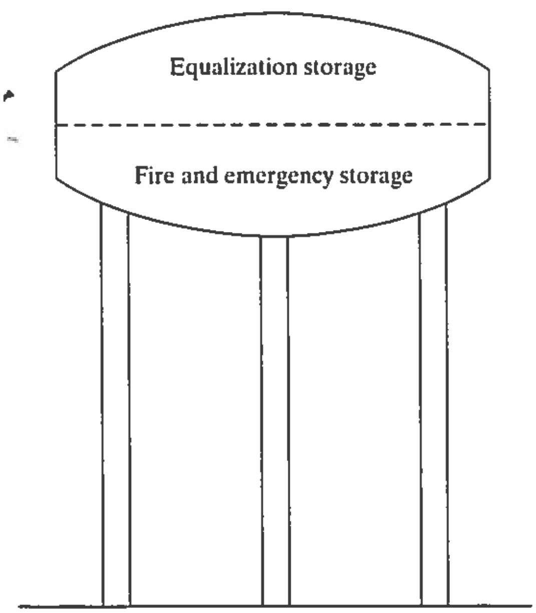


Figure 4.17 Schematic of system storage allocation.

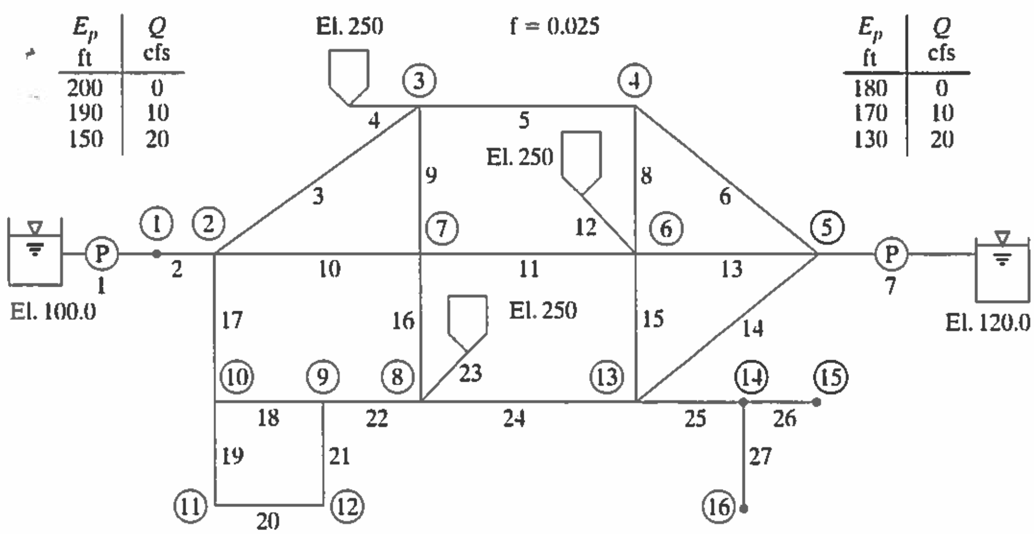
Steady-state water distribution system modeling can be used to evaluate water velocities, pipe headlosses, and system pressure. Generally, water velocities should be less than 5 fps (1.5 mps) and headlosses (expressed as S_f) should be less than 0.01 for small pipes and less than 0.003 for large pipes in the system. Usually the minimum pipe diameter is 6 inches (150 mm) in residential areas and 8 inches (203 mm) in mercantile areas. Time-dependent computer simulations of the water distribution system can be used to evaluate pumping schedules and the operation of elevated storage tanks.

Air-relief valves should be installed at high points in the water distribution system to prevent the accumulation of air, and drain valves should be provided at low points. Gate valves are commonly installed throughout the service area and are used to isolate segments of the system during repairs. Pressure-regulating valves may be used to divide the service area into pressure zones.

Drinking water standards set maximum limits on the level of contaminants that may be hazardous to the health of consumers. Treatment processes commonly used for public water supply do not remove all undesirable contaminants from the raw water. Many of the contaminants originate from industrial, agricultural, and domestic pollution. Safe drinking water standards apply to drinking water systems (publicly or privately owned) that serve at least 25 people (or 15 service connections) for at least 60 days per year. The Safe Drinking Water Act was passed in 1974 and amended in 1986 and 1996, and gives the Environmental Protection Agency the authority to set drinking water standards. Information on the drinking water standards can be obtained from http://www.epa.gov/water/.

Example 4.19(a) Municipal Water Distribution System Simulation

Conduct a 24-hour simulation of the operation of the water distribution system shown below using a 1-hour computational time step. The system includes 27 pipes, 7 closed loops, 5 fixed-grade nodes, and 2 pumps. The demands on the system are adjusted for



Example 4.19a Municipal water distribution system.

each time step using a global demand factor from Fig. 4.16 for the maximum day of the year. In addition, a fire demand is added at nodes 7 and 8 of 3.0 cfs each between the hours of 10 and 12.

The elevated storage tanks in the system are variable-level tanks, and the water level in the tank is computed after each time step. The elevation/storage information for each tank is listed below.

Elevation ft	Storage ft^3
220.0	0
230.0	40,000
240.0	120,000
250.0	200,000
260.0	250,000

The pumps in the system are variable-speed pumps, and the speed of the pump is adjusted after each time step depending on the elevation of the hydraulic grade line (HGL) at a specified junction node. The junction node for changing pump 1 is node 3, and the junction node for changing pump in line 7 is node 6 according to the following.

HGL Elevation	Speed ratio
<240	1.5
250	1.0
>260	0.5

The pump characteristic curve is adjusted using Eqs. 4.18 and 4.19.

The pipe network program (NETWORK.FOR) was modified for variable demands, variable-level elevated storage tanks and variable-speed pumps and is called NETOPSIM.FOR. The input data and summary of the output file are listed below. Comment lines in the program explain the input requirements.

Pipe no.	Node US	Node DS	Length (ft)	Diameter (in)	Minor loss coefficient	Fixed grade (ft)
1	0	1	2,000.0	24.0	0.5	100.0
2	1	2	800.0	24.0	0.0	
3	2	3	5,000.0	18.0	0.0	
4	3	0	700.0	18.0	0.5	250.0
5	3	4	3,700.0	12.0	0.0	
6	5	4	3,900.0	15.0	0.0	
7	0	5	2,100.0	24.0	0.5	120.0
8	6	4	2,500.0	10.0	0.0	
9	3	7	3,100.0	12.0	0.0	
10	2	7	5,500.0	18.0	0.0	
11	6	7	3,700.0	15.0	0.0	
12	0	6	900.0	18.0	0.5	250.0
13	5	6	2,900.0	15.0	0.0	
14	5	13	4,500.0	15.0	0.0	
15	6	13	2,500.0	15.0	0.0	
16	7	8	2,700.0	15.0	0.0	

Pipe no.	Node US	Node DS	Length (ft)	Diameter (in)	Minor loss coefficient	Fixed grade (ft)
17	2	10	3,100.0	18.0	0.0	
18	10	9	1,900.0	15.0	0.0	
19	10	11	1,600.0	8.0	0.0	
20	11	12	1,500.0	6.0	0.0	
21	9	12	1,650.0	8.0	0.0	
22	8	9	2,900.0	15.0	0.0	
23	0	8	1,900.0	18.0	7.5	250.0
24	13	8	3,100.0	15.0	0.0	
25	13	14	1,600.0	8.0	0.0	
26	14	15	1,750.0	6.0	0.0	
27	14	16	1,500.0	6.0	0.0	

Junction no.	Elevation (ft)	Demand (gpm)
1	90.00	0
2	110.00	694
3	95.00	694
4	105.00	2,083
5	100.00	694
6	103.00	2,428
7	97.00	2,083
8	103.00	1,044
9	107.00	0
10	112.00	0
11	115.00	350
12	112.00	350
13	110.00	0
14	120.00	0
15	135.00	175
16	130.00	175

RESULTS AT TIME = 18.000000 HOURS

PIPE	NODES		DISCHARGE	VELOCITY	HEADLOSS
1	0	1	9811.37	6.97	19.24
2	1	2	9811.37	6.97	7.55
3	2	3	2490.56	3.15	12.81
4	3	0	−1025.06	−1.29	−.32
5	3	4	958.79	2.72	10.67
6	5	4	3708.57	6.75	55.11
7	0	5	12657.70	8.99	33.60
8	6	4	540.14	2.21	5.69
9	3	7	821.83	2.34	6.57
10	2	7	2920.58	3.69	19.37
11	6	7	646.91	1.18	1.59
12	0	6	1969.76	2.49	1.49

13	5	6	4072.61	7.41	49.42
14	5	13	3141.52	5.71	45.63
15	6	13	-1214.69	-2.21	-3.79
16	7	8	-818.17	-1.49	-1.86
17	2	10	2665.23	3.37	9.09
18	10	9	1742.27	3.17	5.93
19	10	11	922.95	5.90	32.45
20	11	12	47.95	.55	.35
21	9	12	827.05	5.29	26.87
22	8	9	-915.23	-1.66	-2.50
23	0	8	1461.11	1.85	1.75
24	13	8	1051.83	1.91	3.52
25	13	14	875.00	5.60	29.17
26	14	15	437.50	4.97	33.61
27	14	16	437.50	4.97	28.81

JUNCTION DATA

NODE	DEMAND	GROUND	HGL	PRESSURE
1	.00	90.00	267.18	76.78
2	1735.00	110.00	259.63	64.84
3	1735.00	95.00	246.82	65.79
4	5207.50	105.00	236.16	56.84
5	1735.00	100.00	291.27	82.88
6	6070.00	103.00	241.85	60.17
7	5207.50	97.00	240.26	62.08
8	2610.00	103.00	242.11	60.28
9	.00	107.00	244.61	59.63
10	.00	112.00	250.54	60.03
11	875.00	115.00	218.08	44.67
12	875.00	112.00	217.74	45.82
13	.00	110.00	245.64	58.78
14	.00	120.00	216.47	41.80
15	437.50	135.00	182.86	20.74
16	437.50	130.00	187.66	24.99

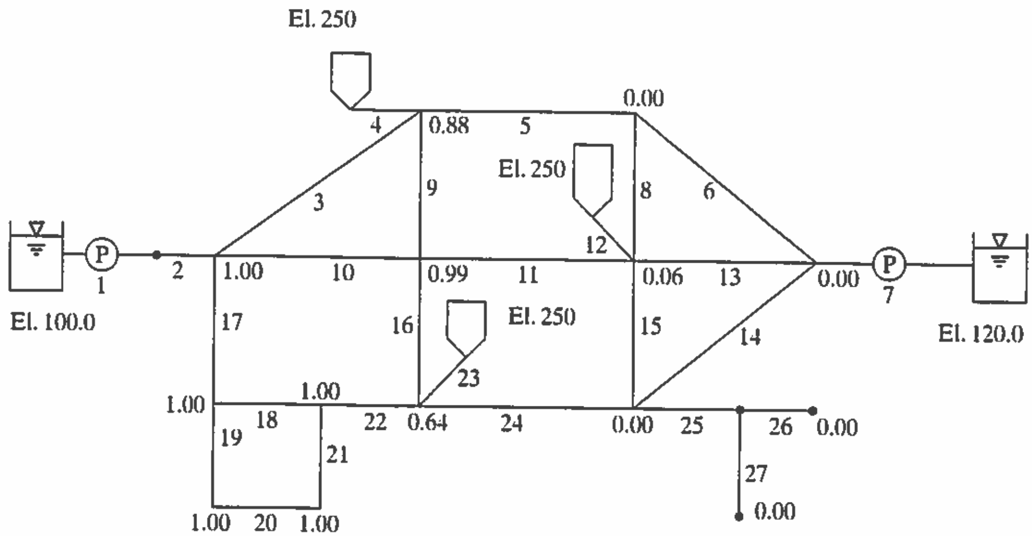
TOTAL DEMANDS ON SYSTEM = 60.1

VARIABLE-LEVEL TANK DATA

TANK	ELEVATION	STORAGE
4	246.11	168880.80
12	241.36	130881.80
23	242.40	139163.80

Example 4.19(b) Water Quality

Assume that the water being pumped into the network at pipe 1 is from an unconfined aquifer. The area adjacent to the well field was an industrial storage site, and the area is contaminated with toxic chemicals. Residents living on the left side of the water distribution system have experienced a high rate of cancers, neurological problems, birth defects, and other health problems. Testing of the groundwater indicates high levels of arsenic and phenols. Determine the fraction of water at each junction node that is from this source.



Example 4.19b Municipal water distribution system concentrations at hour 96.

The variation in concentration along a pipe can be estimated using the one-dimensional water quality model.

$$\frac{\partial C}{\partial t} = -V\frac{\partial C}{\partial x} + Dx\frac{\partial^2 C}{\partial x^2} - \text{losses} \tag{4.19.1}$$

where C is the concentration of the contaminate, V is the average velocity in the pipe, and Dx is the longitudinal dispersion coefficient. The first term on the left is the convective term, the second term on the left is the dispersion term, and the last term is the reaction/decay rate. For the purpose of this study, the losses will be represented using first-order reaction/decay (KC). Equation 4.19.1 can be written

$$C_i^+ = \frac{C_{i+1} + C_{i-1}}{2} - \left[\frac{V(C_{i+1} - C_{i-1})}{2dx} - \frac{Dx(C_{i+1} - 2C_i + C_{i-1})}{dx^2} + KC_i\right]dt \tag{4.19.2}$$

for pipe nodes 2 through $N - 1$ and

$$C_N^+ = C_N - \left[\frac{V(C_N - C_{N-1})}{dx} + KC_N\right]dt \tag{4.19.3}$$

for pipe node N.

The concentration at a junction node (C_J) is the blended concentration of the flow into the node or

$$C_J^+ = \frac{\sum^{\text{in}} C_{N,K}^+ Q_K}{\sum^{\text{in}} Q_K} \tag{4.19.4}$$

where C_J^+ is the concentration of the flow leaving the junction. Concentration of the contaminate in elevated storage tanks (C_t) depends on the concentration of the water flowing into the tank ($C_{N,K}$) and the volume of water in the storage tank ($\forall_t$) or

$$C_i^+ = \frac{C_i \forall_i + C_{N,K}^+ Q_K dt}{\forall_i + Q_K dt} \tag{4.19.5}$$

where Q_K is the discharge rate flowing into the tank and dt is the computation time step. The pipe network operation simulation program was modified to include Eqs. 4.19.2–4.19.5. The modified program is called NETOPCON.FOR. Comment lines in the program explain the input requirements. The figure 4.19b shows the concentration at the junction nodes for hour 96.

4.5.4 Pointer Matrix

The A array in Eq. 4.77 is mainly filled with zero values. Of the 81 values in the 9×9 array, only 28 are nonzero. A large municipal water distribution system may have 1,000 pipes, and the A array would be dimensioned $1{,}000 \times 1{,}000$. If the average node or loop equation only has five terms, more than 99 percent of the elements in the array would be zeros. A pointer matrix is a matrix of integers containing the positions of the nonzero elements in the A array.

An example of a pointer matrix is shown in Fig. 4.18 for the pipe network shown in Fig. 4.13. The square A array has been replaced with an A array

$[A]_{old}$ =

	1	2	3	4	5	6	7	8	9
1	X	X	0	0	X	0	0	0	0
2	0	X	X	0	0	X	0	0	0
3	0	0	X	X	0	0	X	0	0
4	0	0	0	0	X	0	0	X	0
5	0	0	0	0	0	X	0	X	X
6	0	0	0	0	0	0	X	0	X
7	0	X	0	0	X	X	0	X	0
8	0	0	X	0	0	X	X	0	X
9	X	X	X	X	0	0	0	0	0

$[A]_{new}$ =

	1	2	3	4	5
1	X	X	X	0	C_1
2	X	X	X	0	C_2
3	X	X	X	0	C_3
4	X	X	0	0	C_4
5	X	X	X	0	C_5
6	X	X	0	0	C_6
7	X	X	X	X	C_7
8	X	X	X	X	C_8
9	X	X	X	X	C_9

$[P]$ =

	1	2	3	4	5
1	1	2	5	0	3
2	2	3	6	0	3
3	3	4	7	0	3
4	5	8	0	0	2
5	6	8	9	0	3
6	7	9	0	0	2
7	2	5	6	8	4
8	3	6	7	9	4
9	1	2	3	4	4

Figure 4.18 Example of pointer matrix.

dimensioned 5×9 that contains the nonzero values in the old A array and a 5×9 P array that includes the column position index for the nonzero values. An extra column was included in the new A array to store the C vector of constants and an extra column was added to the P array to store the number of nonzero values in each row.

For a municipal water distribution system, the A array and P array (integer) might each be dimensioned $25 \times 1{,}000$. This would handle a 1,000 pipe system with up to 24 pipes in one loop. This would represent a reduction in memory requirements of about 96 percent. The use of the pointer matrix also eliminates the need to multiply most zero elements, and the procedure saves computation time.

4.6 UNSTEADY FLOW

Unsteady flow in pipe systems can result in extremely high or low pressures. High pressures can cause rupture of the pipe or damage to pipeline appurtenances. Excessively low pressure can cause collapse of the pipe or vaporization of the water. When the pressure in the pipe drops below the vapor pressure of water, a vapor cavity is formed causing water column separation. The high pressure generated during vapor cavity closure can also cause pipe failure.

The most common cause of unsteady flow in a pipe system is valve movement. Either opening or closing a valve will cause pressure waves to travel through the system. Severe unsteady flow conditions can occur in a pump system both during pump run-down after a power failure or during pump start-up. Water hammer pressure can also occur during air removal from the pipe system while filling empty lines. Extreme unsteady flow pressures can be prevented by controlling valve movement or installing surge relief valves, surge tanks, air chambers, or air-vacuum valves in a pipe system.

4.6.1 Basic Equations for Unsteady Flow

Unsteady flow occurs in a pipe system during pump start-up, pump shutdown, valve changes, and air removal. Figure 4.19 shows the pressure wave propagation resulting from a sudden valve closure in a single pipeline of length (L). Figure 4.19(a) represents steady-state conditions with a uniform velocity (V) in the pipeline. Because the velocity head is small compared with the water hammer pressure head, only the HGL is shown in Fig. 4.19. At time $= 0$, the valve is closed and a compression wave is generated at the valve. The wave travels upstream toward the reservoir at a velocity $\mathbf{a}$. The water velocity at the valve decreases to zero and the pressure head at the valve increases by ΔH. The increase pressure causes an increase in water density and an expansion of the pipe. At time $L/2\mathbf{a}$, the pressure wave is half-way to the reservoir. Water upstream of the wave front is moving toward the valve at the initial velocity. Velocity reversal occurs when the wave front reaches the reservoir, and the pressure head decreases to that in