CE 3354 Engineering Hydrology
Green-Ampt Loss Model

Contents

1 Green-Ampt Model

This section presents an approximation to the Green-Ampt loss model as a computational algorithm. The computations follow the example presented in CMM starting on page 144.

The purpose is to illustrate the loss model using R, FORTRAN, and MS Excel. Any of the tools is fine, however the program based tools (R and FORTRAN) have advantages if the analyst intends to integrate the procedures into larger projects. The procedure outlined in the text starts with the following inputs.

1. The analyst presumably has a rainfall input time series, preferably uniformly spaced. If non-uniform, then convert to uniform spacing by linear interpolation using R, FOR-TRAN, HEC-HMS utilities, or possibly Excel[1].

2. The analyst needs reasonable guesses for soil properties relevant to the area of interest, as a minimum one would need hydraulic conductivity, K, saturation change across the wetting front, $\Delta\theta$, and an estimate of suction, ψ. The porosity of the soil and the capillary rise are reasonable substitutes for these last two values; the analyst needs to understand that these values will overestimate the $\Delta\theta$ value as well as the ψ value.

3. The analyst needs a reasonable guess of the current position of the wetting front. A value of zero is fine, but represents the case where the soil has lost "memory" of prior events.

---

[1]Making data uniformly spaced in Excel has been a hassle – if you have solved this with some VBA script by all means use your tool.

1.1   Model equations

The following equations from page 142 are used. The algorithm is slightly modified by use
of the `min()` function available in most software.

$$f_t = K(\frac{\psi \Delta \theta}{F_t} + 1) \tag{1}$$

$$F_{t+\Delta t} = F_t + min(i_t, f_t) * \Delta t \tag{2}$$

In words the algorithm is stated as follows:

1. Start at $t = 0$ and $F_0 = 0$ (or some other known value).

2. Compute f_t using Equation 1 and the known value of F_0.

3. Using equation 2 compute $F_{t+\Delta t}$ using the potential rate just computed and the rainfall intensity i_t for the interval.

4. Update all counters and values, and repeat steps 1-3.

1.2   Green-Ampt Computations using R

```
# ************************************************************
# Green-Ampt Infiltration Models (for pedagogical use)
# ************************************************************
# Constants:  PSI: soil suction at interface (L)
#             DELTHETA: change in soil moisture content
#                                 (analog to porosity) (dimensionless)
#             K : saturated hydraulic condutctivity (L/T)
#             DELT: Time step increment (simulation constant)
# Variables: GA_CUM: value of cumulative infiltration for some given time.
#
# Functions:
#       GA_rate: computes potential infiltration rate.
#                Requires GA_CUM specification, returns Inf for GA_CUM=0
#    GA_accum: computes accumulated infiltration from GA_CUM
#             to current value given intensity and GA_rate.
#
GA_rate<-function(K,PSI,DELTHETA,GA_CUM){
 K*((PSI*DELTHETA/GA_CUM)+1)
```

```
 }
GA_accum<-function(intensity,infiltration_rate,DELT,GA_CUM){
min(intensity,infiltration_rate)*DELT+GA_CUM
}
# Usage
GA_rate(1.09,11.01,0.247,0)
GA_accum(1.08,Inf,0.167,0)
# As a nested call
GA_accum(1.08,GA_rate(1.09,11.01,0.247,0),0.167,0)
# ***************************************************
#  Now to work the example: pg 144 Example 5.4.1
# ***************************************************
# First the raw rainfall cumulative values:
accum_rain<-c(0.00,0.18,0.39,0.65,0.97,1.34,1.77,2.41,3.55,6.73,8.38,
9.19,9.71,10.13,10.49,10.77,11.01,11.20,11.37)
# The elapsed time in minutes
elapsed_time<-seq(0,180,10)
# build the other arrays in the example
incr_rain<-accum_rain # make a second vector, fill to get length OK
# set first value to zero
incr_rain[1]<-0
# Now populate the vector (note indexing) and unit conversion (minutes2hours)
for (i in 2:19)
(incr_rain[i]<-accum_rain[i]-accum_rain[i-1])/
 ((elapsed_time[i]-elapsed_time[i-1])/60)
# Same game for intensity
rain_rate<-incr_rain
# set last value to zero
rain_rate[19]<-0
for(i in 1:18)rain_rate[i]<-incr_rain[i+1]/
 ((elapsed_time[i+1]-elapsed_time[i])/60)
# Now the Green-Ampt computations
f_rate<-seq(0,18)# array for rate
F_depth<-seq(0,18)# array for depth
for (i in 1:19){
f_rate[i]<-GA_rate(1.09,11.01,0.247,F_depth[i]);
F_depth[i+1]<-GA_accum(rain_rate[i],f_rate[i],0.167,F_depth[i])
}
# Now plot everything
par(mfrow=c(3,2))
#
```

```
plot(elapsed_time,accum_rain,xlim=c(0,180),ylim=c(0,12),
xlab="Time(minutes)",ylab="Depth(cm)",type="s",main="Accum. Rain")
#
plot(elapsed_time,f_rate,xlim=c(0,180),ylim=c(0,24),
xlab="Time(minutes)",ylab="Rate(cm/hr)",
type="s",main="Infiltration Potential Rate")
#
plot(elapsed_time,incr_rain,xlim=c(0,180),ylim=c(0,4),
xlab="Time(minutes)",ylab="Depth(cm)",type="s",main="Incr. Rain")
#
plot(elapsed_time,F_depth[1:19],xlim=c(0,180),ylim=c(0,12),
xlab="Time(minutes)",ylab="Depth(cm)",type="s",main="Infiltration Depth")
#
plot(elapsed_time,rain_rate,xlim=c(0,180),ylim=c(0,24),
xlab="Time(minutes)",ylab="Rate(cm/hr)",type="s",main="Intensity")
#
plot(elapsed_time,accum_rain-F_depth[1:19],
xlim=c(0,180),ylim=c(0,12),xlab="Time(minutes)",ylab="Depth(cm)",
type="s",main="Excess Rain Depth")
cbind(elapsed_time,accum_rain,accum_rain-trunc(F_depth[1:19]*100)/100)
```

Figure 1 is a plot array showing the input cumulative rainfall, the incremental rainfall, intensity, potential loss rate, computed loss volume and the computed excess cumulative rainfall (i.e. the runoff).

1.3   Green-Ampt Computations using Excel

Similar computations using Excel[2] are illustrated in Figures 2 and 4.

Figure 2 displays cell contents as values – the typical appearance of a spreadsheet. Figure 3 displays the same six plots as illustrated in R.

Figure 4 displays the cell contents as formulas to illustrate the requisite arithmetic and is included to assist the reader in building their own spreadsheets.

---

[2]These are Excel 2008 implementations, the graphics may not be backward compatable, but the computations are elementary and should translate without any changes.
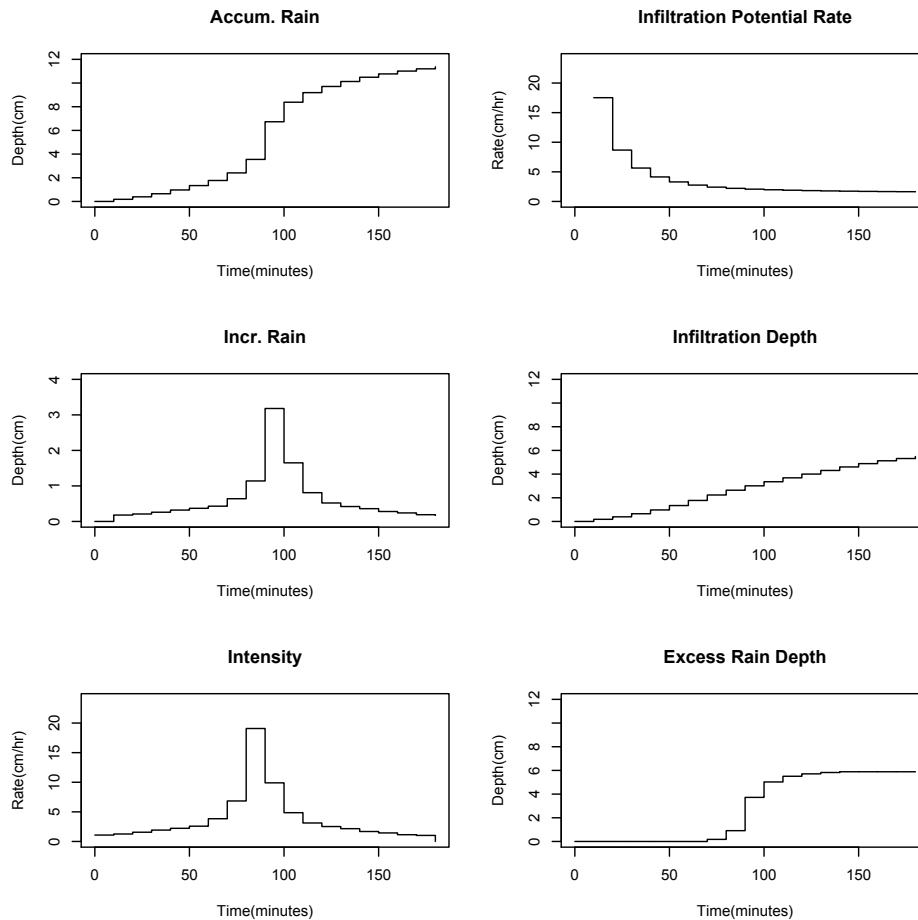
Figure 1: Plots (six panels) of various components of loss model.
Upper left: Cumulative rainfall input.
Upper right: Infiltration potential, Equation 1.
Middle Left: Incremental rainfall input (differencing of cumulative).
Middle Right: Infiltration depth, Equation 2.
Lower Left: Rainfall intensity (ratio of incremental depth to time interval).
Lower Right: Excess rainfall depth (differencing of upper left and middle right, truncation two places to right of decimal before differencing.

	A	B	C	D	E	F	G	H	I
1						Example 5.4.1			
2						Green-Ampt Infiltration Excess Mode			
3									
4	K	1.09	sat. hyd. cond.						
5	PSI	11.01	soil suction						
6	THETA	0.412	effective porosity						
7	Se	0.4	effective saturation						
8									
9	DELTHETA	0.2472							
10									
11									
12	TIME	RAIN_INCR	RAIN_CUM	INTENSITY	f	F	RAIN_E	INTENSITY_EXCESS	
13	0	0	0	1.08	1.00E+09	0	0	0	
14	10	0.18	0.18	1.26	17.57124	0.18	0	0	
15	20	0.21	0.39	1.56	8.696724	0.39	0	0	
16	30	0.26	0.65	1.92	5.654035	0.65	0	0	
17	40	0.32	0.97	2.22	4.148374	0.97	0	0	
18	50	0.37	1.34	2.58	3.303897	1.34	0	0	
19	60	0.43	1.77	3.84	2.766058	1.77	0	0	
20	70	0.64	2.41	6.84	2.419722	2.231	0.179	0.18	
21	80	1.14	3.55	19.08	2.216154	2.634	0.916	0.74	
22	90	3.18	6.73	9.9	2.077671	3.004	3.726	2.81	
23	100	1.65	8.38	4.86	1.975576	3.35	5.03	1.3	
24	110	0.81	9.19	3.12	1.896323	3.679	5.511	0.48	
25	120	0.52	9.71	2.52	1.832537	3.995	5.715	0.2	
26	130	0.42	10.13	2.16	1.779804	4.301	5.829	0.11	
27	140	0.36	10.49	1.68	1.735296	4.597	5.893	0.06	
28	150	0.28	10.77	1.44	1.69825	4.877	5.893	0	
29	160	0.24	11.01	1.14	1.669723	5.117	5.893	0	
30	170	0.19	11.2	1.02	1.648969	5.307	5.893	0	
31	180	0.17	11.37		1.631621	5.477	5.893	0	

Figure 2: Excel worksheet with cells displayed as **VALUES**
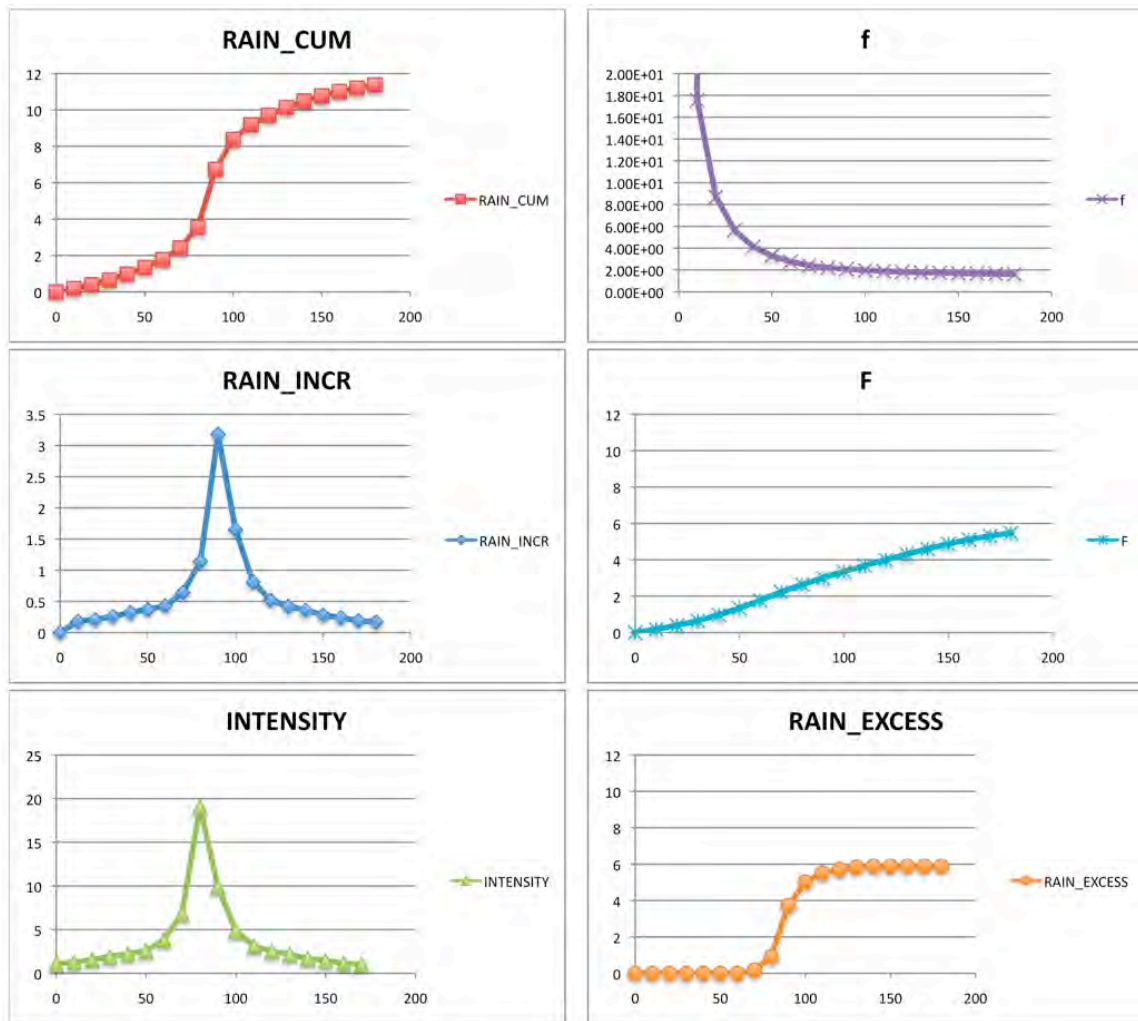
Figure 3: Similar plots as Figure 1, except Excel is used as the plotting tool and the axes are not labeled (i.e. time, depth, and rate labels are missing).

	A	B	C	D	E	F	G	H
1								
2						Example 5.4.1		
3						Green-Ampt Infiltration Excess Model		
4	K	1.09	sat. hyd. cond.					
5	PSI	11.01	soil suction					
6	THETA	0.412	effective porosity					
7	Se	0.4	effective saturation					
8								
9	DELTHETA	=(1-B7)*B6						
10								
11								
12	TIME	RAIN_INCR	RAIN_CUM	INTENSITY	f	F	RAIN_EXCESS	INTENSITY_EXCESS
13	0	0	0	=B14/((A14-A13)/60)	1000000000	0	=C13-F13	0
14	10	0.18	=B14+C13	=B15/((A15-A14)/60)	=$B$4*(($B$5*$B$9/F14)+1)	=MIN(D13,E13)*((A14-A13)/60)+F13	=C14-F14	=IF((G14-G13)>=0,(G14-G13),0)
15	20	0.21	=B15+C14	=B16/((A16-A15)/60)	=$B$4*(($B$5*$B$9/F15)+1)	=MIN(D14,E14)*((A15-A14)/60)+F14	=C15-F15	=IF((G15-G14)>=0,(G15-G14),0)
16	30	0.26	=B16+C15	=B17/((A17-A16)/60)	=$B$4*(($B$5*$B$9/F16)+1)	=MIN(D15,E15)*((A16-A15)/60)+F15	=C16-F16	=IF((G16-G15)>=0,(G16-G15),0)
17	40	0.32	=B17+C16	=B18/((A18-A17)/60)	=$B$4*(($B$5*$B$9/F17)+1)	=MIN(D16,E16)*((A17-A16)/60)+F16	=C17-F17	=IF((G17-G16)>=0,(G17-G16),0)
18	50	0.37	=B18+C17	=B19/((A19-A18)/60)	=$B$4*(($B$5*$B$9/F18)+1)	=MIN(D17,E17)*((A18-A17)/60)+F17	=C18-F18	=IF((G18-G17)>=0,(G18-G17),0)
19	60	0.43	=B19+C18	=B20/((A20-A19)/60)	=$B$4*(($B$5*$B$9/F19)+1)	=MIN(D18,E18)*((A19-A18)/60)+F18	=C19-F19	=IF((G19-G18)>=0,(G19-G18),0)
20	70	0.64	=B20+C19	=B21/((A21-A20)/60)	=$B$4*(($B$5*$B$9/F20)+1)	=MIN(D19,E19)*((A20-A19)/60)+F19	=C20-F20	=IF((G20-G19)>=0,(G20-G19),0)
21	80	1.14	=B21+C20	=B22/((A22-A21)/60)	=$B$4*(($B$5*$B$9/F21)+1)	=MIN(D20,E20)*((A21-A20)/60)+F20	=C21-F21	=IF((G21-G20)>=0,(G21-G20),0)
22	90	3.18	=B22+C21	=B23/((A23-A22)/60)	=$B$4*(($B$5*$B$9/F22)+1)	=MIN(D21,E21)*((A22-A21)/60)+F21	=C22-F22	=IF((G22-G21)>=0,(G22-G21),0)
23	100	1.65	=B23+C22	=B24/((A24-A23)/60)	=$B$4*(($B$5*$B$9/F23)+1)	=MIN(D22,E22)*((A23-A22)/60)+F22	=C23-F23	=IF((G23-G22)>=0,(G23-G22),0)
24	110	0.81	=B24+C23	=B25/((A25-A24)/60)	=$B$4*(($B$5*$B$9/F24)+1)	=MIN(D23,E23)*((A24-A23)/60)+F23	=C24-F24	=IF((G24-G23)>=0,(G24-G23),0)
25	120	0.52	=B25+C24	=B26/((A26-A25)/60)	=$B$4*(($B$5*$B$9/F25)+1)	=MIN(D24,E24)*((A25-A24)/60)+F24	=C25-F25	=IF((G25-G24)>=0,(G25-G24),0)
26	130	0.42	=B26+C25	=B27/((A27-A26)/60)	=$B$4*(($B$5*$B$9/F26)+1)	=MIN(D25,E25)*((A26-A25)/60)+F25	=C26-F26	=IF((G26-G25)>=0,(G26-G25),0)
27	140	0.36	=B27+C26	=B28/((A28-A27)/60)	=$B$4*(($B$5*$B$9/F27)+1)	=MIN(D26,E26)*((A27-A26)/60)+F26	=C27-F27	=IF((G27-G26)>=0,(G27-G26),0)
28	150	0.28	=B28+C27	=B29/((A29-A28)/60)	=$B$4*(($B$5*$B$9/F28)+1)	=MIN(D27,E27)*((A28-A27)/60)+F27	=C28-F28	=IF((G28-G27)>=0,(G28-G27),0)
29	160	0.24	=B29+C28	=B30/((A30-A29)/60)	=$B$4*(($B$5*$B$9/F29)+1)	=MIN(D28,E28)*((A29-A28)/60)+F28	=C29-F29	=IF((G29-G28)>=0,(G29-G28),0)
30	170	0.19	=B30+C29	=B31/((A31-A30)/60)	=$B$4*(($B$5*$B$9/F30)+1)	=MIN(D29,E29)*((A30-A29)/60)+F29	=C30-F30	=IF((G30-G29)>=0,(G30-G29),0)
31	180	0.17	=B31+C30		=$B$4*(($B$5*$B$9/F31)+1)	=MIN(D30,E30)*((A31-A30)/60)+F30	=C31-F31	=IF((G31-G30)>=0,(G31-G30),0)
32								

Figure 4: Excel worksheet with cells displayed as **FORMULAS**

2   Application to Exercise Problem

The exercise presents the following problem statement:

> 5. . . ., except find the excess precipitation predicted for a soil with porosity
> 0.35 and a saturated hydraulic conductivity of 0.5 inches per hour. Assume
> geologically realistic values for suction potential (zero is OK) and an initial depth
> of infiltration (non-zero, otherwise you get infinite gradients)[3]. The rainfall data
> is supplied in Table 1.

---

[3]This problem is intended for the simplified Green-Ampt model, if you choose the classical Green-Ampt,
make appropriate assumptions for missing parameters. I found that with R one needed to use a small
non-zero value for suction to work with zero infiltration depth. In Excel there was no problem because
the initial infiltration potential is specified as a large value – these are computational requirements and in
no way reflect degeneration of the underlying theory.

Table 1: Hyetograph (one-hour increments) - Incremental Values

Time (hours)	Depth (inches)
1	0.16
2	0.17
3	0.19
4	0.20
5	0.21
6	0.24
7	0.27
8	0.31
9	0.37
10	0.46
11	0.66
12	1.29
13	4.63
14	0.85
15	0.54
16	0.41
17	0.33
18	0.29
19	0.26
20	0.23
21	0.21
22	0.19
23	0.18
24	0.17

2.1   Solution Approach Using R

Using R, import the elapsed time, and depths – observe that these are incremental values. Can either use R to accumulate or (in this example) just used Excel to prepare the data files.

```
> #  Now to work the example:
> # First the raw rainfall cumulative values:
> # this file is made from Excel, but could be an ASCII data file
> z<-read.csv(file="ce5361_green_ampt_solution.csv",header=T)
> attach(z)
> accum_rain<-RAIN_CUM
> elapsed_time<-TIME
> # The elapsed time in minutes
>
> # build the other arrays in the example
> incr_rain<-accum_rain # make a second vector, fill to get length OK
> # set first value to zero
> incr_rain[1]<-0
> # Now populate the vector (note indexing) and unit conversion (minutes2hours)
> for (i in 2:25) (incr_rain[i]<-accum_rain[i]-accum_rain[i-1])
/((elapsed_time[i]-elapsed_time[i-1]))
> # Same game for intensity
> rain_rate<-incr_rain
> # set last value to zero
> rain_rate[25]<-0
> for(i in 1:24)rain_rate[i]<-incr_rain[i+1]/((elapsed_time[i+1]-elapsed_time[i]))
> # Now the Green-Ampt computations
> f_rate<-seq(0,24)# array for rate
> F_depth<-seq(0,24)# array for depth
> for (i in 1:25){f_rate[i]<-GA_rate(0.5,0.00001,0.35,F_depth[i]);
F_depth[i+1]<-GA_accum(rain_rate[i],f_rate[i],1.0,F_depth[i])}
> # Now plot everything
> par(mfrow=c(3,2))
# intentional mis-labeled units in this code fragment, verify if student
understands R syntax or just a cut-and-paste.
> plot(elapsed_time,accum_rain,xlim=c(0,24),ylim=c(0,12),xlab=
"Time(minutes)",ylab="Depth(cm)",type="s",main="Accum. Rain")
> plot(elapsed_time,f_rate,xlim=c(0,24),ylim=c(0,24),xlab=
"Time(minutes)",ylab="Rate(cm/hr)",type="s",main="Infiltration Potential Rate")
> plot(elapsed_time,incr_rain,xlim=c(0,24),ylim=c(0,4),xlab=
```

```
"Time(minutes)",ylab="Depth(cm)",type="s",main="Incr. Rain")
> plot(elapsed_time,F_depth[1:25],xlim=c(0,24),ylim=c(0,12),xlab=
"Time(minutes)",ylab="Depth(cm)",type="s",main="Infiltration Depth")
> plot(elapsed_time,rain_rate,xlim=c(0,24),ylim=c(0,24),xlab=
"Time(minutes)",ylab="Rate(cm/hr)",type="s",main="Intensity")
> plot(elapsed_time,accum_rain-F_depth[1:25],xlim=c(0,24),
ylim=c(0,12),xlab="Time(minutes)",ylab="Depth(cm)",type="s",main="Excess Rain Depth")
> cbind(elapsed_time,accum_rain,accum_rain-trunc(F_depth[1:25]*100)/100)
       elapsed_time accum_rain
 [1,]             0      0.00 0.00
 [2,]             1      0.16 0.00
 [3,]             2      0.33 0.00
 [4,]             3      0.52 0.00
 [5,]             4      0.72 0.00
 [6,]             5      0.93 0.00
 [7,]             6      1.17 0.00
 [8,]             7      1.44 0.00
 [9,]             8      1.75 0.00
[10,]             9      2.12 0.00
[11,]            10      2.58 0.00
[12,]            11      3.24 0.16
[13,]            12      4.53 0.95
[14,]            13      9.16 5.08
[15,]            14     10.01 5.43
[16,]            15     10.55 5.47
[17,]            16     10.96 5.47
[18,]            17     11.29 5.47
[19,]            18     11.58 5.47
[20,]            19     11.84 5.47
[21,]            20     12.07 5.47
[22,]            21     12.28 5.47
[23,]            22     12.47 5.47
[24,]            23     12.65 5.47
[25,]            24     12.82 5.47
```

And the all important graphic is in Figure 5. The student should note the error in both the time axis labels (units should be hours!) and the depth and rate axis (length units should be inches).
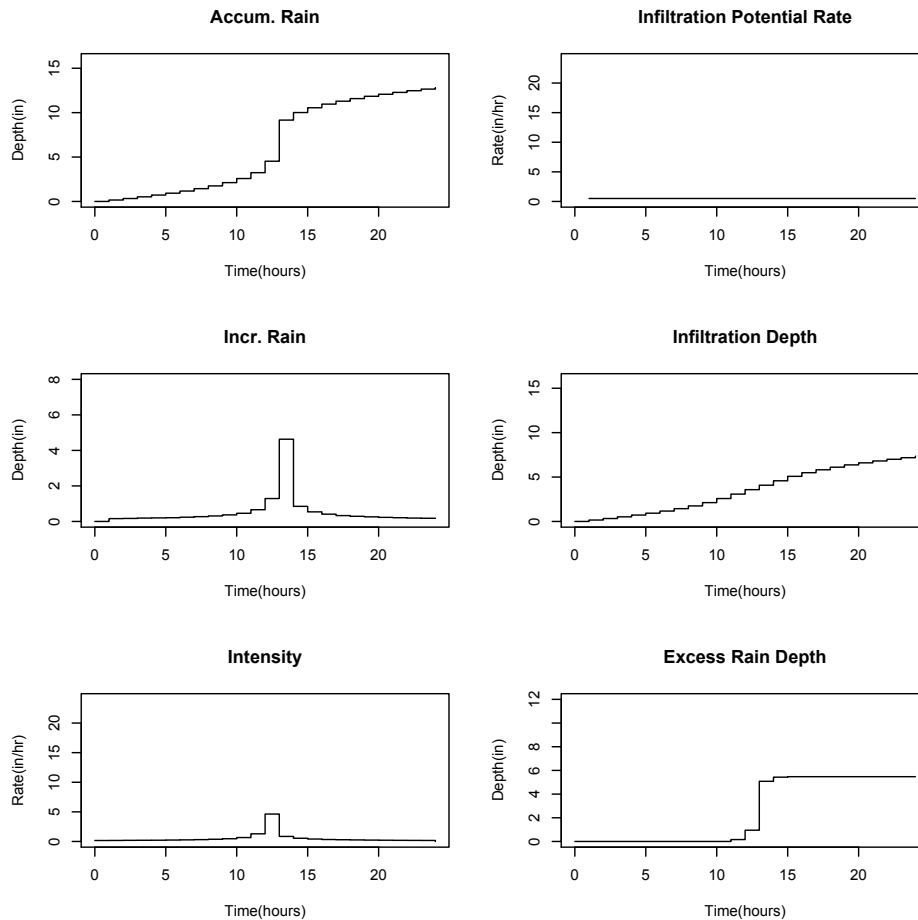
Figure 5: Plots (six panels) of various components of loss model.
Upper left: Cumulative rainfall input.
Upper right: Infiltration potential, Equation 1.
Middle Left: Incremental rainfall input (differencing of cumulative).
Middle Right: Infiltration depth, Equation 2.
Lower Left: Rainfall intensity (ratio of incremental depth to time interval).
Lower Right: Excess rainfall depth (differencing of upper left and middle right, truncation two places to right of decimal before differencing.

2.2   Solution Approach Using Excel

Same concept – supply the rainfall time series. Change material properties as indicated by problem, and adjust dimensions (time and length) as needed. .

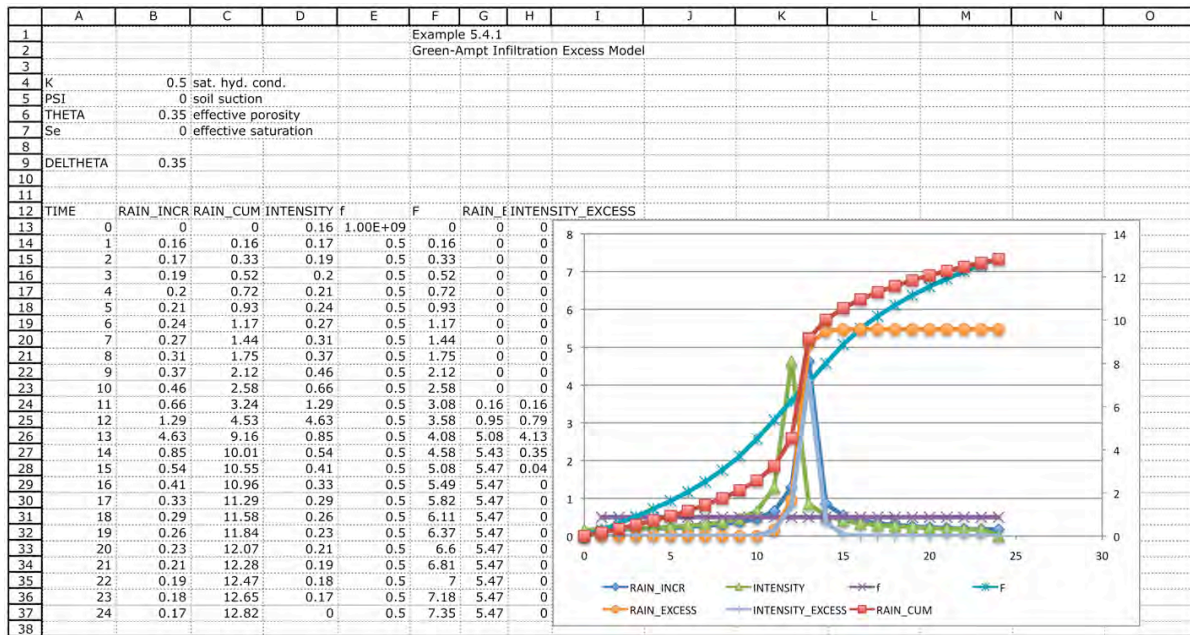|	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1						Example 5.4.1									
2						Green-Ampt Infiltration Excess Model									
3															
4	K	0.5	sat. hyd. cond.												
5	PSI	0	soil suction												
6	THETA	0.35	effective porosity												
7	Se	0	effective saturation												
8															
9	DELTHETA	0.35													
10															
11															
12	TIME	RAIN_INCR	RAIN_CUM	INTENSITY	f	F	RAIN_E	INTENSITY_EXCESS							
13	0	0	0	0.16	1.00E+09	0	0	0							
14	1	0.16	0.16	0.17	0.5	0.16	0	0							
15	2	0.17	0.33	0.19	0.5	0.33	0	0							
16	3	0.19	0.52	0.2	0.5	0.52	0	0							
17	4	0.2	0.72	0.21	0.5	0.72	0	0							
18	5	0.21	0.93	0.24	0.5	0.93	0	0							
19	6	0.24	1.17	0.27	0.5	1.17	0	0							
20	7	0.27	1.44	0.31	0.5	1.44	0	0							
21	8	0.31	1.75	0.37	0.5	1.75	0	0							
22	9	0.37	2.12	0.46	0.5	2.12	0	0							
23	10	0.46	2.58	0.66	0.5	2.58	0	0							
24	11	0.66	3.24	1.29	0.5	3.08	0.16	0.16							
25	12	1.29	4.53	4.63	0.5	3.58	0.95	0.79							
26	13	4.63	9.16	0.85	0.5	4.08	5.08	4.13							
27	14	0.85	10.01	0.54	0.5	4.58	5.43	0.35							
28	15	0.54	10.55	0.41	0.5	5.08	5.47	0.04							
29	16	0.41	10.96	0.33	0.5	5.49	5.47	0							
30	17	0.33	11.29	0.29	0.5	5.82	5.47	0							
31	18	0.29	11.58	0.26	0.5	6.11	5.47	0							
32	19	0.26	11.84	0.23	0.5	6.37	5.47	0							
33	20	0.23	12.07	0.21	0.5	6.6	5.47	0							
34	21	0.21	12.28	0.19	0.5	6.81	5.47	0							
35	22	0.19	12.47	0.18	0.5	7	5.47	0							
36	23	0.18	12.65	0.17	0.5	7.18	5.47	0							
37	24	0.17	12.82	0	0.5	7.35	5.47	0							
38															

Figure 6: Solution worksheet for Exercise 2 using Modified Green-Ampt.