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CHAPTER VII

GROUND WATER[1]

by John G. Ferris[2]

This chapter presents certain phases of the ground-water hydrology with special reference to the physics of movement of ground water. It is not intended to present a complete discussion of either the general or the quantitative phases of ground-water hydrology, which is beyond the scope of this book. For more complete discussions of the occurrence and movements of ground water, the reader is referred to the papers listed at the end of the chapter.

Our study of the waters of the earth progresses from the more familiar fields of atmospheric water and surface water to the third province of hydrology, which deals with the study of subsurface or ground water. Among the many prerequisites necessary to the study of ground-water hydrology, probably the one most neglected in the training of engineers is the subject of geology. Inasmuch as it would be impossible to correct this deficiency in any single chapter, it is necessarily assumed that the student has sufficient background training in this field to recognize the degree to which geology controls the occurrence and movement of ground water.

Although man has long been familiar with the development of small water supplies from wells, it is only in recent years that much thought has been directed to the hydraulics of ground-water flow. The great advances made since the turn of the century in the improvement of well-drilling methods and pumping equipment, particularly in the development of the deep-well turbine pump, have resulted in a marked upward trend in the use of ground water for domestic, rural, municipal, and industrial water supply. It is of interest to note that in 1939 it was estimated[3] that, in the United

[1] Published by permission of the Director of the U. S. Geological Survey in the public interest.

[2] District Engineer, Ground Water Division, U. S. Geological Survey.

[3] Anonymous, Inventory of Water Supply Facilities, *Eng. News-Record*, 1939, **123**, 414.

States, about 9100 public water supplies were derived from ground water and about 3300 from surface sources. Superimpose on this established upward trend the demands of industry awakened to the economic advantages of ground water for air temperature and humidity control and as a relatively constant-quality source lending itself to almost fixed treatment. Notwithstanding the magnitude of the total withdrawal of ground water for all the above uses, this total is exceeded by the present demand for ground water in irrigation.

An ever-increasing number of problems has attended the rapid growth in the use of ground water. Those engaged in the search for answers to these problems are handicapped by the deficiencies in hydrologic research and the lack of trained technicians in this field. Our ground-water reserves have, too frequently, been called inexhaustible. Advances in hydrology show the fallacy of such terminology. Equally unfavorable, however, is the dissemination of discouraging opinions by those who have experienced water shortages that result from overdevelopment. It becomes increasingly evident that a wiser and fuller use of this great national resource can be achieved only by the sound and rational methods of the trained hydrogeologist and hydrogeological engineer.

The earth's crust, composed of its myriad and varied hard rocks and the unconsolidated overburden, serves as a vast underground reservoir for the storage and transmission of percolating ground waters. The rocks comprising the earth's crust are seldom if ever solid throughout. They contain numerous openings called interstices that vary through a wide range of sizes and shapes. Although these interstices may reach cavernous size in some rocks, it should be noted that most of them are very small. Generally, they are interconnected, permitting movement of the percolating waters, but in some rocks they are isolated, preventing the transmission of water between interstices. Accordingly, then, the mode of occurrence of ground water in the rocks of a given area is largely determined by the geology of that area.

Porosity

The physical property of a rock that defines the degree to which it contains interstices is termed its porosity and is expressed quantitatively as the percentage that the interstitial volume is of the total. The porosity of a material is dependent on the interrelation of size, shape, and manner of sorting of its component parts in the case of unconsolidated or pervious sedimentary material; or on the size, shape, and pattern of channeling in the case of relatively soluble rock such as limestone; or on the size, shape, and pattern of fracturing in the dense sedimentary, igneous, and metamorphic rocks. Some idea of the relation of porosity to rock texture and particle sorting may be gained by reference to Fig. 62.

The porosity of rock or unconsolidated material may range from considerably less than 1 per cent to more than 50 per cent. However, a porosity in excess of 40 per cent is rare except in soils or poorly compacted materials. In general, we may consider a porosity greater than 20 per cent as large, between 5 and 20 per cent as medium, and less than 5 per cent as small.

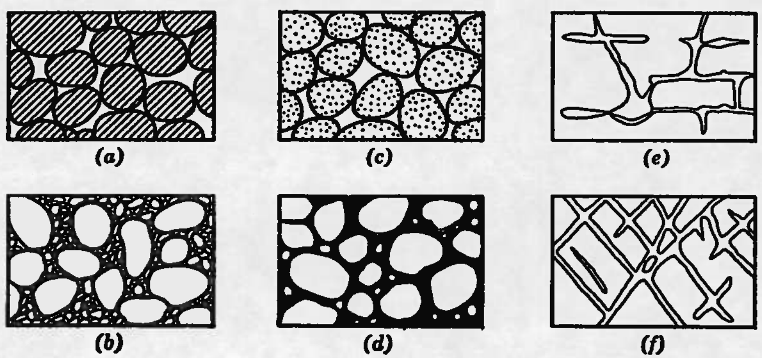


Fig. 62. Diagram showing several types of rock interstices and the relation of rock texture to porosity. *a*, Well-sorted sedimentary deposit having high porosity; *b*, poorly sorted sedimentary deposit having low porosity; *c*, well-sorted sedimentary deposit consisting of pebbles that are themselves porous, so that the deposit as a whole has a very high porosity; *d*, well-sorted sedimentary deposit whose porosity has been diminished by the deposition of mineral matter in the interstices; *e*, rock rendered porous by solution; *f*, rock rendered porous by fracturing. After Meinzer, *U. S. Geological Survey Water-Supply Paper* 489, 1923, Fig. 1, p. 3.

There are a number of methods in use for determining the porosity of rocks or soils which are based on either volumetric or specific-gravity measurements of dry versus saturated samples. The relation of the factors most commonly required for porosity

tests is summarized by the following equation,[1]

$$P = 100\left(\frac{W}{V}\right) = 100\left(\frac{V - v}{V}\right) = 100\left(\frac{S - a}{S}\right) = 100(b - a) \quad (1)$$

where P is porosity, W volume of water required to saturate dry sample of rock or soil, V volume of sample, v aggregate volume of solid particles comprising the sample, S weighted average of specific gravities of minerals composing the soil or rock, a specific gravity of dry samples, and b specific gravity of saturated sample.

Rather elaborate core-sampling apparatus have been devised to obtain samples in an undisturbed condition. However, the removal of any sample from its original environment is certain to disturb the sample to some extent. There is no positive assurance that any laboratory procedure reproduces the original regimen of pressure, temperature, or volume. Further, in the final analysis the sample represents only an infinitesimal section of the soil or rock formation.

Mechanics of Interstitial Flow

As stated in the preceding chapter there are two principal forces that control the movement of water in rocks and unconsolidated material, namely, gravity and capillarity. As to the first, we are all familiar with its action, and it is this force of gravity that is responsible for the entrance and percolation of ground waters from the time they start from the surface, percolate downward, and move laterally in the saturated zone to emerge as springs and seeps making up the base flow of surface streams and at flowing wells, ponds, and lakes. Normally ground-water flow is laminar, but turbulent flow may occur where conditions are favorable, as for example near a stream channel or discharging well.

The second force, capillarity, is generally not accorded proper attention in proportion to its importance. Molecules possess definite size, shape, and fields of force in all states of matter. It is their degree of mobility relative to one another which distinguishes the gas, liquid, or solid state. In a gas they are separated by distances which minimize the attraction between adjacent molecules, and thus unrestrained motion occurs. In a liquid the

[1] O. E. Meinzer, The Occurrence of Ground Water in the United States, *U. S. Geological Survey Water-Supply Paper* 489, 1923, p. 12.

molecules are free to move relative to one another but are held sufficiently close to each other by the cohesive forces to prevent all but a small proportion from escaping as vapor. In a solid the molecules, though not absolutely immobile, are essentially fixed in position.

The spontaneous contraction of a liquid surface toward the shape with minimum surface area is typified in the formation of spherical droplets. A diagrammatic representation of the attractive forces which might obtain for molecules within a liquid is shown by Fig. 63. Each interior molecule is surrounded by others on every side and is in equilibrium with the attractive forces from all directions. The surface molecules, though attracted inward and to each side, are not subjected to a comparable attraction outward because there are very few molecules outside the liquid. Thus, all surface molecules are subject to a resultant inward attraction, perpendicular to the surface. Consequently, the surface molecules move inward until the concentration of interior molecules reaches a maximum and the surface contracts to the minimum area for the given volume.

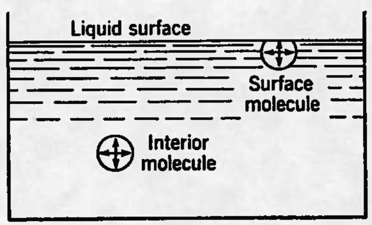


FIG. 63.

Inasmuch as work must be done to extend a surface by moving an interior molecule to the surface against the inward attractive forces, the spontaneous contraction of a liquid surface indicates that there is energy resident in the surface. It follows that, to form a free surface within a given liquid, work must be done against the mutual attraction of the molecules on each side of the interface. If T indicates the energy per unit of surface area, the work W which is required to separate a liquid column will be $2AT$ because two free surfaces of area A are formed. The quantity $W/A = 2T$ is generally termed the work of cohesion.

When a curved surface is extended by moving each element in a radial direction there is an increase in area. To increase the surface area, work must be done by the pressure differential that moves the surface. As shown by Adam,[1] this work may be evaluated by

[1] N. K. Adam, *The Physics and Chemistry of Surfaces*, Oxford Univ. Press, London, third edition, 1941, pp. 1–16.

considering the energy changes which occur in the displacement of the surface. In Fig. 64, $ABCD$ is an infinitesimal part of a surface with sides at right angles. Let the elemental area be extended in a radial direction a distance Δn, with the normals to the boundaries in the displaced position $A'B'C'D'$ the same as the normals in the original position. The normals from A and B meet at O_1; those from B and C meet at O_2. Denote the radius of curvature for arc AB as R_1 and for arc BC as R_2. Angle AO_1B measures AB/R_1 radians, and angle BO_2C is BC/R_2.

Then the area of the displaced surface may be computed as

$$(A'B')(B'C') = \left(AB + \frac{AB\,\Delta n}{R_1}\right) \cdot \left(BC + \frac{BC\,\Delta n}{R_2}\right) \quad (2)$$

Expanding equation 2 and neglecting second-order quantities there follows

$$ABCD\left(1 + \frac{\Delta n}{R_1} + \frac{\Delta n}{R_2}\right)$$

The net increase in area is

$$ABCD\,\Delta n\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

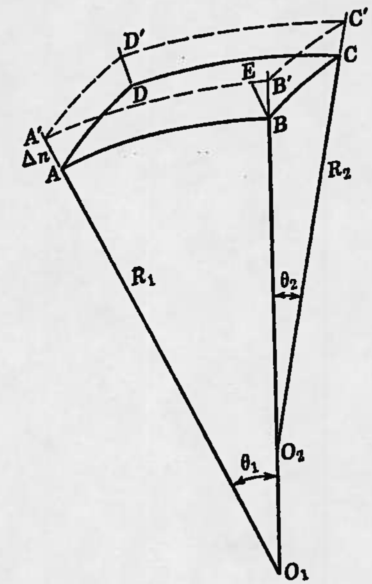


FIG. 64.

The net increase in surface energy is

$$\Delta W = T\Delta A = TABCD\,\Delta n\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad (3)$$

Denoting the pressure on the concave side as p_1 and on the convex as p_2, the work done by this pressure differential is

$$(p_1 - p_2)ABCD\,\Delta n \quad (4)$$

Since no work is done by other forces, the above quantities are equal, and there follows the fundamental equation of capillarity.

$$p_1 - p_2 = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad (5)$$

One of the more important and better-known applications of the forces of adhesion is the capillary rise of liquids in tubes or interstices. When water is touching glass, the glass becomes "wetted" and the angle of contact is very small. Figure 65 shows a section through a circular glass capillary tube having a diameter d. The contact angle between the water and the glass is represented by α. If it is assumed that α is zero and that the meniscus is a perfect hemisphere, $R_1 = R_2 = d/2$. Equation 5 may then be written

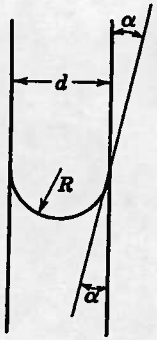


FIG. 65.

$$p_1 - p_2 = \frac{4T}{d} \tag{6}$$

This pressure difference forces the liquid up the tube to a height h, above the plane surface outside the tube, such that the weight of this liquid column just balances the pressure deficiency under the meniscus. Then

$$p_1 - p_2 = wh = \frac{4T}{d}$$

and

$$h = \frac{4T}{dw} \tag{7}$$

where h is height of capillary column, in feet; T surface tension, in pounds per foot; d tube diameter, in feet; and w unit weight, in pounds per cubic foot.

The value of T for water may be taken as 0.005 lb per ft for the range of temperatures encountered in ground water. The capillary rise of water in a glass tube 0.1 in. in diameter may be found from equation 7 to be approximately 0.46 in. As shown by this equation, the rise of water level in a capillary tube is inversely proportional to the diameter of the tube and, consequently, the capillary rise in a tube of 0.01-in. diameter under the above conditions is 10 times as great or 4.6 in. The values of h computed from equation 11 agree quite well with observed values for glass tubes having diameters less than 0.2 in., but for larger tubes observed values are less than computed values.

Inasmuch as the magnitude of the capillary rise depends on the energy resident in the liquid surface, the shape of the tube below

the meniscus does not affect the height of water level in the tube. Reference is made to Fig. 66 which shows the type of interstitial opening that may occur in consolidated or unconsolidated material. It is assumed that the effective diameter of cross section at the meniscus is equal in each case to the bore of the capillary tube. In each of the three examples the water level will be held at the same height if each interstice is filled and if the height of section A above the reservoir level is equal to h as given by equation 7. If the water level declines in either of the irregular-shaped tubes, there will be a large lowering of water level when the water surface reaches a section of enlarged area. A new equilibrium level occurs

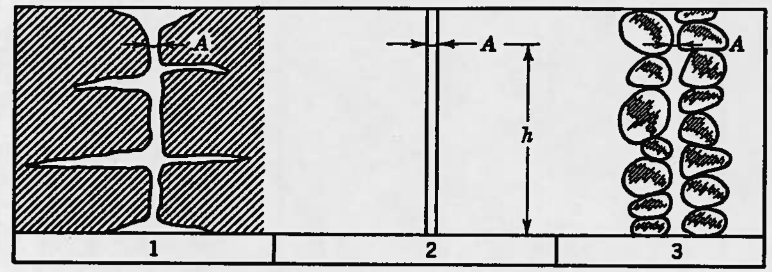


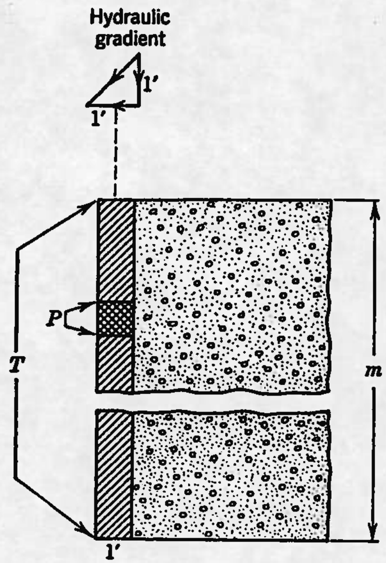
FIG. 66. Diagram showing that during a declining water level, for any given area A at the water surface, the height at which water can be held by capillarity is independent of the size and shape of the tube below the free-water surface. A rise in water level, however, may result in a much lower level in tubes 1 and 3.

when the declining head reaches a section with sufficient surface tension to balance the weight of the residual water column. With a declining water table, the capillary fringe tends to reach a maximum though quite irregular thickness. A restricted section of capillary size, at or below the limit of capillary lift, retains water in the irregular-shaped tubes at a level above the receding water table, which is dependent on the surface energy at the critical cross section. When the water table is rising, the fringe advance is limited to the interstitial openings of continuous capillary size. It has been observed by laboratory experimentation that the height of capillary lift is somewhat greater if the water level in a glass tube recedes from a higher stage to the equilibrium level than if the water level advances toward this level from a lower stage. This phenomenon, which is termed hysteresis, is generally ascribed to frictional forces or differences in wall adhesion under the two conditions.

Permeability and Transmissibility

The vertical percolation of ground water through capillary interstices results in the build-up of a hydraulic gradient with consequent lateral percolation of water through interconnecting interstices. The capacity of a formation for transmitting water is measured by its coefficient of permeability, which is defined by Meinzer[1] as the rate of flow of water in gallons per day through a cross-sectional area of 1 sq ft under a hydraulic gradient of 1 ft per ft at a temperature of 60° F.

The term *coefficient of transmissibility* introduced by Theis[2] is coming into popular usage in ground-water hydrology. The coefficient of transmissibility is defined as the rate of flow of water in gallons per day through a vertical strip of the aquifer 1 ft wide and extending the full saturated height under a hydraulic gradient of 100 per cent at a temperature of 60° F. The difference between the coefficients of permeability and transmissibility is shown in diagrammatic form by Fig. 67.



P = coefficient of permeability in gpd/sq ft
T = coefficient of transmissibility in gpd/ft
$T = P \cdot m$

Fig. 67.

The permeability of granular material varies with the diameter and degree of assortment of the individual particles. A well-sorted gravel has a much higher permeability than a well-sorted coarse sand. However, gravel with a

[1] N. D. Stearns, Laboratory Tests on Physical Properties of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 596, 1928, p. 148.

[2] C. V. Theis, The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage, *Trans. Am. Geophys. Union*, 1935, p. 520.

moderate percentage of medium- and fine-grained material may be considerably less permeable than a uniformly sized coarse sand. In graded material, the particles of moderate size fill the pore spaces between the larger particles, and in turn the resultant pore spaces are filled by the fine materials, thus forming a compactly knit and impervious mass such as is obtained in good concrete.

Measurements of the permeability of rocks and unconsolidated materials may be made by either field or laboratory methods as described by Muskat[1] and by Wenzel.[2] Laboratory determinations of the coefficient of permeability are made by measuring the discharge or the time rate of change in head, for the percolation of measured quantities of water through a known area and volume of soil sample. Devices used for this purpose are termed permeameters and include a supply reservoir or tank from which water is discharged through a percolation cylinder under either a constant or a variable head. The percolation cylinder is accurately machined to a fixed diameter and is equipped with a base screen which supports the soil sample and permits free inflow of water. Manometer tubes in the supply and receiving reservoirs are used to determine the loss of head that occurs for the vertical percolation of known quantities of water through the soil cylinder at measured rates. A schematic representation of the more common types of permeameter is shown by Fig. 68.

The use of permeameters to determine the permeability of unconsolidated material is invalidated to a large degree because of the great errors introduced in repacking a disturbed sample. Inasmuch as the packing arrangement is a critical factor in determining the permeability of an incoherent material it would seem advisable to apply laboratory methods only to consolidated materials or cores of unconsolidated material. Further caution should be exercised because the volume of material used in permeameter tests represents only an infinitesimal sample of a formation that is generally quite heterogeneous. Accordingly, to be of value permeameter programs should include intensive sampling and testing methods on many samples collected at frequent depth intervals and at numerous locations within the area.

[1] Morris Muskat, *Flow of Homogeneous Fluids through Porous Media*, McGraw-Hill, 1937.

[2] L. K. Wenzel and V. C. Fishel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942.

Field determinations of permeability are made by either the velocity or the potential method. In the velocity method one well is used for the injection of salt, dye, or an electrolyte. Two or more

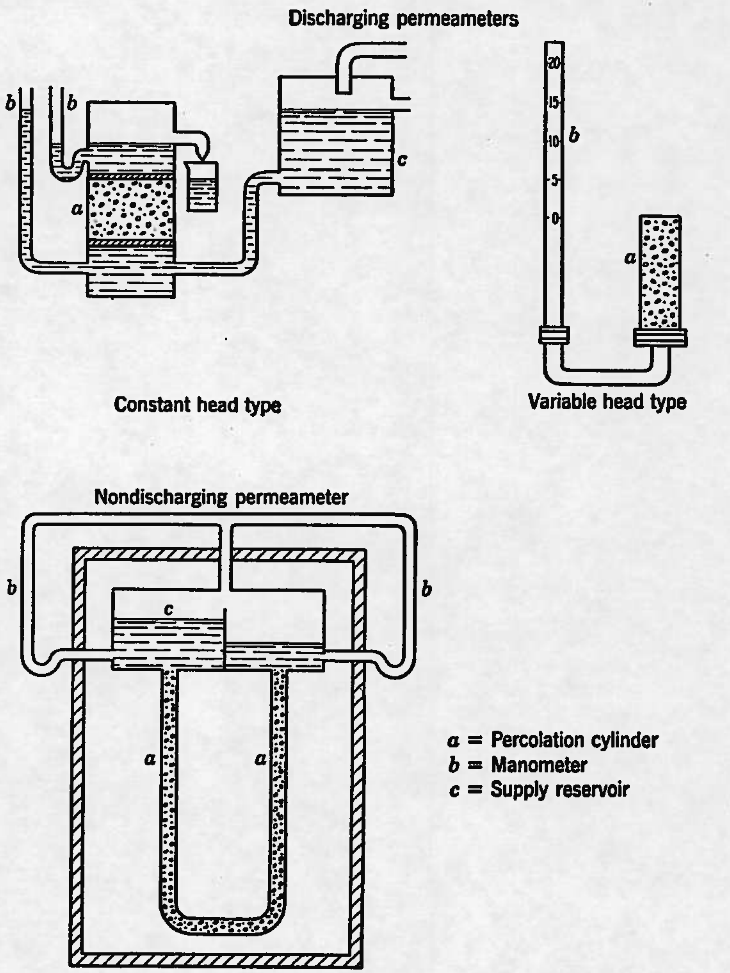


Fig. 68.

wells are used as observation stations to determine the time rate of travel of the injected substance through the water-bearing material. Fluorescein is generally used for the dye method and can be detected by eye or in more dilute form by colorimeter. The chemical or salt method requires periodic sampling and analysis

of water from each observation well to determine the time of arrival of the salted solution. The electrolyte method requires periodic readings of the electric conductivity of the water in each observation well. Measurements of the water-table gradient, the distance between observation wells, and the time of travel of the injected material provide the basis for determining the permeability of the material over the path of travel. A sketch of the equipment setup for one form of the velocity method is shown in Fig. 69.

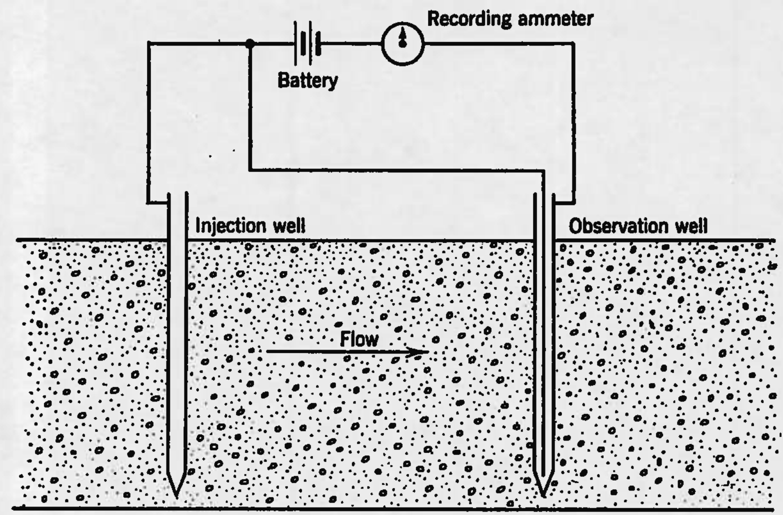


Fig. 69.

Inasmuch as the velocity of flow through most ground-water aquifers is measured in terms of a few feet per day, it is necessary that velocity observations be confined to small areas in order to secure results within a reasonable time. This method measures the velocity of the fastest thread of water that happens to intersect the two wells and not necessarily the average velocity between the wells. This method would be impractical for sampling adequately any heterogeneous aquifer that has large variations in vertical and horizontal permeability.

Potential methods of determining permeability are based on measurements of the amount and rate of drawdown or recovery of water level in observation wells at different distances from a well that is either pumping or recovering from pumping, respectively.

The principles and application of this method are covered in detail under the discussion of ground-water hydraulics. A distinct advantage of the potential method is its ability to sample large areas of the undisturbed aquifer within a limited time and at a minimum of expense.

Specific Yield

The remaining physical characteristic in the hydrology of ground water to be discussed is the specific yield. When saturated rocks or soils are drained under the action of gravity, it is found that the volume of water yielded by draining is less than the volume of void space indicated by the total porosity of the material because of the pellicular water that is retained by molecular attraction. The quantity of water yielded by gravity drainage from saturated water-bearing material is termed the specific yield and is expressed as a percentage of the total volume of the material drained. The quantity of water retained by the material against the pull of gravity is termed the specific retention or field capacity and is again expressed as a percentage of the total volume of the material. A somewhat similar term, moisture equivalent,[1] is frequently used to represent the moisture retained by a saturated sample when subjected to an arbitrary centrifugal force. It is evident that the sum of the specific yield and the specific retention of a material is equal to its porosity.

If evaporation is prevented, the greater part of the water retained by a column of rock or sand and gravel, after draining for 24 hr, will be retained almost indefinitely as a film held by molecular adhesion on the walls of the interstices. The greater the amount of total interstitial surface in a rock or unconsolidated material the greater is its specific retention. As would be expected, it is found that, as the effective diameter of grain decreases, the specific retention generally increases because the total exposed surface area increases with decreasing grain size. (See page 233.)

Although the total porosity of a clay or fine sand might be equivalent to the total porosity of a coarse gravel, it follows from the above that the large specific retention of the clay would result in a very small specific yield, whereas the reverse would be true for the coarse gravel. For practical purposes, a water-bearing formation of coarse gravel would supply large quantities of water to wells, whereas clay formations, although saturated and of high porosity, would be of little value in this respect. Accordingly, we find that specific yield is termed by some as the effective or practical porosity.

[1] L. J. Briggs and J. W. McLane, The Moisture Equivalent of Soils, *U. S. Department of Agriculture Bureau of Soils Bul.* 45, 1907.

Determinations of the specific yield or specific retention by laboratory methods are limited by the difficulties of securing undisturbed and representative samples that were noted under permeability determinations by laboratory methods. In addition, the short sample columns used in the laboratory cannot duplicate the very long capillary tubes that probably exist in the thick sections found in the field. As for permeability, the most satisfactory determinations of specific yield are made in the field through the medium of pumping tests.

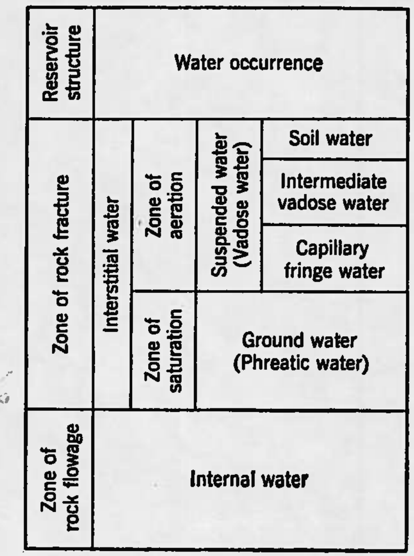


FIG. 70.

The Ground-Water Reservoir

The classification of the earth's crust with reference to its properties as a reservoir for the storage and transmission of percolating ground waters and the subdivision of this reservoir into its component parts is shown by Fig. 70. Interstices are probably absent in the zone of rock flowage, because the stresses are beyond the elastic limit and the rock is in a state of plastic flow. Water in this zone is classified as internal water and is not in the realm of the hydrologist. The depth at which rocks undergo permanent deformation is not known accurately but is generally estimated as many miles.

In the zone of rock fracture, the stresses are below the elastic limit, and interstices can exist in the rocks. Water in this zone is stored in the soil or rock interstices and accordingly is termed interstitial water. Although there is no direct relation between porosity and depth, in general, the porosity decreases with depth, the large openings particularly being absent at great depths. In crystalline rocks most of the water is encountered within 300 ft of the surface.[1] In sedimentary rocks, such as limestone and sandstone, porous zones that yield water readily are encountered in some places at depths of more than 6000 ft, although most wells in these strata find little water below a depth of 2000 ft. The decrease in size of interstices with increased depth is caused in part by the increased pressure at great depth, which tends to close the pore spaces or crevices, and in part by the cementation of interstices by the heavier and more highly mineralized waters.

The zone of aeration is that part of the earth's crust where the water present is not under hydrostatic pressure, except temporarily, and for the most part the interstices are filled with atmospheric gases. Water retained in this zone is held by molecular attraction and is termed pellicular, suspended, or vadose water. The thickness of the zone of aeration varies considerably depending on the geology, hydrology, and topography of the area. It may be virtually nonexistent in lowland areas adjacent to bodies of surface water as in marsh lands, or it may be as much as 1000 ft thick as in arid regions of great topographic relief.

The belt of soil water consists of the soil and other unconsolidated materials in which the root systems of plants, grasses, and trees are developed and from which water is discharged to the atmosphere by evaporation or transpiration. Evaporation occurs largely at the surface except in tight clay soils under prolonged drying where shrinkage cracks develop and permit air circulation to some depth. Although water may be brought to the evaporation areas by capillarity, in general water is not discharged in appreciable quantities by evaporation below depths of a few feet. As to transpiration, note that, although the root penetration of most common grasses and field crops is seldom more than a few feet, records indicate root development for wheat to depths of 7 ft; for alfalfa as much as 30 ft[2]; and for some perennials in arid regions as much as 50 ft.[3]

[1] E. E. Ellis, Occurrence of Water in Crystalline Rocks, *U. S. Geological Survey Water-Supply Paper* 160, 1906, pp. 19–28.
[2] W. W. Burr, The Storage and Use of Soil Moisture, *Nebraska Univ. Research Bull.* 5, 1914, p. 9.
[3] O. E. Meinzer, Plants as Indicators of Ground Water, *U. S. Geological Survey Water-Supply Paper* 577, 1927, p. 77.

The capillary fringe is the belt overlying the zone of saturation and containing interstices, some or all of which are filled with water that is in connection with and is a continuation of the zone of saturation, being held above that zone by capillarity acting against the force of gravity. The thickness of the capillary fringe in granular material is a function of the effective particle size and generally increases as the grain size decreases. The fringe thickness may range from a few inches in coarse gravel to 8 ft in silty material and is probably much greater in very fine-grained sediments. The capillary fringe in a given material may vary slightly in thickness from summer to winter because of changes in water temperature. The surface tension of water increases as the temperature decreases, and within the range from 60° to 32° F it increases about 3 per cent. Although the density of water varies with temperature, this change is negligible. Accordingly, then, the thickness of the capillary fringe would be somewhat greater in late winter and spring, the period of lowered ground-water temperature.

Beneath the capillary fringe lies the zone of saturation. It is this zone that is of importance to the hydraulic engineer and well driller as the source of water for wells and springs. It is of importance to the hydrologist as the reservoir that provides the closing link in the hydrologic cycle by serving as the mechanism for the intake, transport, and return of underground waters from and to the surface and the atmosphere. The upper surface of the zone of saturation is called the water table.

A profile section of a drainage basin is shown by Fig. 71 in diagrammatic form with greatly contracted horizontal scale. In the highland area, water is discharged from the soil belt either by direct evaporation from the soil or by transpiration from the vegetal cover. During a prolonged drought the vegetation in this area is dependent on the moisture retained in the soil belt, which is determined by the specific retention of the material composing this belt. Plant root systems contain many fine rootlets that are capable of absorbing a large part of the water held on the soil particles by molecular attraction. Inasmuch as an adequate water supply is vital to continued growth, the plant develops an extensive network of rootlets to satisfy its moisture demands. Under conditions of gradually diminishing soil water supply, when the leaves of the plant first undergo permanent wilting, minimum soil water content is expressed by the wilting coefficient[1] of the soil, which is defined as the ratio of the weight of water in the soil, under the above-noted conditions, to the weight of the soil when dry. Accordingly, then, the capacity of a soil for storing water available for plant growth is measured by the difference between its specific retention and its wilting coefficient when expressed in comparable units.

In areas where the soil belt is close to or in contact with the capillary fringe, certain types of plants may develop rootlets in the capillary fringe and with the aid of capillarity are able to

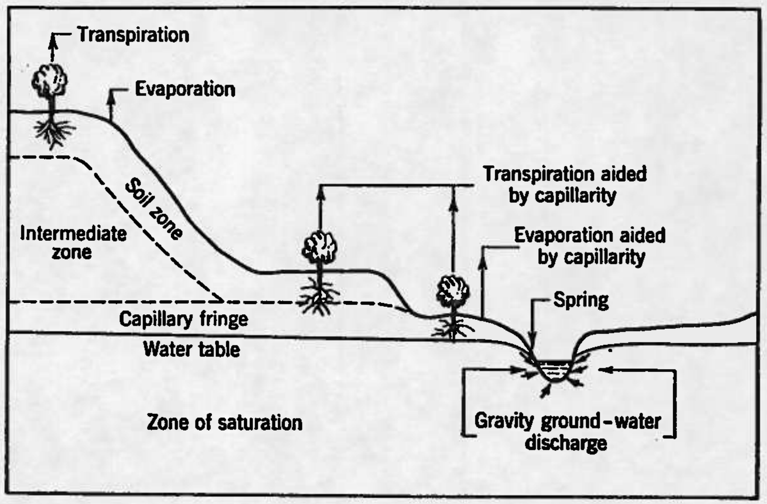


Fig. 71.

pump water from the zone of saturation. Such plants, which are called phreatophytes,[2] can maintain a continuous discharge of water from the water table to the surface and may contribute large quantities of water to the atmosphere.

In regions where the soil belt is very thin, the capillary fringe may be in close contact with the surface and permit direct evaporation discharge to the atmosphere with continuous replenishment from the water table through capillary lift by the fringe material.

[1] L. J. Briggs and H. L. Shantz, The Wilting Coefficient for Different Plants and Its Indirect Determination, *U. S. Department of Agriculture Bureau of Plant Industry Bul.* 230, 1912.

[2] O. E. Meinzer, Outline of Ground-Water Hydrology, *U. S. Geological Survey Water-Supply Paper* 494, 1923, p. 55.

The moisture discharge by evaporation under this condition may be considerable if a large part of the drainage area is represented by land of low relief and small elevation above the surface streams. The rate of evaporation varies greatly in response to the variations in conditions over the soil surface and depends upon the evaporation opportunity, i.e., the ratio of the actual evaporation to the evaporation from a free water surface under existing atmospheric conditions.

A portion of the record obtained from an automatic water-stage recorder in operation on a shallow well near Roscommon, Michigan, is reproduced as Fig. 72. Detailed examination of this diagram shows the consistent daily drawdown of water level through the

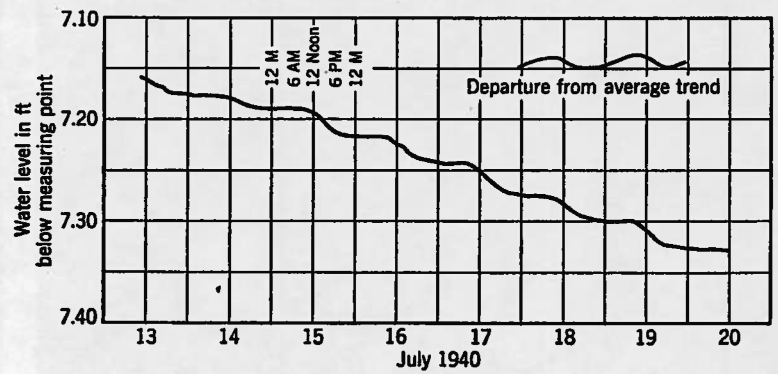


Fig. 72.

sunlight period. A time difference is noted at each end of the cycle, in that the drawdown period generally starts about 8:00 AM each day or about 2½ hr after sunrise, and the recovery of water level or cessation of drawdown starts shortly before 6:00 PM or more than 2 hr before sunset. The maximum rate of drawdown of water level occurs during the noon period when the sun is at the zenith. The periods of so-called recovery are represented by the intervals of decreased drawdown rate and in this example do not imply an actual rise in water level. During the growing season, the general trend of water level is downward, and the transpiration phenomenon is superimposed on this downward trend. To show the effect more clearly, a portion of the graph is reproduced by replotting the departure of the curve from the average downtrend. (See also Figs. 49 and 50, page 160.)

Water Table and Artesian Aquifers

Although the idealized conditions represented by Fig. 71 are found frequently in the field, usually an actual cross section of a valley is more complex than indicated by this diagram. Field reconnaissance may reveal more than one water-bearing formation with considerable variation in the character of each stratum. The many geologic processes involved, coupled with the great variations in intensity and duration of the forces in action during each stage of development leading up to the existing structures, have resulted in an infinite number of variations in the geologic and hydrologic dimensions of the ground-water reservoir. The "hodgepodge" assortment of the drift cover in glaciated areas typifies these complexities. However, the heterogeneous nature of the surficial materials does not invalidate the fundamental principles but merely complicates their application.

A stratum or formation of permeable material that will yield gravity ground water in appreciable quantities is termed an aquifer. The term "appreciable quantity" is relative because, where ground water is obtained with difficulty, even fine-grained, poorly productive materials may be classed as principal aquifers. If an aquifer is overlain by a confining bed of impervious material and if the water level in a tightly cased well penetrating the aquifer rises above the bottom of the confining bed, the aquifer is termed artesian. The overlying confining bed is an aquiclude. The artesian aquifer differs from the water-table aquifer in that the surface, formed by contouring or connecting the heights of the water level in tightly cased wells tapping the aquifer, is not a free surface exposed to the soil atmosphere but is an imaginary pressure surface standing above the body of the aquifer and consequently receives the name of piezometric surface. Although the term piezometric surface can be applied also to the water-table surface, the reverse is not true. Contours drawn on the piezometric surface are referred to as isopiestic lines. A diagrammatic cross section illustrating the application of the above terminology is shown in Fig. 73.

The water level in Well 1, which taps aquifer A, coincides with the water table or surface of the zone of saturation in this aquifer, and consequently Well 1 is a nonartesian or water-table well. The water levels in Wells 2, 3, and 4 stand above the base of the over-

lying aquiclude, and aquifers B and C are artesian. Region a–a is an area of artesian flow, and Well 4 is a flowing well. The lower static level in aquifer B indicates that water is moving from aquifer A or C into aquifer B through a distant break in aquiclude A or B or by vertical leakage through the aquicludes. The high head in Wells 3 and 4 indicates that the recharge or intake area of

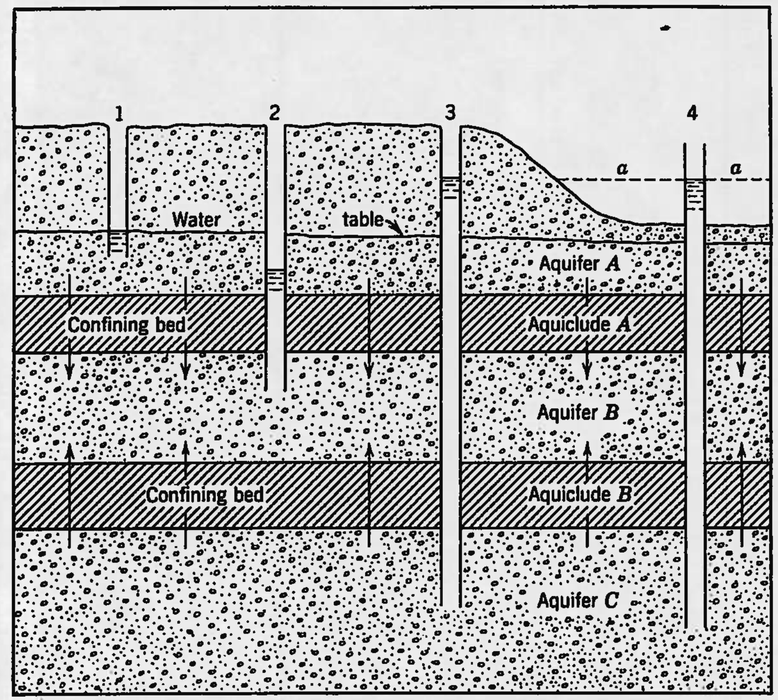
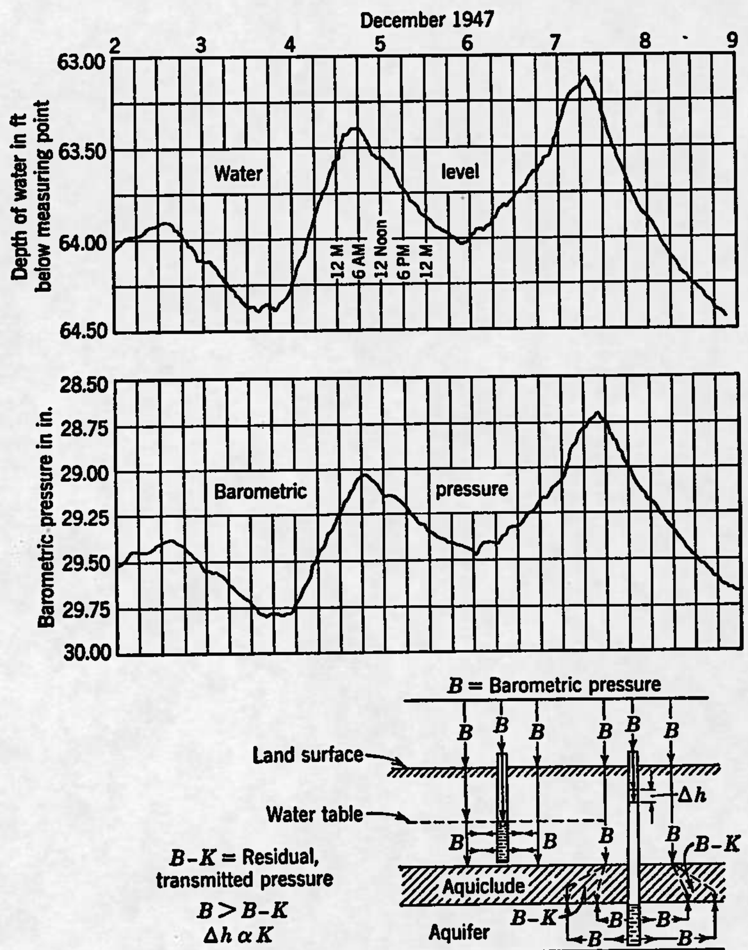


Fig. 73.

aquifer C is at a relatively high elevation, probably above the land surface shown in the cross section. Aquifers like B and C, which contain confined ground water, are pressure conduits and exhibit interesting elastic phenomena.

Elastic Properties of Artesian Aquifers

It is a common observation that wells in some areas will undergo changes of water level during periods of large fluctuation in barometric pressure. A representation of the mechanism causing the change of water level under varying air pressure and a hydrograph from a well exhibiting this effect are shown in Fig. 74. As the barometric pressure increases, the water level in the well casing



Idealized cross section of aquifer showing how bridging action of overlying aquiclude sets up pressure differential in the underlying aquifer which is balanced by a change in hydrostatic head.

Fig. 74.

tends to be depressed. An equal effect is exerted on the soil column and on the shallow water table with a resultant balance of the pressure inside and outside the well tapping the shallow aquifer,

and consequently no net change in water level occurs. In the deeper aquifer, however, the overlying aquiclude is competent to some degree in resisting the load imposed by the rising barometer and does not transmit the full effect of the air pressure change. Consequently, the water level in the well tapping confined ground water is depressed an amount equal to the proportion of the barometric pressure change that is not transmitted by the aquiclude. The ratio of water-level change to the barometric change, in equivalent units, is termed the barometric efficiency of the well. Note that the effect is inverse, that is, as the barometric pressure rises the water level declines.

Inasmuch as the increased hydrostatic pressure in the aquifer, which accompanies a rise in barometric pressure, exceeds the residual pressure transmitted through the aquiclude, a net positive pressure is exerted on the aquiclude. As a result of this pressure the aquiclude is compressed slightly, or, conversely, the aquifer expands a small amount. The slight increase in aquifer volume accommodates the water displaced from the well casing by the increased pressure on the water surface. Changes in barometric pressure are generally of very short duration compared to the time required for the displaced water to move through the formation for any distance. Consequently, barometric fluctuations are recorded in confined aquifers if the overlying aquiclude is competent to resist pressure and extends over an appreciable area. Wells located near an outcrop area or near a break in the aquiclude, where contact with the surface or surface formations occurs within close proximity to the well, will not exhibit barometric effects.

Reports of blowing and sucking wells which exhibit pronounced updraft or downdraft of air at the well mouth may be explained by the above-noted barometric effects. These reports are especially prevalent where an extensive aquiclude occurs some distance above the water table, so that there is a body of air confined between the water table and the aquiclude, which communicates with the atmosphere only through wells. Also the frequent reference to noticeable cloudiness or color in the well water preceding a storm might be explained in part by the rapid rise of water level, which would accompany a barometric low and would bring into the well silty or fine material, as a result of the quick inrush of water through the screen. Some cloudiness may also be caused by gas that escapes from solution when the atmospheric pressure is lowered.

Superimposed loads on the earth's crust also produce changes of water level in wells tapping confined ground water, as indicated by Fig. 75, which shows an autographic record of water-level fluctuation caused by railroad trains passing within 100 ft of the observation well. The alternate loading and unloading of the earth's

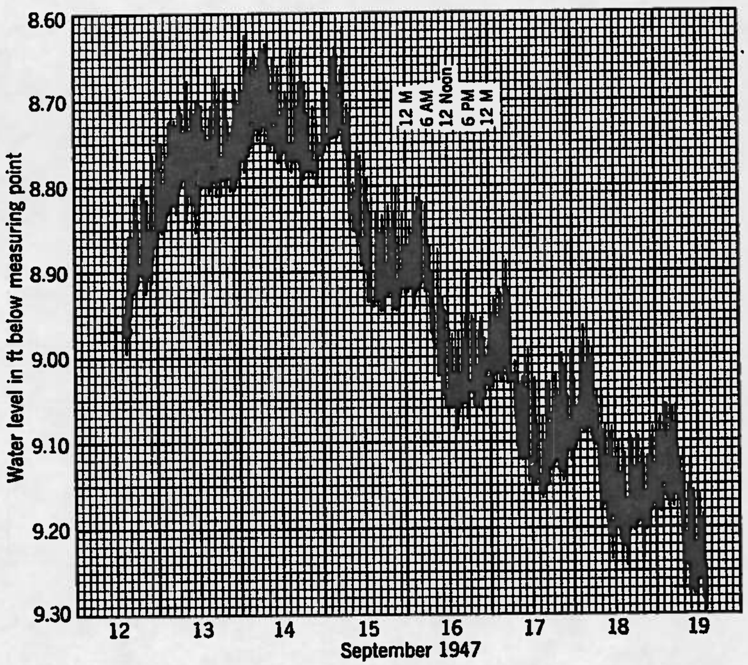


Fig. 75. Hydrograph from automatic water-stage recorder in operation on well tapping Marshall sandstone at Battle Creek, Michigan. Short-period vertical displacements superimposed on curve are water-level fluctuations caused by artesian loading from passing railroad trains.

crust by ocean tides in the coastal areas results in a corresponding cycle of water-level fluctuation in wells as shown by Fig. 76. In these cases, the resultant of the impressed pressure that is transmitted to the aquifer, because of the incompetency of the aquiclude to resist entirely the increase in pressure, causes a rise in water level in the well casing, and the ratio of this rise to the total load impressed is termed the tidal efficiency of the aquifer. Inasmuch as the tidal efficiency is a measure of the incompetency of the aquiclude and the barometric efficiency is a measure of its compe-

tency, it is evident that the sum of the barometric and tidal efficiencies of an aquifer must equal unity, as demonstrated mathematically by Jacob.[1]

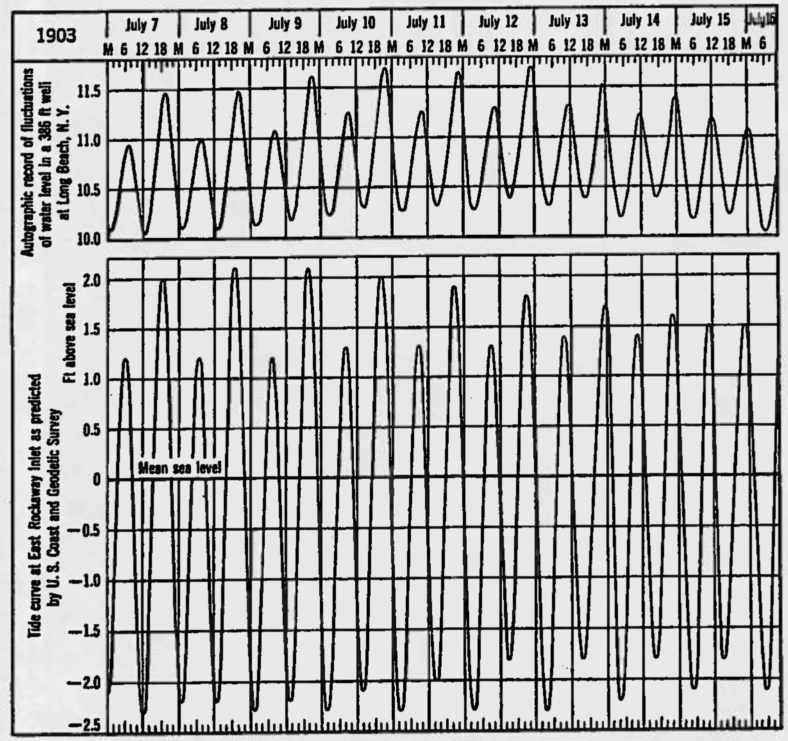


Fig. 76. Hydrograph showing fluctuations of water level in a 386-ft well at Long Beach, N. Y., as compared to the tide at East Rockaway Inlet, N. Y. From Plate 4 of *U. S. Geological Survey Water-Supply Paper* 155.

Subsurface Leakage

The presence of aquicludes or confining layers of considerable thickness and of dense, compact texture has probably served to further the somewhat popular but quite erroneous belief that the artesian aquifers are insulated strata containing connate waters. In this connection, it should be noted that many of our highly developed artesian aquifers would be dry today if such insulation

[1] C. E. Jacob, On the Flow of Water in an Elastic Artesian Aquifer, *Trans. Am. Geophys. Union*, 1940, p. 583.

were general. Fortunately, however, most aquifers receive recharge either through direct infiltration on outcrop areas, through permeable breaks in the confining aquicludes, or by means of leakage through the aquiclude itself. Like many physical terms, the word impervious is only relative and not absolute because air or water will permeate most materials if sufficient time and pressure are involved.

To demonstrate the possible magnitude of aquiclude leakage, there is represented in Fig. 77 an idealized cross section of a

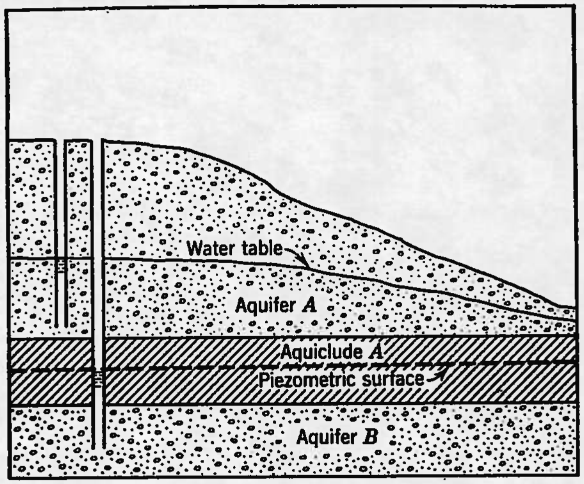


Fig. 77. Generalized cross section of shallow and deep aquifer showing differential hydrostatic head.

geologic condition that is found frequently in the field. It is assumed that the average coefficient of permeability is 2000 gal per day per sq ft for aquifer *B* and 0.2 gal per day per sq ft for aquiclude *A*, or a ratio of 10,000 to 1. The permeability value selected for the aquifer is representative of the average obtained for many sand and gravel formations. The value selected for the aquiclude corresponds to a sample of clayey silt tested in the hydrologic laboratory[1] of the U. S. Geological Survey. The mechanical analysis for this material indicates clay 49.3 per cent,

[1] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, p. 13.

silt 45.3 per cent, and material larger than silt but less than 0.50 mm 5.4 per cent; the porosity was 55.5 per cent. Assume that the water table in the shallow aquifer has a head 50 ft greater than the piezometric surface in the deeper aquifer. Assign a thickness of 50 ft for aquiclude *A*. With the foregoing conditions, it is calculated that the leakage through the aquiclude from the shallow to the deep aquifer would occur at the rate of 0.2 gal per day per sq ft of aquiclude area. This seems a minor item at first consideration, but for each square mile the leakage totals 5.6 million gal per day or enough to supply a community of 56,000 people at an average rate of 100 gal per capita per day. When we consider the many square miles of contributing area available to most large aquifers, it is evident that the assumed aquiclude can contain even less pervious material and still pass appreciable quantities of water.

For the assumed conditions with a porosity of 56 per cent, the movement of water through the aquiclude would occur at the rate of 0.6 in. per day or require about 3 yr for a traverse through the 50-ft section. Accordingly, then, an aquifer recharged only by leakage from adjacent aquicludes will not show water-level fluctuations in response to short period changes in precipitation rate.

In addition to the dewatering problems in subsurface construction where aquifers are exposed by excavation, other difficult problems may arise in deep excavation into an overlying aquiclude. Prior to excavation, the stresses in an aquiclude would be in equilibrium with the total force exerted by the underlying aquifer. Assume that at the site the aquifer has a large hydrostatic head and a high transmissibility. The aquiclude over a long period of time has compacted to a thickness that provides the inherent stability to balance the upward pressure from the aquifer. Although detailed information is not available, it would seem probable that any excavation to appreciable depth in the aquiclude might disturb the force balance to an extent that might result in upheaval of the pit floor and general instability. If rupture of the aquiclude occurred or if permeable zones were exposed, large boils or springs might develop. A condition of this type might be remedied by a few properly spaced wells that penetrate the deep aquifer and are pumped at a rate sufficient to reduce the pressure and restore an equilibrium state.

Underflow

To all who are acquainted with the construction of blind drains the type of ground-water flow termed underflow will strike a familiar chord. The geologic "horse" in sedimentary rock and the buried kames, eskers, alluvium, and outwash channels in the drift mantle are examples of nature's large-scale underdrains. Underflow may occur under either water-table or artesian conditions as shown by Fig. 78. Inasmuch as the word channel is generally used in surface-water terminology for flow with a free surface, the term underflow channel can be assigned for the water-table condition because a free surface exists. In a similar manner, underflow conduit can be used for the artesian condition because the term conduit generally implies confined flow.

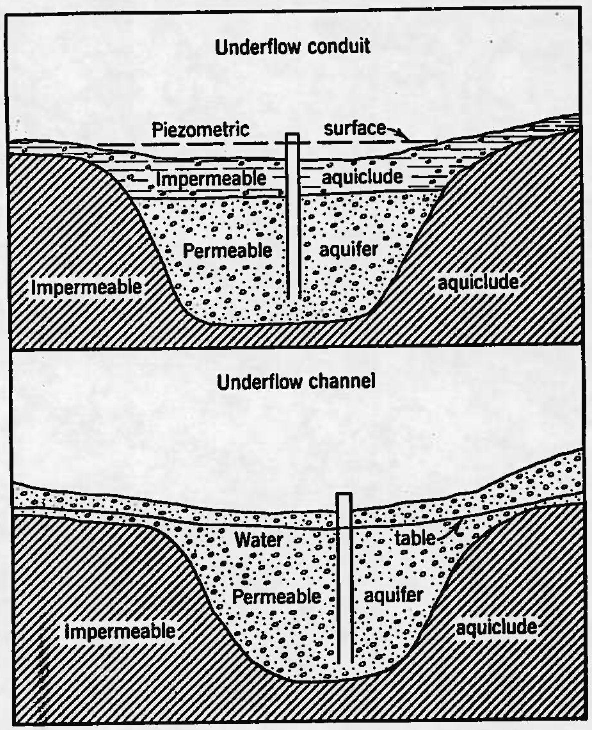


FIG. 78.

In periods of extended drought many stream channels, though dry with reference to surface flow, may carry appreciable quantities of water as underflow. In view of the vast network of preglacial and interglacial stream channels throughout the glaciated areas of the United States, plus the evidence that many of these buried channels are very large, it becomes evident that the underflow in channels filled with very permeable material may be an appreciable part of the base flow from some drainage basins. Although the velocity of such underflow would be very much less than surface flow, the total discharge becomes appreciable if large areas are involved.

Seepage

The movement of water between ground-water aquifers and surface sources can be termed seepage and is further classified as influent seepage, which is recharge from surface bodies of water, and effluent seepage, which is discharge to surface bodies of water. Thus surface streams are influent streams if the stream contributes water to the ground-water reservoir and effluent streams if water is received from the water table. A sketch of conditions existing in each type is shown by Fig. 79. The local build-up of head on the water table underlying an influent stream is termed a ground-water mound. The so-called base flow of surface streams is the effluent seepage from the drainage basin. During periods of prolonged drought, when the total flow of a stream is restricted to the base flow, the stream is functioning solely as a drain. Accordingly, the collection of pertinent data concerning the volume of discharge and the time rate of water-table decline for base-flow periods, which is one phase of present field investigations conducted by the U. S. Geological Survey, will provide in time the basis for application of drain formulas and the ultimate forecast of base flow for major streams.

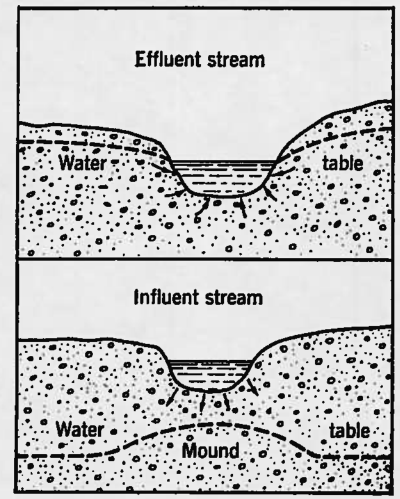


FIG. 79.

Ground-Water Hydraulics

Although the first studies of the flow of water through capillary tubes by Hagen[1] and Poiseuille[2] indicated that the rate of flow is proportional to the hydraulic gradient, it was Darcy[3] who confirmed and applied this law to the flow of water percolating through filter sands. Darcy's law is expressed as follows, in the system of units of general usage,

$$v = \frac{PI}{7.48p} \tag{8}$$

where v is velocity in feet per day, P coefficient of permeability in gallons per day per square foot, I hydraulic gradient in feet per foot, and p porosity in per cent.

In most ground-water problems, the total volume of flow is required, rather than the velocity, and consequently equation 8 is modified to the following form

$$Q_d = PIA \tag{9}$$

where Q_d is discharge in gallons per day, P coefficient of permeability in gallons per day per square foot, I hydraulic gradient in feet per foot, and A area of flow cross section in square feet.

This formula may be adapted for use with the more convenient coefficient of transmissibility by noting the distinction between its definition and that of the coefficient of permeability

$$Q_d = TIW \tag{10}$$

where Q_d and I are defined as above, T is the coefficient of transmissibility in gallons per day per foot, and W the width of flow cross section in feet.

Either equation 9 or 10 may be used for determining the discharge of ground water through underflow channels or conduits or for computing the discharge across a given length of a contour on the water table or the piezometric surface. Most underflow problems can be greatly simplified by assuming an idealized rectangular cross section that closely approximates the actual

[1] G. Hagen, Ueber die Bewegung des Wassers in engen cylindrischen Rohren, *Ann. Physik Chem.*, Leipzig, 1839, **46**, 423–442.

[2] J. L. M. Poiseuille, Recherches expérimentales sur le mouvement des liquides dans les tubes de tres petit diamètre, *Roy. Acad. Sci. Inst. France Math. Phys. Sci. Mem.*, 1846, **9**, 433.

[3] Henri Darcy, *Les Fontaines publiques de la ville de Dijon*, Paris, 1856.

section. An approximation of this type is shown in Fig. 80, with appropriate notations concerning the application of the above terminology. A sample computation is made using the following

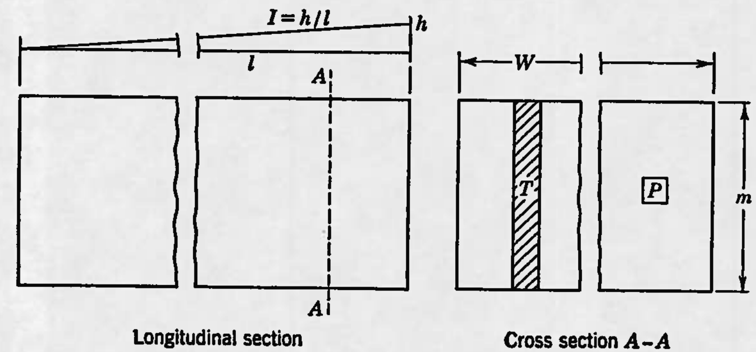


FIG. 80.

assumed values, which are representative of average conditions for a sand and gravel filled channel.

Assumptions:

$$P = 2000 \text{ gal per day per sq ft}$$

$$I = 10 \text{ ft per mile}$$

$$A = mW$$

where $m = 100$ ft, average thickness of aquifer, and $W = 5000$ ft, average width of aquifer; then by equation 9

$$Q_d = PIA$$

$$= 2000 \cdot \left[\frac{10}{5280}\right] \cdot (100 \times 5000)$$

$$Q_d = 1.9 \text{ mgd}$$

or by equation 10

$$Q = TIW$$

where

$$T = P \cdot m = 2000 \cdot 100 = 200{,}000$$

$$Q_d = 200{,}000 \cdot \left[\frac{10}{5280}\right] \cdot 5000$$

$$Q_d = 1.9 \text{ mgd}$$

Although the above formulas are of considerable value in underflow determinations, their application requires a knowledge of the permeability or transmissibility of the aquifer. The first application of Darcy's law to the analysis of the hydraulics of wells was made by Dupuit[1] in 1863. His equation was derived for a discharging well located at the center of a highly idealized circular island and thus quite limited in its application. A modification of the Dupuit analysis was developed by Thiem[2] in 1906 that utilized for the first time two or more observation wells to determine the

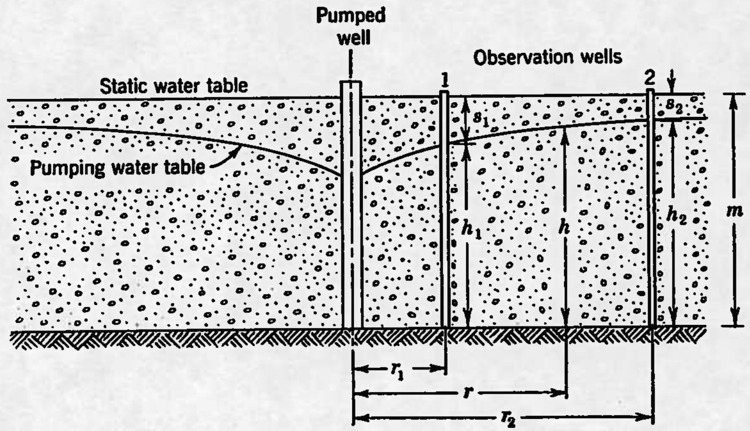


FIG. 81.

field coefficient of permeability. The derivation of Thiem's formula may be set up from the profile section through the cone of depression shown by Fig. 81. From Darcy's law, the flow through any concentric cylindrical section of the water-bearing material is given by equation 9

$$Q_d = PIA$$

Using cylindrical coordinates, we take r as the radius of any cylinder and h as the height of the cone of depression at the distance r from the well. Then

$$I = \frac{dh}{dr} \tag{11}$$

[1] Jules Dupuit, *Études théoretiques et pratiques sur le mouvement des eaux*, Paris, second edition, 1863.
[2] Gunter Thiem, *Hydrologische Methoden*, J. M. Gebhardt, Leipzig, 1906, 56 pp.

and the flow area

$$A = 2\pi rh \tag{12}$$

Therefore

$$Q_d = P \cdot \frac{dh}{dr} \cdot 2\pi rh \tag{13}$$

Rewriting and setting up the integral between the limits r_1 and r_2, the respective distances to observation wells 1 and 2

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi P}{Q_d} \int_{h_1}^{h_2} h\, dh \tag{14}$$

Integrating and inserting the limits

$$\log_e \frac{r_2}{r_1} = \frac{2\pi P}{2Q_d} [h_2^2 - h_1^2] = \frac{\pi P}{Q_d} [h_2^2 - h_1^2] \tag{15}$$

But

$$h_2^2 - h_1^2 = [h_2 + h_1][h_2 - h_1] \tag{16}$$

and

$$h_2 - h_1 = s_1 - s_2 \tag{17}$$

If the amount of drawdown is small compared to the saturated thickness of the water-bearing material then h_2 and h_1 are nearly equal and approximate the saturated thickness, m, or

$$h_2 + h_1 = 2m \tag{18}$$

$$\log_e \frac{r_2}{r_1} = \frac{\pi P}{Q_d} \cdot 2m(s_1 - s_2) = \frac{2\pi Pm}{Q} (s_1 - s_2) \tag{19}$$

but

$$T = Pm$$

therefore

$$\log_e \frac{r_2}{r_1} = \frac{2\pi T}{Q_d} (s_1 - s_2) \tag{20}$$

Converting the logarithm to the base 10, expressing the pumping rate in gallons per minute, and transposing to solve for T

$$T = \frac{527.7Q \log_{10} \frac{r_2}{r_1}}{s_1 - s_2} \tag{21}$$

where T is the coefficient of transmissibility in gallons per day per foot, Q is discharge of pumped well in gallons per minute, r_1 and r_2 are the respective distances of observation well from pumped well in feet, and s_1 and s_2 are the respective drawdown or recovery in observation well in feet.

The derivation of the Thiem formula is based on the following assumptions and its successful application is dependent on the degree to which these qualifications are satisfied by the field conditions: (1) the aquifer is homogeneous, isotropic, and of infinite areal extent; (2) the discharging well penetrates and receives water from the entire thickness of the aquifer; (3) the coefficient of transmissibility is constant at all places and at all times; (4) pumping has continued at a uniform rate for a time sufficient for the hydraulic system to reach an equilibrium stage or a steady flow condition; (5) the flow lines are radial; and (6) flow is laminar. Despite the limiting assumptions, Thiem's formula is widely applicable to ground-water problems and, as will be demonstrated, many of the above limitations can be removed by appropriate adjustment.

As shown by Wenzel,[1] the equilibrium formulas used by Slichter, Turneaure and Russell, Israelsòn, and Muskat are essentially modified forms of Thiem's method. Furthermore, Jacob[2] demonstrated that Wenzel's "limiting formula" and "gradient formula" are also specialized forms of the Thiem method. Accordingly, all the above formulas are limited by the same assumptions used in deriving Thiem's formula. Several of the above equilibrium formulas entail the determination of R, the distance from the pumped well at which the drawdown is inappreciable, and necessarily assume that the drawdown cone has reached a state of equilibrium over the entire area of influence. The use of R arises when observation wells are not available, and the required two points for the equilibrium formula are (1) the pumped well where r and s are measurable and (2) the point of zero drawdown at the radius of influence. Arbitrary values have been assigned to R by several investigators: Slichter[3] gives 600 ft;

[1] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, pp. 79–82.
[2] C. E. Jacob, Notes on Determining Permeability by Pumping Tests under Water-Table Conditions, U. S. Geological Survey, mimeographed report, June 1944.
[3] C. S. Slichter, Theoretical Investigation of the Motion of Ground Water, *U. S. Geological Survey 19th Ann. Rept.*, 1899, Part 2, p. 360.

Muskat[1] 500 ft; and Tolman[2] 1000 ft. Turneaure and Russell[3] calculate R by assuming that the cone of influence ceases development when it has intercepted a width of underflow that contributes a volume of flow equivalent to the discharge of the pumped well. When it is recognized that measurable drawdowns were observed by Leggette[4] at distances as great as 7.1 miles from a pumped well, it appears that under certain conditions the above estimates for R may be low. A ground-water reservoir tends toward a state of equilibrium with natural discharge balancing natural recharge. Consequently, the development of a well disturbs this balance and the new equilibrium state is reached by the propagation of the cone of depression to an extent where the natural recharge is increased or the natural discharge is decreased by an amount equal to the withdrawal by the well. In some instances the radius of the cone necessary to reach the areas of natural recharge or discharge may be many times greater than the values cited or estimated by the above-mentioned investigators.

The Nonequilibrium Formula

A major advancement in ground-water hydraulics was made by Theis[5] in 1935 with his development of the nonequilibrium formula which introduces the time factor and the specific yield or coefficient of storage. This formula was derived by analogy between the flow of ground water and the flow of heat by conduction. Later, Jacob[6] demonstrated the derivation of this formula using hydraulic concepts directly.

A generalized free-body diagram of the flow system in the vicinity of a discharging well is shown by Fig. 82. Assuming that impermeable planes bound the system on top and bottom and all

[1] Morris Muskat, *Flow of Homogeneous Fluids through Porous Media*, McGraw-Hill, 1937, p. 95.
[2] C. F. Tolman, Ground Water, McGraw-Hill, 1937, p. 387.
[3] F. E. Turneaure and H. L. Russell, *Public Water Supplies*, John Wiley, third edition, 1924, p. 258.
[4] R. M. Leggette, The Mutual Interference of Artesian Wells on Long Island, N. Y., *Trans. Am. Geophys. Union*, 1937, p. 493.
[5] C. V. Theis, The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage, *Trans. Am. Geophys. Union*, 1935, pp. 519–524.
[6] C. E. Jacob, On the Flow of Water in an Elastic Artesian Aquifer, *Trans. Am. Geophys. Union*, 1940, pp. 574–586.

flow is radial, we find from the principle of the conservation of matter that the difference in the rate of flow through the inner and

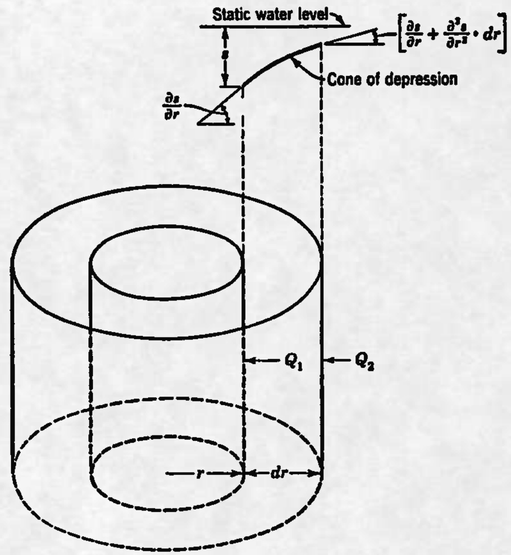


Fig. 82.

outer faces of the cylindrical shell must be drawn from storage within the shell.

$$Q_1 - Q_2 = \frac{dv}{dt} \tag{22}$$

From equation 10 the flow through the inner face is

$$Q_1 = TI_1W_1 = \frac{-T\,\partial s 2\pi r}{\partial r} \tag{23}$$

Since the second derivative defines the rate of change in slope, we can determine the slope or gradient of the piezometric surface at the outer face of the cylinder.

$$I_2 = I_1 + \frac{\partial^2 s}{\partial r^2}dr$$

$$= \frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}dr \tag{24}$$

Then the flow through the outer face is

$$Q_2 = -T\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}dr\right)2\pi(r + dr) \tag{25}$$

The rate of change in volume within the cylindrical shell is expressed as

$$\frac{dv}{dt} = 2\pi r\,dr\,\frac{\partial s}{\partial t}S \tag{26}$$

where S is the coefficient of storage. For water-table conditions, S is equivalent to the specific yield of the materials de-watered by pumping. For artesian conditions, where water is drawn from storage by the compression of the aquifer, S is equal to the water obtained from a column of water-bearing material with a base one foot square and a height equal to the thickness of the aquifer.

Substituting the above values in equation 22

$$-T\frac{\partial s}{\partial r}2\pi r + T\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}dr\right)2\pi(r + dr) = 2\pi r\,dr\,\frac{\partial s}{\partial t}S$$

$$-T\frac{\partial s}{\partial r}2\pi r + T\left[2\pi r\frac{\partial s}{\partial r} + 2\pi r\,dr\frac{\partial^2 s}{\partial r^2} + 2\pi\,dr\frac{\partial s}{\partial r} + 2\pi(dr)^2\frac{\partial^2 s}{\partial r^2}\right]$$

$$= 2\pi r\,dr\,\frac{\partial s}{\partial t}S$$

Dividing through by $2\pi rT\,dr$ and neglecting differentials higher than first order there follows

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \tag{27}$$

This is the differential equation for the radial flow of water in an elastic artesian aquifer. For a constant pumping rate Q, the solution of this equation is given by

$$s = \frac{Q}{4\pi T}\int_{r^2S/4Tt}^{\infty}\frac{e^{-u}}{u}du \tag{28}$$

Expressing Q in gallons per minute and T in gallons per day per foot, equation 28 is written thus

$$s = \frac{114.6Q}{T}\int_{1.87r^2S/Tt}^{\infty}\frac{e^{-u}\,du}{u} \tag{29}$$

where $u = \dfrac{1.87r^2S}{Tt}$ (30)

t = time since pumping started in days

Q = discharge of pumped well in gallons per minute

The expression in equation 29 is not directly integrable as an elementary function, but its value can be computed by the following series.

$$\int_{1.87r^2S/Tt}^{\infty} \frac{e^{-u}}{u}\,du$$
$$= W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \cdots \quad (31)$$

As noted above, the exponential integral is written symbolically as $W(u)$, which in this usage is generally read "well function of u." Tables of the value of this exponential integral have been published.[1] Values of $W(u)$ for values of u from 10^{-15} to 9.9 as tabulated by Wenzel[2] are given in Table 11. Values in the table are values of $W(u)$ for different values u equal to N (Columns 1 and 18), multiplied by 10 to the various powers shown at the top of each remaining column. For example, when u has the value 5.0, $W(u)$ is determined from the line having $N = 5$ and Column 17 as 0.001148, or, when u has the value 0.005, $W(u)$ is 4.7261 (from the same line and Column 14).

From inspection of equations 29 and 30 it is seen that, if s can be measured for one or more values of r and for several values of t and if the discharge Q is known, S and T can be determined. However, the presence of two unknowns and the nature of the exponential integral makes it impossible to effect an exact analytical solution. Inasmuch as one of the unknowns, T, occurs twice in the equation, once in the argument of the function and again as a divisor of the exponential integral, solution by trial would be most laborious. However, a graphical method of superposition, devised by Theis, makes it possible to obtain a simple solution of

[1] *Smithsonian Physical Tables*, Table 32, eighth revised edition, 1933; the values to be used are those given for $Ei(-x)$, with the sign changed.

[2] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, facing p. 89.

the equation. The first step in this method is the plotting of a "type curve," on logarithmic coordinate tracing paper, which represents the evaluation of the exponential integral or series of

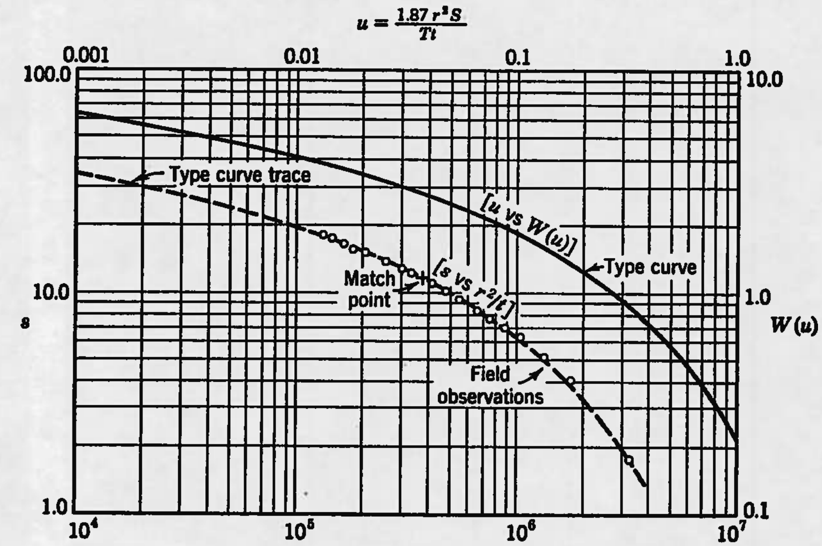


FIG. 83. Logarithmic graph of the exponential integral "type curve."

Q = 250 gpm

Match-point coordinates

$u = 0.1 \quad W(u) = 1.82$

$r^2/t = 3.75 \times 10^5 \quad s = 11.6$

$$T = \frac{114.6Q \cdot W(u)}{s} = \frac{114.6 \times 250 \times 182}{11.6} = 4500 \text{ gpd /ft}$$

$$S = \frac{uT}{1.87r^2/t} = \frac{0.1 \times 4500}{1.87 \times 3.75 \times 10^5} = 6.4 \times 10^{-4}$$

equation 31. From Table 11 values of $W(u)$ are plotted against the argument u to generate the type curve which is shown by a solid line in Fig. 83.

Rearranging equations 29 and 30 there follows

$$s = \left[\frac{114.60}{T}\right] W(u) \quad (32)$$

and

$$\frac{r^2}{t} = \left[\frac{T}{1.87S}\right] u \quad (33)$$

TABLE 11

VALUES OF $W(u)$ FOR NONEQUILIBRIUM FORMULA

1	2	3	4	5	6	7	8	9
N	$N \times 10^{-15}$	$N \times 10^{-14}$	$N \times 10^{-13}$	$N \times 10^{-12}$	$N \times 10^{-11}$	$N \times 10^{-10}$	$N \times 10^{-9}$	$N \times 10^{-8}$
1.0	33.9616	31.6590	29.3564	27.0538	24.7512	22.4486	20.1460	17.8435
1.1	33.8662	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482
1.2	33.7792	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611
1.3	33.6992	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811
1.4	33.6251	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070
1.5	33.5561	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380
1.6	33.4916	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735
1.7	33.4309	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128
1.8	33.3738	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557
1.9	33.3197	31.0171	28.7145	26.4119	24.1094	21.8068	19.5042	17.2016
2.0	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503
2.1	33.2196	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550
2.3	33.1286	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106
2.4	33.0861	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680
2.5	33.0453	30.7427	28.4401	26.1375	23.8349	21.5323	19.2298	16.9272
2.6	33.0060	30.7035	28.4009	26.0983	23.7957	21.4931	19.1905	16.8880
2.7	32.9683	30.6657	28.3631	26.0606	23.7580	21.4554	19.1528	16.8502
2.8	32.9319	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138
2.9	32.8968	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788
3.0	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449
3.1	32.8302	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121
3.2	32.7984	30.4958	28.1932	25.8907	23.5881	21.2855	18.9829	16.6803
3.3	32.7676	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495
3.4	32.7378	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197
3.5	32.7088	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907
3.6	32.6806	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625
3.7	32.6532	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351
3.8	32.6266	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085
3.9	32.6006	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825
4.0	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572
4.1	32.5506	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325
4.2	32.5265	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084
4.3	32.5029	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3848
4.4	32.4800	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174
4.7	32.4140	30.1114	27.8088	25.5062	23.2037	20.9011	18.5985	16.2959
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4596	16.1571
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207
5.7	32.2211	29.9185	27.6159	25.3133	23.0108	20.7082	18.4056	16.1030
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029
6.4	32.1053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717
6.6	32.0745	29.7719	27.4693	25.1667	22.8641	20.5616	18.2590	15.9564
6.7	32.0595	29.7569	27.4543	25.1517	22.8491	20.5465	18.2439	15.9414
6.8	32.0446	29.7421	27.4395	25.1369	22.8343	20.5317	18.2291	15.9265
6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119
7.0	32.0156	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976
7.1	32.0015	29.6989	27.3963	25.0937	22.7911	20.4885	18.1860	15.8834
7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694
7.3	31.9737	29.6711	27.3685	25.0659	22.7633	20.4608	18.1582	15.8556
7.4	31.9601	29.6575	27.3549	25.0523	22.7497	20.4472	18.1446	15.8420
7.5	31.9467	29.6441	27.3415	25.0389	22.7363	20.4337	18.1311	15.8286
7.6	31.9334	29.6308	27.3282	25.0257	22.7231	20.4205	18.1179	15.8153
7.7	31.9203	29.6178	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022
7.8	31.9074	29.6048	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516
8.2	31.8574	29.5548	27.2523	24.9497	22.6471	20.3445	18.0419	15.7393
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034
8.6	31.8098	29.5072	27.2046	24.9020	22.5995	20.2969	17.9943	15.6917
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801
8.8	31.7868	29.4842	27.1816	24.8790	22.5765	20.2739	17.9713	15.6687
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574
9.0	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6243
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135
9.4	31.7208	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509

(From U. S. Geological Survey Water-Supply Paper 887.)

TABLE 11—*(Continued)*

VALUES OF $W(u)$ FOR NONEQUILIBRIUM FORMULA

10	11	12	13	14	15	16	17	18
$N \times 10^{-7}$	$N \times 10^{-6}$	$N \times 10^{-5}$	$N \times 10^{-4}$	$N \times 10^{-3}$	$N \times 10^{-2}$	$N \times 10^{-1}$	N	N
15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194	1.0
15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	.1860	1.1
15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	1.6595	.1584	1.2
15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	1.5889	.1355	1.3
15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	1.5241	.1162	1.4
15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	1.4645	.1000	1.5
15.0709	12.7683	10.4657	8.1634	5.8621	3.5739	1.4092	.08631	1.6
15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	1.3578	.07465	1.7
14.9531	12.6505	10.3479	8.0455	5.7446	3.4581	1.3098	.06471	1.8
14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	1.2649	.05620	1.9
14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	.04890	2.0
14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	1.1829	.04261	2.1
14.7524	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	.03719	2.2
14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	1.1099	.03250	2.3
14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	1.0762	.02844	2.4
14.6246	12.3220	10.0194	7.7172	5.4167	3.1365	1.0443	.02491	2.5
14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	1.0139	.02185	2.6
14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	0.9849	.01918	2.7
14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	.9573	.01686	2.8
14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	.9309	.01482	2.9
14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	.9057	.01305	3.0
14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	.8815	.01149	3.1
14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	.8583	.01013	3.2
14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	.8361	.008939	3.3
14.3171	12.0145	9.7120	7.4097	5.1102	2.8379	.8147	.007891	3.4
14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	.7942	.006970	3.5
14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	.7745	.006160	3.6
14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	.7554	.005448	3.7
14.2059	11.9033	9.6007	7.2985	4.9993	2.7306	.7371	.004820	3.8
14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	.7194	.004267	3.9
14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	.7024	.003779	4.0
14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	.6859	.003349	4.1
14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	.6700	.002969	4.2
14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	.6546	.002633	4.3
14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	.6397	.002336	4.4
14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	.6253	.002073	4.5
14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	.6114	.001841	4.6
13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	.5979	.001635	4.7
13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	.5848	.001453	4.8
13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	.5721	.001291	4.9
13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	.5598	.001148	5.0
13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	.5478	.001021	5.1
13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	.5362	.0009086	5.2
13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	.5250	.0008086	5.3
13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	.5140	.0007198	5.4
13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	.5034	.0006409	5.5
13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	.4930	.0005708	5.6
13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	.4830	.0005085	5.7
13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	.4732	.0004532	5.8
13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	.4637	.0004039	5.9
13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	.4544	.0003601	6.0
13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	.4454	.0003211	6.1
13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	.4366	.0002864	6.2
13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	.4280	.0002555	6.3
13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	.4197	.0002279	6.4
13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	.4115	.0002034	6.5
13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	.4036	.0001816	6.6
13.6388	11.3362	9.0337	6.7317	4.4351	2.1917	.3959	.0001621	6.7
13.6240	11.3214	9.0189	6.7169	4.4204	2.1779	.3883	.0001448	6.8
13.6094	11.3068	9.0043	6.7023	4.4059	2.1643	.3810	.0001293	6.9
13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	.3738	.0001155	7.0
13.5808	11.2782	8.9757	6.6737	4.3775	2.1376	.3668	.0001032	7.1
13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	.3599	.00009219	7.2
13.5530	11.2504	8.9479	6.6460	4.3500	2.1118	.3532	.00008230	7.3
13.5394	11.2368	8.9343	6.6324	4.3364	2.0991	.3467	.00007364	7.4
13.5260	11.2234	8.9209	6.6190	4.3231	2.0867	.3403	.00006583	7.5
13.5127	11.2102	8.9076	6.6057	4.3100	2.0744	.3341	.00005886	7.6
13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	.3280	.00005263	7.7
13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	.3221	.00004707	7.8
13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	.3163	.00004210	7.9
13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	.3106	.00003767	8.0
13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	.3050	.00003370	8.1
13.4367	11.1342	8.8317	6.5298	4.2346	2.0042	.2996	.00003015	8.2
13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	.2943	.00002699	8.3
13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	.2891	.00002415	8.4
13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	.2840	.00002162	8.5
13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	.2790	.00001936	8.6
13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	.2742	.00001733	8.7
13.3661	11.0635	8.7610	6.4592	4.1646	1.9393	.2694	.00001552	8.8
13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	.2647	.00001390	8.9
13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	.2602	.00001245	9.0
13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	.2557	.00001115	9.1
13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	.2513	.000009988	9.2
13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	.2470	.000008948	9.3
13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	.2429	.000008018	9.4
13.2896	10.9870	8.6845	6.3828	4.0887	1.8695	.2387	.000007185	9.5
13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	.2347	.000006439	9.6
13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	.2308	.000005771	9.7
13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	.2269	.000005173	9.8
13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	.2231	.000004637	9.9

If a constant withdrawal rate is maintained, that is, if Q is constant, the bracketed portions of equations 32 and 33 are constant for a given pumping test. Note that s is related to r^2/t in a manner that is similar to the relation of $W(u)$ to u. Consequently, if values of the drawdown or recovery, s, are plotted against r^2/t on logarithmic coordinate tracing paper, to the same scale as the type curve, $W(u)$ versus u, the curve of observed data will be similar to the type curve.

The circles on Fig. 83 represent the successive values of drawdown as computed from periodic measurements of the water-level decline in an observation well 48 ft from a well that was pumped at the constant rate of 250 gpm. In practice, the computed values of s and r^2/t are plotted on a separate sheet of logarithmic tracing paper and this graph of observed data is superimposed on the graph of the type curve. When the coordinate axes of the two curves are held parallel, the data curve is translated to a position which represents the best fit of the field data to the type curve. With both graph sheets at the best match position, an arbitrary point on the top curve is selected and pricked through or otherwise marked on the lower curve. The coordinates of this common point are noted for the upper and the lower graph. The trace of the type curve, for the match position, is shown as a dashed line through the field data, on Fig. 83. The coordinates of the match point are recorded, and the use of these data with equations 32 and 33 to solve for T and S is also shown.

The determination of the coefficients of transmissibility and storage for an aquifer, by the discharging-well method, is somewhat similar to the testing procedure of measuring beam deflections under a given load to determine the elastic properties of a structural material. As in the case of the beam, we can predict the drawdown or deflection of the water level at any point or for any load from a knowledge of its behavior under a known load at known points of observation.

Assume that a well of 300-gpm capacity is to be drilled in the comparatively poor aquifer covered by the pumping test of Fig. 83 and that the following conditions apply for this example.

Total depth of well	= 200 ft
Screen setting	= 170 to 200 ft
Diameter of well	= 12 in.
Proposed yield	= 300 gpm
Static depth to water	= 5 ft below grade

The problem is to predict the performance of this well for 30 days of continuous operation at peak capacity with total withdrawal from storage, that is, no recharge from rainfall or other sources. From the above conditions we may set up the known quantities as follows:

$$Q = 300 \text{ gpm} \qquad r = 6 \text{ in.} = 0.5 \text{ ft}$$
$$t = 30 \text{ days} \qquad T = 4500 \text{ gpd/ft}$$
$$S = 6.4 \times 10^{-4}$$

Then from equation 30

$$u = \frac{1.87(0.5)^2 6.4}{4.5 \times 10^3 \times 30 \times 10^4} = 2.2 \times 10^{-9}$$

The value of $W(u)$ corresponding to the above value of u is read from Table 11 as

$$W(u) = 19.4$$

From equation 32

$$s = \frac{114.6 \times 300 \times 19.4}{4500} = 148 \text{ ft}$$

The pumping level at the end of the 30-day period is

Static level	=	5 ft below grade
Drawdown	=	148 ft
Pumping level	=	153 ft below grade

The estimated pumping level is only 17 ft above the top of the well screen by the end of the 30-day period if the well is pumped continuously at the maximum rate. Consequently, it is advisable to examine the performance of this proposed well over longer periods of pumping. In the manner outlined above for the computation of the 30-day pumping level, the levels for other periods are computed and plotted as the lowermost curve on Fig. 84. As indicated by this curve, the pumping level will reach the top of the well screen after 9 mo of continuous pumping at the 300-gpm rate. We assume that the aquifer is completely penetrated[1] and that the top of the aquifer coincides with the top of the well screen. Accordingly, the lowering of the water level in the pumped well below the top of the aquifer results in partial dewatering of the

[1] The discharge Q will be smaller for a partially penetrating well. For a discussion of the effect of partial penetration on well yield see reference 30 at the end of this chapter.

aquifer. This reduction in saturated thickness proportionately reduces the transmissibility and thereby increases the drawdown and furthers the decline of pumping level for the given discharge rate. Extensive dewatering of an aquifer develops a "vicious cycle" of increased drawdown followed by decreased aquifer thickness, a condition which occasions additional drawdown and further reduction in aquifer capacity until the well fails at the excessive pumping rate. If the rate of pumping is reduced, the water level in the pumped well will recover in proportion to the reduction in discharge. For the above example reducing the pumping rate to

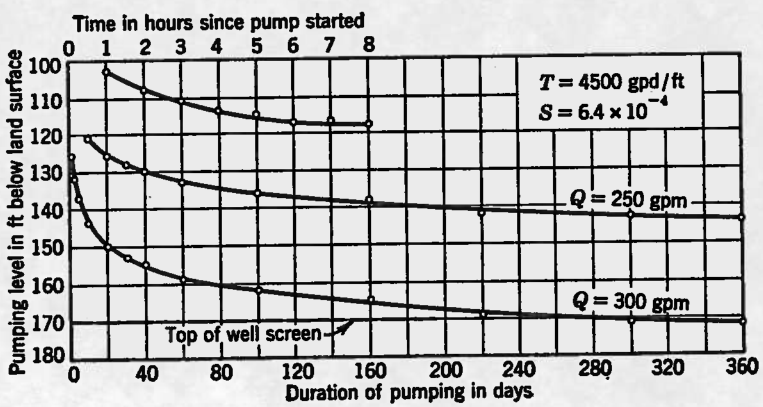


Fig. 84.

250 gpm raises the pumping level to 28 ft above the top of the well screen at the time when the 300-gpm rate would start dewatering the aquifer.

In the above example, the original assumption that all water would be withdrawn from storage without replenishment results in a progressive decline of water level to infinite time. Such calculations are of value in estimating minimum performance of wells under adverse conditions of extended drought. Under normal conditions, as pumping continues, the cone of depression deepens and expands until (1) it intercepts a surface stream which is adequate to support the well discharge under the given conditions, (2) it encompasses an area that will support the well yield under the prevailing rate of surface infiltration, or (3) it intercepts areas of discharge and reduces this discharge by an amount equal to the well withdrawal. Most frequently, the well yield is obtained from a

combination of two or more of these sources. If the total water available from the several sources is less than the pumping rate, progressive decline of water level may occur to a degree determined by the excess of discharge over recharge. Ultimately the net discharge cannot be greater than the available recharge from all sources.

As indicated in the above example, excessive dewatering of an aquifer is to be avoided, but it does not follow that all dewatering is objectionable. In any water-table aquifer it is of course necessary to partially dewater the aquifer to induce flow toward the well. Sometimes it may be advisable to reduce the saturated thickness to provide storage capacity for the intake of percolating waters when infiltration occurs. If the cone of depression is maintained at a high stage and a shallow depth below land surface, then, when recharge occurs, all water in excess of the limited storage capacity will be rejected. The problem of determining proper pumping levels and well yields is one of engineering economics, which must be solved for each particular well field.

It is of interest to note that the majority of well installations to date were rated or sized without benefit of long-term drawdown predictions. The most general and widespread method in use for determining well capacity is to make a brief and generally inadequate pumping test. Measurements are made of the well discharge and drawdown through this short pumping period. Water levels are measured by an air line and altitude gage and are accurate to perhaps the nearest foot. If the pumping level remains relatively steady during the latter part of the test period, as it generally does when the pumping rate is constant because the decline is rather slow after several hours, it is quite common practice to assume that the well will continue to pump at the observed rate for an infinite time. The pumping levels for the above example are computed for an 8-hr period and a discharge of 300 gpm, and plotted as the uppermost curve on Fig. 84. The level from the sixth to the eighth hour inclusive does not decline more than 2 ft and air-line readings would probably show little or no change in drawdown for this period. Thus, in accord with the above-mentioned practice, it is assumed that this well will deliver 300 gpm or more for any period. The fallacy of this reasoning is better understood after examination of the modified form of the nonequilibrium formula.

Modified Nonequilibrium Formula

It was recognized by Jacob[1] that the sum of the terms in the series of equation 31 beyond $\log_e u$ is not of appreciable magnitude when u becomes small. From the form of equation 30 it is noted that u decreases as the time, t, increases. Accordingly, for large values of t, the terms beyond $\log_e u$ in the exponential series may be neglected and equation 32 may be written

$$s = \frac{114.6Q}{T} W(u) = \frac{114.6Q}{T} [-0.5772 - \log_e u]$$

or

$$s = \frac{114.6Q}{T}\left[\log_e\left(\frac{1}{u}\right) - 0.5772\right] \tag{34}$$

but

$$u = \frac{1.87r^2S}{Tt} \qquad \text{and} \qquad \frac{1}{u} = \frac{Tt}{1.87r^2S}$$

then

$$s = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt}{1.87r^2S}\right) - 0.5772\right] \tag{35}$$

In applying the above equation to measurements of the drawdown or recovery of water level in a particular observation well, the distance r will be constant and there follows

at time t_1

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_1}{1.87r^2S}\right) - 0.5772\right]$$

at time t_2

$$s_2 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_2}{1.87r^2S}\right) - 0.5772\right]$$

then the change in drawdown from time t_1 to t_2

$$s_2 - s_1 = \frac{114.6Q}{T}\log_e\left(\frac{t_2}{t_1}\right) \tag{36}$$

Converting to logarithms to the base 10

$$s_2 - s_1 = \frac{264Q}{T}\log_{10}\left(\frac{t_2}{t_1}\right) \tag{37}$$

[1] C. E. Jacob, Drawdown Test to Determine Effective Radius of Artesian Well, *Proc. A.S.C.E.*, vol. 72, No. 5 (May 1946), pp. 629–646.

where Q and T are as previously defined, t_1 and t_2 are time in days since pumping started, and s_1 and s_2 are respective drawdowns at noted times, in feet.

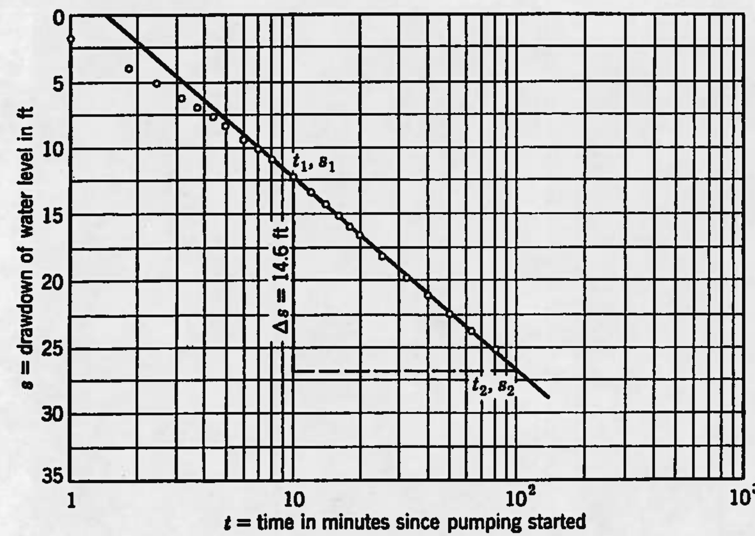


FIG. 85. Semilog graph of pumping test data for application of modified Theis formula.

$$Q = 250 \text{ gpm}$$
$$r = 48 \text{ ft}$$
$$T = \frac{264Q \cdot \log_{10} t_2/t_1}{s_2 - s_1}$$
$$T = \frac{264 \times 250 \times \log_{10} 100/10}{26.8 - 12.2}$$
$$T = 4500 \text{ gpd/ft}$$
$$S = \frac{0.3Tt_0}{r^2}$$
$$S = \frac{0.3 \times 4500 \times \frac{1.46}{1440}}{48^2}$$
$$S = 6.0 \times 10^{-4}$$

The most convenient procedure for application of the above equation is to plot the observational data for each well on semilogarithmic coordinate paper as shown by Fig. 85, which is based on the same test data used in Fig. 83. From this curve make an arbitrary choice of t_1 and t_2 and note the corresponding values of

s_1 and s_2. For convenience t_1 and t_2 are chosen one log cycle apart; then

$$\log_{10}\left(\frac{t_2}{t_1}\right) = 1$$

and

$$s_2 - s_1 = \Delta s = \frac{264Q}{T} \tag{38}$$

or

$$T = \frac{264Q}{\Delta s} \tag{39}$$

where Δs is the drawdown difference per log cycle, in feet.

Although the absolute value of the drawdown increases as the logarithm of the time of pumping, it follows from equation 38 that the drawdown per log cycle of time varies directly with the discharge Q, and inversely as the coefficient of transmissibility T.

Extrapolating the straight line of the semilog curve to its intersection with the zero-drawdown axis permits computation of S, the storage coefficient, as follows.

When $s = 0$, equation 24 yields

$$s = 0 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_0}{1.87r^2S}\right) - 0.5772\right]$$

$$\log_e\left(\frac{Tt_0}{1.87r^2S}\right) = 0.5772 \qquad \text{or} \qquad \frac{Tt_0}{1.87r^2S} = e^{0.5772}$$

and

$$S = \frac{Tt_0}{1.87r^2e^{0.5772}} = \frac{0.3Tt_0}{r^2} \tag{40}$$

where S, T, and r are as previously defined and t_0 is time intercept on zero-drawdown axis, in days.

The curvature of the semilog graph where time t is small indicates that the approximation is not valid in this region. Caution should be exercised in the use of this method to make certain that pumping has continued until t becomes large and all plotted points fall on a straight line. For moderate distances from the pumped well, this condition is generally satisfied within an hour or less for artesian conditions, but for water-table aquifers 12 hr or more may be required because of the time lag due to slow draining of the interstices.

It is advisable for any test to plot both the log-log graph for the type curve application and the semilog plot for the modified formula. The semilog plot, as indicated above, is an approximation method which yields a straight-line graph in the region where this approximation is justified. As shown by Fig. 83, when t is small and u is appreciable, the field observations fall beneath the selected line. As the distance, r, increases, the time required for the field data to become asymptotic to the limiting value of the slope also increases. Thus, for an observation well at a considerable distance from the pumped well, there is a greater risk in selecting the tangent prematurely. Assurance is gained by comparison with the result given by the log-log plot and by mutual agreement with the results from other observation wells. The interference effect of other pumping wells, the limitation of geologic boundaries, and other extraneous effects may distort the plotted data in a manner that makes the straight-line selection unreliable. The semilog plot does, however, present certain advantages in pumping-test analysis and its use is well justified as long as proper caution is observed. If analysis of the data is correct, the values of T and S determined by either method should agree within a small percentage.

In general, it is not possible to determine the storage coefficient, S, from observations within the pumping well, by either of the above methods because the effective radius of the well is not known. For a well finished in rock, the effective radius may approach the nominal radius, but not necessarily because the existence of large crevices, the overreaming action of the drill bit, or local cementation of the bore face may result in an effective radius greater or smaller than the nominal size. In a well finished in unconsolidated materials, the water level in the pumped well is lower than the water level in an equivalent uncased hole by the amount of friction loss through the screen. If development of the well is incomplete, the packing of fine material in the formation adjacent to the screen can greatly reduce the permeability and result in an effective radius which is considerably less than the nominal drilled size. A method of determining the effective radius for any well has been developed by Jacob.[1]

Noting from equation 35 that the drawdown varies with the

[1] C. E. Jacob, Drawdown Test to Determine Effective Radius of Artesian Well, *Proc. A.S.C.E.*, vol. 72, No. 5 (May 1946), pp. 629–646.

logarithm of the time of pumping we can recognize the fallacy in determining well capacity from the short-duration type of pumping test which was outlined previously. Any linear plot of drawdown level versus pumping time, on a time scale extended to fit an 8-hr pumping test as shown by the uppermost curve of Fig. 83 will invariably give the deceptive picture of the approach to a fixed pumping level. This flattening of slope is representative of a logarithmic relation when plotted to rectangular coordinates. If the pumping levels for the 8-hr test were plotted to the compressed time scale of the lower graph, there would appear to be a nearly vertical decline of operating level for the 8-hr period. When one considers that a well user may plan to pump a well continuously for the total of 8760 hr per year and for many years, it is evident that the 8-hr or other short-term acceptance test is not only inadequate but quite misleading unless supported by quantitative methods such as those outlined above.

Adjustment of Test Data for Thin Aquifers

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One of the basic assumptions in the derivation of the Thiem and the Theis formulas is the stipulation of a constant value of transmissibility. However, under water-table conditions the drawdown of water level by a discharging well reduces the saturated thickness of the aquifer, and, if this reduction in thickness is appreciable, the transmissibility is not constant but decreases with time. The following method, described by Jacob,[1] permits the correction of observed data to compensate for this effect. Appreciable reduction in saturated thickness voids the relation expressed by equation 18, and consequently equation 15 must be reduced to the drawdown form in some other manner. From Fig. 81 note that

$$h = m - s$$

then

$$h_2^2 = m^2 - 2ms_2 + s_2^2$$

$$h_1^2 = m^2 - 2ms_1 + s_1^2$$

and

$$h_2^2 - h_1^2 = -2ms_2 + 2ms_1 + s_2^2 - s_1^2$$

$$= 2ms_1 - s_1^2 - (2ms_2 - s_2^2)$$

[1] C. E. Jacob, Notes on Determining Permeability by Pumping Tests under Water-Table Conditions, U. S. Geological Survey, mimeographed report, June 1944.

$$h_2^2 - h_1^2 = 2m\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right] \tag{41}$$

Substituting the above expression in equation 15 there follows

$$\log_e \frac{r_2}{r_1} = \frac{\pi P}{Q_d} 2m\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right]$$

but

$$T = Pm$$

then

$$\log_e \frac{r_2}{r_1} = \frac{2\pi T}{Q_d}\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right] \tag{42}$$

Converting to logarithms to the base 10, replacing Q_d by Q, and transposing equation 42, there follows

$$T = \frac{527.7Q \log_{10} r_2/r_1}{[(s_1 - s_1^2/2m) - (s_2 - s_2^2/2m)]} \tag{43}$$

The above equation should be used in lieu of equation 21 if the saturated thickness of the aquifer is appreciably diminished by the declining water level. Note that if the drawdowns s_1 and s_2 are very small compared to the original saturated thickness, m, the correction fraction may be omitted, and equation 43 reduces to equation 21. Compensation of the drawdown data by subtracting the factor $(s^2/2m)$ should result in a straight-line graph for the semilog plottings of the Thiem or modified Theis method.

The Method of Images

The assumption of infinite areal extent for an aquifer, which is necessary for the development of either the equilibrium or the nonequilibrium formula, is essentially fulfilled by a few major aquifers of sedimentary rock, such as the Dakota sandstone described by Meinzer.[1] However, in most areas the existence of formation boundaries or of folds and faults or the dissection by surface streams serves to limit the continuity of consolidated strata to distances of a few miles or more. In the unconsolidated materials and particularly in the glaciated areas the prerequisite

[1] O. E. Meinzer and H. A. Hard, The Artesian-Water Supply of the Dakota Sandstone in North Dakota, with Special Reference to the Edgeley Quadrangle, *U. S. Geological Survey Water-Supply Paper* 520, 1925, pp. 73–95.

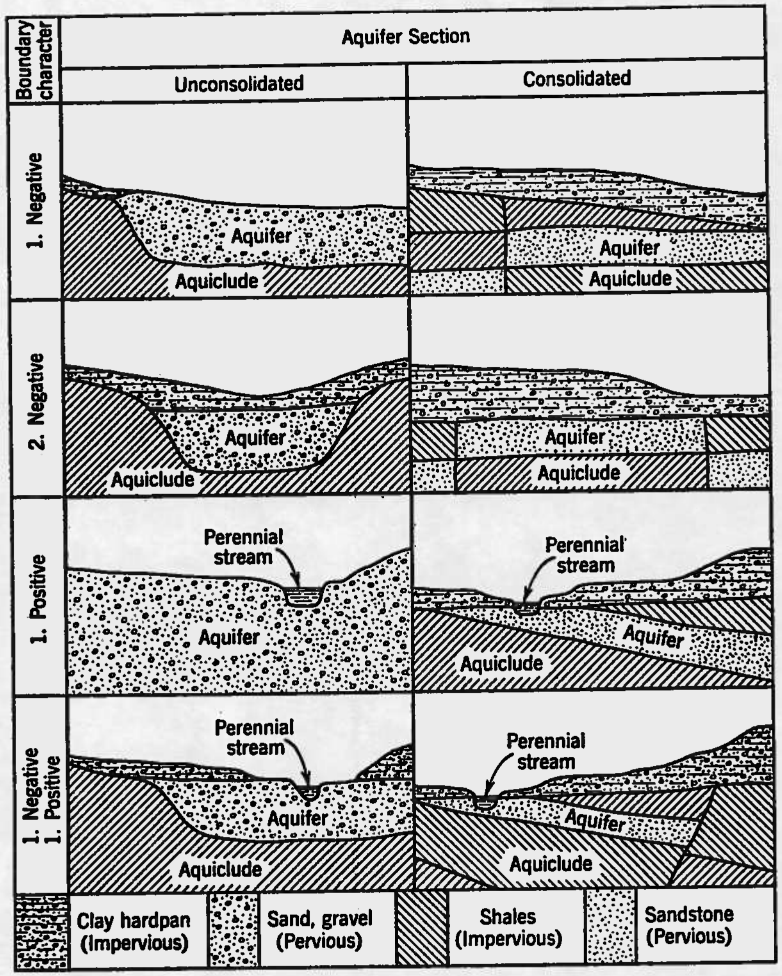


Fig. 86. Idealized examples showing possible geologic structures which form definite bounding conditions for the flow systems in aquifers. The term negative is used for boundaries formed by impermeable materials which do not contribute flow to the system. Positive boundaries refer to the intersection of the aquifer by sources of water which sustain the hydrostatic head and stop the growth of the drawdown cone.

of infinite areal extent is seldom satisfied. Consequently, it is necessary to make appropriate adjustment for the effect of these geologic boundaries before the above formulas can be applied to problems of flow in areally limited aquifers.

Inasmuch as an impervious formation detracts from the contributing area of the aquifer it bounds, we refer to its contact as a negative boundary. In a similar manner, we use the term positive boundary where an aquifer is intersected by a perennial stream or other body of surface water with sufficient flow to prevent development of the cone of depression beyond the surface source. Several possible types of geologic boundaries are shown in generalized form by Fig. 86.

It is recognized that, except for some faulted structures, most geologic boundaries do not occur as abrupt straight-line demarcations but rather as tapered and irregular terminals. In general, however, the area covered by a well-field or pumping-test site is relatively small compared to the area of even the limited aquifers, and it is often possible to treat the geologic boundary as an abrupt discontinuity. The greater the distance to the boundary from the well site the smaller would be the error involved by this approximation. Where conditions permit the assumption of a straight-line demarcation for a geologic boundary, it is possible to solve the flow problem by the substitution of a hypothetical system that satisfies the limits of the real system.

The method of images devised by Lord Kelvin in his work on electrostatic theory is a convenient tool for the solution of boundary problems. An idealized section of an aquifer that is intersected and bounded by a surface stream is shown by Fig. 87. To be effective as a boundary, the stream flow must equal or exceed the withdrawal of the well because any flow below the well yield would result in drying up of the stream and elimination of the boundary. Assume the stream to be of infinitesimal width or the equivalent of a line source. In a rigid analysis it would be necessary also that the stream extend the full depth of the aquifer to justify fully the use of unidimensional method. However, reasonable estimates can be made by this method when observation wells are available at distances that are sufficient to minimize the effect of vertical flow components. If the stream cannot be depleted, the boundary limit requires that there shall be zero drawdown at the line source. Any system that can satisfy this boundary limit is a solution of the real problem. As shown by the central diagram of Fig. 87, the real and bounded aquifer is replaced by an imaginary aquifer of infinite areal extent and an imaginary recharging well is placed on the opposite side of and equidistant from the boundary. As illustrated,

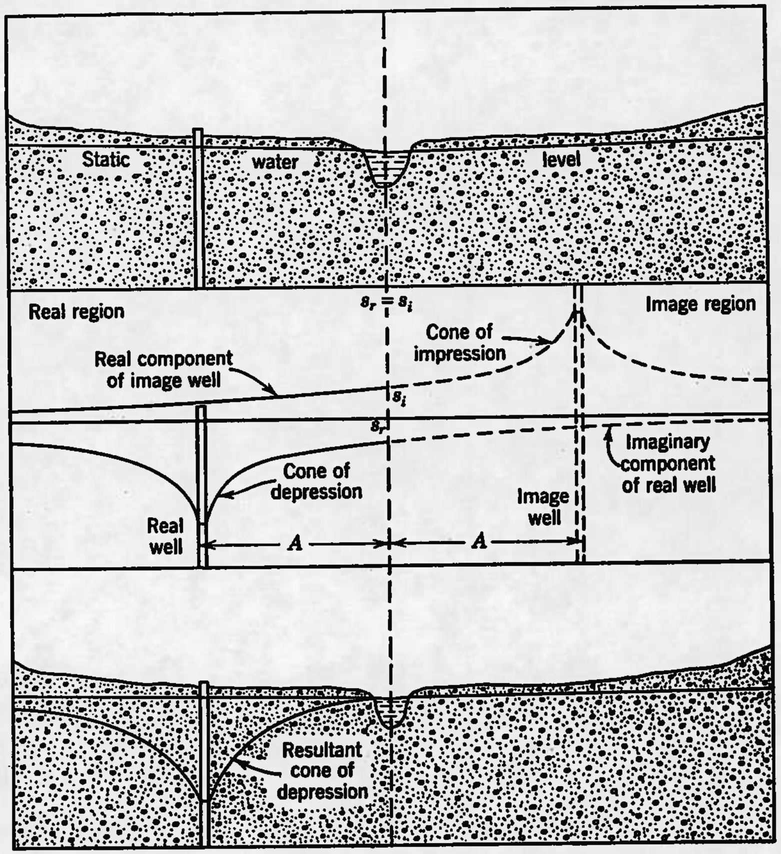


FIG. 87. Idealized section of an aquifer which is intersected by a stream together with a hypothetical well system for the solution of this type of flow problem.

the imaginary recharge well returns water to the aquifer at the same rate as it is withdrawn by the real discharge well. Consequently, the image well produces a build-up of water level at the boundary that is exactly equal to and cancels the drawdown of the real well. This system results in zero drawdown at the boundary which satisfies the limit of the real problem.

The real components of the cone of depression of the real well and the cone of impression of the image well are shown as solid lines in the region of real values. To secure the resultant cone of depression or to evaluate the drawdown at any point in the real

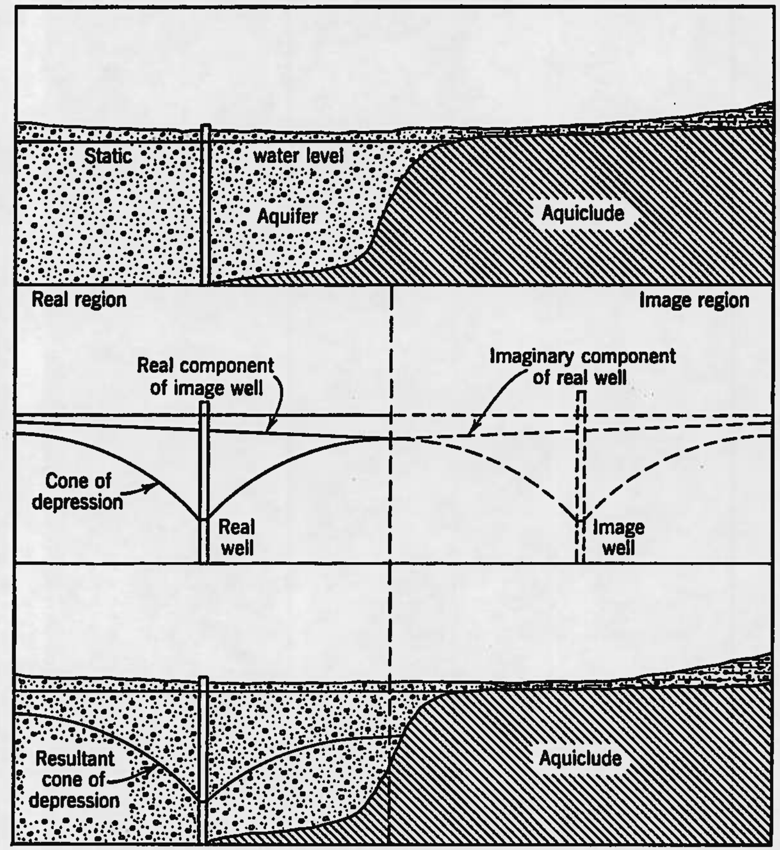


FIG. 88. Idealized section of an aquifer bounded by an impermeable formation together with a hypothetical well system for the solution of this type of flow problem.

region, it is necessary to add algebraically the real components of the several depression cones. The resultant cone of depression is steepened on the riverward side of the well and flattened on the landward side. This point should be recognized in drawing or examining contour maps of the water-table or piezometric surface for aquifers of this type.

An aquifer bounded by impervious strata is shown in idealized section by Fig. 88. The boundary is approximated as before by a sharp line of demarcation. Inasmuch as the impervious strata cannot contribute water to the pumped well, the limit imposed by the barrier is that there shall be no flow across the boundary line. As shown by the image setup in the central diagram, an imaginary discharging well is placed across the boundary at a right angle to and equidistant from the boundary. The drawdown of the image well at the boundary is equal to the effect of the real well, and the symmetrical drawdown cones produce a ground-water divide everywhere along the boundary. The image system produces a normal derivative equal to zero along the divide, and, as there can be no flow across a divide, this image system satisfies the limit of the real problem and is therefore a solution.

The real components of the cones of depression of the image well and the real well are shown as solid lines in the region of real values. Again, the resultant cone of depression or the drawdown at any point in the aquifer is determined by adding the real component of each depression cone. The resultant cone of depression for this example is flattened on the side adjacent to the boundary and steepened on the opposite side of the well.

An example of an aquifer bounded by impervious strata on two sides is shown by the idealized section of Fig. 89. The setup of the primary images to balance the effect of the real well, at each boundary is similar to the previous examples. The real component of each cone of depression is shown as a solid line in the region of real values. Although these primary images balance the effect of the real well at their respective boundaries, each image produces an unbalanced drawdown at the farther boundary. These unbalanced drawdowns at the boundaries theoretically produce a gradient and consequent flow across the boundary. Thus it is necessary to add a secondary set of image wells at the appropriate distances to balance the residual effect of the primary images. Each image well in the secondary set will again disturb the balance at the farther boundary and all successive sets of images to infinity will leave residuals at the boundaries. In practice it is necessary only to add image pairs until the residual effects are negligible in comparison with the total effect.

The modified nonequilibrium formula illustrated in Fig. 85 is particularly advantageous for the analysis of image or boundary

effects because it is easier to recognize changes in slope of a straight line than to detect changes in curvature of a log-log curve. Equation 38 indicates that $\Delta s/1$, the slope of the semilog graph, is dependent only on the pumping rate and the transmissibility of the aquifer.

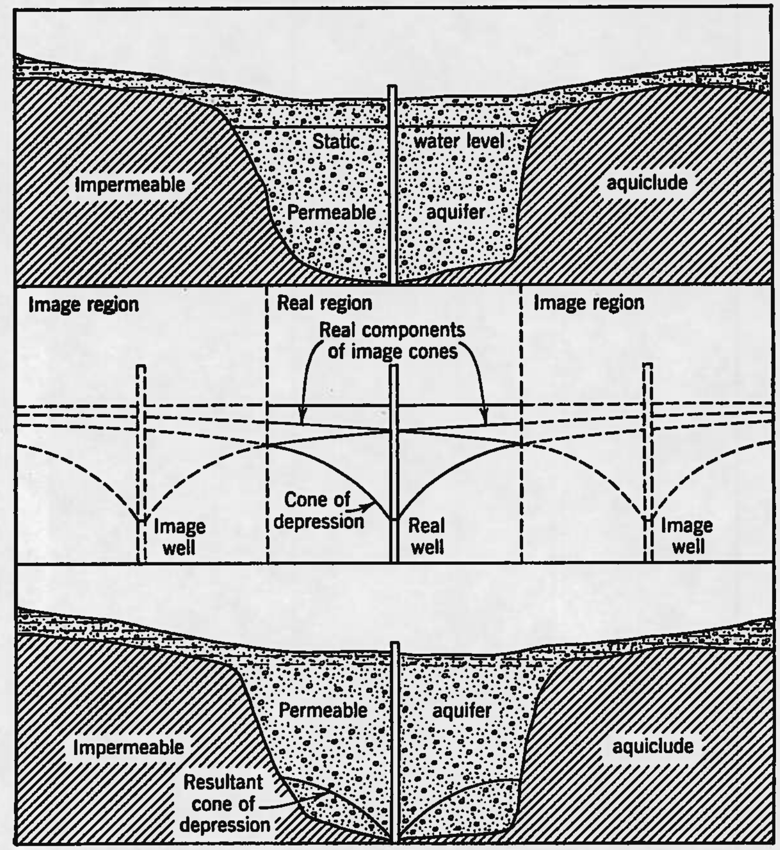
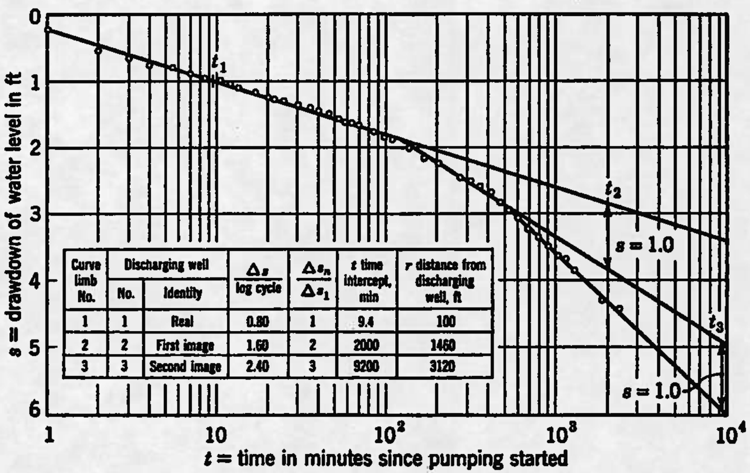


Fig. 89. Idealized section of underflow channel bounded by impermeable formation and setup of hypothetical well system used for solution of flow problems under this condition.

For a specific aquifer the transmissibility is a constant and the rate of pumping may be held constant. Under these conditions the graph of drawdown versus the logarithm of time since the pump started will show successive changes in slope as pumping continues. The water level will draw down at an initial rate under the influence of the real well that is nearest to the observation well. When the cone of depression of the nearer image well affects the observation well, the rate of drawdown will be doubled after r^2/t becomes small, because the total rate of withdrawal is then equal to the rate of the real well plus one image well, or twice the rate of the real well. The effect of the farther image triples the slope of the semilog graph after r^2/t becomes small for this image, because the total rate for the real well plus two image wells is three times as great as the rate of the real well.



Curve limb No.	Discharging well		$\frac{\Delta s}{\text{log cycle}}$	$\frac{\Delta s_n}{\Delta s_1}$	t time intercept, min	r distance from discharging well, ft
	No.	Identity				
1	1	Real	0.80	1	9.4	100
2	2	First image	1.60	2	2000	1460
3	3	Second image	2.40	3	9200	3120

FIG. 90. Graph showing drawdown of water level in observation Well E-1 by pumping tests on Test Well 36 at Hemphill Road and Saginaw Street, at southern city limits of Flint, Mich.

As previously mentioned, the use of Jacob's approximate method should be restricted to small values of u, which occur when the distance r is small and the time t is large. Frequently image distances are large in relation to the distance from an observation well to a pumping well. If a single image is involved, the semilog plotting of drawdown versus time shows a two-limbed graph. The transition from the first limb to the second limb follows a curved path through the region where the values of u for the image well are large. When t becomes sufficiently large in relation to r^2, the value of u for the image well becomes small and the observed data follow the straight line of the second segment of the graph.

If more than one image well occurs and if the image wells are at comparable distances, then the effect of second-, third-, or higher-order image wells may reach an observation well before sufficient time has elapsed for u to become small enough to warrant the application of this method to the effect of the first image well. Under these circumstances, the observed data follow a path of increasing curvature. An example of a semilog graph for an observation well affected by second- and higher-order image wells is shown as Fig. 90.

Approximate values for the location of the image wells are obtained by drawing tangents to the observed-data graph at the appropriate slope values. If the divergence of each line of Fig. 90 were noted, the graph could be separated and replotted as three separate lines with each component having the same slope and intercepting the zero-drawdown axis at its respective value of time. The time intercept on this axis permits the calculation of the storage coefficient by equation 40. Inasmuch as the coefficient of storage, S, is constant for a given aquifer the following relation obtains.

$$S = \frac{0.3Tt_1}{r_1^2} = \frac{0.3Tt_2}{r_2^2} = \frac{0.3Tt_3}{r_3^2}$$

or

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} = \frac{t_3}{r_3^2} \tag{44}$$

It follows from the above equation that, if the time intercepts are known for all wells, real or imaginary, and if the distance from the observation well to the real well is known, the distance to any image well can be determined from the data on the semilog graph.

The determination of the image distance from the time intercept of the semilog graph on the zero-drawdown axis may involve considerable error if the observational data are dispersed and if the slope is small because the intercept is poorly defined for small slopes. In addition, this intercept generally occurs at very small values of time, and consequently small errors in the intercept locus result in appreciable errors in the distance values.

The following method avoids the objections of the intercept method. Assume two observation wells at distances r_1 and r_2 from a discharging well, and from equation 35 the drawdown in each well is calculated as follows.

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_1}{1.87r_1^2S}\right) - 0.5772\right]$$

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_2}{1.87r_2^2S}\right) - 0.5772\right]$$

From the semilog graph for one well record the value of the time for a particular value of s, and from the graph for the second well record the time value for the same value of s. Then when $s_1 = s_2$

$$\log_e\left(\frac{Tt_1}{1.87r_1^2S}\right) = \log_e\left(\frac{Tt_2}{1.87r_2^2S}\right)$$

or

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} \tag{45}$$

This relation is identical with equation 44. From the form of equations 44 and 45 it follows that for a given aquifer the times of occurrence of zero drawdown or of equal drawdown vary directly as the squares of the distances from the observation well to the discharging well and are independent of the rate of pumping. The equal-drawdown method was used for the data on Fig. 90.

The time values at the tangent intersections determine only the approximate distances to the image wells. The time intercepts so determined will be too small and the calculated image distances will therefore be smaller than the correct distances. These preliminary estimates of the image distances may be corrected by trial. The method is to assume locations of the image wells based on the values computed by the first approximation. Using equations 32 and 33, we can determine the drawdown component resulting from each image well. The time-drawdown graph is obtained by adding these components. Successive adjustments may be made in the image distances until the computed and observed graphs are in agreement.

If a pumping test is run without prior knowledge of geologic boundaries and if the semilog graphs for all observation wells show evidence of image reflections, it would be possible to locate such boundaries by calculating the distance from each observation well to each image well. By scribing this distance as an arc from

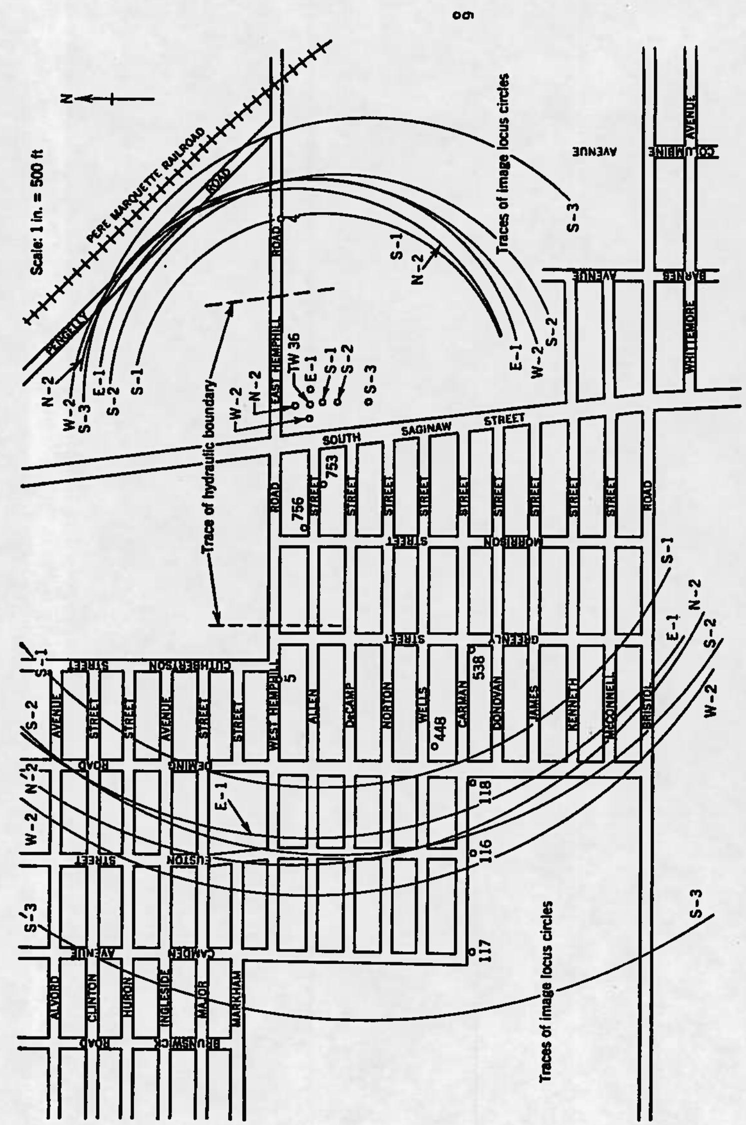


FIG. 91. Map showing location of test wells and pumping-test site near intersection of East Hemphill Road and South Saginaw Street at southern city limits of Flint, Michigan.

the respective observation well, the image well is located at the intersection of the arcs. The boundary is located at the midpoint of a line joining the real well and the image well.

The field layout and the image arc intersections for a test of the above type are shown in Fig. 91. Theoretically the arcs should intersect at a common point, but deviations of the real aquifer from the vertically bounded aquifer assumed in the derivation of the method of images result in a dispersion of the arcs and their intersections. As shown in Fig. 91, the east image well is quite definitely located by the arc intersections in the vicinity of East Hemphill Road, notwithstanding the fact that the observation wells are concentrated in a rather small area. With few or no arc inter-

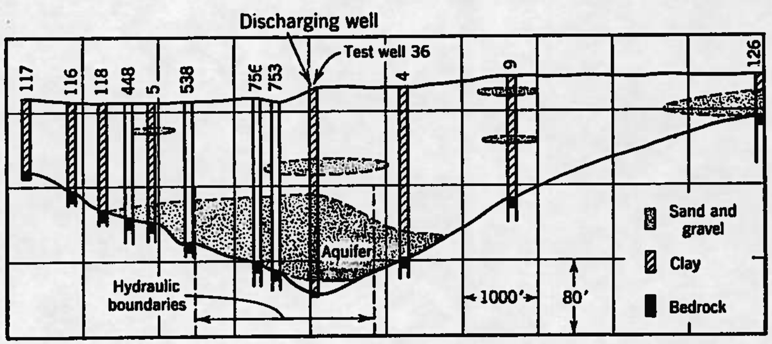


Fig. 92.

sections, the location of the image well may be taken at the center of the arc band where arcs are most closely grouped. It therefore appears from Fig. 91 that the west image well is near the intersection of the arc band with West Hemphill Road. Without a knowledge of the geology of the area, it may be necessary to describe the complete circles in order to determine the location of the narrowest portion of the arc band. The dispersion of the arcs for the west image suggests that the west boundary may be less abrupt than the east boundary. For comparison, Fig. 92 shows the geologic cross section of the aquifer as determined by test drilling and the trace of the computed boundaries are shown thereon as dashed lines. Recall that the boundaries determined by the pumping-test data represent a rectangular aquifer section which is equivalent hydraulically to the real aquifer.

Under favorable conditions, which justify the several assump-

tions embodied in the above methods, these principles properly applied permit the location of geologic boundaries within sufficient limits to reduce greatly the number of test wells required for their confirmation. In any problem, the pumping-test analysis must conform with the geologic evidence if both are sound. Any disagreement between the hydraulic data and the geologic data is untenable and points to incorrect analysis of either or both sets of data. It should also be recognized that long-term extrapolation of pumping-test results hinges on the loose assumption that the conditions observed over a short period of pumping will continue to prevail over the extrapolated period. Consequently, it is advisable for the purpose of long-term predictions to reinforce the data by a number of pumping tests at several sites, to obtain an adequate sampling of the area that will be affected by long-continued pumping.

Application of the Storage Equation to an Aquifer

The general hydrologic equation groups all parts of ground-water movement under the broad term of ground-water increment. With the knowledge of ground-water occurrence and mechanics now available, it is feasible to set up the storage equation for an aquifer as follows,

$$F + R_S + R_U + R_L + R_W = E + D_S + D_U + D_L + D_W + \Delta S \tag{46}$$

where F = recharge from infiltration
R_S = recharge from surface bodies of water
R_U = recharge from lateral underflow
R_L = recharge by leakage through an aquiclude
R_W = recharge by wells, trenches, or other infiltration devices

and

E = discharge by evapo-transpiration
D_S = discharge to surface bodies of water
D_U = discharge by lateral underflow
D_L = discharge by leakage through an aquiclude
D_W = discharge by wells
ΔS = change in storage volume

Quantitative field investigations to evaluate the magnitude of the several terms in equation 46 for our major aquifers represent one phase of the work of the U. S. Geological Survey and its cooperating agencies. The extent of these surveys to date, however, has been so limited that very few areas are adequately covered by records of sufficient length to permit appraisal of all the above factors.

Estimates of the portion of rainfall that may enter an aquifer are generally made by an analysis of fluctuations in ground-water level as the result of specific storms or on the basis of long-term correlations between water-level hydrographs and precipitation records. The intensity, frequency, and time of occurrence of the rainfall greatly influence the amount of ground-water recharge. Short periods of heavy precipitation may result in considerable overland runoff, but their duration may be so short that the rain may do no more than wet the upper part of the soil. Conversely, a rain of light intensity but long duration is conducive to slow, continued percolation that ultimately saturates the soil and permits considerable recharge to the underlying water table. Inasmuch as the soil-moisture deficiency may absorb a large part of the percolating waters as they pass through the soil zone to the underlying water table, it follows that light rainfalls of short duration are of little benefit unless they occur with sufficient frequency to overcome the depletion of soil moisture.

The most favorable period for recharge from precipitation is the nongrowing season, extending from the first killing frosts in fall to the last killing frosts in spring. During this period, the moisture demands of vegetation are generally negligible, and the soil evaporation is greatly reduced. Accordingly, then, a large portion of the precipitation penetrates to the water table until the soil belt is frozen. In this connection, it is of interest to note that snow cover provides relatively good insulation from frigid temperatures. Consequently, early snowfall of sufficient depth may protect the soil zone from frost formation and may permit appreciable percolation into the soil during periods of snow melt. Especially favorable is a spring period of gradual snow melt over unfrozen soil with precipitation occurring at intensities that limit surface runoff.

Topography is also a controlling factor in determining the opportunity and amount of recharge to an aquifer from infiltration. In areas of great relief the steep slopes accelerate the rate of overland runoff. However, in areas having flat slopes the surface runoff is sluggish and there is appreciable ponding or surface storage for lengthy periods and thus greater opportunity for ground-water recharge. In regions where the water table is close to

the surface, the volume of storage space available for recharge intake is limited by the shallow depth to the water table. When the reservoir is filled, the excess recharge is rejected and the water is discharged as surface runoff.

The amount of water-table rise for a given rainfall may vary considerably within any area because of differences in porosity of the aquifer, in both vertical and horizontal directions. In material of low porosity, a moderate rainfall may result in very large rises of water level, although the total volume of water recharged is quite small. Generally, these high heads result in rapid discharge of the

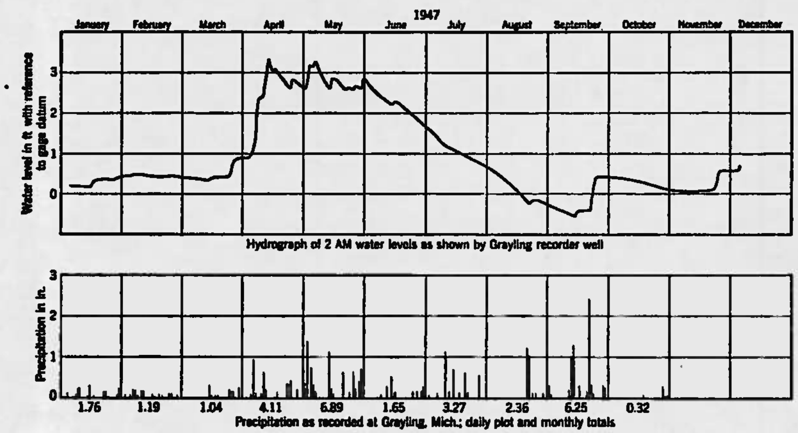


Fig. 93.

stored water after the rain ceases. Throughout a period of recharge, there is continued discharge by the aquifer. Consequently, in correlating records of ground-water level with precipitation, it should be recognized that the recorded rise in water level represents the net difference between the simultaneous recharge and discharge.

A portion of the hydrograph for a shallow observation well located near Grayling, Michigan, is reproduced as Fig. 93. This well is finished in sand and gravel at a depth of 8 ft and is located in an area that is densely covered by small oak and pine trees. It is of interest to note that the water level in this well recovered from a near-record low stage in January 1947 to a near-record high stage by May 1947 as the result of a short period of above-normal precipitation, which occurred during the latter part of the nongrowing season.

The determination of recharge from or discharge to surface bodies of water is generally made by stream-discharge measurements at selected cross sections. For areas where stream-gaging stations are established, it may be possible to secure continuous records of discharge and to utilize these data during base-flow periods. The amount of seepage per foot of channel may be small in comparison with the total discharge of the stream. Consequently, discharge measurements must be made with great precision and repeated on numerous occasions at various stages. It may be necessary to gage the stream flow at several widely distant sections to introduce sufficient seepage length to detect measurable differences in discharge. If gaging sections must be spaced at long intervals, the geologic reconnaissance must be made in sufficient detail to evaluate the effect of underflow channels, faults, or other discontinuities that may intersect the stream between gaging sections.

Considerable attention has been directed to methods of determining and forecasting effluent seepage, or discharge to surface bodies of water, because it represents the base flow of our surface streams. Inasmuch as the water table throughout a drainage basin is a hydraulic unit, it seems probable that a definite law could be developed that would properly weight the geologic characteristics of the basin and would express the rate of effluent seepage in terms of the water-table height at known points in the basin. With such a relationship, it would be feasible to forecast accurately the effluent seepage for any position of the water table at a given point. By selection of appropriate observation wells, which introduce time lag in proportion to their distance from the stream, these forecasts could be made for prolonged periods. A contribution toward this analysis was made by Jacob,[1] who outlined an experimental technique for research into this field. A formula developed by Theis[2] for the flow of water to a drain holds considerable promise for the prediction of effluent seepage from water-table elevation and forms the basis for interpretation of experimental evidence now being collected.

The development of agricultural drainage to lower regional

[1] C. E. Jacob, Correlation of Ground-Water Levels and Precipitation, Long Island, N. Y., *Trans. Am. Geophys. Union*, 1944, p. 939.

[2] C. V. Theis, Ground-Water in the Middle Rio Grande Valley of New Mexico, *U. S. Geological Survey Rio Grande Joint Investigation*, 1937, p. 44.

water tables by increasing effluent seepage may lead to detrimental conditions through a part of the growing season. There are shown in Table 12 the average hydrologic conditions that prevail during

TABLE 12

Summary of Average Hydrologic Conditions That Prevail in Many Areas

Factor	Average Condition Prevailing in the Season Noted: Winter–Spring	Summer–Fall
Soil moisture content	High	Low
Capillary fringe	Thick and at high stage	Thin and at low stage
Water table	High stage	Low stage
Vegetation demand	Minimum	Maximum
Ground-water flow	Cold and viscous water retards flow	Warm and less viscous water accelerates flow
Surface water	High stages and peak flows	Low stages and base flows

the spring and summer seasons. It is evident that the spring season is one in which all or nearly all the factors contributing to poor drainage are at their maximum or worst condition. Drains designed to meet this extreme condition are not only more than adequate through the balance of the year but may even cause overdrainage through the growing season. Through the summer and fall seasons such overdrainage may deplete the regional ground-water body to such an extent that the capillary zone is drawn beyond the reach of the plant rootlets, and large soil-moisture deficiencies may develop. This practice may account for the large loss of top soil evidenced in some overdrained muckland areas. It would seem reasonable that in some areas drainage ditches should be blocked and the water should be used for irrigation in the season of low water table.

The measurement of recharge or discharge by underflow is based on observation of ground-water gradients that prevail across key cross sections of the underflow channel or conduit. From drilling records it may be possible to estimate the size and shape of the underflow channel at several points. If feasible, pumping tests should be conducted to determine the channel capacity at the control section. Observation wells, strategically located, provide the basis for making long-term measurements of the changes in water-table gradients across the control sections. From a knowledge of the channel area, transmissibility, and water-table gradient it is possible to determine the quantity of underflow at each time of water-level observation.

The studies of recharge and discharge from adjacent aquicludes are thus far of limited scope. Considerable research in this field is required, both in the collection of field data and in the development of applicable mathematical techniques. As previously mentioned, the rates of ground-water movement through the aquicludes are small and the collection of experimental data may be handicapped by long time lags. It would seem desirable, however, to obtain water-level records for wells tapping the aquicludes at varying depths and particularly to observe the fluctuations in areas of heavy withdrawal or reduction in aquifer pressure. For the present, the leakage factor can only be estimated and such estimates would be difficult to defend.

The withdrawal of water by wells in any area can be estimated by a comprehensive canvass of all well owners to determine their rate of pumping and the extent to which this rate fluctuates daily, weekly, and annually. Many well installations are not metered, but reasonable estimates can usually be made by correlation of water use with a convenient unit of production that is measured. Where water use is not related to a particular unit of output, it may be correlative with total pay rolls, total operating expenses, or total net or gross income. It is generally possible to secure fair to good records of total pumpage by the above methods. From the pumpage inventory, a map of the type shown by Fig. 94[1] may be drawn to show the amount and distribution of pumping at the time of the inventory. This map serves as a guide to the interpretation of the water-level contour map, as regions of large withdrawal should correspond with the areas of low water level and regions of little or no pumping should be areas of high water level. Deviations from the general correspondence between the pumpage-distribution map and the water-level contours may indicate the presence of zones of natural recharge or discharge or may reveal the existence of geologic boundaries or inhomogeneities. In addition to this areal correlation of pumpage and water level, it is necessary to plot pumpage versus time to determine the seasonal and long-term trends in withdrawal and correlate these data with the water-level hydrographs as shown by Fig. 95.[1]

[1] W. T. Stuart, Ground-Water Resources of the Lansing Area, Michigan, *Progress Report* 13, Michigan Department of Conservation, June 1945.

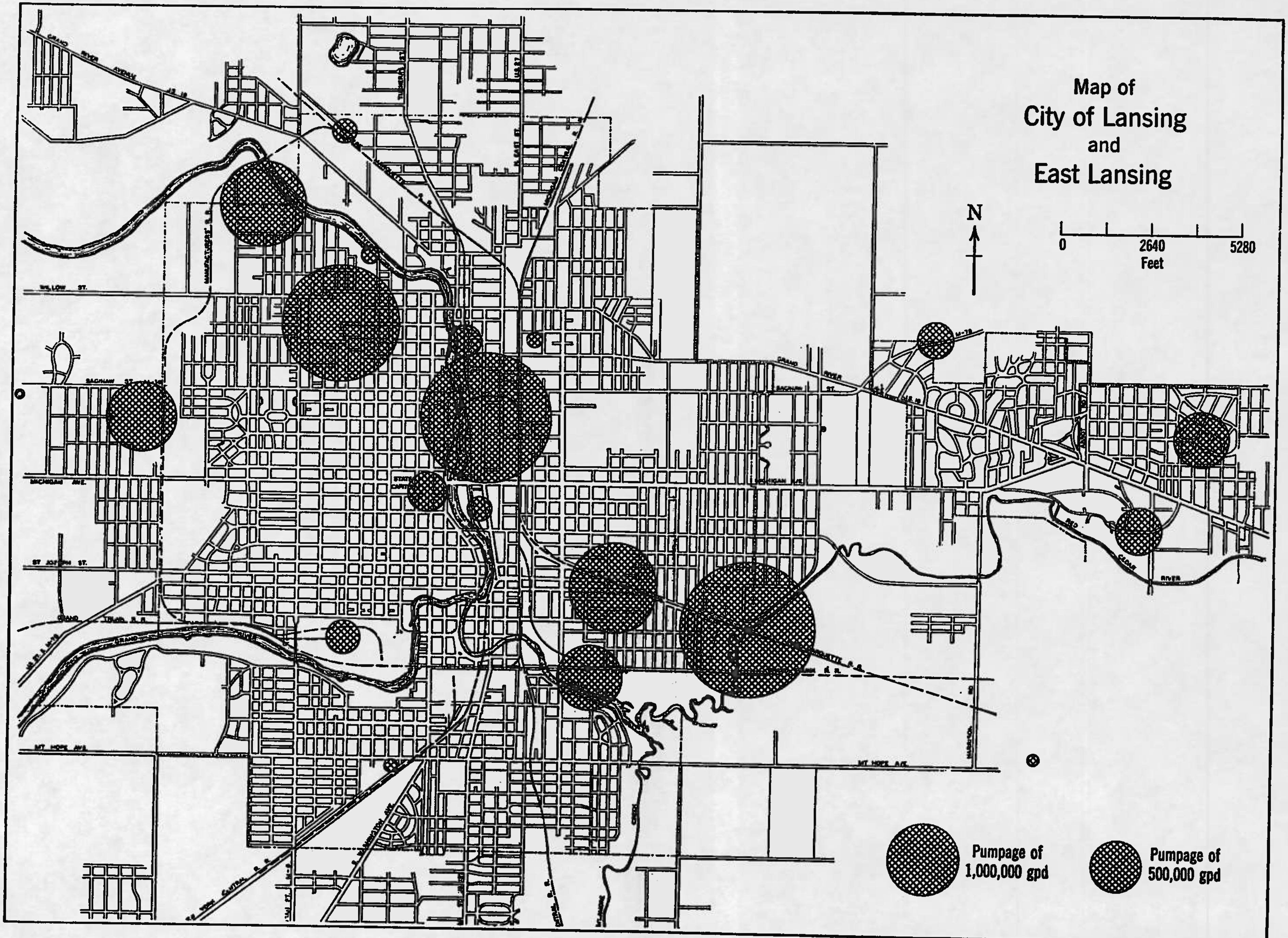


Fig. 94. Distribution and magnitude of pumpage from wells, Lansing, Mich., 1944.

Although artificial recharging of ground-water reservoirs is not a widespread practice at present, it is most probable that the rising trend in the number of recharge installations will steepen as the development of larger ground-water supplies continues. Recharge is effected either by water spreading or by diffusion wells. Water

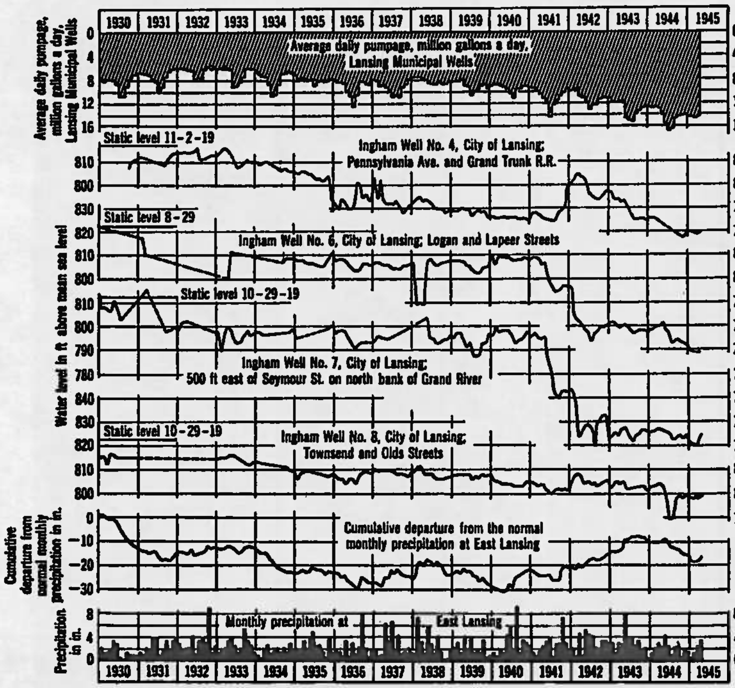


Fig. 95. Graphs showing pumpage, fluctuations of water levels, and precipitation, Lansing, Mich.

spreading may be accomplished by (1) the flooding method, (2) the basin method, or (3) the ditch or furrow method. The flooding method is restricted to topography which lends itself to surface ponding under conditions that permit prolonged retention of the surface waters. The basin and ditch methods require periodic maintenance to remove the accumulated silt from the percolation areas. A plan of the Canyon Basin spreading ground[1] in the San

[1] A. T. Mitchelson and Dean C. Muckel, Spreading Water for Storage Underground, *U. S. Department of Agriculture Tech. Bul.* 578, 1937, pp. 26–31.

Gabriel Valley, California, is shown in Fig. 96. Well hydrographs showing the effect of this spreading operation are shown in Fig. 97.

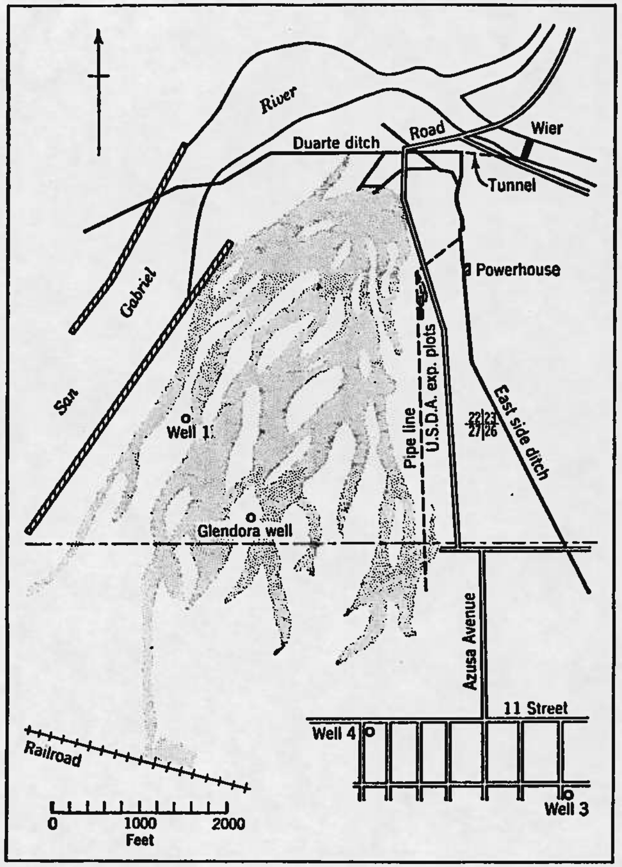


FIG. 96. Spreading ground of Canyon Basin, San Gabriel River, near Azusa, Calif. The general outline of the debris cone overlying the basin is shown. The Duarte ditch which supplies water for artificial spreading, the location of the experimental plots of the Bureau of Agricultural Engineering, and the location of key wells are also shown.

Recharge of underground reservoirs by diffusion wells is practiced on a large scale in Kings and Queens Counties on Long Island, New York, as the result of conservation legislation which requires that new air-conditioning and cooling wells with a capacity

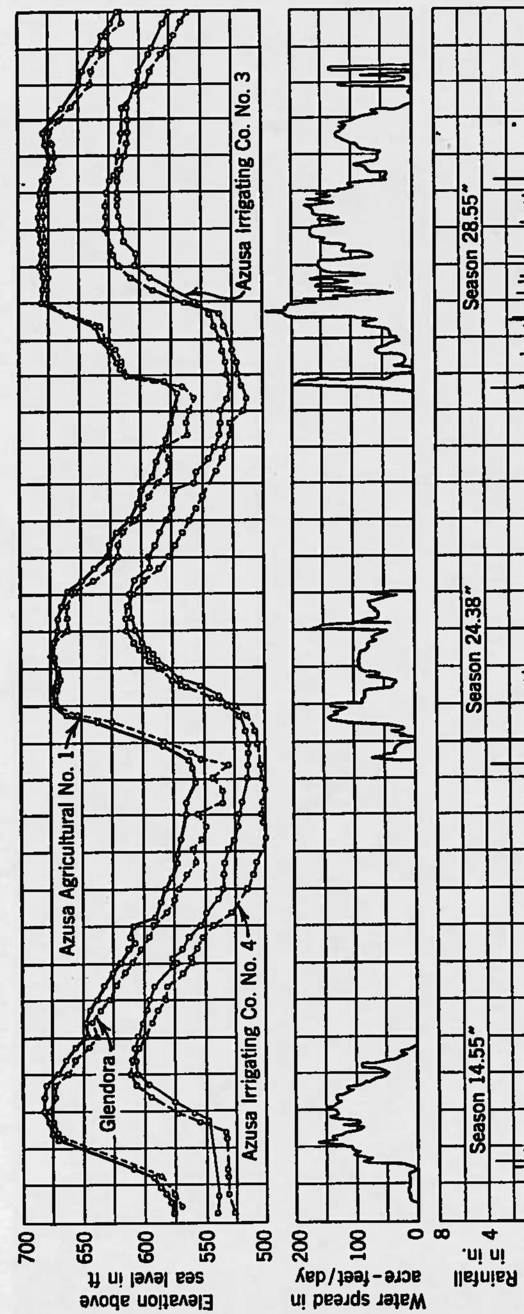


FIG. 97. Hydrographs of wells near San Gabriel River spreading grounds showing effect of spreading on elevation of underground water table, also showing daily total amount of water spread during seasons of 1932–1933, 1933–1934, and 1934–1935, and daily rainfall for these seasons.

greater than 100,000 gpd must return the water to the aquifer from which it is drawn. During the summer of 1944 more than 200 recharge wells and basins returned water at a combined rate of 60,000,000 gpd.[1] Diffusion wells require periodic surging and development to remove the accumulation of silt and other fine materials. Although the unit volume cost of recharging with diffusion wells may greatly exceed that of the spreading method, in the highly industrialized and urbanized areas this may be the only possible method. An example of the solution of a problem of critical water shortage by recharging through diffusion wells is found in the experience of several large distillers in Louisville, Kentucky, during the summer of 1944.[2] Faced with an imminent water shortage the distillers, at the suggestion of the U. S. Geological Survey, converted to city water during cold weather, permitted the wells to rest, and used the cold surface water from the city supply to recharge the aquifer. Recharging started on March 10, 1944, and continued until the latter part of May 1944. By utilizing city water and ground water it was possible for the distilleries to continue full operation during the summer.

Provision is made in the storage equation, page 259, to add the contribution by artificial recharge. This quantity is determined by a pumpage inventory in the case of diffusion wells and by surface-water measurement in the case of water spreading.

The changes in storage volume within an aquifer are based on periodic observations of changes in water level for a selected network of observation wells. Contours of the water table or piezometric surface are drawn and used for estimates of change in storage volume. An example of a map showing contours on the piezometric surface of an artesian aquifer with extensive well development is shown by Fig. 98.[3]

The collection of detailed records of the above nature is a function of the U. S. Geological Survey and many cooperating state agencies. Although the scope of these surveys is still quite limited, the upward trend of investigatory surveys of ground-water conditions in recent years indicates a gradual realization by

[1] M. L. Brashears, Artificial Recharge of Ground Water on Long Island, New York, *Econ. Geol.*, vol. XLI, No. 5, August 1946, pp. 503–516.

[2] W. F. Guyton, Artificial Recharge of Glacial Sand and Gravel with Filtered River Water at Louisville, Kentucky, *Econ. Geol.*, vol. XLI, No. 6, September-October 1946, pp. 644–658.

[3] W. T. Stuart, Ground-Water Resources of the Lansing Area, Michigan, *Progress Report* 13, Michigan Department of Conservation, June 1945.

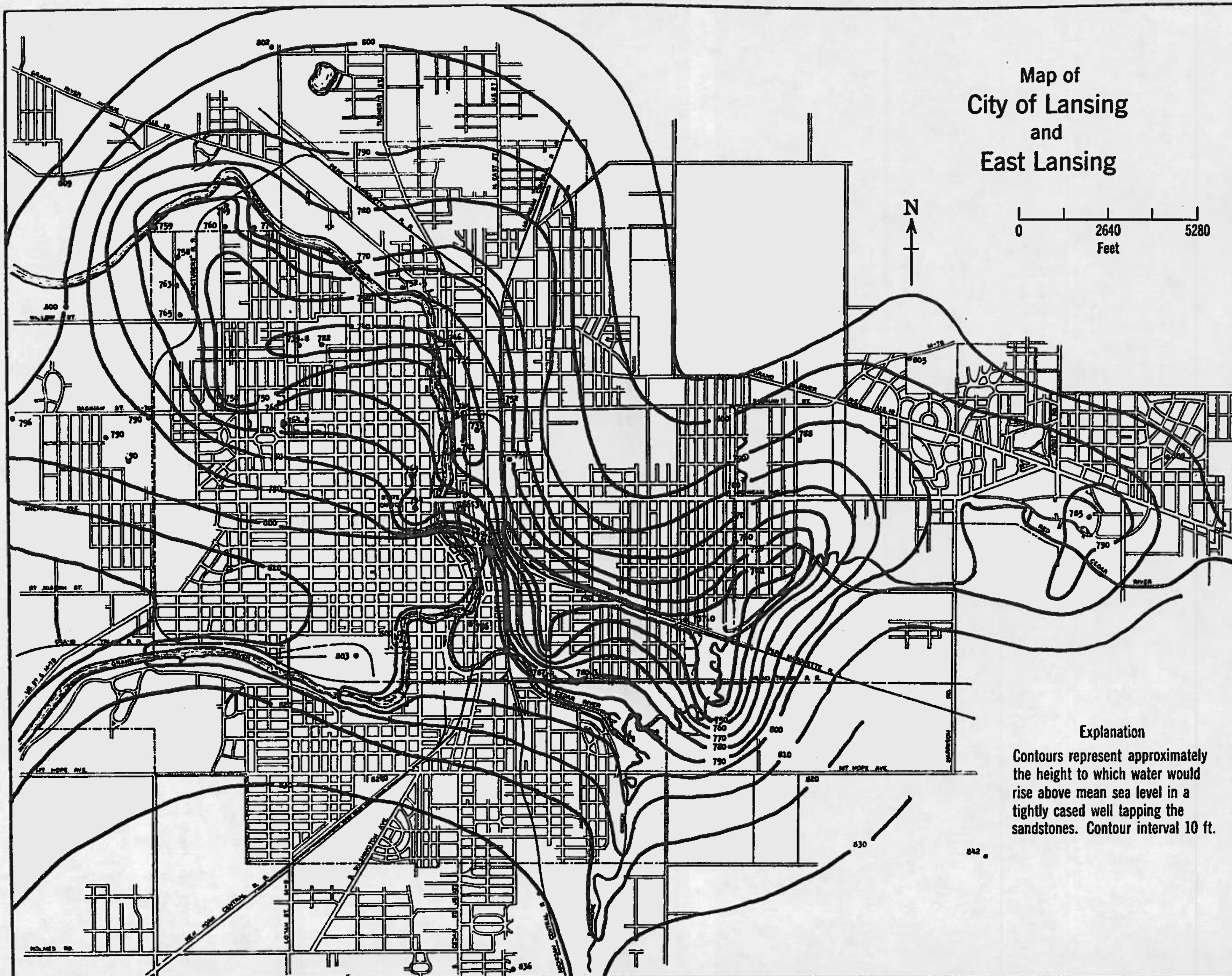

Map of
City of Lansing
and
East Lansing
N
0
2640
5280
Feet
Explanation
Contours represent approximately the height to which water would rise above mean sea level in a tightly cased well tapping the sandstones. Contour interval 10 ft.

Fig. 98.

engineers, geologists, and hydrologists that a real and practical solution of ground-water problems can be made.

It remains a common practice to adjudge the merits of a ground-water supply on the basis of a short capacity test on a single test well, no observational data being recorded except the discharge and drawdown of the well. Frequently, these measurements are made at random intervals and under widely varying conditions. Interruptions in pumping, which permit recovery of the water table, may invalidate the meager data collected and may lead to quite erroneous conclusions, but they seem of little importance to the untrained observer. The influence of geologic boundaries is oftentimes not recognized, and as a result unexpected well failure may occur.

Considerable optimism is warranted in facing future problems when one recognizes the advances made in ground-water hydrology within the short span of years since the development of the non-equilibrium formula by Theis in 1935. Although the mathematical tools are still limited, the awakened interest in the field will in time develop new methods and techniques. The collection of observational data will uncover the required evidence to interpret more clearly many ground-water phenomena that are now recorded but not understood.

When one recognizes that the great majority of our ground-water developments in this country are based on meager and obsolete knowledge of the principles of ground-water hydrology, it is evident that a large field of opportunity lies ahead for adequately trained hydrologists.

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CHAPTER VIII

RUNOFF

The practical objective of the science of hydrology is to provide a means for determining the characteristics of the hydrograph that may be expected for a stream draining any particular basin. The basic principles have been explained in the preceding pages. It remains to be shown how those principles can be best used for the determination of (1) the maximum flood flow that may be expected to occur with any stated frequency for a given basin; (2) the minimum flow that may be anticipated under a given set of conditions; and (3) the monthly, annual, or average long-term yield.

Before the solution of these several problems is taken up, attention should be called to the fact that the first is concerned almost entirely with surface runoff; in the second, ordinarily only ground-water flow is involved; and in the third, it is the sum of the two, without reference to the source.

SURFACE RUNOFF

In the determination of the maximum flood flow or any of the other characteristics of surface runoff, the very nature of the available data depends to a considerable extent upon the size of the area drained. We will, therefore, first consider the surface runoff from small plots because they serve to provide a simple basic understanding of the runoff process. The methods developed for small plots may be used for determining the runoff from subdivisions of drainage systems where no runoff data are available on the larger areas. Then the surface runoff of small watersheds having areas varying from a few acres to perhaps 10 sq miles will be studied. These are of the size encountered in the design of culverts, storm sewers, airports, and small bridges. Finally, methods will be discussed for determining the characteristics of the flood hydrographs that may be expected from larger drainage basins such as those involved in flood studies, power development, irrigation, and water supply.