interior angle of the enlargement walls. Note as $\theta = 180$ (the sudden enlargement) K_L is approximately unity.

When water discharges into a reservoir from a pipe, the entire kinetic energy per pound of fluid is dissipated within the reservoir. Consequently K_L for an exit equals 1.0. Exit head losses are therefore given by,

$$h_L = \frac{V^2}{2g} \qquad (3\text{-}3)$$

Jeppson, R.W. 1977
Analysis of Flow in Pipe Networks
Ann Arbor Science.

Chapter IV
Incompressible Flow in Pipe Networks

Introduction

Analyses and design of pipe networks create a relatively complex problems, particularly if the network consists of a large number of pipes as frequently occurs in the water distribution systems of large metropolitan areas, or natural gas pipe networks. Professional judgment is involved in deciding which pipes should be included in a single analysis. Obviously it is not practical to include all pipe which delivers to all customers of a large city, even though they are connected to the total delivery system. Often only those main trunk lines which carry the fluid between separate sections of the area are included, and if necessary analyses of the networks within these sections may be included. This manual deals only with steady-state solutions. In a water distribution system, the steady-state analysis is a small but vital component of assessing the adequacy of a network. Such an analysis is needed each time changing patterns of consumption or delivery are significant or add-on features, such as supplying new subdivisions, addition of booster pumps, or storage tanks change the system. In addition to steady analyses, studies dealing with unsteady flows or transient problems, operation and control, acquisition of supply, optimization of network performance against cost, and social implications should be given consideration but are beyond the scope of this text.

The steady-state problem is considered solved when the flow rate in each pipe is determined under some specified patterns of supply and consumption. The supply may be from reservoirs, storage tanks and/or pumps or specified as inflow or outflow at some point in the network. From the known flow rates the pressures or head losses throughout the system can be computed. Alternatively, the solution may be initially for the heads at each junction or node of the network and these can be used to compute the flow rates in each pipe of the network.

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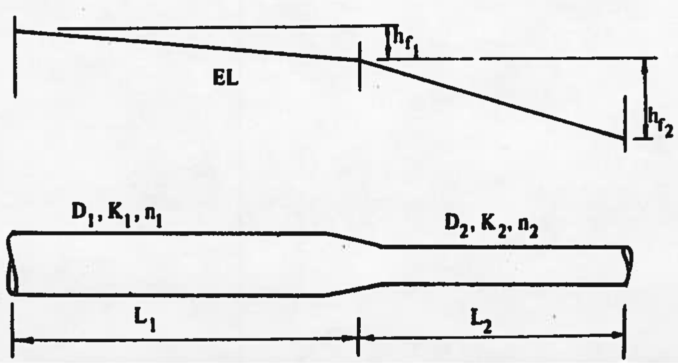
The oldest method for systematically solving the problem of steady ›w in pipe networks is the Hardy Cross method. Not only is this method ited for hand solutions, but it has been programmed widely for mputer solutions, but particularly as computers allowed much larger tworks to be analyzed it become apparent that convergence of the Hardy oss method might be very slow or even fail to provide a solution in some ses. In the past few years the Newton-Raphson method has been utilized solve large networks, and with improvements in algorithms based on e Newton-Raphson method, computer storage requirements are not eatly larger than those needed by the Hardy Cross method. An lditional method called the "linear theory method" has also been oposed, and does not require an initialization as do the other two ethods but more storage. The solution of a system of algebraic equations ves the flow rates in each pipe or the head or pressures throughout the stem described in this chapter. In the subsequent three chapters the plementation of the linear theory method, the Newton-Raphson method id the Hardy Cross method is discussed, in the reverse order of their istorical development.

Reducing Complexity of Pipe Networks

In general, pipe networks may include series pipes, parallel pipes, id branching pipes (i.e. pipes that form the topology of a tree). In ldition, elbows, valves, meters, and other devices which cause local sturbances and minor losses may exist in pipes. All of the above should : combined with or converted to an "equivalent pipe" in defining the etwork to be analyzed. The concept of equivalence is useful in mplifying networks. Methods for defining an equivalent pipe for each of ie above mentioned occurrences follows.

eries pipes

The method for reducing two or more pipes of different size in series ill be explained by reference to the diagram below. The same flow must



pass through each pipe in series. An equivalent pipe is a pipe which will carry this flow rate and produce the same head loss as two or more pipes, or

$$h_{f_e} = \Sigma h_{f_i} \quad \text{(4-1)}$$

Expressing the individual head losses by the exponential formula gives,

$$K_e Q^{n_e} = K_1 Q^{n_1} + K_2 Q^{n_2} + \ldots = \Sigma K_i Q^{n_i} \quad \text{(4-2)}$$

For network analysis K_e and n_e are needed to define the equivalent pipe's hydraulic properties. If the Hazen-Williams equation is used, all exponents $n = 1.852$, and consequently

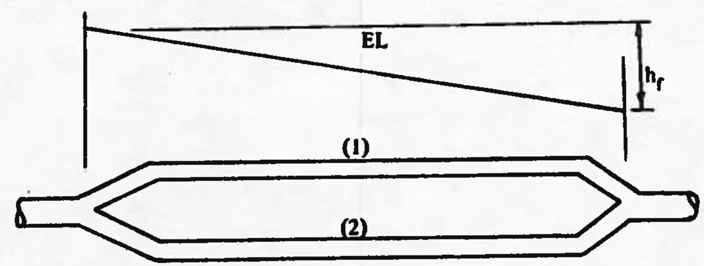
$$K_e = K_1 + K_2 + \ldots = \Sigma K_i \quad \text{(4-3)}$$

or the coefficient K_e for the equivalent pipe equals the sum of K's of the individual pipes in series. If the Darcy-Weisbach equation is used, the exponents n in Eq. 4-2 will not necessarily be equal, but generally these exponents are near enough equal that the n_e for the equivalent pipe can be taken as the average of these exponents and Eq. 4-3 used to compute K_e.

Parallel pipes

An equivalent pipe can also be used to replace two or more pipes in parallel. The head loss in each pipe between junctions where parallel pipes part and join again must be equal, or

$$h_f = h_{f_1} = h_{f_2} = h_{f_3} = \ldots \quad \text{(4-4)}$$



The total flow rate will equal the sum of the individual flow rates or

$$Q = Q_1 + Q_2 + \ldots = \Sigma Q_i \quad \text{(4-5)}$$

Solving the exponential formula $h_f = KQ^n$ for Q and substituting into Eq. 4-5 gives

$$\left(\frac{h_f}{K_e}\right)^{\frac{1}{n_e}} = \left(\frac{h_f}{K_1}\right)^{\frac{1}{n_1}} + \left(\frac{h_f}{K_2}\right)^{\frac{1}{n_2}} + \ldots = \Sigma \left(\frac{h_f}{K_i}\right)^{\frac{1}{n_i}} \quad \ldots\ldots(4\text{-}6)$$

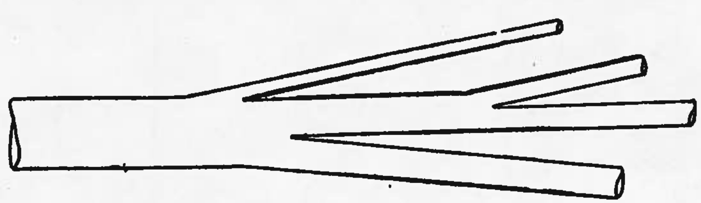
If the exponents are equal as will be the case in using the Hazen-Williams equation, the head loss h_f may be eliminated from Eq. 4-6 giving

$$\left(\frac{1}{K_e}\right)^{\frac{1}{n}} = \left(\frac{1}{K_1}\right)^{\frac{1}{n}} + \left(\frac{1}{K_2}\right)^{\frac{1}{n}} + \ldots = \Sigma \left(\frac{1}{K_i}\right)^{\frac{1}{n}} \quad \ldots\ldots\ldots\ldots(4\text{-}7)$$

When the Darcy-Weisbach equation is used for the analysis, it is common practice to assume n is equal for all pipes and use Eq. 4-7 to compute the K_e for the equivalent pipe.

Branching system

In a branching system a number of pipes are connected to the main to form the topology of a tree. Assuming that the flow is from the main into the smaller laterals it is possible to calculate the flow rate in any pipe as the sum of the downstream consumptions or demands. If the laterals supply water to the main, as in a manifold, the same might be done. In either case by proceeding from the outermost branches toward the main or "root of the tree" the flow rate can be calculated, and from the flow rate in each pipe the head loss can be determined using the Darcy-Weisbach or Hazen-Williams equation. In analyzing a pipe network containing a branching system, only the main is included with the total flow rate specified by summing from the smaller pipes. Upon completing the analysis the pressure head in the main will be known. By substracting individual head losses from this known head, the heads (or pressures) at any point throughout the branching system can be determined.



Minor losses

When a valve, meter, elbow, or other device exists in a pipe causing a minor loss which is not insignificant in comparison to the frictional loss in that pipe, an equivalent pipe should be formed for use in the network

analyses. This equivalent pipe should have the same head loss for any flow rate as the sum of the pipe frictional loss and the minor head loss. The equivalent pipe is formed by adding a length ΔL to the actual pipe length such that the frictional head loss in the added length of pipe equals the minor losses. Computation of ΔL will be slightly different depending upon whether the Darcy-Weisbach or Hazen-Williams equations are to be used.

Most minor losses are computed from the formula Eq. 3-1 $h_L = K_L (V^2/2g)$ or a loss coefficient multiplied by the velocity head as described in Chapter III. The Darcy-Weisbach equation $h_f = [f(L/D)](V^2/2g)$ may also be thought of as the product of a coefficient times the velocity head. Consequently, if the Darcy-Weisbach equation is to be used in the network analyses, the length ΔL can be found by equating these two coefficients with ΔL replacing L in the Darcy-Weisbach coefficient. After solving for ΔL, the length to be added to the actual pipe length is,

$$\Delta L = \frac{K_L D}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4\text{-}8)$$

Since f is generally a function of the flow rate, ΔL also depends upon the flow rate. In practice it is generally adequate to compute ΔL, by using the f values for wholly rough flow, or if knowledge of the approximate flow rate is available the friction factor f corresponding to it may be used in Eq. 4-8. If several devices causing minor losses exist in a single pipe, then the sum of the individual ΔL's is added to the length of the actual pipe.

The coefficient K in the exponential formula for the equivalent pipe is obtained by substituting L + ΣΔL for the length of the pipe in Eq. 2-28 or

$$K_e = \frac{a(L + \Sigma\Delta L)}{2g\,DA^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4\text{-}9)$$

when using the Darcy-Weisbach equation for computing frictional losses. When using the Hazen-Williams formula for this purpose the added pipe length ΔL, due to the device causing the minor loss, can be computed from

<u>ES</u> $\Delta L = 0.00532\, K_L\, Q^{0.148}\, C_{HW}^{1.852}\, D^{0.8703}$

<u>SI</u> $\Delta L = 0.00773\, K_L\, Q^{0.148}\, C_{HW}^{1.852}\, D^{0.8703}$

and the K in the exponential formula is

<u>ES</u> $K_e = 4.73\, \frac{L + \Delta L}{C_{HW}^{1.852}\, D^{4.87}}$ (D and L in feet)

<u>SI</u> $K_e = \frac{10.7\,(L + \Delta L)}{C_{HW}^{1.852}\, D^{4.87}}$ ....(4-11)

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Example Problem in Finding Equivalent Pipes

1. A 12-inch (30.48 cm) cast iron pipe which is 100 ft (30.48 m) long is attached in series to a 10-inch (25.4 cm) dia. iron pipe which is 300 ft (91.44 m) long to carry a flow rate of 5.0 cfs (0.142 cm) of water at 68°F (20°C). Find the length of 10-inch (25.4 cm) pipe which is equivalent to the series system.

Solution:

<u>ES</u>

$V_{12} = 5/0.785 = 6.37$ fps
$V_{10} = 5/0.545 = 9.17$ fps
$Re_{12} = 6.37(1)/1.084 \times 10^{-5} = 5.90 \times 10^5$
$Re_{10} = 9.17(0.833)/1.084 \times 10^{-5} = 7.05 \times 10^5$
$(e/D)_{12} = 0.0102/12 = 0.00085$
$(e/D)_{10} = 0.0102/10 = 0.00102$

from the Moody diagram

$f_{12} = 0.0195, f_{10} = 0.0192$

$h_f = h_{f_{12}} + h_{f_{10}}$

$$h_f = 0.0195 \frac{100}{1} \frac{(6.37)^2}{64.4} + 0.0192 \frac{300}{0.833} \frac{(9.17)^2}{64.4}$$

$h_f = 1.23 + 9.02 = 10.25$ ft.
$L = h_f D(2g)/fV^2 = 340.7$ ft.

<u>SI</u>

$V_{12} = 0.142/0.0729 = 1.95$ m/s
$V_{10} = 0.142/0.0507 = 2.80$ m/s
$Re_{12} = 1.95(0.3048)/1.007 \times 10^{-6} = 5.90 \times 10^5$
$Re_{10} = 2.80(0.254)/1.007 \times 10^{-6} = 7.05 \times 10^5$
$(e/D)_{12} = 0.0259/30.48 = 0.00085$
$(e/D)_{10} = 0.0259/25.4 = 0.00102$

from the Moody diagram

$$h_f = 0.0195 \frac{30.48}{0.3048} \frac{(1.95)^2}{19.62} + 0.0192 \frac{91.44}{0.254} \frac{(2.80)^2}{19.62}$$

$h_f = 0.378 + 2.762 = 3.14$ m
$L = 3.14 (0.254)(19.62)/(0.0192 \times 2.80^2) = 103.9$ m

2. Using the Hazen-Williams formula find the coefficient K_e in the exponential formula and the diameter of an equivalent pipe to replace two 500 ft parallel pipes of 8-inch and 6-inch diameters. $C_{HW} = 120$ for both pipes, and make the equivalent pipe 500 ft long.

Solution:

$$K_8 = \frac{4.73 L}{C_{HW}^{1.852} D^{4.87}} = \frac{4.73 (500)}{7090(0.667)^{4.87}} = 2.403$$

$$K_6 = \frac{4.73 (500)}{7090(0.5)^{4.87}} = 9.754$$

From Eq. 4-7

$$\left(\frac{1}{K_e}\right)^{0.54} = \left(\frac{1}{2.403}\right)^{0.54} + \left(\frac{1}{9.754}\right)^{0.54} = 0.915$$

$K_e = 1.178$ ft

$$D_e = \left[\frac{4.73 L}{C_{HW}^{1.852} K_e}\right]^{1/4.87} = \left[\frac{4.73 (500)}{7090 (1.178)}\right]^{0.2053}$$

$= 0.772$ ft
$= 9.26$ inches

3. An 800 ft long 8-inch cast iron pipe contains an open globe valve. Determine the length of the equivalent pipe if the flow rate is approximately 700 gpm.

Solution:

Using the procedure for determining f described in Chapter II, f = 0.0218. From Eq. 4-8

$$\Delta L = \frac{K_L D}{f} = \frac{10(8/12)}{0.0218} = 306 \text{ ft}$$

$$L_e = L + \Delta L = 1106 \text{ ft}$$

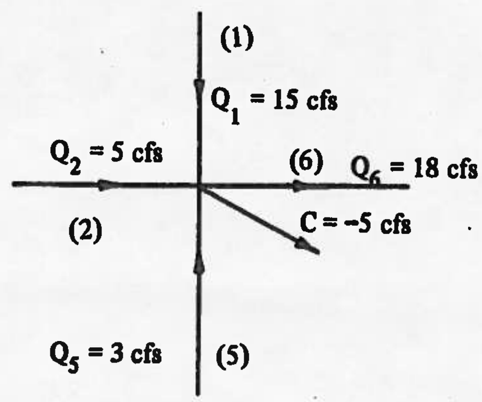
Systems of Equations Describing Steady Flow in Pipe Networks

Flow rates as unknowns

The analysis of flow in networks of pipes is based on the continuity and energy laws as described in Chapter I. To satisfy continuity, the mass, weight, or volumetric flow rate into a junction must equal the mass, weight, or volumetric flow rate out of a junction. If the volumetric flow rate is used this principle, as discussed in Chapter I, can be expressed mathematically as,

$$(\Sigma Q_i)_{out} - (\Sigma Q_i)_{in} = C \quad \ldots\ldots\ldots\ldots (4\text{-}12)$$

in which C is the external flow at the junction (commonly called consumption or demand). C is positive if flow is into the junction and negative if it is out from the junction. For example if four pipes meet at a junction as shown in the sketch, Eq. 4-12 at this junction is



$$Q_6 - Q_1 - Q_2 - Q_5 = -5$$
$$18 - 15 - 5 - 3 = -5$$

If a pipe network contains J junctions (also called nodes) and all external flows are known then J-1 independent continuity equations in the form of Eq. 4-12 can be written. The last, or the J^{th} continuity equation, is not independent; that is, it can be obtained from some combination of the first J-1 equations. Note in passing that each of these continuity equations is linear, i.e., Q appears only to the first power.

In addition to the continuity equations which must be satisfied, the energy principle provides equations which must be satisfied. These additional equations are obtained by noting that if one adds the head losses around a closed loop, taking into account whether the head loss is positive or negative, that upon arriving at the beginning point the net head losses equals zero. Mathematically, the energy principle gives L equations of the form

$$\sum_{\ell}^{I} h_{f_\ell} = 0$$

$$\sum_{\ell}^{II} h_{f_\ell} = 0 \quad \ldots\ldots\ldots\ldots (4\text{-}13)$$

$$\vdots$$

$$\sum_{\ell}^{L} h_{f_\ell} = 0$$

in which L represents the number of non-overlapping loops (also referred to as natural loops) in the network, and the summation on small ℓ is over the pipes in the loops I, II, ..., L. By use of the exponential formula $h_f = KQ^n$, Eqs. 4-13 can be written in terms of the flow rate, or

$$\sum_{\ell}^{I} K_\ell Q_\ell^{n_\ell} = 0$$

$$\sum_{\ell}^{II} K_\ell Q_\ell^{n_\ell} = 0 \quad \ldots\ldots\ldots\ldots (4\text{-}14)$$

$$\vdots$$

$$\sum_{\ell}^{L} K_\ell Q_\ell^{n_\ell} = 0$$

A pipe network consisting of J junctions and L non-overlapping loops and N pipes will satisfy the equation

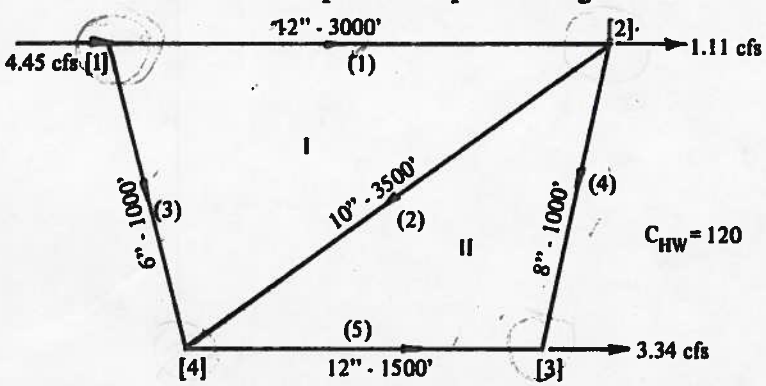
$$N = (J - 1) + L \quad \ldots\ldots\ldots\ldots (4\text{-}15)$$

(If all of the external flows are not known, then all J junction equations are independent and available for use as will be discussed in the next chapter.) Since the flow rate in each pipe can be considered unknown, there will be N unknowns. The number of independent equations which can be obtained for a network as described above are (J-1) + L. Consequently the number of independent equations is equal in number to the unknown flow rates in the N pipes. The (J-1) continuity equations are linear and the L energy (or head losses) equations are nonlinear. Since large networks may consist of hundreds of pipes, systematic methods which utilize computers

FIGURE 4

are needed for solving this system of simultaneous equations. Such methods are described in subsequent chapters.

As an example in defining the system of N equations which must be satisfied in solving for the N unknown volumetric flow rates in the N pipes of a network, consider the simple two loop network given below. In this



example there are five pipes and therefore five unknown flow rates. There are four junctions and therefore three independent continuity equations and two energy equations for the head losses around the two basic loops can be written. On the sketch of this network the pipe numbers are enclosed by parentheses, the junction or node numbers are within [ ] brackets and the loops are denoted by Roman Numerals I and II. Arrow heads denote assumed directions of flows in the pipes. Flow rates, etc., will be denoted by a subscript corresponding to the pipe number in which that flow rate occurs. This same notation will be followed throughout the remainder of the manual. Also considerations of space prevent duplicating solutions in ES and SI units.

The J-1 = 3 continuity equations at the three consecutive junctions 1, 2, and 3 are

$$Q_1 + Q_3 = 4.45$$
$$-Q_1 + Q_2 + Q_4 = -1.11$$
$$-Q_4 - Q_5 = -3.34$$

The continuity equation at junction 4 is $-Q_3 - Q_2 + Q_5 = 0$. However, this equation is not independent of the above three equations since it can be obtained as minus the sum of these three equations. The Hazen-Williams equation will be used to define the head losses in each pipe. In expressing these head-losses the exponential equation will be used. From Eq. 2-22 the K coefficients for the exponential formula are: $[K = 4.73L/(C_{HW}^{1.852}D^{4.87})]$:

$$K_1 = 2.018, K_2 = 5.722, K_3 = 19.674, K_4 = 4.847, K_5 = 1.009$$

The energy loss equations around the two loops are:

$$2.018\,Q_1^{1.852} + 5.722\,Q_2^{1.852} - 19.674\,Q_3^{1.852} = 0$$
$$4.847\,Q_4^{1.852} - 1.009\,Q_5^{1.852} - 5.722\,Q_2^{1.852} = 0$$

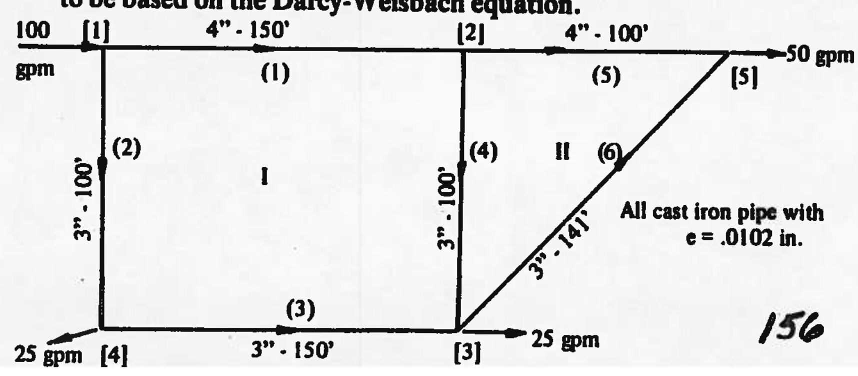
These two energy equations are obtained by starting at junctions 1 and 2, respectively, and moving around the respective loops I and II in a clockwise direction. If the assumed direction of flow is opposite to this clockwise movement a minus precedes the head loss for that pipe. Simultaneous equations such as those above will be called *Q-equations.*

A solution to the five unknown flow rates from the five simultaneous equations above, by the procedure described subsequently in Chapter V is: $Q_1 = 3.350$ cfs, $Q_2 = 0.897$ cfs, $Q_3 = 1.104$ cfs, $Q_4 = 1.340$ cfs, $Q_5 = 2.001$ cfs. This solution can be verified by substituting into each of the above equations. It is relatively easy to determine flows in individual pipes which also satisfy the J-1 continuity equations. However, the correct flow rates which simultaneously satisfy the L energy equations are virtually impossible to obtain by trial and error if the system is large.

After solving the system of equations for the flow rate in each pipe, the head losses in each pipe can be determined. From a known head or pressure at one junction it is then a routine computation to determine the heads and pressures at each junction throughout the network, or at any point along a pipe, by substracting the head loss from the head at the upstream junction, plus accounting for differences in elevations if this be the case. In some problems the external flows may not be known as was assumed in the above example. Rather, the supply of water may be from reservoirs and/or pumps. The amount of flow from these individual sources will not only depend upon the demands, but also will depend upon the head losses throughout the system. Methods for incorporating pumps and reservoirs into a network analysis in which the flow rates in the individual pipes of the network are initially considered the unknowns will be dealt with in Chaper V in conjunction with the linear theory method of solution.

Example Problem in Writing Flow Rate Equations

1. Write the system of equations whose solution provides the flow rates in the six pipes of the network shown below. The energy equations are to be based on the Darcy-Weisbach equation.



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Solution:

Before the energy equations can be defined it is necessary that K and n for each pipe be determined for the range of flow rates expected in that pipe by the procedures described in Chapter II. This might be accomplished by a computer program which determines f for the specified flow rate plus an incremental flow rate and f for the specified flow rate minus an incremental flow rate, and from these f's and Q's compute a and b in Eq. 2-26 and thereafter K and n for the exponential formula. In solving a pipe network problem a computer algorithm for doing this might be called upon whenever the flow rates being used in the solution are outside the range for which K and n are applicable. A listing of a FORTRAN program for accomplishing these computations follows along with the input data required for this problem. Values for K and n for each pipe is given below the listing.

```
100 FORMAT(8F10.5)
    ELOG=9.35*ALOG10(2.71828183)
 20 READ(5,100,END=99) Q,D,FL,E,DQP,VIS
    DEQ=Q*DQP
    ED=E/D
    D=D/12.
    A=.78539392*D**2
    Q1=Q-DEQ
    Q2=Q+DEQ
    V1=Q1/A
    V2=Q2/A
    RE1=V1*D/VIS
    RE2=V2*D/VIS
    ARL=FL/(64.4*D*A**2)
    F=1./(1.14-2.*ALOG10(ED))**2
    RE=RE1
    MM=0
 57 MCT=0
 52 FS=SQRT(F)
    FZ=.5/(F*FS)
    ARG=ED+9.35/(RE*FS)
    FF=1./FS-1.14+2.*ALOG10(ARG)
    DF=FZ+ELOG*FZ/(ARG*RE)
    DIF=FF/DF
    F=F+DIF
    MCT=MCT+1
    IF(ABS(DIF) .GT. .00001 .AND. MCT .LT. 15) GO TO 52
    IF(MM .EQ. 1) GO TO 55
    MM=1
    RE=RE2
    F1=F
    GO TO 57
 55 F2=F
    BE=(ALOG(F1)-ALOG(F2))/(ALOG(Q2)-ALOG(Q1))
    AE=F1*(Q-DEQ)**BE
    EP=2.-BE
    CK=AE*ARL
    WRITE(6,101) Q,D,BE,AE,EP,CK
```



```
101 FORMAT(1H ,5F12.5,E16.6)
    GO TO 20
 99 STOP
    END
```

Input data for above problem.

Q	D	L	e	ΔQ ratio	ν
0.12	4.	150.	0.0102	0.1	0.00001217
0.10	3.	100.	0.0102	0.1	0.00001217
0.05	3.	150.	0.0102	0.1	0.00001217
0.05	3.	100.	0.0102	0.1	0.00001217
0.1	4.	100.	0.0102	0.1	0.00001217
0.05	3.	141.	0.0102	0.1	0.00001217

Pipe No.	1	2	3	4	5	6
Q(cfs)	0.12	0.10	0.05	0.05	0.10	0.05
K	21.0	63.9	85.2	56.8	13.6	80.1
n	1.90	1.92	1.88	1.88	1.89	1.88

The equations which will provide a solution are:

$$Q_1 + Q_2 = 0.223$$
$$-Q_1 + Q_4 + Q_5 = 0$$
$$-Q_3 - Q_4 + Q_6 = -0.056$$
$$-Q_2 + Q_3 = -0.056$$

$$21.0\,Q_1^{1.90} + 56.8\,Q_4^{1.88} - 85.2\,Q_3^{1.88} - 63.9\,Q_2^{1.92} = 0$$
$$13.6\,Q_5^{1.89} - 80.1\,Q_6^{1.88} - 56.8\,Q_4^{1.88} = 0$$

Heads at Junctions as Unknowns

If the head (either the total head or the piezometric head, since the velocity head is generally ignored in determining heads or pressure in pipe networks) at each junction is initially considered unknown instead of the flow rate in each pipe, the number of simultaneous equations which must be solved can be reduced in number. The reduction in number of equations, however, is at the expense of not having some linear equations in the system.

To obtain the system of equations which contain the heads at the junctions of the network as unknowns, the J-1 independent continuity equations are written as before. Thereafter the relationship between the flow rate and head loss is substituted into the continuity equations. In writing these equations it is convenient to use a double subscript for the flow rates. These subscripts correspond to the junctions at the ends of the pipe. The first subscript is the junction number from which the flow comes and the second is the junction number to which the flow is going. Thus Q_{12} represents the flow in the pipe connecting junctions 1 and 2 assuming the flow is from junction 1 to junction 2. If the flow is actually in this direction Q_{12} is positive and Q_{21} equals minus Q_{12}. Solving for Q from the exponential formula (using the double subscript notation) gives

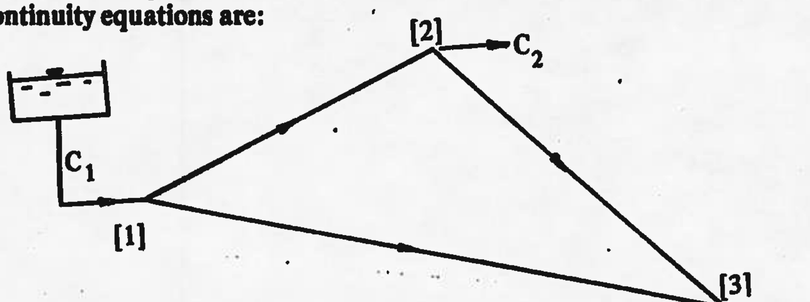
$$Q_{ij} = (h_{L_{ij}}/K_{ij})^{1/n_{ij}} = \left(\frac{H_i - H_j}{K_{ij}}\right)^{1/n_{ij}} \quad \text{.........(4-16)}$$

If Eq. 4-16 is substituted into the junction continuity equations (Eq. 4-12), the following equation results:

$$\left[\Sigma\left(\frac{H_i - H_j}{K_{ij}}\right)^{\frac{1}{n_{ij}}}\right]_{out} - \left[\Sigma\left(\frac{H_i - H_j}{K_{ij}}\right)^{\frac{1}{n_{ij}}}\right]_{in} = C \quad \text{........(4-17)}$$

Upon writing an equation of the form of Eq. 4-17 at J-1 junctions, a system of J-1 nonlinear equations is produced.

As an illustration of this system of equations with the heads at the junctions as the unknowns, consider the simple one loop network below which consists of three junctions and three pipes. In this network two independent continuity equations are available and consequently the head at one of the junctions must be known. In this case at [1]. The two continuity equations are:



$$Q_{12} + Q_{13} = C_1 = C_2 + C_3$$

$$Q_{21} + Q_{23} = -C_2 \quad (\text{or}, -Q_{12} + Q_{23} = -C_2)$$

Note that even though in the second equation the flow in pipe 1-2 is toward the junction, the flow rate Q_{21} is not preceded by a minus sign since the notation 2-1 takes care of this. Alternatively the equations could have been written at junctions 2 and 3 instead of 1 and 2. Substituting Eq. 4-16 into these continuity equations gives the following two equations to solve for the heads, H_2 and H_3 (H_1 is known and the subscripts of the H's denote the junction numbers):

$$\left(\frac{H_1 - H_2}{K_{12}}\right)^{1/n_{12}} + \left(\frac{H_1 - H_3}{K_{13}}\right)^{1/n_{13}} = C_2 + C_3$$

$$-\left(\frac{H_1 - H_2}{K_{12}}\right)^{1/n_{12}} + \left(\frac{H_2 - H_3}{K_{23}}\right)^{1/n_{13}} = -C_2$$

Since a negative number cannot be raised to a power a minus sign must precede any term in which the subscript notation is opposite to the direction of flow, i.e. the second form of equation as given in parentheses is used. Systems of these equations will be referred to as H-equations.

Upon solving this nonlinear system of equations, the pressure at any junction j can be computed by substracting the junction elevation from the head h_j and then multiplying this difference by γ the specific weight of the fluid. To determine the flow rates in the pipes of the network, the now known heads are substituted into Eq. 4-16.

Corrective flow rates around loops of network considered unknowns

Since the number of junctions minus 1 (i.e. J-1) will be less in number than the number of pipes in a network by the number of loops L in the network, the last set of H-equations will generally be less in number than the system of Q-equations. This reduction in number of equations is not necessarily an advantage since all of the equations are nonlinear, whereas in the system of Q-equations only the L energy equations were nonlinear. A system which generally consists of even fewer equations can be written for solving a pipe network, however. These equations consider a corrective flow rate in each loop as the unknowns. This latter system will be referred to as the ΔQ-equations. Since there are L basic loops in a network the ΔQ-equations consist of L equations, all of which are nonlinear.

It is not difficult to establish an initial flow in each pipe which satisfies the J-1 junction continuity equation (which must also satisfy the J^{th} junction continuity equation). These initial flow estimates generally will not simultaneously satisfy the L head loss equations. Therefore they must be corrected before they equal the true flow rates in the pipes. A flow rate adjustment can be added (accounting for sign) to the initially assumed flow in each pipe forming a loop of the network without violating continuity at the junctions. This fact suggests establishing L energy (or head loss) equations around the L loops of the network in which the initial flow plus the corrective loop flow rate ΔQ is used as the true flow rate in the head loss equations. Upon satisfying these head loss equations by finding the appropriate corrective loop flow rates, the J-1 continuity equations would remain satisfied as they initially were. The corrective loop flow rates ΔQ_l may be taken positive in the clockwise or counterclockwise direction around the basic loops, but the sign convention must be consistent around any particular loop, and preferably in the same direction of all loops of the network. Clockwise directions will be considered positive in this book.

In order to develop the mathematics of this possible system of ΔQ-equations, the initially assumed flows, which satisfy the junction continuity equations, will contain an o subscript as well as an i to denote the pipe number. Thus Q_{oi}, $i = 1,2 \ldots N$ represents the initially assumed flow rates in the N pipes. The corrective loop flow rates will be denoted by $\Delta Q\ell$. Thus ΔQ_ℓ, $\ell = 1,2 \ldots L$ are corrective flow rates around the L loops of the system which must be added to Q_o for a given pipe to get the actual flow rate in that pipe. Using this notatin the L energy equations around the basic loops can be written as,

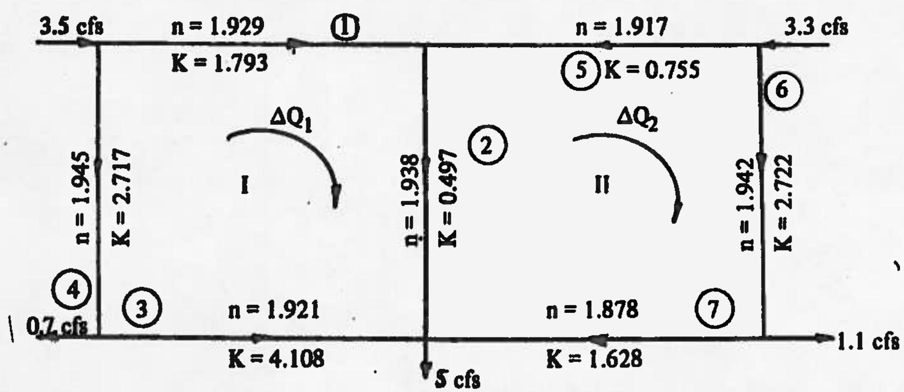
$$\sum_i^{I} K_i (Q_{o_i} + \Delta\vec{Q}_1)^{n_i} = 0 \quad \text{(head loss around loop I)}$$

$$\sum_i^{II} K_i (Q_{o_i} + \Delta\vec{Q}_2)^{n_i} = 0 \quad \text{(head loss around loop II)}$$

$$\vdots \qquad \qquad \ldots\ldots\ldots\ldots (4\text{-}18)$$

$$\sum_i^{L} K_i (Q_{o_i} + \Delta\vec{Q}_L)^{n_i} = 0 \quad \text{(head loss around loop L)}$$

in which each summation includes only those pipes in the loop designated by the Roman numeral I, II, ... L, and $\vec{\Delta Q}\ell$ always includes $\Delta Q\ell$ and also any other ΔQ's flowing through the pipe for which the term applies.



The system of equations, Eq. 4-18, will be set up for the two-loop network shown below. Values for K and n in the exponential formula for the expected flow rates are given by each pipe in the network. The two corrective loop flow rates ΔQ_1 and ΔQ_2 are taken as positive in the clockwise direction. The first step is to provide initial estimates of the flow rate in each pipe which satisfy the junction continuity equations. The estimates are: $Q_{01} = 1.75$ cfs, $Q_{02} = 3.55$ cfs, $Q_{03} = 1.05$ cfs, $Q_{04} = 1.75$ cfs, $Q_{05} = 1.8$ cfs, $Q_{06} = 1.5$ cfs, $Q_{07} = 0.4$ cfs in the directions shown by the arrows on the sketch. The head loss equations around the two loops are:

$$F_1 = 1.793\,(1.75 + \Delta Q_1)^{1.929} + 0.497\,(3.55 + \Delta Q_1 - \Delta Q_2)^{1.938}$$
$$- 4.108\,(1.05 - \Delta Q_1)^{1.921} - 2.717(1.75 - \Delta Q_1)^{1.945} = 0$$

$$F_2 = -0.755\,(1.8 - \Delta Q_2)^{1.917} + 2.722\,(1.5 + \Delta Q_2)^{1.942}$$
$$+ 1.628\,(0.4 + \Delta Q_2)^{1.878} - 0.497\,(3.55 - \Delta Q_2 + \Delta Q_1)^{1.938} = 0$$

Upon obtaining the solution to these two equations for the two unknowns ΔQ_1 and ΔQ_2, the flow rates in each pipe can easily be determined by adding these corrective loop flow rates to the initially assumed flow rates.From these flow rates the head losses in each pipe are determined.

The nonlinearities in this system of equations, as well as the previous two systems discussed, make solution difficult. In the next three chapters methods for obtaining solutions are discussed. The Newton method and the Hardy Cross method (which is the Newton method applied to one equation at a time) are well adapted for the corrective loop flow rate equations, and also the junction head equations. These methods are described respectively in Chapters VI and VII. The Q-equations or the equations which consider the flow in each pipe unknown, can be solved by the linear theory method as discussed in the next chapter, Chapter V. This flow rate system of equations can be solved by the Newton method also. In fact the linear theory method is a variation of the Newton method.