forces is given by

$$\frac{dW_p}{dt} = \int_A p\mathbf{v}\cdot\mathbf{n}\,dA \tag{2.49}$$

where W_p is the work done against external pressure forces. The work done by a fluid in the control volume is typically separated into work done against external pressure forces, W_p, plus work done against rotating surfaces, W_s, commonly referred to as the *shaft work*. The rotating element is called a *rotor* in a gas or steam turbine, an *impeller* in a pump, and a *runner* in a hydraulic turbine. The rate at which work is done by a fluid system, dW/dt, can therefore be written as

$$\frac{dW}{dt} = \frac{dW_p}{dt} + \frac{dW_s}{dt} = \int_A p\mathbf{v}\cdot\mathbf{n}\,dA + \frac{dW_s}{dt} \tag{2.50}$$

Combining Equation 2.50 with the steady-state energy equation (Equation 2.47) leads to

$$\frac{dQ_h}{dt} - \frac{dW_s}{dt} = \int_A \rho\left(\frac{p}{\rho} + e\right)\mathbf{v}\cdot\mathbf{n}\,dA \tag{2.51}$$

Substituting the definition of the internal energy, e, given by Equation 2.48 into Equation 2.51 yields

$$\frac{dQ_h}{dt} - \frac{dW_s}{dt} = \int_A \rho\left(h + gz + \frac{v^2}{2}\right)\mathbf{v}\cdot\mathbf{n}\,dA \tag{2.52}$$

where h is the enthalpy of the fluid defined by

$$h = \frac{p}{\rho} + u \tag{2.53}$$

Denoting the rate at which heat is being added to the fluid system by $\dot{Q}$, and the rate at which work is being done against moving impervious boundaries (shaft work) by $\dot{W}_s$, the energy equation can be written in the form

$$\dot{Q} - \dot{W}_s = \int_A \rho\left(h + gz + \frac{v^2}{2}\right)\mathbf{v}\cdot\mathbf{n}\,dA \tag{2.54}$$

Considering the terms $h + gz$, where

$$h + gz = \frac{p}{\rho} + u + gz = g\left(\frac{p}{\gamma} + z\right) + u \tag{2.55}$$

and γ is the specific weight of the fluid, Equation 2.55 indicates that $h + gz$ can be assumed to be constant across the inflow and outflow openings illustrated in Figure 2.4, since a hydrostatic pressure distribution across the inflow/outflow boundaries guarantees that $p/\gamma + z$ is constant across the inflow/outflow boundaries normal to the flow direction, and the internal energy, u, depends only on the temperature, which can be assumed constant across each boundary. Since $\mathbf{v}\cdot\mathbf{n}$ is equal to zero over the impervious boundaries in contact with the fluid system, Equation 2.54 can be integrated

$$\dot{Q} - \dot{W}_s = (h_1 + gz_1)\int_{A_1} \rho \mathbf{v}\cdot\mathbf{n}\, dA + \int_{A_1} \rho\frac{v^2}{2}\mathbf{v}\cdot\mathbf{n}\, dA + (h_2 + gz_2)\int_{A_2}$$

$$+ \int_{A_2} \rho\frac{v^2}{2}\mathbf{v}\cdot\mathbf{n}\, dA$$

$$= -(h_1 + gz_1)\int_{A_1} \rho v_1\, dA - \int_{A_1} \rho\frac{v_1^3}{2}\, dA + (h_2 + gz_2)\int_{A_2} \rho v_2$$

$$+ \int_{A_2} \rho\frac{v_2^3}{2}\, dA$$

where the subscripts 1 and 2 refer to the inflow and outflow boundaries, resp and the negative signs result from the fact that the unit normal points ou control volume, causing $\mathbf{v}\cdot\mathbf{n}$ to be negative on the inflow boundary and po: the outflow boundary.

Equation 2.56 can be simplified by noting that the assumption of ste: requires that rate of mass inflow to the control volume is equal to the mass rate and, denoting the mass flow rate by $\dot{m}$, the continuity equation requires t

$$\dot{m} = \int_{A_1} \rho v_1\, dA = \int_{A_2} \rho v_2\, dA$$

Furthermore, the constants α_1 and α_2 can be defined by the equations

$$\int_{A_1} \rho\frac{v^3}{2}\, dA = \alpha_1\rho\frac{V_1^3}{2}A_1$$

$$\int_{A_2} \rho\frac{v^3}{2}\, dA = \alpha_2\rho\frac{V_2^3}{2}A_2$$

where A_1 and A_2 are the areas of the inflow and outflow boundaries, respe and V_1 and V_2 are the corresponding mean velocities across these boundari constants α_1 and α_2 are determined by the velocity profile across the flow bour and these constants are called *kinetic energy correction factors*. If the vel constant across a flow boundary, then it is clear from Equation 2.58 that the energy correction factor for that boundary is equal to unity; for any other distribution, the kinetic energy factor is greater than unity. Combining Equatic to 2.59 leads to

$$\dot{Q} - \dot{W}_s = -(h_1 + gz_1)\dot{m} - \alpha_1\rho\frac{V_1^3}{2}A_1 + (h_2 + gz_2)\dot{m} + \alpha_2\rho\frac{V_2^3}{2}A_2$$

Invoking the continuity equation requires that

$$\rho V_1 A_1 = \rho V_2 A_2 = \dot{m}$$

and combining Equations 2.60 and 2.61 leads to

$$\dot{Q} - \dot{W}_s = \dot{m}\left[\left(h_2 + gz_2 + \alpha_2\frac{V_2^2}{2}\right) - \left(h_1 + gz_1 + \alpha_1\frac{V_1^2}{2}\right)\right] \tag{2.62}$$

which can be put in the form

$$\frac{\dot{Q}}{\dot{m}g} - \frac{\dot{W}_s}{\dot{m}g} = \left(\frac{p_2}{\gamma} + \frac{u_2}{g} + z_2 + \alpha_2\frac{V_2^2}{2g}\right) - \left(\frac{p_1}{\gamma} + \frac{u_1}{g} + z_1 + \alpha_1\frac{V_1^2}{2g}\right) \tag{2.63}$$

and can be further rearranged into the useful form

$$\left(\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2\right) + \left[\frac{1}{g}(u_2 - u_1) - \frac{\dot{Q}}{\dot{m}g}\right] + \left[\frac{\dot{W}_s}{\dot{m}g}\right] \tag{2.64}$$

Two key terms can be identified in Equation 2.64: the (shaft) work done by the fluid per unit weight, h_s, defined by the relation

$$h_s = \frac{\dot{W}_s}{\dot{m}g} \tag{2.65}$$

and the energy loss per unit weight, commonly called the head loss, h_L, defined by the relation

$$h_L = \frac{1}{g}(u_2 - u_1) - \frac{\dot{Q}}{\dot{m}g} \tag{2.66}$$

Combining Equations 2.64 to 2.66 leads to the most common form of the steady-state *energy equation*

$$\boxed{\left(\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2\right) + h_L + h_s} \tag{2.67}$$

where a positive head loss indicates an increase in internal energy (manifested by an increase in temperature) and/or a loss of heat, and a positive value of h_s is associated with work being done by the fluid, such as in moving a turbine runner. Many practitioners incorrectly refer to Equation 2.67 as the *Bernoulli equation*, which bears some resemblance to Equation 2.67 but is different in several important respects. Fundamental differences between the energy equation and the Bernoulli equation are that the Bernoulli equation is derived from the momentum equation, which is independent of the energy equation, and the Bernoulli equation does not account for fluid friction.

Energy and hydraulic grade lines. The *total head*, h, of a fluid at any cross section of a pipe is defined by

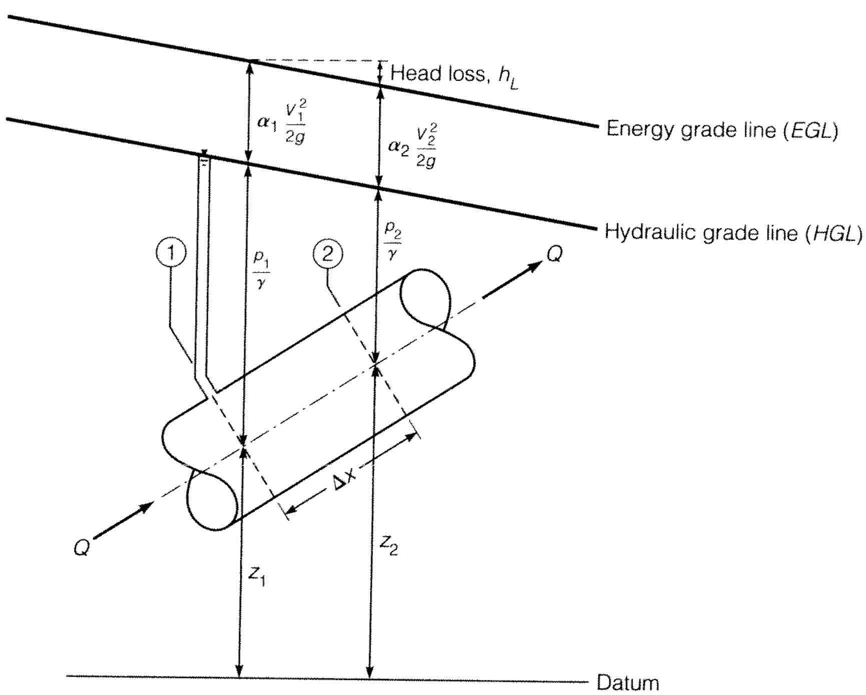
$$\boxed{h = \frac{p}{\gamma} + \alpha\frac{V^2}{2g} + z} \tag{2.68}$$

where p is the pressure in the fluid at the centroid of the cross section, γ is th weight of the fluid, α is the kinetic energy correction factor, V is the averag across the pipe cross section, and z is the elevation of the centroid of the p section. The total head measures the average energy per unit weight of flowing across a pipe cross section. The energy equation, Equation 2.67, s changes in the total head along the pipe are described by

$$h(x + \Delta x) = h(x) - (h_L + h_s)$$

where x is the coordinate measured along the pipe centerline, Δx is the between two cross sections in the pipe, h_L is the head loss, and h_s is the sh done by the fluid over the distance Δx. The practical application of Equa is illustrated in Figure 2.5, where the head loss, h_L, between two sections a Δx apart is indicated. At each cross section, the total energy, h, is plotted r a defined datum, and the locus of these points is called the *energy grade* energy grade line at each pipe cross section is located a distance p/γ + vertically above the centroid of the cross section, and between any two cross the elevation of the energy grade line falls by a vertical distance equal to the l caused by pipe friction, h_L, plus the shaft work, h_s, done by the fluid. The *h grade line* measures the hydraulic head $p/\gamma + z$ at each pipe cross section. It i a distance p/γ above the pipe centerline and indicates the elevation to which would rise in an open tube connected to the wall of the pipe section. The h grade line is therefore located a distance $\alpha V^2/2g$ below the energy grade line water-supply applications the velocity heads are negligible and the hydrau line closely approximates the energy grade line.

FIGURE 2.5: Head loss along pipe

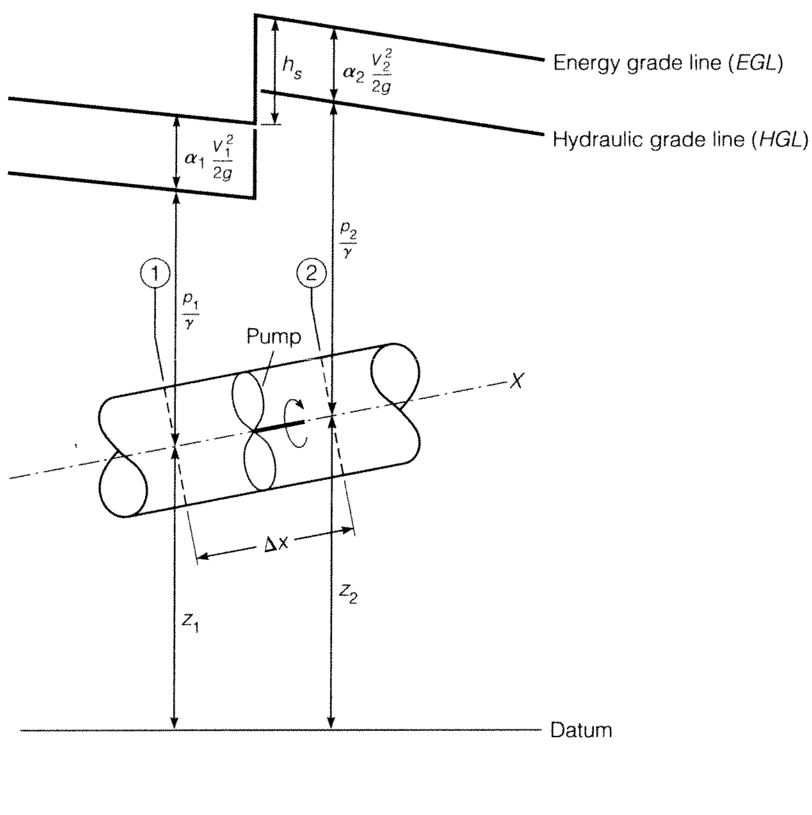


Both the hydraulic grade line and the energy grade line are useful in visualizing the state of the fluid as it flows along the pipe and are frequently used in assessing the performance of fluid-delivery systems. Most fluid-delivery systems, for example, require that the fluid pressure remain positive, in which case the hydraulic grade line must remain above the pipe. In circumstances where additional energy is required to maintain acceptable pressures in pipelines, a pump is installed along the pipeline to elevate the energy grade line by an amount h_s, which also elevates the hydraulic grade line by the same amount. This condition is illustrated in Figure 2.6. In cases where the pipeline upstream and downstream of the pump are of the same diameter, the velocity heads $\alpha V^2/2g$ both upstream and downstream of the pump are the same, and the head added by the pump, h_s, goes entirely to increase the pressure head, p/γ, of the fluid. It should also be clear from Figure 2.5 that the pressure head in a pipeline can be increased by simply increasing the pipeline diameter, which reduces the velocity head, $\alpha V^2/2g$, and thereby increases the pressure head, p/γ, to maintain the same approximately total energy at the pipe section. Expansion losses will cause a reduction in total energy.

Velocity profile. The momentum and energy correction factors, α and β, depend on the cross-sectional velocity distribution. The velocity profile in both smooth and rough pipes of circular cross section can be estimated by the semi-empirical equation

$$\boxed{v(r) = \left[(1 + 1.326\sqrt{f}) - 2.04\sqrt{f}\log\left(\frac{R}{R - r}\right)\right]V} \tag{2.70}$$

2.6: Pump effect on pipeline



where $v(r)$ is the velocity at a radial distance r from the centerline of the pipe radius of the pipe, f is the friction factor, and V is the average velocity across

The velocity distribution given by Equation 2.70 agrees well with veloc surements in both smooth and rough pipes. This equation, however, is not ap within the small region close to the centerline of the pipe and is also not ap in the small region close to the pipe boundary. This is apparent since at th the pipe dv/dr must be equal to zero, but Equation 2.70 does not have a zero $r = 0$. At the pipe boundary v must also be equal to zero, but Equation 2.7 velocity of zero at a small distance from the wall, with a velocity of $-\infty$ at $r =$ energy and momentum correction factors, α and β, derived from the velocit are (Moody, 1950)

$$\alpha = 1 + 2.7f$$

$$\beta = 1 + 0.98f$$

Another commonly used equation to describe the velocity distribution i lent pipe flow is the empirical *power law* equation given by

$$v(r) = V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

where V_0 is the centerline velocity and n is a function of the Reynolds numl Values of n typically range between 6 and 10 and can be approximated by (l McDonald, 1992; Schlichting, 1979)

$$n = 1.83 \log \text{Re} - 1.86$$

The power law is not applicable within $0.04R$ of the wall, since the power la an infinite velocity gradient at the wall. Although the profile fits the data clos centerline of the pipe, it does not give zero slope at the centerline. The kinetic coefficient, α, derived from the power law equation is given by

$$\alpha = \frac{(1 + n)^3(1 + 2n)^3}{4n^4(3 + n)(3 + 2n)}$$

For n between 6 and 10, α varies from 1.08 to 1.03. In most engineering appli α and β are taken as unity (see Problem 2.14).

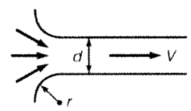
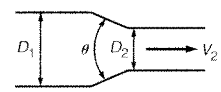
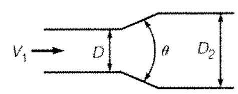
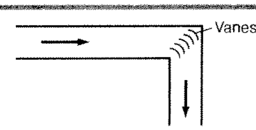
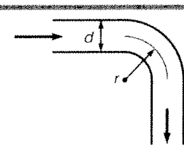
Head losses in transitions and fittings. The head losses in straight pipes of c diameter are caused by friction between the moving fluid and the pipe bound are estimated using the Darcy–Weisbach equation. Flow through pipe fittings, bends, and through changes in pipeline geometry causes additional head los that are quantified by an equation of the form

$$h_0 = \sum K \frac{V^2}{2g}$$

where K is a loss coefficient that is specific to each fitting and transition, an the average velocity at a defined location within the transition or fitting. Tl

2.7: Loss coefficients for ns and fittings

Roberson and Crowe

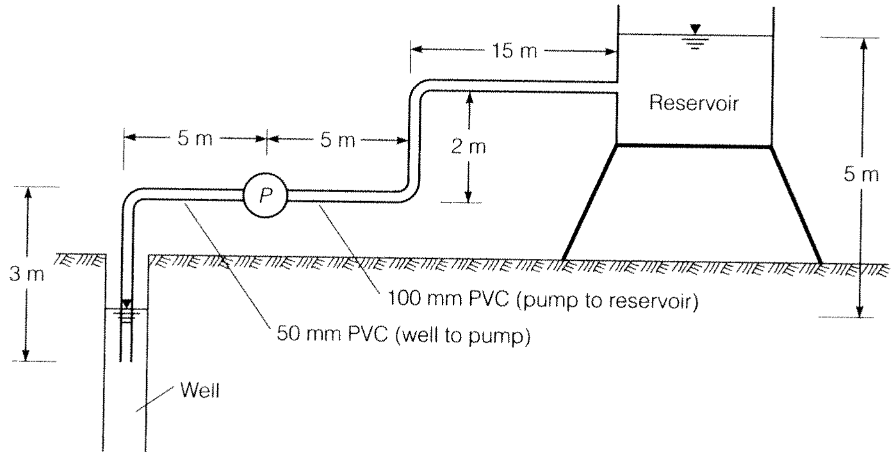
Description	Sketch	Additional Data	K	
Pipe entrance		r/d	K	
		0.0	0.50	
		0.1	0.12	
		>0.2	0.03	
Contraction		D_2/D_1	K $\theta = 60°$	K $\theta = 180°$
		0.0	0.08	0.50
		0.20	0.08	0.49
		0.40	0.07	0.42
		0.60	0.06	0.27
		0.80	0.06	0.20
		0.90	0.06	0.10
Expansion		D_1/D_2	K $\theta = 20°$	K $\theta = 180°$
		0.0		1.00
		0.20	0.30	0.87
		0.40	0.25	0.70
		0.60	0.15	0.41
		0.80	0.10	0.15
90° miter bend		Without vanes	$K = 1.1$	
		With vanes	$K = 0.2$	
90° smooth bend		r/d	K $\theta = 90°$	
		1	0.35	
		2	0.19	
		4	0.16	
		6	0.21	
Threaded pipe fittings			K	
	Globe valve — wide open		10.0	
	Angle valve — wide open		5.0	
	Gate valve — wide open		0.2	
	Gate valve — half open		5.6	
	Return bend		2.2	
	Tee			
	straight-through flow		0.4	
	side-outlet flow		1.8	
	90° elbow		0.9	
	45° elbow		0.4	

coefficients for several fittings and transitions are shown in Figure 2.7. Head losses in transitions and fittings are also called *local head losses* or *minor head losses*. The latter term should be avoided, however, since in some cases these head losses are a significant portion of the total head loss in a pipe. Detailed descriptions of local head losses in various valve geometries can be found in Mott (1994), and additional data on local head losses in pipeline systems can be found in Brater and colleagues (1996).

EXAMPLE 2.6

A pump is to be selected that will pump water from a well into a storage reservoir. In order to fill the reservoir in a timely manner, the pump is required to deliver 5 L/s when the water level in the reservoir is 5 m above the water level in the well. Find the head that must be added by the pump. The pipeline system is shown in Figure 2.8. Assume that the local loss coefficient for each of the bends is equal to 0.25 and that the temperature of the water is 20°C.

FIGURE 2.8: Pipeline system



Solution Taking the elevation of the water surface in the well to be equal to
proceeding from the well to the storage reservoir (where the head is equal
the energy equation (Equation 2.67) can be written as

$$0 - \frac{V_1^2}{2g} - \frac{f_1 L_1}{D_1}\frac{V_1^2}{2g} - K_1\frac{V_1^2}{2g} + h_p - \frac{f_2 L_2}{D_2}\frac{V_2^2}{2g} - (K_2 + K_3)\frac{V_2^2}{2g} - \frac{V}{2}$$

where V_1 and V_2 are the velocities in the 50-mm ($=D_1$) and 100-mm ($=D_2$
respectively; L_1 and L_2 are the corresponding pipe lengths; f_1 and f_2 are th
sponding friction factors; K_1, K_2, and K_3 are the loss coefficients for each of tl
bends; and h_p is the head added by the pump. The cross-sectional areas of eac
pipes, A_1 and A_2, are given by

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.05)^2 = 0.001963 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.10)^2 = 0.007854 \text{ m}^2$$

When the flowrate, Q, is 5 L/s, the velocities V_1 and V_2 are given by

$$V_1 = \frac{Q}{A_1} = \frac{0.005}{0.001963} = 2.54 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.005}{0.007854} = 0.637 \text{ m/s}$$

PVC pipe is considered smooth ($k_s \approx 0$) and therefore the friction factor, f,
estimated using the Jain equation

$$f = \frac{0.25}{\left[\log_{10}\frac{5.74}{\text{Re}^{0.9}}\right]^2}$$

where Re is the Reynolds number. At 20°C, the kinematic viscosity, ν, is equal to 1.00×10^{-6} m^2/s and for the 50-mm pipe

$$\text{Re}_1 = \frac{V_1 D_1}{\nu} = \frac{(2.54)(0.05)}{1.00 \times 10^{-6}} = 1.27 \times 10^5$$

which leads to

$$f_1 = \frac{0.25}{\left[\log_{10} \frac{5.74}{(1.27 \times 10^5)^{0.9}}\right]^2} = 0.0170$$

and for the 100-mm pipe

$$\text{Re}_2 = \frac{V_2 D_2}{\nu} = \frac{(0.637)(0.10)}{1.00 \times 10^{-6}} = 6.37 \times 10^4$$

which leads to

$$f_2 = \frac{0.25}{\left[\log_{10} \frac{5.74}{(6.37 \times 10^4)^{0.9}}\right]^2} = 0.0197$$

Substituting the values of these parameters into the energy equation yields

$$0 - \left[1 + \frac{(0.0170)(8)}{0.05} + 0.25\right] \frac{2.54^2}{(2)(9.81)} + h_p$$
$$- \left[\frac{(0.0197)(22)}{0.10} + 0.25 + 0.25 + 1\right] \frac{0.637^2}{(2)(9.81)} = 5$$

which leads to

$$h_p = 6.43 \text{ m}$$

Therefore the head to be added by the pump is 6.43 m.

Local losses are frequently neglected in the analysis of pipeline systems. As a general rule, neglecting local losses is justified when, on average, there is a length of 1000 diameters between each local loss (Streeter et al., 1998).

Head losses in noncircular conduits. Most pipelines are of circular cross section, but flow of water in noncircular conduits is commonly encountered. The hydraulic radius, R, of a conduit of any shape is defined by the relation

$$R = \frac{A}{P} \tag{2.77}$$

where A is the cross-sectional area of the conduit and P is the wetted perimeter. For circular conduits of diameter D, the hydraulic radius is given by

$$R = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} \tag{2.78}$$

or

$$D = 4R \tag{2.79}$$

Using the hydraulic radius, R, as the length scale of a closed conduit in D, the frictional head losses, h_f, in noncircular conduits can be estimated ι Darcy–Weisbach equation for circular conduits by simply replacing D by 4, yields

$$h_f = \frac{fL}{4R}\frac{V^2}{2g}$$

where the friction factor, f, is calculated using a Reynolds number, Re, define

$$\text{Re} = \frac{\rho V(4R)}{\mu}$$

and a relative roughness defined by $k_s/4R$.

Characterizing a noncircular conduit by the hydraulic radius, R, is ne approximate, since conduits of arbitrary cross section cannot be describe single parameter. Secondary currents that are generated across a noncircular cross section to redistribute the shears are another reason why noncircular (cannot be completely characterized by the hydraulic radius (Liggett, 1994). H using the hydraulic radius as a basis for calculating frictional head losses in nor conduits is usually accurate to within 15% for turbulent flow (Munson et al White, 1994). This approximation is much less accurate for laminar flows the accuracy is on the order of ±40% (White, 1994). Characterization of non conduits by the hydraulic radius can be used for rectangular conduits where t of sides, called the *aspect ratio*, does not exceed about 8:1 (Olson and Wrigh although some references state that aspect ratios must be less than 4:1 (Po Wiggert, 2001).

EXAMPLE 2.7

Water flows through a rectangular concrete culvert of width 2 m and depth 1 r length of the culvert is 10 m and the flowrate is 6 m^3/s, estimate the head loss the culvert. Assume that the culvert flows full.

Solution The head loss can be calculated using Equation 2.80. The hydraulic R, is given by

$$R = \frac{A}{P} = \frac{(2)(1)}{2(2+1)} = 0.333 \text{ m}$$

and the mean velocity, V, is given by

$$V = \frac{Q}{A} = \frac{6}{(2)(1)} = 3 \text{ m/s}$$

At 20°C, $\nu = 1.00 \times 10^{-6}$ m^2/s, and therefore the Reynolds number, Re, is gi

$$\text{Re} = \frac{V(4R)}{\nu} = \frac{(3)(4 \times 0.333)}{1.00 \times 10^{-6}} = 4.00 \times 10^6$$

A median equivalent sand roughness for concrete can be taken as k_s = (Table 2.1), and therefore the relative roughness, $k_s/4R$, is given by

$$\frac{k_s}{4R} = \frac{1.6 \times 10^{-3}}{4(0.333)} = 0.00120$$

Substituting Re and $k_s/4R$ into the Jain equation (Equation 2.39) for the friction factor gives

$$f = \frac{0.25}{\left[\log\left(\frac{ks/4R}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}$$

$$= \frac{0.25}{\left[\log\left(\frac{0.00120}{3.7} + \frac{5.74}{(4.00\times10^6)^{0.9}}\right)\right]^2}$$

which yields

$$f = 0.0206$$

The frictional head loss in the culvert, h_f, is therefore given by the Darcy–Weisbach equation as

$$h_f = \frac{fL}{4R}\frac{V^2}{2g} = \frac{(0.0206)(10)}{(4 \times 0.333)}\frac{3^2}{2(9.81)} = 0.0709 \text{ m}$$

The head loss in the culvert can therefore be estimated as 7.1 cm.

Empirical friction-loss formulae. Friction losses in pipelines should generally be calculated using the Darcy–Weisbach equation. However, a minor inconvenience in using this equation to relate the friction loss to the flow velocity results from the dependence of the friction factor on the flow velocity; therefore, the Darcy–Weisbach equation must be solved simultaneously with the Colebrook equation. In modern engineering practice, computer hardware and software make this a very minor inconvenience. In earlier years, however, this was considered a real problem, and various empirical head-loss formulae were developed to relate the head loss directly to the flow velocity. Those most commonly used are the *Hazen–Williams formula* and the *Manning formula*.

The *Hazen–Williams formula* (Williams and Hazen, 1920) is applicable only to the flow of water in pipes and is given by

$$\boxed{V = 0.849C_H R^{0.63} S_f^{0.54}} \tag{2.82}$$

where V is the flow velocity (in m/s), C_H is the Hazen–Williams roughness coefficient, R is the hydraulic radius (in m), and S_f is the slope of the energy grade line, defined by

$$S_f = \frac{h_f}{L} \tag{2.83}$$

where h_f is the head loss due to friction over a length L of pipe. Values of C_H for a variety of commonly used pipe materials are given in Table 2.2. Solving Equations 2.82 and 2.83 yields the following expression for the frictional head loss:

$$h_f = 6.82\frac{L}{D^{1.17}}\left(\frac{V}{C_H}\right)^{1.85} \tag{2.84}$$

TABLE 2.2: Pipe Roughness Coefficients

	C_H		n	
Pipe material	Range	Typical	Range	Ty
Ductile and cast iron:				
New, unlined	120–140	130	—	0.
Old, unlined	40–100	80	—	0.'
Cement lined and seal coated	100–140	120	0.011–0.015	0.'
Steel:				
Welded and seamless	80–150	120	—	0.(
Riveted	—	110	0.012–0.018	0.l
Mortar lining	120–145	130	—	–
Asbestos cement		140	—	0.(
Concrete	100–140	120	0.011–0.015	0.(
Vitrified clay pipe (VCP)	—	110	0.012–0.014	–
Polyvinyl chloride (PVC)	135–150	140	0.007–0.011	0.(
Corrugated metal pipe (CMP)	—	—	—	0.(

Sources: Velon and Johnson (1993); Wurbs and James (2002).

where D is the diameter of the pipe. The Hazen–Williams equation is ap to the flow of water at 16°C in pipes with diameters between 50 mm and 1. and flow velocities less than 3 m/s (Mott, 1994). Outside of these conditi of the Hazen–Williams equation is strongly discouraged. To further suppc quantitative limitations, Street and colleagues (1996) and Liou (1998) hav that the Hazen–Williams coefficient has a strong Reynolds number depende is mostly applicable where the pipe is relatively smooth and in the early its transition to rough flow. Furthermore, Jain and colleagues (1978) have that an error of up to 39% can be expected in the evaluation of the vel the Hazen–Williams formula over a wide range of diameters and slopes. In these cautionary notes, the Hazen–Williams formula is frequently used in the States for the design of large water-supply pipes without regard to its limited applicability, a practice that can have very detrimental effects on pipe design ar potentially lead to litigation (Bombardelli and García, 2003). In some cases, er have calculated correction factors for the Hazen–Williams roughness coeffi account for these errors (Valiantzas, 2005).

A second empirical formula that is sometimes used to describe flow in the Manning formula, which is given by

$$\boxed{V = \frac{1}{n} R^{\frac{2}{3}} S_f^{\frac{1}{2}}}$$

where V, R, and S_f have the same meaning and units as in the Hazen–W formula, and n is the Manning roughness coefficient. Values of n for a va commonly used pipe materials are given in Table 2.2. Solving Equations 2.85 a yields the following expression for the frictional head loss:

$$h_f = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}}$$

The Manning formula applies only to rough turbulent flows, where the frictional head losses are controlled by the relative roughness. Such conditions are delineated by Equation 2.37.

EXAMPLE 2.8

Water flows at a velocity of 1 m/s in a 150-mm diameter new ductile iron pipe. Estimate the head loss over 500 m using: (a) the Hazen–Williams formula, (b) the Manning formula, and (c) the Darcy–Weisbach equation. Compare your results and assess the validity of each head-loss equation.

Solution

(a) The Hazen–Williams roughness coefficient, C_H, can be taken as 130 (Table 2.2), $L = 500$ m, $D = 0.150$ m, $V = 1$ m/s, and therefore the head loss, h_f, is given by Equation 2.84 as

$$h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85} = 6.82 \frac{500}{(0.15)^{1.17}} \left(\frac{1}{130} \right)^{1.85} = 3.85 \text{ m}$$

(b) The Manning roughness coefficient, n, can be taken as 0.013 (approximation from Table 2.2), and therefore the head loss, h_f, is given by Equation 2.86 as

$$h_f = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} = 6.35 \frac{(0.013)^2(500)(1)^2}{(0.15)^{\frac{4}{3}}} = 6.73 \text{ m}$$

(c) The equivalent sand roughness, k_s, can be taken as 0.26 mm (Table 2.1), and the Reynolds number, Re, is given by

$$\text{Re} = \frac{VD}{\nu} = \frac{(1)(0.15)}{1.00 \times 10^{-6}} = 1.5 \times 10^5$$

where $\nu = 1.00 \times 10^{-6}$ m²/s at 20°C. Substituting k_s, D, and Re into the Colebrook equation yields the friction factor, f, where

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right] = -2 \log \left[\frac{0.26}{3.7(150)} + \frac{2.51}{1.5 \times 10^5 \sqrt{f}} \right]$$

which yields

$$f = 0.0238$$

The head loss, h_f, is therefore given by the Darcy–Weisbach equation as

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.0238 \frac{500}{0.15} \frac{1^2}{2(9.81)} = 4.04 \text{ m}$$

It is reasonable to assume that the Darcy–Weisbach equation yields the most accurate estimate of the head loss. In this case, the Hazen–Williams formula gives a head loss 5% less than the Darcy–Weisbach equation, and the Manning formula yields a head loss 67% higher than the Darcy–Weisbach equation.

From the given data, Re $= 1.5 \times 10^5$, $D/k_s = 150/0.26 = 577$, and Equat
gives the limit of rough turbulent flow as

$$\frac{1}{\sqrt{f}} = \frac{\text{Re}}{200(D/k_s)} = \frac{1.5 \times 10^5}{200(577)} \quad \rightarrow \quad f = 0.591$$

Since the actual friction factor ($=0.0238$) is much less than the minimum frictio for rough turbulent flow ($=0.591$), the flow is not in the rough turbulent regi the Manning equation is not valid. Since the pipe diameter ($=150$ mm) is b 50 mm and 1850 mm, and the velocity ($=1$ m/s) is less than 3 m/s, the Hazen–V formula is valid. The Darcy–Weisbach equation is unconditionally valid. Give results, it is not surprising that the Darcy–Weisbach and Hazen–Williams form in close agreement, with the Manning equation giving a significantly differen These results indicate why application of the Manning equation to closed- flows is strongly discouraged.

2.3 Pipe Networks

Pipe networks are commonly encountered in the context of water-distribution s The performance criteria of these systems are typically specified in terms of m flow rates and pressure heads that must be maintained at the specified point network. Analyses of pipe networks are usually within the context of: (1) desi new network, (2) designing a modification to an existing network, and/or (3) eva the reliability of an existing or proposed network. The procedure for anal pipe network usually aims at finding the flow distribution within the netwo the pressure distribution being derived from the flow distribution using the equation. A typical pipe network is illustrated in Figure 2.9, where the bo conditions consist of inflows, outflows, and constant-head boundaries such as reservoirs. Inflows are typically from water-treatment facilities, and outflov consumer withdrawals or fires. All outflows are assumed to occur at network ju

The basic equations to be satisfied in pipe networks are the continuity and equations. The continuity equation requires that, at each node in the netwo sum of the outflows is equal to the sum of the inflows. This requirement is ex

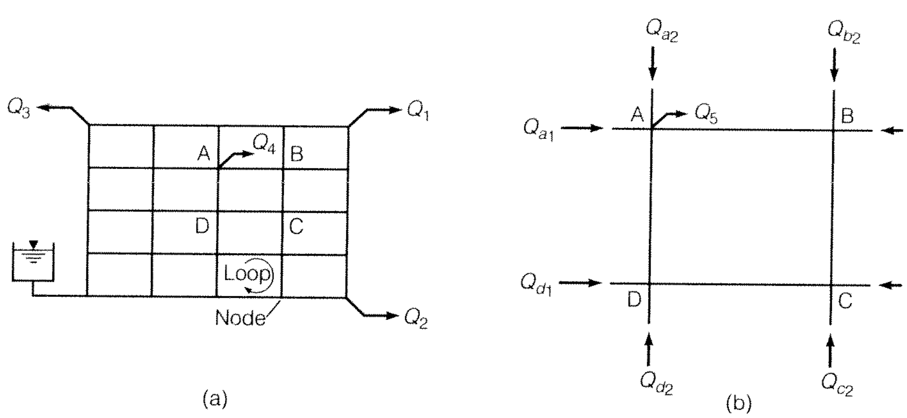


FIGURE 2.9: Typical pipe network

by the relation

$$\sum_{i=1}^{NP(j)} Q_{ij} - F_j = 0, \qquad j = 1, \ldots, NJ \tag{2.87}$$

where $NP(j)$ is the number of pipes meeting at junction j; Q_{ij} is the flowrate in pipe i at junction j (inflows positive); F_j is the external flow rate (outflows positive) at junction j; and NJ is the total number of junctions in the network. The energy equation requires that the heads at each of the nodes in the pipe network be consistent with the head losses in the pipelines connecting the nodes. There are two principal methods of calculating the flows in pipe networks: the nodal method and the loop method. In the nodal method, the energy equation is expressed in terms of the heads at the network nodes, while in the loop method the energy equation is expressed in terms of the flows in closed loops within the pipe network.

2.3.1 Nodal Method

In the nodal method, the energy equation is written for each pipeline in the network as

$$h_2 = h_1 - \left(\frac{fL}{D} + \sum K_m\right)\frac{Q|Q|}{2gA^2} + \frac{Q}{|Q|}h_p \tag{2.88}$$

where h_2 and h_1 are the heads at the upstream and downstream ends of a pipe; the terms in parentheses measure the friction loss and local losses, respectively; and h_p is the head added by pumps in the pipeline. The energy equation given by Equation 2.88 has been modified to account for the fact that the flow direction is in many cases unknown, in which case a positive flow direction in each pipeline must be assumed, and a consistent set of energy equations stated for the entire network. Equation 2.88 assumes that the positive flow direction is from node 1 to node 2. Application of the nodal method in practice is usually limited to relatively simple networks.

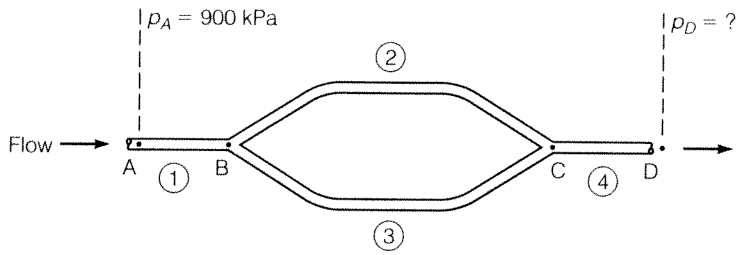
EXAMPLE 2.9

The high-pressure ductile-iron pipeline shown in Figure 2.10 becomes divided at point B and rejoins at point C. The pipeline characteristics are given in the following tables.

Pipe	Diameter (mm)	Length (m)
1	750	500
2	400	600
3	500	650
4	700	400

Location	Elevation (m)
A	5.0
B	4.5
C	4.0
D	3.5

FIGURE 2.10: Pipe network



If the flowrate in Pipe 1 is 2 m^3/s and the pressure at point A is 900 kPa, calc pressure at point D. Assume that the flows are fully turbulent in all pipes.

Solution The equivalent sand roughness, k_s, of ductile-iron pipe is 0.26 mm. pipe and flow characteristics are as follows:

Pipe	Area (m^2)	Velocity (m/s)	k_s/D	f
1	0.442	4.53	0.000347	0.0154
2	0.126	—	0.000650	0.0177
3	0.196	—	0.000520	0.0168
4	0.385	5.20	0.000371	0.0156

where it has been assumed that the flows are fully turbulent. Taking γ = 9.79 the head at location A, h_A, is given by

$$h_A = \frac{p_A}{\gamma} + \frac{V_1^2}{2g} + z_A = \frac{900}{9.79} + \frac{4.53^2}{(2)(9.81)} + 5 = 98.0 \text{ m}$$

and the energy equations for each pipe are as follows

$$\text{Pipe 1:} \quad h_B = h_A - \frac{f_1 L_1}{D_1}\frac{V_1^2}{2g} = 98.0 - \frac{(0.0154)(500)}{0.75}\frac{4.53^2}{(2)(9.81)}$$
$$= 87.3 \text{ m}$$

$$\text{Pipe 2:} \quad h_C = h_B - \frac{f_2 L_2}{D_2}\frac{Q_2^2}{2gA_2^2} = 87.3 - \frac{(0.0177)(600)}{0.40}\frac{Q_2^2}{(2)(9.81)(0.12}$$
$$= 87.3 - 85.2Q_2^2$$

$$\text{Pipe 3:} \quad h_C = h_B - \frac{f_3 L_3}{D_3}\frac{Q_3^2}{2gA_3^2} = 87.3 - \frac{(0.0168)(650)}{0.50}\frac{Q_3^2}{(2)(9.81)(0.19}$$
$$= 87.3 - 29.0Q_3^2$$

$$\text{Pipe 4:} \quad h_D = h_C - \frac{f_4 L_4}{D_4}\frac{Q_4^2}{2gA_4^2} = h_C - \frac{(0.0156)(400)}{0.70}\frac{Q_4^2}{(2)(9.81)(0.385)}$$
$$= h_C - 3.07Q_4^2$$

and the continuity equations at the two pipe junctions are

$$\text{Junction B:}\quad Q_2 + Q_3 = 2\ \text{m}^3/\text{s} \tag{2.93}$$

$$\text{Junction C:}\quad Q_2 + Q_3 = Q_4 \tag{2.94}$$

Equations 2.90 to 2.94 are five equations in five unknowns: h_C, h_D, Q_2, Q_3, and Q_4. Equations 2.93 and 2.94 indicate that

$$Q_4 = 2\ \text{m}^3/\text{s}$$

Combining Equations 2.90 and 2.91 leads to

$$87.3 - 85.2Q_2^2 = 87.3 - 29.0Q_3^2$$

and therefore

$$Q_2 = 0.583Q_3 \tag{2.95}$$

Substituting Equation 2.95 into Equation 2.93 yields

$$2 = (0.583 + 1)Q_3$$

or

$$Q_3 = 1.26\ \text{m}^3/\text{s}$$

and from Equation 2.95

$$Q_2 = 0.74\ \text{m}^3/\text{s}$$

According to Equation 2.91

$$h_C = 87.3 - 29.0Q_3^2 = 87.3 - 29.0(1.26)^2 = 41.3\ \text{m}$$

and Equation 2.92 gives

$$h_D = h_C - 3.07Q_4^2 = 41.3 - 3.07(2)^2 = 29.0\ \text{m}$$

Therefore, since the total head at D, h_D, is equal to 29.0 m, then

$$29.0 = \frac{p_D}{\gamma} + \frac{V_4^2}{2g} + z_D = \frac{p_D}{9.79} + \frac{5.20^2}{(2)(9.81)} + 3.5$$

which yields

$$p_D = 236\ \text{kPa}$$

Therefore, the pressure at location D is 236 kPa.

This problem has been solved by assuming that the flows in all pipes are fully turbulent. This is generally not known a priori, and therefore a complete solution would require repeating the calculations until the assumed friction factors are consistent with the calculated flowrates.

2.3.2 Loop Method

In the loop method, the energy equation is written for each loop of the net which case the algebraic sum of the head losses within each loop is equal to z requirement is expressed by the relation

$$\sum_{j=1}^{NP(i)} (h_{L,ij} - h_{p,ij}) = 0, \qquad i = 1, \ldots, NL$$

where NP(i) is the number of pipes in loop i, $h_{L,ij}$ is the head loss in pipe j ($h_{p,ij}$ is the head added by any pumps that may exist in line ij, and NL is the of loops in the network. Combining Equations 2.87 and 2.96 with an expre calculating the head losses in pipes, such as the Darcy–Weisbach equation, pump characteristic curves, which relate the head added by the pump to the through the pump, yields a complete mathematical description of the flow p Solution of this system of flow equations is complicated by the fact that the e are nonlinear, and numerical methods must be used to solve for the flow dist in the pipe network.

Hardy Cross method. The Hardy Cross method (Cross, 1936) is a simple te for hand solution of the loop system of equations governing flow in pipe n This iterative method was developed before the advent of computers, and mu efficient algorithms are now used for numerical computations. In spite of Hardy Cross method is presented here to illustrate the iterative solution of equations in pipe networks. The Hardy Cross method assumes that the head in each pipe is proportional to the discharge, Q, raised to some power n, in wh

$$h_L = rQ^n$$

where typical values of n range from 1 to 2, where $n = 1$ corresponds to visc and $n = 2$ to fully turbulent flow. The proportionality constant, r, depends c head-loss equation is used and the types of losses in the pipe. Clearly, if all he are due to friction and the Darcy–Weisbach equation is used to calculate t losses, then r is given by

$$r = \frac{fL}{2gA^2D}$$

and $n = 2$. If the flow in each pipe is approximated as $\hat{Q}$, and ΔQ is the errc estimate, then the actual flowrate, Q, is related to $\hat{Q}$ and ΔQ by

$$Q = \hat{Q} + \Delta Q$$

and the head loss in each pipe is given by

$$\begin{aligned} h_L &= rQ^n \\ &= r(\hat{Q} + \Delta Q)^n \\ &= r\left[\hat{Q}^n + n\hat{Q}^{n-1}\Delta Q + \frac{n(n-1)}{2}\hat{Q}^{n-2}(\Delta Q)^2 + \cdots + (\Delta Q)^n\right] \end{aligned}$$

If the error in the flow estimate, ΔQ, is small, then the higher-order terms in ΔQ can be neglected and the head loss in each pipe can be approximated by

$$h_L \approx r\hat{Q}^n + rn\hat{Q}^{n-1}\Delta Q \tag{2.101}$$

This relation approximates the head loss in the flow direction. However, in working with pipe networks, it is required that the algebraic sum of the head losses in any loop of the network (see Figure 2.9) must be equal to zero. We must therefore define a positive flow direction (such as clockwise), and count head losses as positive in pipes when the flow is in the positive direction and negative when the flow is opposite to the selected positive direction. Under these circumstances, the sign of the head loss must be the same as the sign of the flow direction. Further, when the flow is in the positive direction, positive values of ΔQ require a positive correction to the head loss; when the flow is in the negative direction, positive values in ΔQ also require a positive correction to the calculated head loss. To preserve the algebraic relation among head loss, flow direction, and flow error (ΔQ), Equation 2.101 for each pipe can be written as

$$h_L = r\hat{Q}|\hat{Q}|^{n-1} + rn|\hat{Q}|^{n-1}\Delta Q \tag{2.102}$$

where the approximation has been replaced by an equal sign. On the basis of Equation 2.102, the requirement that the algebraic sum of the head losses around each loop be equal to zero can be written as

$$\sum_{j=1}^{NP(i)} r_{ij}Q_j|Q_j|^{n-1} + \Delta Q_i \sum_{j=1}^{NP(i)} r_{ij}n|Q_j|^{n-1} = 0, \qquad i = 1, \ldots, NL \tag{2.103}$$

where $NP(i)$ is the number of pipes in loop i, r_{ij} is the head-loss coefficient in pipe j (in loop i), Q_j is the estimated flow in pipe j, ΔQ_i is the flow correction for the pipes in loop i, and NL is the number of loops in the entire network. The approximation given by Equation 2.103 assumes that there are no pumps in the loop, and that the flow correction, ΔQ_i, is the same for each pipe in each loop. Solving Equation 2.103 for ΔQ_i leads to

$$\boxed{\Delta Q_i = -\frac{\sum_{j=1}^{NP(i)} r_{ij}Q_j|Q_j|^{n-1}}{\sum_{j=1}^{NP(i)} nr_{ij}|Q_j|^{n-1}}} \tag{2.104}$$

This equation forms the basis of the Hardy Cross method.

The steps to be followed in using the Hardy Cross method to calculate the flow distribution in pipe networks are:

1. Assume a reasonable distribution of flows in the pipe network. This assumed flow distribution must satisfy continuity.
2. For each loop, i, in the network, calculate the quantities $r_{ij}Q_j|Q_j|^{n-1}$ and $nr_{ij}|Q_j|^{n-1}$ for each pipe in the loop. Calculate the flow correction, ΔQ_i, using Equation 2.104. Add the correction algebraically to the estimated flow in each pipe. [*Note*: Values of r_{ij} occur in both the numerator and denominator of Equation 2.104; therefore, values proportional to the actual r_{ij} may be used to calculate ΔQ_i.]
3. Repeat step 2 until the corrections (ΔQ_i) are acceptably small.

The application of the Hardy Cross method is best demonstrated by an exam

EXAMPLE 2.10

Compute the distribution of flows in the pipe network shown in Figure 2.11(a the head loss in each pipe is given by

$$h_L = rQ^2$$

and the relative values of r are shown in Figure 2.11(a). The flows are ı dimensionless for the sake of illustration.

Solution The first step is to assume a distribution of flows in the pipe netw satisfies continuity. The assumed distribution of flows is shown in Figure 2.11(l with the positive-flow directions in each of the two loops. The flow correction loop is calculated using Equation 2.104. Since $n = 2$ in this case, the flow cc formula becomes

$$\Delta Q_i = -\frac{\sum_{j=1}^{NP(i)} r_{ij}Q_j|Q_j|}{\sum_{j=1}^{NP(i)} 2r_{ij}|Q_j|}$$

The calculation of the numerator and denominator of this flow correction for loop I is tabulated as follows:

Loop	Pipe	Q	$rQ\|Q\|$	$2r\|Q\|$
I	4–1	70	29,400	840
	1–3	35	3675	210
	3–4	−30	−4500	300
			28,575	1350

The flow correction for loop I, ΔQ_I, is therefore given by

$$\Delta Q_I = -\frac{28,575}{1350} = -21.2$$

and the corrected flows are

Loop	Pipe	Q
I	4–1	48.8
	1–3	13.8
	3–4	−51.2

Moving to loop II, the calculation of the numerator and denominator of correction formula for loop II is given by

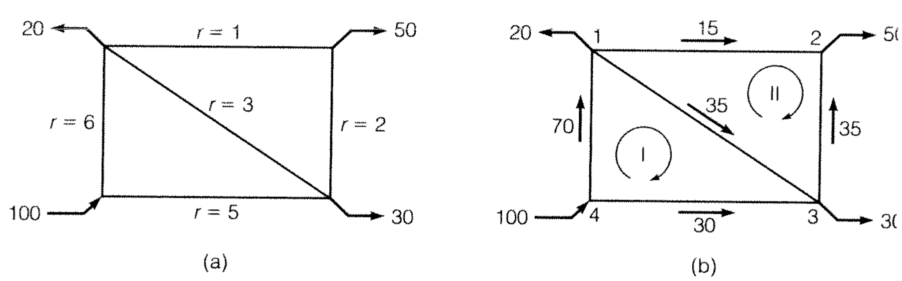


FIGURE 2.11: Flows in pipe network

Loop	Pipe	Q	$rQ\|Q\|$	$2r\|Q\|$
II	1–2	15	225	30
	2–3	−35	−2450	140
	3–1	−13.8	−574	83
			−2799	253

The flow correction for loop II, ΔQ_{II}, is therefore given by

$$\Delta Q_{II} = -\frac{-2799}{253} = 11.1$$

and the corrected flows are

Loop	Pipe	Q
II	1–2	26.1
	2–3	−23.9
	3–1	−2.7

This procedure is repeated in the following table until the calculated flow corrections do not affect the calculated flows, to the level of significant digits retained in the calculations.

Iteration	Loop	Pipe	Q	$rQ\|Q\|$	$2r\|Q\|$	ΔQ	Corrected Q
2	I	4–1	48.8	14,289	586		47.7
		1–3	2.7	22	16		1.6
		3–4	−51.2	−13,107	512		−52.3
				1204	1114	−1.1	
	II	1–2	26.1	681	52		29.1
		2–3	−23.9	−1142	96		−20.9
		3–1	−1.6	−8	10		1.4
				−469	157	3.0	
3	I	4–1	47.7	13,663	573		47.7
		1–3	1.4	6	8		1.4
		3–4	−52.3	−13,666	523		−52.3
				3	1104	0.0	
	II	1–2	29.1	847	58		29.2
		2–3	−20.9	−874	84		−20.8
		3–1	1.4	6	8		1.5
				−21	150	0.1	
4	I	4–1	47.7	13,662	573		47.7
		1–3	1.5	7	9		1.5
		3–4	−52.3	−13,668	523		−52.3
				1	1104	0.0	
	II	1–2	29.2	853	58		29.2
		2–3	−20.8	−865	83		−20.8
		3–1	1.5	7	9		1.5
				−5	150	0.0	

The final flow distribution, after four iterations, is given by

Pipe	Q
1–2	29.2
2–3	−20.8
3–4	−52.3
4–1	47.7
1–3	−1.5

It is clear that the final results are fairly close to the flow estimates after iteration.

As the above example illustrates, complex pipe networks can gen treated as a combination of simple loops, with each balanced in turn until co flow conditions exist in all loops. Typically, after the flows have been com all pipes in a network, the elevation of the hydraulic grade line and the are computed for each junction node. These pressures are then assessed re acceptable operating pressures.

2.3.3 Practical Considerations

In practice, analyses of complex pipe networks are usually done using c programs that solve the system of continuity and energy equations that gc flows in the network pipelines. These computer programs, such as EPANET (F 2000), generally use algorithms that are computationally more efficient than tl Cross method, such as the linear theory method, the Newton–Raphson met the gradient algorithm (Lansey and Mays, 1999).

The methods described in this text for computing steady-state flows and in water distribution systems are useful for assessing the performance of under normal operating conditions. Sudden changes in flow conditions, pump shutdown/startup and valve opening/closing, cause hydraulic transi can produce significant increases in water pressure—a phenomenon call *hammer*. The analysis of transient conditions requires a computer program to a numerical solution of the one-dimensional continuity and momentum equ flow in pipelines, and is an essential component of water-distribution syste (Wood, 2005d). Transient conditions will be most severe at pump stations an valves, high-elevation areas, locations with low static pressures, and locations far from elevated storage reservoirs (Friedman, 2003). Appurtenances used to the effects of water hammer include valves that prevent rapid closure, pressu valves, surge tanks, and air chambers. Detailed procedures for transient ar pipeline systems can be found in Martin (2000).

2.4 Pumps

Pumps are hydraulic machines that convert mechanical energy to fluid ener can be classified into two main categories: (1) positive displacement pun (2) rotodynamic or kinetic pumps. Positive displacement pumps deliver a fixed of fluid with each revolution of the pump rotor, such as with a piston or cylind rotodynamic pumps add energy to the fluid by accelerating it through the ac