CHAPTER 2

Flow in Closed Conduits

2.1 Introduction

Flow in closed conduits includes all cases where the flowing fluid comple the conduit. The cross sections of closed conduits can be of any shape or the conduits can be made of a variety of materials. Engineering applicatior principles of flow in closed conduits include the design of municipal wate systems and transmission lines. The basic equations governing the flow of closed conduits are the continuity, momentum, and energy equations, and t useful forms of these equations for application to pipe flow problems are in this chapter. The governing equations are presented in forms that are ap to any fluid flowing in a closed conduit, but particular attention is given to of water.

The computation of flows in pipe networks is a natural extension of tl in single pipelines, and methods of calculating flows and pressure distribu pipeline systems are also described here. These methods are particularly appli the analysis and design of municipal water-distribution systems, where the e is frequently interested in assessing the effects of various modifications to the Because transmission of water in closed conduits is typically accomplishe pumps, the fundamentals of pump operation and performance are also prese this chapter. A sound understanding of pumps is important in selecting the app pump to achieve the desired operational characteristics in water-transmission s The design protocol for municipal water-distribution systems is presente example of the application of the principles of flow in closed conduits. Meth estimating water demand, design of the functional components of distribution s network analysis, and the operational criteria for municipal water-distribution are all covered.

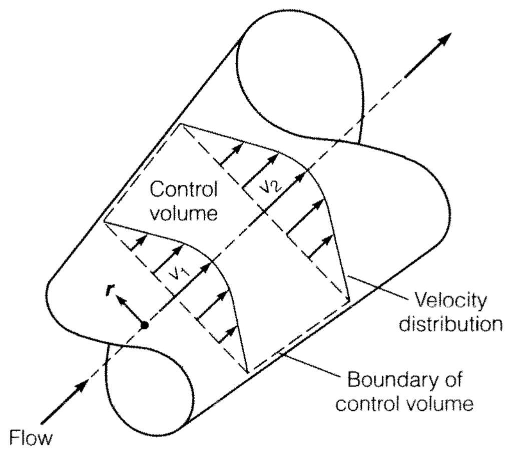
2.2 Single Pipelines

The governing equations for flows in pipelines are derived from the conservati of mass, momentum, and energy, and the forms of these equations that are mos for application to closed-conduit flow are derived in the following sections.

2.2.1 Steady-State Continuity Equation

Consider the application of the continuity equation to the control volume illust Figure 2.1. Fluid enters and leaves the control volume normal to the control su with the inflow velocity denoted by $v_1(\mathbf{r})$ and the outflow velocity by $v_2(\mathbf{r})$, wh the radial position vector originating at the centerline of the conduit. Both the and outflow velocities vary across the control surface. The steady-state co

2.1: Flow through
onduit



equation for an incompressible fluid can be written as

$$\int_{A_1} v_1 \, dA = \int_{A_2} v_2 \, dA \tag{2.1}$$

Defining V_1 and V_2 as the average velocities across A_1 and A_2, respectively, where

$$V_1 = \frac{1}{A_1} \int_{A_1} v_1 \, dA \tag{2.2}$$

and

$$V_2 = \frac{1}{A_2} \int_{A_2} v_2 \, dA \tag{2.3}$$

the steady-state continuity equation becomes

$$\boxed{V_1 A_1 = V_2 A_2 \; (= Q)} \tag{2.4}$$

The terms on each side of Equation 2.4 are equal to the volumetric flowrate, Q. The steady-state continuity equation simply states that the volumetric flowrate across any surface normal to the flow is a constant.

EXAMPLE 2.1

Water enters a pump through a 150-mm diameter intake pipe and leaves through a 200-mm diameter discharge pipe. If the average velocity in the intake pipeline is 1 m/s, calculate the average velocity in the discharge pipeline. What is the flowrate through the pump?

Solution In the intake pipeline, $V_1 = 1$ m/s, $D_1 = 0.15$ m and

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.15)^2 = 0.0177 \text{ m}^2$$

In the discharge pipeline, $D_2 = 0.20$ m and

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.20)^2 = 0.0314 \text{ m}^2$$

According to the continuity equation,

$$V_1 A_1 = V_2 A_2$$

Therefore,

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = (1)\left(\frac{0.0177}{0.0314}\right) = 0.56 \text{ m/s}$$

The flowrate, Q, is given by

$$Q = A_1 V_1 = (0.0177)(1) = 0.0177 \text{ m}^3\text{/s}$$

The average velocity in the discharge pipeline is 0.56 m/s, and the flowrate the pump is 0.0177 m^3/s.

2.2.2 Steady-State Momentum Equation

Consider the application of the momentum equation to the control volume ill in Figure 2.1. Under steady-state conditions, the component of the mo equation in the direction of flow (x-direction) can be written as

$$\sum F_x = \int_A \rho v_x \mathbf{v} \cdot \mathbf{n} \, dA$$

where $\sum F_x$ is the sum of the x-components of the forces acting on the flui control volume, ρ is the density of the fluid, v_x is the flow velocity in the x-d and $\mathbf{v} \cdot \mathbf{n}$ is the component of the flow velocity normal to the control surface. S unit normal vector, $\mathbf{n}$, in Equation 2.5 is directed outward from the control the momentum equation for an incompressible fluid (ρ = constant) can be w

$$\sum F_x = \rho \int_{A_2} v_2^2 \, dA - \rho \int_{A_1} v_1^2 \, dA$$

where the integral terms depend on the velocity distributions across the inf outflow control surfaces. The velocity distribution across each control sı generally accounted for by the *momentum correction coefficient*, β, definec relation

$$\boxed{\beta = \frac{1}{AV^2} \int_A v^2 \, dA}$$

where A is the area of the control surface and V is the average velocity over the surface. The momentum coefficients for the inflow and outflow control surf and A_2, are given by β_1 and β_2, where

$$\beta_1 = \frac{1}{A_1 V_1^2} \int_{A_1} v_1^2 \, dA$$

$$\beta_2 = \frac{1}{A_2 V_2^2} \int_{A_2} v_2^2 \, dA$$

Substituting Equations 2.8 and 2.9 into Equation 2.6 leads to the following form of the momentum equation

$$\sum F_x = \rho\beta_2 V_2^2 A_2 - \rho\beta_1 V_1^2 A_1 \tag{2.10}$$

Recalling that the continuity equation states that the volumetric flowrate, Q, is the same across both the inflow and outflow control surfaces, where

$$Q = V_1 A_1 = V_2 A_2 \tag{2.11}$$

then combining Equations 2.10 and 2.11 leads to the following form of the steady-state momentum equation

$$\sum F_x = \rho\beta_2 Q V_2 - \rho\beta_1 Q V_1 \tag{2.12}$$

or

$$\sum F_x = \rho Q(\beta_2 V_2 - \beta_1 V_1) \tag{2.13}$$

In many cases of practical interest, the velocity distribution across the cross section of the closed conduit is approximately uniform, in which case the momentum coefficients, β_1 and β_2, are approximately equal to unity and the steady-state momentum equation becomes

$$\sum F_x = \rho Q(V_2 - V_1) \tag{2.14}$$

Consider the common case of flow in a straight pipe with a uniform circular cross section illustrated in Figure 2.2, where the average velocity remains constant at each cross section,

$$V_1 = V_2 = V \tag{2.15}$$

then the steady-state momentum equation becomes

$$\sum F_x = 0 \tag{2.16}$$

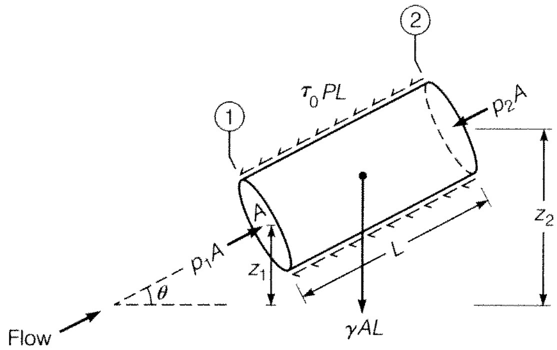
The forces that act on the fluid in a control volume of uniform cross section are illustrated in Figure 2.2. At Section 1, the average pressure over the control surface is equal to p_1 and the elevation of the midpoint of the section relative to a defined datum is equal to z_1, at Section 2, located a distance L downstream from Section 1, the pressure is p_2, and the elevation of the midpoint of the section is z_2. The average shear stress exerted on the fluid by the pipe surface is equal to τ_0, and the total shear force opposing flow is $\tau_0 PL$, where P is the perimeter of the pipe. The fluid weight acts vertically downward and is equal to γAL, where γ is the specific weight of the fluid and A is the cross-sectional area of the pipe. The forces acting on the fluid system that have components in the direction of flow are the shear force, $\tau_0 PL$; the weight of the fluid in the control volume, γAL; and the pressure forces on the upstream and downstream faces, $p_1 A$ and $p_2 A$, respectively. Substituting the expressions for the forces into the momentum equation, Equation 2.16, yields

$$p_1 A - p_2 A - \tau_0 PL - \gamma AL \sin\theta = 0 \tag{2.17}$$

where θ is the angle that the pipe makes with the horizontal and is given by the relation

$$\sin\theta = \frac{z_2 - z_1}{L} \tag{2.18}$$

FIGURE 2.2: Forces on flow in closed conduit



Combining Equations 2.17 and 2.18 yields

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - z_2 + z_1 = \frac{\tau_0 PL}{\gamma A}$$

Defining the *total head*, or energy per unit weight, at Sections 1 and 2 as h_1 where

$$h_1 = \frac{p_1}{\gamma} + \frac{V^2}{2g} + z_1$$

and

$$h_2 = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2$$

then the *head loss* between Sections 1 and 2, Δh, is given by

$$\Delta h = h_1 - h_2 = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

Combining Equations 2.19 and 2.22 leads to the following expression for hea

$$\Delta h = \frac{\tau_0 PL}{\gamma A}$$

In this case, the head loss, Δh, is entirely due to pipe friction and is commonly by h_f. In the case of pipes with circular cross sections, Equation 2.23 can be w

$$h_f = \frac{\tau_0(\pi D)L}{\gamma(\pi D^2/4)} = \frac{4\tau_0 L}{\gamma D}$$

where D is the diameter of the pipe. The ratio of the cross-sectional area, A perimeter, P, is defined as the *hydraulic radius*, R, where

$$R = \frac{A}{P}$$

and the head loss can be written in terms of the hydraulic radius as

$$h_f = \frac{\tau_0 L}{\gamma R}$$

The form of the momentum equation given by Equation 2.26 is of limited utility in that the head loss, h_f, is expressed in terms of the boundary shear stress, τ_0, which is not an easily measurable quantity. However, the boundary shear stress, τ_0, can be expressed in terms of measurable flow variables using dimensional analysis, where τ_0 can be taken as a function of the mean flow velocity, V; density of the fluid, ρ; dynamic viscosity of the fluid, μ; diameter of the pipe, D; characteristic size of roughness projections, ε; characteristic spacing of the roughness projections, ε'; and a (dimensionless) form factor, m, that depends on the shape of the roughness elements on the surface of the conduit. This functional relationship can be expressed as

$$\tau_0 = f_1(V, \rho, \mu, D, \varepsilon, \varepsilon', m) \tag{2.27}$$

According to the Buckingham pi theorem, this relationship between eight variables in three fundamental dimensions can also be expressed as a relationship between five nondimensional groups. The following relation is proposed:

$$\frac{\tau_0}{\rho V^2} = f_2\left(\text{Re}, \frac{\varepsilon}{D}, \frac{\varepsilon'}{D}, m\right) \tag{2.28}$$

where Re is the Reynolds number defined by

$$\text{Re} = \frac{\rho V D}{\mu} \tag{2.29}$$

The relationship given by Equation 2.28 is as far as dimensional analysis goes, and experiments are necessary to determine an empirical relationship between the nondimensional groups. Nikuradse (1932; 1933) conducted a series of experiments in pipes in which the inner surfaces were roughened with sand grains of uniform diameter, ε. In these experiments, the spacing, ε', and shape, m, of the roughness elements (sand grains) were constant and Nikuradse's experimental data fitted to the following functional relation:

$$\frac{\tau_0}{\rho V^2} = f_3\left(\text{Re}, \frac{\varepsilon}{D}\right) \tag{2.30}$$

It is convenient for subsequent analysis to introduce a factor of 8 into this relationship, which can then be written as

$$\frac{\tau_0}{\rho V^2} = \frac{1}{8} f\left(\text{Re}, \frac{\varepsilon}{D}\right) \tag{2.31}$$

or simply

$$\frac{\tau_0}{\rho V^2} = \frac{f}{8} \tag{2.32}$$

where the dependence of the *friction factor*, f, on the Reynolds number, Re, and relative roughness, ε/D, is understood. Combining Equations 2.32 and 2.24 leads to the following form of the momentum equation for flows in circular pipes:

$$\boxed{h_f = \frac{fL}{D}\frac{V^2}{2g}} \tag{2.33}$$

This equation, called the *Darcy–Weisbach equation*,* expresses the frictioı loss, h_f, of the fluid over a length L of pipe in terms of measurable par including the pipe diameter (D), average flow velocity (V), and the frictic (f) that characterizes the shear stress of the fluid on the pipe. Some re name Equation 2.33 simply as the Darcy equation; however, this is ina ate, since it was Julius Weisbach who first proposed the exact form of E 2.33 in 1845, with Darcy's contribution on the functional dependence of and D in 1857 (Brown, 2002; Rouse and Ince, 1957). The differences laminar and turbulent flow were later quantified by Osbourne Reynolds† (Reynolds, 1883).

Based on Nikuradse's (1932, 1933) experiments on sand-roughene Prandtl and von Kármán established the following empirical formulae for es the friction factor in turbulent pipe flows:

$$\text{Smooth pipe}\left(\frac{k}{D} \approx 0\right): \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\text{Rough pipe}\left(\frac{k}{D} \gg 0\right): \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{k/D}{3.7}\right)$$

where k is the roughness height of the sand grains on the surface of the p variables k and ε are used equivalently to represent the roughness height, alth more used in the context of an equivalent roughness height and ε as an actual rc height. Turbulent flow in pipes is generally present when Re $>$ 4000; tran turbulent flow begins at about Re $=$ 2300. The pipe behaves like a *smooth pi* the friction factor does not depend on the height of the roughness projec the wall of the pipe and therefore depends only on the Reynolds number. I *pipes*, the friction factor is determined by the relative roughness, k/D, and I independent of the Reynolds number. The smooth-pipe case generally o lower Reynolds numbers, when the roughness projections are submerged wi viscous boundary layer. At higher values of the Reynolds number, the thickne viscous boundary layer decreases and eventually the roughness projections p sufficiently far outside the viscous boundary layer that the shear stress of ı boundary is dominated by the hydrodynamic drag associated with the ro projections into the main body of the flow. Under these circumstances, the the pipe becomes *fully turbulent*, the friction factor is independent of the R number, and the pipe is considered to be (hydraulically) rough. The flow is turbulent under both smooth-pipe and rough-pipe conditions, but the flow is *fully turbulent* when the friction factor is independent of the Reynolds ı Between the smooth- and rough-pipe conditions, there is a transition region i the friction factor depends on both the Reynolds number and the relative roı Colebrook (1939) developed the following relationship that asymptotes to the

*Henry Darcy (1803–1858) was a nineteenth-century French engineer; Julius Weisbach (1806– a German engineer of the same era. Weisbach proposed the use of a dimensionless resistance c and Darcy carried out the tests on water pipes.

†Osbourne Reynolds (1842–1912).

and von Kármán relations:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{2.35}$$

This equation is commonly referred to as the *Colebrook equation* or *Colebrook–White* equation. Equation 2.35 can be applied in the transition region between smooth-pipe and rough-pipe conditions, and values of friction factor, f, predicted by the Colebrook equation are generally accurate to within 10–15% of experimental data (Finnemore and Franzini, 2002; Alexandrou, 2001). The accuracy of the Colebrook equation deteriorates significantly for small pipe diameters, and it is recommended that this equation not be used for pipes with diameters smaller than 2.5 mm (Yoo and Singh, 2005).

Commercial pipes differ from Nikuradse's experimental pipes in that the heights of the roughness projections are not uniform and are not uniformly distributed. In commercial pipes, an *equivalent sand roughness*, k_s, is defined as the diameter of Nikuradse's sand grains that would cause the same head loss as in the commercial pipe. The equivalent sand roughness, k_s, of several commercial pipe materials is given in Table 2.1. These values of k_s apply to clean new pipe only; pipe that has been in service for a long time usually experiences corrosion or scale buildup that results in values of k_s orders of magnitude larger than the values given in Table 2.1 (Echávez, 1997; Gerhart et al., 1992). The rate of increase of k_s with time depends primarily on the quality of the water being transported, and the roughness coefficients for older water mains are usually determined through field testing (AWWA, 1992). The expression for the friction factor derived by Colebrook (Equation 2.35) was plotted by Moody (1944) in what is commonly referred to as the *Moody diagram*,* reproduced in Figure 2.3. The Moody diagram indicates that for Re $\leq$ 2000, the flow is laminar and the friction factor is given by

$$f = \frac{64}{\mathrm{Re}} \tag{2.36}$$

which can be derived theoretically based on the assumption of laminar flow of a Newtonian fluid (Daily and Harleman, 1966). For 2000 $<$ Re $\leq$ 4000 there is no fixed relationship between the friction factor and the Reynolds number or relative roughness, and flow conditions are generally uncertain (Wilkes, 1999). Beyond a Reynolds number of 4000, the flow is turbulent and the friction factor is controlled by the thickness of the laminar boundary layer relative to the height of the roughness projections on the surface of the pipe. The dashed line in Figure 2.3 indicates the boundary between the fully turbulent flow regime, where f is independent of Re, and the transition regime, where f depends on both Re and the relative roughness, k_s/D. The equation of this dashed line is given by (Mott, 1994)

$$\frac{1}{\sqrt{f}} = \frac{\mathrm{Re}}{200(D/k_s)} \tag{2.37}$$

*This type of diagram was originally suggested by Blasius in 1913 and Stanton in 1914 (Stanton and Pannell, 1914). The Moody diagram is sometimes called the *Stanton diagram* (Finnemore and Franzini, 2002).

TABLE 2.1: Typical Equivalent Sand Roughness for Various New Materials

Material	Equivalent sand roughness, k_s (mm)
Asbestos cement:	
Coated	0.038
Uncoated	0.076
Brass	0.0015–0.003
Brick	0.6
Concrete:	
General	0.3–3.0
Steel forms	0.18
Wooden forms	0.6
Centrifugally spun	0.13–0.36
Copper	0.0015–0.003
Corrugated metal	45
Glass	0.0015–0.003
Iron:	
Cast iron	0.19–0.26
Ductile iron	0.26
Lined with bitumen	0.12
Lined with spun concrete	0.030–0.038
Galvanized iron	0.15
Wrought iron	0.046 –0.06
Lead	0.0015
Plastic (PVC)	0.0015–0.03
Steel	
Coal-tar enamel	0.0048
New unlined	0.045–0.076
Riveted	0.9–9.0
Wood stave	0.18

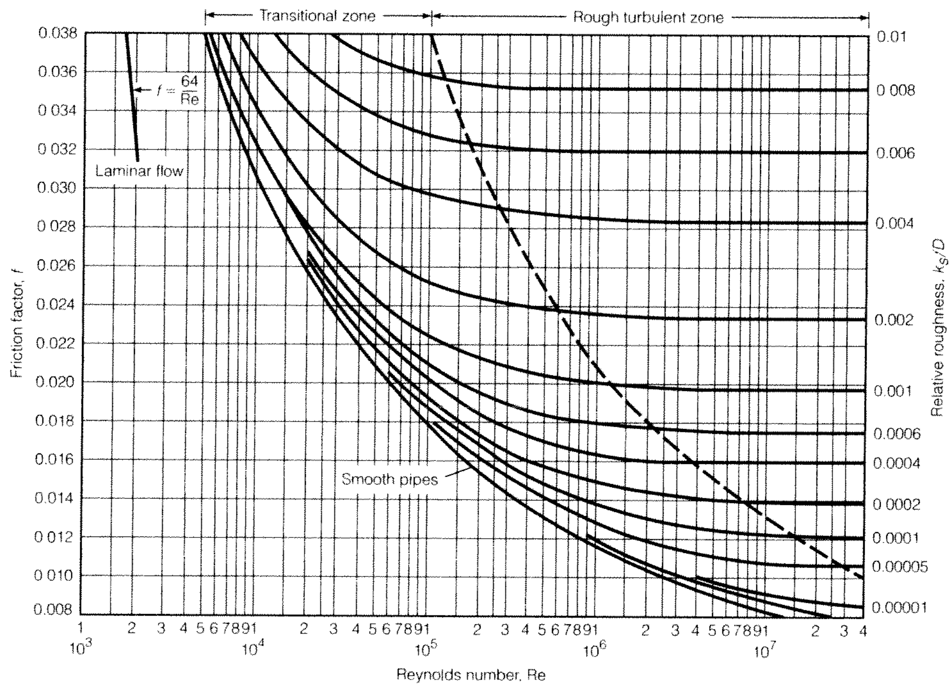
Sources: Haestad Methods, Inc. (2002), Moody (1944), Sanks (1998).

The line in the Moody diagram corresponding to a relative roughness of zero d the friction factor for pipes that are hydraulically smooth.

Although the Colebrook equation (Equation 2.35) can be used to calcu friction factor in lieu of the Moody diagram, this equation has the drawback an *implicit equation* for the friction factor and must be solved iteratively. Th inconvenience was circumvented by Jain (1976), who suggested the following equation for the friction factor:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right), \quad 10^{-6} \leq \frac{k_s}{D} \leq 10^{-2},\ 5000 \leq \text{Re} \leq 10^8$$

where, according to Jain (1976), Equation 2.38 deviates by less than 1% f Colebrook equation within the entire turbulent-flow regime, provided that the tions on k_s/D and Re are honored. The Jain equation (Equation 2.38) can l



2.3: Moody diagram Moody (1944).

conveniently written as

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}, \qquad 10^{-6} \leq \frac{k_s}{D} \leq 10^{-2},\ 5000 \leq \mathrm{Re} \leq 10^8 \tag{2.39}$$

According to Franzini and Finnemore (1997) and Granger (1985), values of the friction factor calculated using the Colebrook equation are generally accurate to within 10% to 15% of experimental data. Uncertainties in relative roughness and in the data used to produce the Colebrook equation make the use of several-place accuracy in pipe flow problems unjustified. As a rule of thumb, an accuracy of 10% in calculating friction losses in pipes is to be expected (Munson et al., 1994; Gerhart et al., 1992).

EXAMPLE 2.2

Water from a treatment plant is pumped into a distribution system at a rate of 4.38 m^3/s, a pressure of 480 kPa, and a temperature of 20°C. The pipe has a diameter of 750 mm and is made of ductile iron. Estimate the pressure 200 m downstream of the treatment plant if the pipeline remains horizontal. Compare the friction factor estimated using the Colebrook equation to the friction factor estimated using the Jain equation. After 20 years in operation, scale buildup is expected to cause the equivalent sand roughness of the pipe to increase by a factor of 10. Determine the effect on the water pressure 200 m downstream of the treatment plant.

Solution According to the Darcy–Weisbach equation, the difference in total head, Δh, between the upstream section (at exit from treatment plant) and the downstream section (200 m downstream from the upstream section) is given by

$$\Delta h = \frac{fL}{D}\frac{V^2}{2g}$$

where f is the friction factor, L is the pipe length between the upstrε downstream sections (= 200 m), D is the pipe diameter (= 750 mm), and velocity in the pipe. The velocity, V, is given by

$$V = \frac{Q}{A}$$

where Q is the flowrate in the pipe (= 4.38 m^3/s) and A is the area of the pi section given by

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.75)^2 = 0.442 \text{ m}^2$$

The pipeline velocity is therefore

$$V = \frac{Q}{A} = \frac{4.38}{0.442} = 9.91 \text{ m/s}$$

The friction factor, f, in the Darcy–Weisbach equation is calculated using tl brook equation:

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right]$$

Here Re is the Reynolds number and k_s is the equivalent sand roughness o iron (= 0.26 mm). The Reynolds number is given by

$$\text{Re} = \frac{VD}{\nu}$$

where ν is the kinematic viscosity of water at 20°C, which is equal to 1.00 m^2/s. Therefore

$$\text{Re} = \frac{VD}{\nu} = \frac{(9.91)(0.75)}{1.00 \times 10^{-6}} = 7.43 \times 10^6$$

Substituting into the Colebrook equation leads to

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{0.26}{(3.7)(750)} + \frac{2.51}{7.43 \times 10^6\sqrt{f}}\right]$$

or

$$\frac{1}{\sqrt{f}} = -2\log\left[9.37 \times 10^{-5} + \frac{3.38 \times 10^{-7}}{\sqrt{f}}\right]$$

This is an implicit equation for f, and the solution is

$$f = 0.016$$

The head loss, Δh, between the upstream and downstream sections can now be calculated using the Darcy–Weisbach equation as

$$\Delta h = \frac{fL}{D}\frac{V^2}{2g} = \frac{(0.016)(200)}{0.75}\frac{(9.91)^2}{(2)(9.81)} = 21.4 \text{ m}$$

Using the definition of head loss, Δh,

$$\Delta h = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

where p_1 and p_2 are the upstream and downstream pressures, γ is the specific weight of water, and z_1 and z_2 are the upstream and downstream pipe elevations. Since the pipe is horizontal, $z_1 = z_2$ and Δh can be written in terms of the pressures at the upstream and downstream sections as

$$\Delta h = \frac{p_1}{\gamma} - \frac{p_2}{\gamma}$$

In this case, $p_1 = 480$ kPa, $\gamma = 9.79$ kN/m^3, and therefore

$$21.4 = \frac{480}{9.79} - \frac{p_2}{9.79}$$

which yields

$$p_2 = 270 \text{ kPa}$$

Therefore, the pressure 200 m downstream of the treatment plant is 270 kPa. The Colebrook equation required that f be determined from an implicit equation, but the explicit Jain approximation for f is given by

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right]$$

Substituting for k_s, D, and Re gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{0.26}{(3.7)(750)} + \frac{5.74}{(7.43 \times 10^6)^{0.9}}\right]$$

which leads to

$$f = 0.016$$

This is the same friction factor obtained using the Colebrook equation within an accuracy of two significant digits.

After 20 years, the equivalent sand roughness, k_s, of the pipe is 2.6 mm, the (previously calculated) Reynolds number is 7.43×10^6, and the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{2.6}{(3.7)(750)} + \frac{2.51}{7.43 \times 10^6\sqrt{f}}\right]$$

or

$$\frac{1}{\sqrt{f}} = -2\log\left[9.37 \times 10^{-4} + \frac{3.38 \times 10^{-7}}{\sqrt{f}}\right]$$

which yields

$$f = 0.027$$

The head loss, Δh, between the upstream and downstream sections is givei Darcy–Weisbach equation as

$$\Delta h = \frac{fL}{D}\frac{V^2}{2g} = \frac{(0.027)(200)}{0.75}\frac{(9.91)^2}{(2)(9.81)} = 36.0 \text{ m}$$

Hence the pressure, p_2, 200 m downstream of the treatment plant is giver relation

$$\Delta h = \frac{p_1}{\gamma} - \frac{p_2}{\gamma}$$

where $p_1 = 480$ kPa, $\gamma = 9.79$ kN/m^3, and therefore

$$36.0 = \frac{480}{9.79} - \frac{p_2}{9.79}$$

which yields

$$p_2 = 128 \text{ kPa}$$

Therefore, pipe aging over 20 years will cause the pressure 200 m downstreaı treatment plant to decrease from 270 kPa to 128 kPa. This is quite a significa and shows why velocities of 9.91 m/s are not used in these pipelines, even f lengths of pipe.

The problem in Example 2.2 illustrates the case where the flowrate a pipe is known and the objective is to calculate the head loss and pressu over a given length of pipe. The approach is summarized as follows: (1) c the Reynolds number, Re, and the relative roughness, k_s/D, from the givı (2) use the Colebrook equation (Equation 2.35) or Jain equation (Equation calculate f; and (3) use the calculated value of f to calculate the head lo the Darcy–Weisbach equation (Equation 2.33), and the corresponding pressu from Equation 2.22.

Flowrate for a given head loss. In many cases, the flowrate through a pip controlled but attains a level that matches the pressure drop available. For e the flowrate through faucets in home plumbing is determined by the gage pre the water main, which is relatively insensitive to the flow through the faucet. A approach to this problem that uses the Colebrook equation has been suggestec (1994), where the first step is to calculate $\text{Re}\sqrt{f}$ using the rearranged Darcy–W equation

$$\text{Re}\sqrt{f} = \left(\frac{2gh_f D^3}{\nu^2 L}\right)^{\frac{1}{2}}$$

Using this value of $\text{Re}\sqrt{f}$, solve for Re using the rearranged Colebrook equation

$$\text{Re} = -2.0(\text{Re}\sqrt{f}) \log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \tag{2.41}$$

Using this value of Re, the flowrate, Q, can then be calculated by

$$Q = \frac{1}{4}\pi D^2 V = \frac{1}{4}\pi D \nu \text{Re} \tag{2.42}$$

This approach must necessarily be validated by verifying that Re > 2300, which is required for application of the Colebrook equation. Swamee and Jain (1976) combine Equations 2.40 to 2.42 to yield

$$\boxed{Q = -0.965 D^2 \sqrt{\frac{gDh_f}{L}} \ln\left(\frac{k_s/D}{3.7} + \frac{1.784\nu}{D\sqrt{gDh_f/L}}\right)} \tag{2.43}$$

EXAMPLE 2.3

A 50-mm diameter galvanized iron service pipe is connected to a water main in which the pressure is 450 kPa gage. If the length of the service pipe to a faucet is 40 m and the faucet is 1.2 m above the main, estimate the flowrate when the faucet is fully open.

Solution The head loss, h_f, in the pipe is estimated by

$$h_f = \left(\frac{p_{\text{main}}}{\gamma} + z_{\text{main}}\right) - \left(\frac{p_{\text{outlet}}}{\gamma} + z_{\text{outlet}}\right)$$

where $p_{\text{main}} = 450$ kPa, $z_{\text{main}} = 0$ m, $p_{\text{outlet}} = 0$ kPa, and $z_{\text{outlet}} = 1.2$ m. Therefore, taking $\gamma = 9.79$ kN/m^3 (at 20°C) gives

$$h_f = \left(\frac{450}{9.79} + 0\right) - (0 + 1.2) = 44.8 \text{ m}$$

Also, since $D = 50$ mm, $L = 40$ m, $k_s = 0.15$ mm (from Table 2.1), and $\nu = 1.00 \times 10^{-6}$ m^2/s (at 20°C), the Swamee–Jain equation (Equation 2.43) yields

$$\begin{aligned}
Q &= -0.965 D^2 \sqrt{\frac{gDh_f}{L}} \ln\left(\frac{k_s/D}{3.7} + \frac{1.784\nu}{D\sqrt{gDh_f/L}}\right) \\
&= -0.965(0.05)^2 \sqrt{\frac{(9.81)(0.05)(44.8)}{40}} \ln\left[\frac{0.15/50}{3.7} + \frac{1.784(1.00 \times 10^{-6})}{(0.05)\sqrt{(9.81)(0.05)(44.8)/40}}\right] \\
&= 0.0126 \text{ m}^3\text{/s} = 12.6 \text{ L/s}
\end{aligned}$$

The faucet can therefore be expected to deliver 12.6 L/s when fully open.

Diameter for a given flowrate and head loss. In many cases, an engineer m a size of pipe to provide a given level of service. For example, the maximum and maximum allowable pressure drop may be specified for a water deliv and the engineer is required to calculate the minimum diameter pipe that w these design constraints. Solution of this problem necessarily requires an procedure. The following steps are suggested (Streeter and Wylie, 1985)

1. Assume a value of f.
2. Calculate D from the rearranged Darcy–Weisbach equation,

$$D = \sqrt[5]{\left(\frac{8LQ^2}{h_f g \pi^2} f\right)}$$

   where the term in parentheses can be calculated from given data.
3. Calculate Re from

$$\text{Re} = \frac{VD}{\nu} = \left(\frac{4Q}{\pi \nu}\right)\frac{1}{D}$$

   where the term in parentheses can be calculated from given data.
4. Calculate k_s/D.
5. Use Re and k_s/D to calculate f from the Colebrook equation.
6. Using the new f, repeat the procedure until the new f agrees with the the first two significant digits.

EXAMPLE 2.4

A galvanized iron service pipe from a water main is required to deliver 200 L a fire. If the length of the service pipe is 35 m and the head loss in the pipe exceed 50 m, calculate the minimum pipe diameter that can be used.

Solution

Step 1. Assume $f = 0.03$
Step 2. Since $Q = 0.2$ m^3/s, $L = 35$ m, and $h_f = 50$ m, then

$$D = \sqrt[5]{\left[\frac{8LQ^2}{h_f g \pi^2}\right] f} = \sqrt[5]{\left[\frac{8(35)(0.2)^2}{(50)(9.81)\pi^2}\right](0.03)} = 0.147\ \text{r}$$

Step 3. Since $\nu = 1.00 \times 10^{-6}$ m^2/s (at 20°C), then

$$\text{Re} = \left[\frac{4Q}{\pi \nu}\right]\frac{1}{D} = \left[\frac{4(0.2)}{\pi(1.00 \times 10^{-6})}\right]\frac{1}{0.147} = 1.73 \times 1($$

Step 4. Since $k_s = 0.15$ mm (from Table 2.1, for new pipe), then

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.147} = 0.00102$$

Step 5. Using the Colebrook equation (Equation 2.35) gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$
$$= -2\log\left(\frac{0.00102}{3.7} + \frac{2.51}{1.73 \times 10^6\sqrt{f}}\right)$$

which leads to

$$f = 0.020$$

Step 6. $f = 0.020$ differs from the assumed f ($= 0.03$), so repeat the procedure with $f = 0.020$.
Step 2. For $f = 0.020$, $D = 0.136$ m
Step 3. For $D = 0.136$ m, $\mathrm{Re} = 1.87 \times 10^6$
Step 4. For $D = 0.136$ m, $k_s/D = 0.00110$
Step 5. $f = 0.020$
Step 6. The calculated f ($= 0.020$) is equal to the assumed f. The required pipe diameter is therefore equal to 0.136 m or 136 mm. A commercially available pipe with the closest diameter larger than 136 mm should be used.

The iterative procedure demonstrated in the previous example converges fairly quickly, and does not pose any computational difficulty. Swamee and Jain (1976) have suggested the following explicit formula for calculating the pipe diameter, D:

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \tag{2.46}$$

$$3000 \leq \mathrm{Re} \leq 3 \times 10^8,\ 10^{-6} \leq \frac{k_s}{D} \leq 2 \times 10^{-2}$$

Equation 2.46 will yield a D within 5% of the value obtained by the method using the Colebrook equation. This method is illustrated by repeating the previous example.

EXAMPLE 2.5

A galvanized iron service pipe from a water main is required to deliver 200 L/s during a fire. If the length of the service pipe is 35 m, and the head loss in the pipe is not to exceed 50 m, use the Swamee–Jain equation to calculate the minimum pipe diameter that can be used.

Solution Since k_s = 0.15 mm, L = 35 m, Q = 0.2 m^3/s, h_f = 50 m, ν = 10^{-6} m^2/s, the Swamee–Jain equation gives

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left\{(0.00015)^{1.25}\left[\frac{(35)(0.2)^2}{(9.81)(50)}\right]^{4.75} + (1.00\times 10^{-6})(0.2)^{9.4}\left[\frac{35}{(9.81)(50)}\right.\right.$$

$$= 0.140\text{ m}$$

The calculated pipe diameter (140 mm) is about 3% higher than calculate(Colebrook equation (136 mm).

2.2.3 Steady-State Energy Equation

The steady-state energy equation for the control volume illustrated in Figu given by

$$\frac{dQ_h}{dt} - \frac{dW}{dt} = \int_A \rho e\,\mathbf{v}\cdot\mathbf{n}\,dA$$

where Q_h is the heat added to the fluid in the control volume, W is the work the fluid in the control volume, A is the surface area of the control volume, density of the fluid in the control volume, and e is the internal energy per unit fluid in the control volume given by

$$e = gz + \frac{v^2}{2} + u$$

where z is the elevation of the fluid mass having a velocity v and internal ener convention, the heat added to a system and the work done by a system are quantities. The normal stresses on the inflow and outflow boundaries of the volume are equal to the pressure, p, with shear stresses tangential to the bou of the control volume. As the fluid moves across the control surface with vel the power (= rate of doing work) expended by the fluid against the external p

FIGURE 2.4: Energy balance in closed conduit