

Interior of powerhouse of Lower Granite Dam on Snake River near Pullman, Washington (Photo by Kevin G. Coulton)

## Hydraulic Machinery

## 8-1 An Introduction to Pumps and Turbines

### *Use of Pumps*

Water lifting devices were probably the first machines to be built by man, and today only the electric motor surpasses the pump for numbers of machines in use throughout the world. In hydraulic engineering, we are primarily interested in pumps for irrigation, flood control, water supply, wastewater, and thermal power plant cooling systems. The design of the pump is primarily dictated by the discharge rate and head to be developed by the pump. Another design consideration is the clarity of the water to be pumped; that is, is it clear water from a lake or well, or is it wastewater that may contain sediment particles, debris, or corrosive products? In this chapter, we present various aspects of the problems associated with pump design and pump station design.

### *Use of Turbines*

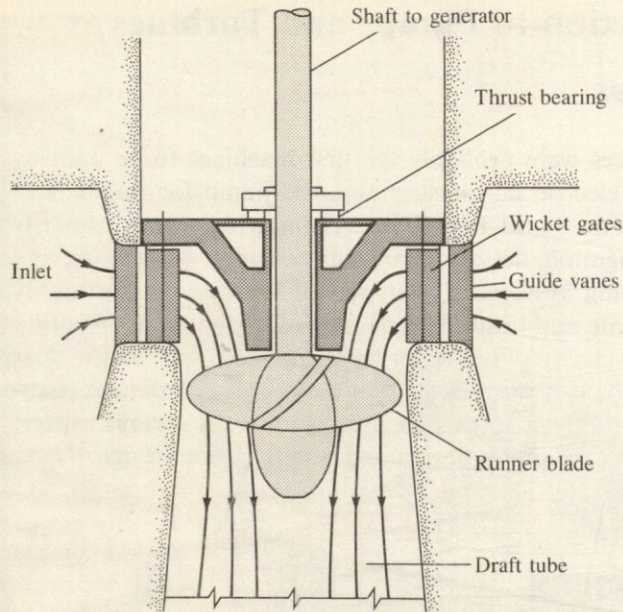
Hydraulic turbines are used to convert the power of flowing water into usable electrical or mechanical power. The turbine design is dictated by the head on the turbine and the discharge through the turbine.

In low-head plants (6–100 ft) with moderate to high discharge, the propeller type of turbine is most often used. Some propeller type turbines have adjustable blades to effect higher efficiencies over a wider range of flow conditions; these are called Kaplan turbines. Kaplan turbines are typically used on run of the river hydroelectric plants, as found on the lower Columbia and Ohio rivers in the United States or on the Danube and Rhine rivers in Europe. A typical section of a Kaplan turbine and flow passages is shown in Fig. 8-1a. A Kaplan turbine runner used in a hydroelectric plant on the Danube River is shown in Fig. 8-1b.

For medium-head (90–1500 ft) power plants, the Francis type of turbine is usually used. Here the water approaches the turbine impeller in a radial direction and leaves the impeller parallel to the propeller's axis. In the Francis turbine, the impeller blades are fixed, but vanes that guide the water into the impeller are adjustable so that high efficiencies can be realized over a wide range of discharge. Francis type turbines are found in the power plants of Grand Coulee Dam in Washington, Shasta Dam in California, and Hoover Dam in Nevada. Figure 8-2 shows a typical Francis turbine installation.

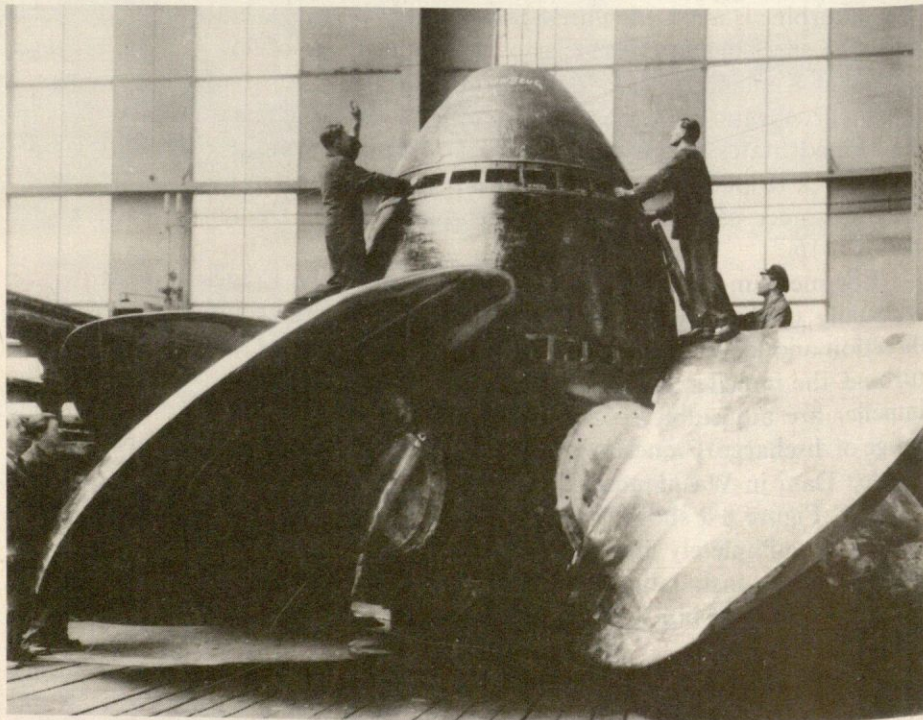
Water completely fills the flow passage of the Francis and propeller type turbines. These installations allow maximum use of head between the upstream and downstream pool levels. They also can accommodate a relatively large discharge of water.

For high-head hydropower installations ( $H > 1500$  ft), the impulse turbine is used. In this type of turbine, water is most often conveyed to the turbine



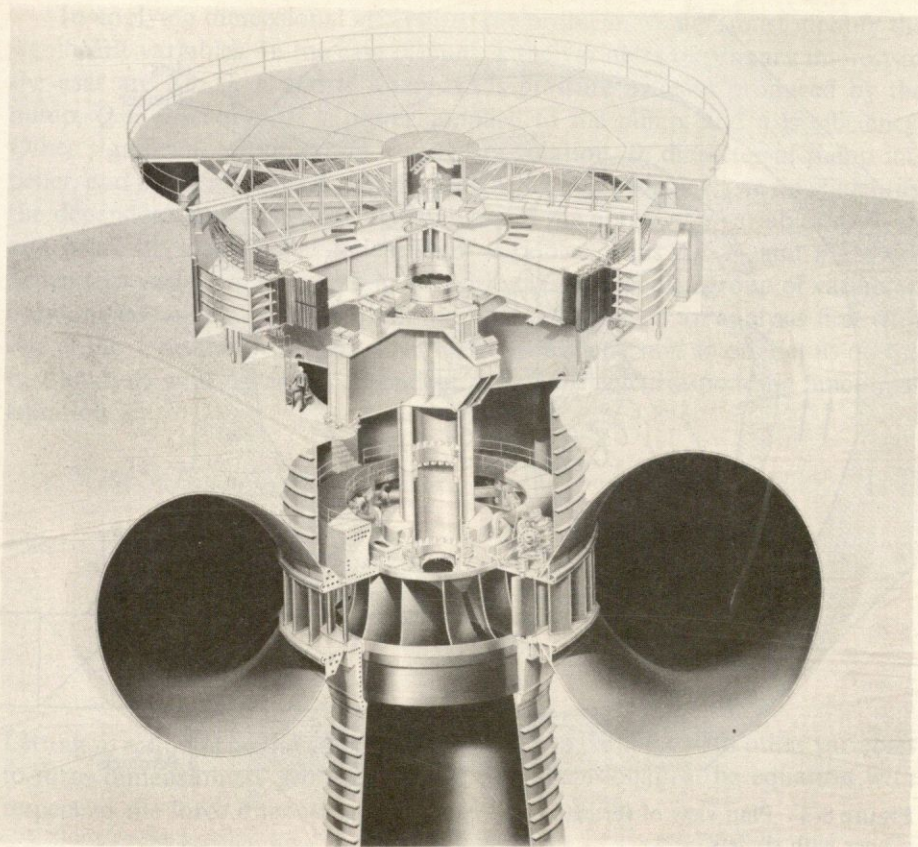
(a) Schematic of section of unit in place.

Figure 8-1 Kaplan turbine (Courtesy of Voith Hydro Inc.)



(b) A Kaplan turbine runner

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**Figure 8-2** Schematic view of Francis-type turbine used in Grand Coulee Dam (Courtesy of Voith Hydro Inc.)

through a pipe called a penstock and then through a nozzle from which a high velocity jet of water issues. This jet impinges on curved vanes (buckets) placed on the periphery of the turbine wheel, thus causing the wheel to rotate and to generate power (see Fig. 8-26). In that application, the axis of the wheel is horizontal. To develop more power from a single wheel, a wheel having a vertical axis with multiple jets is used (see Fig. 8-3). Although the impulse turbine is the logical choice of turbine type for high-head installations, it is also suitable for many lower head sites if discharge is relatively small. Thus many small-scale hydropower plants use impulse turbines. The primary advantages of the impulse turbine are its simplicity and ease of maintenance. One of its disadvantages is that the impeller must be placed so that it is always above the highest level of the downstream pool; therefore, in run of the river plants, much head would be wasted when the river discharge is low and the downstream pool level is at low elevation.

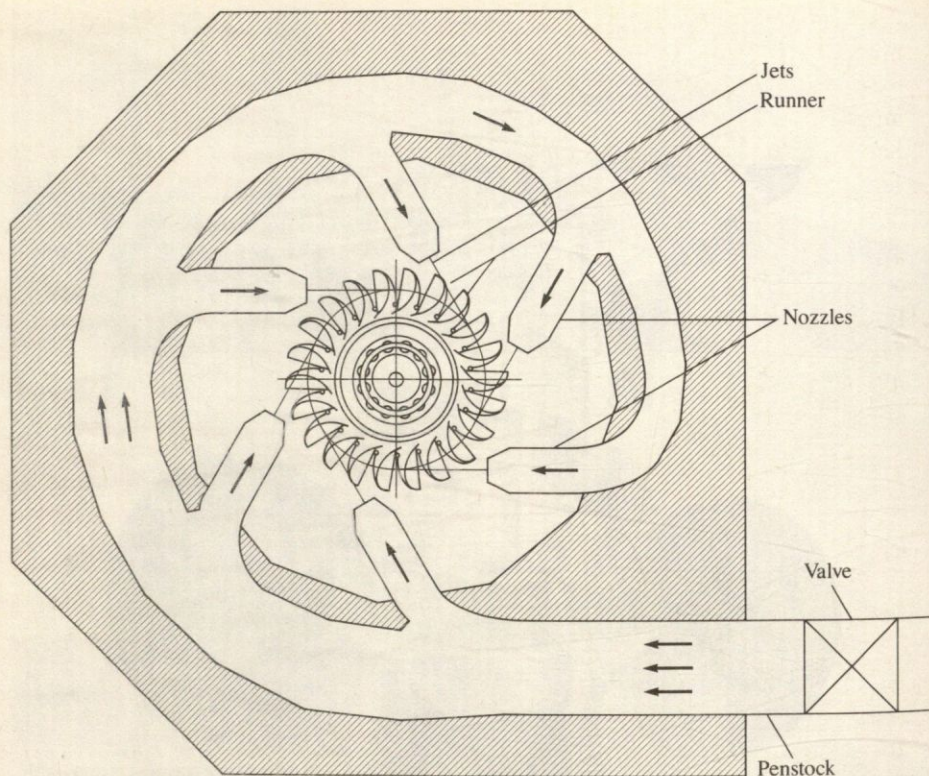


Figure 8-3 Plan view of vertical-axis-impulse-turbine runner with six jets

## 8-2 Dimensionless Parameters for Turbomachines

When designing turbomachines, the designer must apply the basic equations of fluid mechanics to shape the vanes and flow passages of the machine so that separation-free flow will occur. Through this as well as through previous experience and testing a final design for a pump or turbine is achieved. Once a pump or turbine is produced, performance tests are made to determine the machine's actual operating characteristics. The tests yield relationships between dimensionless parameters for that particular machine. In the following paragraphs, we develop by means of dimensional analysis the relevant dimensionless parameters used with turbomachines.\*

\* As shown in Roberson (12), these parameters could also be developed by approaching the problem from the fundamental theory of lift and drag of an airfoil; however, for the sake of brevity, we use the dimensional analysis approach.

In applying dimensional analysis to the problem, we first must identify the significant variables. In the case of pumps, the variables of primary interest to the user are  $\Delta p$ ,  $Q$ ,  $P$  and  $\eta$ , where  $\Delta p$  is pressure increase produced by the pump,  $Q$  is discharge,  $P$  is power supplied to the pump, and  $\eta$  is efficiency. Other significant variables are  $n$ , speed of rotation,  $D$ , diameter of pump impeller, and  $\rho$ , the mass density of the fluid being pumped. Next, we must identify the dependent and independent variables. Of the above, the variables  $n$ ,  $D$ ,  $Q$ , and  $\rho$  are all identified as independent variables. Thus  $\Delta p$ ,  $P$ , and  $\eta$  are the dependent variables. However, in dimensional analysis of a group of variables, only one dependent variable is allowed. We therefore do an analysis first with one of the dependent variables, then with another one, and so on. Let us do the first analysis with  $\Delta p$  as the dependent variable. The corresponding functional equation is

$$\Delta p = f(\rho, n, D, Q) \quad (8-1)$$

where

$$\begin{aligned} [\Delta p]^* &= F/L^2 \\ [\rho] &= FT^2/L^4 \\ [n] &= T^{-1} \\ [D] &= L \\ [Q] &= L^3/T \end{aligned}$$

Letting  $\rho$ ,  $n$ , and  $D$  be the variables that we use to combine with other variables to form dimensionless groups, first, we nondimensionalize the equation with respect to the force dimension,  $F$ . So doing we obtain

$$\frac{\Delta p}{\rho} = f(n, D, Q) \quad (8-2)$$

Next, we nondimensionalize it in time,  $T$ :

$$\frac{\Delta p}{\rho n^2} = f\left(D, \frac{Q}{n}\right) \quad (8-3)$$

Finally, we make the functional equation completely dimensionless by combining powers of  $D$  with the remaining groups of variables:

$$\frac{\Delta p}{\rho n^2 D^2} = f\left(\frac{Q}{nD^3}\right) \quad (8-4)$$

In working with pumps, we often focus on the change in head,  $\Delta H$ , rather than  $\Delta p$ . Therefore, in Eq. (8-4), if we let  $\rho = \gamma/g$  and  $\Delta p = \gamma\Delta H$  and cancel out the

\* The brackets around the variable mean dimensions of.

$\gamma$ 's we obtain

$$\frac{\Delta H}{n^2 D^2 / g} = f\left(\frac{Q}{n D^3}\right) \quad (8-5)$$

The dimensionless parameter on the left-hand side of Eq. (8-5) is the *head coefficient*,  $C_H$ , and the parameter inside the parentheses on the right-hand side is the *discharge coefficient*,  $C_Q$ . Thus

$$C_H = \frac{\Delta H}{n^2 D^2 / g} \quad \text{and} \quad C_Q = \frac{Q}{n D^3} \quad (8-6)$$

where  $C_H = f(C_Q)$  (8-6)

By applying dimensional analysis to the variables  $P$ ,  $\rho$ ,  $n$ ,  $D$ , and  $Q$  it can be shown that

$$\frac{P}{\rho D^5 n^3} = f\left(\frac{Q}{n D^3}\right) \quad (8-7)$$

or  $C_P = f(C_Q)$  (8-8)

where  $C_P$  is defined as the *power coefficient*. Similarly, it can be shown that  $\eta = f(C_Q)$ .

Summarizing, the dimensionless parameters used in similarity analyses of pumps are as follows:

$$C_H = \frac{\Delta H}{D^2 n^2 / g} \quad (8-9)$$

$$C_P = \frac{P}{\rho D^5 n^3} \quad (8-10)$$

$$C_Q = \frac{Q}{n D^3} \quad (8-11)$$

where  $C_H$  and  $C_P$  are functions of  $C_Q$  for a given type of pump.

### 8-3 Axial-Flow Pumps

Figure 8-4 is a set of curves of  $C_H$ ,  $C_P$ , and  $\eta$  versus  $C_Q$  for a typical axial-flow pump. Dimensional curves (head, power, and efficiency versus  $Q$  for a constant speed of rotation) from which Fig. 8-4 was developed are shown in Fig. 8-5. Curves like those shown in Figs. 8-4 and 8-5 characterize the pump's performance, so they are often called *characteristic curves* or *performance curves*. These curves are obtained by experiment.

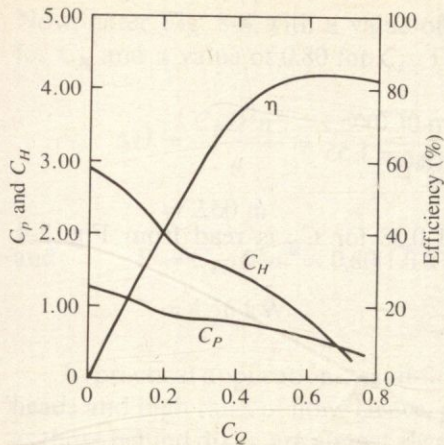


Figure 8-4 Dimensionless performance curves for a typical axial-flow pump (15)

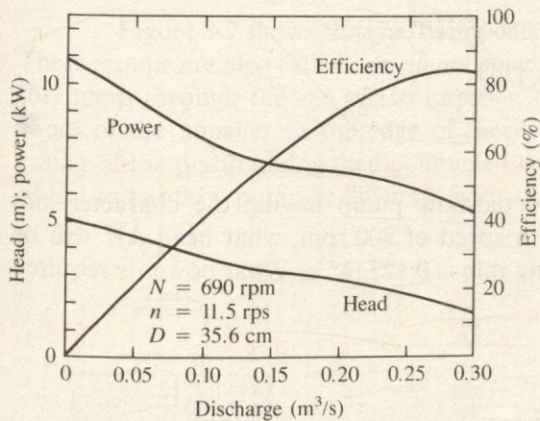


Figure 8-5 Performance characteristics of a typical axial-flow pump (15)

Performance curves are used to predict prototype operation from model tests or the effect of change of speed of the pump. Two examples of these applications follow.

**EXAMPLE 8-1** For the pump represented by Figs. 8-4 and 8-5, what discharge in cubic meters per second will occur when the pump is operating against a 2-m head and at a speed of 600 rpm? What power in kilowatts is required for these conditions?



**SOLUTION** First compute  $C_H$ . Here,

$$D = 35.6 \text{ cm} \quad \text{and} \quad n = 10 \text{ rps}$$

$$\text{Then, } C_H = \frac{2 \text{ m}}{(0.356 \text{ m})^2(10^2 \text{ s}^{-2})/(9.81 \text{ m/s}^2)} = 1.55$$

With a value of 1.55 for  $C_H$ , a value of 0.38 for  $C_Q$  is read from Fig. 8-4. Hence,  $Q$  is calculated as follows:

$$C_Q = 0.38 = \frac{Q}{nD^3}$$

$$\begin{aligned} \text{or } Q &= 0.38(10 \text{ s}^{-1})(0.356 \text{ m})^3 \\ &= 0.171 \text{ m}^3/\text{s} \end{aligned}$$

From Fig. 8-4, the value of  $C_P$  is 0.80 for  $C_Q = 0.38$ , then,

$$\begin{aligned} P &= 0.80 \rho D^5 n^3 \\ &= 0.80(1.0 \text{ kN} \cdot \text{s}^2/\text{m}^4)(0.356 \text{ m})^5(10 \text{ s}^{-1})^3 \\ &= 4.57 \text{ km} \cdot \text{N/s} = 4.57 \text{ kJ/s} \\ &= 4.57 \text{ kW} \end{aligned}$$

**EXAMPLE 8-2** If a 30-cm axial-flow pump having the characteristics shown in Fig. 8-4 is operated at a speed of 800 rpm, what head  $\Delta H$  will be developed when the water pumping rate is  $0.127 \text{ m}^3/\text{s}$ ? What power is required for this operation?

**SOLUTION** First compute

$$C_Q = \frac{Q}{nD^3}$$

$$\text{where } Q = 0.127 \text{ m}^3/\text{s}$$

$$n = \frac{800}{60} = 13.3 \text{ rps}$$

$$D = 30 \text{ cm}$$

$$\begin{aligned} \text{Then, } C_Q &= \frac{0.127 \text{ m}^3/\text{s}}{13.3 \text{ s}^{-1}(0.30 \text{ m})^3} \\ &= 0.354 \end{aligned}$$

Now, enter Fig. 8-4 with a value of  $C_Q = 0.354$ , and read off a value of 1.60 for  $C_H$  and a value of 0.80 for  $C_P$ . Then,

$$\Delta H = \frac{C_H D^2 n^2}{g} = \frac{1.60(0.30 \text{ m})^2(13.3 \text{ s}^{-1})^2}{(9.81 \text{ m/s}^2)}$$

$$= 2.60 \text{ m}$$

and

$$P = C_P \rho D^5 n^3 = 0.80(1.0 \text{ kN} \cdot \text{s}^2/\text{m}^4)(0.30 \text{ m})^5(13.3 \text{ s}^{-1})^3$$

$$= 4.56 \text{ kW}$$

In practical applications, axial-flow pumps are best suited for relatively low heads and high rates of flow. Hence, pumps used for dewatering lowlands, such as those behind dikes, are almost always of the axial-flow type. Figure 8-6 shows a typical setup for an axial-flow pump. For larger heads, radial- or mixed-flow machines are more efficient.

## 8-4 Radial- and Mixed-Flow Pumps

Figure 8-7 shows the type of impeller used for many radial-flow pumps. These pumps are also called *centrifugal pumps*. Liquid from the inlet pipe enters the pump through the eye of the impeller, then travels outward between the vanes of the impeller to the edge of the impeller, where the fluid enters the casing of the pump and is then conducted to the discharge pipe. The principle of the radial-flow pump is different from that of the axial-flow pump in that

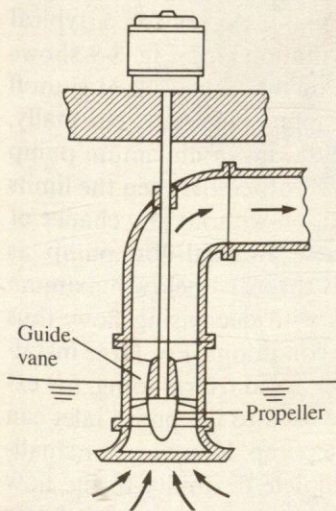


Figure 8-6 Axial-flow pump

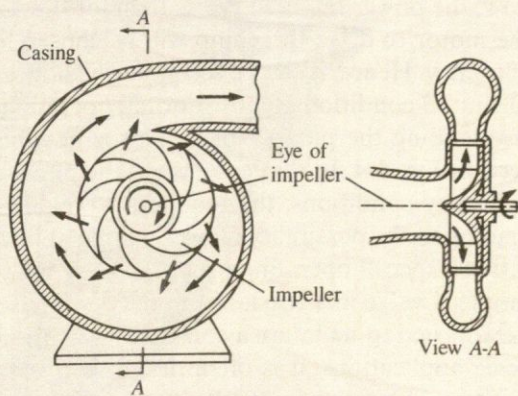


Figure 8-7 Centrifugal pump

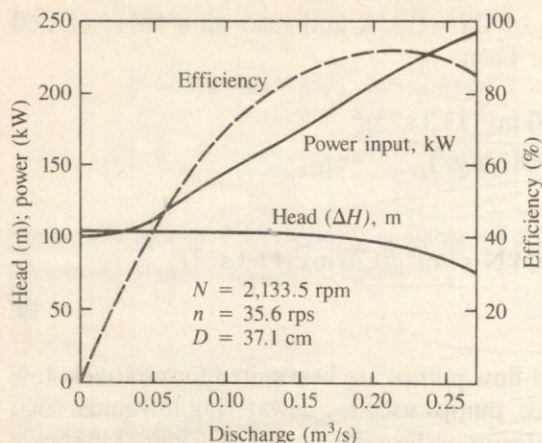
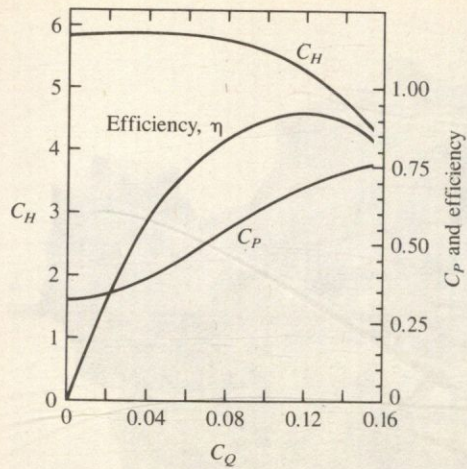


Figure 8-8 Typical performance curves for a centrifugal pump (3)

the change in pressure largely results by rotary action (pressure increasing outward like that of a rotating tank of water). Additional pressure increase is produced in the radial-flow pump when the high velocity flow leaving the impeller is reduced in the expanding section of the casing.

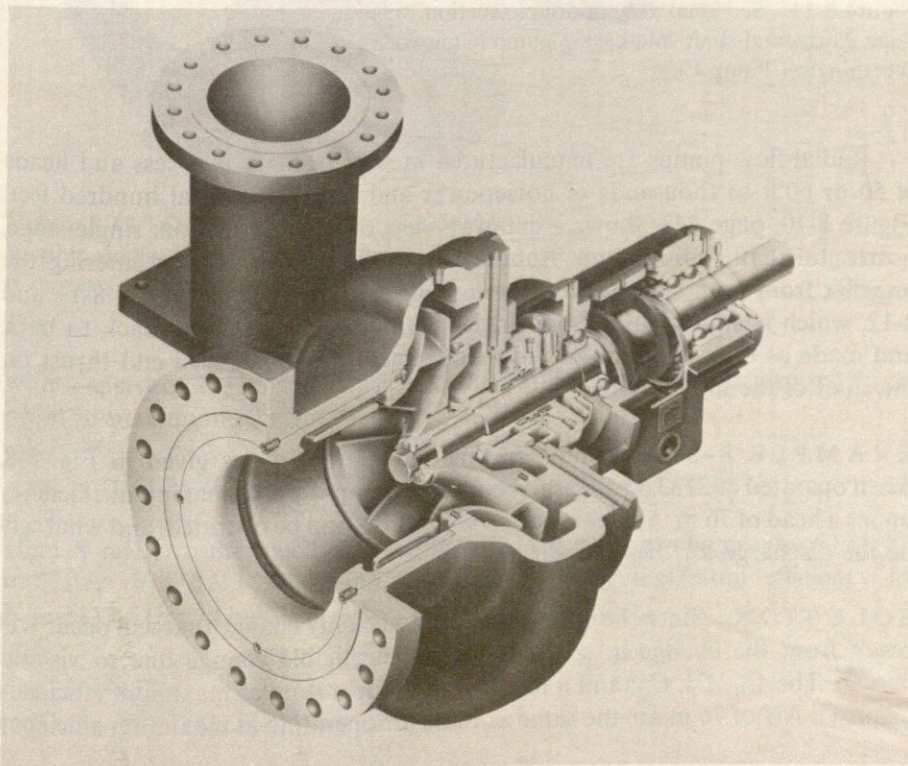
Although the basic designs of the radial- and axial-flow pumps are different, it can be shown that the same similarity parameters ( $C_Q$ ,  $C_P$ , and  $C_H$ ) apply for both types. Thus the methods we discussed for relating size, speed, and discharge in axial-flow machines also apply to the radial-flow machine.

The major practical difference between the axial- and radial-flow pumps, so far as the user is concerned, is in the performance characteristics of the two pumps. Figure 8-8 shows the dimensional performance curves for a typical radial-flow pump operating at a constant speed of rotation, and Fig. 8-9 shows the dimensionless performance curves for the same pump. Note that at shutoff flow, the power required is less than for flow at maximum efficiency. Normally, the motor to drive the pump will be chosen for conditions of maximum pump efficiency. Hence, it can be seen that the flow can be throttled between the limits of shutoff condition and the normal operating condition without any chance of overloading the pump motor. This is not the case for an axial-flow pump, as seen in Fig. 8-4. In that case, when the pump flow is throttled below maximum efficiency conditions, the required power increases with decreasing flow, thus leading to the possibility of overloading at low-flow conditions. For large installations, special operating procedures are followed to avoid overloading; for example, a valve in a bypass from the pump discharge back to the pump inlet can be adjusted to maintain a constant flow through the pump. However, for small-scale applications, it is often desirable to have complete flexibility in the flow control without the complexity of special operating procedures. In this latter case, a radial-flow pump offers a distinct advantage.



**Figure 8-9** Dimensionless performance curves for a centrifugal pump, from data given in Figure 8-8 (3)

**Figure 8-10** Cutaway view of a single-suction, single-stage, horizontal-shaft radial pump (Courtesy of Ingersol Rand Co.)



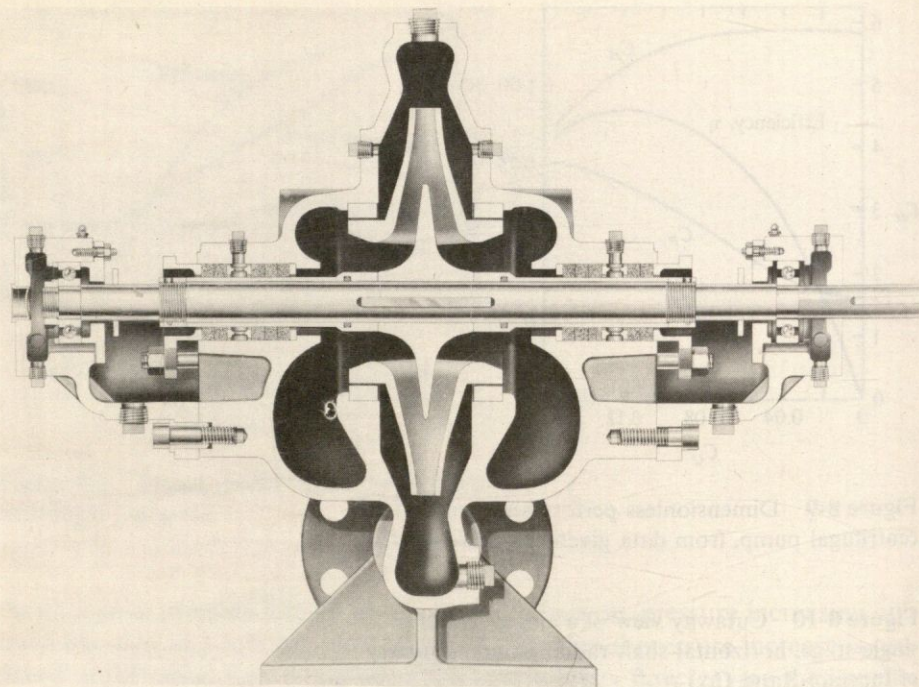
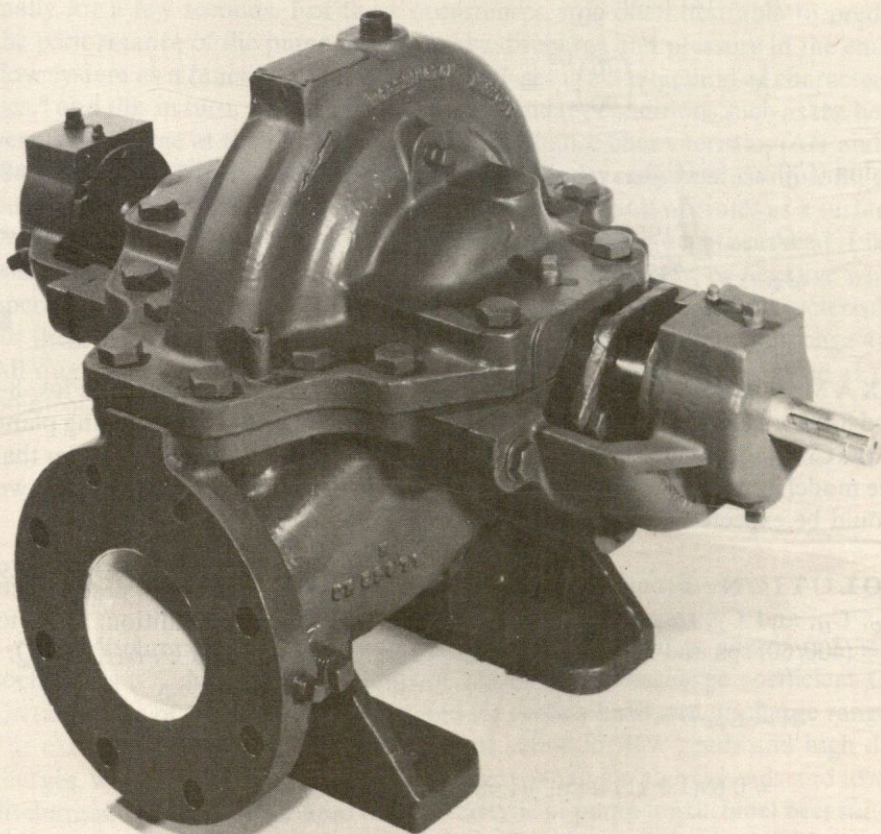


Figure 8-11 Sectional view of double-suction, single-stage, horizontal-shaft split-casing pump (Courtesy of Worthington Pump Co.)

Radial-flow pumps are manufactured in sizes from 1 hp or less and heads of 50 or 60 ft to thousands of horsepower and heads of several hundred feet. Figure 8-10, page 443, shows a cutaway view of a single-suction, single-stage, horizontal-shaft radial pump. Another common design has flow entering the impeller from both sides (*double-suction impeller*), as shown in Figs. 8-11 and 8-12, which is equivalent to two single-suction impellers placed back to back and made as a single casting. This arrangement gives balanced end thrust on the shaft of the impeller.

**EXAMPLE 8-3** The pump having the characteristics given in Fig. 8-8, when operated at 2133.5 rpm, is to be used to pump water at maximum efficiency under a head of 76 m. At what speed should the pump be operated, and what will be the discharge for these conditions?

**SOLUTION** Since the diameter is fixed, the only change that will occur will result from the change in speed (assuming negligible change due to viscous effects). The  $C_H$ ,  $C_P$ ,  $C_Q$ , and  $\eta$  for this pump operating at maximum efficiency against a  $\Delta H$  of 76 m are the same as these for operating at maximum efficiency



**Figure 8-12** Overall outside view of a double-suction, single-stage, horizontal-shaft split-casing pump (Courtesy of Worthington Pump Co.)

with a speed of 2133.5 rpm, since both operating conditions correspond to the point of maximum efficiency in Fig. 8-8. Thus we can write

$$(C_H)_N = (C_H)_{2133.5 \text{ rpm}}$$

Here  $N$  refers to the speed of rotation with  $\Delta H = 76$  m. The graph of Fig. 8-8 indicates that  $\Delta H = 90$  m and  $Q = 0.225$  m<sup>3</sup>/s at maximum efficiency for  $N = 2133.5$  rpm. Thus

$$\frac{76 \text{ m}}{N^2} = \frac{90 \text{ m}}{(2133.5)^2}$$

$$N^2 = (2133.5)^2 \frac{76}{90}$$

$$N = 2133.5 \left( \frac{76}{90} \right)^{1/2} = 1960 \text{ rpm}$$

Using  $(C_Q)_{1960} = (C_Q)_{2133.5 \text{ rpm}}$  and solving for the ratio of discharge, we have

$$\frac{Q_{1960}}{Q_{2133.5}} = \frac{1960}{2133.5} = 0.919$$

$$Q_{1960} = 0.207 \text{ m}^3/\text{s} \quad \blacksquare$$

**EXAMPLE 8-4** The pump having the characteristics shown in Figs. 8-8 and 8-9 is a model of a pump that was actually used in one of the pumping plants of the Colorado River Aqueduct (3). For a prototype that is 5.33 times larger than the model and operates at a speed of 400 rpm, what head, discharge, and power would be expected at maximum efficiency?

**SOLUTION** From Fig. 8-9, we pick off values of 0.115, 5.35, and 0.69 for  $C_Q$ ,  $C_H$ , and  $C_P$ , respectively, for the maximum efficiency condition. Then for  $n = (400/60)$  rps and  $D = 0.371 \times 5.33 = 1.98$  m, we solve for  $P$ ,  $\Delta H$ , and  $Q$ :

$$P = C_P \rho D^5 n^3$$

$$= 0.69(1.0 \text{ kN} \cdot \text{s}^2/\text{m}^4)(1.98 \text{ m})^5 \left( \frac{400}{60} \text{ s}^{-1} \right)^3$$

$$= 6200 \text{ kW}$$

$$\Delta H = \frac{C_H D^2 n^2}{g} = \frac{5.35(1.98 \text{ m})^2 (400/60 \text{ s}^{-1})^2}{(9.81 \text{ m/s}^2)}$$

$$= 95.0 \text{ m}$$

$$Q = C_Q n D^3$$

$$= 0.115 \left( \frac{400}{60} \text{ s}^{-1} \right) (1.98 \text{ m})^3 = 5.95 \text{ m}^3/\text{s} \quad \blacksquare$$

## 8-5 Performance Characteristics Under Abnormal Operating Conditions

The performance curves we have presented up to now are for pumps operating at a speed and discharge to yield the highest efficiency. However, there

are cases, such as during a power outage, when the pump will operate abnormally for a few seconds. For these occurrences, it is often desirable to predict the performance of the pump as well as the discharge and pressure in the entire flow system as a function of time. These analyses use the method of characteristics,\* and this in turn, requires the input of boundary conditions such as the head versus discharge at the pump. Thus the performance characteristics ( $\Delta H$  and  $Q$  for a wide range of pump speed) are required. Under abnormal conditions, the pump speed may reduce to zero and then reverse so that it operates as a turbine. When operating as a turbine, the pump is said to have a negative speed. Likewise, the discharge will be positive when operating normally, or negative when operating as a turbine when the flow reverses. The performance characteristics for these operating conditions can be presented as shown in Fig. 8-13, page 448. All quantities on this graph are expressed as a percentage of the value at the point of best efficiency.

## 8-6 Specific Speed

A pump's performance is given by the values of its power and head coefficients ( $C_P$  and  $C_H$ ) for a range of values of the discharge coefficient  $C_Q$ . Certain types of machines are best suited for certain head and discharge ranges. For example, an axial-flow machine is best suited for low heads and high discharges, whereas a radial-flow machine is best suited for higher heads and lower discharges. The parameter used to pick the type of pump (or turbine) best suited for a given application is specific speed  $n_s$ . Specific speed is obtained by combining  $C_H$  and  $C_Q$  in such a manner that the diameter  $D$  is eliminated:

$$n_s = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{(Q/nD^3)^{1/2}}{[\Delta H/(D^2n^2/g)]^{3/4}} = \frac{nQ^{1/2}}{g^{3/4} \Delta H^{3/4}} \quad (8-12)$$

Thus specific speed relates different types of pumps without reference to size.

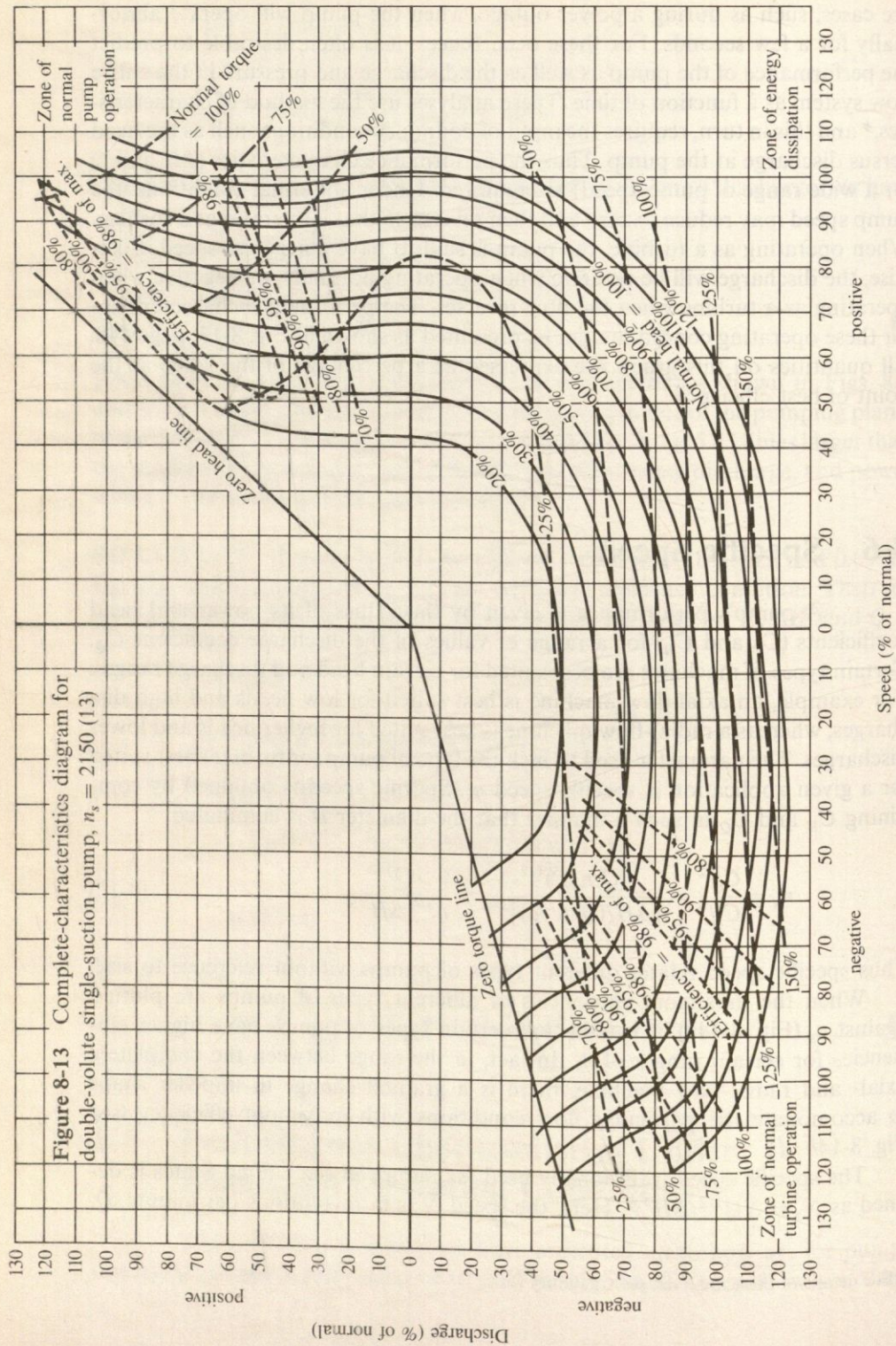
When the maximum efficiencies of different types of pumps are plotted against  $n_s$  (Fig. 8-14a), it is seen that certain types of pumps have higher efficiencies for certain ranges of  $n_s$ . In fact, in the range between the completely axial- and radial-flow machine, there is a gradual change in impeller shape to accommodate the particular flow conditions with maximum efficiency (see Fig. 8-14b-d).

The specific speed traditionally used for pumps in the United States is defined as  $N_s = NQ^{1/2}/\Delta H^{3/4}$ . Here, the speed  $N$  is in revolutions per minute,  $Q$ ,

\* For details of these analyses, see Chaudhry (1).



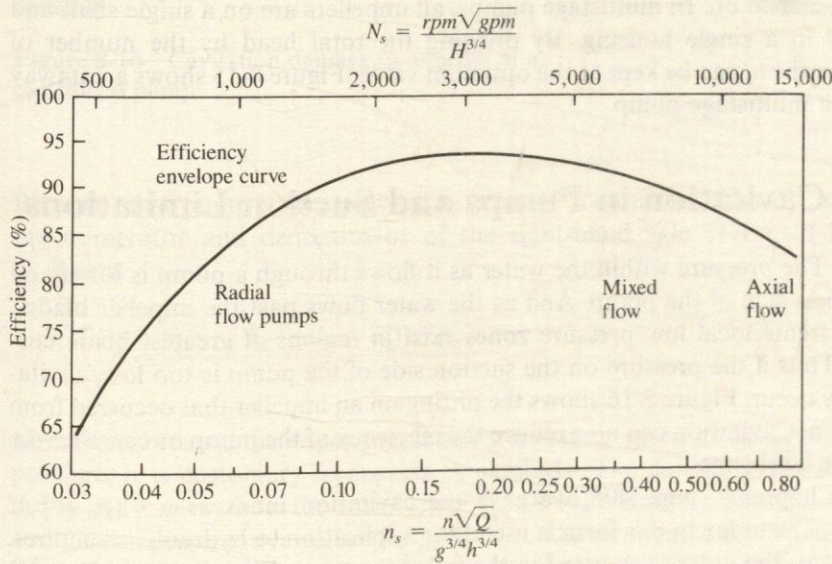
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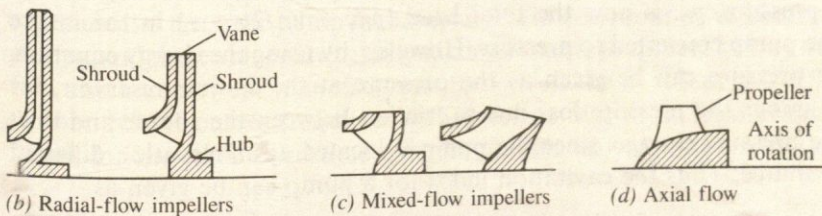
the discharge per suction inlet, is in gallons per minute, and  $\Delta H$  is in feet. This form is not dimensionless, and its values are much larger than those found for  $n_s$  (the conversion factor is 17,200). Most texts and references published before the introduction of the SI system of units use the traditional definition for specific speed.

### 8-7 Multistage Pumps

Because  $n_s$  varies inversely with  $\Delta H^{3/4}$ ,  $n_s$  will obviously decrease as the head  $\Delta H$  increases. It may also be seen (Fig. 8-14) that the efficiency is small for low values of  $n_s$ . If only one stage (one impeller) is used to pump water with a very large head difference, the efficiency would be quite low. Therefore, to maintain the  $n_s$  near values for which the efficiency is high multistage pumps are manufactured. That is, the stages are in series, wherein the discharge from the



(a) Optimum efficiency and impeller designs versus specific speed  $n_s$



**Figure 8-14** Optimum efficiency and impeller designs versus specific speed

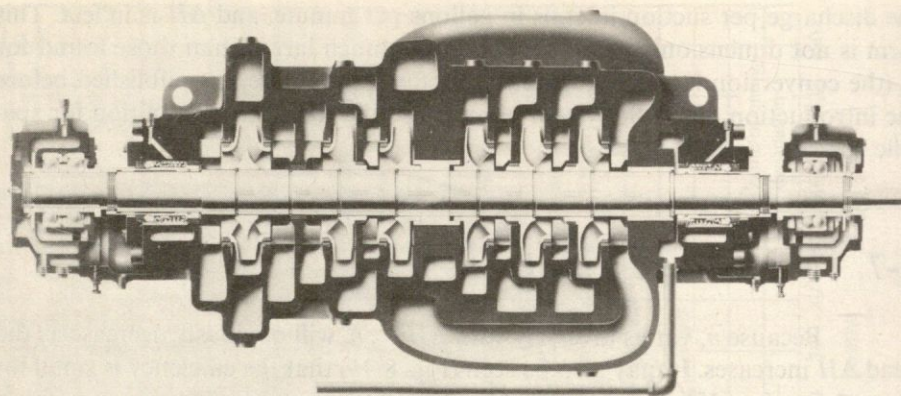


Figure 8-15 Cutaway view of multistage pump  
(Courtesy of Worthington Pump Co.)

first stage (first impeller) discharges directly into the suction side of the second impeller and so on. In multistage pumps, all impellers are on a single shaft and enclosed in a single housing. By dividing the total head by the number of impellers, the  $n_s$  can be kept at the optimum value. Figure 8-15 shows a cutaway view of a multistage pump.

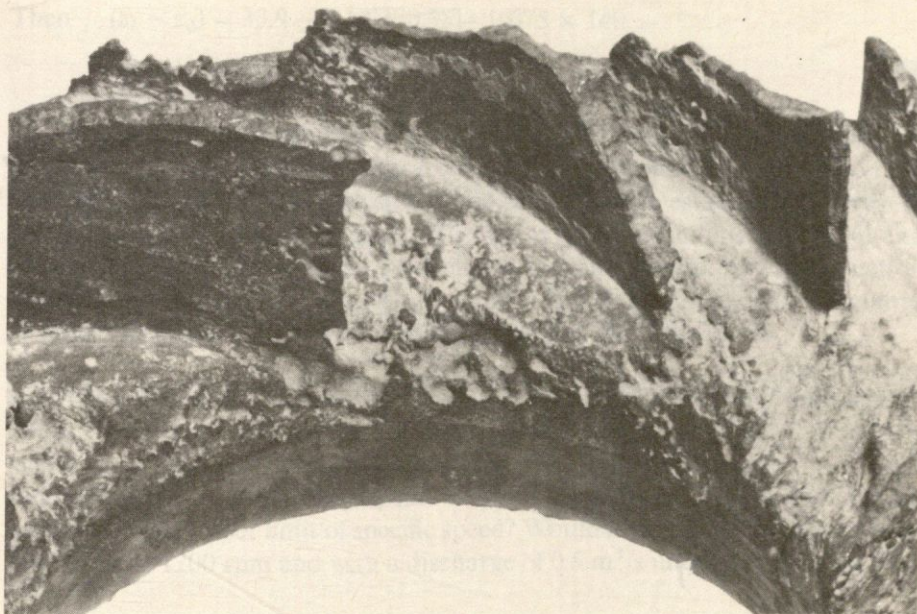
## 8-8 Cavitation in Pumps and Suction Limitations

The pressure within the water as it flows through a pump is lowest on the suction side of the pump. And as the water flows past the impeller blades, more extreme local low pressure zones exist in regions of greatest blade curvature. Thus if the pressure on the suction side of the pump is too low, cavitation may occur. Figure 8-16 shows the pitting on an impeller that occurred from cavitation. Cavitation can also reduce the efficiency of the pump or cause severe vibration problems.

In Chapter 7, page 405, we gave the cavitation index as  $\sigma = (p_0 - p_v) / (\rho V_0^2 / 2)$ . The index in this form is useful for application to hydraulic structures. For pumps, the index is changed in the following ways. First, instead of  $\rho V_0^2 / 2$  in the denominator, the pressure produced by the pump,  $\Delta p$ , is used. Next, the ambient pressure,  $p_0$ , is now the total head  $(p_i / \gamma + V_i^2 / 2g + z_i)$  in the intake pipe of the pump converted to pressure. However, by using the energy equation, this total pressure can be given as the pressure at the source (reservoir, for example) minus the pressure loss due to friction between the source and inlet minus the pressure change, since the pump is located at an elevation different from the source. Thus the cavitation index for a pump can be given as

$$\sigma = \frac{p_0 - \gamma h_L - \gamma(z_i - z_0) - p_v}{\Delta p} \quad (8-13)$$

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**Figure 8-16** Cavitation damage to impeller of a centrifugal pump

Because most pump data are given in terms of head rather than  $\Delta p$ , we divide the numerator and denominator of the right-hand side of Eq. (8-13) by  $\gamma$  to obtain

$$\sigma = \frac{p_0/\gamma - h_L - (z_i - z_0) - p_v/\gamma}{\Delta h} \quad (8-14)$$

Moreover, because the vapor pressure,  $p_v$ , is usually given in terms of absolute pressure, it is customary to express  $p_0$  similarly. The definition sketch shown in Fig. 8-17 shows how the variables of Eq. (8-14) relate to a physical situation and gives representative values for  $p_0$  and  $p_v$ . The numerator on the right-hand side of Eq. (8-14) is the net positive suction head (NPSH), and the value of  $\sigma$  when significant cavitation is first observed is the critical value,  $\sigma_c$ .<sup>\*</sup> Through experimental tests, the pump manufacturer will determine  $\sigma_c$  values for different pumps, and these values are usually made available to those who buy the pumps. It is also common for manufacturers to convert the  $\sigma_c$  values to NPSH values so that NPSH curves (required NPSH versus  $N_s$ ), or at least critical NPSH values for the rated conditions (maximum efficiency condition), are available.

<sup>\*</sup> Cavitation can be produced in an experimental test program by operating the pump at a given speed and then gradually lowering the pressure on the suction side of the pump (for example, the head could be lowered in the reservoir shown in Fig. 8-17) until cavitation occurs.

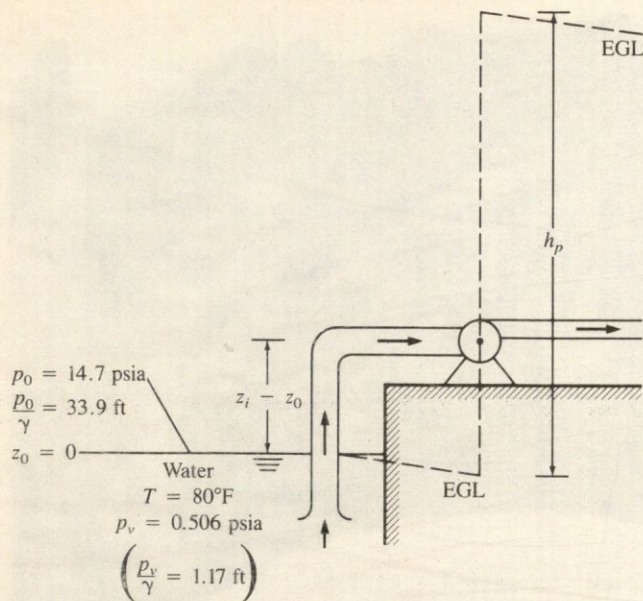


Figure 8-17 Definition sketch

**EXAMPLE 8-5** Tests on a given centrifugal pump yielded a value for  $\sigma_c$  of 0.075 when the pump was operating at maximum efficiency. For this maximum efficiency condition,  $h_p = 140$  ft and  $Q = 1.35$  cfs. If the pump is to be installed in a setting such as shown in Fig. 8-17, what is the maximum value of  $(z_i - z_0)$  that can be used with cavitation-free operation? Assume it will be operating at maximum efficiency. Further assume the intake-pipe diameter is 8 in. and the total head loss coefficient (sum of loss coefficients for inlet, pipe friction, bend, and so on) has a value of 1.5.

**SOLUTION** From Eq. (8-14), we have

$$\frac{p_0}{\gamma} - h_L - (z_i - z_0) - \frac{p_v}{\gamma} = \sigma \Delta H$$

$$\text{or} \quad (z_i - z_0) = \frac{p_0}{\gamma} - h_L - \frac{p_v}{\gamma} - \sigma \Delta H$$

$$\text{But} \quad h_L = \frac{KV^2}{2g} = 1.5 \frac{(Q^2/A^2)}{2g} = 0.35 \text{ ft}$$

Assume  $T = 80^\circ\text{F}$  and standard sea-level atmospheric pressure ( $p_0 = 14.7$  psia and  $p_0/\gamma = 33.9$  ft). Also  $p_v = 0.506$  psia (from Appendix Table A-4, page 648) from which  $p_v/\gamma = 1.17$  ft.

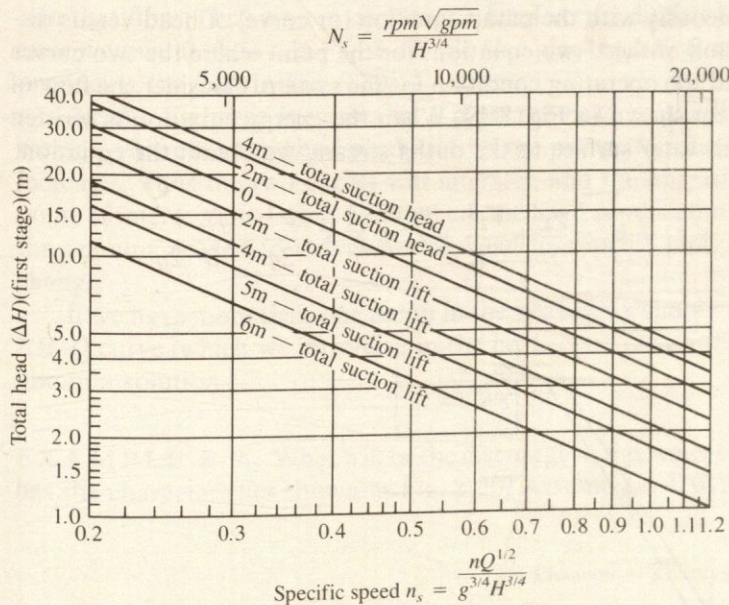
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$$\begin{aligned} \text{Then } (z_i - z_o) &= 33.9 - 1.17 - 0.35 - 0.075 \times 140 \\ &= 21.9 \text{ ft} \end{aligned}$$

Because most centrifugal pumps used in a given range of  $n_s$  have about the same shape and performance characteristics, certain general limitations on the flow conditions on the suction side of the pump may be established to prevent cavitation. These limitations are published by the Hydraulic Institute (6) and are given in terms of maximum  $\Delta H$  versus  $n_s$  for different suction lifts or suction heads. For example, a chart for single-suction mixed-flow and axial-flow pumps is shown in Fig. 8-18. Example 8-6 illustrates the use of the chart.

**EXAMPLE 8-6** An axial-flow pump is to be used to lift water from a main irrigation canal to a smaller irrigation canal at a higher level. If the total head (elevation difference plus head losses in the pipe) is to be 11 m, and if the total suction head is to be 2 m (the impeller is below the water level in the main canal), what is the safe upper limit of specific speed? Would it be safe to operate a pump at a speed of 1200 rpm and with a discharge of  $0.5 \text{ m}^3/\text{s}$  under these conditions?

**SOLUTION** We enter Fig. 8-18 with a total head of 11 m and a total suction head of 2 m and read a value of  $n_s$  of 0.51. Thus the safe upper limit of specific speed is 0.51.



**Figure 8-18** Specific speed limitations for single-suction mixed-flow and axial-flow pumps\* (6)

\* Pumping clear water,  $30^\circ\text{C}$  at sea level

By definition, we have

$$n_s = \frac{nQ^{1/2}}{g^{3/4} \Delta H^{3/4}}$$

Hence, for  $N = 1200$  rpm or  $n = 20$  rps,  $Q = 0.50$  m<sup>3</sup>/s, and  $\Delta H = 11$  m. We compute  $n_s$  as follows:

$$n_s = \frac{20(0.50)^{1/2}}{(9.81)^{3/4}(11)^{3/4}} = 0.42$$

The  $n_s$  computed here is less than the allowable  $n_s$ ; thus the stated operating conditions are *within the safe range*. ■

## 8-9 Pumps Operating in a Pipe System

In Chapter 5, we considered several problems involving head loss for a given discharge or vice versa. In this chapter, we have considered how the head developed by a pump is related to the discharge through the pump. We now link the two to see what head and discharge will prevail when a given pump is operated in a given pipe system. The solution (that is, the flow rate for a given system) is obtained when the system equation (or curve) of head versus discharge is solved simultaneously with the pump equation (or curve) of head versus discharge. The solution of these two equations (or the point where the two curves intersect) will yield the operating condition for the system. Consider the flow of water in the system shown in Fig. 8-19. When the energy equation is written from the reservoir water surface to the outlet stream, we obtain the equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum K_L \frac{V^2}{2g} + \frac{fL}{D} \frac{V^2}{2g}$$

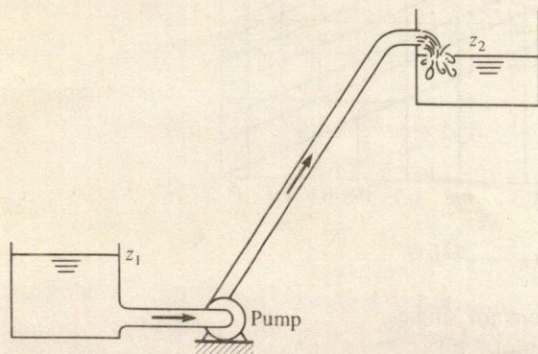


Figure 8-19 Pump and pipe combination

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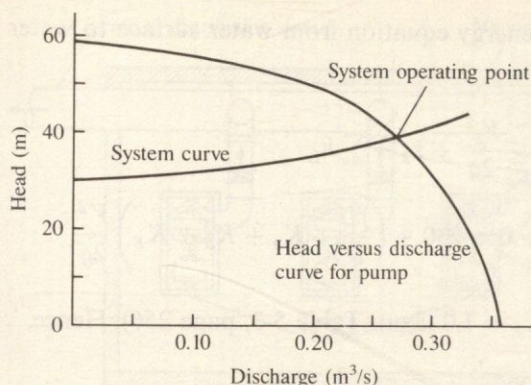


Figure 8-20 Pump and system curves

This equation simplifies to

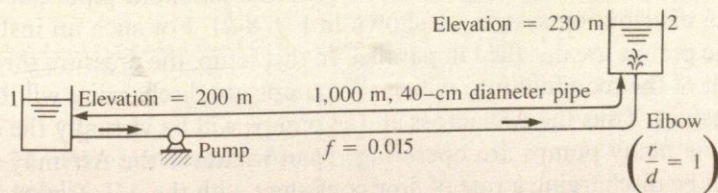
$$h_p = (z_2 - z_1) + \frac{V^2}{2g} \left( 1 + \sum K_L + \sum \frac{fL}{D} \right) \quad (8-15)$$

Hence, for any given discharge, a certain head  $h_p$  must be supplied to maintain that flow. Thus we can construct a head versus discharge curve, called the *system curve*, as shown in Fig. 8-20. We also plot the  $\Delta H$ - $Q$  curve for the pump producing the flow in Fig. 8-20.

As the discharge increases in a pipe, the head required for flow also increases. However, the head produced by the pump decreases as the discharge increases. Thus the two curves will intersect, and the operating point is at the point of intersection—that point where the head produced by the pump is just the amount needed to overcome the head loss in the pipe and the elevation change.

If we have more than one pump in the system, we simply use the composite  $\Delta H$ - $Q$  curve (which we introduce in the next section) with the system curve to obtain a solution.

**EXAMPLE 8-7** What will be the discharge in this water system if the pump has the characteristics shown in Fig. 8-20? Assume  $f = 0.015$ .





**SOLUTION** First, write the energy equation from water surface to water surface.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$0 + 0 + 200 + h_p = 0 + 0 + 230 + \left( \frac{fL}{D} + K_e + K_b + K_E \right) \frac{V^2}{2g}$$

where  $K_e = 0.5$ ,  $K_b = 0.35$ , and  $K_E = 1.0$  (from Table 5-3, page 256). Hence,

$$h_p = 30 + \frac{Q^2}{2gA^2} \left[ \frac{0.015(1000)}{0.40} + 0.5 + 0.35 + 1 \right]$$

$$= 30 + \frac{Q^2}{2 \times 9.81 \times [(\pi/4) \times 0.4^2]^2} (39.3) = 30 \text{ m} + 127 Q^2 \text{ m}$$

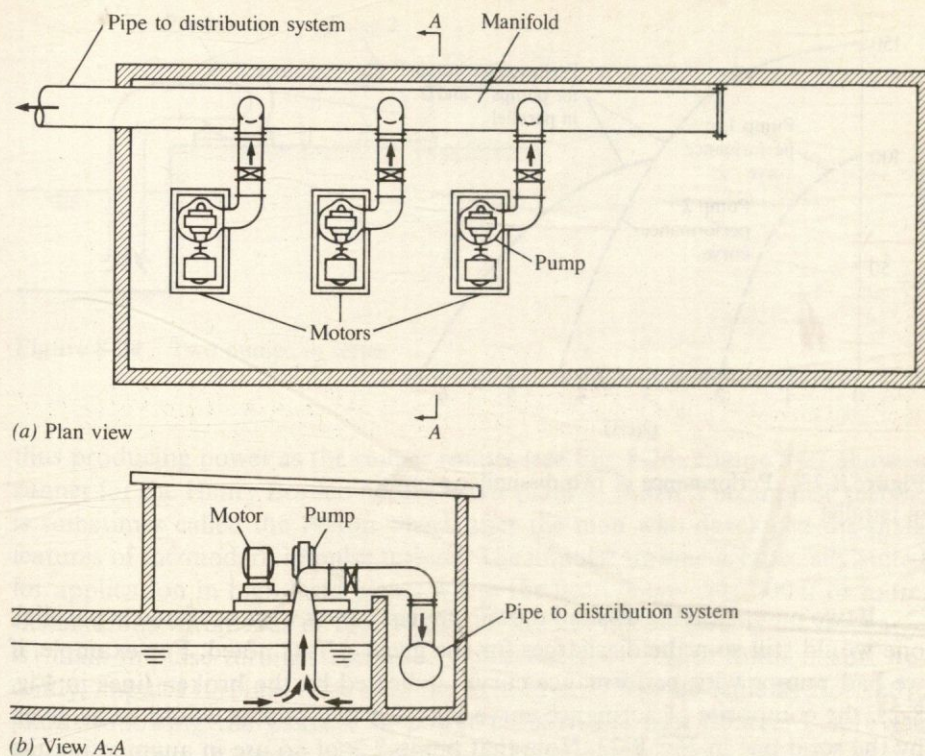
Now, we make a table of  $Q$  versus  $h_p$  to give values to produce a system curve that will be plotted with the pump curve. When the system curve is plotted on the same graph as the pump curve, it is seen (Fig. 8-20) that the operating condition occurs at  $Q = 0.27 \text{ m}^3/\text{s}$ .

$Q, \text{ m}^3/\text{s}$	$Q^2, \text{ m}^6/\text{s}^2$	$127Q^2$	$h_p = 30 \text{ m} + 127Q^2 \text{ m}$
0	0	0	30
0.1	$1 \times 10^{-2}$	1.3	31.3
0.2	$4 \times 10^{-2}$	5.1	35.1
0.3	$9 \times 10^{-2}$	11.4	41.4

## 8-10 Pumps Operated in Combination

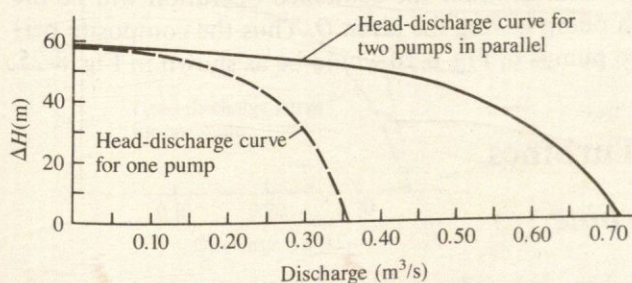
### *Parallel Installation*

When pumps are used to supply a system in which the demand may vary considerably, installing several pumps but using only those needed to satisfy the demand at any particular time is customary. Therefore, to determine the discharge in the system, one must construct pump performance curves that represent the multiple operation. For example, it is common practice to have three or more pumps discharging into a common manifold pipe that supplies water to a distribution system, as shown in Fig. 8-21. For such an installation, we say the pumps are installed in parallel. In this setup, the pressure throughout the length of the manifold into which the pumps are discharging will be essentially constant. Thus the  $\Delta H$  across all the pumps will be virtually the same no matter how many pumps are operating. Then whatever the  $\Delta H$  may be, each pump will be discharging a rate of flow consistent with the  $\Delta H$ - $Q$  curve for that



**Figure 8-21** Parallel pump installation

particular pump, and the total discharge from all the pumps will be the sum of those discharges. One can develop a  $\Delta H-Q$  curve for multiple pump operation by taking different  $H$ 's and determining the corresponding  $\sum Q$  for each  $\Delta H$ . If the pumps have identical performance characteristics, then the  $\sum Q$  would simply be  $nQ$ , where  $n$  is the number of pumps in operation. The  $\Delta H-Q$  curve for two pumps having the performance curve shown in Fig. 8-20 operating in parallel would be as shown in Fig. 8-22.



**Figure 8-22** Performance curve for two identical pumps in parallel

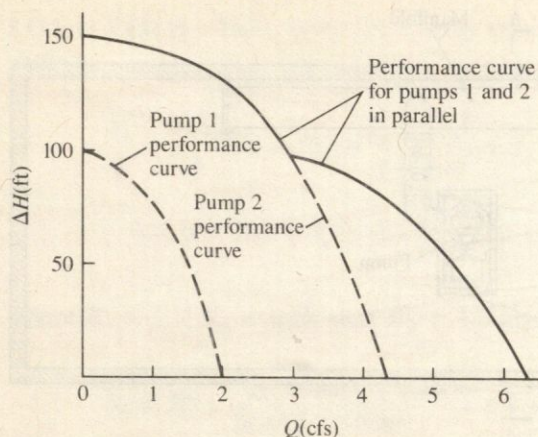


Figure 8-23 Performance of two dissimilar pumps in parallel

If two pumps having dissimilar performance curves were operated in parallel, one would still sum the discharges for the given  $\Delta H$  as noted. For example, if we had pumps with performance curves indicated by the broken lines in Fig. 8-23, the composite performance curve for these two pumps would be as given by the solid line in Fig. 8-23. Note that pump 2 is of no use in augmenting the discharge if  $\Delta H$  is greater than its shutoff head ( $\Delta H = 100$  ft in this case). The composite curve is constructed with the assumption that a check valve prevents flow from passing back through pump 1 when the  $\Delta H$  exceeds 100 ft. If there were no check valve, there would be a negative contribution from pump 1 at the higher heads. Thus one must be careful about installing and operating pumps in combination having dissimilar performance curves; otherwise, the combined operation may not be any better than the operation of a single pump.

**SERIES OPERATION** Although less common than parallel pump installations, the series pump installations are sometimes desirable. Figure 8-24 shows two pumps in series. For a series installation, the discharge will be the same through each pump, so the total head for the combined operation will be the summation of  $\Delta H$  for each pump having the given  $Q$ . Thus the composite performance curve for the two pumps of Fig. 8-20 would be as shown in Fig. 8-25.

## 8-11 Hydraulic Turbines

### *Impulse Turbine*

In the impulse turbine, a jet of water issuing from a nozzle impinges on vanes (sometimes called buckets) of the turbine wheel (often called runner).

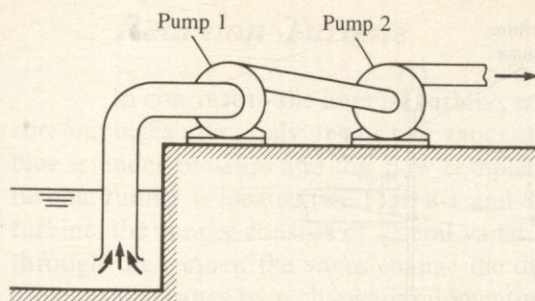


Figure 8-24 Two pumps in series

thus producing power as the runner rotates (see Fig. 8-26). Figure 8-27 shows a runner for the Henry Borden hydroelectric plant in Brazil. The impulse turbine is sometimes called the Pelton wheel, after the man who developed the main features of the modern impulse turbine. The impulse turbine is especially suited for application in high-head plants where the head is typically 500 ft or more. Installations with heads of 1000 to 2000 ft are not uncommon. If the discharge is small, impulse turbines can also be effectively used with lower heads. Recently, impulse turbines have been installed in several small-scale hydroelectric plants following the increase in power costs in the past decade. Before the increased costs in power, the small-scale hydropower plants were often not economically feasible.

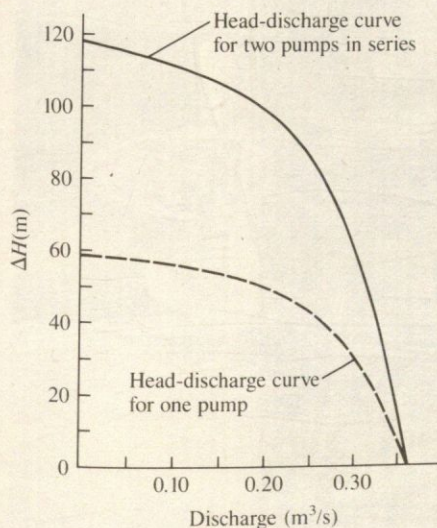


Figure 8-25 Performance curve for two identical pumps in series

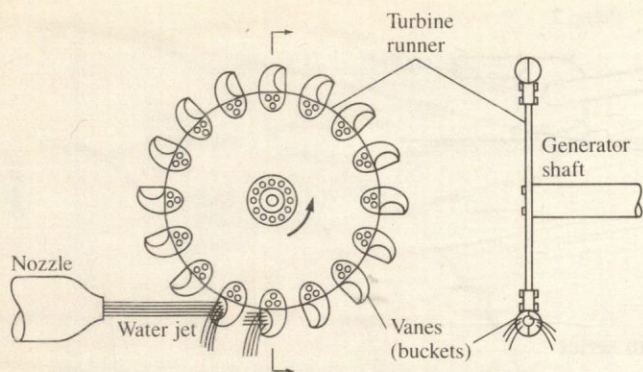


Figure 8-26 Impulse turbine wheel

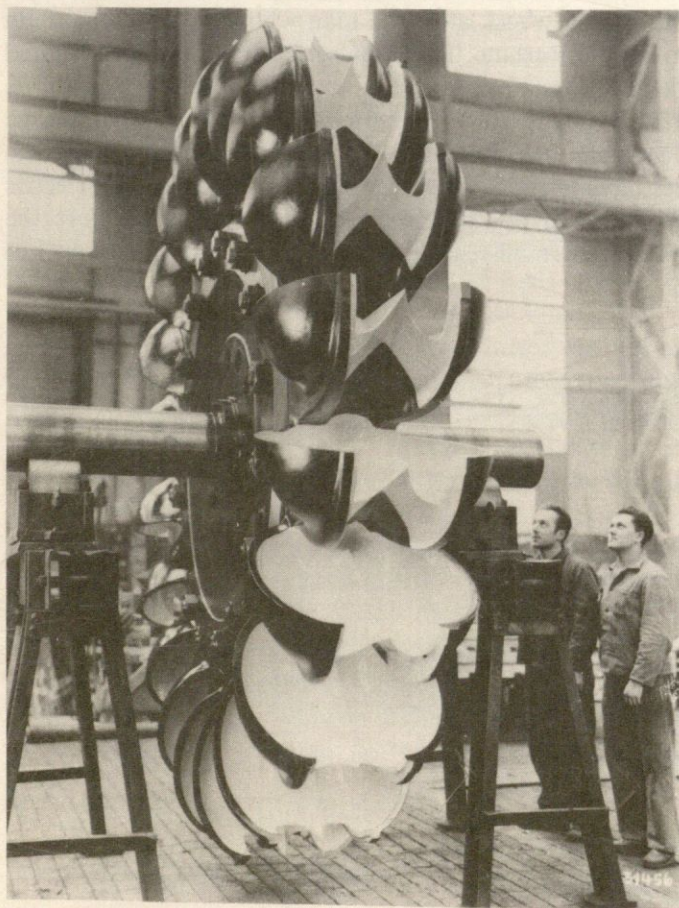


Figure 8-27 Spare runner for the Henry Borden power plant in Brazil. (Courtesy of Voith Hydro Inc.)

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## Reaction Turbine

In contrast to the impulse turbine, where a jet under atmospheric pressure impinges upon only one or two vanes at a time, the flow in a reaction turbine is under pressure and this flow completely fills the chamber in which the turbine runner is located (see Figs. 8-1 and 8-2, pages 434–35). In the reaction turbine, the runner consists of several vanes attached to a hub. As flow passes through the runner, the vanes change the direction of flow, thus producing a force on the vanes by a change in momentum. The force causes the runner to rotate, and power is generated. Because flow in a reaction turbine continuously acts on all the vanes (in contrast to the impulse turbine where only one or two buckets are acted on by the jet at any time), a runner of given size will develop more power per unit head than the impulse turbine.

Reaction turbines can be designed to operate under quite low heads (only 3 or 4 ft). But some have been installed to operate efficiently at heads of more than 1000 ft. The runner of a medium- to high-head turbine has a different shape than a low-head turbine. For example, the most effective type of runner for a low-head ( $3 \text{ ft} < H < 100 \text{ ft}$ ) turbine is a propeller or *axial-flow* type shown in Fig. 8-1. The propeller blades may be fixed; however, for higher efficiencies

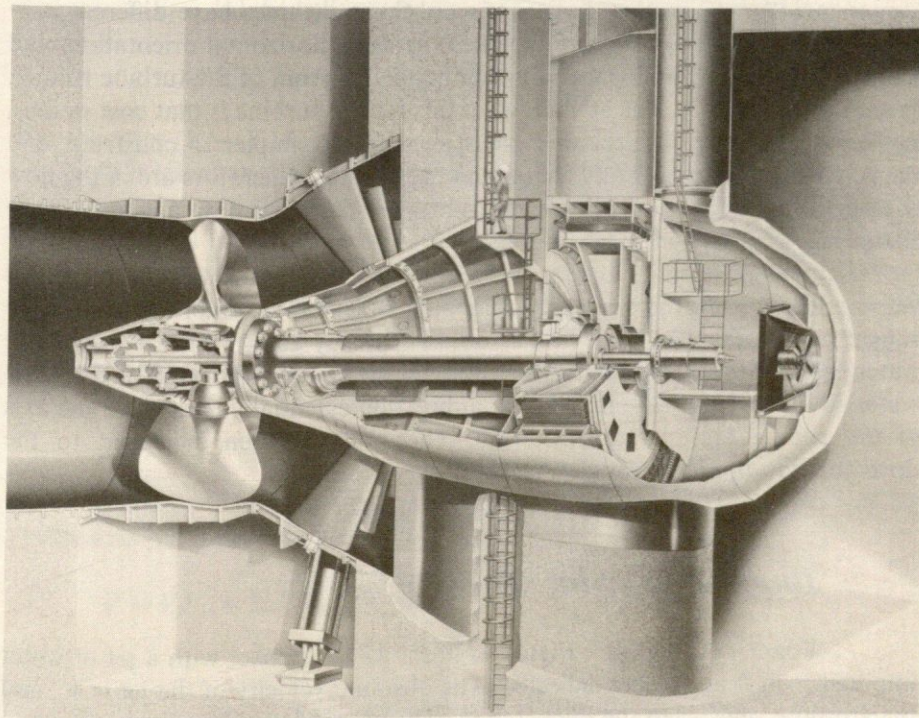


Figure 8-28 Schematic view of bulb turbine  
(Courtesy of Voith Hydro Inc.)

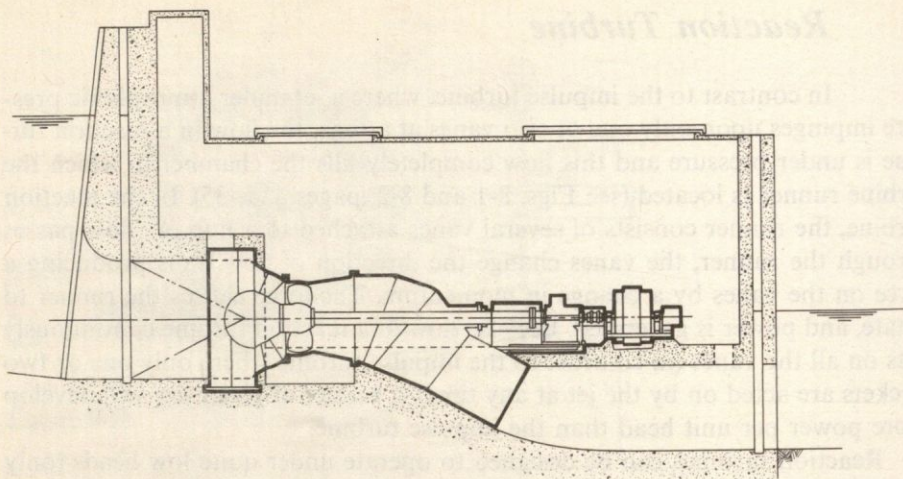


Figure 8-29 Schematic view of an S-turbine

over a wider range of load, the adjustable blade or Kaplan type of turbine is favored. The conventional configuration for the axial-flow turbine is a vertical turbine shaft that drives the generator above the turbine, as shown in Fig. 8-1. However, other innovative designs of axial-flow machines have different configurations. The axis of the *bulb*-type turbine has a horizontal orientation, and the generator is in a bulb-shaped housing just upstream of the turbine runner, as shown in Fig. 8-28. The advantage of this type of turbine is that cost savings can be achieved because the exit flow passages are simpler to construct, and the power-house is essentially eliminated because the generators are in the flow passages within the dam. Another variation of the axial-flow machine is the S-turbine, as shown in Fig. 8-29.

The reaction turbine in Fig. 8-2 is called a Francis turbine, after the man who perfected it. In the Francis turbine, the water before entering the turbine runner is prerotated by the spiral *scroll case* and by guide vanes. Then, as the water passes through the runner, the action of the vanes of the impeller turns the water so that it leaves the impeller in a direction essentially parallel to the axis of the runner and without rotation. The water is then conveyed to the downstream *tail race* through a *draft tube*.

### *Impulse Turbine Theory*

**FORCE ON BUCKET** Figure 8-30 shows the bucket with a jet of water impinging on it and being deflected. The absolute velocity of the jet is  $V_j$ , and the absolute velocity of the bucket is  $V_B$ . Therefore, if the momentum equation is applied to the control volume as shown and we let it move with the bucket,

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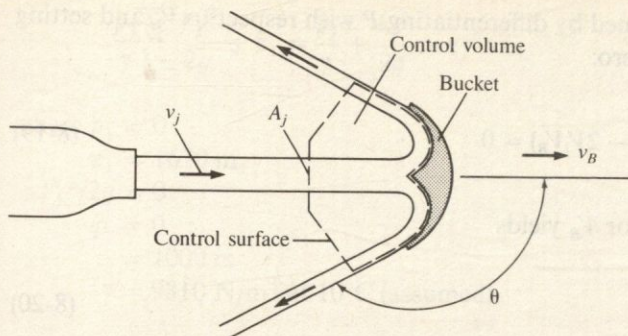


Figure 8-30 Deflection of jet by vane

it can be shown that the force on the bucket will be

$$F_{Bx} = -\rho(V_j - V_B)A_j[(V_j - V_B)_{2x} - (V_j - V_B)_{1x}] \quad (8-16)$$

where  $(V_j - V_B)$  is the velocity of the jet relative to the bucket, and the subscripts 2x and 1x refer to the x components of the relative jet velocity exiting and entering the control volume, respectively.  $(V_j - V_B)_{2x} = (V_j - V_B) \cos \theta$ . Thus, for a given  $(V_j - V_B)$ ,  $F_B$  will be maximized if  $\theta = 180^\circ$ . Then Eq. (8-16) becomes

$$F_B = 2\rho(V_j - V_B)A_j(V_j - V_B) \quad (8-17)$$

The quantity  $(V_j - V_B)A_j$  represents the discharge of water turned by a single bucket. It is less than the discharge issuing from the nozzle of the turbine. However, when we consider the discharge turned by the whole runner it can be appreciated that it will be the same as the discharge from the nozzle. Physically this can be explained by the fact that as a given bucket intercepts the jet from the nozzle not only is it deflecting that flow but there will also be a small cylinder of water being turned by the bucket just ahead of the given bucket (see Fig. 8-26). Therefore, the total force acting on the buckets will be

$$F_B = 2\rho V_j A_j (V_j - V_B) \quad (8-18)$$

**POWER** The power developed will be the product of the speed of the bucket and the force acting on it, or

$$\begin{aligned} P &= V_j F_B = 2\rho V_j V_B A_j (V_j - V_B) \\ &= 2\rho A_j (V_j^2 V_B - V_j V_B^2) \end{aligned}$$

Note that if  $V_B = 0$  or if  $V_B = V_j$ , the power will be zero. There must be an optimum bucket speed between these two limits that will maximize the power.



This speed can be determined by differentiating  $P$  with respect to  $V_B$  and setting the differential equal to zero:

$$\frac{dP}{d(V_B)} = 2\rho A_j(V_j^2 - 2V_jV_B) = 0 \quad (8-19)$$

Then, solving Eq. (8-19) for  $V_B$  yields

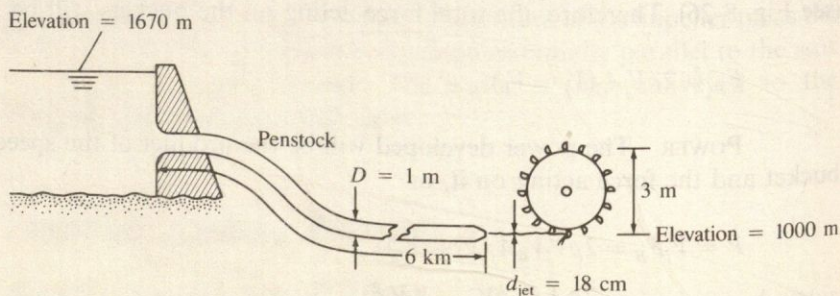
$$V_B = \frac{1}{2} V_j \quad (8-20)$$

and 
$$P = Q\gamma \left( \frac{V_j^2}{2g} \right) \quad (8-21)$$

Thus it has been shown that the maximum power will be developed when the bucket speed is one half the jet speed. And Eq. (8-21) shows that the power thus developed is the same as the maximum theoretical power because  $V_j^2/2g$  is the total head in the jet.

**PRACTICAL CONSIDERATION** From a practical standpoint, the jet is usually turned less than  $180^\circ$  because of interference of the exiting jet with the incoming jet. Experience indicates that the optimum  $\theta$  should be about  $165^\circ$ . Experience also shows that the desired speed ratio,  $V_B/V_j$ , is about 0.45 instead of 0.5. This is due to the hydraulic friction between the jet and the bucket surfaces. The efficiency of large impulse turbines is near 90% (2).

**EXAMPLE 8-8** What power in kilowatts can be developed by the impulse turbine shown if the turbine efficiency is 85%? Assume the resistance coefficient  $f$  of the penstock is 0.015 and the head loss in the nozzle itself is nil. What will be the angular speed of the wheel assuming ideal conditions ( $V_j = 2V_{\text{bucket}}$ ), and what torque will be exerted on the turbine shaft?



**SOLUTION** First determine the jet velocity by applying the energy equation from the reservoir to the free jet before it strikes the turbine buckets.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_j}{\gamma} + \frac{V_j^2}{2g} + z_j + h_L$$

where  $p_1 = 0$

$$z_1 = 1670 \text{ m}$$

$$V_1^2/2g = 0$$

$$p_j = 0$$

$$z_j = 1000 \text{ m}$$

$$\gamma = 9810 \text{ N/m}^3 \text{ at } 10^\circ\text{C (assumed)}$$

The penstock water velocity is

$$V_{\text{penstock}} = \frac{V_j A_j}{A_{\text{penstock}}} = 0.0324 V_j$$

$$\text{Then } h_L = \frac{fL}{D} \frac{V^2}{2g} = \frac{0.015 \times 6000}{1} (0.0324)^2 \frac{V_j^2}{2g} = 0.094 \frac{V_j^2}{2g}$$

Now solving the energy equation for  $V_j$  yields

$$V_j = \left( \frac{2g \times 670}{1.094} \right)^{1/2}$$

$$= 109.6 \text{ m/s}$$

The gross power is

$$P = Q\gamma \frac{V_j^2}{2g} = \frac{\gamma A_j V_j^3}{2g}$$

$$\text{or } P = \frac{9810(\pi/4)(0.18)^2(109.6)^3}{2 \times 9.81}$$

$$= 16,760 \text{ kW}$$

The power output of turbine is

$$P = 16,760 \times \text{efficiency} = 14,245 \text{ kW}$$

The tangential bucket speed will be  $\frac{1}{2}V_j$ ; therefore,

$$V_{\text{bucket}} = \frac{1}{2} 109.6 \text{ m/s} = 54.8 \text{ m/s}$$

$$\text{or } r\omega = 54.8 \text{ m/s}$$

$$\text{Thus } \omega = \frac{54.8 \text{ m/s}}{1.5 \text{ m}} = 36.53 \text{ rad/s}$$

The wheel speed is

$$N = (36.53 \text{ rad/s}) \frac{1 \text{ rev}}{2\pi \text{ rad}} 60 \text{ s/min} = 349 \text{ rpm}$$

$$\text{Power} = T\omega$$

$$\text{Thus } T = \frac{\text{Power}}{\omega} = 14,245 \text{ kW}/36.53 \text{ rad/s} = 390 \text{ kN}\cdot\text{m} \quad \blacksquare$$

### *Reaction Turbine Theory*

**CHARACTERISTICS OF THE REACTION TURBINE** In contrast to the impulse turbine, where a jet under atmospheric pressure impinges on only one or two vanes at a time, the flow in a reaction turbine is under pressure, and this flow completely fills the chamber in which the impeller is located (see Fig. 8-31). There is a drop in pressure from the outer radius of the impeller,  $r_1$ , to the inner radius  $r_2$ . This is also different from the impulse turbine, where the pressure is the same for the entering and exiting flow. The original form of the reaction turbine, first extensively tested by J.B. Francis, had a complete radial-flow impeller (Fig. 8-32). That is, the flow passing through the impeller had velocity components only in a plane normal to the axis of the runner. More recent impeller designs such as the mixed-flow and axial-flow types are still called reaction turbines.

**TORQUE AND POWER RELATIONS FOR THE REACTION TURBINE** We will use the angular-momentum equation to develop formulas for the torque and power.\* The segment of turbine runner shown in Fig. 8-32 depicts the flow conditions that occur for the entire runner. We can see that guide vanes outside the runner itself cause the fluid to have a tangential component of velocity around the entire circumference of the runner. Thus the fluid will have an initial amount of angular momentum with respect to the turbine axis when it approaches the turbine runner. As the fluid passes through the passages of the runner, the runner vanes effect a change in the magnitude and direction of velocity. Thus the angular momentum of the fluid is changed, which produces a torque on the runner. This torque drives the runner, which in turn, generates power. To quantify the above, we let  $V_1$  and  $\alpha_1$  represent the incoming velocity

\* The angular momentum approach, yielding equations of the same form, is also applicable for radial-flow pumps.

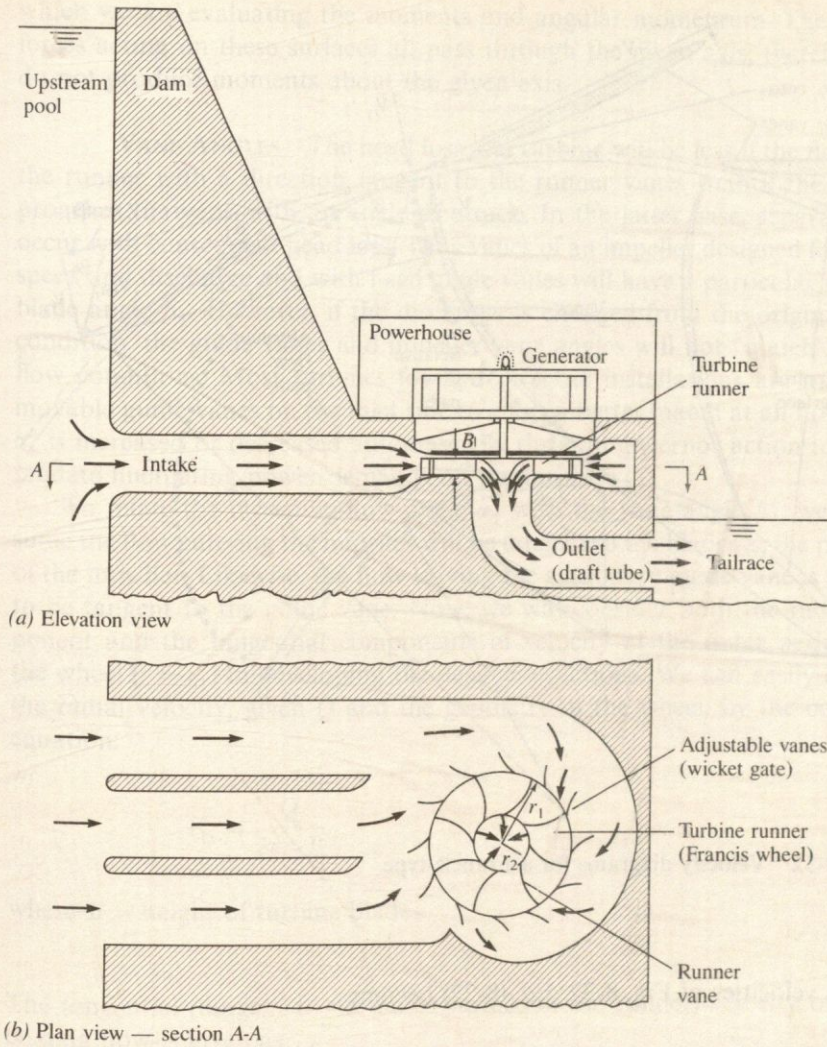


Figure 8-31 Schematic view of reaction turbine installation

and angle of the velocity vector with respect to a tangent to the runner, respectively. Similar terms at the inner-runner radius are  $V_2$  and  $\alpha_2$ .

To obtain the torque on the turbine shaft, the angular-momentum equation is applied to a control volume. Then for steady flow, we have

$$\sum \mathbf{M} = \sum_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \mathbf{A}$$

or 
$$\mathbf{T}_{\text{shaft}} = \sum_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \mathbf{A} \quad (8-22)$$

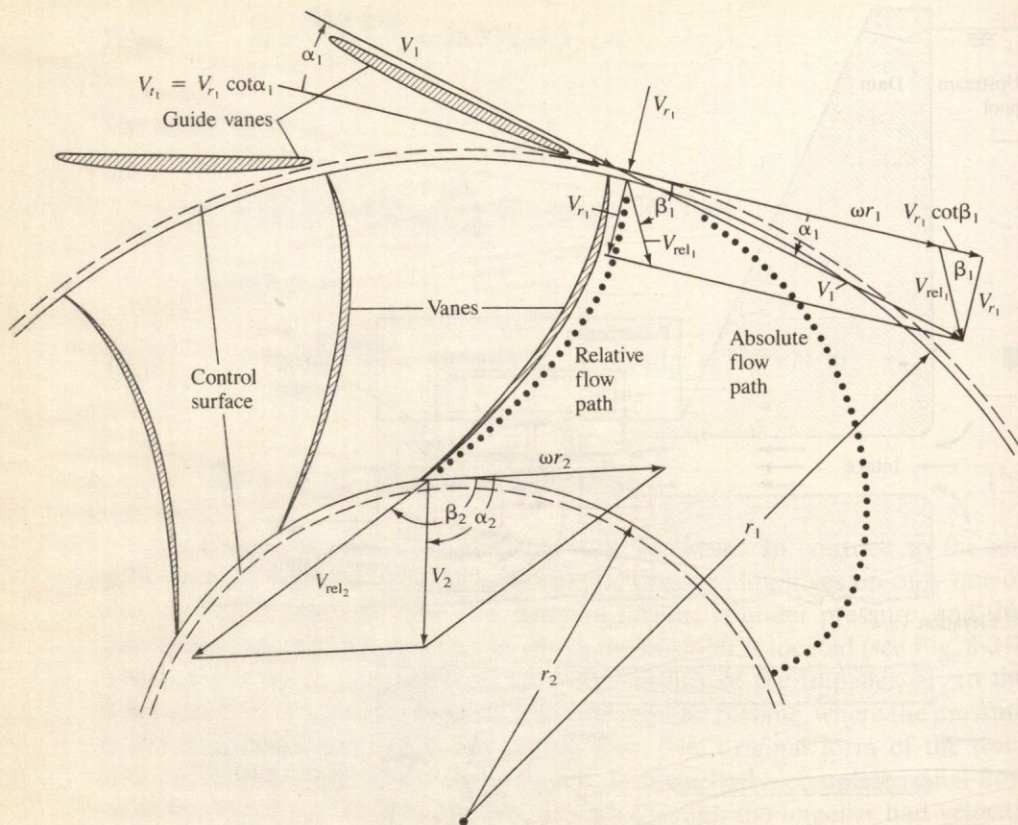


Figure 8-32 Velocity diagrams for a Francis-type runner

For the velocities of Fig. 8-32, Eq. (8-22) becomes

$$T = (-r_1 V_1 \cos \alpha_1) \rho (-Q) + (-r_2 V_2 \cos \alpha_2) \rho (+Q)$$

$$= \rho Q (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2)$$

The power from this turbine will be  $T\omega$ , or

$$P = \rho Q \omega (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2) \quad (8-23)$$

Equation (8-23) shows that the power production is a function of the direction of the flow velocities entering and leaving the impeller,  $\alpha_1$  and  $\alpha_2$ .

It may be noted that even though the pressure varies within the flow in the reaction turbine, it does not enter into the expressions we have derived using the angular-momentum equation. The reason it does not appear is that the outer and inner control surfaces that we chose are concentric with the axis about

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which we are evaluating the moments and angular momentum. The pressure forces acting on these surfaces all pass through the given axis; therefore, they do not produce moments about the given axis.

**VANE ANGLES** The head loss in a turbine will be less if the flow enters the runner with a direction tangent to the runner vanes than if the flow approaches the vane with an angle of attack. In the latter case, separation will occur with consequent head loss. Thus vanes of an impeller designed for a given speed and discharge and with fixed guide vanes will have a particular optimum blade angle  $\beta_1$ . However, if the discharge is changed from the original design condition, the guide vanes and impeller vane angles will not "match" the new flow conditions. Most turbines for hydroelectric installations are made with movable guide vanes on the inlet side to effect a better match at all flows. Thus  $\alpha_1$  is increased or decreased automatically through governor action to accommodate fluctuating power demands on the turbine.

To relate the incoming-flow angle  $\alpha_1$  with the vane angle  $\beta_1$ , we first assume the flow entering the impeller will be tangent to the blades at the periphery of the impeller. Likewise, the flow leaving the stationary guide vane is assumed to be tangent to the guide vane. Now, we will consider both the radial component and the tangential components of velocity at the outer periphery of the wheel ( $r = r_1$ ) in developing the desired equations. We can easily compute the radial velocity, given  $Q$  and the geometry of the wheel, by the continuity equation:

$$V_{r_1} = \frac{Q}{2\pi r_1 B}$$

where  $B$  = height of turbine blades.

The tangential (tangent to the outer surface of the runner) velocity of the incoming flow is given as

$$V_{t_1} = V_{r_1} \cot \alpha_1 \quad (8-24)$$

However, in relation to the flow through the runner, this same tangential velocity is equal to the tangential component of relative velocity in the runner,  $V_{r_1} \cot \beta_1$ , plus the velocity of the runner itself,  $\omega r_1$ . Thus the tangential velocity when viewed with respect to the runner motion is

$$V_{t_1} = r_1 \omega + V_{r_1} \cot \beta_1 \quad (8-25)$$

Now, on eliminating  $V_{t_1}$  between Eqs. (8-24) and (8-25), we have

$$V_{r_1} \cot \alpha_1 = r_1 \omega + V_{r_1} \cot \beta_1 \quad (8-26)$$

Equation (8-26) can be rearranged to yield

$$\alpha_1 = \operatorname{arccot} \left( \frac{r_1 \omega}{V_{r_1}} + \cot \beta_1 \right) \quad (8-27)$$

**EXAMPLE 8-9** A Francis turbine is to be operated at a speed of 600 rpm and with a discharge of  $4.0 \text{ m}^3/\text{s}$ . If  $r_1 = 0.60 \text{ m}$ ,  $\beta_1 = 110^\circ$ , and the blade height  $B$  is 10 cm, what should be the guide vane angle  $\alpha_1$  for a nonseparating flow condition at the runner entrance?

**SOLUTION**

$$\alpha_1 = \operatorname{arccot} \left( \frac{r_1 \omega}{V_{r_1}} + \cot \beta_1 \right)$$

where  $r_1 \omega = 0.6 \text{ m} \times 600 \text{ rpm} \times 2\pi \text{ rad/rev} \times \frac{1}{60} \text{ min/s} = 37.7 \text{ m/s}$

$$V_{r_1} = \frac{Q}{2\pi r_1 B} = \frac{4.00 \text{ m}^3/\text{s}}{2\pi \times 0.6 \text{ m} \times 0.10 \text{ m}} = 10.61 \text{ m/s}$$

$$\cot \beta_1 = -0.364$$

Then,  $\alpha_1 = \operatorname{arccot}(3.55 - 0.364)$   
 $= 17.4^\circ$  ■

**PERIPHERAL SPEED OF TURBINE RUNNER AS A FUNCTION OF  $\Delta H$**  In our discussion on the impulse turbine, we showed that the bucket speed must be about 0.5 of the speed of the jet that drives the turbine for maximum efficiency. For reaction turbines, a similar relationship exists. That is, design experience shows that the ratio of the peripheral speed of the runner to  $\sqrt{2g\Delta H}$  must be in a particular range of values to yield maximum efficiency. This ratio is identified by the symbol  $\phi$  or

$$\phi = \frac{u}{\sqrt{2g\Delta H}} \quad (8-28)$$

where  $u = r\omega = \pi DN/60$

$\Delta H$  = change in head across the turbine.

The denominator of the term on the right-hand side of Eq. (8-28) is often called the *spouting velocity*, and  $\phi$  is called the *speed ratio*. The ranges of speed ratios for maximum efficiency for the different types of turbines are impulse turbine, 0.43–0.47; Francis turbine, 0.5–1.0; propeller turbine, 1.5–3.0.

Equation (8-28) along with representative values of  $\phi$  for different types of turbines shows that the speed of a given turbine will be a function of  $\Delta H$ . Because

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turbines are usually designed to operate at constant speed (which is necessary to maintain constant 60-cycle current), the optimum speed and size of turbine are obviously closely linked for most efficient operation. For example, if we were considering a turbine for a medium-head hydropower site, we would undoubtedly use a Francis turbine, and the average  $\phi$  value would be about 0.75. Then, when this value is substituted into Eq. (8-28), we would have

$$0.75 = \frac{\pi DN/60}{\sqrt{2g \Delta H}}$$

or

$$D = \frac{60 \times 0.75 \sqrt{2g \Delta H}}{\pi N} \quad (8-29)$$

Thus  $D$  is essentially fixed for a given  $N$  and  $\Delta H$ . The actual speed  $N$  for a specific installation must be a synchronous speed for the generator. For a 60-cycle frequency, it can be shown that the speed in rpm is given as

$$N = 7200/n \quad (8-30)$$

where  $n$  is the number of poles in the generator and must be an even integer. In Europe and South America, where the frequency is 50 cycles/s, the speed is given as  $N = 6000/n$ .

**TURBINE OPERATION WITH VARIABLE LOAD** For a turbine and generator to supply power under varying load demands at a constant speed, control gates must be adjusted to increase or decrease the discharge to match the load. The flow conditions for a given gate opening will be somewhat different from the flow conditions for other gate openings; therefore, efficiency and overall performance characteristics will vary somewhat with the gate opening. Figure 8-33 shows power and efficiency versus gate opening for a typical reaction turbine operating at a speed of  $N = 138.6$  rpm. In Fig. 8-33, data are given for various heads so that such information might be available in an installation where the head might also vary, such as when a reservoir water surface elevation changes over time.

**TURBINE SPECIFIC SPEED** As in the case of pumps, *specific speed* is a dimensionless parameter that is useful in selecting the proper type of turbine for a particular set of  $\Delta H$  and  $Q$ . It is also useful for correlating results of cavitation tests on turbines. For pumps, the specific speed was defined as

$$n_s = \frac{nQ^{1/2}}{g^{3/4} \Delta H^{3/4}} \quad (8-31)$$

However, for turbines, we are more interested in the power of the turbine rather than  $Q$ . Therefore, the turbine  $n_s$  is expressed in terms of power,  $P$ , by



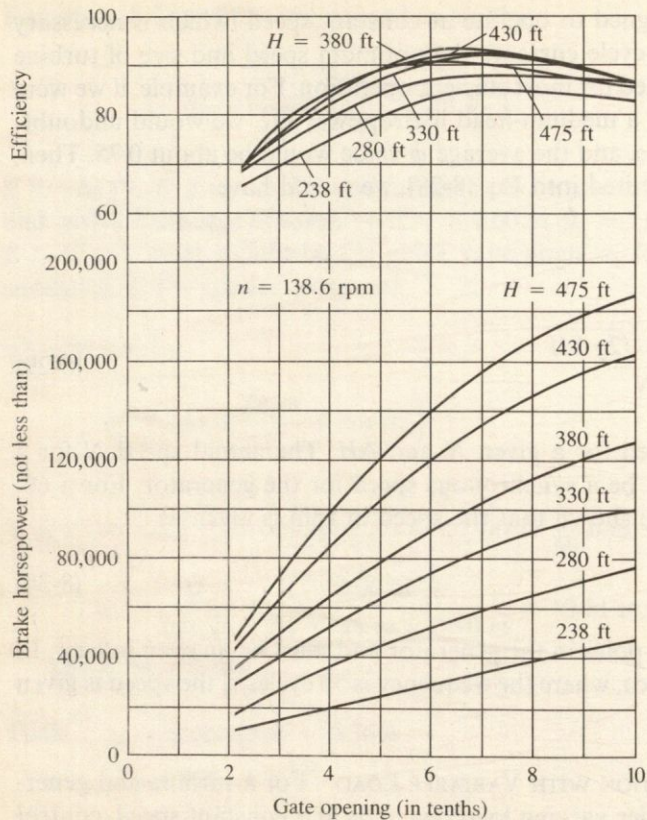


Figure 8-33 Power and efficiency versus gate opening for a turbine (13)

multiplying and dividing Eq. (8-31) by  $\gamma^{1/2} \Delta H^{1/2}$  to obtain

$$n_s = \frac{nQ^{1/2} \gamma^{1/2} \Delta H^{1/2}}{g^{3/4} \Delta H^{3/4} \gamma^{1/2} \Delta H^{1/2}} \tag{8-32}$$

But  $Q\gamma \Delta H = P$ , so Eq. (8-32) can be expressed as

$$n_s = \frac{nP^{1/2}}{\gamma^{1/2} g^{3/4} \Delta H^{5/4}} \tag{8-33}$$

This is the dimensionless form of  $n_s$ . A dimensional form of specific speed has customarily been used by the hydraulic turbine industry, and this is given as

$$N_s = \frac{NP^{1/2}}{\Delta H^{5/4}} \tag{8-34}$$

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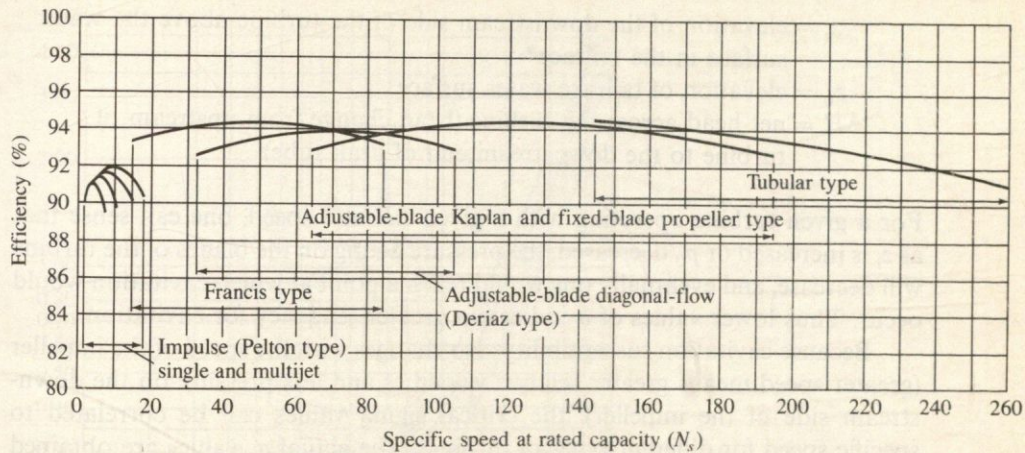


Figure 8-34 Typical peak efficiencies of various turbines in relation to specific speed

where  $N$  = rotational speed in rpm

$P$  = power in horsepower

$\Delta H$  = head on the turbine in feet.

For a given  $Q$  and  $\Delta H$ , the actual speed is directly proportional to specific speed, as can be seen in Eq. (8-34). Moreover, higher values of speed result in reduced diameters (see Eq. 8-29) and weights of the generator and turbine, as well as the reduced size and cost of powerhouse. Therefore, to take advantage of these possible lower costs, the design engineer always desires to choose turbines with high specific speeds even if there is some sacrifice in efficiency.

Figure 8-34 shows typical peak efficiencies for various types of turbines as a function of specific speed. A plot like this is helpful in determining the type of turbine to use for a particular hydropower site.

**CAVITATION IN TURBINES** Like pumps, turbines are also susceptible to cavitation. The region of the flow where cavitation is most likely to occur is on the downstream side of the turbine impeller blades. The cavitation index for turbines is the same as for pumps (Eq. 8-14); however, the elevation of the turbine refers to the tailrace water surface elevation, and the head loss is assumed to be negligible. Thus the cavitation index is given as

$$\sigma = \frac{p_0/\gamma - p_v/\gamma - (z_t - z_0)}{\Delta H} \quad (8-35)$$

where  $p_0$  = absolute atmospheric pressure

$p_v$  = absolute vapor pressure of the water

$z_t$  = elevation of the downstream side of the turbine above the water surface in the tailrace\*

$z_0$  = elevation of tailrace water surface

$\Delta H$  = net head across the turbine (head change from upstream of turbine to the downstream end of draft tube)

For a given turbine operating with a given  $\Delta H$  and speed, one can sense that as  $z_t$  is increased or  $p_0$  decreased, the pressure acting on the blades of the turbine will decrease, and eventually one would reach a point at which cavitation would occur. Thus lower values of  $\sigma$  indicate a greater tendency for cavitation.

Because cavitation susceptibility also changes with the speed of the impeller (greater speed means greater relative velocities and less pressure on the downstream side of the impeller), the critical sigma values can be correlated to specific speed for different types of turbines. The actual  $\sigma_c$  values are obtained experimentally. Results of experimental tests are shown in Fig. 8-35.

If one considers a given water temperature (also given  $p_v$ ), one can solve for  $z_t$  (allowable elevation of turbine) for given  $\Delta H$  and atmospheric pressure,  $p_0$ . For standard atmospheric pressure and 80°F water temperature, such computations yield the plots shown in Fig. 8-36, which is similar to Fig. 8-18 for pumps.

**EXAMPLE 8-10** Select the type, speed, and size of turbine for a site where the net head is 330 ft and  $Q = 4300$  cfs.† Determine also the elevation of the turbine with reference to the water surface in the tailrace. Assume the turbine will drive a 60-cycle generator.

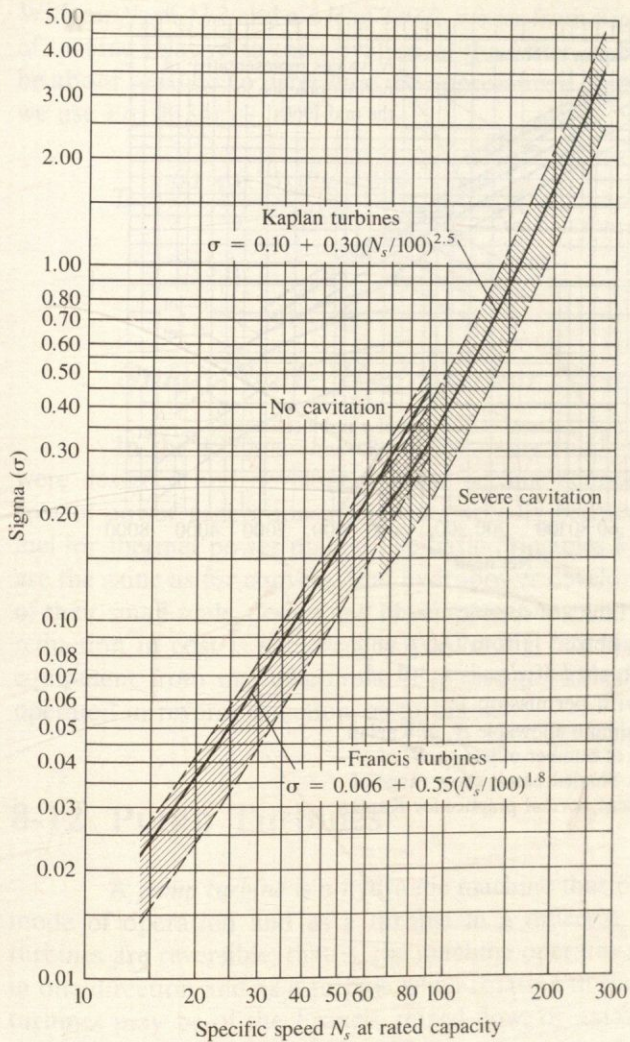
**SOLUTION** From Fig. 8-36 for a net head,  $\Delta H$ , of 330 ft, we see that  $N_s$  is about 45 for the normal practice of the 1960s. We also see from this figure and Fig. 8-34 that the turbine should be a Francis type. From Fig. 8-34, we can expect the turbine to have a peak efficiency of about 94%. Then the maximum developed power for the turbine will be

$$\begin{aligned} P &= \frac{Q\gamma\Delta H \times \text{efficiency}}{550} \\ &= \frac{4300 \text{ ft}^3/\text{s} \times 62.4 \text{ lb}/\text{ft}^3 \times 330 \text{ ft} \times 0.94}{550} \\ &= 151,330 \text{ hp} \end{aligned}$$

Because  $N_s = NP^{1/2}/H^{5/4}$ , one can solve for an approximate value of  $N$ :

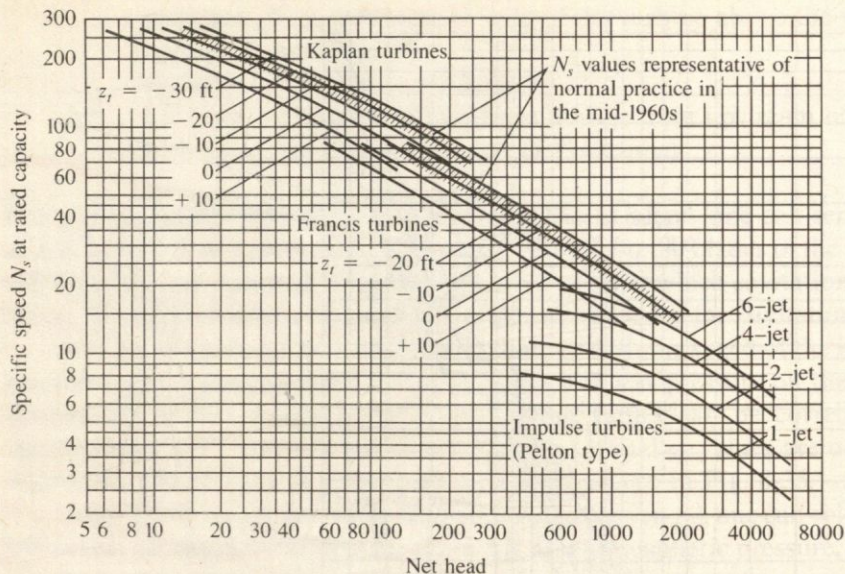
\* The *tailrace* is the channel (canal or natural stream) that the flow from the draft tube discharges into.

† The head and discharge values are those for one of several turbines installed at Grand Coulee Dam in Washington state.



**Figure 8-35** Sigma values for various specific speeds\*  
 [From Davis and Sorensen (4), *Handbook of Applied Hydraulics*, 3rd ed., McGraw Hill, 1969. Used with permission.]

\* Shaded bands show approximate range of critical sigma of Francis and Kaplan turbines; curves indicate minimum plant sigma recommended as a guide for preliminary selection purposes.



**Figure 8-36** Approximate limits of specific speed for various turbines and net heads\* [From Davis and Sorensen (4), *Handbook of Applied Hydraulics*, 3rd ed. McGraw Hill, 1969. Used with permission.]

\* Effect of draft head  $z_r$  on maximum allowable  $N_s$  of Kaplan and Francis-turbines and effect of number of jets on  $N_s$  of impulse turbines are illustrated. Shaded bands show range of  $N_s$  values representative of present normal practice for Kaplan and Francis turbines.

$$N = \frac{N_s H^{5/4}}{P^{1/2}}$$

$$= \frac{45 \times (300)^{5/4}}{(151,330)^{1/2}} = 162.7 \text{ rpm}$$

However, it is necessary that the turbine operate at a synchronous speed for the generator that will be obtained from Eq. (8-30) or  $N = 7200/n$ . Using 46 poles (the number of poles that yields the synchronous speed closest to but not exceeding  $N = 162.7$  rpm), we have

$$N = \frac{7200}{46} = 156.5 \text{ rpm}$$

Then a new value of  $N_s$  is calculated based on this speed:

$$N_s = \frac{156.5 \times (151,330)^{1/2}}{(330)^{5/4}} = 43.3$$

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With an  $N_s$  of 43.3 and a  $\Delta H$  of 330 ft, we see from Fig. 8-36 that  $z_t$  (the elevation of turbine relative to the elevation of the water surface in the tailrace) should be about -10 ft. To determine the approximate diameter of the Francis wheel, we use Eq. (8-34):

$$D = \frac{60 \times 0.75 \sqrt{2g \times 330}}{\pi \times 156.5}$$

$$= 13.3 \text{ ft}$$

### *Small-Scale Hydropower Systems*

In the 1970s and early 1980s, many small-scale hydropower systems were designed and developed. These became economically feasible when the cost of power generation increased markedly because of the increased cost of fuel for thermal power plants. The basic principles for these small-scale plants are the same as for conventional hydropower developments. However, because of their small scale, design and development costs have to be minimized. Some reduction in cost is achieved by purchasing standardized turbines and other equipment from manufacturers. It is also possible to use centrifugal pumps, operated in reverse direction, as turbines.\*

## 8-12 Pump Turbines

A *pump turbine* is a hydraulic machine that operates as a pump in one mode of operation and as a turbine in a different mode. Almost all pump turbines are reversible; that is, the machine operates as a pump when rotating in one direction and as a turbine when rotating in the reverse direction. Pump turbines may be of the Francis, mixed-flow, or axial-flow type depending on the head the machine is designed for.

### *Use of Pump Turbines*

Pump turbines are used in pumped storage installations to level the peaks and valleys of a typical electrical utility load curve. For example, a typical electrical utility will have high loads during the day and low loads at night. If a pumped storage installation is used in the system, then during periods of low demand, some of the "excess" energy can be used to pump water (using the

\* For more on small-scale hydropower systems, see Warnick (18), Haroldsen (5), Kittredge (8), Mayo (10), and Shafer (14).

pumping mode of the pump turbine) from a low elevation to a storage reservoir at higher elevation. Thus energy is stored in the reservoir. Then during periods of high demand, the water from the high reservoir is run back through the pump turbine using the turbine mode to generate power. Pumped storage installations are especially desirable when used with a power plant that is most effectively operated at constant power for long periods, such as a nuclear power plant. The pumped storage part of the system does not produce any additional energy — it simply transfers energy produced during lower demand periods to high demand periods. The overall efficiency (including mechanical, electrical, and hydraulic losses) for a modern pumped storage system is about 75%.

### *Head Ranges for Different Types of Pump Turbines*

The propeller or axial-flow type of pump turbine is generally designed to operate under fairly low heads (3 to 100 ft) and is often designed with adjustable blades so that it may be operated at high efficiency over wide ranges of head and discharge. This latter aspect is especially suited for use in tidal power installations.

For intermediate heads (35 to 300 ft), the mixed-flow type of impeller with adjustable blades is most often used. The radial-flow or Francis type runner with fixed vanes is used with high heads. Single-stage Francis type pump turbines have been designed and built for heads from 75 to 2000 ft. If heads greater than about 2000 ft are encountered, a more complex multistage machine must be used.\*

### *Cavitation in Pump Turbines*

Pump turbines are generally more susceptible to cavitation than a turbine or pump with the same specific speed. Therefore, to avoid cavitation, pump-turbine impellers have to be placed at an elevation lower than the tail-water elevation. The lower elevation produces a higher pressure on the impeller, which suppresses the onset of cavitation.

## 8-13 Viscous Effects

In earlier sections, we developed similarity parameters to predict prototype results from model tests, but we neglected to discuss the viscous effects in model pumps. Although viscous effects are usually small, they are not negligible, especially if the model is quite small.

\* For more details on pump turbines, see Webb (19).

To minimize the viscous effects in modeling pumps, the Hydraulic Institute standards (6) recommend that the size of the model be such that the model impeller is not less than 30 cm in diameter. These same standards state that "the model should have complete geometric similarity with the prototype, not only in the pump proper, but also in the intake and discharge conduits."

Even with complete geometric similarity, one can expect the model to be slightly less efficient than the prototype. An empirical formula proposed by Moody that is used for estimating prototype efficiencies of radial- and mixed-flow pumps and turbines from model efficiencies is

$$\frac{1 - e_1}{1 - e} = \left(\frac{D}{D_1}\right)^{1/5} \quad (8-36)$$

where  $e_1$  is the efficiency of the model, and  $e$  is the efficiency of the prototype.

**EXAMPLE 8-11** A model having an impeller diameter of 45 cm is tested and found to have an efficiency of 85%. If a geometrically similar prototype has an impeller diameter of 1.80 m, estimate its efficiency when it is operating under conditions dynamically similar to those in the model test ( $C_{Q \text{ model}} = C_{Q \text{ prototype}}$ ).

**SOLUTION** We apply Eq. (8-36) with the condition that  $e_1 = 0.85$  and  $D/D_1 = 4$ . Then

$$\begin{aligned} e &= 1 - \frac{1 - e_1}{(D/D_1)^{1/5}} \\ &= 1 - \frac{0.15}{1.32} = 1 - 0.11 = 0.89 \end{aligned}$$

The efficiency of the prototype is estimated to be 89%. ■

## 8-14 Other Types of Pumps

The pumps and turbines we have discussed so far in this chapter are all classified as turbomachines. In turbomachines, the exchange of energy is accomplished by means of hydrodynamic forces developed between a moving fluid and the rotating and stationary parts of the machine. For example, in the axial-flow pump, the lift force of the rotating blades of the impeller produces the pressure increase of the pump.

Another entirely different class of pump is the positive displacement type. All positive displacement pumps have parts that interact in such a way that definite volumes of fluid are conveyed in the desired pumping direction essentially in proportion to the speed of operation of the pump. One of the simplest



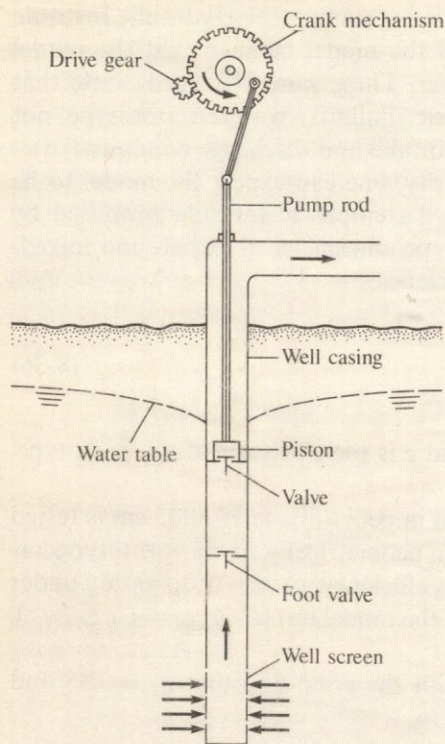


Figure 8-37 Reciprocating piston pump

positive displacement pumps is the reciprocating piston pump shown in Fig. 8-37. In this pump, as the pump rod and piston are raised by the crank mechanism, the valve in the piston is closed so that the piston draws water into the well and up through the well pipe (well casing). On this upstroke, the foot valve remains open. On the downstroke of the piston, the foot valve closes, but the valve in the piston opens. Thus one can see that for each cycle of the crank that drives the piston a definite volume of water will be "lifted" from the well and through the outlet pipe. If the speed of the crank is doubled, the rate of pumping would also be doubled (neglecting leakage past seals of the piston). The work required to pump the water can be expressed in terms of the essentially static force applied to the piston to lift the water times the distance through which the piston acts when water is being lifted.

Many other types of positive displacement pumps have configurations different from that of the simple piston pump. Several of these pumps are the *gear pump*, *two-lobe rotary pump*, and *screw pump*.

Besides the broad categories of turbomachines and positive displacement pumps, *jet pumps* and *hydraulic rams* have limited but important use in special situations.

Descriptions of the aforementioned pumps are given under separate headings below.

### *Gear Pump*

Figure 8-38 is a section through a spur-gear pump. The gears rotate in the direction indicated, and these gears have very close clearance with the casing of the pump. Where the gear teeth contact, they form a tight liquid seal. Thus as the gears rotate, liquid flows in between the gear teeth on the suction side in very much the same way that liquid is drawn into the cylinder of a piston pump when the piston is on the suction stroke. As the gears rotate, the liquid is trapped between the teeth and the casing and is carried around to the discharge side of the pump, where the liquid is forced out as the teeth of the gears mesh together.

Gear pumps are just one class of rotary pumps that are used for pumping various kinds of liquids over a wide range of pressure, viscosities, and temperatures. Several applications of rotary pumps are

1. Chemical processing
2. Food handling
3. Tank truck loading and unloading
4. Machine tool coolants
5. Pressure lubrication
6. Hydraulic power transmission
7. General transfer of liquids

The efficiency depends on the viscosity of the liquid being pumped, but it may be as high as 70% for low viscosity liquids. Rotary pumps can be designed to develop pressures up to 5000 psi, and some have capacities as high as 5000 gpm.

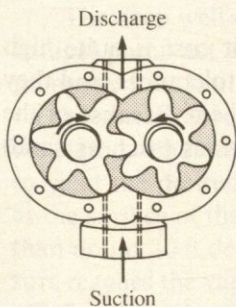


Figure 8-38 Spur-gear pump

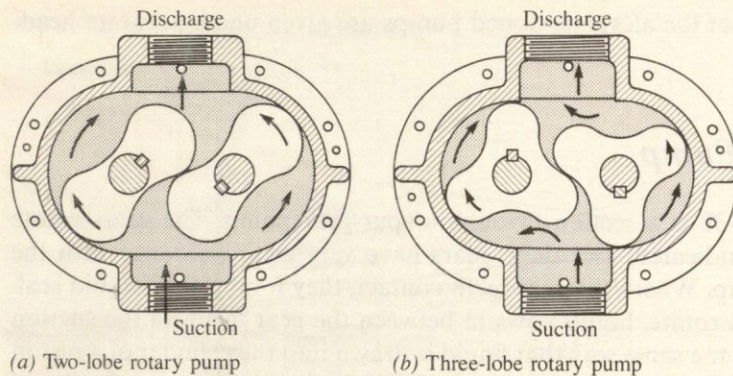


Figure 8-39 Lobe pumps

### *Lobe Pumps*

Figure 8-39 shows two-lobe and three-lobe rotary pumps. These pumps operate exactly like gear pumps. However, because of the smaller number but larger volume of "chambers" that produce the pumping action, there may be more of a pulsating flow from the lobe pump than from the gear pump.

### *Screw Pumps*

Screw pumps are similar to gear and lobe pumps in that pumping occurs as the elements of the pump rotate and mesh. In the screw pump, the liquid is carried between screw threads on one or more rotors and is displaced axially as the screws rotate.

### *Disadvantages of Rotary Pumps*

The main disadvantages of rotary pumps are their cost is quite high because of the precise machining required to produce close tolerances, and they are not suited for pumping liquids that have abrasives in them. Because of the close clearances in rotary pumps, liquids containing abrasives (such as sand) will usually cause rapid wear of the surfaces.

### *Jet Pumps*

*Jet pumps* derive their pumping action from a high velocity jet of fluid that then becomes entrained with the fluid it is pumping. The high momentum

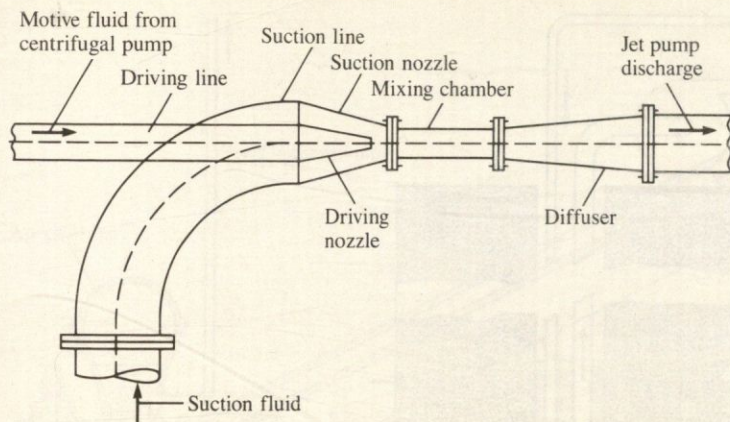


Figure 8-40 Jet pump

of the jet is converted to pressure in a diffuser. Liquid jet pumps are sometimes also called *eductors*. Figure 8-40 shows the essential features of a jet pump. There are many advantages of the jet pump, for example, it is self priming; it has no moving parts; and it can be made from any machinable materials, glass, and fiberglass. The main disadvantage of the jet pump is its relatively low efficiency. The entrainment process inherent in its operation produces large head losses that account for this low efficiency. Despite its low efficiency, it has several uses, including

1. Deep-well pumping
2. Bilge pumping on ships
3. Providing circulation in rearing tanks of fish hatcheries (absence of moving mechanical parts do not injure fish)
4. Chemical processing mixing
5. Pumping out wells, pits, sumps where there is an accumulation of sand or mud

The deep well application is illustrated in Fig. 8-41. The jet pump and centrifugal pump act as a two-stage pumping unit. In the pumping process, the jet pump near the bottom of the well produces enough pressure so that the pressure on the suction side of the centrifugal pump is well above the vapor pressure of the liquid. Thus the centrifugal pump provides the remaining necessary head to yield the desired results. Without the jet pump, the centrifugal pump alone at the surface of the ground would not be able to pump water from a well more than about 30 ft deep because the water would vaporize when the suction pressure reaches the vapor pressure of the water (equivalent to about -33 ft of head at normal temperatures). The jet pump is well suited for this kind of application because it can be designed to be a relatively compact unit that can be easily installed in a well. A typical commercial deep well unit has a 1-in. pressure pipe

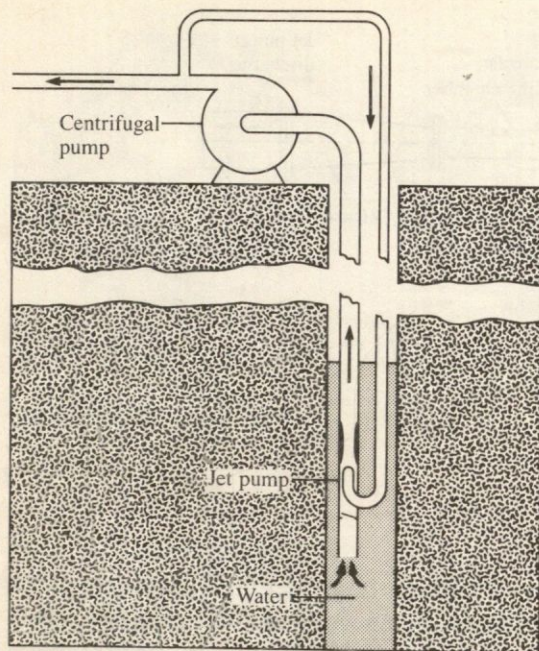


Figure 8-41 Jet pump in combination with a centrifugal pump for pumping water from a well

with a  $1\frac{1}{4}$ -in. discharge pipe and, depending on the depth of the well, several available nozzle and diffuser combinations (7).

### Hydraulic Ram

The hydraulic ram was first developed in England in about 1800. It uses a relatively large flow of water under low head to pump a much smaller amount of water to a much higher elevation. Figure 8-42 shows the essential features of a hydraulic ram. Valve *W* is the waste valve, and valve *C* is a check valve. Assuming the cycle of operation starts with zero velocity in the drive pipe with

Table 8-1 Data on Selected Hydraulic Rams in the United States

Location	Discharge (cfs)		Head (ft)		Strokes per minute
	to ram	to reservoir	drive pipe	pump head	
U.S. Naval Coaling Station, Bradford, Rhode Island	1.29	0.52	37	84	130
Seattle Water Works, Seattle, Washington	1.63	0.55	49	131	65

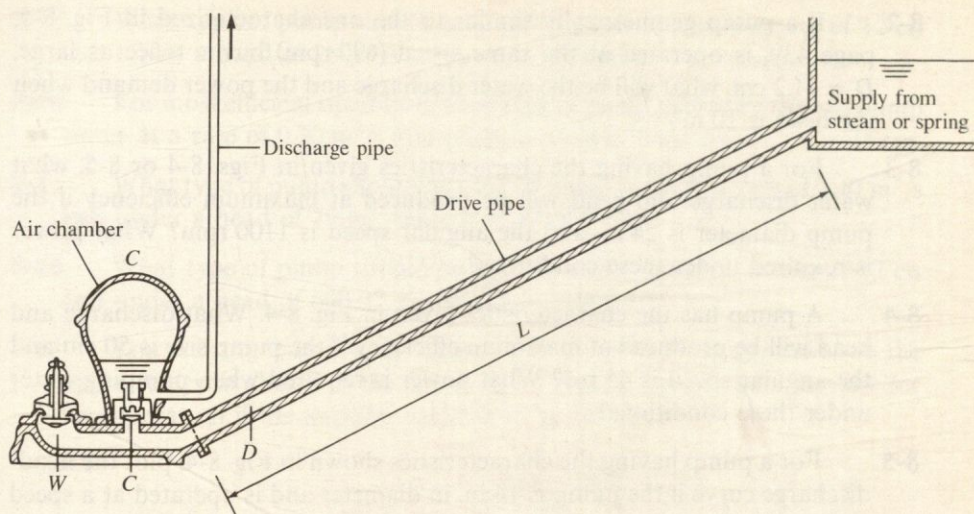


Figure 8-42 Hydraulic ram

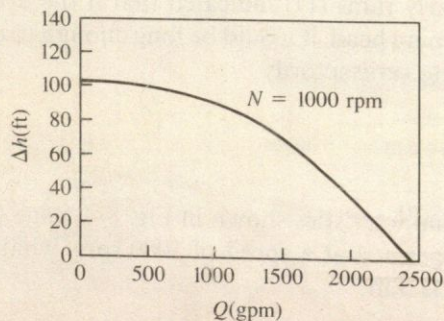
valve  $W$  open and valve  $C$  closed, flow starts past valve  $W$  and, because of continuity, water also starts moving in the drive pipe. The flow through valve  $W$  and the drive pipe will accelerate until the velocity past the valve is so great that valve  $W$  closes quickly. The valve closure is initiated because the drag on the valve overcomes the weight of the valve that tends to keep it open. Once valve  $W$  closes, it produces a water hammer pressure in the drive pipe and in the body of the ram. This pressure will be large enough to open the check valve  $C$ , and some water will be forced into the air chamber. The pressure in the air chamber increases and further compresses the air. More water goes into the air chamber and, because of this increase of pressure, flow occurs in the discharge pipe. After a short time, the water hammer pressure in the drive pipe and ram subsides so that valve  $C$  closes. A short time later, the pressure in the drive pipe and ram is further relieved (the relief wave of water hammer starts) and valve  $W$  opens. Once valve  $W$  is opened, the cycle repeats itself.

In the early 1900s, many hydraulic rams were used for municipal water supplies as well as for individual farms. Table 8-1 gives data for two of these early rams. A study of some of these early rams (11) indicated that if the drive pipe was three times as long as the pumping head, it would be long enough to develop water hammer pressures to operate satisfactorily.

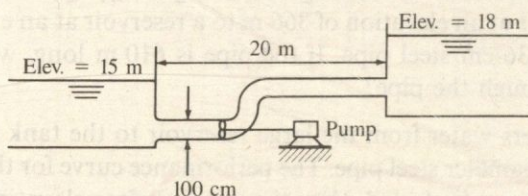
## PROBLEMS

- 8-1 If the pump having the characteristics shown in Fig. 8-4, page 439, has a diameter of 40 cm and is operated at a speed of 1000 rpm, what will be the discharge when the head is 3 m?

- 8-2 If a pump geometrically similar to the one characterized in Fig. 8-5, page 439, is operated at the same speed (690 rpm) but is twice as large,  $D = 71.2$  cm, what will be the water discharge and the power demand when the head is 10 m?
- 8-3 For a pump having the characteristics given in Figs. 8-4 or 8-5, what water discharge and head will be produced at maximum efficiency if the pump diameter is 24 in. and the angular speed is 1100 rpm? What power is required under these conditions?
- 8-4 A pump has the characteristics given in Fig. 8-4. What discharge and head will be produced at maximum efficiency if the pump size is 50 cm and the angular speed is 45 rps? What power is required when pumping water under these conditions?
- 8-5 For a pump having the characteristics shown in Fig. 8-4, plot the head-discharge curve if the pump is 14 in. in diameter and is operated at a speed of 900 rpm.
- 8-6 For a pump having the characteristics shown in Fig. 8-4, plot the head-discharge curve if the pump diameter is 60 cm and the speed is 690 rpm.
- 8-7 If the pump having the characteristics in Fig. 8-8, page 442, is operated at a speed of 30 rps, what will be the shutoff head?
- 8-8 If the pump having the characteristics in Fig. 8-9, page 443, is 40 cm in diameter and is operated at a speed of 25 rps, what will be the discharge when the head is 50 m?
- 8-9 If the pump having the characteristics in Fig. 8-8 is doubled in size but halved in speed, what will be the head and discharge at maximum efficiency?
- 8-10 For a pump having the characteristics shown in Fig. 8-9, plot the head-discharge curve if the pump diameter is 1.52 m and the speed is 500 rpm.
- 8-11 If a pump having the characteristics given in Figs. 8-8 or 8-9 is operated at a speed of 1500 rpm, what will be the discharge when the head is 160 ft?
- 8-12 If the pump having the performance curve shown is operated at a speed of 1500 rpm, what will be the maximum possible head developed?

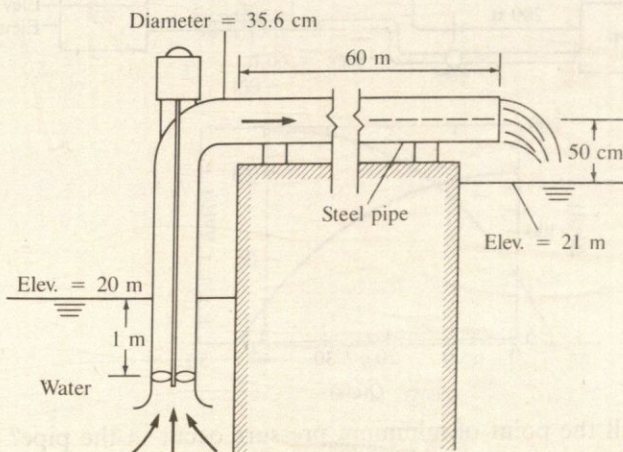


- 8-13 What type of pump should be used to pump water at a rate of 12 cfs and under a head of 25 ft? Assume  $N = 1500$  rpm.
- 8-14 For most efficient operation, what type of pump should be used to pump water at a rate of  $0.30 \text{ m}^3/\text{s}$  and under a head of 8 m? Assume  $n = 25$  rps.
- 8-15 What type of pump should be used to pump water at a rate of  $0.40 \text{ m}^3/\text{s}$  and under a head of 70 m? Assume  $N = 1100$  rpm.
- 8-16 What type of pump should be used to pump water at a rate of 12 cfs and under a head of 600 ft? Assume  $N = 1100$  rpm.
- 8-17 You want to pump water at a rate of  $1.0 \text{ m}^3/\text{s}$  from the lower to the upper reservoir shown in the figure. What type of pump would you use for this operation if the impeller speed is to be 600 rpm?



PROBLEM 8-17

- 8-18 The pump used in the system shown has the characteristics given in Fig. 8-5, page 439. What discharge will occur under the conditions shown, and what power is required?

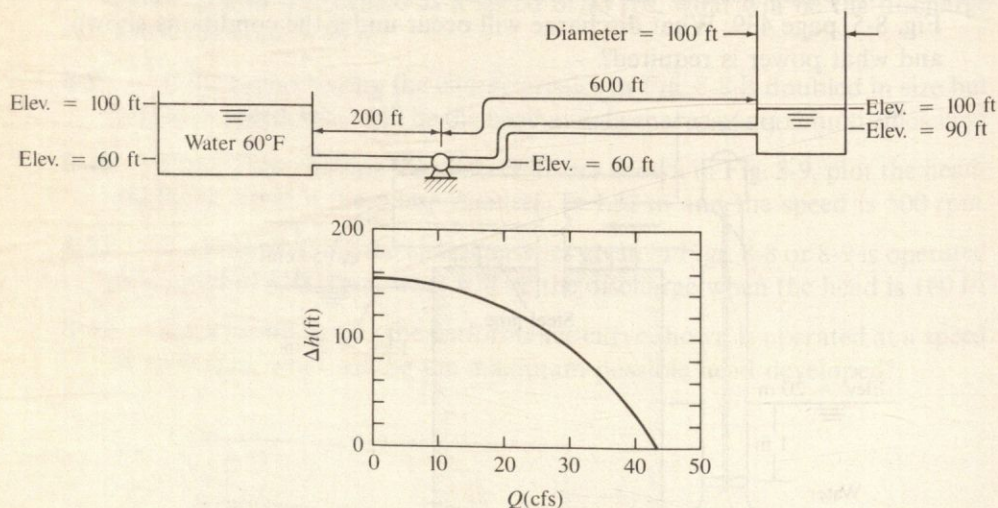


PROBLEMS 8-18, 8-19

- 8-19 If the conditions are the same as in Prob. 8-18 except that the speed is increased to 900 rpm, what discharge will occur, and what power is required for the operation?

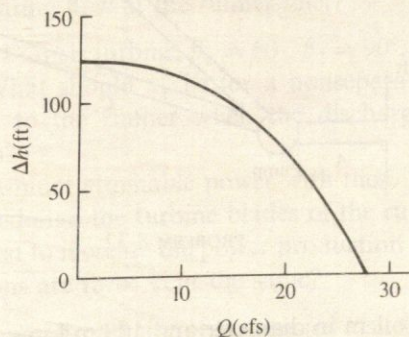
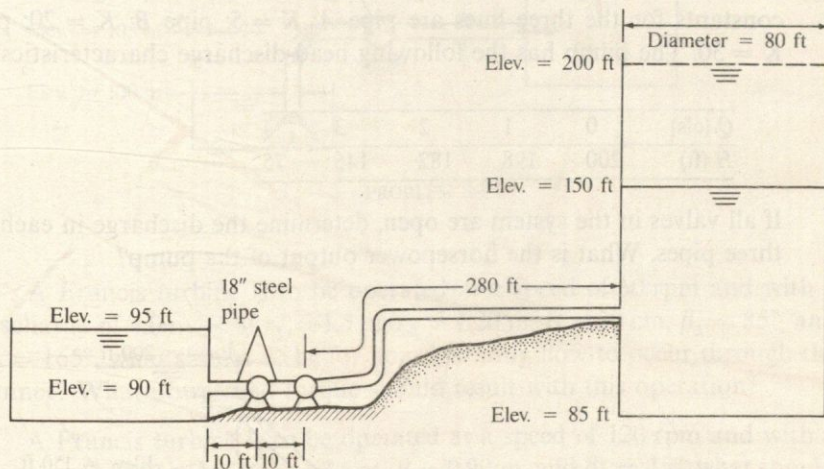


- 8-20 An axial-flow pump is to be used to lift water against a head (friction and static) of 5.0 m. If the discharge is to be  $0.40 \text{ m}^3/\text{s}$ , what maximum speed in revolutions per minute is allowed if the suction head is 1.5 m?
- 8-21 What is the specific speed for the pump operating under the conditions given in Prob. 8-18? Is this a safe operation with respect to the susceptibility to cavitation?
- 8-22 What is the specific speed for the pump operating under the conditions of Prob. 8-19? Is this a safe operation with respect to the susceptibility to cavitation?
- 8-23 For the conditions of Prob. 8-17, would the pump operate without cavitation?
- 8-24 A pump having the characteristics given in Fig. 8-8, page 442, pumps water from a reservoir at an elevation of 366 m to a reservoir at an elevation of 450 m through a 36-cm steel pipe. If the pipe is 610 m long, what will be the discharge through the pipe?
- 8-25 The pump delivers water from the large reservoir to the tank through the 800-ft long, 2-ft diameter steel pipe. The performance curve for the pump is also shown. The pump is started when the water surface elevation in the tank is 100 ft and continues to operate until the water surface elevation is 200 ft.



- Where will the point of minimum pressure occur in the pipe? What is the magnitude of the minimum pressure?
- What power must be supplied to the pump when the water surface elevation in the tank is 100 ft (a minute or so after the pump is turned on) if the pump efficiency is 70%?
- Estimate the time required to fill the reservoir to the 200-ft level.

- 8-26 If two pumps like the one given in Prob. 8-25 had been installed and operated in parallel for the conditions of Prob. 8-25, what would be the initial discharge?
- 8-27 If two pumps like the one given in Prob. 8-25 had been installed in series for the conditions of Prob. 8-25, what would be the initial discharge?
- 8-28 Two pumps having the performance curve shown are operated in series in the 18-in. diameter steel pipe. When both are operating, estimate the time to fill the tank from the 150-ft level to the 200-ft level. Estimate the maximum pressure in the pipe during the filling phase. Where will this maximum pressure occur? What would have been the initial discharge if the pumps had been installed in parallel?



PROBLEM 8-28

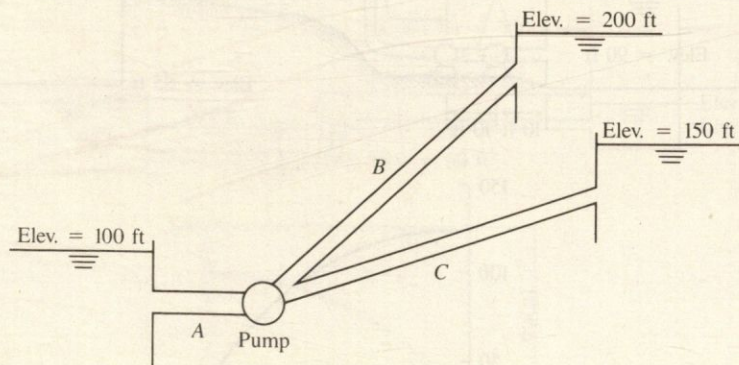
- 8-29 The pump of Prob. 8-12 is used to pump water from reservoir *A* to reservoir *B*. The pump is installed in a 2-mi long, 12-in. pipe joining the two reservoirs. There are two bends in the pipe ( $r/D = 1.0$ ), and two gate

valves are open when pumping. When the water surface elevation in reservoir *B* is 30 ft above the water surface in reservoir *A* at what rate will water be pumped?

- 8-30 Work Prob. 8-29 but have two pumps like that of Prob. 8-12 operating in parallel.
- 8-31 Work Prob. 8-29 but have two pumps like that of Prob. 8-12 operating in parallel and have an 18-in. pipe instead of a 12-in. pipe.
- 8-32 The pump in the system shown discharges water into a wye connection and thus into the two pipes shown. The levels in the three reservoirs remain constant. With all regulating valves open, head losses in each of the three pipes can be expressed as  $H_L = KQ^2$ , where  $H_L$  is the head loss in feet,  $K$  is a constant of proportionality, and  $Q$  is in cfs. The magnitudes of the constants for the three lines are pipe *A*:  $K = 5$ ; pipe *B*:  $K = 20$ ; pipe *C*:  $K = 30$ . The pump has the following head-discharge characteristics:

$Q$ (cfs)	0	1	2	3	4
$H$ (ft)	200	198	182	145	75

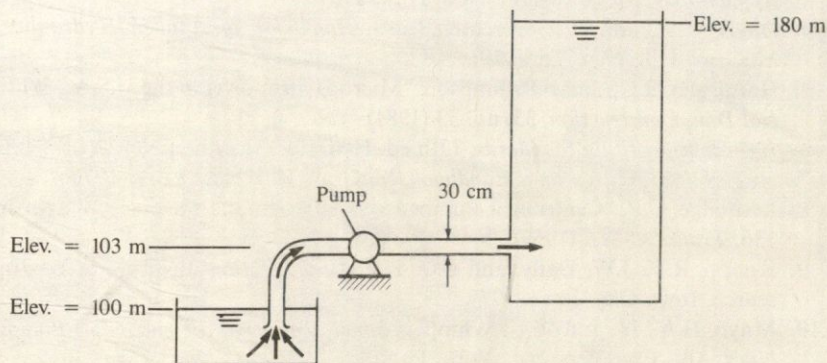
If all valves in the system are open, determine the discharge in each of the three pipes. What is the horsepower output of the pump?



PROBLEM 8-32

- 8-33 A penstock 1 m in diameter and 10 km long carries water from a reservoir to an impulse turbine. If the turbine is 83% efficient, what power can be produced by the system if the upstream reservoir elevation is 650 m above the turbine jet, and the jet diameter is 16.0 cm? Assume  $f = 0.016$ , and neglect head losses in the nozzle. What should the diameter of the turbine wheel be if it is to have an angular speed of 360 rpm? Assume ideal conditions for the bucket design ( $V_{\text{bucket}} = \frac{1}{2}V_j$ ).

- 8-34 Assume the characteristic curves shown in Fig. 8-8, page 442, are for a single-stage, single-suction centrifugal pump. This pump is to be installed in the system shown below. Do you think the pump will cavitate? Assume head loss in the system (from reservoir to reservoir) is negligible. Show computations to justify your answer.



PROBLEM 8-34

- 8-35 A Francis turbine is to be operated at a speed of 60 rpm and with a discharge of  $4.0 \text{ m}^3/\text{s}$ . If  $r_1 = 1.5 \text{ m}$ ,  $r_2 = 1.20 \text{ m}$ ,  $B = 30 \text{ cm}$ ,  $\beta_1 = 85^\circ$ , and  $\beta_2 = 165^\circ$ , what should  $\alpha_1$  be for nonseparating flow to occur through the runner? What power and torque should result with this operation?
- 8-36 A Francis turbine is to be operated at a speed of 120 rpm and with a discharge of  $113 \text{ m}^3/\text{s}$ . If  $r_1 = 2.5 \text{ m}$ ,  $B = 0.90 \text{ m}$ , and  $\beta_1 = 45^\circ$ , what should  $\alpha_1$  be for nonseparating flow at the runner inlet?
- 8-37 a. For a given Francis turbine,  $\beta_1 = 60^\circ$ ,  $\beta_2 = 90^\circ$ ,  $r_1 = 5 \text{ m}$ ,  $r_2 = 3 \text{ m}$ , and  $B = 1 \text{ m}$ . What should  $\alpha_1$  be for a nonseparating flow condition at the entrance to the runner when the discharge rate is  $126 \text{ m}^3/\text{s}$  and  $N = 60 \text{ rpm}$ ?
- b. What is the maximum attainable power with these conditions?
- c. If you were to redesign the turbine blades of the runner, what changes would you suggest to increase the power production if the discharge and overall dimensions are to be kept the same?
- 8-38 Select the type, speed, and size of turbine for a site where the net head is 600 ft and  $Q = 10 \text{ cfs}$ .
- 8-39 Select the type, speed, and size of turbine for a site where the net head is 200 ft and  $Q = 1000 \text{ cfs}$ .
- 8-40 Select the type, speed, and size of turbine for a site where the net head is 50 ft and  $Q = 3000 \text{ cfs}$ .

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