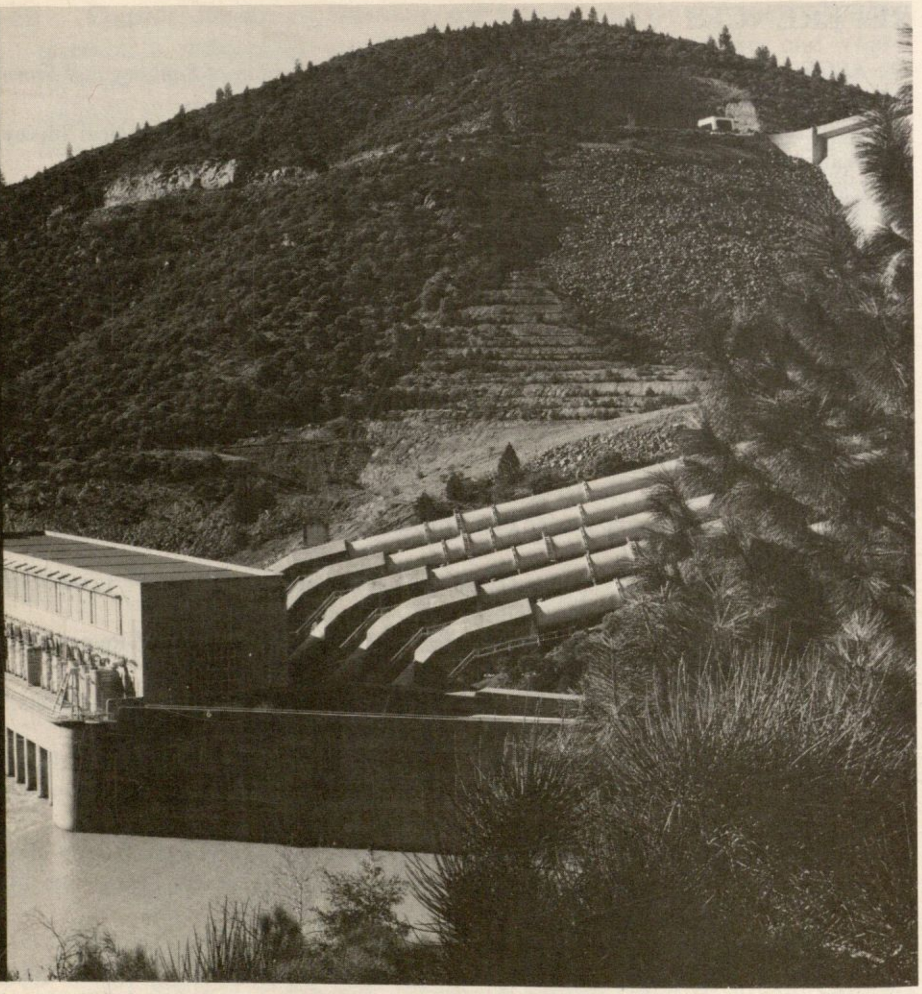
5



These five 15-ft diameter penstocks deliver water from the reservoir behind Shasta Dam to the powerhouse. At design flow the discharge in each penstock is 3,800 cfs and the head on each turbine is 380 feet yielding a power output of 140,000 horsepower for each turbine. The steel plate thickness of the penstocks is 2 inches, and the anchor blocks just upstream of the powerhouse each weigh over 2 million pounds. (Courtesy of the U.S. Bureau of Reclamation).

Closed Conduit Flow

5-1 General Considerations

In the design and operation of a pipeline, the main considerations are head losses, forces and stresses acting on the pipe material, and discharge. Head loss for a given discharge relates to flow efficiency; that is, an optimum size of pipe will yield the least overall cost of installation and operation for the desired discharge. Choosing a small pipe results in low initial costs; however, subsequent costs may be excessively large because of high energy cost from large head losses. Forces and stresses mainly result from fluid pressure in a pipe. Forces are also created by momentum change associated with flow around bends or through other types of pipe fittings.

The basic continuity, energy, and momentum equations of fluid mechanics are used in the solution of pipe-flow problems. For example, to design a pipe, you would use the continuity and energy equations to obtain the required pipe diameter. Then applying the momentum equation will yield the forces acting on bends for a given discharge. Applications of the aforementioned equations in the design and analysis of conduits are treated in this chapter. The energy equation is considered first.

5-2 Energy Equation

The initial design of a conduit involves determining the size of the conduit with the least cost for a required discharge. This cost includes first cost plus operating and maintenance costs. We only briefly discuss the economic aspect of conduit design. The hydraulic aspects of the problem require applying the one-dimensional steady flow form of the energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \tag{5-1}$$

where p/γ = pressure head
$\alpha V^2/2g$ = velocity head
z = elevation
h_p = head supplied by a pump
h_t = head supplied to a turbine
h_L = head loss between sections 1 and 2

A typical graphical representation of the terms of Eq. (5-1) is shown in Fig. 5-1. We give further explanation of the terms of Eq. (5-1) in the following paragraphs.

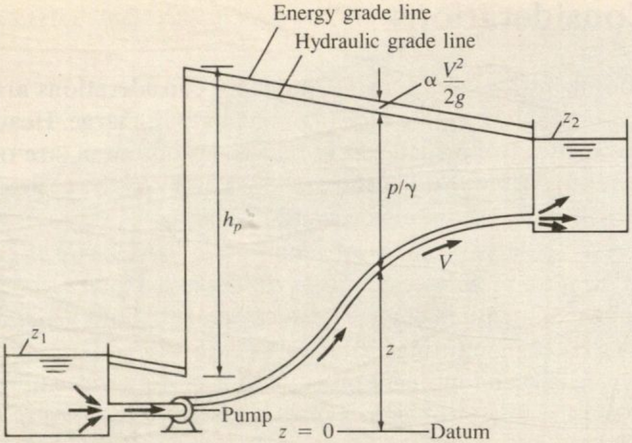


Figure 5-1 Definition sketch for terms in the energy equation

Velocity Head

In $\alpha V^2/2g$, the velocity V is the mean velocity in the conduit at a given section and is obtained by $V = Q/A$, where Q is the discharge, and A is the cross-sectional area of the conduit. The kinetic energy correction factor is given by α, and its definition is

$$\alpha = \frac{\int_A u^3\,dA}{V^3 A} \tag{5-2}$$

where u = velocity at any point in the section.

In Eq. (5-2), the integration is carried out over the cross section of the pipe. It can be shown that α has a minimum value of unity when the velocity is uniform across the section, and that it has values greater than unity depending on the degree of velocity variation across a section. It can also be shown that if laminar flow occurs in a pipe, the velocity distribution across the section will be parabolic, and α will have a value of 2.0 (36). However, if the flow is turbulent, as is the usual case for water flow in large conduits, the velocity is fairly uniform over most of the conduit section, and α has a value near unity (typically: $1.04 < \alpha < 1.06$). Therefore, in hydraulic engineering, for ease of application in pipe flow, the value of α is usually assumed to be unity, and the velocity head is then simply $V^2/2g$.

Pump or Turbine Head

The head supplied by a pump is directly related to the power supplied to the flow, as given in Eq. (5-3).

$$P = Q\gamma h_p \tag{5-3}$$

Likewise, if head is supplied to a turbine, the power supplied to the turbine will be $P = Q\gamma h_t$. In the preceding two equations, P refers to the power supplied directly to or taken directly from the flow. If you want to relate that to electrical or mechanical energy of the pump or turbine, you must include an efficiency factor. For example, the power that could be obtained from a turbine would be $P = eQ\gamma h_t$, where e is the efficiency of the turbine generator.

Head-Loss Term

The head-loss term h_L accounts for the conversion of mechanical energy to internal energy (heat). When this conversion occurs, the internal energy is not readily converted back to useful mechanical energy; therefore, it is called *head loss*. Head loss results from viscous resistance to flow (friction) at the conduit wall or from the viscous dissipation of turbulence usually occurring with separated flow, such as in bends, fittings, or outlet works.

5-3 Head Loss

Variables Affecting Head Loss

Head loss in a straight length of pipe is due to dissipation of energy caused by the resistance of the pipe wall. In the case of laminar flow, which generally occurs with the Reynolds number less than 2000, the head loss is all due to viscous resistance. The head loss is a function of the first power of the velocity. If the flow is turbulent, the head loss is related to the dissipation of the kinetic energy of turbulence, which produces a more complicated relationship between head loss and velocity. If the conduit is rough, still more variables involving the characteristics of roughness are needed to define the head loss.

Laminar-Flow Head Loss and Velocity Distribution

It can be shown that for steady-uniform flow in a pipe, the shear-stress distribution will vary linearly from a maximum at the pipe wall to zero at the

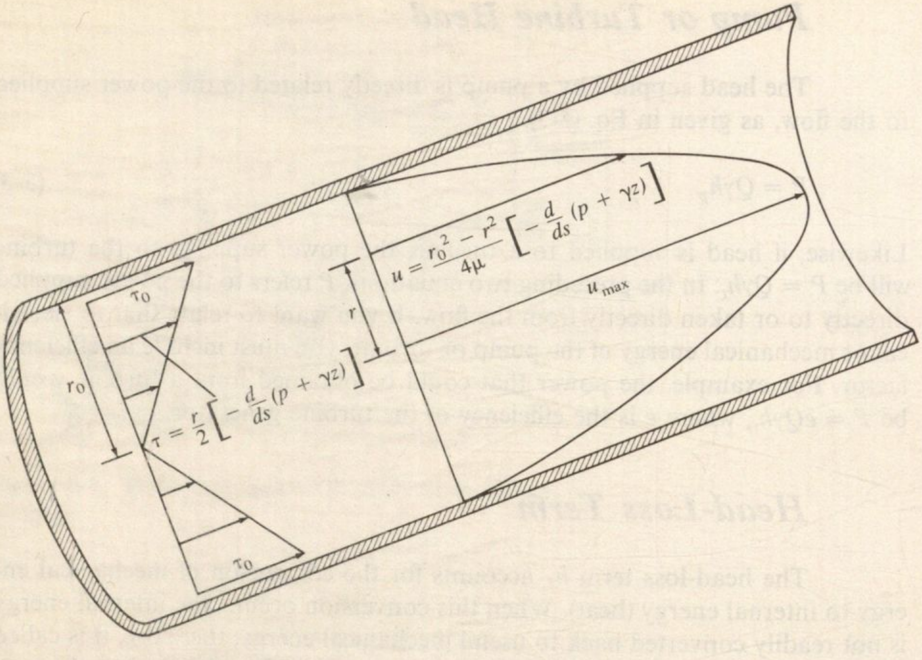


Figure 5-2 Distribution of shear stress and velocity for laminar flow in a pipe

center (36). If the flow is laminar, the velocity distribution will be parabolically distributed, as given by Eq. (5-4) and shown in Fig. 5-2.

$$u = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{5-4}$$

where r_0 is the pipe radius, and s is the coordinate axis parallel to the pipe axis and in the direction of flow.

By integrating the velocity across the section and using Eq. (5-1), it can be shown that the head loss for laminar flow is given by

$$h_L = \frac{32\mu LV}{\gamma D^2} \tag{5-5}$$

Turbulent-Flow Head Loss and Velocity Distribution

SMOOTH PIPES When the pipe flow-Reynolds number, $VD\rho/\mu$, is greater than about 3000, one can expect the flow to be turbulent, and in this case, the shear stress is primarily in the form of Reynolds stress, which varies linearly across the pipe section (increasing from zero at the center of the pipe) except near the pipe wall in the viscous sublayer, where the Reynolds stress decreases, and a true viscous shear stress takes over.

The velocity distribution takes different forms depending on the relative distance from the pipe wall. In the viscous sublayer, the velocity distribution is given by

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \qquad \text{for } 0 < \frac{u_* y}{\nu} < 5 \tag{5-6}$$

where u = velocity
y = distance from pipe wall
ν = kinematic viscosity
u_* = shear velocity = $\sqrt{\tau_0/\rho}$
τ_0 = shear stress at pipe wall

Immediately outside the viscous sublayer, the velocity distribution is of the logarithmic form

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.5 \qquad \text{for } 20 < \frac{u_* y}{\nu} \leqslant 10^5 \tag{5-7}$$

Figure 5-3 is a plot of Eqs. (5-6) and (5-7) as well as an indication of the spread of experimental data from various sources. It has also been found that

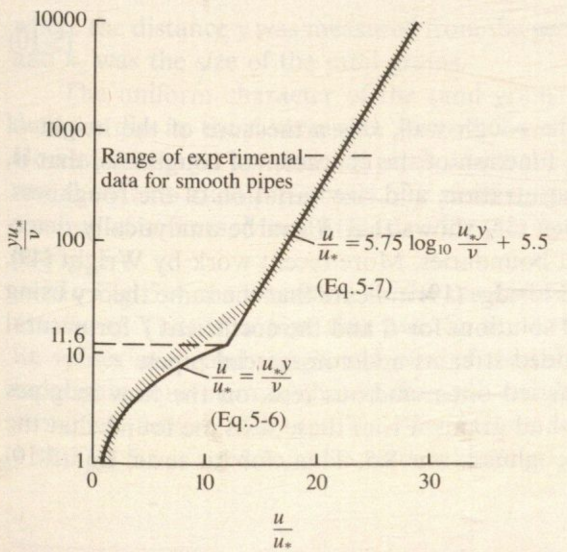


Figure 5-3 Velocity distribution for smooth pipes (37)

Table 5-1 Exponents for Power-Law Equation (37)

Re →	4×10^3	2.3×10^4	1.1×10^5	1.1×10^6	3.2×10^6
m →	$\frac{1}{6.0}$	$\frac{1}{6.6}$	$\frac{1}{7.0}$	$\frac{1}{8.8}$	$\frac{1}{10.0}$

the velocity distribution for turbulent flow can be approximated quite well with a power law formula. This is given as

$$\frac{u}{u_{\max}} = \left(\frac{y}{r_0}\right)^m \tag{5-8}$$

where $u_{\max}$ = velocity at the pipe center
r_0 = radius of pipe
m = exponent that varies with the Reynolds number, Re

The variation of m with Re is given in Table 5-1.

The head loss for turbulent flow in pipes is given by the Darcy-Weisbach formula as

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{5-9}$$

Rough Pipes Numerous tests on flow in rough pipes show that a semilogarithmic velocity distribution is valid over most of the pipe section (36, 37). This relationship is given as

$$\frac{u}{u_*} = 5.75 \log_{10}\frac{y}{k} + B \tag{5-10}$$

where y is the distance from the rough wall, k is a measure of the height of roughness elements, and B is a function of the character of roughness, that is, B is a function of the type, concentration, and size variation of the roughness. Research by Roberson and Chen (35) shows that B can be analytically determined for artificially roughened boundaries. More recent work by Wright (49), Calhoun (15), Kumar (25), and Eldridge (19) indicate that the same theory using a numerical approach will yield solutions for B and the coefficient f for natural roughness as found in rock-bedded streams and commercial pipes.

In 1933, Nikuradse (32) carried out numerous tests on the flow in pipes roughened with uniform-sized sand grains. From these tests, he found that the value for B with this kind of roughness was 8.5. Thus, for his tests, Eq. (5-10) becomes

$$\frac{u}{u_*} = 5.75 \log_{10}\frac{y}{k_s} + 8.5 \tag{5-11}$$

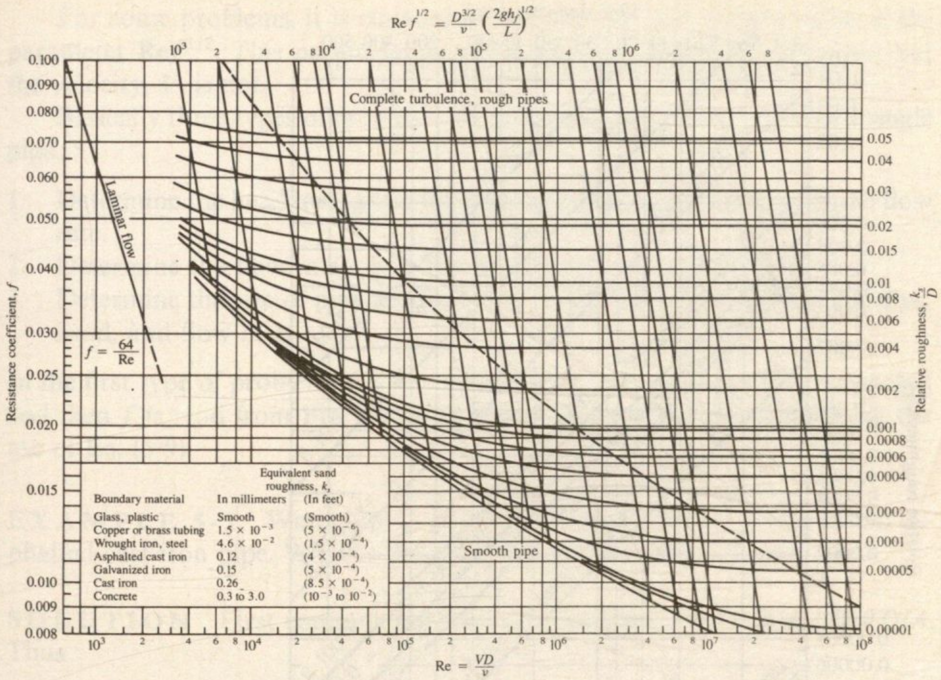


Figure 5-4 Resistance coefficient f versus Re (31)

where the distance y was measured from the geometric mean of the wall surface, and k_s was the size of the sand grains.

The uniform character of the sand grains used in Nikuradse's tests produces a dip in the f-versus-Re curve before reaching a constant value of f. However, tests on commercial pipes where the roughness is somewhat random reveal that no such dip occurs. By plotting data for commercial pipe from a number of sources, Moody (31) developed a design chart similar to that shown in Fig. 5-4.

In Fig. 5-4, the variable k_s is the symbol used to denote the *equivalent sand roughness.* That is, a pipe that has the same resistance characteristics at high Re values as a sand-roughened pipe of the same size is said to have a size of roughness equivalent to that of the sand-roughened pipe. Figure 5-4 gives approximate values of k_s and k_s/D for various kinds of pipe. This figure along with Fig. 5-5 is used to solve certain kinds of pipe-flow problems.*

* Besides the k_s values given in Fig. 5.5, see Sec. 5-9, where we give further information on k_s values for very large conduits.

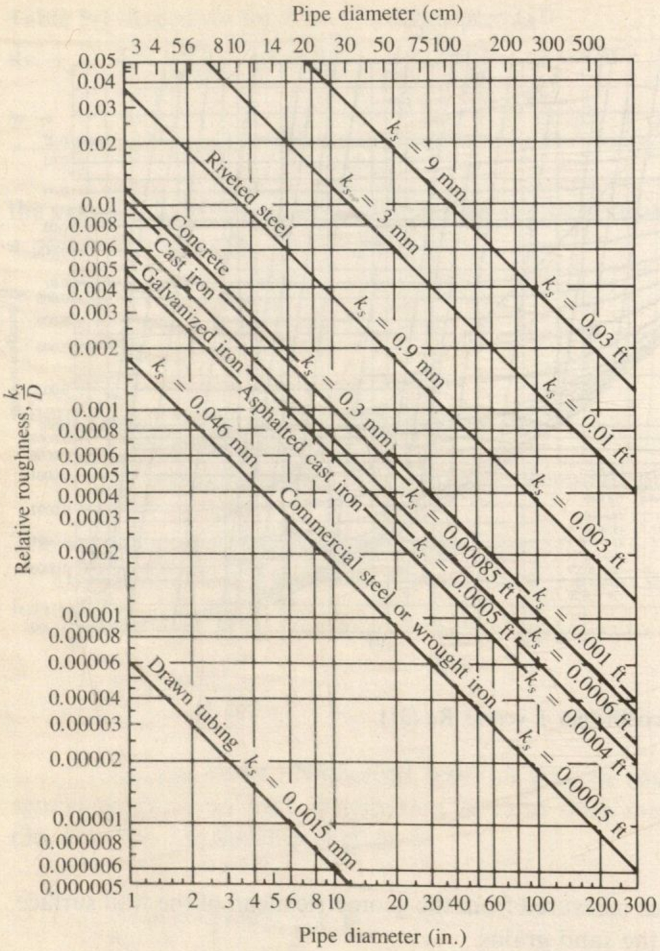


Figure 5-5 Relative roughness for various kinds of pipe (31)

In Fig. 5-4, the abscissa (labeled at the bottom) is the Reynolds number, Re, and the ordinate (labeled at the left) is the resistance coefficient f. Each solid curve is for a constant relative roughness, k_s/D, and the values of k_s/D are given on the right at the end of each curve. To find f, given Re and k_s/D, go to the right to find the correct relative-roughness curve; then look at the bottom of the chart to find the given value of Re and, with this value of Re, move vertically upward until you reach the given k_s/D curve. Finally, from this point, move horizontally to the left scale to read the value of f. If the curve for the given value of k_s/D is not plotted in Fig. 5-4, simply find the proper position on the graph by interpolation between curves of k_s/D, which bracket the given k_s/D.

For some problems, it is convenient to enter Fig. 5-4 using a value of the parameter $\mathrm{Re}f^{1/2}$. This parameter is useful when h_f and k_s/D are known but the velocity, V, is not.

Basically three types of problems are involved with uniform flow in a single pipe.

1. Determine the head loss, given the kind and size of pipe along with the flow rate.
2. Determine the flow rate, given the head, kind, and size of pipe.
3. Determine the size of pipe needed to carry the flow, given the kind of pipe, head, and flow rate.

In the first type of problem, the Reynolds number and k_s/D are first computed and then f is read from Fig. 5-4, after which the head loss is obtained by the use of Eq. (5-9).

EXAMPLE 5-1 Water, 20°C, flows at a rate of 0.05 m³/s in a 20-cm asphalted cast-iron pipe. What is the head loss per kilometer of pipe?

SOLUTION First compute the Reynolds number, VD/ν, where $V = Q/A$. Thus

$$V = \frac{0.05\ \mathrm{m^3/s}}{(\pi/4)(0.20^2\ \mathrm{m^2})} = 1.59\ \mathrm{m/s}$$

$$\nu = 1.0 \times 10^{-6}\ \mathrm{m^2/s}$$

Then $$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.59\ \mathrm{m/s})(0.20\ \mathrm{m})}{10^{-6}\ \mathrm{m^2/s}}$$

$$= 3.18 \times 10^5$$

From Fig. 5-5, $k_s/D = 0.0007$. Then from Fig. 5-4 on page 247, using the values obtained for k_s/D and Re, we find $f = 0.019$. Finally the head loss is computed from the Darcy-Weisbach equation:

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} = 0.019\left(\frac{1{,}000\ \mathrm{m}}{0.20\ \mathrm{m}}\right)\left(\frac{1.59^2\ \mathrm{m^2/s^2}}{2(9.81\ \mathrm{m/s^2})}\right)$$

$$= 12.2\ \mathrm{m}$$

The head loss per kilometer is 12.2 m. ■

In the second type of problem, if you know the value of h_f, then k_s/D and the value of $(D^{3/2}/\nu)\sqrt{2gh_f/L}$ can be computed so that the top scale can be used to enter the design chart of Fig. 5-4. The lines that slope down and to the right are lines of equal value of the parameter $(D^{3/2}/\nu)\sqrt{2gh_f/L}$. Then, once f is read

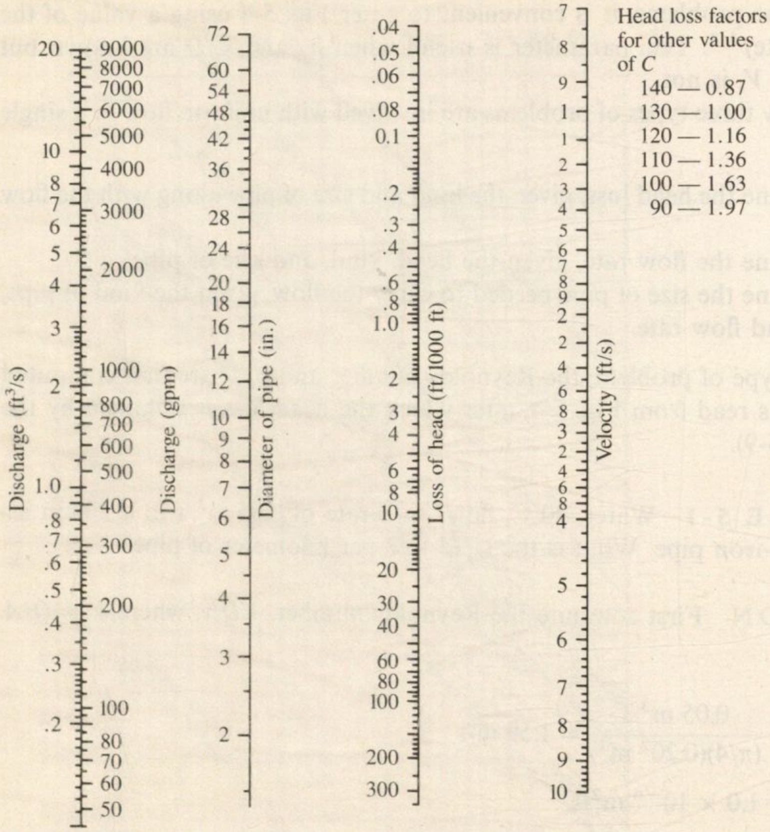


Figure 5-6 Nomograph for solving the Hazen-Williams formula with $C_h = 130$ (5)

from the chart, the velocity from Eq. (5-9) is solved and the discharge is computed from $Q = VA$. This procedure yields a direct solution for Q.

However, many problems for which the discharge Q is desired cannot be solved directly. For example, a problem in which water flows from a reservoir through a pipe and into the atmosphere is of this type. Here, part of the available head is lost to friction in the pipe, and part of the head remains in kinetic energy in the jet as it leaves the pipe. Therefore, at the outset, one does not know how much head loss occurs in the pipe itself. To effect a solution, you must iterate on f. The energy equation is written and an initial value for f is guessed; then the velocity, V, is solved. With this value of V, a Reynolds number is computed that allows a better value of f to be determined through the use of Fig. 5-4 and so on. This type of solution usually converges quite rapidly because f changes more slowly than Re.

In the third type of problem, it is usually best to first assume a value of f and then solve for D, after which a better value of f is computed based on the first estimate of D. This iterative procedure is continued until a valid solution is obtained. A trial-and-error procedure is necessary because without D you cannot compute k_s/D or Re to enter the Moody diagram.

EXAMPLE 5-2 What size asphalted cast-iron pipe is needed to carry, water at a discharge of 12 cfs and with a head loss of 4 ft per 1000 ft of pipe?

SOLUTION First assume $f = 0.015$. Then

$$h_f = \frac{fL}{D} \cdot \frac{V^2}{2g} = \frac{fL}{D} \cdot \frac{Q^2/A^2}{2g} = \frac{fLQ^2}{2g(\pi/4)^2 D^5}$$

or $$D^5 = \frac{fLQ^2}{0.785^2(2gh_f)}$$

or, for this example,

$$D^5 = \frac{0.015(1000\text{ ft})(12\text{ ft}^3/\text{s})^2}{0.615(64.4\text{ ft/s}^2)(4\text{ ft})} = 13.63\text{ ft}^5$$

$$D = 1.69\text{ ft} = 20.3\text{ in.}$$

Now compute a more accurate value of f:

$$\frac{k_s}{D} = 0.00025 \qquad V = \frac{Q}{A} = \frac{12\text{ ft}^3/\text{s}}{0.785(2.86\text{ ft}^2)} = 5.34\text{ ft/s}$$

Then $$\text{Re} = \frac{VD}{\nu} = \frac{5.34\text{ ft/s}(1.69\text{ ft})}{1.21(10^{-5}\text{ ft}^2/\text{s})} = 7.47 \times 10^5$$

From Fig. 5-5, $f = 0.0155$. Now recompute D by applying the ratio of f's to previous calculations for D^5:

$$D^5 = \frac{0.0155}{0.015}(13.63\text{ ft}^5) = 14.08\text{ ft}^5$$

$$D = 1.70\text{ ft} = 20.4\text{ in.}$$

Use a 22-in. diameter pipe.

Note: In actual design practice, if a nonstandard size of pipe is called for as a result of the design calculation, it is customary to choose the next standard size larger that is available commercially. By doing so, the cost is less than that for an odd-sized pipe, and the pipe will be more than large enough to carry the flow. In this case, the 22-in. pipe is the next standard size larger. ■

Head Loss Using the Hazen-Williams Formula

The head loss formulas we have presented up to now are general because they are applicable for any fluid and any system of units. Other more restrictive empirical equations are also useful for their limited range of application. The most notable one, used for decades by waterworks engineers in the United States, is the Hazen-Williams formula. In English units, the formula is given in Eq. (5-12):

$$V = 1.318C_h R^{0.63} S^{0.54} \tag{5-12}$$

where V = mean velocity in ft/s
C_h = Hazen-Williams friction coefficient (depends on pipe roughness)
R = hydraulic radius in ft
$S = h_f/L$ (slope of energy grade line)

To solve for head loss using the Hazen-Williams equation, a little algebraic manipulation of Eq. (5-12) yields

$$h_f = 3.02LD^{-1.167}\left(\frac{V}{C_h}\right)^{1.85} \tag{5-13}$$

The resistance coefficient C_h depends on the surface characteristics of the pipe

Table 5-2 Hazen-Williams C_h Values for Different Kinds of Pipe (5)

Character of Pipe	C_h
New or in excellent condition cast-iron and steel pipe with cement or bituminous linings centrifugally applied, concrete pipe centrifugally spun, cement-asbestos pipe, copper tubing, brass pipe, plastic pipe, and glass pipe	140
Older pipe listed above in good condition, and cement mortar-lined pipes in place with good workmanship, larger than 24 in. in diameter	130
Cement mortar-lined pipe in place, small diameter with good workmanship or large diameter with ordinary workmanship; wood stave; tar dipped cast-iron pipe new or old in inactive water	120
Old unlined or tar-dipped cast-iron pipe in good condition	100
Old cast-iron pipe severely tuberculated, or any pipe with heavy deposits	10–80

wall. Representative values of C_h for various kinds and conditions of pipe are given in Table 5-2.

Because of the widespread use of the Hazen-Williams formula, charts and tables have been developed for easy solution of the formula. One of these charts, in nomograph form, is shown in Fig. 5-6.

The Hazen-Williams formula and several other empirical head loss formulas are applicable only for water flow and for the usual range of pipe sizes and discharges found in water distribution systems. Therefore, the Hazen-Williams formula may yield erroneous results for fluids other than water and for pipe diameters smaller than 2 in. and larger than 6 ft.

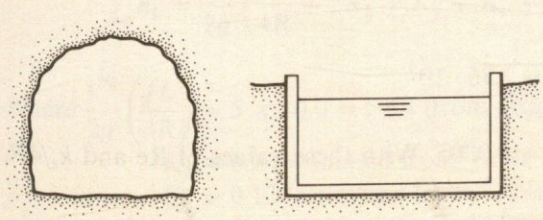
Head Loss in Noncircular Conduits

TYPES OF NONCIRCULAR CONDUITS One type of noncircular closed conduit commonly used in water resources projects is the tunnel. The tunnel cross section is typically rounded at the top and flat on the bottom, a horseshoe shape, as shown in Fig. 5-7*a*. Another noncircular cross section often used in hydraulic engineering is the *rectangular section*. The rectangular conduit may be used as a closed conduit; however, it is most often used as an open channel, as shown in Fig. 5-7*b*. In either case, the method for calculating head loss in these noncircular conduits is the same.

HYDRAULIC RADIUS CONCEPT If it is assumed that the wall shear stress, τ_0, is uniformly distributed around the part of the perimeter of the conduit in contact with the flowing liquid (called the *wetted perimeter*), it can be shown that the Darcy-Weisbach equation has the following form (36):

$$h_f = \frac{f}{4} \cdot \frac{L}{A/P} \cdot \frac{V^2}{2g} \tag{5-14}$$

where A = cross-sectional area of flow section
P = wetted perimeter



(*a*) Tunnel cross section (*b*) Rectangular cross section

Figure 5-7 Noncircular conduits

The ratio A/P is called the *hydraulic radius R*; therefore, the Darcy-Weisbach equation reduces to

$$h_f = \frac{fL}{4R} \cdot \frac{V^2}{2g} \tag{5-15}$$

Equation (5-15) is the same as the original form of the equation, Eq. (5-9), except that the diameter, D, is replaced by $4R$. Experimental evidence shows that we can solve for the head loss in noncircular conduits if we apply the same methods and equations that we used for pipes but use $4R$ in place of D. Thus the relative roughness is $k_s/4R$, and the Reynolds number is defined as $V(4R)/\nu$.

EXAMPLE 5-3 A concrete-lined tunnel has a cross section described as follows. The top part of the tunnel is a 20-ft diameter semicircle, and the bottom part is a rectangular section 20 ft wide by 10 ft high. Estimate the head loss in 1-mi length of tunnel when water is flowing in it with a mean velocity of 12 ft/s.

SOLUTION The head loss is given as $h_f = f(L/4R)V^2/2g$, where f is a function of Re and $k_s/4R$. First solve for the hydraulic radius R:

$$R = \frac{A}{P}$$

$$= \frac{(\pi 10^2/2) + (20 \cdot 10)}{20 + 2 \cdot 10 + \pi \cdot 10}$$

$$= \frac{357}{71.4} = 5.00 \text{ ft}$$

Next solve for the Reynolds number:

$$\text{Re} = V \cdot \frac{4R}{\nu}$$

Assume the water temperature is 60°F, so $\nu = 1.22 \cdot 10^{-5}$ ft²/s (from Table A-4, page 648).

Then $$\text{Re} = 12 \cdot 4 \cdot \frac{5.00}{1.22 \cdot 10^{-5}} = 1.96 \cdot 10^7$$

Assume $k_s = 0.01$ ft. Then $k_s/4R = 0.0005$. With these values of Re and $k_s/4R$, we obtain $f = 0.017$.

The head loss is then computed:

$$h_f = 0.017 \cdot \frac{5280}{(4 \times 5.0)} \cdot \frac{12^2}{(2 \times 32.2)} = 10.0 \text{ ft/mi}$$ ■

Head Loss Due to Transitions and Fittings

Besides the head loss due to the conduit itself, other losses are caused by the inlet, outlet, bends, and other appurtenances that alter the uniform flow regime in the conduit. Physically, all these head losses occur because additional turbulence is created by the particular conduit fitting, and the energy associated with the turbulence is finally dissipated into heat that produces the head loss. The head loss produced by transitions and fittings is expressed as

$$h_L = K\frac{V^2}{2g} \tag{5-16}$$

where V is the mean velocity in the conduit, and K is the loss coefficient for the particular fitting involved. Table 5-3 on the next page gives the loss coefficients, determined by experimentation, for various transitions and fittings.

EXAMPLE 5-4 The conduit of Example 5-3 is used to convey water from a reservoir (water surface elevation 5000 ft) through hydroturbines and from there to another reservoir (water surface elevation 3000 ft). The tunnel is 5 mi long and has two long-radius 45° bends in it plus two wide-open gate valves and well-designed inlets and outlets. What power can be delivered to the turbines if we assume the flow passages associated with the turbines themselves have a loss coefficient of 0.20? Further assume the water velocity in the tunnel is the same as in Example 5-3 and that the head loss for the gate valve is negligible.

SOLUTION The energy equation is first written with $\alpha = 1$:

$$\frac{p_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2 + h_t + \sum h_L$$

In this example, let point 1 be at the upper reservoir water surface, and let point 2 be at the lower reservoir water surface. The head loss will be given as

$$\sum h_L = \frac{V^2}{2g}\left(\frac{fL}{4R} + 2K_b + K_e + K_o + 0.2\right)$$

where $\dfrac{V^2}{2g}\left(\dfrac{fL}{4R}\right) = 5 \times 10.0 = 50$ ft (from Example 5-3)

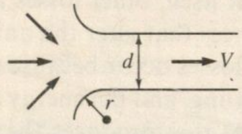
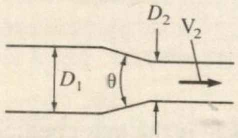
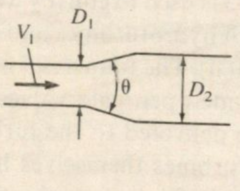
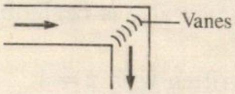
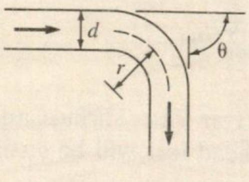
$K_b \approx 0.1$ (estimated from Table 5-3)

$K_e = 0.12$ (estimated from Table 5-3)

$K_{\text{outlet}} = K_E = 0.15$ (estimated from Table 5-3, assuming $\theta = 10°$)

Then $$\sum h_L = \frac{12^2}{2 \times 32.2}(2 \times 0.1 + 0.12 + 0.15 + 0.2) + 50.0 = 51.5 \text{ ft}$$

Table 5-3 Loss Coefficients for Various Transitions and Fittings

Description	Sketch	Additional Data	K		Source
Pipe entrance		r/d	K_e		(7)
		0.0	0.50		
		0.1	0.12		
$h_L = K_e V^2/2g$		>0.2	0.03		
Contraction		D_2/D_1	K_C $\theta = 60°$	K_C $\theta = 180°$	(7)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.32	
		0.80	0.05	0.18	
$h_L = K_C {V_2}^2/2g$		0.90	0.04	0.10	
Expansion		D_1/D_2	K_E $\theta = 10°$	K_E $\theta = 180°$	(7)
		0.0		1.00	
		0.20	0.13	0.92	
		0.40	0.11	0.72	
		0.60	0.06	0.42	
$h_L = K_E {V_1}^2/2g$		0.80	0.03	0.16	
90° miter bend		Without vanes	$K_b = 1.1$		(42)
		With vanes	$K_b = 0.2$		(42)
Smooth bend		r/d	K_b $\theta = 45°$	K_b $\theta = 90°$	(14)
		1	0.10	0.35	(22)
		2	0.09	0.19	and
		4	0.10	0.16	(30)
		6	0.12	0.21	
Threaded pipe fittings	Globe valve—wide open		$K_v = 10.0$		
	Angle valve—wide open		$K_v = 5.0$		
	Gate valve—wide open		$K_v = 0.2$		
	Gate valve—half open		$K_v = 5.6$		
	Return bend		$K_b = 2.2$		
	Tee		$K_t = 1.8$		
	90° elbow		$K_b = 0.9$		
	45° elbow		$K_b = 0.4$		

The head given up to the turbines h_t is then

$$h_t = 5000 - 3000 - 51.5 = 1948.5 \text{ ft}$$

Finally $$P = \frac{Q\gamma h_t}{550}$$

$$= 12 \cdot \left[\left(\pi \cdot \frac{10^2}{2}\right) + (20 \cdot 10)\right] \cdot 62.4 \cdot \frac{1948.5}{550}$$

$$= 947{,}000 \text{ hp}$$ ■

Explicit Equations for h_f, Q, and D

On pages 249 to 251, we presented methods by which h_f, Q, and D can be calculated. All these methods involve using the Moody diagram (Fig. 5-4). With the advent of computers and programmable calculators, it is most desirable to be able to solve similar problems without having to resort to the Moody diagram. By using the Colebrook-White formula, from which the Moody diagram was developed, Swamee and Jain (44) developed explicit formulas relating f, h_f, Q, and D. It is reported that their formulas yield results that deviate no more than 3% from those obtained from the Moody diagram for the following ranges of k_s/D and Re: $10^{-5} < k_s/D < 2 \times 10^{-2}$ and $4 \times 10^3 < \text{Re} < 10^8$. The formulas for f and Q are

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \tag{5-17}$$

$$Q = -2.22D^{5/2}\sqrt{gh_f/L}\,\log\left(\frac{k_s}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_f/L}}\right) \tag{5-18}$$

They also developed a formula for the explicit determination of D. A modified version of that formula, given by Streeter and Wylie (41), is

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \tag{5-19}$$

If you want to solve for head loss given Q, L, D, k_s, and ν, simply solve for f by Eq. (5-17) and compute h_f with the Darcy-Weisbach equation, Eq. (5-9). Straightforward calculations for Q and D can also be made if h_f is known. However, for problems involving head losses in addition to h_f, an iterative solution is required. For computing Q, you can assume an f and solve for Q from the energy equation after substituting Q/A in that equation. Then compute

Re and use that in Eq. (5-17) to get a better estimate of f and so on, until Q converges analogous to the procedure for determining Q using the Moody diagram. In this case, however, Eq. (5-17) is substituted for the Moody diagram. Similarly, D can be determined given Q, v, the change in pressure or head, and the geometric configuration.

EXAMPLE 5-5 Determine the diameter of steel pipe needed to deliver water (20°C) at a rate of 2 m^3/s from a reservoir with water surface at an elevation of 60 m to a reservoir 200 m away with a water surface elevation of 30 m. Assume a square-edged inlet and outlet and no bends in the pipe. Further assume there are two open gate valves in the pipe.

SOLUTION Writing the energy equation from the upper to lower reservoir, we have

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + \sum h_L$$

$$0 + 0 + z_1 = 0 + 0 + z_2 + \left(k_e + 2k_v + k_E + \frac{fL}{D}\right)\frac{V^2}{2g}$$

$$0 = z_2 - z_1 + \left(k_e + 2k_v + k_E + \frac{fL}{D}\right)\frac{Q^2}{2gA^2}$$

Assume $k_s = 0.046$ mm, $k_e = 0.5$, $k_v = 0.2$, $k_E = 1.0$, $f = 0.02$, and let $A = \pi D^2/4$. Then

$$0 = 30 - 60 + \frac{[1.9 + (0.02 \times 200/D)]}{2^2/[2g(\pi^2/16)D^4]}$$

Solving this for D yields $D = 0.56$ m, $V = 8.12$ m/s, and $\text{Re} \approx 4.5 \times 10^6$. Then from Eq. (5-17), we have

$$f = 0.25/[\log(0.000046/3.7 \times 0.56) + (5.74/9.82 \times 10^5)]^2$$

$$f = 0.0092 \qquad \text{Re} = 4.5 \times 10^6$$

Substituting this value of f back into the energy equation and solving for a better value of D yields $D = 0.52$ m. Another iteration still yields

$$D = 0.52 \text{ m}$$ ■

In Example 5-5, we used the same iterative procedure introduced in Example 5-3 except that we replaced the Moody diagram by Eq. (5-17) and included the energy equation because head losses other than pipe friction losses were significant. Another more rapid iterative solution, presented by Streeter and Wylie (41), for D uses Eq. (5-18) when other than pipe friction losses are present. These are expressed as an equivalent length of pipe, and h_f is the total difference in head between the sections under consideration. Thus in Example 5-5, we would obtain the equivalent length of pipe for the minor losses as

$$f\left(\frac{L_e}{D}\right)\left(\frac{V^2}{2g}\right) = 1.9\,\frac{V^2}{2g}$$

where L_e = equivalent pipe length = $1.9D/f$. Then

$$L = L_{\text{pipe}} + L_e = L + 1.9D/f$$

You could then solve Eq. (5-19) by iteration. That is, first assume f, then solve for D, after which a better value of f is obtained from Eq. (5-19), and so on.

EXAMPLE 5-6 Solve Example 5-5 using Eq. (5-19).

SOLUTION First assume $f = 0.02$.

Then $L = L + L_e = 200\text{ m} + \dfrac{1.9\,D}{0.02}$

$$= 200 + 95D$$

Letting $h_f = 30$ m and $L = 200 + 95\,D$ in Eq. (5-19) and solving for D yields

$$D = 0.66\left[0.000046\left(\frac{(200 + 95D) \times 2^2}{9.81 \times 30}\right)^{4.75} + 10^{-6} \times 2^{9.4}\left(\frac{200 + 95D}{9.81 \times 30}\right)^{5.2}\right]^{0.04} = 0.509\text{ m}$$

First iteration:

With $D = 0.51$ m, Re $= 5.00 \times 10^6$, $f = 0.0122$ (from Eq. 5-17)

$L_e = 1.9D/0.0122 = 155.7D \qquad D = 0.509$ m

$L = 200 + 156D \qquad D = 0.506$ m

Second iteration:

With $D = 0.506$ m, Re $= 5 \times 10^6$, $f = 0.0122$

$L_e = 156D$

Since there is no significant change in either f or L_e, the diameter will be the same: $D = 0.51$ m.

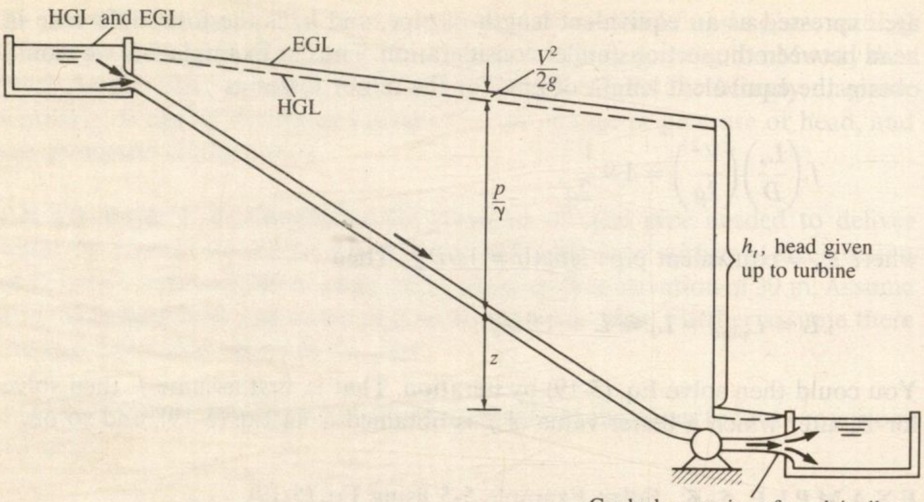


Figure 5-8 Drop in EGL and HGL due to turbine

Hydraulic and Energy Grade Lines

As we noted, the terms of Eq. (5-1) have linear dimension (feet or meters); thus we can attach a useful physical relationship to them, as shown in Fig. 5-1, page 242. If you were to tap a piezometer into the pipe of Fig. 5-1, the liquid would rise to a height p/γ above the pipe; hence the reason for the name *hydraulic grade line* (*HGL*). The total head $(p/\gamma + V^2/2g + z)$ in the system is greater than $p/\gamma + z$ by an amount $\alpha V^2/2g$; thus the *energy grade line* (*EGL*) is above the HGL a distance $\alpha V^2/2g$. The engineer who develops a visual concept of the energy equation as we explained earlier will find it much easier to sense trouble spots (usually points of low pressure) in the system.

Some hints for drawing hydraulic grade lines and energy grade lines are as follows:

1. By definition, the EGL is positioned above the HGL an amount equal to the velocity head. Thus if the velocity is zero, as in a lake or reservoir, the HGL and EGL will coincide with the liquid surface (see Fig. 5-8).
2. Head loss for flow in a pipe or channel always means the EGL will slope downward in the direction of flow. The only exception to this rule occurs when a pump supplies energy (and pressure) to the flow. Then an abrupt rise in the EGL occurs from the upstream side to the downstream side of the pump.

3. In point 2, we noted that a pump can cause an abrupt rise in the EGL because energy is introduced into the flow by the pump. Similarly, if energy is abruptly taken out of the flow by, for example, a turbine, the EGL and HGL will drop abruptly as in Fig. 5-8. Figure 5-8 also shows that much of the velocity head can be converted to pressure head if there is a gradual expansion such as at the outlet. Thus the head loss at the outlet is reduced, making the turbine installation more efficient. If the outlet to a reservoir is an abrupt expansion, all the kinetic energy is lost; thus the EGL will drop an amount $\alpha V^2/2g$ at the outlet.
4. In a pipe or channel where the pressure is zero, the HGL is coincident with the water in the system because $p/\gamma = 0$ at these points. This fact can be used to locate the HGL at certain points in the physical system, such as at the outlet end of a pipe, where the liquid discharges into the atmosphere, or at the upstream end, where the pressure is zero in the reservoir (see Fig. 5-8).
5. For steady flow in a pipe that has uniform physical characteristics (diameter, roughness, shape, and so on) along its length, the head loss per unit of length will be constant; thus the slope $(\Delta h_L/\Delta L)$ of the EGL and HGL will be constant along the length of pipe.
6. If a flow passage changes diameter, such as in a nozzle or a change in pipe size, the velocity therein will also change; hence the distance between the EGL and HGL will change. Moreover, the slope on the EGL will change because the head loss per length will be larger in the conduit with the larger velocity.
7. If the HGL falls below the pipe, p/γ is negative, thereby indicating subatmospheric pressure (see Fig. 5-9). If the pressure head of water is less than the vapor pressure head of the water (approximately -33 ft at standard atmospheric pressure and $T = 60°\text{F}$), cavitation will occur. Generally, cavitation in conduits is undesirable. It increases the head loss and can cause structural damage to the conduit from excessive vibration and pitting of the conduit walls. If the pressure at a section in the pipe decreases to the vapor pressure and stays that low, a large vapor cavity can form leaving a gap of water vapor with columns of water on either side of the cavity. As the cavity

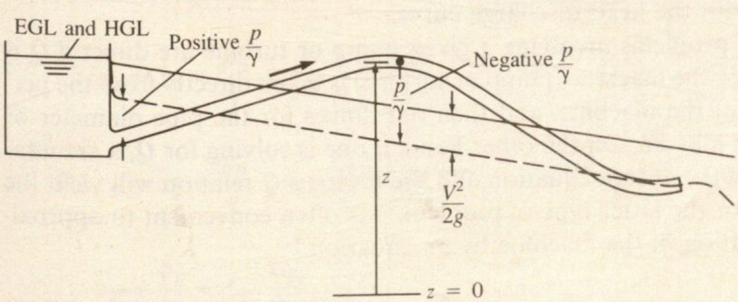


Figure 5-9 Subatmospheric pressure when pipe is above the HGL

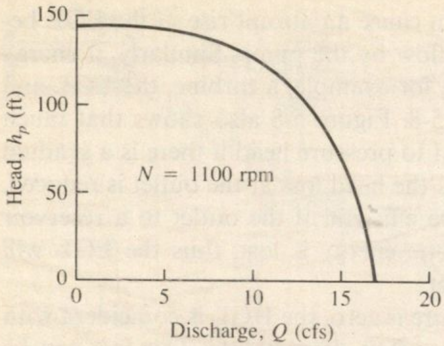


Figure 5-10 Typical performance curve for a centrifugal pump

grows in size, the columns of water move away from each other. Often these columns of water will rejoin later, and when they do, a very high dynamic pressure (water hammer) can be generated, possibly rupturing the pipe. Furthermore, if the pipe is relatively thin walled, such as thin-walled steel pipe, subatmospheric pressure can cause the pipe wall to collapse. Therefore, the design engineer should be extremely cautious about negative pressure heads in the pipe.*

5-4 Head-Discharge Relations for Pump or Turbine

In the energy equation, Eq. (5-1), the terms h_p and h_t are the heads supplied by a pump or given up to a turbine, respectively, and these heads are a function of the discharge for a machine (pump or turbine) that is operating at a given speed. Figure 5-10 is a typical plot of h_p versus Q for a centrifugal pump. Such a plot is one of the *performance* or *characteristic* curves of the machine. Other performance curves, such as efficiency and power versus discharge, are often included with the head discharge curve.

Solutions of problems involving a given pump or turbine are direct if Q is given; the head for the machine (pump or turbine) is taken directly from the performance curve of the machine, and then one solves for the pipe diameter or head, as the case may be. On the other hand, if one is solving for Q, a simultaneous solution of the energy equation and the h versus Q relation will yield the desired result. For the latter type of problem, it is often convenient to approximate the hQ relation of the machine by an equation.†

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* For a more detailed description of water hammer with methods of analyses, see Chapter 11.

† For more detail about performance curves, pumps, and turbines, see Chapter 8.

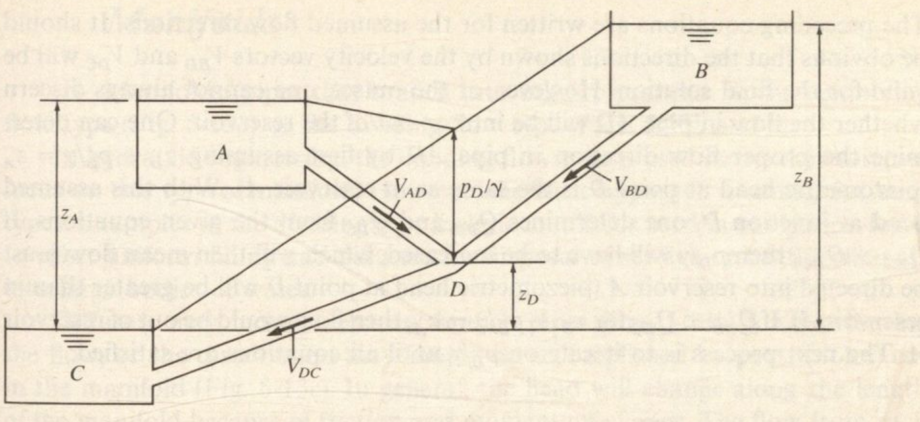


Figure 5-11 Flow in a branched-pipe system

5-5 Conduit Systems

So far, we have considered only problems involving pipes in series. Other applications include branching pipes, parallel pipes, manifolds, and pipe networks.

Branching Pipes

Consider the case shown in Fig. 5-11, where three reservoirs are connected by a branched-pipe system. The problem is to determine the discharge in each pipe and the head at the junction point D. There are four unknowns (V_{AD}, V_{BD}, V_{DC}, and p_D/γ), and a solution is obtained by solving the energy equations for the pipes (neglecting velocity heads and including only pipe losses) and the continuity equation. These equations are given as

$$z_A = z_D + \frac{p_D}{\gamma} + f_{AD}\frac{L_{AD}}{D_{AD}}\frac{V_{AD}^2}{2g} \tag{5-20}$$

$$z_B = z_D + \frac{p_D}{\gamma} + f_{BD}\frac{L_{BD}}{D_{BD}}\frac{V_{BD}^2}{2g} \tag{5-21}$$

$$z_D + \frac{p_D}{\gamma} = f_{DC}\frac{L_{DC}}{D_{DC}}\frac{V_{DC}^2}{2g} \tag{5-22}$$

$$V_{AD}A_{AD} + V_{BD}A_{BD} = V_{DC}A_{DC} \tag{5-23}$$

The preceding equations are written for the assumed flow directions. It should be obvious that the directions shown by the velocity vectors V_{BD} and V_{DC} will be valid for the final solution. However, at the outset, one cannot always discern whether the flow in pipe AD will be into or out of the reservoir. One can determine the proper flow direction in pipe AD by first assuming $z_D + p_D/\gamma = z_A$ (piezometric head at point D is the same as in reservoir A). With this assumed head at junction D, one determines Q_{BD} and Q_{DC} from the given equations. If $Q_{BD} > Q_{DC}$, then p_D/γ will have to be increased, which will then mean flow must be directed into reservoir A (piezometric head at point D will be greater than in reservoir A). If $Q_{BD} < Q_{DC}$ for $z_D + p_D/\gamma = z_A$, then flow would be out of reservoir A. The next process is to iterate on p_D/γ until all equations are satisfied.

Parallel Pipes

Consider a pipe that branches into two parallel pipes and then rejoins, as in Fig. 5-12. A problem involving this configuration might be to determine the division of flow in each pipe given the total flow rate. It can be seen that the head loss must be the same in each pipe because the pressure difference is the same. Thus we can write

$$h_{L1} = h_{L2}$$

$$f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Then
$$\left(\frac{V_1}{V_2}\right)^2 = \frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}$$

or
$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2}\right)^{1/2}$$

If f_1 and f_2 are known, the division of flow can be easily determined; however, some trial-and-error analysis may be required if f_1 and f_2 are in the range where they are functions of the Reynolds number.

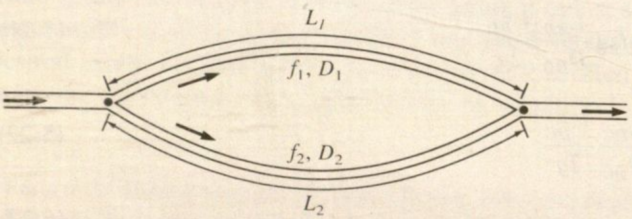


Figure 5-12 Flow in parallel pipes

Manifolds

Manifolds (pipes that branch into other pipes) can be the combining flow type (Fig. 5-13*a*) or the dividing flow type (Figs. 5-13*b* and *c*).

In hydraulic engineering, the dividing flow type is used more; therefore, we focus on this type of manifold. However, the basic approach is similar for both types. Examples of the dividing type include manifolds used in navigation locks to effect uniform filling and diffusers for disposal of sewage or heated effluents into large bodies of water.

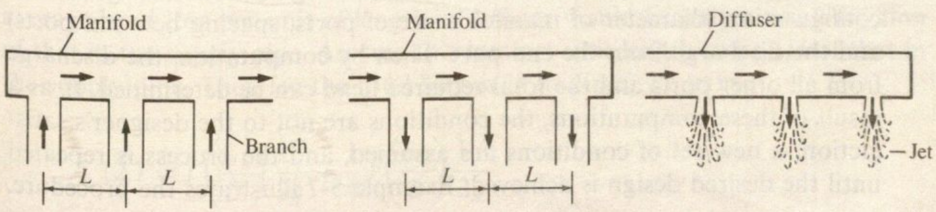
Diffusers may include separate branch pipes (Fig. 5-13*b*) for distributing the flow, or the distribution may simply be accomplished by ports (holes) cut in the manifold (Fig. 5-13*c*). In general, the head will change along the length of the manifold because of friction and momentum change. The flow from each port or branch pipe also is affected by the magnitude of the velocity in the diffuser. Thus the discharge from each branch will in general be different depending on its location along the manifold. This assumes all branches are the same size.

In most manifold design problems, the objective is to distribute a given discharge fairly uniformly along the length of the manifold. The method of analysis and design of manifolds follows essentially the procedure given by Rawn (34). The basic assumptions and fluid mechanics principles involved in a manifold design problem are as follows:

1. The discharge from each port or branch pipe can be expressed as

$$q = Ka\sqrt{2gE} \tag{5-24}$$

where K = flow coefficient
a = cross-sectional area of branch pipe or port
$E = V^2/2g + \Delta h$
V = mean velocity in manifold
$\Delta h = [(p_m/\gamma) + z_m] - [(p_o/\gamma) + z_o]$, and m and o are subscripts that refer to conditions inside and outside the manifold, respectively, at the section where the branch or port is located.



(*a*) Combining flow (*b*) Dividing flow types (*c*)

Figure 5-13 Flow manifolds

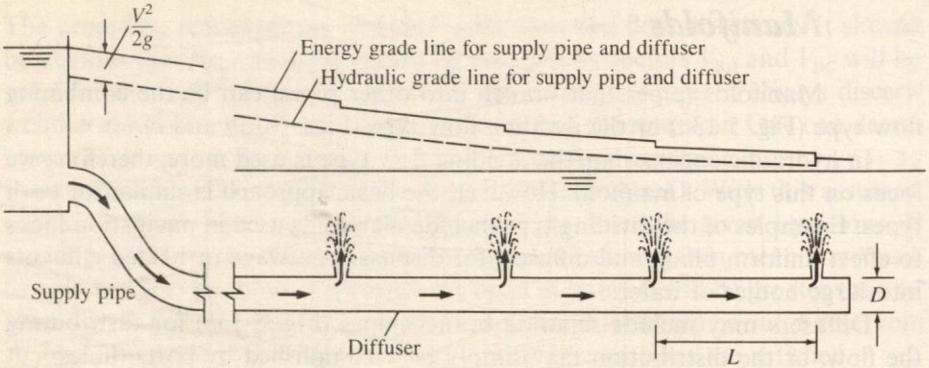


Figure 5-14 Schematic view of a diffuser

Thus Eq. (5-24) is a discharge equation for the branch pipe or port, and E is the difference in head between the manifold and the ambient fluid at the outlet of the branch pipe or port.

2. The branches or ports are spaced at intervals along the manifold, and between the branches or ports in the manifold a change in head will occur due to the head loss in the manifold. This is given as

$$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g}$$

where L = port or branch spacing (see Fig. 5-13)
D = diameter of manifold
V = mean velocity in the manifold.

There may also be a small change in head across the section where a port or branch is located (from upstream of port or branch to downstream of it); however, accepted design practice assumes this change in head is negligible (34).

The design of a manifold is described, by example, for a diffuser, such as is used in the discharge of wastewater into the sea. The physical setup is shown in Fig. 5-14. The objective in the manifold design is to distribute a given total discharge of effluent fairly uniformly along the length of the manifold. The design process is iterative. The design assumes the geometric configuration (diameter of manifold, size of ports, spacing between ports) and the discharge from the end port. Then by computation, the discharge from all other ports and the total required head can be determined. If, as a result of these computations, the conditions are not to the designer's satisfaction, a new set of conditions are assumed, and the process is repeated until the desired design is achieved. Example 5-7 illustrates the procedure.

EXAMPLE 5-7 Wastewater, after primary treatment, is to be discharged into a large body of water by means of a diffuser. Design a manifold type of diffuser to discharge effluent at a rate of 4 cfs. Assume the design criteria limit the total head (above that of lake level) at the upstream end of the supply pipe that feeds the diffuser to 25 ft, and that the difference in discharge between the upstream and downstream ports is to be no greater than 10% of the discharge from the downstream port. Further assume the supply pipe will be 400 ft long.

SOLUTION Assume the manifold will be made of PVC pipe, the average discharge from each port $\bar{q}$ will be 0.20 cfs, and the spacing between ports will be 4 ft. Thus there will have to be approximately 20 ports [4.0 cfs/(0.20 cfs/port)], and the total length of diffuser itself will be 80 ft. Assume the velocity from the downstream end port will be 19.0 ft/s. Therefore, the head, E_{end}, at the dead end of the diffuser will be given by

$$E_{\text{end}} = \left(\frac{V^2}{2g}\right) + \Delta h$$

However, $V^2/2g = 0$ at the dead end. Therefore,

$$E_{\text{end}} = \Delta h = \left(\frac{q^2}{K^2 a^2}\right)\frac{1}{2g}$$

where $q^2/a^2 = (19.0)^2\ \text{ft}^2/\text{s}^2$. K is the flow coefficient for the orifice, and as noted by Subramanya (43), it can be given as

$$K = 0.675\sqrt{1 - \frac{V^2}{2gE}} \tag{5-25}$$

Then for the end section where $V \approx 0$ ($K = 0.675$), the total energy head is $E_{\text{end}} = (19.0^2/0.675^2)/(2 \times 32.2) = 12.3$ ft.

Next, as a first approximation, assume the diameter of the manifold pipe and feeder pipe are the same size and are based on the head available as given by the design criteria. That is, the total head available at the inlet to the supply pipe is given as 25 ft, and the head at the dead end of the diffuser (just calculated above) is 12.3 ft. Therefore, the head available for flow in the supply pipe and the manifold is equal to 25 − 12.3, or 12.7 ft. Because the velocity in the diffuser averaged over its length is only about one half the supply pipe velocity, the average head loss per unit of length in diffuser would be only a small fraction of that in the supply pipe. Assume this loss per unit length to be 1/4 of that in the supply pipe. Then for the total loss in both pipes, we have

$$12.7 = f\left(\frac{L_1}{D}\right)\frac{V^2}{2g} + \frac{1}{4}f\left(\frac{L_2}{D}\right)\frac{V^2}{2g}$$

where $L_1 = 400$ ft
$L_2 = 80$ ft

Assume $f = 0.015$ (first approximation). Then

$$12.7 = 0.015(420/D)\frac{Q^2}{(2gA^2)}$$

$$= 0.015(420/D)\frac{4^2}{\left(64.4 \cdot \frac{\pi^2}{4^2} \cdot D^4\right)}$$

$$D^5 = 0.200 \text{ ft}^5 \qquad D = 0.725 \text{ ft} = 8.7 \text{ in.}$$

The 8.7-in. size is not a standard size; therefore, choose the next standard size larger, which is a 10.0-in. size. To complete the initial geometric characteristics of the diffuser, we determine the port size. This is found from Eq. (5-25) for $q = 0.20$ cfs and assuming $E \approx 12.3$ ft and $K = 0.675$.

Now we solve for a and d_{port}.

$$a = \frac{q}{(K\sqrt{2gE})} = \frac{0.2}{(0.675\sqrt{64.4 \times 12.3})}$$

$$= 0.0102 \text{ ft}^2$$

$$d = 0.1142 \text{ ft} = 1.37 \text{ in.}$$

Thus $d = 0.1142$ ft is used for the initial port diameter.

Now that the basic geometric configurations have been assumed along with q at the end port, we can analyze the flow in the diffuser to determine the discharge distribution from the ports and the head required at the inlet to the supply pipe.

The analysis starts at the downstream end of the diffuser and works upstream (step by step) until the head at the upstream end of the diffuser is obtained. Then the head at the inlet to the supply pipe is obtained. This procedure follows that given by Rawn (34) and by Vigander (48). Fig. 5-15 shows the

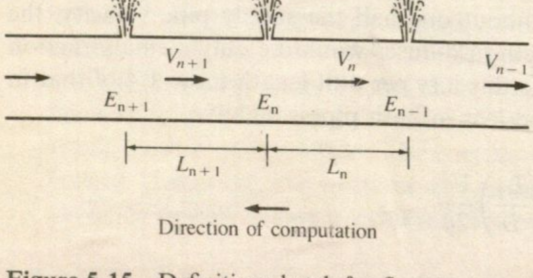


Figure 5-15 Definition sketch for flow in a manifold

diffuser pipe and the notation used in the computations. For each step of the computation, the following equations are solved in the order shown.

$$V_{n-1} = \frac{Q_{n-1}}{A_{\text{diffuser}}} \qquad \text{(}\textit{Note}\text{: This is zero at the dead end.)}$$

$$K_n = 0.675\sqrt{1 - \frac{V_{n-1}^2}{2gE_n}} \qquad \text{(}\textit{Note}\text{: This has a value of 0.675 for the first port.)}$$

$$q_n = K_n a_n \sqrt{2gE_n}$$

$$\Delta V_n = \frac{q_n}{(\pi D^2/4)}$$

$$V_n = V_{n-1} + \Delta V_n$$

$$\text{Re}_n = V_n \frac{D}{\nu}$$

$$f_n = \frac{0.25}{\left[\log\left(\frac{k_s}{3.7\,D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}$$

$$h_{f_n} = f\left(\frac{L}{D}\right)\left(\frac{V_n^2}{2g}\right)$$

$$E_{n+1} = E_n + h_{f_n}$$

where V_n is the mean velocity in the manifold at the nth computational step, E_n is the total energy head at the nth step, a_n is the cross-sectional area of the nth port, and ΔV_n is the change in velocity in the manifold due to the discharge from the nth port. It was assumed that $k_s = 0$ for the PVC pipe.

The table on page 270 is a summary of the computations for this diffuser.

The table shows that 20 ports provide a total discharge of 7.069 ft/s × $(\pi/4) \times (10/12)^2 = 3.86$ cfs. The percent difference in discharge between the upstream and downstream ports is given as

$$\left(\frac{0.1945 - 0.1893}{0.1945}\right) \times 100 = 2.67\%$$

The total head required at the inlet to the supply pipe is the total head at the upstream end of the diffuser plus the head loss in the supply pipe, or

$$E = E_{20} + h_{f_{\text{supply pipe}}}$$

$$= 12.4 + f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

Port	V_{n-1} (ft/s)	q_n (ft^3/s)	ΔV_n (ft/s)	V_n (ft/s)	Re	f_n	h_{f_n} (ft)	E_{n+1} (ft)	K_n
1	0.000	0.1945	0.357	0.357	2.0×10^4	0.0250	0.0003	12.300	0.675
2	0.357	0.1945	0.357	0.714	4.0	0.0215	0.0006	12.300	0.675
3	0.714	0.1945	0.357	1.071	6.4	0.0196	0.00168	12.300	0.675
4	1.071	0.1944	0.357	1.428	8.6	0.0185	0.00281	12.302	0.675
5	1.428	0.1943	0.356	1.784	1.1×10^5	0.0176	0.0042	12.305	0.674
6	1.784	0.1942	0.356	2.140	1.3	0.0170	0.0058	12.306	0.674
7	2.140	0.1940	0.356	2.496	1.5	0.0169	0.0076	12.308	0.673
8	2.496	0.1939	0.355	2.851	1.7	0.0160	0.0097	12.310	0.672
9	2.851	0.1936	0.355	3.206	1.9	0.0157	0.01199	12.312	0.672
10	3.206	0.1934	0.355	3.561	2.1	0.0153	0.0145	12.314	0.671
11	3.561	0.1931	0.354	3.915	2.3	0.0151	0.0172	12.317	0.670
12	3.915	0.1928	0.354	4.268	2.6	0.0148	0.0201	12.320	0.668
13	4.268	0.1925	0.353	4.621	2.8	0.0146	0.0232	12.322	0.667
14	4.621	0.1921	0.352	4.973	3.0	0.0144	0.0265	12.326	0.666
15	4.973	0.1917	0.351	5.324	3.2	0.0142	0.0300	12.329	0.664
16	5.324	0.1913	0.351	5.675	3.4	0.0140	0.0337	12.332	0.663
17	5.675	0.1908	0.350	6.025	3.6	0.0139	0.0376	12.336	0.661
18	6.025	0.1904	0.349	6.374	3.8	0.0137	0.0416	12.340	0.659
19	6.374	0.1898	0.348	6.722	4.0	0.0136	0.0458	12.344	0.658
20	6.722	0.1893	0.347	7.069	4.2	0.0135	0.0502	12.348	0.656

$$E = 12.4 + 0.0134\left(\frac{400}{(10/12)}\right)\left(\frac{7.759^2}{64.4}\right)$$

$$= 18.4 \text{ ft}$$

The total head at the upstream end of the inlet pipe is well within the original design criteria, as is the limit on the distribution of discharge from the ports. However, the total discharge from the 20 ports (3.86 cfs) is just short of the design discharge of 4.0 cfs. To determine the total head required at the inlet to yield a discharge of 4.00 cfs, multiply the 18.4 ft of head by $(4.00/3.86)^2$:

$$E = 18.4 \times \left(\frac{4.00}{3.86}\right)^2 = 19.76 \text{ ft}$$

This calculation assumes all head losses are a function of V^2 or Q^2/A^2, which is essentially the case here. Similarly, for a total head of 25 ft at the inlet, one can estimate the discharge through the system to be

$$Q = 3.86\left(\frac{25}{18.4}\right)^{1/2} = 4.50 \text{ cfs} \qquad \blacksquare$$

The designer may also wish to see if an 8-in. pipe size would produce results within the established criteria. If so, the same process would be repeated with that size.

All the preceding computations are easily programmed for computer solution; therefore, additional runs for different conditions are easily made. And as in all engineering design problems, the total cost of construction for each condition can be estimated. Thus the designer, by iteration, can find a solution that will satisfy the technical requirements for a minimum overall cost, which is the essence of engineering design.

Pipe Networks

The most common pipe networks are the water-distribution systems for municipalities. These systems have one or more sources (discharge of water into the system) and numerous loads: one for each household and commercial establishment. For purposes of simplification, the loads are usually lumped throughout the system. Figure 5-16 shows a simplified distribution system with two sources and seven loads.

The engineer is often engaged to design the original system or to recommend an economical expansion to the network. An expansion may involve additional housing or commercial developments, or it may be to handle increased loads within the existing area. In any case, the engineer is required to predict pressures throughout the network for various operating conditions, that is, for various combinations of sources and loads. The solution of such a problem must satisfy three basic requirements:

1. Continuity must be satisfied. That is, the flow into a junction of the network must equal the flow out of the junction. This must be satisfied for all junctions.

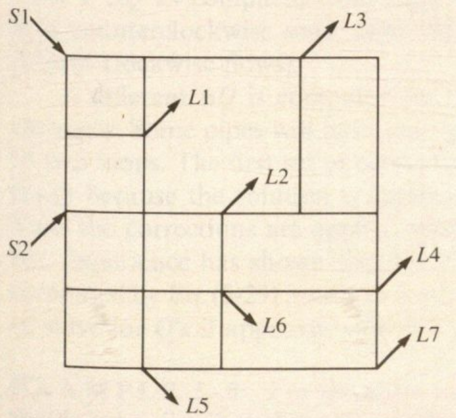


Figure 5-16 Pipe network

2. The head loss between any two junctions must be the same regardless of the path in the series of pipes taken to get from one junction point to the other. This requirement results because pressure must be continuous throughout the network (pressure cannot have two values at a given point). This condition leads to the conclusion that the algebraic sum of head losses around a given loop must be equal to zero. Here the sign (positive or negative) for the head loss in a given pipe is given by the sense of the flow with respect to the loop, that is, whether the flow has a clockwise or counterclockwise sense.
3. The flow and head loss must be consistent with the appropriate velocity-head-loss equation.

Only a few years ago, these solutions were made by a trial-and-error hand computation, but recent applications using digital computers have made the older methods obsolete. Even with these advances, however, the engineer charged with the design or analysis of such a system must understand the basic fluid mechanics of the system to be able to interpret the results properly and to make good engineering decisions based on the results. Therefore, the method of solution first given by Cross (17) is presented as follows:

The flow is first distributed throughout the network so that the continuity requirement (requirement 1) is satisfied for all junctions. This first guess at the flow distribution obviously will not satisfy requirement 2 regarding head loss; therefore, corrections are applied. For each loop of the network, a discharge correction is applied to yield a zero net head loss around the loop. For example, consider the isolated loop in Fig. 5-17. In this loop, the loss of head in the clockwise sense will be given by

$$\sum h_{L_c} = h_{L_{AB}} + h_{L_{BC}}$$
$$= \sum kQ_c^{\,n} \qquad (5\text{-}26)$$

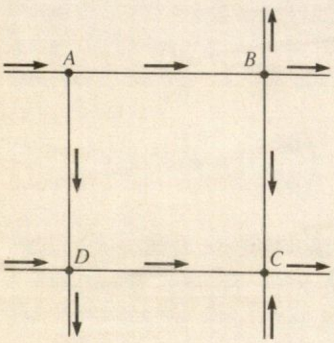


Figure 5-17 A typical loop of a network

The loss of head for the loop in the counterclockwise sense is

$$\sum h_{L_{cc}} = \sum_{cc} kQ_{cc}^n \tag{5-27}$$

For a solution, the clockwise and counterclockwise head losses have to be equal or

$$\sum h_{L_c} = \sum h_{L_{cc}}$$
$$\sum kQ_c^n = \sum kQ_{cc}^n$$

As we noted, the first guess for flow in the network is undoubtedly in error; therefore, a correction in discharge, ΔQ, will have to be applied to satisfy the head loss requirement. If the clockwise head loss is greater than the counterclockwise head loss, ΔQ would have to be applied in the counterclockwise sense. That is, subtract ΔQ from the clockwise flows and add it to the counterclockwise flows:

$$\sum k(Q_c - \Delta Q)^n = \sum k(Q_{cc} + \Delta Q)^n \tag{5-28}$$

Expanding the summation on either side of Eq. (5-28) and including only two terms of the expansion, we obtain

$$\sum k(Q_c^n - nQ_c^{n-1}\Delta Q) = \sum k(Q_{cc}^n + nQ_{cc}^{n-1}\Delta Q)$$

Then solving for ΔQ, we get

$$\Delta Q = \frac{\sum kQ_c^n - \sum kQ_{cc}^n}{\sum nkQ_c^{n-1} + \sum nkQ_{cc}^{n-1}} \tag{5-29}$$

Thus if ΔQ as computed from Eq. (5-29) is positive, the correction is applied in a counterclockwise sense (add ΔQ to counterclockwise flows and subtract it from clockwise flows).

A different ΔQ is computed for each loop of the network and applied to the pipes. Some pipes will have two ΔQ's applied because they will be common to two loops. The first set of corrections usually will not yield the final desired result because the solution is approached only by successive approximations. Thus the corrections are applied successively until the corrections are negligible. Experience has shown that for most loop configurations, applying ΔQ as computed by Eq. (5-29) produces too large a correction. Fewer trials are required to solve for Q's if approximately 0.6 of the computed ΔQ is used.

EXAMPLE 5-8 For the given source and loads shown in Fig. A, how will the flow be distributed in the simple network, and what will be the pressures

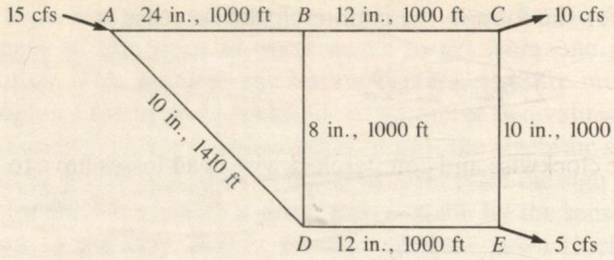


Figure A

at the load points if the pressure at the source is 60 psi? Assume horizontal pipes and $f = 0.012$ for all pipes.

SOLUTION An assumption is made for the discharge in all pipes making certain that the continuity equation is satisfied at each junction. Figure B shows the network with assumed flows.

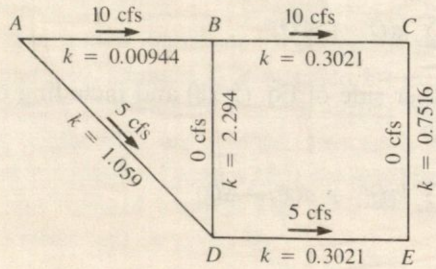


Figure B

The Darcy-Weisbach equation is used for computing the head loss; therefore, we have

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$= 8\left(\frac{fL}{gD^5\pi^2}\right)Q^2$$

$$= kQ^2$$

where $k = 8\left(\dfrac{fL}{gD^5\pi^2}\right)$.

The loss coefficient, k, for each pipe is computed and shown in Fig. B. Next, the flow corrections for each loop are calculated as shown in the accompanying table. Since $n = 2$ (exponent on Q), $nkQ^{n-1} = 2kQ$. When the corrections obtained in the table are applied to the two loops, we get the pipe discharges shown in Fig. C. Then with additional iterations, we get the final distribution of flow as shown in Fig. D. Finally, the pressures at the load points are calculated (see page 276):

Loop *ABC*

Pipe	$h_f = kQ^2$	$2kQ$
AB	+0.944	0.189
AD	−26.475	10.590
BD	0	0
	$\sum kQ_c^2 - \sum kQ_{cc}^2 = -25.53$	$\sum 2kQ = 10.78$

$$\Delta Q = -25.53/10.78 = -2.40 \text{ cfs}$$

Loop *BCDE*

Pipe	h_f	$2kQ$
BC	+30.21	6.042
BD	0	0
CE	0	0
DE	− 7.55	3.02
	+22.66	9.062

$$\Delta Q = 22.66/9.062 = 2.50 \text{ cfs}$$

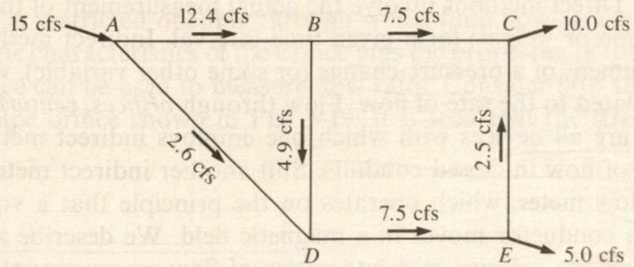


Figure C

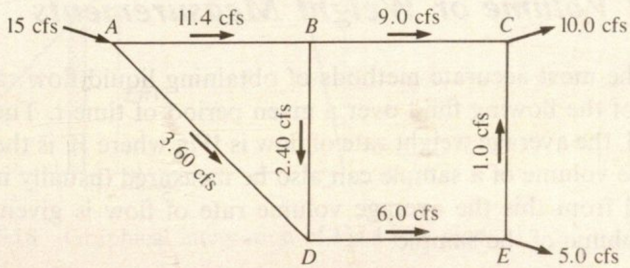


Figure D

$$p_C = p_A - \gamma(k_{AB}Q_{AB}^2 + k_{BC}Q_{BC}^2)$$
$$= 60 \text{ psi} \times 144 \text{ psf/psi} - 62.4(0.00944 \times 11.4^2 + 0.3021 \times 9.0^2)$$
$$= 8640 \text{ psf} - 1603 \text{ psf}$$
$$= 7037 \text{ psf}$$
$$= 48.9 \text{ psi}$$
$$p_E = 8640 - \gamma(k_{AD}Q_{AD}^2 + k_{DE}Q_{DE}^2)$$
$$= 8640 - 62.4(1.059 \times 3.5^2 + 0.3021 \times 6^2)$$
$$= 7105 \text{ psf}$$
$$= 49.3 \text{ psi}$$

■

5-6 Instruments and Procedures for Discharge Measurement

Direct and Indirect Methods of Flow Measurements

The methods of flow measurement can broadly be classified as either direct or indirect. Direct methods involve the actual measurement of the quantity of flow (volume or weight) for a given time interval. Indirect methods involve the measurement of a pressure change (or some other variable), which in turn is directly related to the rate of flow. Flow through *orifices*, *venturi meters*, and *flow nozzles* are all devices with which one employs indirect methods to measure the rate of flow in closed conduits. Still another indirect meter is the *electromagnetic* flow meter, which operates on the principle that a voltage is generated when a conductor moves in a magnetic field. We describe all these methods, as well as the *velocity-area integration* of flow measurement, in this section.

Direct Volume or Weight Measurements

One of the most accurate methods of obtaining liquid-flow rate is to collect a sample of the flowing fluid over a given period of time t. Then if the sample is weighed, the average weight rate of flow is W/t, where W is the weight of the sample. The volume of a sample can also be measured (usually in a calibrated tank), and from this the average volume rate of flow is given as $\forall/t$, where $\forall$ is the volume of the sample.

Velocity-Area-Integration Method

If the velocity in a pipe is symmetrical, the distribution of the velocity along a radial line can be used to determine the volume rate of flow (discharge) in the pipe. The discharge is obtained by numerically or graphically integrating $V\,dA$ over the cross-sectional area of the pipe. Thus a velocity traverse across the flow section provides the primary data from which the discharge is evaluated. The velocity can be measured by a pitot tube or some other suitable velocity meter. We give one procedure for evaluating this discharge in the following paragraph.

From test data of V versus r, compute $2\pi Vr$ for various values of r; then when $2\pi Vr$ versus r is plotted, the area under the resulting curve (Fig. 5-18), will yield the discharge. This is so because $dQ = V\,dA = V(2\pi r\,dr)$, which is given by an elemental strip of area in Fig. 5-18. Hence the total area will yield the total discharge. This procedure involving the velocity-area-integration method is applicable to pipes when the velocity distribution is symmetrical with the axis of the pipe. However, even for unsymmetrical flows, it should be obvious that by summing $V\Delta A$ over a flow section, you can obtain the total flow rate. Such a procedure is commonly used to obtain the discharge in streams and rivers, as we noted in Chapter 4.

Orifice

A restricted opening through which fluid flows is an *orifice*, and if the geometric characteristics of the orifice plus the properties of the fluid are known, the orifice can be used to measure flow rates. Consider flow through the sharp-edged pipe orifice shown in Fig. 5-19. It is seen that the streamlines continue

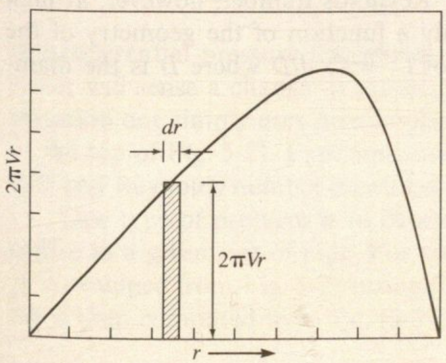


Figure 5-18 Graphical integration of $V\,dA$ in a pipe

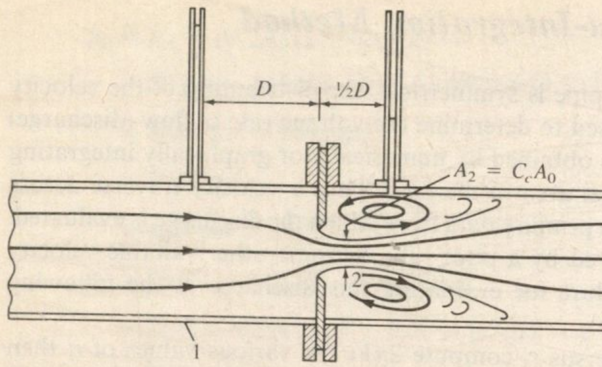


Figure 5-19 Flow through a pipe orifice

to converge a short distance downstream of the plane of the orifice; hence the minimum-flow area is actually smaller than the area of the orifice. To relate the minimum-flow area, A_j, often called the contracted area of the jet or *vena contracta*, to the area of the orifice A_o, we use the contraction coefficient, which is defined as

$$A_j = C_c A_o$$

$$C_c = \frac{A_j}{A_o}$$

Then, for a circular orifice, with diameter d,

$$C_c = \frac{(\pi/4)d_j^2}{(\pi/4)d^2} = \left(\frac{d_j}{d}\right)^2$$

Because d_j and d_2 are identical, we also have $C_c = (d_2/d)^2$. At low values of the Reynolds number, C_c is a function of the Reynolds number; however, at high values of the Reynolds number, C_c is only a function of the geometry of the orifice. Figure 5-20 shows the variation of C_c with d/D where D is the diam-

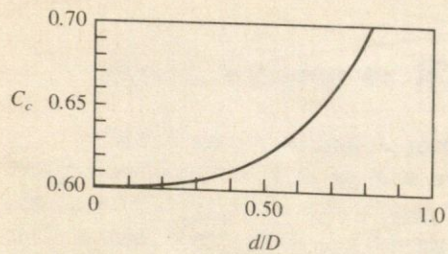


Figure 5-20 Contraction coefficient as a function of d/D for $\mathrm{Re}_d = 10^6$

eter of the pipe for a Reynolds number (Vd/ν) of 10^6.* The discharge equation for the orifice is derived by writing the Bernoulli equation between sections 1 and 2 in Fig. 5-19 and then eliminating V_1 by means of the continuity equation $V_1A_1 = V_2A_2$. Then solving for V_2 and multiplying by the flow area, C_cA_o, we obtain the discharge equation

$$Q = \frac{C_cA_o}{\sqrt{1 - C_c^2A_o^2/A_1^2}}\sqrt{2g(h_1 - h_2)} \tag{5-30}$$

Equation (5-30) is the discharge equation for the flow of an incompressible inviscid fluid through an orifice. However, this is valid only at relatively high Reynolds numbers. For low and moderate values of the Reynolds number, viscous effects may be significant and an additional coefficient is applied to the discharge equation to relate the ideal to the actual flow. This is called the *coefficient of velocity* C_v; thus for viscous flow in an orifice, we have the following discharge equation:

$$Q = \frac{C_vC_cA_o}{\sqrt{1 - C_c^2A_o^2/A_1^2}}\sqrt{2g(h_1 - h_2)}$$

The product C_vC_c is called the *discharge coefficient* C_d, and the combination $C_vC_c/(1 - C_c^2A_o^2/A^2)^{1/2}$ is called the *flow coefficient* K. Thus we have $Q = KA_o\sqrt{2g(h_1 - h_2)}$, where

$$K = \frac{C_d}{\sqrt{1 - C_c^2A_o^2/A_1^2}}$$

If Δh is defined as $h_1 - h_2$, the final form of the discharge equation for an orifice reduces to

$$Q = KA_o\sqrt{2g\,\Delta h} \tag{5-31}$$

If a differential pressure transducer is connected across the orifice, the transducer will sense a change in pressure equivalent to $\gamma\Delta h$. Therefore, in this application one simply uses $\Delta p/\gamma$ in place of Δh in Eq. (5-31) and in the parameter at the top of Fig. 5-21. Experimentally determined values of K as a function of d/D and Reynolds number based on orifice size, $4Q/\pi d\nu$, are given in Fig. 5-21.

One type of problem is to determine Δh for a given discharge through an orifice in a given size of pipe. For such a problem Re_d is equal to $4Q/\pi d\nu$ and K is obtained from Fig. 5-21 (using the vertical lines and the bottom scale), and Δh is then computed from Eq. (5-31). When Q is to be determined, we use the

* These were obtained by using the values of K from Fig. 5-21 and values of C_v given by Lienhard (27) and then calculating to obtain C_c.

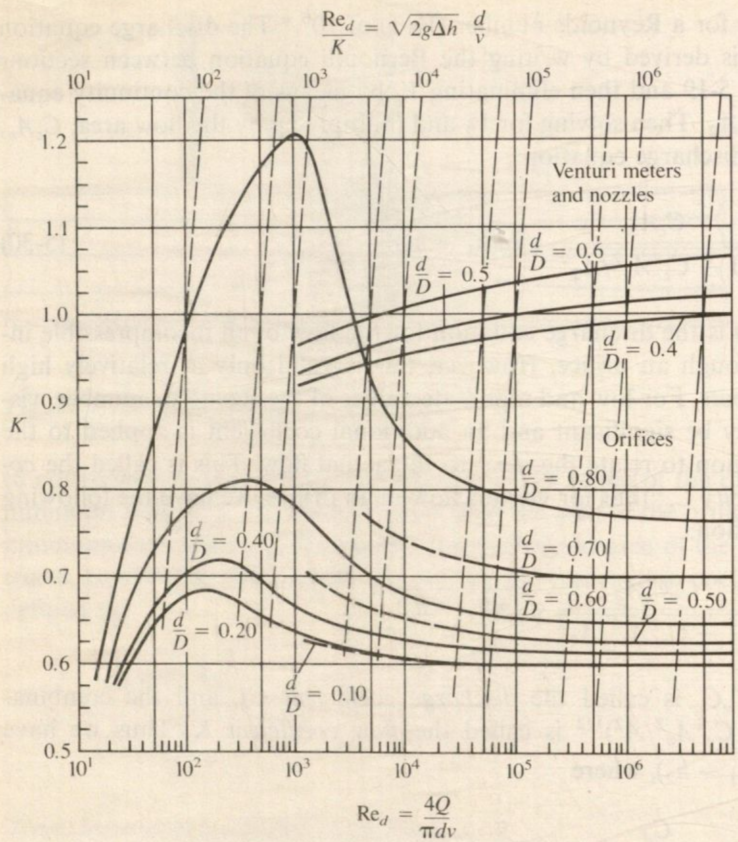


Figure 5-21 Flow coefficient K and Re_d/K versus the Reynolds number for orifices, nozzles, and venturi meters (20, 23)

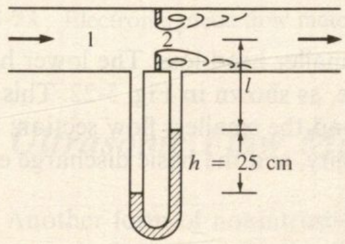
top scale with the slanted lines to determine K for given values of d, D, Δh and v. With K, we can then solve for Q from Eq. (5-31).

The literature on orifice flow contains many discussions concerning the optimum placement of pressure taps on both the upstream and downstream side of the orifice. The data given in Fig. 5-21 are for "corner taps." That is, on the upstream side, the pressure readings were taken immediately upstream of the plate orifice (at the corner of the orifice plate and the pipe wall), and the downstream tap was at a similar downstream location. However, pressure data from flange taps (1 in. upstream and 1 in. downstream) and from the taps shown in Fig. 5-20 all yield virtually the same values for K—the differences are no greater than the deviations involved in reading Fig. 5-21.*

* For more precise values of K with specific types of taps, see the ASME report on fluid meters (20).

EXAMPLE 5-9 A 15-cm orifice is located in a horizontal 24-cm water pipe, and a water-mercury manometer is connected to either side of the orifice. When the deflection on the manometer is 25 cm, what is the discharge in the system? Assume the water temperature is 20°C.

SOLUTION The discharge is given by Eq. (5-31): $Q = KA_o\sqrt{2g\,\Delta h}$. To either enter Fig. 5-21 or use Eq. (5-31), we will need to first evaluate Δh, the change in piezometric head in meters of fluid that is flowing. This is obtained by applying the equation of hydrostatics to the manometer shown as follows.



Writing the manometer equation from point 1 to point 2, we get

$$\Delta h = 0.25\text{ m}\,(13.6 - 1)$$
$$= 3.15\text{ m of water}$$

The kinematic viscosity of water at 20°C is $1.0 \times 10^{-6}\text{ m}^2/\text{s}$; so we now can compute $d\sqrt{2g\,\Delta h}/\nu$, the parameter needed to enter Fig. 5-21:

$$\frac{d\sqrt{2g\,\Delta h}}{\nu} = \frac{0.15\text{ m}\,\sqrt{2(9.81\text{ m/s}^2)(3.15\text{ m})}}{1.0 \times 10^{-6}\text{ m}^2/\text{s}} = 1.2 \times 10^6$$

From Fig. 5-21 with $d/D = 0.625$, we read K to be 0.66 (interpolated). Hence

$$Q = 0.66A_o\sqrt{2g\,\Delta h}$$
$$= 0.66\,\frac{\pi}{4}\,d^2\sqrt{2(9.81\text{ m/s}^2)(3.15\text{ m})}$$
$$= 0.66(0.785)(0.15^2\text{ m}^2)(7.87\text{ m/s})$$
$$= 0.092\text{ m}^3/\text{s}$$ ■

Venturi Meter

The orifice is a simple and accurate device for the measurement of flow; however, the head loss for the orifice is quite large. It is like an abrupt enlargement in a pipe: $h_L = (V_2 - V_1)^2/2g$. The venturi meter operates on the same

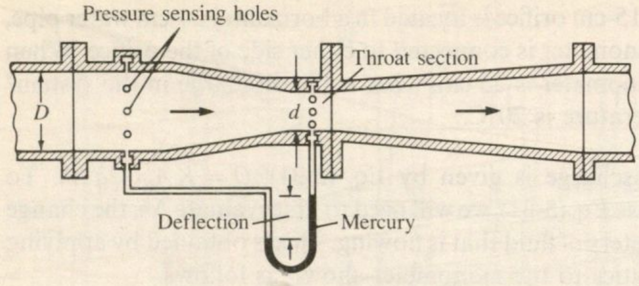


Figure 5-22 Typical venturi meter

principle as the orifice but with a much smaller head loss. The lower head loss results from streamlining the flow passage, as shown in Fig. 5-22. This streamlining eliminates any jet contraction beyond the smallest flow section; thus the coefficient of contraction has a value of unity, and the basic discharge equation for the venturi meter is

$$Q = \frac{A_2 C_d}{\sqrt{1-(A_2/A_1)^2}} \sqrt{2g(h_1 - h_2)} \tag{5-32}$$

$$= K A_2 \sqrt{2g\,\Delta h} \tag{5-33}$$

The discharge equation for the venturi meter, Eq. (5-33), is the same as for the orifice, Eq. (5-31). However, K for the venturi meter approaches unity at high values of the Reynolds number and small d/D ratios. This trend can be seen in Fig. 5-21, where values of K for the venturi meter are plotted along with similar data for the orifice.

Electromagnetic Flow Meter

All the flow meters introduced to this point require that some sort of obstruction be placed in the flow. The electromagnetic flow meter neither obstructs the flow nor requires pressure taps, which are subject to clogging. Its basic principle is that a conductor that moves in a magnetic field produces an electromotive force. Hence liquids having a degree of conductivity will generate a voltage between the electrodes as in Fig. 5-23, and this voltage will be proportional to the velocity of flow in the conduit.

The main advantages of the electromagnetic flow meter are that the output signal varies linearly with the flow rate, and the meter causes no resistance to the flow. The major disadvantage is its high cost.*

* For a summary of the theory and application of the electromagnetic flow meter, see Shercliff (39).

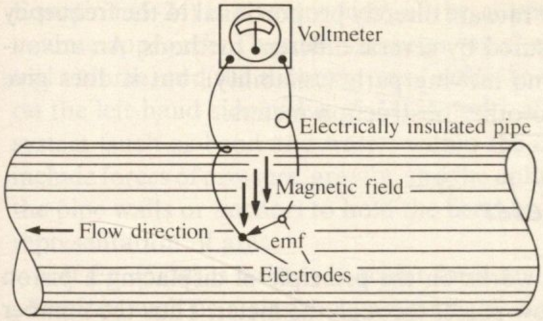


Figure 5-23 Electromagnetic flow meter

Ultrasonic Flow Meter

Another form of nonintrusive flow meter used in diverse applications is the ultrasonic flow meter. Basically, there are two different modes of operation for ultrasonic flow meters. One mode involves measuring the difference in travel time for a sound wave traveling upstream and downstream between two measuring stations. The difference in travel time is proportional to flow velocity. The second mode of operation is based on the Doppler effect. When an ultrasonic beam is projected into an inhomogeneous fluid, some acoustic energy is scattered back to the transmitter at a different frequency (Doppler shift). The measured frequency difference is related directly to the flow velocity.

Turbine Flow Meter

The turbine flow meter consists of a wheel with a set of curved vanes (blades) mounted inside a duct. The volume rate of flow through the meter is related to the rotational speed of the wheel, and this rotational rate is generally measured by a blade passing an electromagnetic pickup mounted in the casing. The meter must be calibrated for the flow conditions of interest. The turbine meter has an accuracy of better than 1% over a wide range of flow rates and operates with small head loss.

Vortex Flow Meter

The vortex flow meter consists of a cylinder mounted across the duct, which sheds vortices and gives rise to an oscillatory flow field. By proper design of the cylindrical element, the Strouhal number for vortex shedding ($S = nd/V_0$) will be constant for Reynolds numbers from 10^4 to 10^6. Over this flow range, the fluid velocity and volume flow rate are directly proportional to the frequency of oscillation, which can be measured by several different methods. An advantage of this meter is that it has no moving parts (reliability), but it does give rise to a head loss comparable to other obstruction meters.

Displacement Meter

The displacement meter works on the principle of displacing a piston or other mechanical part when flow passes through the meter. Thus the number of oscillations of the piston or disc (as in the nutating disc meter) can be monitored, which in turn will be proportional to the quantity of flow through the meter. Displacement meters are used extensively for measuring the quantity of water used by households or businesses from municipal water systems.*

Salt-Velocity Method

This method of discharge was first developed by Allen (1). The method, which is based on the increased electrical conductivity of a salt solution, is used in the following manner: A concentrated dose of salt solution is injected into the conduit at a given time and location in the conduit. Then by means of electrodes and associated instruments, the conductivity is recorded at a station farther downstream. The "instant" of passage of the salt solution is assumed to be at the center of gravity of the conductivity-time trace (the conductivity will increase and then decrease over a considerable length of time because of mixing due to turbulence). Thus the mean velocity can be determined from the length of travel and elapsed time of travel of the salt solution.

5-7 Forces and Stresses in Pipes and Bends

Forces on Bends and Transitions

BENDS Because of the change in momentum that occurs with flow around a bend, the momentum equation is used to calculate the forces acting on bends and transitions. The general momentum equation for steady one-dimensional flow is

$$\sum \mathbf{F}_{\text{syst}} = \sum_{\text{c.s.}} \mathbf{V}\rho\mathbf{V}\cdot\mathbf{A} \tag{5-34}$$

* For more complete information on displacement meters, as well as the other types of meters we have discussed, see the American Water Works manual on water meters (13).

Equation (5-34) is a vector form of the momentum equation using the control volume approach. Thus the subscript "syst" refers to everything inside the control volume, and the subscript "c.s." refers to the control surface. The $\sum \mathbf{F}$ term on the left-hand side of Eq. (5-34) includes all the external forces acting on the system (such as bend and water) within the control volume. Such forces could include forces of pressure, gravity, and the unknown force (usually acting through the pipe walls or anchor) to hold the bend or transition in place. **A** is the vector representation of area.

If Eq. (5-34) is written in scalar form and simplified for a single-stream application, such as a single stream of water flowing through a bend, we obtain

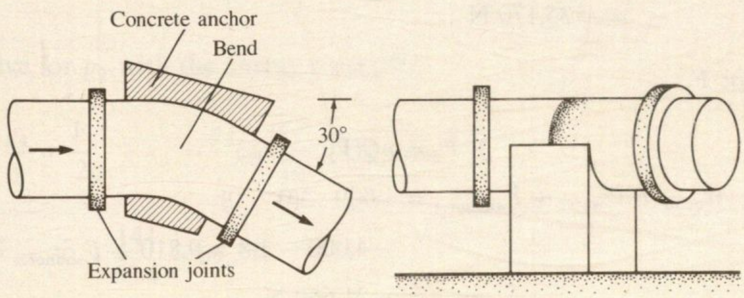
$$\sum F_x = \rho Q(V_{2x} - V_{1x}) \qquad (5\text{-}34a)$$

$$\sum F_y = \rho Q(V_{2y} - V_{1y}) \qquad (5\text{-}34b)$$

$$\sum F_z = \rho Q(V_{2z} - V_{1z}) \qquad (5\text{-}34c)$$

Example 5-10 illustrates an application of Eq. (5-34).

EXAMPLE 5-10 A 1-m diameter pipe has a 30° horizontal bend in it, as shown in Fig. A, and carries water (10°C) at a rate of 3 m^3/s. If we assume the pressure in the bend is uniform at 75 kPa gauge, the volume of the bend is 1.8 m^3, and the metal in the bend weighs 4 kN, what forces must be applied to the bend by the anchor to hold the bend in place? Assume expansion joints prevent any force transmittal through the pipe walls of the pipes entering and leaving the bend.



(a) Plan view *(b)* Elevation view

Figure A

SOLUTION Since only a single stream of water is involved in this problem, we can use Eqs. (5-34) for the solution. Consider the control volume shown in Fig. B, and first solve for the x component of force:

Then $\sum F_x = \rho Q(V_{2x} - V_{1x})$

$$p_1 A_1 - p_2 A_2 \cos 30^\circ + F_{\text{anchor},x} = 1{,}000 \cdot 3(V_{2x} - V_{1x})$$

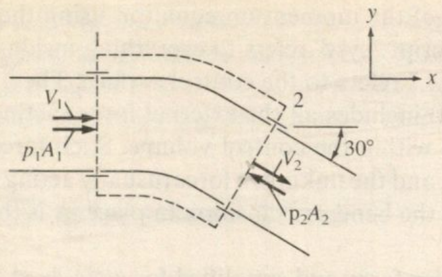


Figure B

where $p_1 = p_2 = 75{,}000$ Pa

$$A_1 = A_2 = (\pi/4)D^2 = 0.785 \text{ m}^2$$
$$V_{2x} = (Q/A_2)\cos 30° = 3.31 \text{ m/s}$$
$$V_{1x} = Q/A_1 = 3.82 \text{ m/s}$$

$$F_{\text{anchor},x} = 3{,}000(3.31 - 3.82) + 75{,}000 \times 0.785(0.866 - 1)$$
$$= -9{,}420 \text{ N}$$

Solve for F_y:

$$\sum F_y = \rho Q(V_{2y} - V_{1y})$$
$$F_{\text{anchor},y} = 1{,}000 \cdot 3(-3.82 \sin 30° - 0) - p_2 A_2 \sin 30°$$
$$= -35{,}170 \text{ N}$$

Solve for F_z:

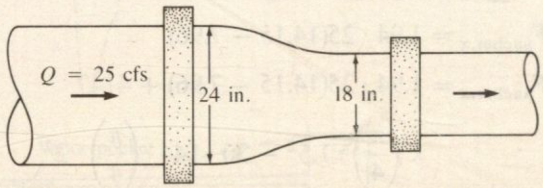
$$\sum F_z = \rho Q(V_{2z} - V_{1z})$$
$$W_{\text{bend}} + W_{\text{water}} + F_{\text{anchor},z} = 1{,}000 \cdot 3(0 - 0)$$
$$-4{,}000 - 1.8 \times 9{,}810 + F_{\text{anchor},z} = 0$$
$$F_{\text{anchor},z} = +21{,}660 \text{ N}$$

Then the total force that the anchor will have to exert on the bend will be

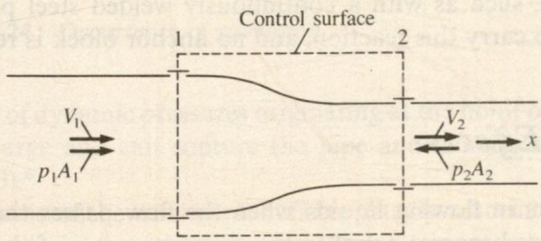
$$\mathbf{F}_{\text{anchor}} = -9{,}420\mathbf{i} - 35{,}170\mathbf{j} + 21{,}660\mathbf{k} \text{ N}$$ ■

TRANSITIONS The fitting between two pipes of different size is a *transition.* Because of the change in flow area and change in pressure, a longitudinal force will act on the transition. To determine the force required to hold the transition in place, the energy, momentum, and continuity equations are all applied. Example 5-11 illustrates the principles.

EXAMPLE 5-11 Water flows through the contraction at a rate of 25 cfs. The head loss coefficient for this particular contraction is 0.20 based on the velocity head in the smaller pipe. What longitudinal force (such as from an anchor) must be applied to the contraction to hold it in place? We assume the upstream pipe pressure is 30 psig, and expansion joints prevent force transmittal between the pipe and the contraction.



SOLUTION Let the x direction be in the direction of flow, and let the control surface surround the transition as shown in the figure.



First solve for p_2 with the energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

where $\dfrac{p_1}{\gamma} = 30 \times \dfrac{144}{62.4} = 69.2 \text{ ft}$

$$V_1 = \frac{Q}{A_1} = \frac{25}{(\pi/4)2^2} = 7.96 \text{ ft/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{25}{(\pi/4) \times 1.5^2} = 14.15 \text{ ft/s}$$

$$z_1 = z_2$$

$$h_L = 0.20\,\frac{V_2^2}{(2g)}$$

Then
$$\frac{p_2}{\gamma} = 69.2 \text{ ft} + \frac{7.96^2}{2g} - \frac{14.15^2}{2g}(1 + 0.2)$$

$$\frac{p_2}{\gamma} = 66.45 \text{ ft}$$

or
$$p_2 = 4147 \text{ psf}$$

Now solve for the anchor force:

$$\sum F_x = \rho Q(V_{2x} - V_{1x})$$

$$p_1A_1 - p_2A_2 + F_{\text{anchor},x} = 1.94 \cdot 25(14.15 - 7.96)$$

$$F_{\text{anchor},x} = 1.94 \cdot 25(14.15 - 7.96) + 4147$$

$$\times \left(\frac{\pi}{4}\right) \cdot 1.5^2 - 30 \cdot 144 \cdot \left(\frac{\pi}{4}\right) \cdot 2^2$$

$$= -5{,}943 \text{ lb}$$

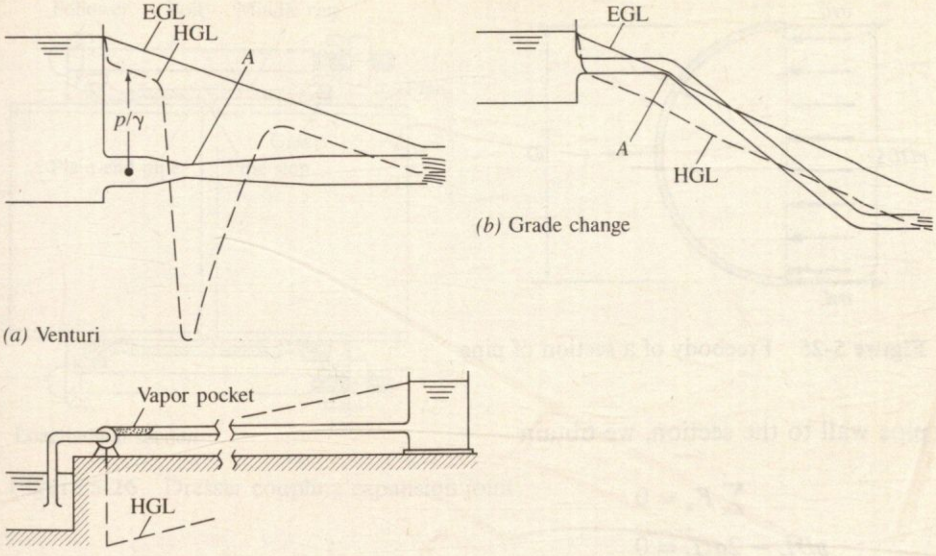
The anchor must exert a force of 5,943 lb in the negative x direction on the transition.

Note: In many cases, such as with a continuously welded steel pipe, the pipe walls are designed to carry this reaction, and no anchor block is required. ■

Cavitation Effects

Cavitation occurs in flowing liquids when the flow passes through a zone in which the pressure becomes equal to the vapor pressure of the liquid, and then the flow continues on to a region of higher pressure. In the vapor pressure zone, vapor bubbles are formed (the liquid boils), and then when the liquid and bubbles enter the higher pressure zone, the bubbles collapse thereby producing dynamic effects that can often lead to decreased efficiency or equipment failure. Figure 5-24 shows three setups that could produce cavitation. In Fig. 5-24*a*, the high velocity flow through the venturi is accompanied by a reduced pressure, and if this pressure is as low as the vapor pressure of the liquid, cavitation will occur at point A inside the venturi section. In Fig. 5-24*b*, it is the combination of pipe elevation change, head loss along the pipe, and locally high velocity along the inside of the bend that creates the lowest pressure at point A in the pipe, and this is where cavitation would first occur.

In some systems, such as one using a pump (Fig. 5-24*c*), vapor pressure can arise because of deceleration of the water column if power to the pump is interrupted. In these cases, the pump will retard the flow, and a vapor cavity will form downstream of the pump. The vapor cavity will grow in size and then decrease. Just before the cavity vanishes, the columns of water on either side of the vapor pocket will be accelerating toward each other. Finally, when the vapor cavity disappears, the two columns of water will impact each other with



(a) Venturi

(b) Grade change

(c) Power outage to pump

Figure 5-24 Occurrence of cavitation

creation of dynamic pressures originating at the point of impact. These pressures can be large and can rupture the pipe and damage the pump (see Fig. 11-1, page 573).*

Because of the detrimental effects usually associated with cavitation, good hydraulic design practice will normally exclude any possibility of its occurrence. With a normal range of water temperature (40°F to 80°F), cavitation will occur if the pressure head gets as low as -33 ft (the hydraulic grade line is at an elevation 33 ft below the point in the question). Thus good design practice does not allow the pressure to be this low. In fact, to be conservative, the U.S. Bureau of Reclamation has recommended (46) that the pressure head throughout the pipe system should be greater than -10 ft. For important systems, special detailed analyses are required.†

Pipe Stress Due to Internal Pressure

In thin-walled pipes ($t/D < 0.1$), the formula for hoop tension stress due to pressure within the pipe is derived by considering a freebody of one half of a pipe section, as shown in Fig. 5-25. Taking a length (normal to page) of pipe L and applying the equation of equilibrium where σ is hoop stress in the

* For more details on the method of analyzing this phenomenon, see Chapter 11.

† For more information on cavitation in pipes, see Knapp (24) and Pearsall (33).

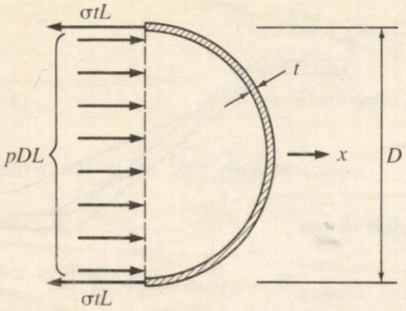


Figure 5-25 Freebody of a section of pipe

pipe wall to the section, we obtain

$$\sum F_x = 0$$

$$pDL - 2\sigma tL = 0$$

or $$\sigma = \frac{pD}{2t} \tag{5-35}$$

In the derivation of Eq. (5-35), it was assumed there is a uniform distribution of stress across the wall because it was assumed to be thin. However, for thick-walled pipes, it can be shown (38) that the circumferential or hoop stress is given by

$$\sigma = \frac{p r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) \tag{5-36}$$

where r = radius to a point in pipe wall
r_i = inside radius of pipe
r_o = outside radius of pipe

Temperature Stress and Strain

Temperature stresses develop when temperature changes occur after the pipe is installed and rigidly held in place. For example, if a pipe is restrained from expanding when the temperature changes $+\Delta T^\circ$, the pipe, in effect, would be subjected to a compressive longitudinal deflection of

$$\Delta L = \Delta T \alpha L \tag{5-37}$$

where α = coefficient of thermal expansion

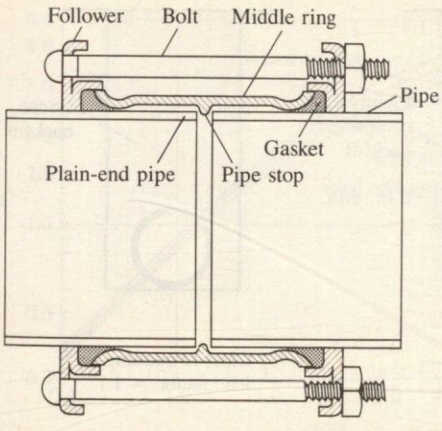


Figure 5-26 Dresser coupling expansion joint

Then the resulting effective longitudinal strain would be $\varepsilon = \Delta L/L = \Delta T\alpha$, and the resulting temperature stress would be

$$\begin{aligned}\sigma &= E\varepsilon \\ &= E\Delta T\alpha\end{aligned} \tag{5-38}$$

where E = elastic modulus

To eliminate the temperature stress, expansion joints are used, as shown in Fig. (5-26). These joints can be placed at regular intervals and must allow the pipe to expand a distance ΔL as given by Eq. (5-37), where L is the spacing between expansion joints.

External Loading

Pipes that are laid in a trench must be designed for external loading (soil forces) as well as for internal pressure. This is especially true for cases in which the internal pressure is low. Because the external loading on the pipe is in general not uniformly distributed around the circumference of the pipe, bending stresses develop within the pipe wall. These bending stresses are a function of the placement of the pipe (whether in a trench or under an open fill), as well as the supporting condition (bedding) under the pipe. Thus if the loading and bedding conditions were completely defined, it would be possible to solve for the resulting stresses in the pipe. However, because of the complexity of the usual underground pipe installations, a simpler more empirical design procedure

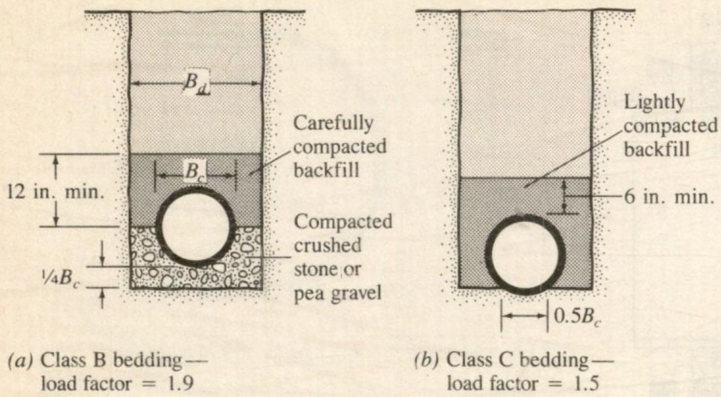


Figure 5-27 Load factors for different bedding conditions

is standard practice. Briefly, the procedure involves determining the design load on the pipe, and then a pipe is chosen that will withstand the given loading for a given bedding condition. In the following sections, we illustrate the procedure involved in the design of pipes placed in trenches and backfilled with soil.

Figure 5-27 shows a typical pipe installation with three different types of bedding conditions. The load on the pipe is equal to the weight of the prism of soil above the pipe in the trench minus the shear force between the trench wall and soil prism. Marston and Spangler carried out a number of tests for various pipe sizes, depths of cover, and different types of soil. Their research was summarized by Spangler (40) in 1948. This work led to Marston's general formula for soil load on pipe, which is $W = C\gamma_s B^2$, where W is the vertical load acting on the pipe per unit of length of pipe (N/m or lb/ft), γ_s is the unit weight of the soil (N/m^3 or lb/ft^3), B is the trench width or conduit width depending on installation conditions, and C is a coefficient that is a function of the relative fill height and soil type.

Rigid Pipe Installation If the pipe is rigid (for example, concrete), Marston's general formula is given as

$$W = C\gamma_s B_d^{\,2} \tag{5-39}$$

where B_d = trench width

In Fig. 5-28, the coefficient C has been plotted as a function of the relative height of soil cover, H/B_d.

Flexible Pipe Installation If the pipe is flexible (for example, steel or plastic) and the soil at the sides is well compacted, the side walls will carry

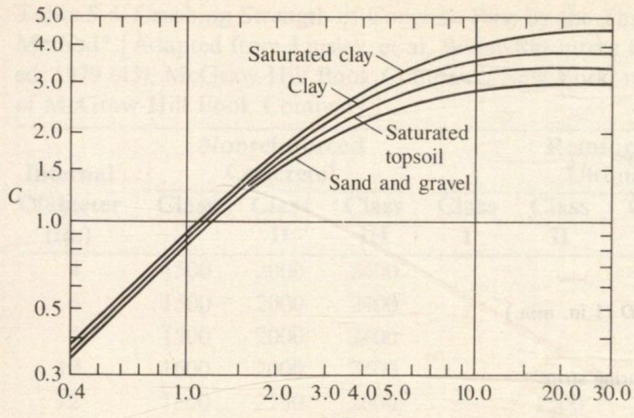


Figure 5-28 Load coefficients for various types of soil (4)

a significant portion of the total load; therefore, the load formula is modified as follows:

$$W = C\gamma_s B_d B_c \tag{5-40}$$

In Eq. (5-40), the coefficient C is obtained as before from Fig. 5-28, and B_c is the conduit diameter.

This procedure for load determination is applicable when pipes are laid in relatively narrow trenches ($B_d \leq 2B_c$). However, if the trench is greater than about twice the pipe diameter, the calculated load will be too large, and a different procedure must be used. If live loads (wheel loads from trucks or trains) are a factor, they, too, must be considered.*

Strength of Rigid Pipes

The strength of rigid pipe such as concrete or clay is determined by a three-edge bearing test such as shown in Fig. 5-29. These laboratory tests establish the basic load carrying capacity (strength) of the pipe (see Table 5-4, page 295, for representative strengths of concrete pipe). However, as we noted, the field strength of the pipe will depend on bedding conditions; therefore, the pipe strength based on the three-edge bearing test is modified by a load factor relating to the type of bedding. For example, if the three-edge bearing strength were 6500 lb/ft, the *field supporting strength* of this conduit with a class C type

* For these and other conditions, see the ASCE manual on sanitary storm sewers (4).

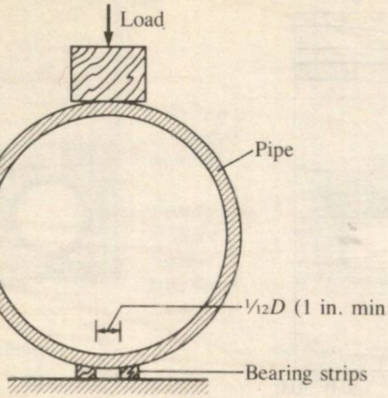


Figure 5-29 Three-edge bearing test

of bedding (see Fig. 5-27) would be 6500 × 1.5 = 9750 lb/ft. Besides the bedding factor, a factor of safety is usually applied to account for the variability of load distribution and other conditions not already accounted for. Thus the safe working strength of the conduit is given as

$$S_{\text{safe}} = \frac{S_{\text{3-edge}} \cdot F_{\text{load}}}{F_{\text{safety}}} \tag{5-41}$$

where S_{safe} = safe supporting strength of conduit
$S_{\text{3-edge}}$ = three-edge bearing strength of conduit
F_{load} = bedding load factor
F_{safety} = factor of safety

The factor of safety is usually taken as 1.5 or greater.

5-8 Pipe Materials

The most common pipe materials are steel, ductile iron, concrete, asbestos cement, and plastic.

Steel

The type of steel in pipes used for civil engineering projects is generally of medium carbon content, which has high strength as well as high ductility. Because of these desirable physical qualities, steel pipe is used in a variety of applications from small pipes in household water systems to 12-ft (and larger) diameter penstocks or pipelines. The AWWA standard recommends (12) the use of a tensile stress for design of steel pipe equal to 50% of the yield point

Table 5-4 Crushing Strength of Concrete Pipe by the Three-Edge Bearing Method* [Adapted from Linsley, et al, Water-Resources Engineering, 3rd. ed. 1979 (43), McGraw-Hill Book Company, New York; used with permission of McGraw-Hill Book Company.]

Internal Diameter (in.)	Nonreinforced Concrete† Class I	Class II	Class III	Reinforced Concrete† Ultimate Strength Class I	Class II	Class III	Class IV	Class V
4	1500	2000	2400					
6	1500	2000	2400					
8	1500	2000	2400					
10	1600	2000	2400					
12	1800	2250	2600	—	1500	2000	3000	3750
15	2000	2600	2900	—	1875	2500	3750	4690
18	2200	3000	3300	—	2250	3000	4500	5620
21	2400	3300	3850	—	2625	3500	5250	6560
24	2600	3600	4400	—	3000	4000	6000	7500
27	2800	3950	4600	—	3375	4500	6750	8440
30	3000	4300	4750	—	3750	5000	7500	9380
33	3150	4400	4875	—	4125	5500	8250	10,220
36	3300	4500	5000	—	4500	6000	9000	11,250
39	—	—	—	—	4825	6500	9750	
42	—	—	—	—	5250	7000	10,500	13,120
48	—	—	—	—	6000	8000	12,000	15,000
54	—	—	—	—	6750	9000	13,500	16,880
60	—	—	—	6000	7500	10,000	15,000	18,750
66	—	—	—	6600	8250	11,000	16,500	20,620
72	—	—	—	7200	9000	12,000	18,000	22,500

* Data from 1977 Annual Book of ASTM Standards as follows: nonreinforced concrete, Specification C14; reinforced concrete, Specification C76, American Society for Testing and Materials, Philadelphia, Pa.

† All strengths in pounds per linear foot. Pipe strength in kilonewtons per meter can be obtained by multiplying the values in the table by 0.0146.

stress. The yield point stress ranges between about 25,000 and 45,000 psi depending on the grade of steel used.

Because steel is susceptible to corrosion, special coatings are used to provide corrosion resistance. Commonly used coatings are coal tar enamels, various kinds of polymers, plastics, cement mortar, and zinc (galvanized pipe). Special situations may require cathodic protection.*

* For more details on both protective coatings and electrical protection to resist corrosion, see AWWA manuals (10, 11) and NBS publication (45).

For drain systems such as highway culverts and sewers, the wall thickness needed to withstand hydraulic design pressures is small; therefore, such thin-walled pipes are very susceptible to collapse under the action of the imposed external soil loads. To prevent this failure, corrugated steel pipe was developed. Corrugated steel pipe is available in diameters from 4 in. to 144 in.

Steel will expand about 3/4 in. for every 100 ft of length for a temperature increase of 100°F; therefore, expansion joints are needed in many installations to prevent excessive temperature stresses. A common type of expansion joint is the Dresser coupling, as shown on page 291 in Fig. 5-26.*

Ductile Iron

Ductile iron pipe is manufactured in diameters from 4 in. to 48 in. From 4-in. to 20-in. diameters, the standard commercial sizes are available in 2-in. steps, and from 24-in. to 48-in. diameters, the sizes are available in 6-in. steps. Ductile iron is relatively resistant to corrosion and can withstand relatively large external loads (such as soil pressure); therefore, it is used extensively for water and sewer systems. For added protection against corrosion, the outside of the pipe is usually coated with an asphaltic compound and the inside, with either a cement lining or asphaltic coating. The pipe can be obtained in thickness to withstand working pressures up to 350 psi (2.41 MPa).

The most common pipe joints are either the push-on bell and spigot type or the mechanical flange type. The bell and spigot is sealed by a rubber gasket. Dresser couplings can also be used on ductile iron pipe.†

Ductile iron is generally more costly than steel; however, because of its rigid characteristic, it is less apt than steel to collapse under negative pressure.

Concrete

Some concrete pipe was first installed in the 1800s, but the greatest use began near the beginning of the twentieth century. Concrete pipe is now used for storm and sanitary sewers, highway and railroad culverts, and pressure pipes. For small culverts and low-pressure irrigation systems, the pipe is often unreinforced; however, for the larger sizes or higher-pressure systems, reinforcing is required.‡

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* For more details on other types of expansion joints, as well as accepted procedures for design and installation of steel pipe, see AWWA manual no. 11 (12).

† For more details on ductile iron pipe, see the Cast Iron Pipe Association (16).

‡ For details on availability, strength, design, and installation of concrete pipe, see publications of the American Concrete Pipe Association (2, 3), AWWA manual no. M9 (9), and the ASTM manual on concrete pipe strength (8).

Asbestos Cement

This type of water pipe is composed of a nonmetallic mixture of asbestos, Portland cement, and silica, so it does not corrode in the usual sense. It is similar to ductile iron in that it is not as resistant to impact loading as steel. Asbestos cement water pipe is available in sizes from 3-in. to 36-in. diameter.

Plastic

The most common type of plastic pipe is polyvinylchloride (PVC). The main advantages of PVC pipe are its corrosion resistance, smoothness (less resistance to flow), and ease of field assembly. PVC pipe is used extensively for irrigation and sewer systems. Where plastic pipe must operate under high pressure, it is common to use the type reinforced with fiberglass for added strength.

5-9 Large Conduit Design

Scope and Use of Large Conduits

Large pressure conduits (6 ft and larger) are required in a variety of applications; however, the most typical cases are water-supply aqueducts for municipal or irrigation use, penstocks in hydropower installations, and outlet works associated with dams. In outlet works, the conduit is often a tunnel used to divert water around the dam during the project's construction. Then after completion of the dam, the tunnel is often used as part of the spillway system. Large conduits are usually made of steel or reinforced concrete. When the conduit is a tunnel, it is generally not economical to make it less than 6 ft in diameter because of the excessive cost of removal of material when excavating the tunnel. Tunnels in rock are constructed by two methods: drilling and blasting and boring. The tunnels may be unlined in sound rock, or they may be lined with reinforced concrete in fractured rock.

On some projects, there may be a combination of tunnel and pressure pipe in series depending on the specific topographic and geologic conditions of the project. Likewise, some tunnels may be lined with concrete, and others may be unlined, depending on the condition of the rock it is driven through. For example, the Apalachia project includes a conduit consisting of both tunnel and pressure pipes in series. The conduit is 8 mi (13 km) long and consists of about 2 mi of 22-ft diameter unlined tunnel, 4 mi of 18-ft concrete-lined tunnel, 1 mi of steel-lined tunnel, and several short lengths of 16-ft steel pipe in areas where the conduit was not tunneled.*

* For details of this project, see Goodhue (21) and Elder (18).

Table 5-5 Values of k_s for Different Conditions—Concrete Pipe (47)

Condition of Pipe	k_s (ft)	k_s (mm)
New pipe, unusually smooth surface, steel forms, smooth joints	0.0001	(0.030)
Fairly new pipe, smooth pipe, steel forms, average workmanship, some pockets on concrete surface, smooth joints	0.0005	(0.15)
Granular or brushed surface in good condition, good joints	0.0010	(0.30)
Centrifugally cast concrete pipe	0.0010	(0.30)
Rough surface eroded by sharp materials in transit; marks visible from wooden forms or spalling of surface	0.002	(0.61)
Unusually rough wood form work, erosion of surface, poor alignment at joints	0.003	(0.91)

Head Loss in Large Conduits

The same basic theory for head loss in conduits presented in Sec. 5-3, pages 243–62 applies to large conduits. However, certain features of large conduits need further consideration in this section. The economics of the design of large conduits is very important, and economic optimization involves the cost required to achieve flow at reduced head loss (smoother conduit) as opposed to the cost of a rougher but larger conduit. Also, if power production is a factor, as in the case of penstocks or tunnels for hydropower plants, the increased revenue that can be obtained with less head loss must be considered. Because of the costs of large conduits, much more attention must be given to their resistance coefficients than for many of the installations that use smaller pipes. The basic approach for estimating head loss is to first estimate the equivalent sand roughness, k_s, and then with the relative roughness, k_s/D, and the Reynolds number, we can obtain the resistance coefficient f from Fig. 5-4, page 247, or Eq. (5-17), page 257. We discuss the special roughness conditions that must be considered for different kinds of large conduits as follows.

Concrete Pipe The roughness of the pipe will of course depend on the surface finish of the pipe, which depends on the type of forms used in construction; however, other factors such as the degree of smoothness at the joints must also be considered. Sometimes concrete will erode with use, thereby causing the roughness to increase with age. Table 5-5 gives values of k_s for different conditions of concrete pipe.

Steel Pipe In steel pipe, the possibility of corrosion and incrustation of the surface with age must also be considered. Table 5-6 gives the values of k_s for different conditions of steel butt-welded pipe.

Table 5-6 Values of k_s for Different Conditions—Steel Butt-Welded Pipe (47)

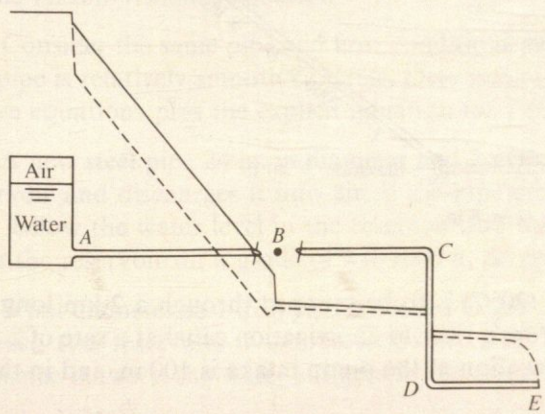
Condition of Pipe	k_s (ft)	k_s (mm)
New smooth pipe with centrifugally applied enamel on surface	0.0001	(0.03)
Hot asphalt dipped pipe	0.0003	(0.09)
Steel pipe with centrifugally applied concrete lining	0.0003	(0.09)
Light rust on surface	0.001	(0.30)
Heavy brush coated application of lining of asphalt or enamel	0.002	(0.61)
General tuberculation (1 to 3 mm in size) of surface	0.006	(1.8)
Severe tuberculation and incrustation	0.02	(6.1)

UNLINED TUNNELS The roughness varies with the degree of excess overbreak and the kind of rock that determines the angularity of the roughness. Therefore, it is difficult to make definitive recommendations for k_s; however, some guidelines can be given based on measurements in existing tunnels. Analysis of 20 unlined tunnels (6) shows that the minimum k_s was about 0.5 ft, and the maximum k_s was about 3.0 ft. For tunnels less than 15 ft in diameter, the average k_s was about 0.85 ft; for tunnels greater than 15 ft in diameter, the average k_s was about 1.42 ft.

These guidelines are for tunnels that were drilled and blasted. Tunnels constructed using a boring machine have much smoother walls. For example, equivalent sand roughness values (k_s's) for granite quartzite and other hard rocks are about 0.04 ft, whereas for mudstone, it can be as low as 0.005 ft.

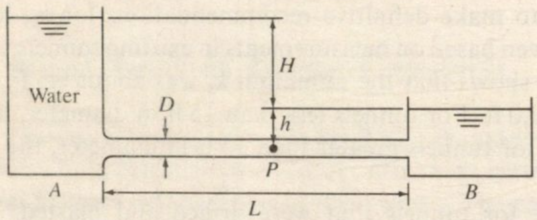
PROBLEMS

5-1 a. Shown below are the HGL and EGL for a pipeline. Indicate which is the HGL and which is the EGL. (continued on next page)

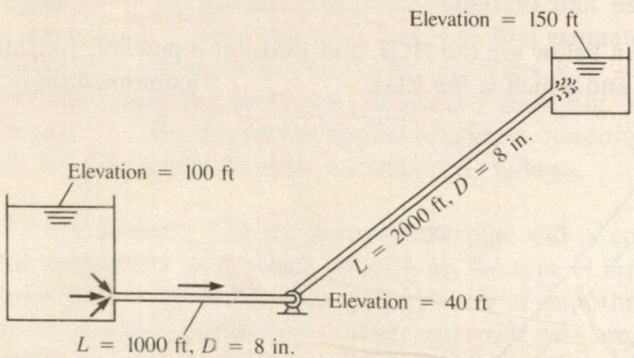


b. Are all pipes the same size? If not, which is the smallest?
c. Is there any region in the pipe where the pressure is below atmospheric pressure? If so, where?
d. Where is the point of maximum pressure in the system?
e. Where is the point of minimum pressure in the system?
f. What do you think is located at the end of the pipe at point *E*?
g. Is the pressure in the air in the tank above or below atmospheric pressure?
h. What do you think is located at point *B*?

5-2 Water flows from reservoir *A* to *B*. The water temperature in the system is 10°C, the pipe diameter *D* is 1 m, and the pipe length *L* is 300 m. If $H = 16$ m, $h = 2$ m, and if the pipe is steel, what will be the discharge in the pipe? In your solution, sketch hydraulic and energy grade lines. What will be the pressure at point *P* halfway between the two reservoirs?

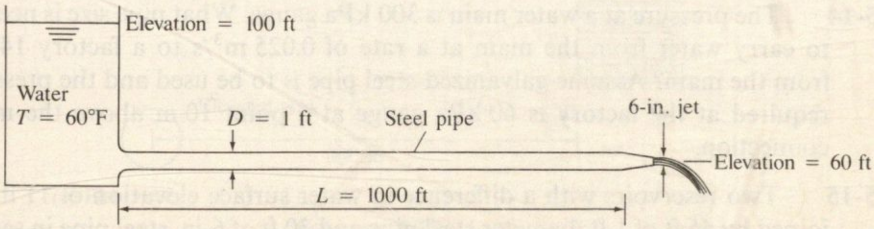


5-3 What horsepower must be supplied to the water to pump 2.5 cfs at 68°F from the lower to the upper reservoir? Assume the pipe is steel. Sketch the hydraulic and energy grade lines.



5-4 Irrigation water (20°C) is to be pumped through a 2-km long by 1-m diameter steel pipe from a river to an irrigation canal at a rate of 2.50 m^3/s. The water surface elevation at the pump intake is 100 m, and in the canal, it is 150 m. What power should be supplied to the pump if the pump efficiency is 82% and the inlet and outlet head losses are nil?

5-5 Water flows from the reservoir through a pipe and then discharges from a nozzle as shown. What is the discharge of water? Also draw the HGL and EGL for the system.



PROBLEM 5-5

5-6 Points *A* and *B* are 3 mi apart along a 24-in. new cast-iron pipe carrying water. Point *A* is 30 ft higher than *B*. If the pressure at *B* is 20 psi greater than at *A*, determine the direction and amount of flow. $T = 50°F$.

5-7 Compare the head loss for water flow ($Q = 4$ cfs) in a 12 in., 3000-ft long steel pipe using the Darcy-Weisbach equation with Fig. 5-4, page 247, and using the Hazen-Williams formula.

5-8 Compare the head loss for water flow ($Q = 200$ cfs) in a 72 in., 10,000-ft long steel pipe using the Darcy-Weisbach equation with the Hazen-Williams formula and with the Swamee-Jain formula.

5-9 Consider flow in a 5-ft diameter steel pipe that is coated with a very smooth enamel finish. Assume water ($T = 60°F$) flows in it with a velocity of 15 ft/s. Determine the head loss per 1000 ft using

a. the Darcy-Weisbach equation
b. the Mannings equation
c. the Hazen-Williams equation

5-10 Consider the same pipe and flow conditions as in Prob. 5-9 except that the pipe is relatively smooth concrete. Determine the head loss with all the above equations plus the explicit equation for f (Swamee's equation).

5-11 A new steel pipe 24 in. in diameter and 2 mi long carries water from a reservoir and discharges it into air. If the pipe comes out of the reservoir 10 ft below the water level in the reservoir and the pipe slopes downward from the reservoir on a grade of 2 ft/1000 ft, determine the discharge.

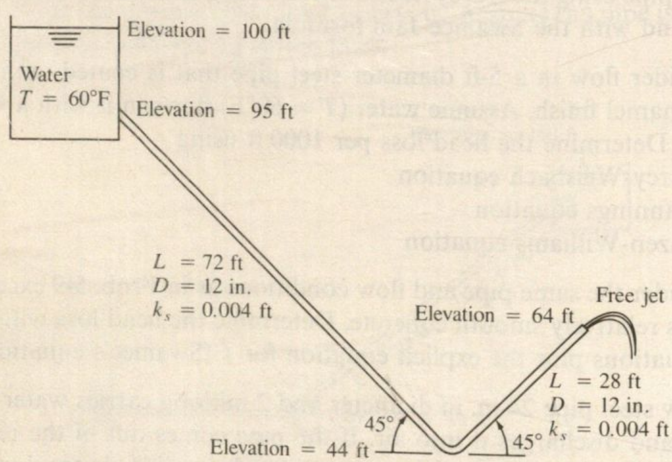
5-12 What diameter cast-iron pipe is needed to carry water at a rate of 10 cfs between two reservoirs if the reservoirs are 2 mi apart and the elevation difference between the water surfaces in the reservoirs is 20 ft?

5-13 Referring to Fig. 5-1, page 242, if the pipe diameter is 2.0 ft, its length is 500 ft, and the pump is made to operate as a turbine with $Q = 32$ cfs, what horsepower will be generated by the turbine? Assume the upper reservoir water surface is at elevation 200 ft and the water surface elevation in the lower one is 100 ft. Assume the turbine efficiency is 85% and the pipe is steel. Also draw the HGL and EGL for this system.

5-14 The pressure at a water main is 300 kPa gauge. What pipe size is needed to carry water from the main at a rate of 0.025 m^3/s to a factory 140 m from the main? Assume galvanized-steel pipe is to be used and the pressure required at the factory is 60 kPa gauge at a point 10 m above the main connection.

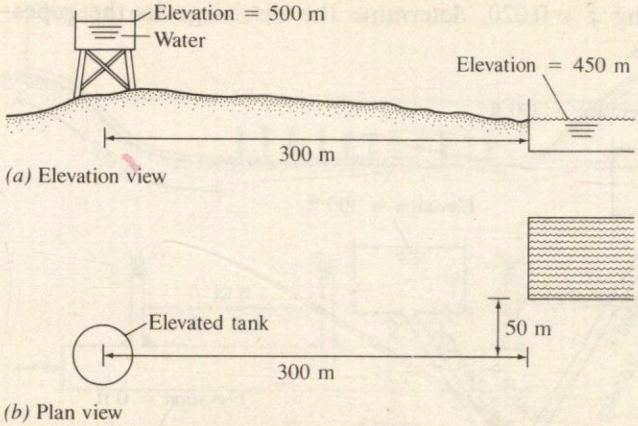
5-15 Two reservoirs with a difference in water surface elevation of 11 ft are joined by 45 ft of 1-ft diameter steel pipe and 30 ft of 6-in. steel pipe in series. The 1-ft line contains three bends ($r/D = 1$), and the 6-in. line contains two bends ($r/D = 4$). If the 1-ft and 6-in. lines are joined by an abrupt contraction, determine the discharge. $T = 60°$F.

5-16 a. Determine the discharge of water through the system shown.
b. Draw the hydraulic and energy grade lines for the system.
c. Locate the point of maximum pressure.
d. Locate the point of minimum pressure.
e. Calculate the maximum and minimum pressures in the system.



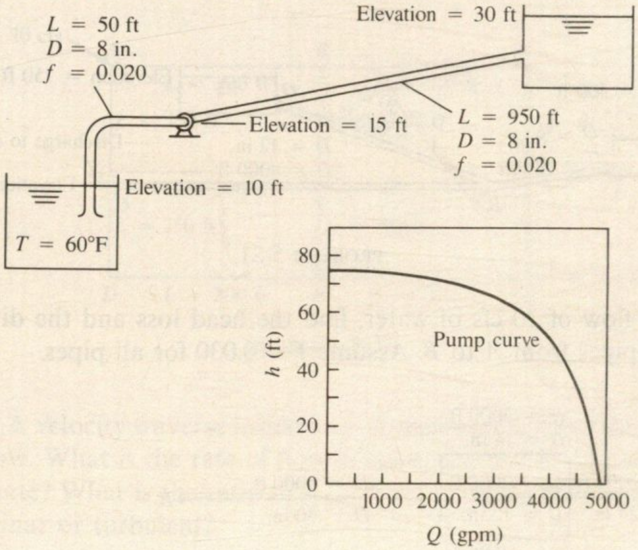
PROBLEM 5-16

5-17 Design a pipe system to supply water flow from the elevated tank to the reservoir at a discharge rate of 2.5 m^3/s.



PROBLEM 5-17

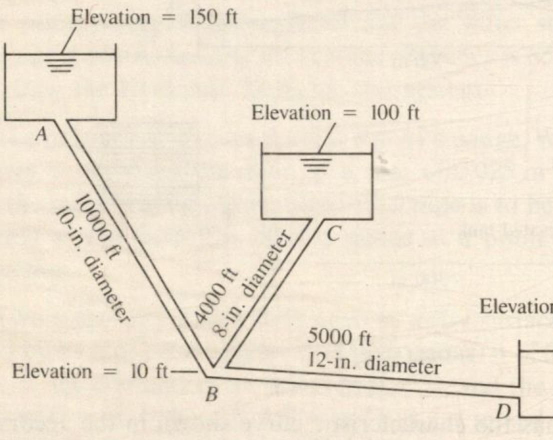
5-18 A pump that has the characteristic curve shown in the accompanying graph is to be installed in the system shown. What will be the discharge of water in the system?



PROBLEM 5-18

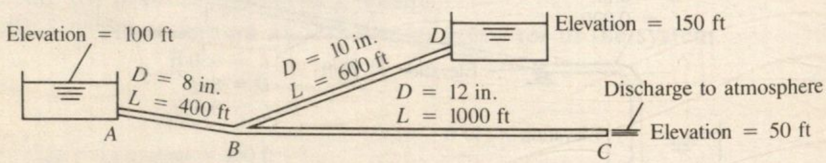
5-19 Solve for the distribution of flow in the network of Example 5-8, pages 273–76, if the loads at points D, E, and C are 0 cfs, 5 cfs, and 20 cfs, respectively. The sole source is at point A. Also, what are the pressures throughout the system if the pressure and elevation at point A are 500 ft and 50 psi, respectively? The elevations at points B, C, D, and E are 450 ft, 430 ft, 440 ft, and 480 ft, respectively. Assume $f = 0.012$.

5-20 Assuming $f = 0.020$, determine the discharge in the pipes. Neglect minor losses.



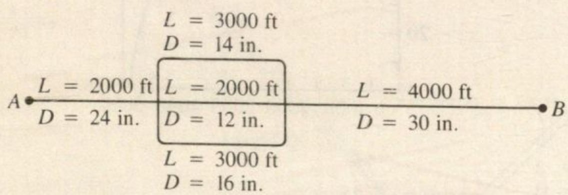
PROBLEM 5-20

5-21 Determine the water discharges in the pipes if they are all asphalted cast iron.



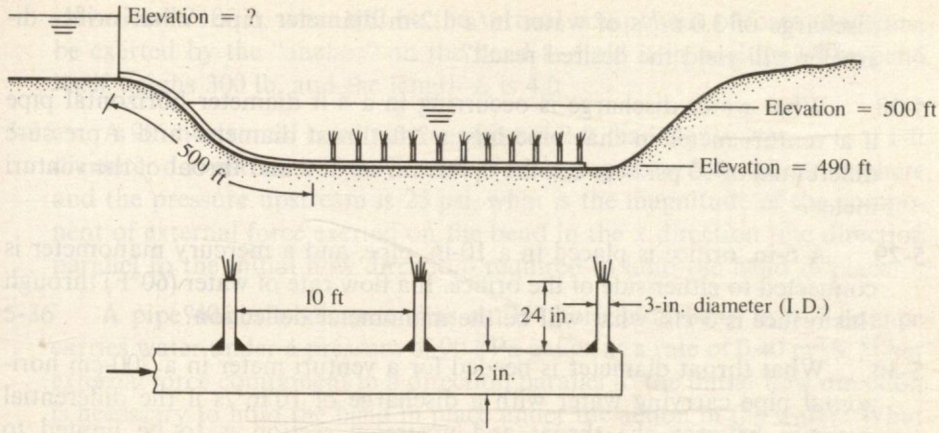
PROBLEM 5-21

5-22 With a flow of 20 cfs of water, find the head loss and the division of flow in the pipes from A to B. Assume $f = 0.030$ for all pipes.



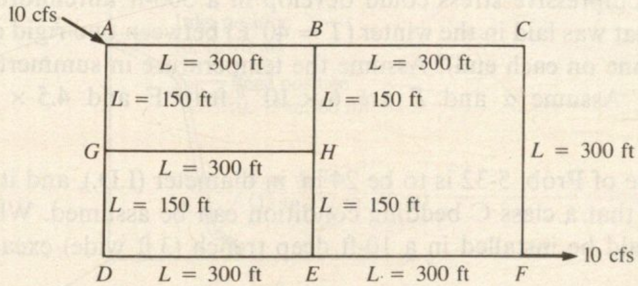
PROBLEM 5-22

5-23 This manifold is used to discharge heated effluent from a power plant into the Columbia River. There are ten discharge pipes spaced 10 ft apart (as shown), and the end pipe is to discharge water at a rate of 2.00 cfs. Determine the water surface elevation required in the reservoir and the total discharge.



PROBLEM 5-23

5-24 Determine the distribution of flow for the given system. All pipes are 1 ft in diameter, and assume $f = 0.015$. Also, what is the pressure at load F if the pressure at source A is 60 psig?



PROBLEM 5-24

5-25 A velocity traverse inside a 16-in. water pipe yields the data in the table below. What is the rate of flow in cubic feet per second and cubic feet per minute? What is the ratio of V_{max} to V_{mean}? Does it appear that the flow is laminar or turbulent?

y^* (in.)	0.0	0.1	0.2	0.4	0.6	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0	8.0
V (ft/s)	0.0	7.2	7.9	8.8	9.3	10.0	10.6	11.0	11.7	12.2	12.6	12.9	13.2	13.5

* Distance from pipe wall.

5-26 What size of orifice is needed to produce a change in head of 8 m for a discharge of 2 m^3/s of water in a 1-m diameter pipe.

5-27 An orifice is to be designed to have a change in pressure of 50 kPa across the orifice (measured with a differential pressure transducer) for a discharge of 3.0 m^3/s of water in a 1.2-m diameter pipe. What orifice diameter will yield the desired result?

5-28 What water discharge is occurring in a 4-ft diameter horizontal pipe if a venturi meter in that pipe has a 2-ft throat diameter and a pressure differential of 10 psi between the upstream section and throat of the venturi meter?

5-29 A 6-in. orifice is placed in a 10-in. pipe, and a mercury manometer is connected to either side of the orifice. If a flow rate of water (60°F) through this orifice is 3 cfs, what will be the manometer deflection?

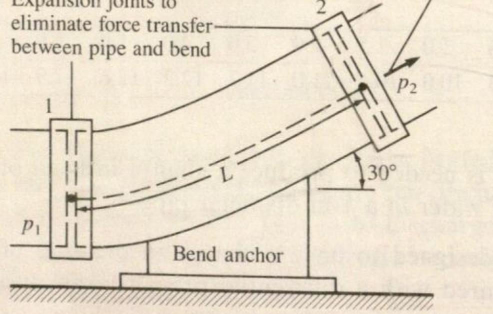
5-30 What throat diameter is needed for a venturi meter in a 200-cm horizontal pipe carrying water with a discharge of 10 m^3/s if the differential pressure between the throat and upstream section is to be limited to 200 kPa at this discharge?

5-31 Water flows through a venturi meter that has a 30-cm throat. The venturi meter is in a 60-cm pipe. What deflection will occur on a mercury-water manometer connected between the upstream and throat sections if the discharge is 0.57 m^3/s? Assume $T = 20°C$.

5-32 What compressive stress could develop in a 500-ft unreinforced concrete pipe that was laid in the winter ($T = 40°F$) between two rigid concrete structures (one on each end)? Assume the temperature in summertime can reach 80°F. Assume α and E are 6×10^{-6} ft/ft/°F and 4.5×10^6 psi, respectively.

5-33 The pipe of Prob. 5-32 is to be 24 in. in diameter (I.D.), and it is to be installed so that a class C bedding condition can be assumed. What class of pipe should be installed in a 10-ft deep trench (3 ft wide) excavated in clay.

5-34 This 30° vertical bend in a pipe having a 2-ft diameter carries water at a rate of 31.4 cfs. If the pressure p_1 is 10 psi at the lower end of the bend where the elevation is 100 ft, and p_2 is 8 psi at the upper end where the

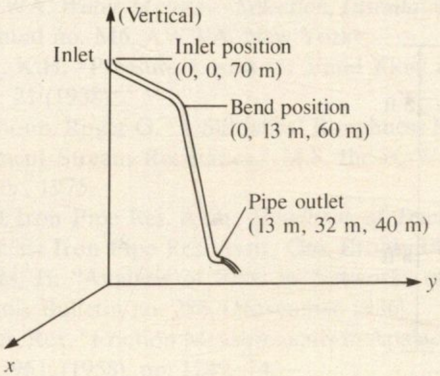


elevation is 103 ft, what will be the vertical component of force that must be exerted by the "anchor" on the bend to hold it in position? The bend itself weighs 300 lb, and the length L is 4 ft.

5-35 A 90° horizontal bend narrows from a 2-ft diameter upstream to a 1-ft diameter downstream. If the bend is discharging water into the atmosphere and the pressure upstream is 25 psi, what is the magnitude of the component of external force exerted on the bend in the x direction (the direction parallel to the initial flow direction) required to hold the bend in place?

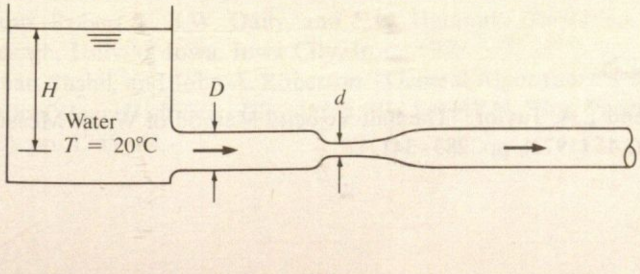
5-36 A pipe 40 cm in diameter has a 135° horizontal bend in it. The pipe carries water under a pressure of 90 kPa gauge at a rate of 0.40 m^3/s. What external force component in a direction parallel to the initial flow direction is necessary to hold the bend in place under the action of the water? What horizontal force component normal to the initial direction of flow is required to hold the bend?

5-37 This 130-cm overflow pipe from a small hydroelectric plant conveys water from the 70-m elevation to the 40-m elevation. The pressures in the water at the bend entrance and exit are 20 kPa and 25 kPa, respectively.

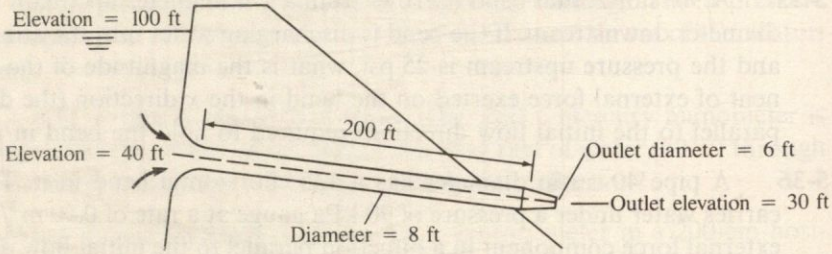


The bend interior volume is 3 m^3, and the bend itself weighs 10 kN. Determine the force that a thrust block must exert on the bend to secure it if the discharge is 15 m^3/s.

5-38 The pipe diameter D is 30 cm, d is 15 cm, and the atmospheric pressure is 100 kPa. What is the maximum allowable discharge before cavitation occurs at the throat of the venturi meter if $H = 5$ m?

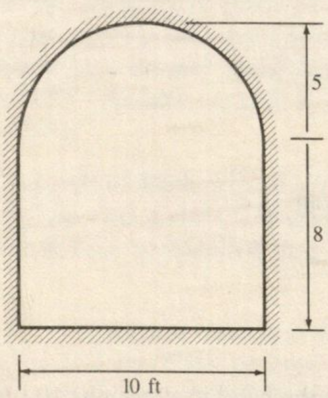


5-39 The sluiceway is steel lined and has a nozzle at its downstream end. What discharge may be expected under the given conditions? What force will be exerted on the joint that joins the nozzle and sluiceway lining?



5-40 An 18-in. pipe abruptly expands to a 24-in. size. These pipes are horizontal, and the discharge of water from the smaller size to the larger is 25 cfs. What horizontal force is required to hold the transition in place if the pressure in the 18-in. pipe is 10 psi? Also, what is the head loss?

5-41 Determine the head loss per 1000 ft in this tunnel that is lined with concrete and is to have a water discharge of 1000 cfs.



PROBLEM 5-41

5-42 Estimate the discharge of water in a tunnel 50% larger (linear dimension) than the one in Prob. 5-41 for a head loss per 1000 ft of 2 ft. Assume the tunnel is unlined, and make your own assumption(s) about the tunnel's degree of roughness.

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