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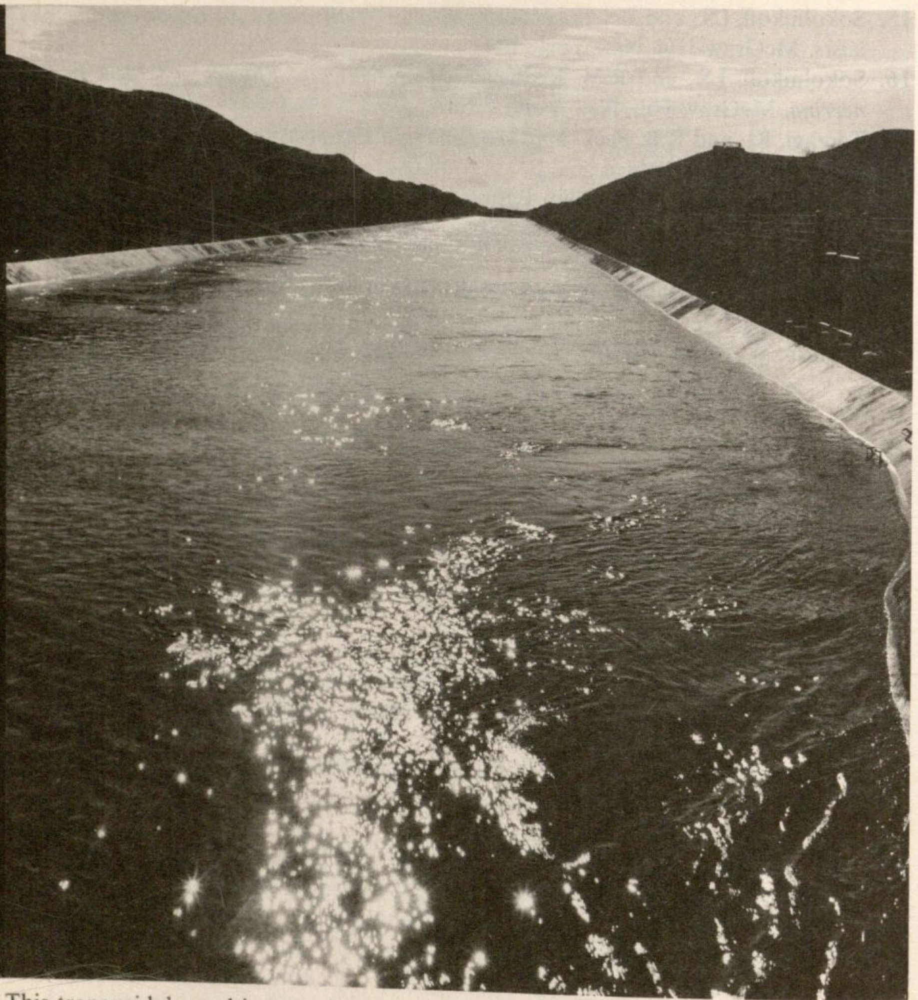


This trapezoidal canal is part of the Colorado River Aqueduct that conveys water to Southern California. This canal is 55 feet across at the top, 25 feet wide at the bottom width, 11 feet deep and delivers water at a maximum rate of 1,850 cfs. (Courtesy of Metropolitan Water District of Southern California.)

Open Channel Flow

4-1 General Considerations

Open channel flow is flow of a liquid in a conduit in which the upper surface of the liquid (that is, the free surface) is in contact with the atmosphere. Water flow in rivers and streams are obvious examples of open channel flow in natural channels. Other occurrences of open channel flow are flow in irrigation canals, sewer lines that flow partially full, storm drains, and street gutters.

The engineer may be required to solve problems having to do with either natural or manmade channels. In natural channels, the problem may be predicting the water surface profile along an extended reach of the river or stream given a certain discharge. Or one may be asked to estimate velocity and depth of flow in a local region of the channel. A more complex type of problem would be one in which the discharge, velocity, and depth along the channel are simultaneously a function of time and distance.

The same types of problems exist for already constructed manmade channels. However, one of the most challenging problems is the design and construction of the channel itself. In this case, as in all designs, the task is to produce a structure that will accomplish the desired result with minimum cost. For example, one might be asked to design a canal to convey water with a given maximum discharge from a reservoir to a power plant at some distance from the reservoir. Although it may be easy to design the canal for the stated conditions, the designer must consider all aspects of the problem, and many may not be envisioned at the outset. The design engineer will often have to consider complex situations that result from unusual natural hazards or abnormal operating procedures—for example, landslides into the reservoir or canal, accidental gate operation, or a breach of a canal embankment. Thus, the designer must envision the whole problem as well as find a satisfactory solution to the primary design objective.

4-2 Steady-Uniform Flow in Open Channels

Definition and Description of Uniform Flow

Uniform flow in a channel exists when there is no change of velocity along the channel. Under this condition, the convective acceleration is zero, and the streamlines are straight and parallel. Because the velocity does not change, the velocity head will be constant; therefore, the energy grade line and water surface will have the same slope as the channel bottom. For the flow to be uniform, the channel must be straight and without change in slope or cross section along the length of the channel. Such a channel is called a *prismatic* channel. When flow is uniform, the depth in the channel is called *normal depth.* Figure 4-1 depicts this condition.

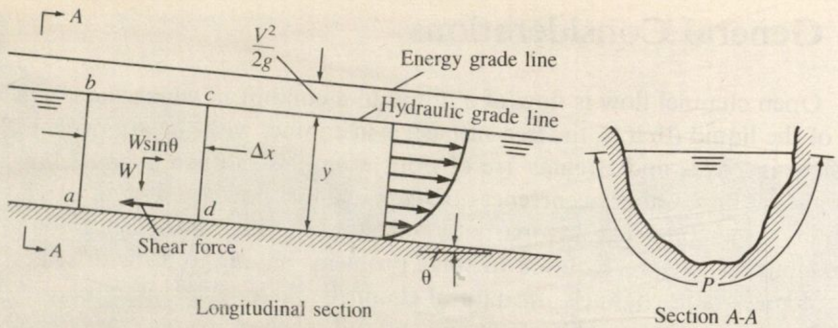


Figure 4-1 Uniform flow in a channel

Because the acceleration of fluid is zero for uniform flow, the net force acting on a mass of fluid, *abcd*, as shown in Fig. 4-1, will be zero. That is, the motive force (gravitational force component) will be equal and opposite to the resisting force of the channel bottom and wall. The motive force is equal to the component of weight of the fluid mass in the direction of flow ($W \sin \theta$). The resistance is equal to the product of the shear stress, τ_0, and the surface area of the channel ($P\,\Delta x$) that is in contact with the liquid. Thus, the force balance equation is

$$W \sin \theta - \tau_0 P\,\Delta x = 0$$

or

$$\gamma A\,\Delta x \sin \theta - \tau_0 P\,\Delta x = 0$$

$$\tau_0 = \gamma \frac{A}{P} \sin \theta \tag{4-1}$$

In Eq. (4-1), A/P is the hydraulic radius, R, and $\sin \theta$ is the slope of the channel S_0. The shear stress, τ_0, can be expressed as a function of the mass density, ρ, the mean velocity, V_0, and a resistance coefficient, c_f: $\tau_0 = c_f \rho (V_0{}^2/2)$. Therefore, Eq. (4-1) can be written as

$$c_f \rho \frac{V_0{}^2}{2} = \gamma R S_0$$

or

$$V_0 = \sqrt{\frac{2g}{c_f}} \sqrt{R S_0} \tag{4-2}$$

Equation (4-2) can also be given in the form

$$V = C \sqrt{R S_0} \tag{4-3}$$

This equation was first developed by Chezy, a French engineer of the eighteenth century.

Resistance Effects Using the Friction Factor

The resistance coefficient, c_f, and the flow coefficient C are both functions of the roughness of the channel bottom and wall and the depth of flow whose values are determined experimentally. In pipe flow, the resistance coefficient, c_f, is one fourth the value of the Darcy-Weisbach friction factor $f(c_f = f/4)$; therefore, Eq. (4-2) can be expressed as

$$V = \sqrt{\frac{8g}{f}}\sqrt{RS} \tag{4-4}$$

Equation (4-4) is a form of the Darcy-Weisbach equation applied to flow in open channels. In Eq. (4-4), the friction factor f is a function of the Reynolds number Re and the relative roughness, $k_s/4R$, and is usually given in graphical form, such as in the Moody diagram (see Fig. 5-5 on page 248). To obtain the discharge equation for open channels, simply multiply both sides of Eq. (4-4) by the cross-sectional area A, yielding

$$VA = \sqrt{\frac{8g}{f}}A\sqrt{RS_0}$$

or

$$Q = \sqrt{\frac{8g}{f}}A\sqrt{RS_0} \tag{4-5}$$

For fairly straight rock-bedded streams, the larger rocks produce most of the resistance to flow. Limerinos (13) has shown that the resistance coefficient f can be given in terms of the size of rock in the stream bed as

$$f = \frac{1}{\left(1.2 + 2.03\log\left(\frac{R}{d_{84}}\right)\right)^2} \tag{4-6}$$

where d_{84} is a measure of the rock size.*

EXAMPLE 4-1 Determine the discharge in a long, rectangular concrete channel that is 5 ft wide, that has a slope of 0.002, and in which the water depth is 2 ft.

* Most river-worn rocks are somewhat elliptical in shape. Limerinos (13) showed that the intermediate dimension correlates best with f. The d_{84} refers to the size of rock (intermediate dimension) for which 84% of the rocks in the random sample are smaller than the d_{84} size. Details for choosing the sample are given by Wolhman (25). The basic procedure entails sampling at least 100 rocks on the channel bottom. For example, a grid with 100 points could be laid out on the channel bottom, and the rock under each grid point would be a rock of the sample from which the d_{84} is determined.

SOLUTION Assume $k_s = 5 \times 10^{-3}$ ft. This is an intermediate value of roughness as given in Chapter 5 (page 248). The hydraulic radius is

$$R = \frac{A}{P} = \frac{(2 \times 5)}{[(2 \times 2) + 5]} = 1.11 \text{ ft}$$

Then $$\frac{k_s}{4R} = \frac{(5 \times 10^{-3})}{(4 \times 1.11)} = 1.13 \times 10^{-3}.$$

Using the Moody diagram (see Fig. 5.5), the f value is found to be about 0.020 for a Reynolds number of about 10^6. Use this f for the first computation of Q:

$$Q = \sqrt{\frac{8 \times 32.2}{0.020}} \times (2 \times 5) \times 1.11 \times 0.002$$

$$Q = 53.5 \text{ cfs} \qquad V = Q/A = 5.35 \text{ ft/s}$$

Assume the water temperature is 60°F, then, the kinematic viscosity will be 1.22×10^{-5} ft²/s, and the Reynolds number $V \times 4R/v$ will be found to be 1.95×10^6. On checking the Moody diagram with the Reynolds number of 1.95×10^6, we find that f is indeed 0.020; therefore, with the given assumptions, the discharge will be 53.5 cfs. ■

The Manning Equation

The discharge equation most often used by hydraulics engineers is the Manning equation, named after an Irish engineer of the nineteenth century. In the Manning equation, using the English system of units, the Chezy coefficient of Eq. (4-3) is given as $C = (1.49/n) \times R^{1/6}$, where n is a resistance factor. Thus, the Manning discharge equation is

$$Q = \frac{1.49}{n} A R^{2/3} S_0^{1/2} \tag{4-7a}$$

In the SI system of units,

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} \tag{4-7b}$$

The resistance factor, n, is a function of a number of variables, the primary one being the roughness of the channel.* To assist the engineer in choosing n, Table 4-1 gives n values for various types and conditions of channels. Figures 4-2, 4-3, and 4-4 are photos of actual channels along with measured n values.

* For a more complete discussion of n values, see Chow (4).

Table 4-1 Typical Values of the Roughness Coefficient n

Lined Canals	n
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, troweled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight, with some grass	0.026
Earth, winding, no vegetation	0.030
Earth, winding	0.050

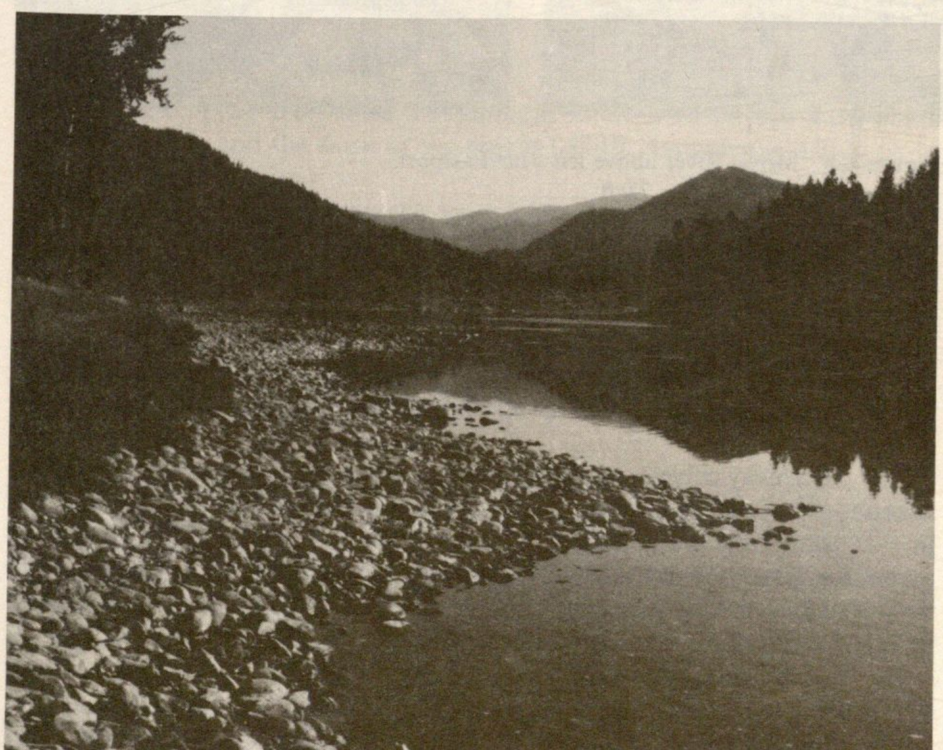
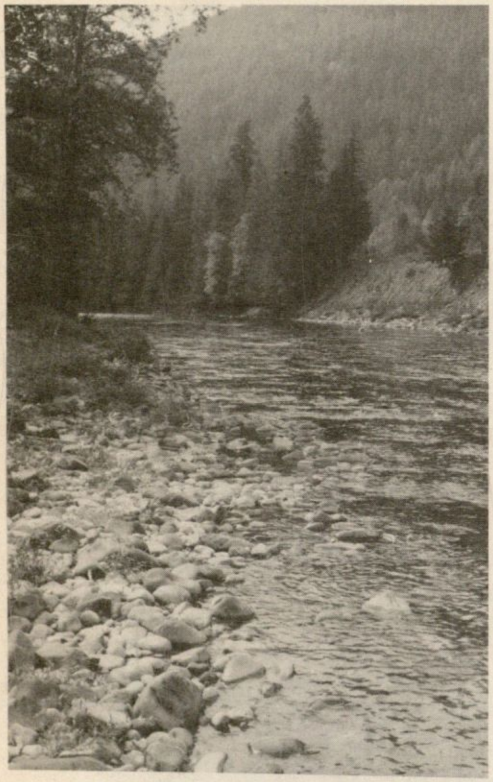


Figure 4-2 Clark Fork River near St. Regis, Montana — $n = 0.028$ for $R = 16$ ft, $d_{84} = 205$ mm



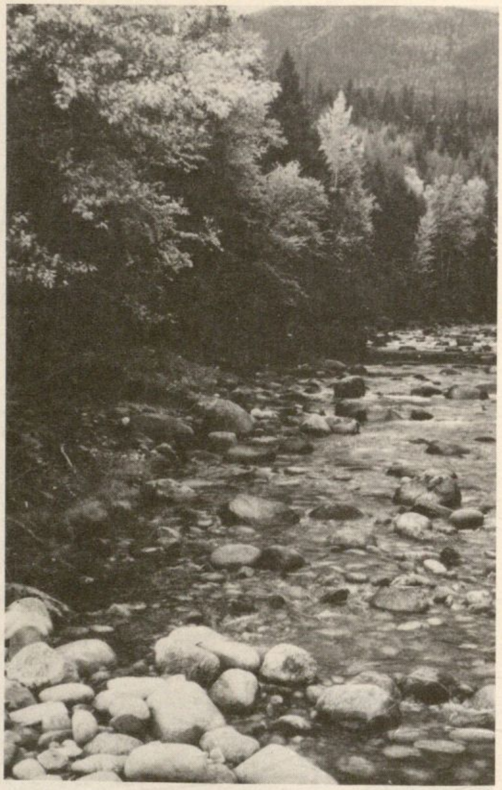


Figure 4-3 Moyie River above left near Eastport, Idaho—$n = 0.038$ for $R = 7.0$ ft

Figure 4-4 Boundary Creek above right near Porthill, Idaho—$n = 0.073$ for $R = 4.0$ ft, $d_{84} = 375$ mm

Flow in Conduits of Circular Cross Section

Highway culverts and city sewers are common examples of open channel conduits of circular cross section. For a given slope and resistance coefficient (n = constant), the discharge will be proportional to $AR^{2/3}$, as can be seen by inspecting Eqs. (4-7a and 4-7b). Since $V = Q/A$, it can also be deduced that the velocity will be proportional to $R^{2/3}$. Therefore, one can easily determine $Q/Q_0 = (AR^{2/3})/(A_0R_0^{2/3})$, where Q is the discharge for a given depth of flow, and Q_0 is the discharge for the completely full conduit. Figure 4-5 is a plot of the relative discharge (Q/Q_0) versus relative depth (y/d_0) for a circular conduit. The relative velocity is obtained by dividing the relative discharge by the relative area (A/A_0). The relative velocity is also shown in Fig. 4-5. *Note*: The maximum discharge and velocity occur at a depth less than that for full flow condition. This occurs

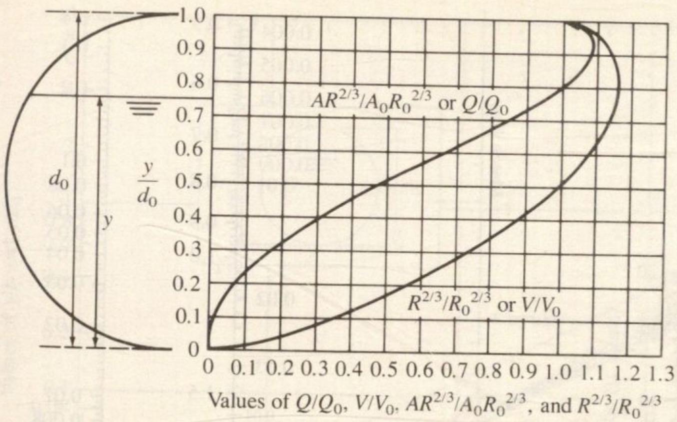


Figure 4-5 Flow characteristics of a circular section assuming constant n value

because as the conduit becomes nearly full, the perimeter increases much faster than the increase in cross-sectional area, thus decreasing the hydraulic radius and discharge.

Figure 4-5 along with Fig. 4-6, page 172, (a nomograph for solving Manning's equation) can be used for easily solving uniform flow problems in circular conduits.

EXAMPLE 4-2 Determine the discharge in a 3-ft sewer pipe if the depth of flow is 1.00 ft, and the slope of the pipe is 0.0019. Assume $n = 0.012$.

SOLUTION For this example we use Fig. 4-6 by drawing a straight line through the points for $n = 0.012$ and $S = 0.0019$. Note where the straight line intersects the match line. Then draw a line through the point of intersection on the match line and the 3 ft diameter point. It is noted that this line intersects the discharge scale at $Q_0 = 30$ cfs which is the discharge for the full flow condition. The relative depth y/d_0 is $1.00/3 = 0.333$; therefore, $Q/Q_0 = 0.20$ (from Fig. 4-5), so $Q = 0.20 \times 30 = 6.0$ cfs. ■

Flow in Conduits of Trapezoidal Cross Section

To assist in solving problems involving the flow in trapezoidal channels, the factor $AR^{2/3}$ of Eq. (4-7) is plotted in Fig. 4-7, page 173, in dimensionless form as a function of the relative depth (y/b), where b is the bottom width of the channel.

EXAMPLE 4-3 Determine the normal depth for a trapezoidal channel with side slopes of 1 vertical to 2 horizontal, a bottom width of 8 ft, discharge of 200 cfs, channel bottom slope of 1.0 ft in 1000 ft, and $n = 0.012$.

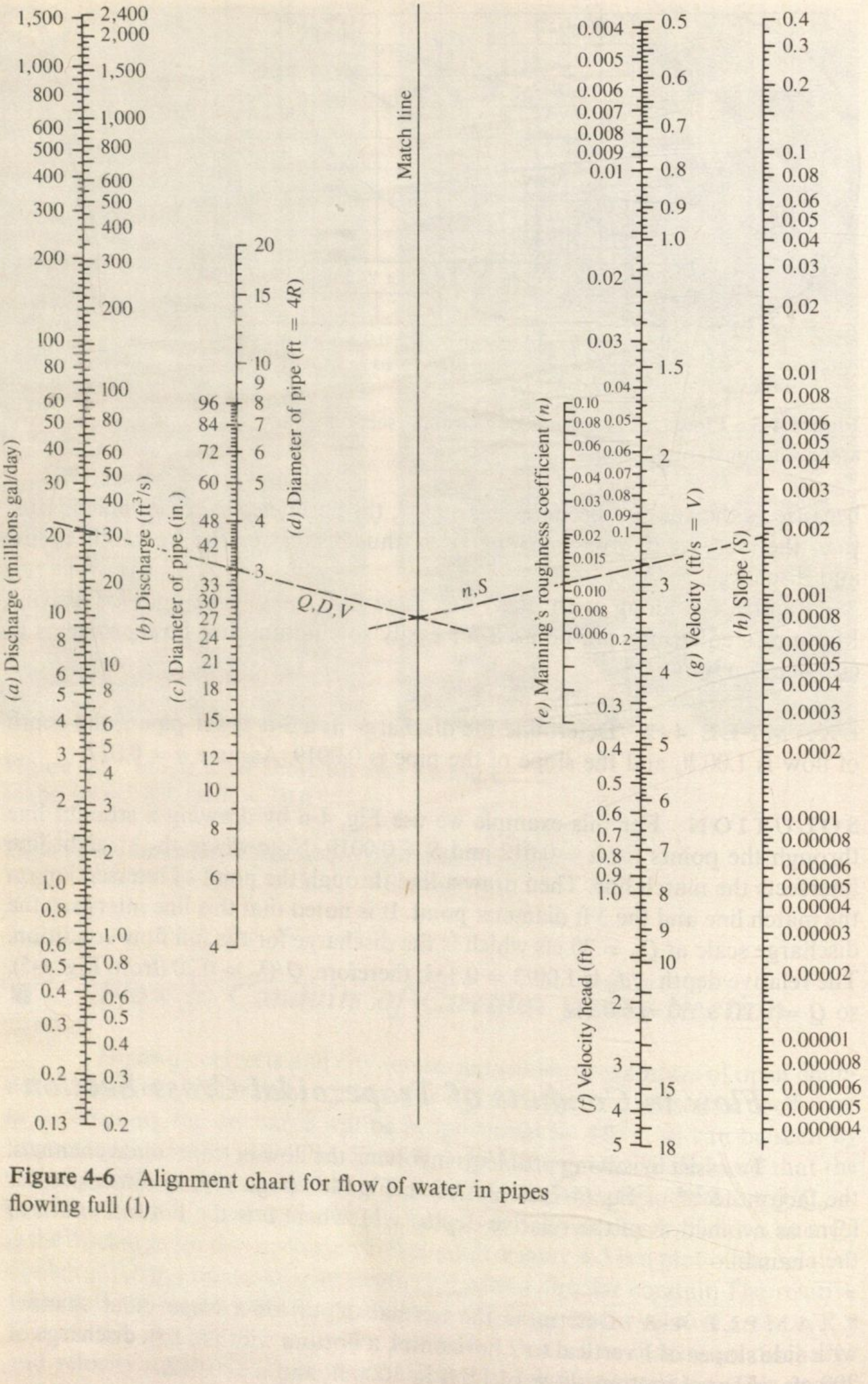


Figure 4-6 Alignment chart for flow of water in pipes flowing full (1)

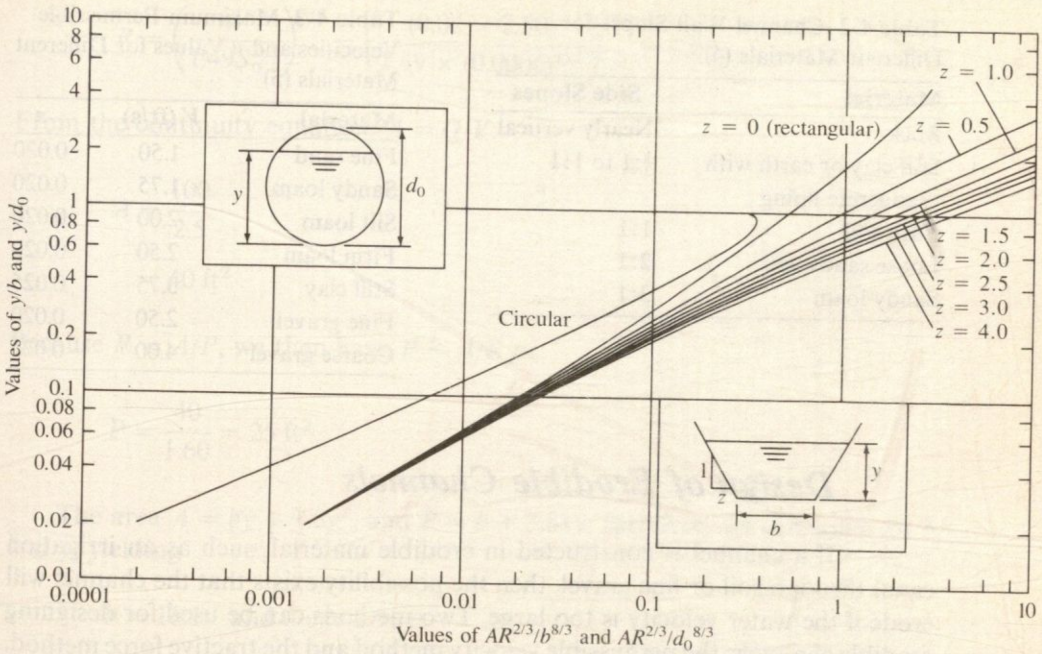


Figure 4-7 Curves for determining the normal depth [adapted from *Open Channel Hydraulics* by Chow (4) Copyright © 1959, McGraw-Hill Book Company, New York; used with permission of McGraw-Hill Book Company.]

SOLUTION With a little algebraic manipulation so that we can use Fig. 4-7, Eq. (4-7a) can be written as

$$AR^{2/3} = \frac{Qn}{(1.49\, S_0^{1/2})}$$

or

$$\frac{AR^{2/3}}{b^{8/3}} = \frac{Qn}{1.49\, S_0^{1/2} b^{8/3}} \tag{4-8}$$

Since $Q = 200$ cfs, $n = 0.012$, $S_0 = 0.001$, and $b = 8$ ft, we can evaluate the right-hand side of Eq. (4-8):

$$\frac{AR^{2/3}}{b^{8/3}} = \frac{200 \times 0.012}{1.49 \times (0.001)^{1/2} \times 8^{8/3}} = 0.199$$

Then for $AR^{2/3}/b^{8/3} = 0.199$, one determines that $y/b = 0.33$ (from Fig. 4-7). Thus,

$$y_n = 0.33b \quad \text{or} \quad y_n = 2.64 \text{ ft}$$

■

Table 4-2 Channel Wall Slopes for Different Materials (4)

Material	Side Slopes
Rock	Nearly vertical
Stiff clay or earth with concrete lining	$\frac{1}{2}$:1 to 1:1
Firm soil	1:1
Loose sandy soil	2:1
Sandy loam	3:1

Table 4-3 Maximum Permissible Velocities and *n* Values for Different Materials (5)

Material	*V* (ft/s)	*n*
Fine sand	1.50	0.020
Sandy loam	1.75	0.020
Silt loam	2.00	0.020
Firm loam	2.50	0.020
Stiff clay	3.75	0.025
Fine gravel	2.50	0.020
Coarse gravel	4.00	0.025

Design of Erodible Channels

If a channel is constructed in erodible material, such as an irrigation canal through soil or fine gravel, then the possibility exists that the channel will erode if the water velocity is too large. Two methods can be used for designing erodible channels: the permissible velocity method and the tractive force method. In this text, we will discuss only the *permissible velocity method*.*

Assuming that the channel will be of trapezoidal cross section, the first decision the designer must make is to choose the appropriate side slope for the channel. A slope should be chosen that will be stable under all conditions. Given the type of material, one can apply basic soil mechanics to determine a suitable slope. Table 4-2 gives approximate permissible side slopes for different materials, and Table 4-3 gives approximate permissible velocities for different materials.

Once Q, V, n, S_0, and the basic channel shape have been determined, we can solve for the depth and width of the channel. Example 4-4 illustrates that procedure.

EXAMPLE 4-4 An unlined irrigation canal is to be constructed in a firm loam soil. The slope is to be 0.0006, and it is to carry a water flow of 100 cfs. Determine an appropriate cross section for this canal.

SOLUTION Using Table 4-2 as a guide, choose a side slope of $1\frac{1}{2}$ horizontal to 1 vertical (this is a conservative choice). From Table 4-3, choose a maximum permissible $V = 2.50$ ft/s, and $n = 0.020$. Now we get the hydraulic radius R from the Manning velocity equation:

$$V = \frac{1.49}{n} R^{2/3} S_0^{1/2}$$

* For information on the more sophisticated *tractive force method*, see Chow (4).

$$R = \left(\frac{nV}{1.49S_0^{1/2}}\right)^{3/2} = \left(\frac{(0.02 \times 2.50)}{1.49 \times (0.0006)^{1/2}}\right)^{3/2} = 1.60$$

From the continuity equation, $A = Q/V$:

$$A = \frac{100}{2.5}$$

$$= 40 \text{ ft}^2$$

Because $R = A/P$, we then have $P = A/R$ or

$$P = \frac{40}{1.60} = 25 \text{ ft}^2$$

The area $A = by + 1.5y^2$, and $P = b + 3.61y$; therefore, we can solve for b and y, yielding

$$b = 18.1 \quad \text{and} \quad y = 1.91 \text{ ft}$$

For ease of construction, use $b = 18$ ft and $y = 2.0$ ft. ■

Best Hydraulic Section

The quantity $AR^{2/3}$ in Manning's equation (Eq. 4-7) is called the section factor in which $R = A/P$; therefore, the section factor relating to uniform flow is given by $A(A/P)^{2/3}$. Thus, for a channel of given resistance and slope, the discharge will increase with increasing cross-sectional area but decrease with increasing wetted perimeter P. For a given area, A, and a given shape of channel, for example, rectangular cross section, there will be a certain ratio of depth to width (y/b) for which the section factor will be maximum. Such a ratio establishes the *best hydraulic section*. That is, the best hydraulic section is the channel proportion that yields a minimum wetted perimeter for a given cross-sectional area.

EXAMPLE 4-5 Determine the best hydraulic section for a rectangular channel.

SOLUTION For the rectangular channel, $A = by$, and $P = b + 2y$. Let A be constant then let us minimize P. But

$$P = b + 2y$$

or

$$P = \frac{A}{y} + 2y$$

Thus, we see that the perimeter varies only with y for the given conditions. If we differentiate P with respect to y and set the differential equal to zero, we will have the condition for minimizing P for the given area A:

$$\frac{dP}{dy} = \frac{-A}{y^2} + 2 = 0$$

or

$$\frac{A}{y^2} = 2$$

But $A = by$, so

$$\frac{by}{y^2} = 2 \quad \text{or} \quad y = \frac{1}{2}b$$

Thus, the best hydraulic section for a rectangular channel occurs when the depth is one half the width of the channel. ■

It can be shown that the best hydraulic section for a trapezoidal channel is half a hexagon; for the circular section, it is the half circle, and for the triangular section, it is half of a square. Of all the various shapes, the half circle has the best hydraulic section.

The best hydraulic section can be relevant to the cost of the channel. For example, if a trapezoidal channel were to be excavated and if the water surface were to be at ground level, the minimum amount of excavation would result if the channel of best hydraulic section were used, and the minimum cost would result if only excavation were involved. However, many other factors are involved in designing the most economical channel. For example, if the water surface lies below ground level, the best hydraulic section will not result in minimum volume of excavation. Thus, the best hydraulic section should be used only as a guide or starting point in designing a channel.

Project Scope

In the preceding sections, we discussed methods for considering uniform flow in channels of various shapes and roughness. However, water-resources projects may include other structures in addition to the channels. For example, consider an irrigation project for which the irrigation water is drawn from a river and distributed to the irrigable land. For a project like this, *intake works, flumes, checks, drops,* and *transitions* may all be included as part of the water distribution system. The next paragraphs scope the need for such structures.

Figure 4-8 shows the layout of a typical irrigation project. Across the river, a *diversion dam* is constructed so as to maintain a water level high enough in the river to be able to always divert the required flow of water into the main irri-

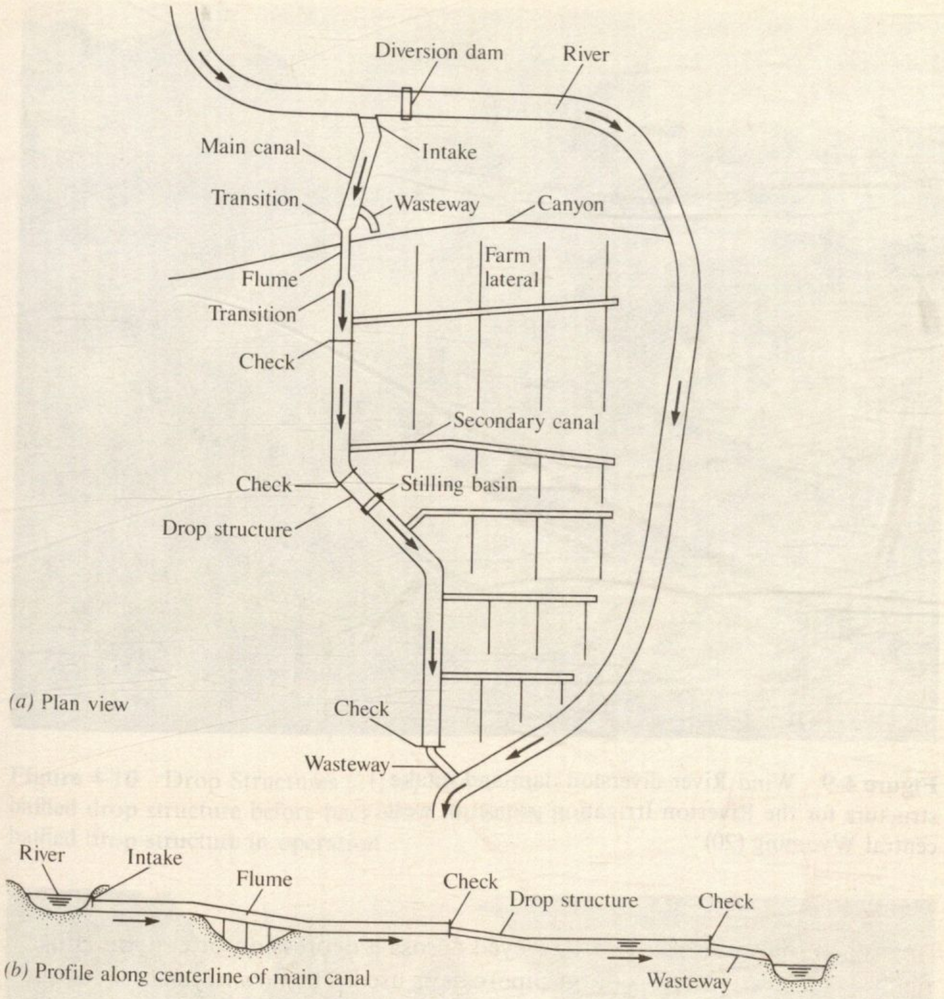


Figure 4-8 Layout of irrigation system (a) Plan view (b) Profile along centerline of main canal

gation canal. The intake to the main canal consists of a canal entrance structure, including gates for controlling the discharge into the canal. Figure 4-9 on the next page is a photograph of a diversion dam and intake structure for the Riverton Irrigation Project in Wyoming.

Wasteways are often provided at intervals along the main canal to prevent overtopping of the canal if an emergency develops where downstream water use is stopped. Then the unused water is wasted into a natural channel such as in the canyon shown in Fig. 4-8.

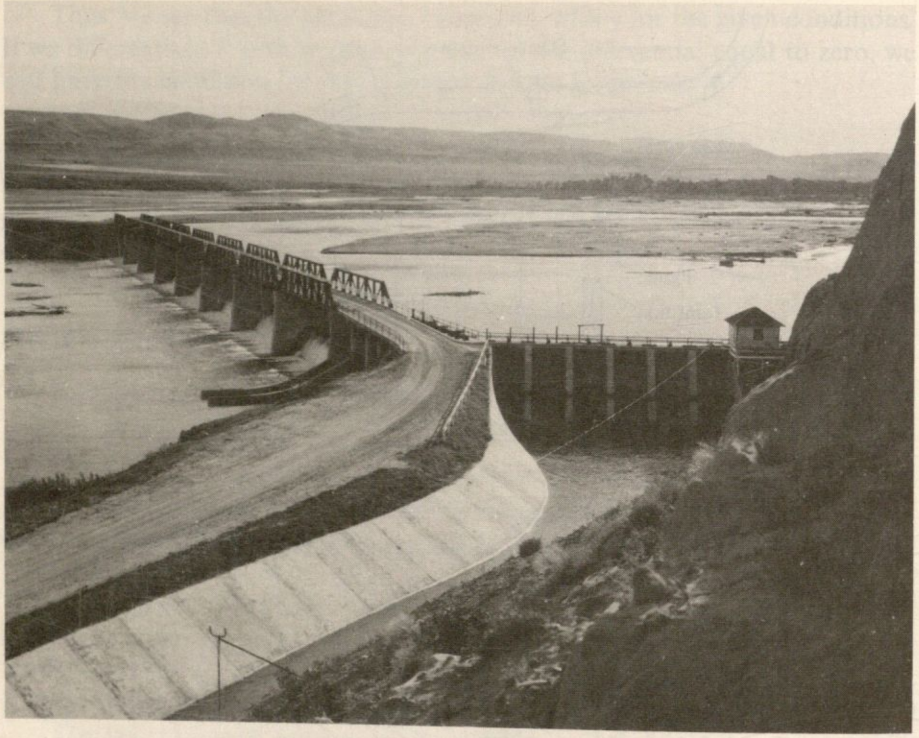


Figure 4-9 Wind River diversion dam and intake structure for the Riverton Irrigation project in west central Wyoming (20)

When the water must be conveyed across a depression or canyon either a flume or an *inverted siphon* (large pipe) can be used. Then *transitions* are required to provide a smooth passage of flow from the canal to the flume or inverted siphon and back again.

A *check structure* is a concrete lined part of the canal in which a gate or stop logs are installed to maintain a high enough water surface level in the canal so that water can be diverted into a secondary canal. Some checks are equipped with automatic water level devices.

If there is a significant drop in the land surface, increased slope of a canal in soil may lead to undesirable erosion. In these cases, a concrete *drop structure* may be needed. Figure 4-10 shows the basic features of a baffled drop structure.

The secondary canals draw water from the main canal through their own intake structures, and farm laterals take water from the secondary canals.

The preceeding paragraphs are to help you understand how various structures can be incorporated to achieve a complete workable project. The example chosen (irrigation system) includes the basic structures that would be encountered in almost any open channel project even though it may not be agricul-

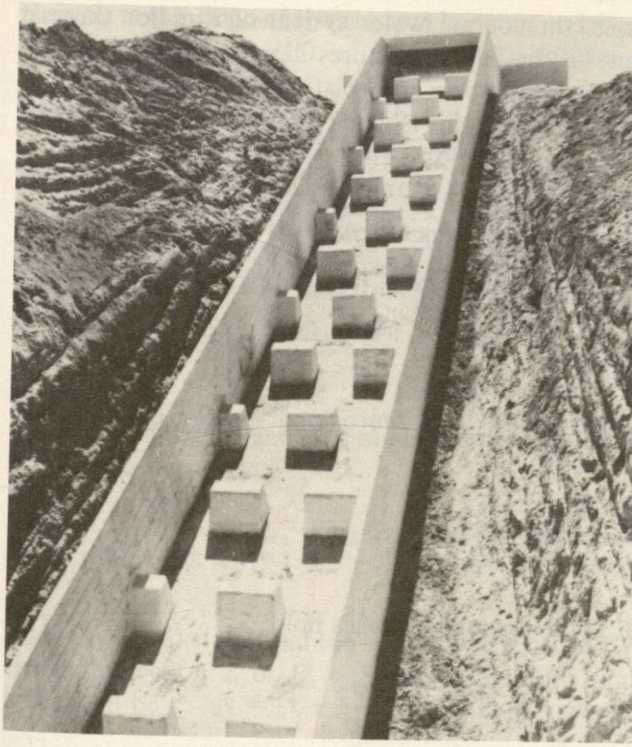
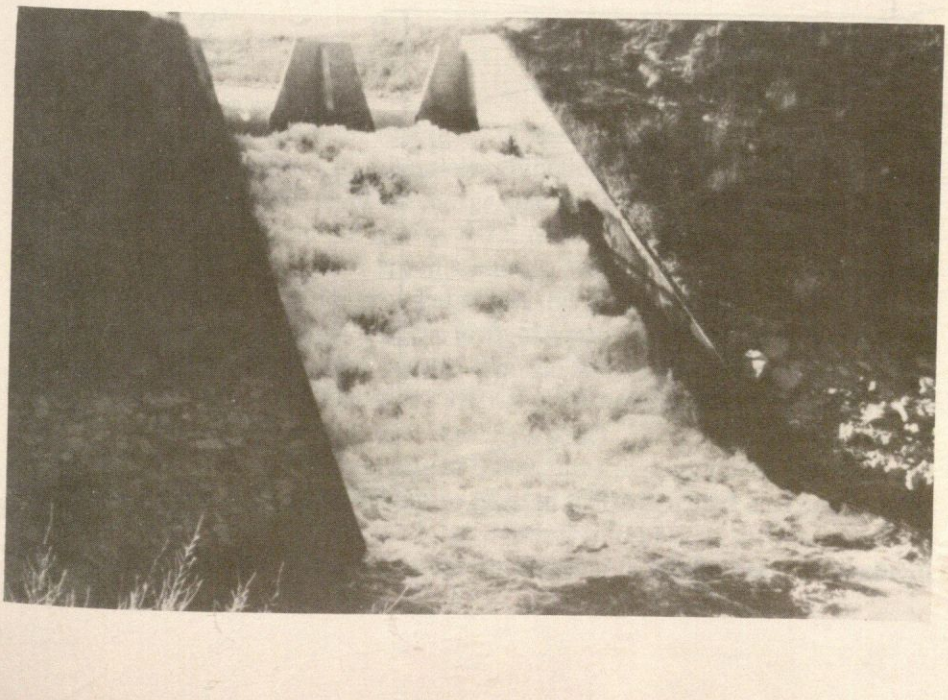


Figure 4-10 Drop Structures (21) (a) Above is a baffled drop structure before backfilling (b) Below is a baffled drop structure in operation



turally oriented. For example, the cooling water system of a major thermal power plant would use many of the basic structures described for the irrigation system. Therefore, if you can develop an appreciation for the basic function of each structure rather than view it as an isolated part of a specific project then your awareness of design considerations will be greatly expanded.

We discuss further details about open channel structures in Chapter 7.

4-3 Steady-Nonuniform Flow in Open Channels

Energy Relations

THE ENERGY EQUATION The one-dimensional energy equation for open channels (see Fig. 4-11) is

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \tag{4-9}$$

We see from Fig. 4-11 that the following equalities hold:

$$\frac{p_1}{\gamma} + z_1 = y_1 + S_0\,\Delta x \quad \text{and} \quad \frac{p_2}{\gamma} + z_2 = y_2$$

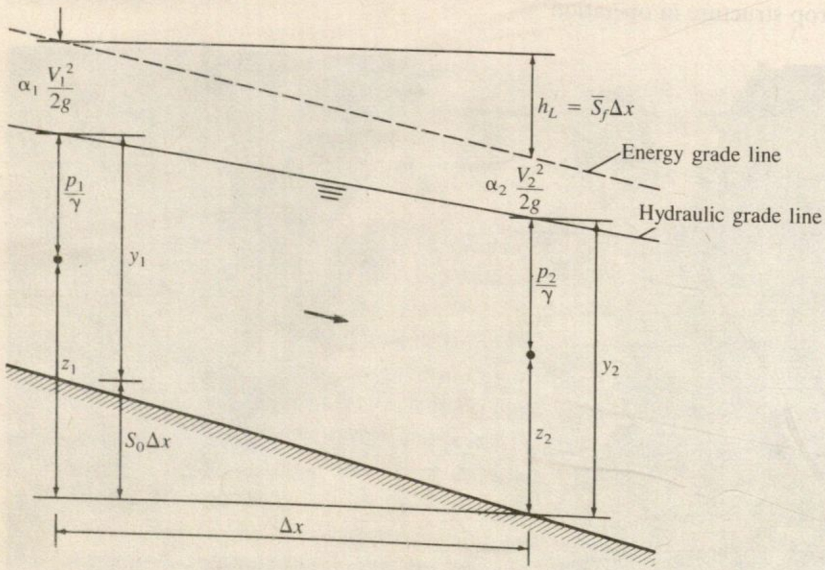


Figure 4-11 Definition sketch for flow in open channels

Here S_0 is the slope of the channel bottom, and y is the depth of flow. Then if we assume $\alpha_1 = \alpha_2 = 1.0$, we can write Eq. (4-9) as

$$y_1 + \frac{V_1^{\,2}}{2g} + S_0\,\Delta x = y_2 + \frac{V_2^{\,2}}{2g} + h_L \tag{4-10}$$

Now, if we consider the special case where the channel bottom is horizontal ($S_0 = 0$), and the head loss is zero ($h_L = 0$), Eq. (4-10) becomes

$$y_1 + \frac{V_1^{\,2}}{2g} = y_2 + \frac{V_2^{\,2}}{2g} \tag{4-11}$$

SPECIFIC ENERGY The sum of the depth of flow and the velocity head is the specific energy:

$$E = y + \frac{V^2}{2g} \tag{4-12}$$

Thus Eq. (4-11) states that the specific energy at section 1 is equal to the specific energy at section 2, or $E_1 = E_2$. The continuity equation between sections 1 and 2 will be

$$A_1 V_1 = A_2 V_2 = Q \tag{4-13}$$

Therefore, Eq. (4-11) can be expressed as

$$y_1 + \frac{Q^2}{2gA_1^{\,2}} = y_2 + \frac{Q_2}{2gA_2^{\,2}} \tag{4-14}$$

Because A_1 and A_2 are both functions of the depth y, the magnitude of the specific energy at section 1 or 2 is solely a function of the depth at each section. If, for a given channel and given discharge, one plots depth versus specific energy, a relationship such as shown in Fig. 4-12, page 182, is obtained. By studying Fig. 4-12 for a given value of specific energy, we can see that the depth may be either large or small. In a physical sense, this means that for the low depth, the bulk of the energy of flow is in the form of kinetic energy ($Q^2/2gA^2$); whereas for a larger depth, most of the energy is in the form of potential energy. Flow under a *sluice gate* (Fig. 4-13, page 182) is an example of flow in which two depths occur for a given value of specific energy. The large depth and low kinetic energy occurs upstream of the gate; the low depth and large kinetic energy occurs downstream. The depths as used here are called *alternate depths*. That is, for a given value of E, the large depth is alternate to the low depth, or vice versa. Returning to the flow under the sluice gate, we find that if we main-

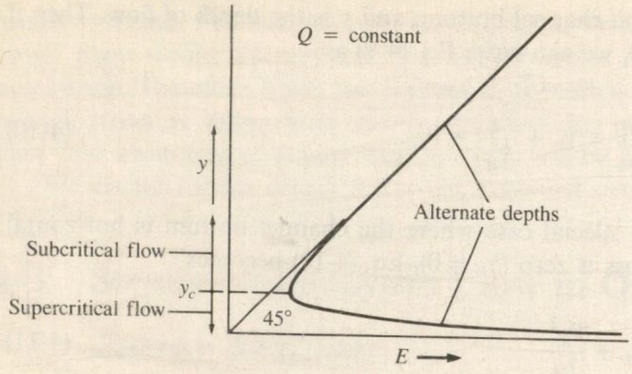


Figure 4-12 Relation of depth versus specific energy

tain the same rate of flow but set the gate with a larger opening, as in Fig. 4-13*b*, the upstream depth will drop, and the downstream depth will rise. Thus we have different alternate depths and a smaller value of specific energy than before. This is consistent with the diagram in Fig. 4-12.

Finally, it can be seen in Fig. 4-12 that a point will be reached where the specific energy is minimum and only a single depth occurs. At this point, the flow is termed *critical*. Thus one definition of critical flow is the flow that occurs when the specific energy is minimum for a given discharge. The flow for which the depth is less than critical (velocity is greater than critical) is termed *supercritical flow*, and the flow for which the depth is greater than critical (velocity is less than critical) is termed *subcritical flow*. Using this terminology, we can see

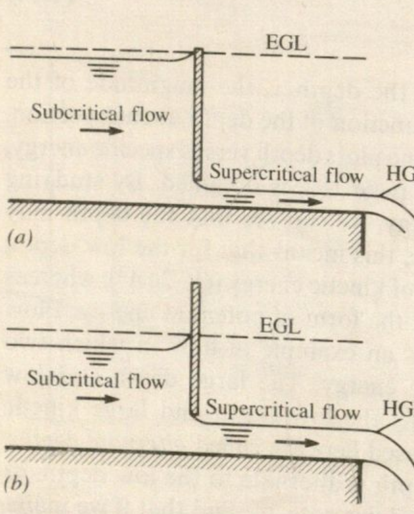


Figure 4-13 Flow under a sluice gate

that subcritical flow occurs upstream and supercritical flow occurs downstream of the sluice gate in Fig. 4-13. We will consider other aspects of critical flow in the next section.

Characteristics of Critical Flow

We have already seen that critical flow occurs when the specific energy is minimum for a given discharge. The depth for this condition may be determined if we solve for dE/dy from $E = y + Q^2/2gA^2$ and set dE/dy equal to zero:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} \tag{4-15}$$

However, $dA = T\,dy$, where T is the width of the channel at the water surface as shown in Fig. 4-14. Then Eq. (4-15), with $dE/dy = 0$, will reduce to

$$\frac{Q^2 T_c}{gA_c{}^3} = 1 \tag{4-16}$$

or

$$\frac{A_c}{T_c} = \frac{Q^2}{gA_c{}^2} \tag{4-17}$$

$$\frac{A_c}{T_c} = \frac{V_c^2}{g} \tag{4-18}$$

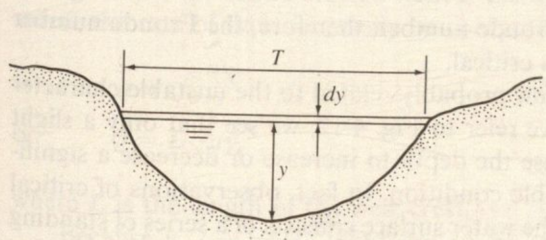


Figure 4-14 Open channel relations

EXAMPLE 4-6 Determine the critical depth in a trapezoidal channel for a discharge of 500 cfs. The width of the channel bottom is 20 ft, and the sides slope upward at an angle of 45°.

SOLUTION Starting with Eq. (4-16),

$$\frac{Q^2 T_c}{gA_c{}^3} = 1$$

or $$\frac{A_c^3}{T_c} = \frac{Q^2}{g}$$

Then, for $Q = 500$ cfs

$$\frac{A_c^3}{T_c} = \frac{500^2}{32.2} = 7764 \text{ ft}^2$$

For this channel, $A = y(b + y)$ and $T = b + 2y$. Then by iteration (choose y and compute A^3/T), we can find y that will yield an A^3/T equal to 7764 ft^2. Such a solution yields $y_c = 2.57$ ft. ■

If the channel is of rectangular cross section, then A_c/T_c is the critical depth, and $Q^2/A_c^2 = q^2/y_c^2$, so the formula for critical depth (Eq. 4-17) becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{4-19}$$

where q is the discharge per unit width of channel.

If we apply Eq. (4-18) to a rectangular channel, divide it by $A_c/T_c = y_c$, and then take the square root of both sides, we obtain

$$\frac{V_c}{\sqrt{gy_c}} = 1 \tag{4-20}$$

The left side of Eq. (4-20) is the Froude number; therefore, the Froude number is equal to unity when the flow is critical.

Originally, the term *critical flow* probably related to the unstable character of the flow for the condition. If we refer to Fig. 4-12, we see that only a slight change in specific energy will cause the depth to increase or decrease a significant amount; this is a very unstable condition. In fact, observations of critical flow in open channels show that the water surface consists of a series of standing waves. Because of the unstable nature of the depth in critical flow, designing canals so that normal depth is either well above or well below critical depth is usually best. The flow in canals and rivers is usually subcritical; however, the flow in steep chutes or over spillways is supercritical.

Occurrence of Critical Depth

Critical flow occurs when a liquid passes over a broad-crested weir (Fig. 4-15*a*). The principle of the broad-crested weir is illustrated by first considering a closed sluice gate that prevents water from being discharged from

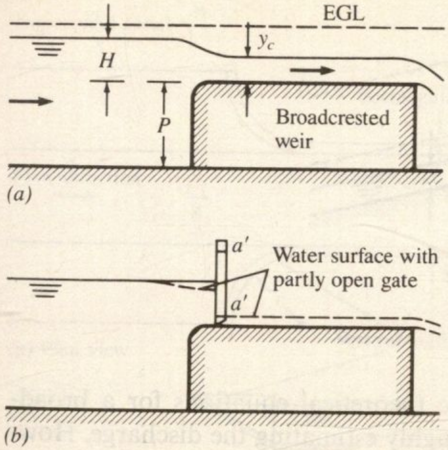


Figure 4-15 Flow over a broad-crested weir

the reservoir (Fig. 4-15*b*). If the gate is opened a small amount (gate position $a' - a'$), the flow upstream of the gate will be subcritical, and the flow downstream will be supercritical (like the condition first introduced in Fig. 4-13). As the gate is opened further, a point is finally reached where the depths upstream and downstream are the same. This is the critical condition. At this gate opening and beyond, the gate has no influence on the flow; this is the condition shown in Fig. 4-15*a*, the broad-crested weir. If the depth of flow over the weir is measured, the rate of flow can easily be computed from Eq. (4-19):

$$q = \sqrt{g y_c^{\,3}}$$

or $$Q = L\sqrt{g y_c^{\,3}} \qquad (4\text{-}21)$$

where L is the length of the weir crest.

Because $y_c/2 = V_c^2/2g$ [from Eq. (4-20)], it is easily shown that $y_c = 2/3E$, where E is the total head above the crest $(H + V_{\text{approach}}^2/2g)$; hence Eq. (4-21) can be rewritten as

$$Q = L\sqrt{g}\left(\frac{2}{3}\right)^{3/2} E^{3/2}$$

or $$Q = 0.385L\sqrt{2g}E_c^{3/2} \qquad (4\text{-}22)$$

For high weirs, the upstream velocity of approach is almost zero. Hence Eq. (4-22) can be expressed as

$$Q_{\text{theor}} = 0.385L\sqrt{2g}H^{3/2} \qquad (4\text{-}23)$$

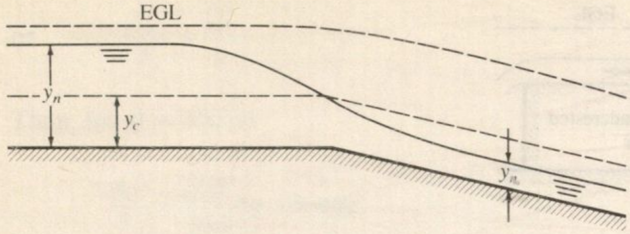


Figure 4-16 Critical depth at break in grade

Equations (4-22) and (4-23) are the basic theoretical equations for a broad-crested weir, and they may be used for roughly estimating the discharge. However, because the discharge will also be influenced by head loss and the shape of the weir, a coefficient of discharge should be used to reflect these effects.*

The depth also passes through a critical stage in channel flow where the slope changes from a mild one to a steep one. Here, a *mild slope* is a slope for which the normal depth y_n is greater than y_c. Likewise, a *steep slope* is one for which $y_n < y_c$. This condition is shown in Fig. 4-16. Note that y_c is the same for both slopes in the figure because y_c is a function of the discharge only. However, normal depth (uniform flow depth) for the mild upstream channel is greater than critical, whereas the normal depth for the steep downstream channel is less than critical; hence it is obvious that the depth must pass through a critical stage. Experiments show that critical depth occurs a very short distance upstream of the intersection of the two channels.

Another place where critical depth occurs is upstream of a free overfall at the end of the channel with a mild slope (Fig. 4-17). Critical depth will occur at a distance of 3 to 4 y_c upstream of the brink. Such occurrences of critical depth (at a break in grade or at a brink) are useful in computing surface profiles because they provide a point for starting surface-profile calculations.†

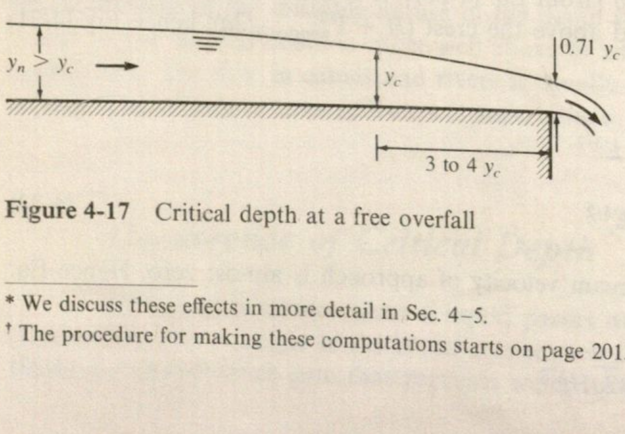


Figure 4-17 Critical depth at a free overfall

* We discuss these effects in more detail in Sec. 4–5.

† The procedure for making these computations starts on page 201.

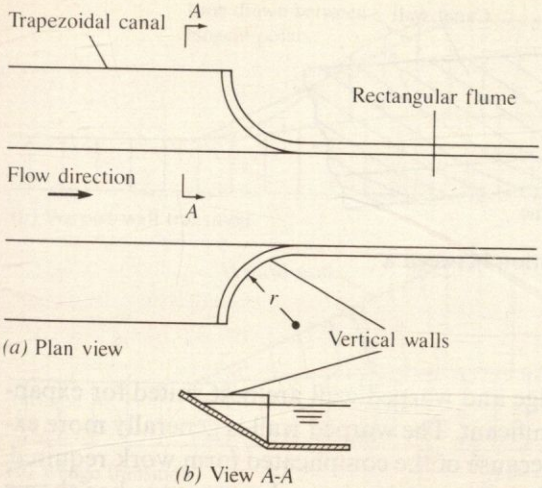


Figure 4-18 Cylinder quadrant inlet transition from trapezoidal canal to rectangular channel

Channel Transitions

Purpose and Types of Transitions A structure designed to convey water smoothly from a conduit of one shape to one of a different shape is called a *transition.* A common transition for open channel flow is used between a canal of trapezoidal cross section and a flume of rectangular section, as shown in Fig. 4-18. Transitions are also used between open channels and inverted siphons (pipe used to convey water across depressions or under highways). If the transition is from a conduit of large cross section to one of smaller cross section, it is an *inlet transition* or a *contraction.* If the transition is from a smaller one to a larger one, it is an *expansion.* In this text, we consider only subcritical flow transitions.*

The simplest type of transition is a straight wall constructed normal to the flow direction, as shown in Fig. 4-19, page 188. This type of transition can work, but it will produce excessive head loss because of the abrupt change in cross section and ensuing separation that would occur. To prevent excessive head losses and to reduce the possibility of erosion in the case of an expansion to an erodible channel, a more gradual type of transition is usually used. Three common types of gradual transitions are the *cylinder-quadrant*, the *wedge* (often called *broken-back*), and the *warped-wall transition* (Fig. 4-20, page 188). All three of these can be successfully used for inlet transitions, but because they are

* For more information on transitions in general and supercritical flow in particular, see Chow (4) and Rouse (18).

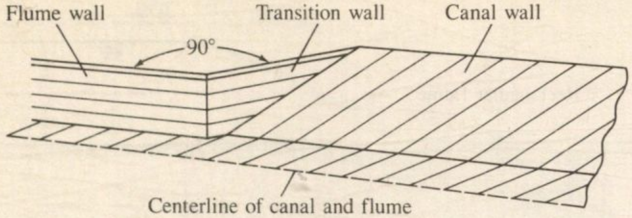


Figure 4-19 Simplest type of transition between a canal and a flume

more gradual expansions the wedge and warped wall are best suited for expansions if head loss or erosion is significant. The warped wall is generally more expensive to build than the others because of the complicated form work required.

If either the wedge or warped-wall transition is used, experience has shown that the expansion should have a more gradual change in section than the inlet transition (9, 21). Quantitatively, the recommendations are manifested in the value of the expansion or contraction angle θ as shown in Fig. 4-21. For the wedge transition, the recommended angle is 27.5° and 22.5°, respectively, for inlets and expansions. For the warped-wall transition, the recommended angle, is 12.5° for both the inlet and expansion.

DESIGN OF TRANSITION TO JOIN CANAL AND FLUME Before the transition itself is designed, one must be given the depth and velocity in both the flume and canal and the water surface elevation in the upstream channel for the case of an inlet (downstream water surface elevation for expansion). Then

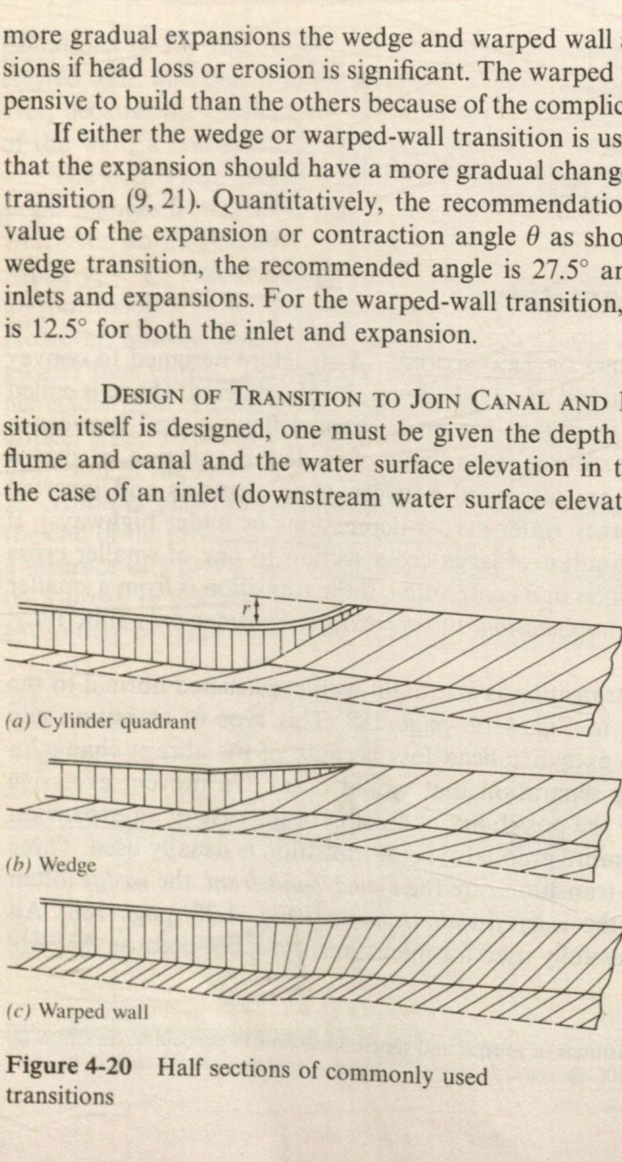
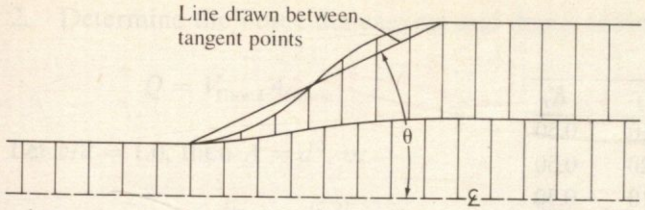
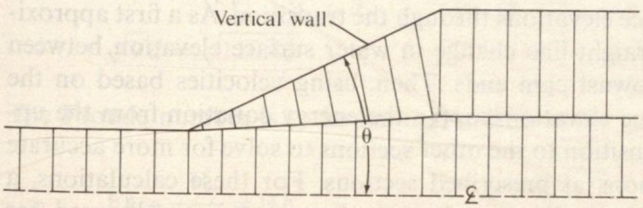


Figure 4-20 Half sections of commonly used transitions



(a) Warped-wall transition



(b) Wedge transition

Figure 4-21 Expansion (or contraction) angles

the transition can be designed step by step as follows:

1. Choose the type of transition to be used (cylinder quadrant, wedge, or warped wall).
2. For an inlet transition, calculate the water surface elevation at the downstream end of the transition. For an expansion, calculate the upstream water surface elevation. This is done by applying the energy equation between the upstream and downstream ends of the transition. The energy equation will include a head loss term for the transition. For an inlet transition, the head loss is given as $K_I V^2/2g$, where K_I is the head loss coefficient for the transition, and V is the velocity in the downstream conduit (the highest mean velocity). For an expansion, the head loss is given as $K_E(V_1{}^2 - V_2{}^2)/2g$, where K_E is the expansion loss coefficient, V_1 is the mean velocity at the upstream end of the expansion, and V_2 is the mean velocity at the downstream end. Table 4-4 shown on the next page lists typical loss coefficients for various types of transitions.
3. For an inlet transition, calculate the downstream invert elevation. For an expansion, calculate the upstream invert elevation.* The invert elevation is simply the water surface elevation at that section minus the depth of water in the flume.
4. Establish invert elevations along the transition by making a straight-line elevation change between the upstream and downstream ends of the transition.

---

* This assumes that the invert elevation (elevation of the bottom of the conduit) of the canal is essentially fixed. Therefore, the invert elevation of the flume must be solved for.

Table 4-4 Transition Loss Coefficients

Type of Transition	K_I	K_E
Cylinder quadrant	0.10	0.50
Wedge	0.20	0.50
Warped wall	0.10	0.30

5. Establish water surface elevations through the transition. As a first approximation, assume a straight-line change in water surface elevation between the upstream and downstream ends. Then, using velocities based on the assumed water surface elevations, apply the energy equation from the upstream end of the transition to the other sections to solve for more accurate water elevations at prescribed sections. For these calculations, a head loss in proportion to distance along the transition is usually used (for example, the head loss halfway down the transition would be assumed to be one half of that for the entire transition).

EXAMPLE 4-7 A transition is needed between a trapezoidal canal carrying water (depth = 3.00 ft; velocity = 2.30 ft/s) and a flume of rectangular cross section. The flume will convey the water around a steep hill. The canal has a bottom width of 10 ft and side slopes of 1 vertical to 2 horizontal. The invert elevation of the canal at its downstream end (upstream end of transition) is 1000 ft. The flume velocity is to be 5.90 ft/s. Determine the proportions for the flume (width and wall height) to keep the Froude number below 0.50, and design a transition to join the canal and flume.

SOLUTION

1. Choose the type of transition.

For this case, let us use a wedge transition. With this type, we use a head loss coefficient of 0.20 ($K_I = 0.20$). Figure A shows the plan view of the transition. The cross-sectional flow area in the channel is $A = 10 \times 3 + 6 \times 3 = 48\ \text{ft}^2$.

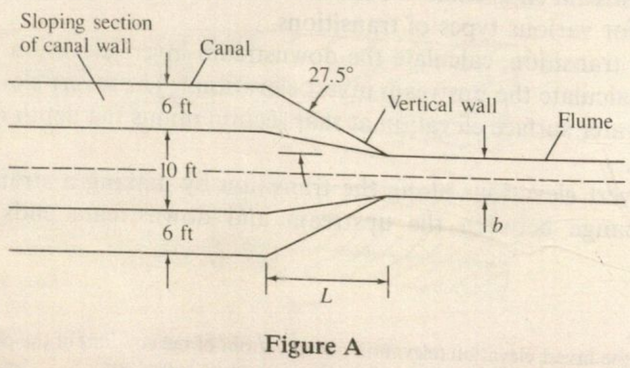


Figure A

2. Determine the flume dimensions and depth of flow in the flume.

$$Q = V_{\text{flume}} A_{\text{flume}}$$

Let $b/d = 1.0$, then $A = d^2$, or

$$d = \sqrt{\frac{Q}{V}} = \sqrt{\frac{2.3 \times 48}{V}} = \sqrt{\frac{110.40}{V}}$$

$$\sqrt{V}d = \sqrt{110.40} \qquad (4\text{-}24)$$

The Froude number is to be limited to 0.50, so

$$\text{Fr} = \frac{V}{\sqrt{gd}} = 0.5$$

or $$V = 0.5\sqrt{gd} \qquad (4\text{-}25)$$

Solving Eqs. (4-24) and (4-25) for d yields $d = 4.326$ ft.

For design, let $b_{\text{flume}} = 4.40$ ft. Then

$$d_{\text{flume}} = \frac{Q}{b \times 5.9} = 4.25 \text{ ft}$$

3. Determine the length L of transition.

$$\tan 27.5° = \frac{11 - 0.50 \times 4.40}{L}$$

or $$L = 16.90 \text{ ft}$$

For design, let $L = 17.0$ ft.

4. Determine the water surface elevation in the flume.

Use the energy equation written from canal to flume and assume that $\alpha_1 = \alpha_2 = 1.1$.

$$z_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + \alpha_2 \frac{V_2^2}{2g} + \sum h_L$$

$$1003.00 + 1.1 \times \frac{2.30^2}{64.4} = z_2 + 1.1 \times \frac{5.9^2}{64.4} + 0.2 \times \frac{5.9^2}{64.4}$$

$$z_2 = 1002.39 \text{ ft}$$

5. Determine the invert elevation of the flume.

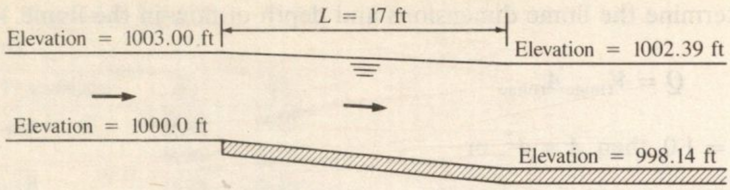


Figure B

$$\text{Invert elevation} = \text{water surface elevation} - d_{\text{flume}}$$
$$= 1002.39 - 4.25$$
$$= 998.14 \text{ ft}$$

Figure B shows the water surface and invert profiles.

6. Now check the velocities at sections A and B where these two sections are 5 ft and 10 ft, respectively, downstream of the upstream end of the transition. Figure C shows the plan view of the transition along with dimensions at sections A and B.*

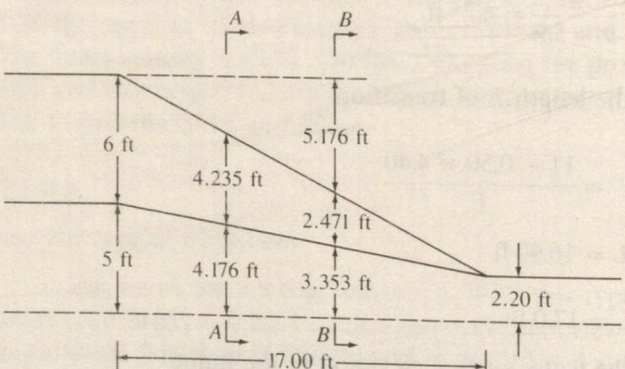


Figure C

First assume a plane water surface between the upstream end of the transition and the downstream end. With this assumption, we can calculate the depths at sections A and B.

* In Fig. C, the transition wall height should include freeboard (extra height added to wall as a safety factor) to account for uncertainties in the design (for example, the assumed loss coefficient may not be exactly the same as the "as built" coefficient) and to provide for waves that were not considered in the design calculations. The height of freeboard is usually from 0.5 to 1.0 ft and will usually match the freeboard used for the canal and flume.

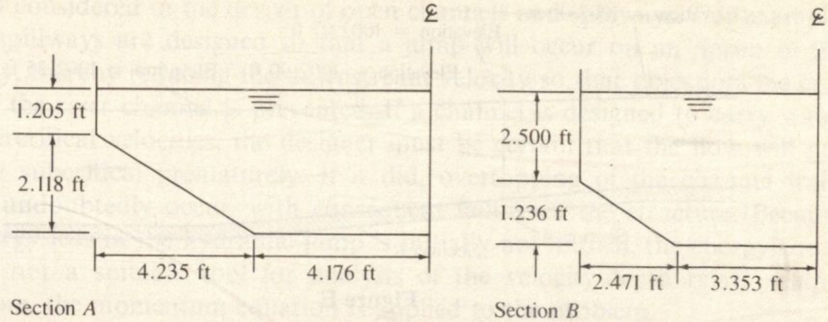


Figure D

$$d_A = 3.00 + \left(\frac{5}{17}\right)[(1002.39 - 998.14) - (1003.00 - 1000.00)]$$

$$= 3.368 \text{ ft}$$

In a similar manner, d_B is calculated.

$$d_B = 3.735 \text{ ft}$$

Figure D shows the cross-sectional flow area (half section) at section A.

$$A_A = 2\left[(4.176 \times 3.368) + (1.205 \times 4.235) + \frac{1}{2} \times (2.118 \times 4.235)\right]$$

$$= 47.306 \text{ ft}^2$$

Then $V_A = \dfrac{Q}{A_A} = \dfrac{2.3 \times 48}{47.306} = 2.33 \text{ ft/s}$

Similar calculations for section B yield:

$$A_B = 40.456 \text{ ft}^2$$

$$V_b = 2.729 \text{ ft/s}$$

7. With the velocities obtained above, we can more accurately determine the water surface elevation at sections A and B by using the energy equation and assuming that the head loss is linearly distributed along the transition.

$$z_1 + \alpha_1 \frac{V_1^{\,2}}{2g} = z_A + \alpha_A \frac{V_A^{\,2}}{2g} + h_{L_{1 \to A}}$$

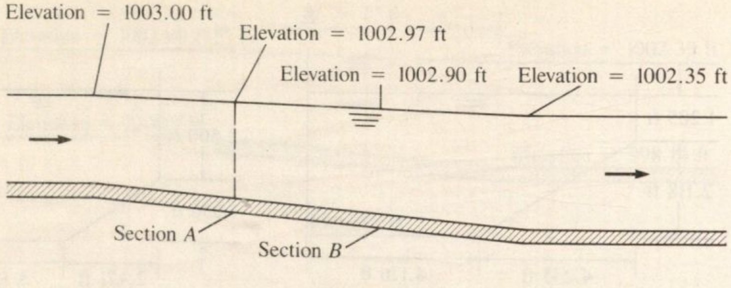


Figure E

$$1003.0 + 1.1 \times \frac{2.3^2}{64.4} = z_A + 1.1 \times \frac{2.33^2}{64.4} + \left(\frac{5}{17}\right) \times 0.20 \times \frac{5.9^2}{64.4}$$

$$1003.0 + 0.090 = z_A + 0.093 + 0.0318$$

$$z_A = 1002.97 \text{ ft}$$

A similar calculation for z_B yields $z_B = 1002.90$ ft.

Figure E shows the corrected water surface profile. ■

The Hydraulic Jump

OCCURRENCE OF THE HYDRAULIC JUMP When the flow is supercritical in an upstream section of a channel and is then forced to become subcritical in a downstream section (the change in depth can be forced by a sill in the channel or by just the prevailing depth in the stream further downstream), a rather abrupt change in depth usually occurs and considerable energy loss accompanies the process. This flow phenomenon, called the *hydraulic jump* (Fig. 4-22),

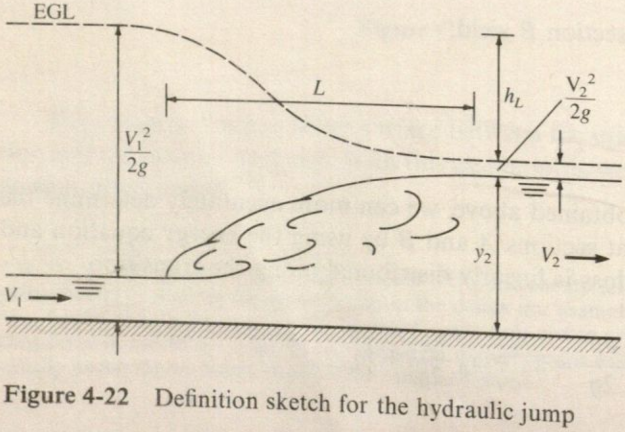


Figure 4-22 Definition sketch for the hydraulic jump

is often considered in the design of open channels and spillways. For example, many spillways are designed so that a jump will occur on an apron of the spillway, thereby reducing the downstream velocity so that objectionable erosion of the river channel is prevented. If a channel is designed to carry water at supercritical velocities, the designer must be certain that the flow will not become subcritical prematurely. If it did, overtopping of the channel walls would undoubtedly occur, with consequent failure of the structure. Because the energy loss in the hydraulic jump is initially not known, the energy equation is not a suitable tool for analysis of the velocity-depth relationships. Therefore, the momentum equation is applied to the problem.

Derivation of Depth Relationships Consider flow as shown in Fig. 4-22. Here it is assumed that uniform flow occurs both upstream and downstream of the jump and that the resistance of the channel bottom is negligible. The derivation is for a horizontal channel, but experiments show that the results of the derivation will apply to all channels of moderate slope ($S_0 < 0.02$). We start the derivation by applying the momentum equation in the x direction to the control volume shown in Fig. 4-23.

$$\sum F_x = \sum_{\text{cs}} V_x \rho \mathbf{V} \cdot \mathbf{A}$$

The forces are the hydrostatic forces on each end of the system, thus the following is obtained:

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho V_1(-V_1 A_1) + \rho V_2(V_2 A_2)$$

or
$$\bar{p}_1 A_1 + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2 \tag{4-26}$$

In Eq. (4-26), $\bar{p}_1$ and $\bar{p}_2$ are the pressures at the centroids of the respective areas A_1 and A_2.

A representative problem might be to determine the downstream depth y_2 given the discharge and upstream depth. The left-hand side of Eq. (4-26) would be known because V, A, and p, are all functions of y and Q, and the right-hand side is a function of y_2; therefore, y_2 can be solved for.

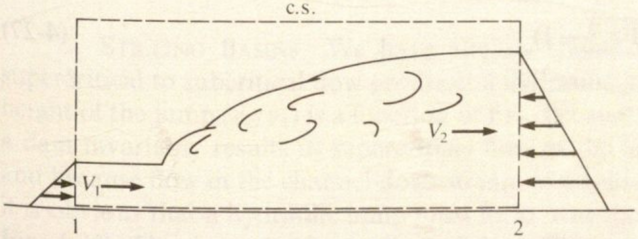


Figure 4-23 Control-volume analysis for the hydraulic jump

EXAMPLE 4-8 Water flows in a trapezoidal channel at a rate of 300 cfs. The channel has a bottom width of 10 ft and side slopes of 1 to 1. If a hydraulic jump is forced to occur where the upstream depth is 1.00 ft, what will be the downstream depth and velocity?

SOLUTION For the upstream section, the area A_1 is equal to 11 ft^2. The depth of the centroid of A_1 is found to be 0.470 ft; therefore, the pressure at the centroid is 62.4 lb/ft^3 × 0.470 ft = 29.3 lb/ft^2. Further $V = Q/A_1 = 27.3$ ft/s. Substituting these values into Eq. (4-26), we get (to 4 significant digits):

$$29.3 \times 11 + 1.94 \times 300 \times 27.3 = \bar{p}_2 A_2 + \rho Q V_2$$

or

$$\bar{p}_2 A_2 + \rho Q V_2 = 16{,}210$$

$$\gamma \bar{y}_2 A_2 + \frac{\rho Q^2}{A_2} = 16{,}210$$

where $A_2 = by_2 + {y_2}^2$

$$\bar{y}_2 = \frac{\sum A_i \bar{y}_i}{A_2} = \left[b\left(\frac{y_2}{2}\right) + \left(\frac{1}{3}\right){y_2}^2 \right] \Big/ (b + y_2)$$

By trial and error, we can solve for A_2 and y_2. The solution gives

$$y_2 = 5.75 \text{ ft}$$

and $$V_2 = \frac{Q}{A_2} = 3.31 \text{ ft/s}$$

■

Head Loss Due to a Hydraulic Jump Because of the intense turbulent mixing that occurs in a hydraulic jump, the head loss due to the jump is relatively large. This is shown in Fig. 4-22. One can determine the head loss due to the hydraulic jump by writing the energy equation across the jump and solving for h_L.*

Hydraulic Jump in a Rectangular Channel If a jump occurs in a rectangular channel, the solution of Eq. (4-26) yields a formula for y_2 as a function of y_1 and the Froude number of the upstream flow:

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8{\text{Fr}_1}^2} - 1\right) \tag{4-27}$$

where $\text{Fr}_1 = \dfrac{V_1}{\sqrt{g y_1}}$.

* For a discussion of other head loss characteristics associated with the hydraulic jump, see Sec. 7-2 of Chapter 7.

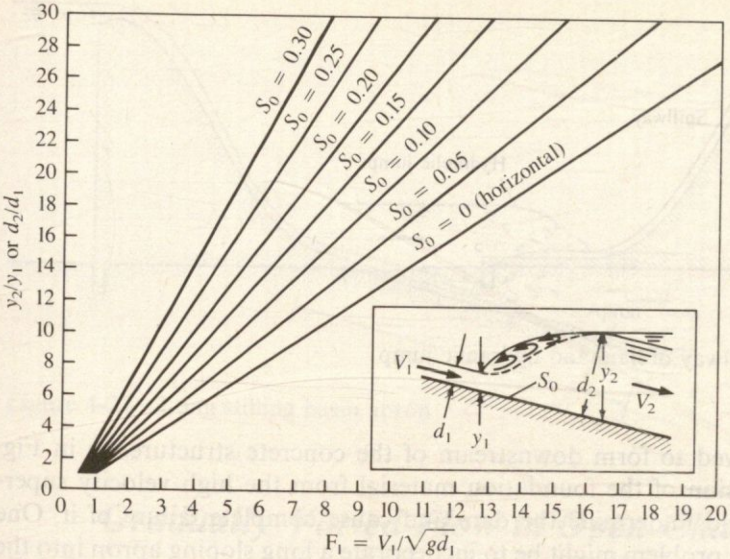


Figure 4-24 Experimental relations between F_1 and y_2/y_1 or d_2/d_1 for jumps in sloping channels [Adapted from *Open Channel Hydraulics* by Chow (4) Copyright 1959, McGraw-Hill Book Company, New York; used with permission of McGraw-Hill Book Company.]

A graph of Eq. (4-27) is shown in Fig. 4-24 (the case for $S_0 = 0$). Figure 4-24 also shows the relations between depth and Froude numbers for hydraulic jumps in channels that are not horizontal.

LENGTH OF THE HYDRAULIC JUMP The length of the hydraulic jump is the distance measured from the front face of the jump to a point on the surface immediately downstream of the roller, as shown in Fig. 4-22. No general theoretical solution exists for this length. However, experiments in rectangular channels (4) show that $L = 6y_2$ for $4 < \mathrm{Fr}_1 < 20$. For Froude numbers outside this range, the length is somewhat less than $6y_2$.

STILLING BASINS We have already shown that the transition from supercritical to subcritical flow produces a hydraulic jump, and that the relative height of the jump (y_2/y_1) is a function of Fr_1. Because flow over the spillway of a dam invariably results in supercritical flow at the lower end of the spillway, and because flow in the channel downstream of a spillway is usually subcritical, it is obvious that a hydraulic jump must form near the base of the spillway (see Fig. 4-25). The downstream portion of the spillway must be designed so that the hydraulic jump always forms on the concrete structure itself. If the hydraulic

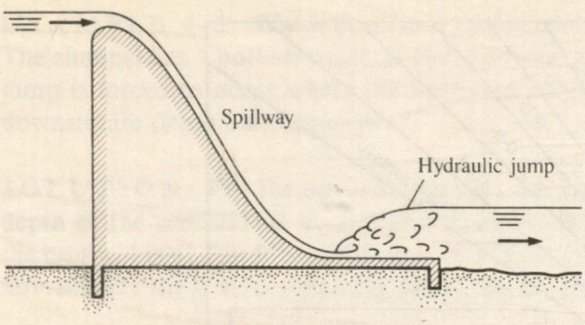


Figure 4-25 Spillway of dam and hydraulic jump

jump were allowed to form downstream of the concrete structure, as in Fig. 4-26, severe erosion of the foundation material from the high velocity supercritical flow could undermine the dam and cause complete failure of it. One way to solve this problem might be to incorporate a long sloping apron into the design of the spillway, as shown in Fig. 4-27. A design like this would work very satisfactorily from the hydraulics point of view. For all combinations of Fr_1 and water surface elevation in the downstream channel, the jump would always form on the sloping apron. However, its main drawback is cost of construction. Construction costs will be reduced as the length (L) of the stilling basin is reduced. Much research has been devoted to the design of stilling basins that will operate properly for all upstream and downstream conditions and yet be relatively short to reduce the cost of the project. Research by the U.S. Bureau of Reclamation and the U.S. Corps of Engineers has resulted in sets of standard designs that can be used.*

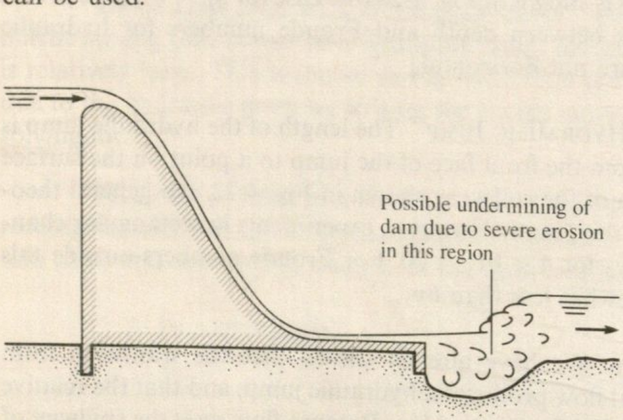


Figure 4-26 Hydraulic jump occurring downstream of spillway apron

* For details of some of these designs, see Sec. 7-2, Chapter 7.

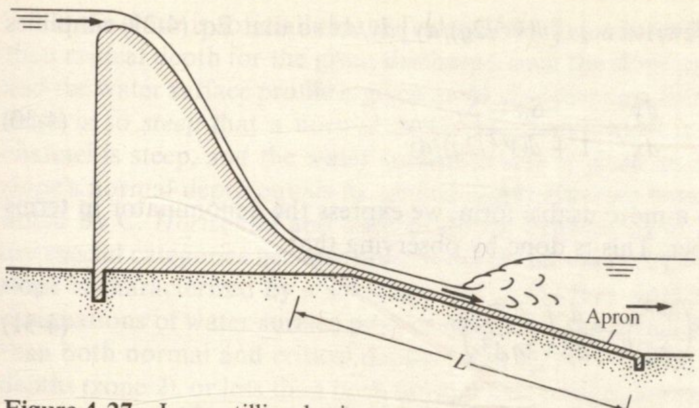


Figure 4-27 Long stilling basin apron

Gradually Varied Flow in Open Channels

Basic Differential Equation for Gradually Varied Flow There are a number of cases of open channel flow in which the change in water surface profile is so gradual that it is possible to integrate the relevant differential equation from one section to another to obtain the desired change in depth. This may be either an analytical integration or, more commonly, a numerical integration. One advantage of the latter procedure is that the complete water surface profile is defined during integration. Considering the energy equation (Eq. 4-10) written for a reach of Δx in length, and defining S_f as the friction slope, h_f/L, Eq. (4-10) becomes

$$y_1 + \frac{V_1^2}{2g} + S_0\,\Delta x = y_2 + \frac{V_2^2}{2g} + S_f\,\Delta x \tag{4-28}$$

Now, let $\Delta y = y_2 - y_1$ and

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$$

Eq. (4-28) then becomes

$$\Delta y = S_0\,\Delta x - S_f\,\Delta x - \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$$

Dividing through by Δx and taking the limit as Δx approaches zero gives us

$$\frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = S_0 - S_f \tag{4-29}$$

The second term is rewritten as $[d(V^2/2g)/dy]\, dy/dx$ so that Eq. (4-29) simplifies to

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + d(V^2/2g)/dy} \tag{4-30}$$

To put Eq. (4-30) in a more usable form, we express the denominator in terms of the Froude number. This is done by observing that

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) \tag{4-31}$$

After differentiating the right side of Eq. (4-31), the equation becomes

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-2Q^2}{2gA^3} \cdot \frac{dA}{dy}$$

But $dA/dy = T$(top width), and $A/T = D$ (hydraulic depth); therefore,

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-Q^2}{gA^2D}$$

or

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = -\mathrm{Fr}^2$$

Hence, when $-\mathrm{Fr}^2$ is substituted for $d(V^2/2g)/dy$ in Eq. (4-30), we obtain

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2} \tag{4-32}$$

This, the general differential equation for gradually varied flow, is used to describe the various types of water-surface profiles that occur in open channels. In the derivation of the equation, S_0 and S_f were taken as positive when sloping downward in the direction of flow. Since y is measured from the bottom of the channel, $dy/dx = 0$ if the slope of the water surface is equal to the slope of the channel bottom, and dy/dx is positive if the water surface slope is less than the channel slope for positive S_0. With these definitions in mind, we will now consider the different forms of water surface profiles.

Classification of Surface Profiles Water surface profiles are classified two different ways: according to the slope of the channel (*mild, steep, critical, horizontal,* or *adverse*) and according to the actual depth of flow in relation to the critical and normal depths (zone 1, 2, or 3). The first letter of the type of slope (M, S, C, H, or A) in combination with 1, 2, or 3 defines the type of surface profile.

If the slope is so small that the normal depth (uniform flow depth) is greater than critical depth for the given discharge, then the slope of the channel is *mild*, and the water surface profile is given an M classification. Similarly, if the channel slope is so *steep* that a normal depth less than critical is produced, then the channel is steep, and the water surface profile is given an S designation. If the slope's normal depth equals its critical depth, then we have a *critical slope*, denoted by C. *Horizontal* and *adverse* slopes, denoted by H and A, respectively, are special categories because normal depth does not exist for them. An adverse slope is characterized by a slope upward in the flow direction. The 1, 2, and 3 designations of water surface profiles indicate if the actual flow depth is greater than both normal and critical depths (zone 1), between the normal and critical depths (zone 2), or less than both normal and critical depths (zone 3). The basic shapes of the various possible profiles are shown in Fig. 4-28, page 202. Figure 4-29 on page 203 shows typical examples of physical situations that produce the various profiles.

With the foregoing introduction to the classification of surface profiles, we can now refer to Eq. (4-32) to describe the shape of the profiles. For example, if we consider the M3 profile, it is known that $\mathrm{Fr} > 1$ because the flow is supercritical ($y < y_c$) and $S_f > S_0$ because the velocity is greater than normal velocity. Inserting these relative values into Eq. (4-32), we see that both the numerator and denominator are negative; thus, dy/dx must be positive (the depth increases in the direction of flow). As critical depth is approached, the Froude number approaches unity; hence, the denominator of Eq. (4-32) approaches zero. Therefore, as the depth approaches critical depth, $dy/dx \to \infty$. What actually occurs when the critical depth is approached in supercritical flow is that a hydraulic jump forms, and a discontinuity in profile is thereby produced.

Certain general features of profiles, as shown in Fig. 4-28, are evident. First, as the depth becomes very great the velocity of flow approaches zero, and $\mathrm{Fr} \to 0$ and $S_f \to 0$. Hence it follows from Eq. (4-32) that dy/dx approaches S_0. That is, the depth increases at the same rate that the channel bottom drops away from the horizontal. Thus, the water surface approaches the horizontal. The curves that tend this way are M1, S1, and C1. A physical example of the M1 curve is that of the water surface profile behind the dam, as shown in Fig. 4-29. This is often called a *backwater curve*. The second general feature of several of the profiles is that the depth approaches normal depth asymptotically. This is shown in the S2, S3, M1, and M2 profiles. For these cases, it is seen that in supercritical flow $y \to y_n$ going downstream, and in subcritical flow $y \to y_n$ going upstream. In Fig. 4-28, profiles approaching critical depth are shown by broken lines. This is done because near critical depth discontinuities develop (hydraulic jump), or the streamlines are very curved (such as near a brink); therefore, the surface profiles cannot be accurately predicted because Eq. (4-32) is based on one-dimensional flow, which in these regions is invalid.

Quantitative Evaluation of Surface Profiles In practice, most surface profiles are generated by numerical integration, that is, by dividing the

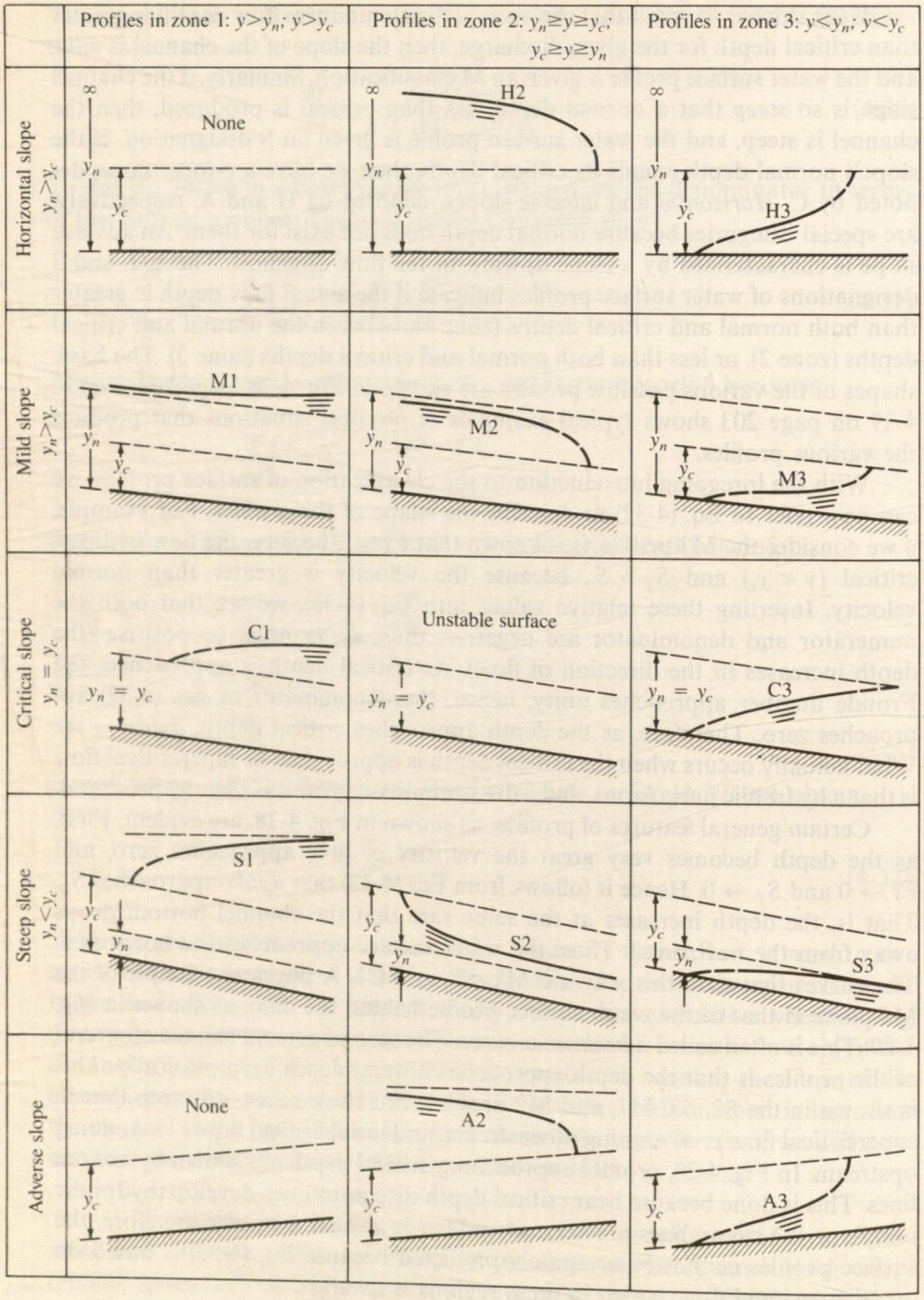


Figure 4-28 Classification of water-surface profiles of gradually varied flow [Adapted from *Open Channel Hydraulics* by Chow (4) Copyright 1959, McGraw-Hill Book Company, New York; used with permission of McGraw-Hill Book Company.]

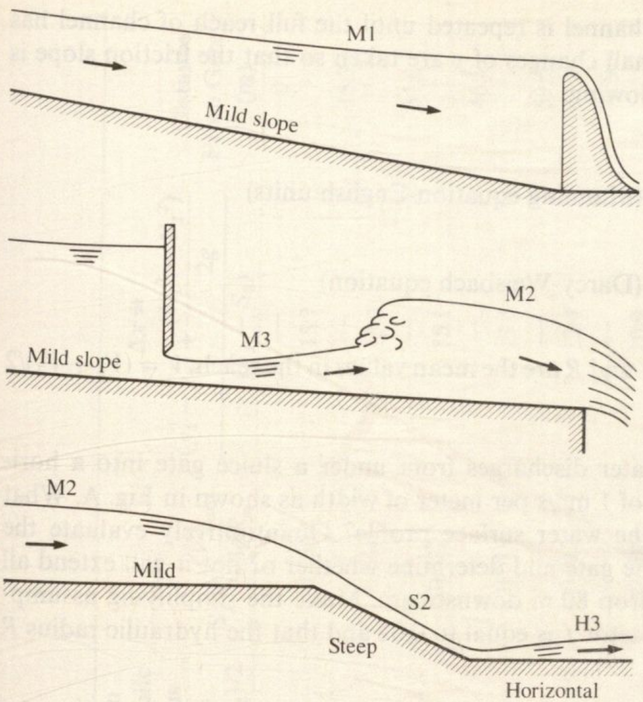


Figure 4-29 Water surface profiles associated with flow behind a dam, flow under a sluice gate, and flow in a channel with a change in grade

channel into short reaches and carrying the computation for water surface elevation from one end of the reach to the other. The two most common approaches are the direct step method and the standard step method.

In the *direct step method*, the depth and velocity are known at a given section of the channel (one end of the reach), and one arbitrarily chooses the depth at the other end of the reach. Then the length of the reach is solved for. The applicable equation is Eq. (4-28). In that equation, if we let $y_1 + \alpha_1 \dfrac{V_1^2}{2g} = E_1$ and $y_2 + \alpha_2 \dfrac{V_2^2}{2g} = E_2$ and solve for Δx, we get

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0} \tag{4-33}$$

The procedure for evaluating a profile is to first ascertain the type of profile that applies to the given reach of channel (we use the methods of the preceding section). Then, starting from a known depth, a finite value of Δx is computed for an arbitrarily chosen change in depth. The process of computing Δx, step by step in the upstream direction (negative Δx) or in the downstream direction (positive Δx) along the channel is repeated until the full reach of channel has been covered. Usually small changes of y are taken so that the friction slope is approximated by the following:

$$S_f = \frac{n^2 V^2}{2.22 R^{4/3}} \quad \text{(Manning equation-English units)}$$

or

$$S_f = \frac{f V^2}{8 g R} \quad \text{(Darcy-Weisbach equation)}$$

In the above equations, V and R are the mean values in the reach; $V = (V_1 + V_2)/2$ and $R = (R_1 + R_2)/2$.

EXAMPLE 4-9 Water discharges from under a sluice gate into a horizontal channel at a rate of 1 m^3/s per meter of width as shown in Fig. A. What is the classification of the water surface profile? Quantitatively evaluate the profile downstream of the gate and determine whether or not it will extend all the way to the abrupt drop 80 m downstream. Make the simplifying assumption that the resistance factor f is equal to 0.02 and that the hydraulic radius R is equal to the depth y.

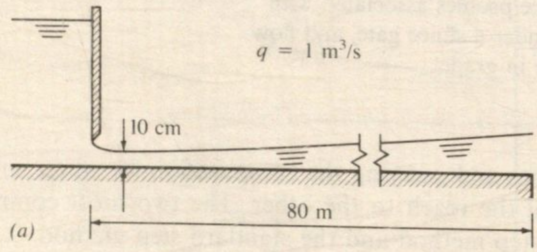


Figure A

SOLUTION First, determine the critical depth y_c:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\left(\frac{1^2\ m^4/s^2)}{(9.81\ m/s^2}\right)\right]^{1/3} = 0.467\ \text{m}$$

The depth of flow from the sluice gate is less than the critical depth. Hence the water surface profile is classified as type H3.

To solve for the depth versus distance along the channel, we apply Eq. (4-33) using a numerical approach. In this example we chose to make the change in depth 0.04 m. The results of the computations are shown on the next page. From the numerical results, we plot the profile shown in Fig. B on page 206, and we see that the *profile extends to the abrupt drop*.

Section Number Downstream of Gate	Depth y (m)	Velocity at Section V (m/s)	Mean Velocity in Reach $(V_1 + V_2)/2$	V^2	Mean Hydraulic Radius $R_m = (y_1 + y_2)/2$	$S_f = \frac{fV^2_{mean}}{8gR_m}$	$\Delta x = \frac{(y_1 - y_2) + \frac{(V_1^2 - V_2^2)}{2g}}{(S_f - S_0)}$	Distance From Gate x (m)
1 (at gate)	0.1	10	—	100	—	—	—	0
	—	—	8.57	73.4	0.12	0.156	15.7	
2	0.14	7.14	—	51.0	—	—	—	15.7
	—	—	6.35	40.3	0.16	0.064	15.3	
3	0.18	5.56	—	30.9	—	—	—	31.0
	—	—	5.05	25.5	0.20	0.032	15.1	
4	0.22	4.54	—	20.6	—	—	—	46.1
	—	—	4.19	17.6	0.24	0.019	13.4	
5	0.26	3.85	—	14.8	—	—	—	59.5
	—	—	3.59	12.9	0.28	0.012	12.4	
6	0.30	3.33	—	11.1	—	—	—	71.9
	—	—	3.13	9.8	0.32	0.008	10.9	
7	0.34	2.94	—	8.6	—	—	—	82.8

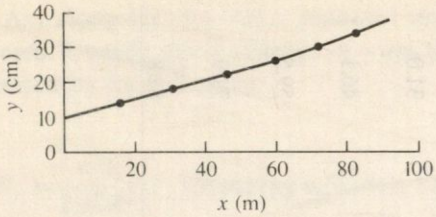


Figure B

The direct step method is ideally suited for prismatic channels because the channel cross section is independent of position in the reach. However, in non-prismatic channels such as in natural channels, the channel cross sections are not independent of distance along the channel. In these channels, a field survey is usually made to provide the data required at the sections considered in the computation. The computations are made between the sections for which the data are available. Therefore, the computation procedure now involves the determination of depth for a given Δx. This computational procedure is called the *standard step method.*

To develop the procedure for the standard step method, we use Eq. (4-28) with the kinetic energy correction factors included plus another head loss term, h_ℓ, where h_ℓ are head losses in the reach in addition to losses resulting from surface resistance alone. These additional losses may be caused by such things as abrupt expansions, contractions, or bends. Thus the relevant equation is

$$y_1 + \frac{\alpha_1 V_1^2}{2g} + S_0 \Delta x = y_2 + \frac{\alpha_2 V_2^2}{2g} + S_f \Delta x + h_\ell \tag{4-34}$$

The method of solution is an iterative one. All information is known at one section (section 1), and one assumes a depth for the other section (section 2). Then with this assumed depth, V_2 can be calculated along with S_f and h_ℓ. After the quantities on the right-hand side of Eq. (4-34) have been calculated, a check is made to see if the equation is satisfied. If not, a new value of y_2 is assumed, and so on. The process is continued until a value for y_2 is found that satisfies Eq. (4-34). Because of the repetitive nature of water surface profile computations,

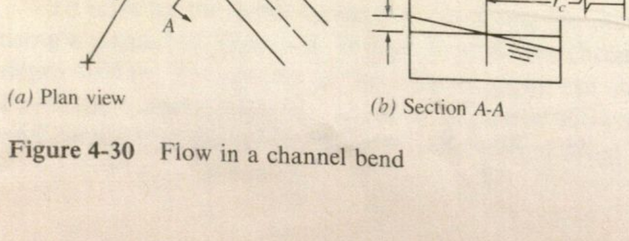


Figure 4-30 Flow in a channel bend

solution by computer is often done. In recent years, a number of sophisticated computer programs have been written to solve water surface profiles. One popular program developed by the Hydrologic Engineering Center of the U.S. Army Corps of Engineers is HEC-2. It uses the standard step method of solution.

For either the direct step method or the standard step method, the direction in which the computation proceeds is upstream for subcritical flows and downstream for supercritical flows.

Flow in Bends With Subcritical Flow

Water Surface Elevation Change Whenever there is a bend in a channel, the water is accelerated toward the center of curvature of the bend; therefore, a force must be applied to the flowing water toward the center of curvature. This force is produced by a rise in the water surface toward the outside of the bend and a drop in the water surface toward the inside (Fig. 4-30). Therefore, when designing a bend, the outside wall of the channel must be made high enough to accommodate the increase in water surface elevation due to the bend. The amount of elevation change is determined by using Euler's equation applied along a radial line:

$$-\frac{\partial h}{\partial r} = \frac{a_r}{g} \tag{4-35}$$

In Eq. (4-35), the acceleration, a_r, is equal to V^2/r toward the center of curvature (in the negative r direction), and h is the piezometric head $(p/\gamma + z)$; therefore, if we consider the change in elevation of the water surface $(p = 0)$, we obtain

$$\frac{\partial z}{\partial r} = \frac{V^2}{gr}$$

Thus it can be shown that the increase in water surface elevation from the channel centerline to the outer wall will be approximately

$$\Delta z = \frac{V^2}{gr_c} \cdot \frac{T}{2} \tag{4-36}$$

where T = water surface width in channel
r_c = radius of curvature of centerline of channel.

Head Loss in Bends The head loss due to the bend (in addition to the normal head loss due to friction) can be expressed as

$$h_{L(\text{bend})} = C_{\text{bend}} \frac{V^2}{2g} \tag{4-37}$$

For most canals, the depth is considerably less than the width of the canal, and the radius of curvature is quite large. Therefore, the head loss in addition to the normal head loss due to friction is quite small. For example, the state of California (6) used values of 0.01 and 0.02 for the bend coefficient, C, for 90° bends with radii of curvature of $5T$ and $2T$, respectively. For 45° bends, half these values of C were used.

SUPERCRITICAL FLOW IN BENDS In supercritical flow in bends, cross waves can develop thereby producing a greater change in water surface elevation than one would predict for subcritical flow in bends. Banking of the channel bottom (used for rectangular channels) or carefully designed spiral curve transitions can be used to prevent or suppress cross waves.*

4-4 Measurement of Discharge in Open Channels

Velocity-Area-Integration Method

The discharge past a flow section is given as $Q = VA$. Here the mean velocity, V, is the component of mean velocity normal to the section area, A. In a river, the velocity varies with depth and position across the river. Therefore, to measure the discharge in a river, standard practice is to make velocity measurements at various stations across the river and to apportion to each station the flow-section area that is closest to that particular station. Then the total discharge in the river is given by

$$Q = \sum V_i A_i$$

where V_i = mean velocity at a particular station
A_i = the section area assigned to the station.

In rivers, theory and experimental evidence both show that the mean velocity in a vertical section is closely approximated by the average of the velocity taken at 0.2 depth and 0.8 depth below the surface. Moreover, if the stream is quite shallow so that it may be difficult to measure the velocity at 0.8 depth, a single measurement of velocity at 0.6 depth is a good approximation to the mean velocity in the vertical section. Figure 4-31 depicts a section in a river where velocity measurements at 0.2 and 0.8 depths have been made in a number of vertical sections across the river. At section 4, the velocities at 0.2 depth and 0.8 depth were measured to be 8.5 ft/s and 6.6 ft/s, respectively; therefore, the mean velocity for that vertical section would be 7.55 ft/s, and the area apportioned to the that section would be 8 ft × [(20/2) + (15/2)] = 140 ft². Then the

* For details on the design of bends for supercritical flow see Chow (4) and Rouse (18).

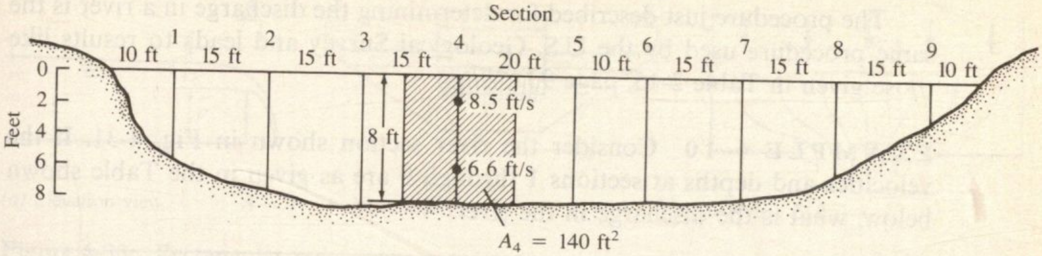


Figure 4-31 River section

ΔQ for section 4 would be given as V_4A_4 or $7.55 \times 140 = 1057$ cfs. The total river discharge is determined by summing all the V_iA_i's across the section.

The most common velocity meter used in rivers in the United States is the Price current meter (Fig. 4-32). Cups on a wheel mounted on a vertical axis cause the wheel to rotate when water flows past it. The meter is calibrated so that the frequency of rotation can be converted to velocity in ft/s or m/s. The meter can be hand held on the end of a rod, or for deep rivers it can be suspended by a cable from a bridge, cable car, or boat. When used in this latter manner, a lead weight is attached to the bottom of the meter (see Fig. 4-32) to stabilize it in the flowing stream.

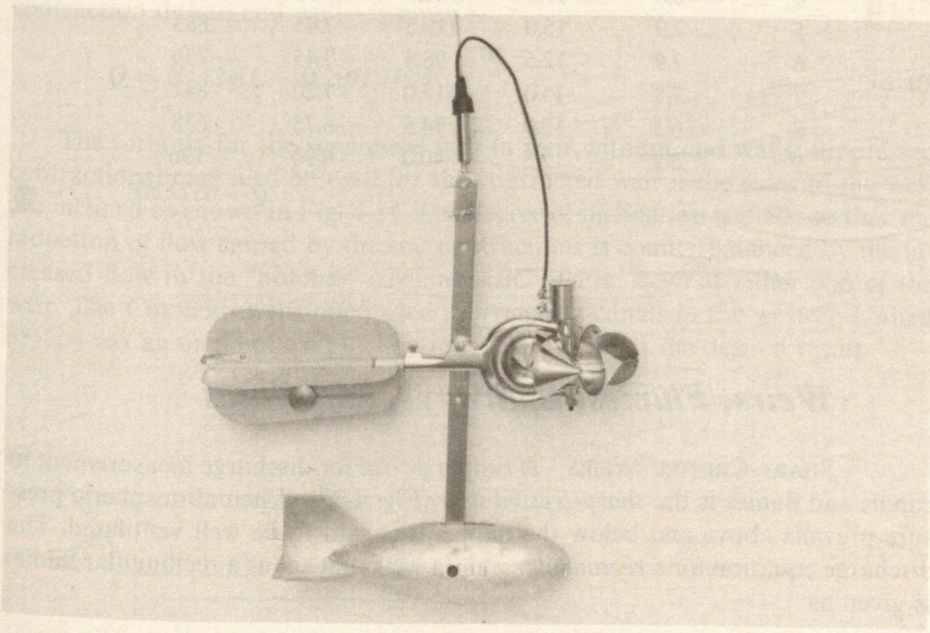


Figure 4-32 Current meter with weight (Courtesy of Teledyne Gurley, Troy, New York)

The procedure just described for determining the discharge in a river is the same procedure used by the U.S. Geological Survey and leads to results like those given in Table 2-15, page 93.

EXAMPLE 4-10 Consider the river section shown in Fig. 4-31. If the velocities and depths at sections 1 through 9 are as given in the Table shown below, what is the discharge in the river?

	Section								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Depth (ft)	5.5	7.3	8.5	8.0	7.9	7.9	7.8	6.3	3.2
Velocity at 0.2 depth (ft/s)	7.7	8.2	8.4	8.5	8.4	8.3	8.1	7.7	
Velocity at 0.8 depth (ft/s)	6.0	6.4	6.5	6.6	6.5	6.6	6.3	5.8	
Velocity at 0.6 depth (ft/s)									4.9

SOLUTION The solution is done in tabular form as shown below.

Section	Depth (ft)	Width (ft)	ΔA (ft^2)	Avg. V (ft/s)	ΔQ (ft/s)
1	5.5	12.5	68.8	6.85	471
2	7.3	15.0	109.5	7.30	799
3	8.5	15.0	127.5	7.45	950
4	8.0	17.5	140.0	7.55	1057
5	7.9	15.0	118.5	7.45	883
6	7.9	12.5	98.8	7.45	736
7	7.8	15.0	117.0	7.20	842
8	6.3	15.0	94.5	6.75	638
9	3.2	12.5	40.0	4.90	196
					$Q =$ 6572 cfs

■

Weirs, Flumes, Spillways, and Gates

SHARP-CRESTED WEIRS A simple device for discharge measurement in canals and flumes is the *sharp-crested weir* (Fig. 4-33). When atmospheric pressure prevails above and below the nappe, it is said to be well ventilated. The discharge equation for a *rectangular weir* (a weir that spans a rectangular flume) is given as

$$Q = K\sqrt{2g}LH^{3/2} \tag{4-38}$$

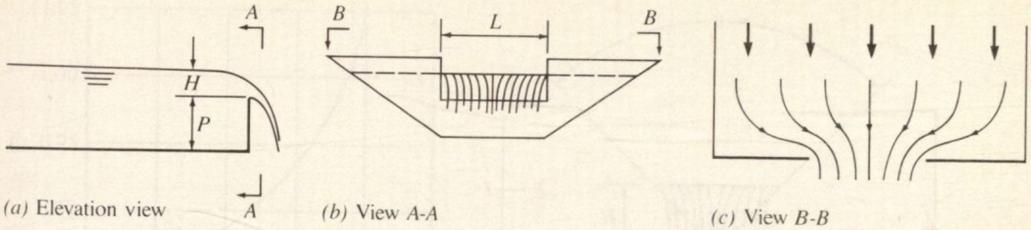


Figure 4-33 Rectangular weir

In Eq. (4-38), K is the flow coefficient of the weir and is given as

$$K = 0.40 + 0.05\frac{H}{P} \tag{4-39}$$

Based on experimental work by Kindsvater (11), this is valid up to an H/P value of about 10.

Often the weir section does not span the entire width of the channel (Fig. 4-33). Therefore, there will be a contraction of the flow section just downstream of the weir so that the effective length of the weir will be somewhat less than L. Experiments show that this effective reduction in length is approximately equal to $0.20H$ when $L/H > 3$. Thus the formula for a *contracted weir* (one with flow contraction due to end walls) is given as

$$Q = K\sqrt{2g}(L - 0.20H)H^{3/2} \tag{4-40}$$

The formula for the *suppressed weir* (a weir without end walls; suppressed contractions) may also be used for the contracted weir if the sides of the weir are inclined as shown in Fig. 4-34. The degree of inclination is made so that the reduction of flow caused by the end contractions is counterbalanced by the increased flow in the "notches" (regions ABC of Fig. 4-34) at either end of the weir. The Cipolletti weir (often used in irrigation canals in the western United States) has an angle of wall inclination of 28° to effect the desired result.

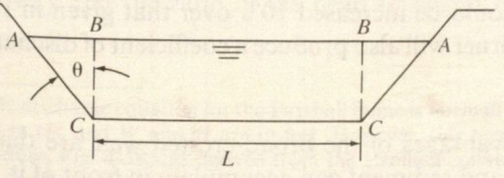


Figure 4-34 Trapezoidal weir

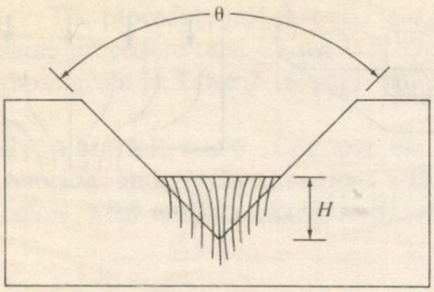


Figure 4-35 Triangular weir

With low flow rates, it is common to use a triangular weir (Fig. 4-35). The basic weir equation is given as

$$Q = \frac{8}{15} K\sqrt{2g}\tan\left(\frac{\theta}{2}\right) H^{5/2} \tag{4-41}$$

where K is the flow coefficient that is primarily a function of the head H. For weirs with θ values of 60° and 90°, Lenz (12) showed that the flow coefficient values varied from 0.60 to 0.57 as the head varied from 0.20 to 2.0 ft.

BROAD-CRESTED WEIRS If the weir is long in the direction of flow so that the flow leaves the weir in essentially a horizontal direction, the weir is a *broad-crested weir* (see Fig. 4-15, page 185). The basic theoretical equation for the broad-crested weir is shown in (Eq. 4-23, page 185). However, to account for head loss and shape of weir, a discharge coefficient should be applied to Eq. (4-23). If the ratio of the actual discharge, Q, to the theoretical discharge, Q_{theor}, is given by C, then

$$Q = 0.385CL\sqrt{2g}H^{3/2} \tag{4-42}$$

For low weirs, the velocity of approach can be significant, and this effect will tend to make C greater than unity. However, the frictional resistance over the length of the weir will tend to make C less than unity. When these two effects are combined, the resulting C values are as shown in Fig. 4-36. These are for a weir with a vertical upstream face and a sharp corner at the intersection of the upstream face and the weir crest. If the upstream face is sloping at a 45° angle, the discharge coefficient should be increased 10% over that given in Fig. 4-36. Rounding of the upstream corner will also produce a coefficient of discharge as much as 3% greater.

VENTURI FLUME Disadvantages of the broad-crested weir are that it produces considerable head loss, and sediment can accumulate in front of it. To reduce both of these detrimental effects, the Venturi flume was developed and calibrated by Parshall (15). Critical flow is produced by reducing the width of

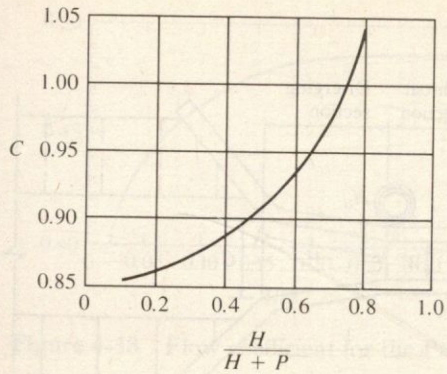


Figure 4-36 Form and resistance coefficient for a broad-crested weir for $0.1 < H/L < 0.8$ (16)

the channel (the Venturi effect) and by increasing the slope of the bottom in the contracted section (Fig. 4-37, pages 214–15). Thus the contracted section serves as a *control* and a predictable head-discharge relationship exists if the depth downstream of the contracted section is low enough to allow "free flow" through the contracted section. The criterion for free flow is that the ratio of downstream head to upstream head, H_d/H_u, shall not exceed 0.70. Then with this condition the discharge through the flume is given as

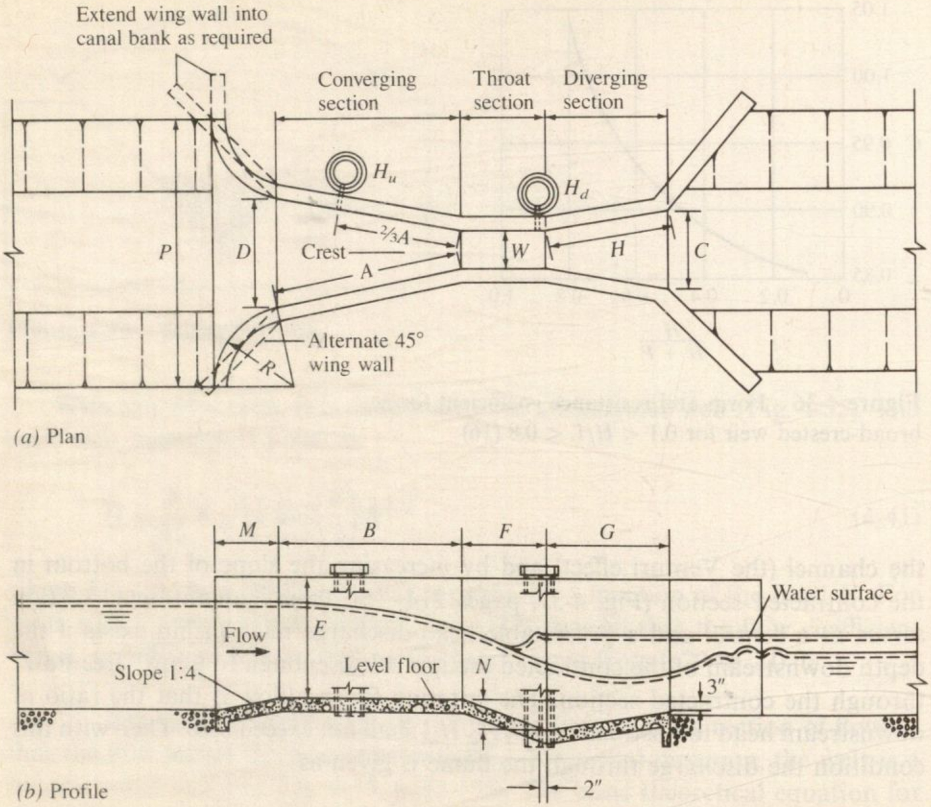
$$Q = K\sqrt{2g}WH_u^{3/2} \tag{4-43}$$

where H_u is the head above the floor level of the flume measured at the location shown in Fig. 4-37, and K is a function of H_u/W as given in Fig. 4-38, page 215.* Equation (4-43) is valid for flumes with throat widths (W) from 1 ft (0.31 m) to 8 ft (2.4 m)†.

Spillways on Dams The spillway crest serves as a control section; therefore, it can be used for discharge measurements. The general form of the discharge equation is the same as for weirs, $Q = KL\sqrt{2g}H^{3/2}$. The head, H, is measured from the crest of the spillway (Fig. 4-39, page 215), and L is the length of the spillway. The value of the flow coefficient depends on the shape of the spillway and the ratio of the head to height of dam.

* The discharge equation for the Parshall flume is normally expressed as $Q = 4BH_u^{1.522W^{0.026}}$, where Q is in cfs, and W and H are in feet. However, this equation is not dimensionally homogenous; therefore, Fig. 4-38 was derived from the Parshall equation, which is dimensionally homogenous and can be applied with any system of units.

† For information on the use of Parshall flumes outside the range of size noted above or for use under submerged conditions ($H_d/H_u > 0.70$), see Parshall (15) or Chow (4).



W (ft)	W (in.)	A (ft)	A (in.)	$\frac{2}{3}A$ (ft)	$\frac{2}{3}A$ (in.)	B (ft)	B (in.)	C (ft)	C (in.)	D (ft)	D (in.)	E (ft)	E (in.)
0	6	2	$\frac{7}{16}$	1	$4\frac{5}{16}$	2	0	1	$3\frac{1}{2}$	1	$3\frac{3}{8}$	2	0
	9	2	$10\frac{5}{8}$	1	$11\frac{1}{8}$	2	10	1	3	1	$10\frac{5}{8}$	2	6
1	0	4	6	3	0	4	$4\frac{7}{8}$	2	0	2	$9\frac{1}{4}$	3	0
1	6	4	9	3	2	4	$7\frac{7}{8}$	2	6	3	$4\frac{3}{8}$	3	0
2	0	5	0	3	4	4	$10\frac{7}{8}$	3	0	3	$11\frac{1}{2}$	3	0
3	0	5	6	3	8	5	$4\frac{3}{4}$	4	0	5	$1\frac{7}{8}$	3	0
4	0	6	0	4	0	5	$10\frac{3}{8}$	5	0	6	$4\frac{1}{4}$	3	0
5	0	6	6	4	4	6	$4\frac{1}{2}$	6	0	7	$6\frac{5}{8}$	3	0
6	0	7	0	4	8	6	$10\frac{3}{8}$	7	0	8	9	3	0
7	0	7	6	5	0	7	$4\frac{1}{4}$	8	0	9	$11\frac{3}{8}$	3	0
8	0	8	0	5	4	7	$10\frac{1}{8}$	9	0	11	$1\frac{3}{4}$	3	0

Figure 4-37 Standard Parshall Flume dimensions (21)

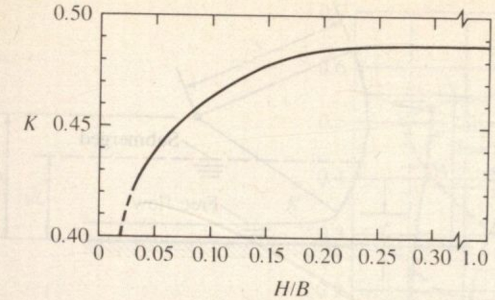


Figure 4-38 Flow coefficient for the Parshall flume

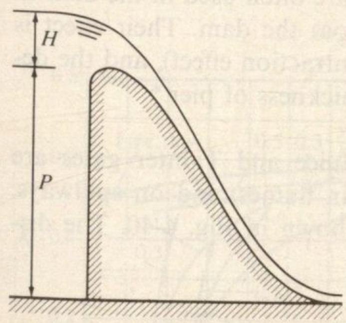
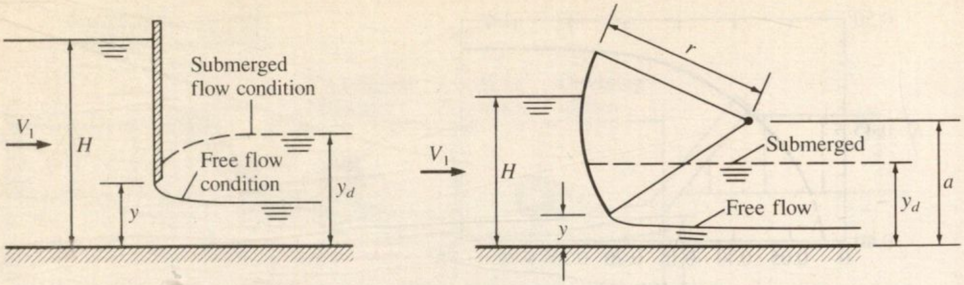


Figure 4-39 Flow over spillway of a dam

F		G		M		N		P		R		Free-Flow Capacity	
												Minimum	Maximum
(ft)	(in.)	(ft)	(in.)	(ft)	(in.)	(ft)	(in.)	(ft)	(in.)	(ft)	(in.)	(cfs)	(cfs)
1	0	2	0	1	0	0	$4\frac{1}{2}$	2	$11\frac{1}{2}$	1	4	0.05	3.9
1	0	1	6	1	0		$4\frac{1}{2}$	3	$6\frac{1}{2}$	1	4	0.09	8.9
2	0	3	0	1	3		9	4	$10\frac{3}{4}$	1	8	0.11	16.1
2	0	3	0	1	3		9	5	6	1	8	0.15	24.6
2	0	3	0	1	3		9	6	1	1	8	0.42	33.1
2	0	3	0	1	3		9	7	$3\frac{1}{2}$	1	8	0.61	50.4
2	0	3	0	1	6		9	8	$10\frac{3}{4}$	2	0	1.3	67.9
2	0	3	0	1	6		9	10	$1\frac{1}{4}$	2	0	1.6	85.6
2	0	3	0	1	6		9	11	$3\frac{1}{2}$	2	0	2.6	103.5
2	0	3	0	1	6		9	12	6	2	0	3.0	121.4
2	0	3	0	1	6		9	13	$8\frac{1}{4}$	2	0	3.5	139.5



(a) Sluice gate (b) Tainter gate

Figure 4-40 Flow below underflow gates

The discharge is also influenced by piers that are often used in the dam to support gates and for supporting a roadway across the dam. Their effect is usually to cause some reduction of flow (it is a contraction effect), and the degree of reduction is a function of the shape and thickness of pier.*

SLUICE AND TAINTER GATES Both the sluice and Tainter gates are used extensively for controlling water in canals, in flumes, and on spillways. They fall in the category of *underflow gates*, as shown in Fig. 4-40. The discharge through underflow gates can be given as

$$Q = C_c C_v L y \sqrt{2g} \sqrt{\left(H + \frac{V_1^2}{2g} - C_c y\right)} \tag{4-44}$$

where C_c, the contraction coefficient, is a function of the relative gate opening and the shape of the gate. The velocity coefficient C_v has a value slightly less than unity. For convenience Eq. (4-44) is simplified to

$$Q = KLy\sqrt{2gH} \tag{4-45}$$

where K is the flow coefficient that is a function of the same parameters that C_c is a function of. If the downstream jet is submerged, as shown in Fig. 4-40, then the discharge rate is a function of the downstream depth, y_d, as well. The functional relationships between K and the other relevant parameters are shown in Fig. 4-41 for both the sluice gate and the Tainter gate. In Fig. 4-41*a*, Fr_0 is the Froude number based on the velocity through the gate opening ($Fr_0 = V/\sqrt{gy}$).†

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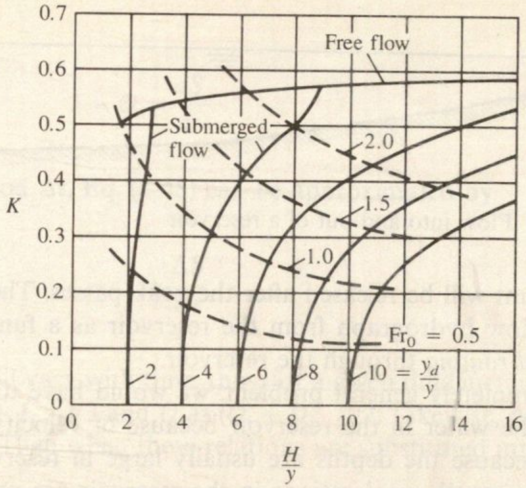
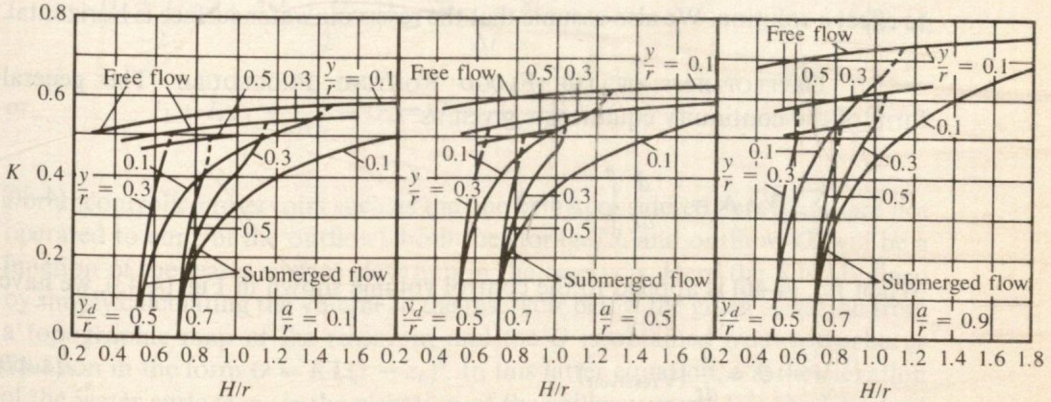
* We discuss these effects and other characteristics of flow over spillways such as design details in Chapter 7.

† For more details, about underflow gates, see Henry (8) and Toch (19). Chapter 7 also includes information about the use of Tainter gates on spillways.



(a) Discharge coefficient for vertical sluice gate (8)



(b) Flow coefficient for Tainter gate (19)

Figure 4-41 Flow coefficient for underflow gates

4-5 Unsteady-Nonuniform Flow in Open Channels

Flood Routing Through a Reservoir

QUALITATIVE DESCRIPTION Consider the reservoir shown in Fig. 4-42. Flow is coming into the reservoir from a stream and leaving it over a spillway. If a flood were to occur on the incoming stream, then as that flood flow comes into the reservoir, some of the flood volume is stored in the reservoir while the outflow discharge increases as the reservoir rises. However, the outflow rate is never as great as the peak inflow rate because of the attenuation of the inflow due to the fact that much of the flow is temporarily stored in the reservoir. The

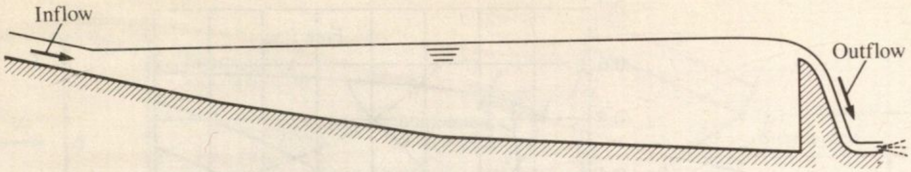


Figure 4-42 Flow into and out of a reservoir

stored amount will be released after the peak passes. The quantitative evaluation of outflow hydrograph from the reservoir as a function of the inflow is termed *flood routing* through the reservoir.

In a completely general problem, we would have to consider local acceleration of the water in the reservoir because of velocity changes with time.* However, because the depths are usually large in reservoirs and because the velocities are small, accelerations in the reservoir are negligible. Therefore, in reservoir flood routing, we simply use the general form of the continuity equation to effect a solution. We also assume that the reservoir water surface is horizontal.

DEVELOPMENT OF THE FLOOD ROUTING PROCEDURE The general form of the continuity equation is given as

$$\sum_{cs} \rho \mathbf{V} \cdot \mathbf{A} = -\frac{d}{dt} \int_{cv} \rho \, d\forall \tag{4-46}$$

When Eq. (4-46) is applied to the control volume shown in Fig. (4-43), we have

$$I - O = \frac{d}{dt} (\forall_{\text{reservoir}}) \tag{4-47}$$

Equation (4-47) states that the volume rate of inflow of water to the reservoir minus the outflow rate is equal to the rate of change of the volume of water in the reservoir. If we call the volume of water in the reservoir storage, S, then Eq.

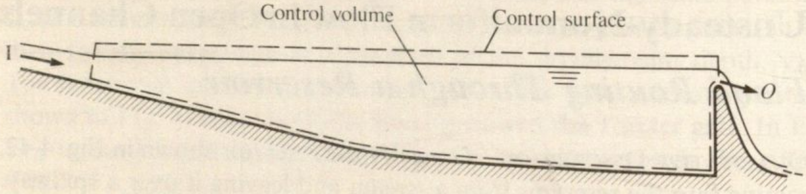


Figure 4-43 Control volume for a reservoir

* We discuss the more general problem in Chapter 12.

(4-47) becomes

$$I - O = \frac{dS}{dt} \tag{4-48}$$

For a finite time period Δt, Eq. (4-48) can be approximated by

$$I - O = \frac{\Delta S}{\Delta t} \tag{4-49}$$

In general, I and O will vary with time; thus, for a given time increment, we can approximate I as $(I_i + I_{i+1})/2$ and O as $(O_i + O_{i+1})/2$. Likewise, ΔS can be expressed as $S_{i+1} - S_i$. Then when these relations are substituted into Eq. (4-49), we get

$$\frac{(I_i + I_{i+1})}{2} - \frac{(O_i + O_{i+1})}{2} = \frac{S_{i+1} - S_i}{\Delta t} \tag{4-50}$$

or

$$I_i + I_{i+1} + \frac{2S_i}{\Delta t} - O_i = O_{i+1} + \frac{2S_{i+1}}{\Delta t} \tag{4-51}$$

For uncontrolled reservoirs such as the one being considered here (gates are not operated to control the outflow), both the storage, S, and outflow, O, will be a function of the water surface elevation in the reservoir. Here the S is obtained by simply calculating the volume in the reservoir below the given elevation from a topographic map of the reservoir, and the O is obtained from a discharge equation in the form $O = KL(z - z_0)^n$. In this latter equation, z is the elevation of the water surface, z_0 is the elevation of the spillway crest, L is the length of the spillway, and K is a flow coefficient for the spillway. Since O and S are specific functions of the water surface elevation in the reservoir (reservoir stage), it should be obvious that O will be functionally related to S. Moreover, if O is functionally related to S, then O will be functionally related to $2S/\Delta t$. That is, for any uncontrolled reservoir, one can determine a specific relationship between O and $2S/\Delta t$. For example, Fig. 4-44 on page 220 shows one such relationship for about a 100-acre reservoir.

We start routing a flood through the reservoir when both inflow and reservoir stage are known (therefore, we also know outflow at the start of the routing process). All values of inflow are also known for the period of routing. Therefore, referring to Eq. (4-51), we see that at the start of the routing process (and at the beginning of the first time increment) all the values for the left-hand side of Eq. (4-51) will be known. Thus, we will also know the value of the sum on the right-hand side of Eq. (4-51). Then from a relationship between O and $(2S/\Delta t) + O$, such as given in Fig. 4-44, we can evaluate the outflow at the end of the time

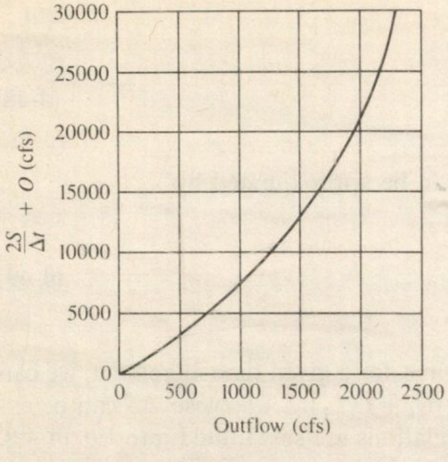


Figure 4-44 Typical relationship between O and $2S/\Delta t + O$

increment. We then use this outflow and the $(2S_i/\Delta t) - O_i$ for the next time increment. Example 4-11 explains the procedure in detail.

EXAMPLE 4-11 Using the storage-outflow relations for a reservoir (Fig. 4-44), route the hypothetical inflow (shown by the triangular hydrograph in Fig. A) through the reservoir.

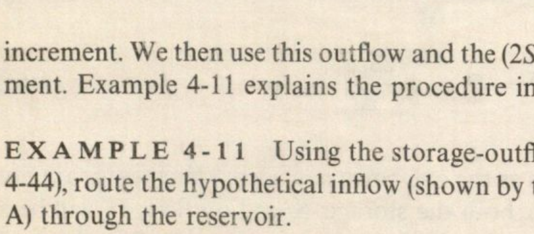


Figure A

SOLUTION The solution is carried out in the accompanying table. The numbers in column 2 are indices for time increments. For example, $i = 3$ is the time at the start of the third hourly increment. Thus, I_i, O_i, S_i for $i = 3$ are the inflow, outflow, and storage, respectively, at the beginning of time increment number 3 or, in this case, at time $= 2$ hr (120 min). The arrows in rows $i = 1$ and 2 show how the calculation proceeds. The procedure is explained as follows:

1. Starting with $i = 1$, we calculate the value in column 4 by summing I_1 and I_2.

(1) Time (hr)	(2) i	(3) I_i (cfs)	(4) $I_i + I_{i+1}$	(5) $\frac{2S_i}{\Delta t} - O_i$	(6) $\frac{2S_{i+1}}{\Delta t} + O_{i+1}$	(7) O_i (cfs)
0	1	0	500	0	500	0
1	2	500	1,500	280	1,780	110
2	3	1,000	2,500	1,200	3,700	290
3	4	1,500	3,500	2,640	6,140	530
4	5	2,000	4,500	4,500	9,000	820
5	6	2,500	5,500	6,680	12,180	1,160
6	7	3,000	5,750	9,300	15,050	1,440
7	8	2,750	5,250	11,690	16,940	1,680
8	9	2,500	4,750	13,320	18,070	1,810
9	10	2,250	4,250	14,290	18,540	1,890
10	11	2,000	3,750	14,700	18,450	1,920
11	12	1,750	3,250	14,650	17,900	1,900
12	13	1,500	2,750	14,140	16,890	1,880
13	14	1,250	2,250	13,250	15,500	1,820
14	15	1,000	1,750	12,100	13,850	1,700
15	16	750	1,250	10,670	11,920	1,590
16	17	500	750	9,100	9,850	1,410
17	18	250	250	7,410	7,660	1,220
18	19	0	0	5,660	5,660	1,000
19	20	0	0	4,100	4,100	780
20	21	0	0	2,900	2,900	600
21	22	0	0	2,000	2,000	450
22	23	0	0	1,320	1,320	340
23	24	0	0	820	820	250
24	25	0	0	500	500	160

2. The value of O_1 is known ($O_1 = O$); therefore, $2S_1/\Delta t$ can be found from Fig. 4-44, which for $i = 1$, yields $(2S_1/\Delta t) = 0$. Thus $(2S_1/\Delta t) - O_1 = 0$. Thus we record a zero in the first row of column 5.
3. The sum of values in columns 4 and 5 yields the value for the left-hand side of Eq. (4-51), and this is equal to $2S_2/\Delta t + O_2$, the value recorded in the first row of column 6 or $(2S_2/\Delta t) + O_2 = 500$. Once this quantity in column 6 is calculated, we go to Fig. 4-44.
4. With a value of $2S_2/\Delta t = 500$, we read off a value of 110 cfs from Fig. 4-44. The 110 cfs is the outflow rate at the end of time period 1, and it is also the outflow rate at the beginning of time period 2 ($i = 2$). Therefore, we record 110 cfs in row 2 of column 7.
5. For time step 2 ($i = 2$), we get $(2S_{i+2}/\Delta t) - O_{i+2}$ by subtracting $2O_2$ in column 7 from the value of $(2S_{i+1}/\Delta t) + O_{i+1}$ in column 6 of row 1. That

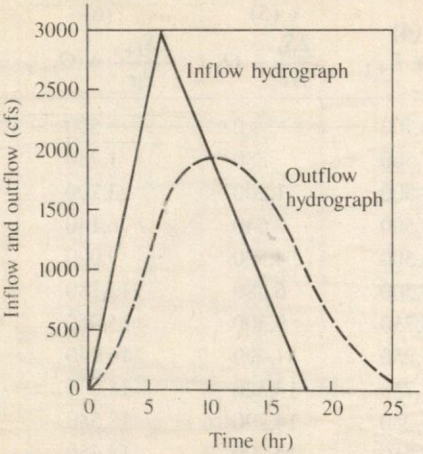


Figure B

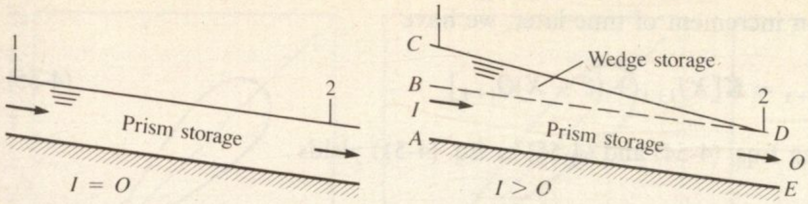
is: $(2S_2/\Delta t) - O_2 = (2S_2/\Delta t) + O_2 - 2O_2$. In this case, we get $(2S_2/\Delta t) - O_2 = 500 - 2(110) = 280$ cfs.

6. We continue this process, row by row, until the routing is completed.

The inflow flood hydrograph and outflow hydrograph are shown in Fig. B. As a result of routing the flood through the reservoir, the peak discharge has been reduced about 35%. ■

Flood Routing Through Channels

Description of the Problem When routing floods through reservoirs, both the storage, S, and outflow, O, are a function of reservoir stage. Thus, the storage could also be given as a function of outflow ($S = f(O)$); therefore, the numerical form of the continuity equation could be solved. The basic continuity equation (Eq. 4-46) is also valid for channel flow. However, because the water surface elevation is not so directly linked to outflow, the problem becomes more complicated. The physical aspect of the problem is revealed if we look at a channel that has steady-uniform flow in it and compare it to the same channel when a flood wave is passing through it (Fig. 4-45). For the case of steady-uniform flow, the discharge will be consistent with the channel slope and depth that prevails. Moreover, because the water surface slope is the same as the channel bed slope, obtaining the storage in the channel is relatively easy. Thus, for any uniform flow, we could obtain the storage for the channel at the given discharge ($S = f(O)$). However, in routing a flood through the channel, the flow is not uniform; therefore, we have the situation shown in Fig. 4-45*b*, where the water surface slope is different from the channel slope. The case shown in Fig. 4-45*b* is for the rising phase of the flood wave. In general, for the flood



(*a*) Steady-uniform flow (*b*) Flood-wave flow

Figure 4-45 Flow in channel

routing situation, $I \neq O$, and $A_1 \neq A_2$. In relation to storage, we may visualize the storage *ABDE* as being closely related to the downstream depth and thus to the outflow, *O*. This storage is sometimes called *prism* storage. Besides the prism storage, there is that part (volume *BCD* in Fig. 4-45*b*) associated with the rising part of the flood wave (or the falling part in a receding flood). This part is called *wedge* storage. We may visualize that the wedge storage is associated with the inflow, *I*, as well as the outflow.

Muskingum Method of Flood Routing As we noted, when a flood wave is passing through a reach of a channel, the storage is a function of both inflow and outflow. In 1938, a method for channel routing was presented by McCarthy (14), which was later used by U.S. Corps of Engineers (23). The method was developed in connection with flood control work on the Muskingum river and is therefore called the *Muskingum method* of flood routing. The critical point of this method is that the storage is expressed as a function of both inflow and outflow as follows:

$$S = KO + KX(I - O) \qquad (4\text{-}52)$$

In Eq. (4-52), *K* is a constant having units of time, and *X* is a dimensionless weighting factor that relates to the amount of wedge storage. In fact, we can think of the first term on the right-hand side of Eq. (4-52) as prism storage and the second term as wedge storage. This concept is reinforced because *K* is equivalent to the time required for an elemental discharge wave to travel the reach. The time increment in Eq. (4-51) should also be approximately this same length of time; therefore, $K \approx \Delta t$. Thus, $KO \approx \Delta t O = \Delta t V A_o = L A_o$ = prism storage.

Equation (4-52) can be written as

$$S = K[XI + (1 - X)O] \qquad (4\text{-}53)$$

Then, when the storage for a given time (let the index = *i*) is determined, we have

$$S_i = K[XI_i + (1 - X)O_i] \qquad (4\text{-}54)$$

Then for an increment of time later, we have

$$S_{i+1} = K[XI_{i+1} + (1 - X)O_{i+1}] \tag{4-55}$$

Substituting Eqs. (4-54) and (4-55) in Eq. (4-51) yields

$$I_i + I_{i+1} + \frac{2K}{\Delta t}[XI_i + (1 - X)O_i] - O_i = \frac{2K}{\Delta t}[XI_{i+1} + (1 - X)O_{i+1}] + O_{i+1} \tag{4-56}$$

On solving Eq. (4-56) for O_{i+1}, we obtain

$$O_{i+1} = C_0 I_{i+1} + C_1 I_i + C_2 O_i \tag{4-57}$$

where
$$C_0 = \frac{0.5\,\Delta t - KX}{K(1 - X) + 0.5\,\Delta t} \tag{4-58}$$

$$C_1 = \frac{0.5\,\Delta t + KX}{K(1 - X) + 0.5\,\Delta t} \tag{4-59}$$

$$C_2 = \frac{K(1 - X) - 0.5\,\Delta t}{K(1 - X) + 0.5\,\Delta t} \tag{4-60}$$

From Eq. (4-57), we can see that if K, X, I_i, I_{i+1}, and O_i are known, we can easily solve for O_{i+1}. This outflow can be used for the starting outflow, O_i, for the next time increment. Then by taking succession time increments, the outflow can be obtained for the entire flood period.

DETERMINATION OF K AND X We have shown that a flood could be routed through a given reach of channel if K, X, inflow, and initial outflow are known. In practice, one often wants to route a flood through a channel that already has gauging stations along its length. For such a channel, records of historical floods would be available. Therefore, the flood routing method is reversed so as to obtain K and X; that is, given the inflow and outflow hydrographs, solve for K and X. We can do this by solving Eq. (4-56) for K:

$$K = \frac{0.5\,\Delta t[(I_i + I_{i+1}) - (O_i + O_{i+1})]}{X(I_{i+1} - I_i) + (1 - X)(O_{i+1} - O_i)} \tag{4-61}$$

The numerator in Eq. (4-61) is the increment of storage that has accumulated in time Δt, and the denominator is termed the weighted inflow and outflow. Let N be the numerator of Eq. (4–61) and D be the denominator. Then for each time increment, the historical I and O hydrographs may be analyzed to obtain N and D (then, in turn, $K = N/D$) for a given value of X. Actually, we want to find one value of K and one value of X that will be valid for the entire

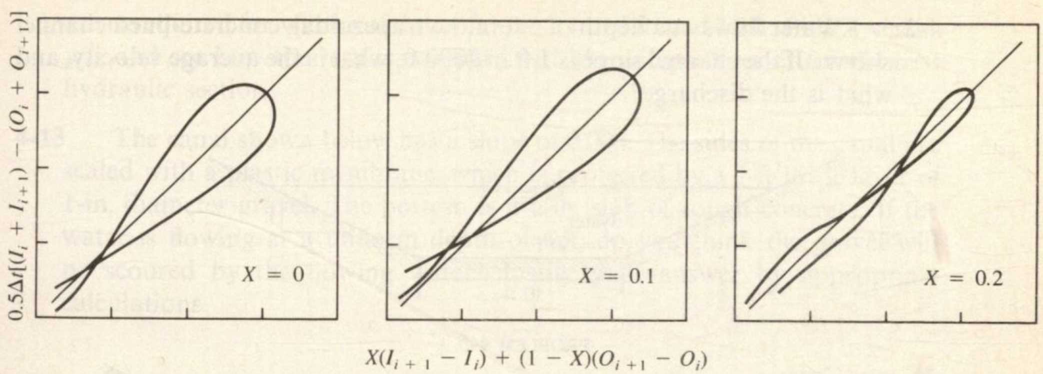


Figure 4-46 Plots for evaluating X and K

hydrograph. To ascertain the "best" values of K and X, the accumulated value of D is plotted against the accumulated value of N, as shown in Fig. 4-46. Obviously, the plot will be different depending on the value of X used in Eq. (4–61). We choose different values of X (say 0.10, 0.20, 0.30, 0.40) and make a plot for each X value. The plot giving a curve that is most nearly a straight line is the best. Thus, that value of X is the one to use, and the slope of the curve will yield the K value that should be used.*

PROBLEMS

4-1 A rectangular concrete channel is 12 ft wide, and water flows in it uniformly at a depth of 4 ft. If the channel drops 10 ft in a length of 800 ft, what is the discharge? Assume $T = 60°F$.

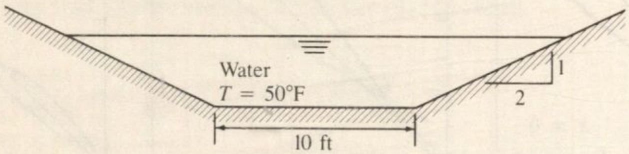
4-2 A concrete sewer pipe 4 ft in diameter is laid so it has a drop in elevation of 0.90 ft per 1000 ft of length. If sewage (assume the properties are the same as those of water) flows at a depth of 2 ft in the pipe, what will be the discharge?

4-3 A rectangular concrete channel 4 m wide on a slope of 0.004 is designed to carry water ($T = 10°C$) at a discharge of 25 m^3/s. Estimate the uniform flow depth for these conditions.

4-4 A rectangular troweled concrete channel 12 ft wide with a slope of 10 ft in 8000 ft is designed for a discharge of 600 cfs. For a water temperature of 40°F, estimate the depth of flow.

* For more detailed information on this subject, see Hjelmfelt (10). The preceding sections on unsteady flow do not explicitly consider the momentum effects in the solutions for flow through reservoirs or channels. A more accurate method of solution considers these effects; however, much more detailed sets of data of the channel geometry are required to bring about a solution. We discuss the more general approach in Chapter 12.

4-5 Water flows at a depth of 6 ft in the trapezoidal, concrete-lined channel shown. If the channel slope is 1 ft in 2000 ft, what is the average velocity, and what is the discharge?



PROBLEM 4-5

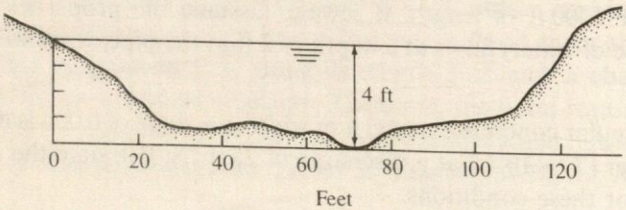
4-6 A concrete-lined trapezoidal channel having a bottom width of 10 ft and side slopes of 1 vertical to 2 horizontal is designed to carry a flow of 3000 cfs. If the slope of the channel is 0.001, what would be the depth of flow in the channel?

4-7 Estimate the discharge in a rock-bedded stream ($d_{84} = 30$ cm) that has an average depth of 2.21 m, a slope of 0.0037, and a width of 46 m. Assume $k_s = d_{84}$.

4-8 Determine the discharge in a 5-ft diameter concrete sewer pipe on a slope of 0.01 that is carrying water at a depth of 4 ft.

4-9 What will be the depth of flow in a trapezoidal concrete-lined channel that has a water discharge of 1000 cfs in it? The channel has a slope of 1 ft in 500 ft. The bottom width of the channel is 10 ft, and the side slopes are 1 V to 1 H.

4-10 Estimate the discharge in the Moyie River near Eastport, Idaho, when the depth is 4 ft, as shown in the figure below. Assume $S_0 = 0.0032$.

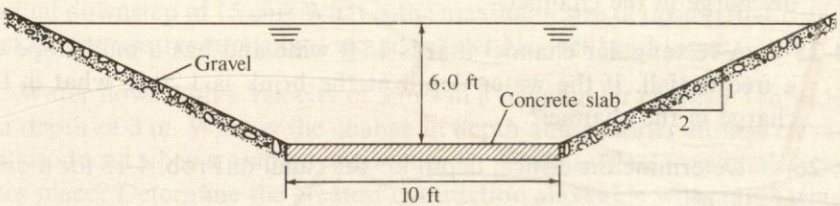


PROBLEM 4-10

4-11 What discharge of water will occur in a trapezoidal channel that has a bottom width of 10 ft and sides slope 1 to 1 if the slope of the channel is 5 ft/mi and the depth is to be 5 ft? The channel will be lined with concrete.

4-12 Consider channels of rectangular cross section carrying 100 cfs of water flow. The channels have a slope of 0.001. Determine the cross-sectional areas required for widths of 2 ft, 4 ft, 6 ft, 8 ft, 10 ft, and 15 ft. Plot A versus y/b, and see how the results compare with the accepted result for the best hydraulic section.

4-13 The canal shown below has a slope of 0.004. The sides of the canal are sealed with a plastic membrane, which is protected by a 1 ft thick layer of 1-in. diameter gravel. The bottom is a 6-in. slab of rough concrete. If the water is flowing at a uniform depth of 6 ft, do you think the gravel will be scoured by the flowing water? Justify your answer by appropriate calculations.



PROBLEM 4-13

4-14 A trapezoidal irrigation canal is to be excavated in soil and lined with coarse gravel. The canal is to be designed for a discharge of 200 cfs, and it will have slope of 0.0016. What should be the magnitude of the cross-sectional area and hydraulic radius for the canal if it is to be designed so that erosion of the canal will not occur? Choose a canal cross section that will satisfy the limitations.

4-15 Determine the cross section of an unlined canal excavated from sandy loam soil that is to carry 30 cfs. Assume the canal slope is to be 0.0005.

4-16 The water discharge in a rectangular channel 5 m wide is 15 m^3/s. If the depth of water is 1 m, is the flow subcritical or supercritical?

4-17 The discharge in a rectangular channel 5 m wide is 10 m^3/s. If the water velocity is 1.0 m/s, is the flow subcritical or supercritical?

4-18 Water flows at a rate of 10 m^3/s in a rectangular channel 3 m wide. Determine the Froude number and the type of flow (subcritical, critical, or supercritical) for depths of 30 cm, 1.0 m and 2.0 m. What is the critical depth?

4-19 For the discharge and channel of Prob. 4-18, what is the alternate depth to the 30-cm depth? What is the specific energy for these conditions?

4-20 Water flows at a depth of 10 cm with a velocity of 6 m/s in a rectangular channel. Is the flow subcritical or supercritical? What is the alternate depth?

4-21 Water flows at the critical depth in a rectangular channel with a velocity of 2 m/s. What is the depth of flow?

4-22 Water flows uniformly at a rate of 9.0 m^3/s in a rectangular channel that is 4.0 m wide and has a bottom slope of 0.005. If n is 0.014, is the flow subcritical or supercritical?

4-23 A rectangular channel is 6 m wide, and the discharge of water in it is 18 m^3/s. Plot depth versus specific energy for these conditions. Let specific energy range from E_{min} to $E = 7$ m. What are the alternate and sequent depths to the 30-cm depth?

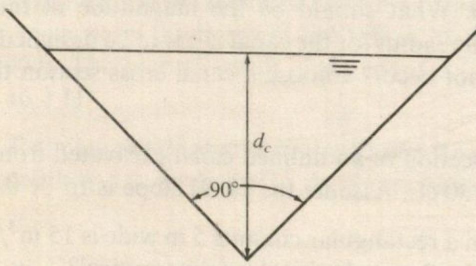
4-24 A long rectangular channel that is 3 m wide and has a mild slope ends in a free outfall. If the water depth at the brink is 0.250 m, what is the discharge in the channel?

4-25 A rectangular channel that is 15 ft wide and has a mild slope ends in a free outfall. If the water depth at the brink is 1.20 ft, what is the discharge in the channel?

4-26 Determine the critical depth for the canal of Prob. 4-13 for a discharge of 700 cfs.

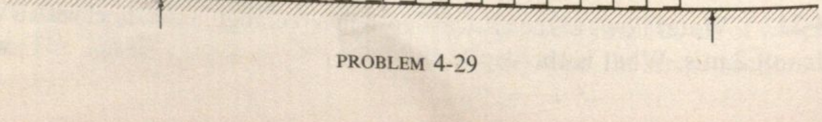
4-27 A 48-in. concrete pipe culvert carries a discharge of water of 25 cfs. Determine the critical depth.

4-28 Derive a formula for critical depth, d_c, in the V-shaped channel shown below.



PROBLEM 4-28

4-29 A 10 ft wide rectangular channel is very smooth except for a small reach that is roughened with angle irons attached to the bottom of the channel (see figure below). Water flows in the channel at a rate of 200 cfs and at a depth of 1.00 ft. Assume frictionless flow except over the roughened part where the total drag of all the roughness (all the angle irons) is assumed to be 2000 lb. Determine the depth at the end of the roughness elements for the assumed conditions.



PROBLEM 4-29

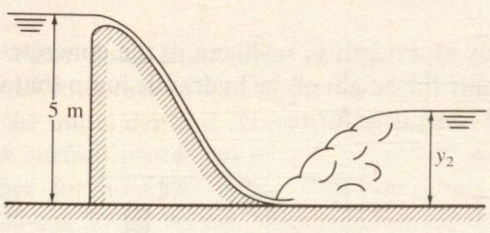
4-30 Water flows with a velocity of 3 m/s and at a depth of 3 m in a rectangular channel. What is the change in depth and in water surface elevation produced by a gradual upward change in bottom elevation (upstep) of 30 cm? What would be the depth and elevation changes if there were a gradual downstep of 30 cm? What is the maximum size of upstep that could exist before upstream depth changes would result? Neglect head losses.

4-31 Water flows with a velocity of 2 m/s and at a depth of 3 m in a rectangular channel. What is the change in depth and in water surface elevation produced by a gradual upward change in bottom elevation (upstep) of 60 cm? What would be the depth and elevation changes if there were a gradual downstep of 15 cm? What is the maximum size of upstep that could exist before upstream changes would result? Neglect head losses.

4-32 Water flows with a velocity of 3 m/s in a rectangular channel 3 m wide at a depth of 3 m. What is the change in depth and in water surface elevation produced when a gradual contraction in the channel to a width of 2.6 m takes place? Determine the greatest contraction allowable without altering the specified upstream conditions. Neglect head losses.

4-33 Design a cylinder quadrant inlet transition to join a trapezoidal concrete-lined channel and a concrete flume of rectangular cross section. Assume the channel is on a slope of 0.0005, has side slopes 1 vertical to 2 horizontal, and the depth of flow therein is 5 ft. The bottom width of the channel is 10 ft. The depth in the flume is to be equal to the width of flume. Assume that the Froude number in the flume is to be equal to or less than 0.60.

4-34 The spillway shown has a discharge of 2.0 m^3/s per meter of width occurring over it. What depth y_2 will exist downstream of the hydraulic jump? Assume negligible energy loss over the spillway.



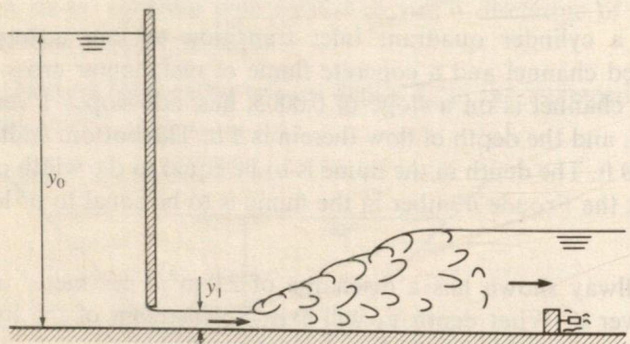
PROBLEM 4-34

4-35 The flow of water downstream from a sluice gate in a horizontal channel has a depth of 30 cm and a flow rate of 1.8 m^3/s per meter of width. Could a hydraulic jump be caused to form downstream of this section? If so, what would be the depth downstream of the jump?

4-36 Consider the dam and spillway shown in Fig. 4-25, page 198. Water is discharging over the spillway of the dam as shown. The elevation difference between upstream pool level and the floor of the apron of the dam is 100 ft. If the head on the spillway is 5 ft and if a hydraulic jump forms on the horizontal apron, what is the depth of flow on the apron just downstream of the jump? Assume that the velocity just upstream of the jump is 95% of the maximum theoretical velocity. *Note*: The discharge over the spillway is given as $Q = KL\sqrt{2g}H^{3/2}$, where L is the length of the spillway, K is a coefficient (assume it has a value of 0.5), and H is the head on the spillway.

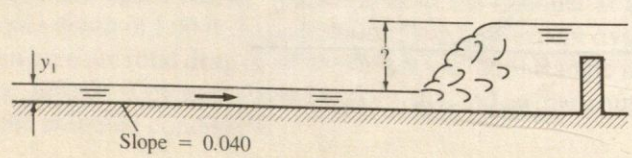
4-37 A hydraulic jump occurs in a wide rectangular channel. If the depths upstream and downstream are 15 cm and 4.0 m, respectively, what is the discharge per foot of width of channel?

4-38 Water is flowing as shown under the sluice gate in a horizontal rectangular channel that is 5 ft wide. The depths y_0 and y_1 are 65 ft and 1 ft, respectively. What will be the horsepower lost in the hydraulic jump?



PROBLEM 4-38

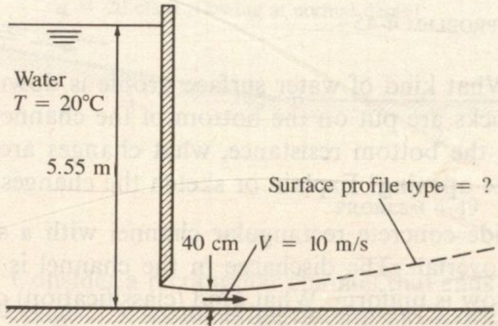
4-39 Water flows uniformly at a depth $y_1 = 40$ cm in the concrete channel, which is 10 m wide. Estimate the height of the hydraulic jump that will form when a sill is installed to force it to form.



PROBLEM 4-39

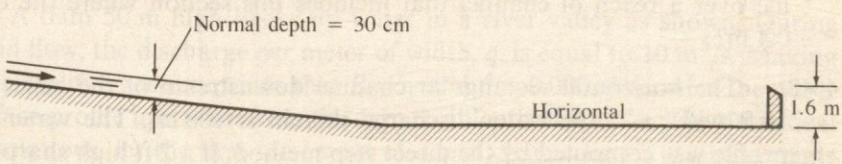
4-40 For the derivation of Eq. (4-27), it is assumed that the bottom shearing force is negligible. For the conditions of Prob. 4-39, estimate the magnitude of the shearing force F_s associated with the hydraulic jump, and then determine F_s/F_H, where F_H is the net hydrostatic force on the hydraulic jump.

4-41 The normal depth in the channel downstream of the sluice gate shown is 1 m. What type of water surface profile occurs downstream of the sluice gate?



PROBLEM 4-41

4-42 The partial water surface profile shown is for a rectangular channel that is 3 m wide and has water flowing in it at a rate of 5 m^3/s. Sketch in the missing part of water surface profile and identify the type(s).

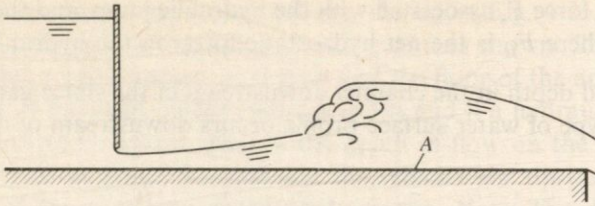


PROBLEM 4-42

4-43 Water flows from under a sluice gate into a horizontal rectangular channel at a rate of 3 m^3/s per meter of width. The channel is concrete, and the initial depth is 20 cm. Apply Eq. (4-33), page 203, to construct the water surface profile up to a depth of 60 cm. In your solution, compute reaches for adjacent pairs of depths given in the following sequence: d = 20 cm, 30 cm, 40 cm, 50 cm, and 60 cm. Assume that f is constant with a value of 0.02. Plot your results.

4-44 A horizontal rectangular concrete channel terminates in a free outfall. The channel is 4 m wide and carries a discharge of water of 12 m^3/s. What is the water depth 300 m upstream from the outfall?

4-45 Given the hydraulic jump shown on the next page for the long horizontal rectangular channel, what kind of water surface profile (classification) is



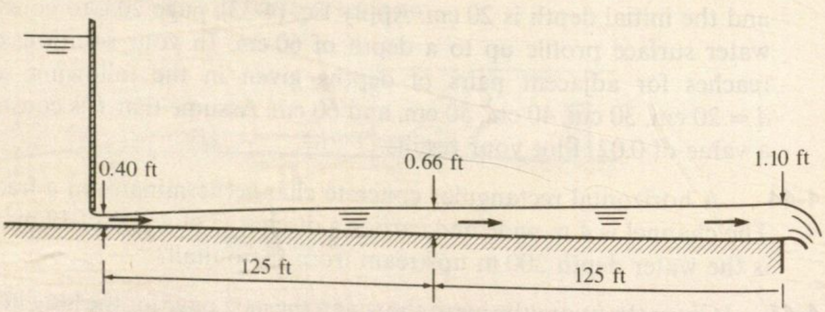
PROBLEM 4-45

upstream of the jump? What kind of water surface profile is downstream of the jump? If baffle blocks are put on the bottom of the channel in the vicinity of *A* to increase the bottom resistance, what changes are apt to occur given the same gate opening? Explain or sketch the changes.

4-46 A very long 10 ft wide concrete rectangular channel with a slope of 0.0001 ends with a free overfall. The discharge in the channel is 120 cfs. One mile upstream the flow is uniform. What kind (classification) of water surface profile occurs upstream of the brink?

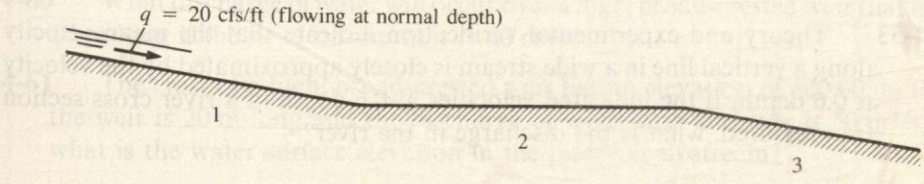
4-47 The discharge in a 2 m wide very rough rectangular channel in which the water flows 1 m deep is 10.5 m^3/s. The channel has a slope of 0.10, and the flow is uniform at this depth. A certain structure causes the flow at another part of the same channel to be locally only 0.40 m deep. At this different section of the channel, the channel slope, roughness, and cross section are the same as before. What do you conclude about the water surface profile over a reach of channel that includes this section where the depth is 0.4 m?

4-48 The horizontal rectangular channel downstream of the sluice gate is 10 ft wide, and the water discharge therein is 108 cfs. The water surface profile was computed by the direct step method. If a 2 ft high sharp-crested weir is installed at the end of the channel, do you think a hydraulic jump would develop in the channel? If so, approximately where would it be located? Justify your answers by appropriate calculations. Label any water surface profiles that can be classified.



PROBLEM 4-48

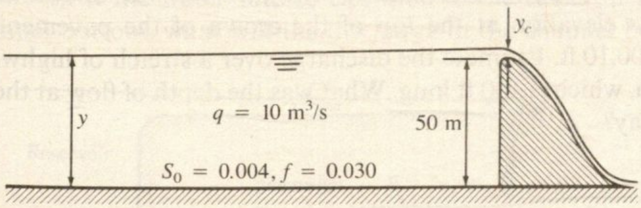
4-49 The discharge per foot of width in this rectangular channel is 20 cfs. The normal depths for parts 1 and 3 are 0.5 ft and 1.00 ft, respectively. The slope for part 2 is 0.001 (sloping upward in the direction of flow). Sketch all possible water surface profiles for flow in this channel, and label each part with its classification.



PROBLEM 4-49

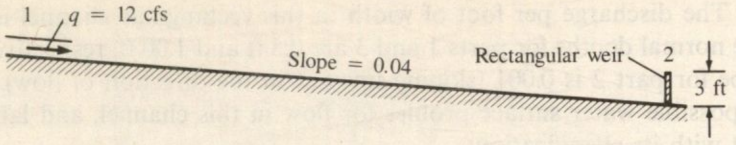
4-50 Consider a rectangular channel that ends with a dam, as shown in Fig. 4-29, page 203. Assume that the slope (S_0), discharge (Q), channel roughness (n) or (k_s), height of dam (Z), and length of spillway (L), are all given. Write a computer program to solve for the backwater curve (M1 curve) for this flow situation. Solve for the backwater curve (water surface elevation versus X) for $S_0 = 0.004$, $Q = 20{,}000$ cfs, $Z = 50$ ft, and $L = 200$ ft, and plot the results. Submit your program with your solution. Make your own assumption about roughness. Assume the channel has the same width as spillway length.

4-51 A dam 50 m high backs up water in a river valley as shown. During flood flow, the discharge per meter of width, q, is equal to 10 m^3/s. Making the simplifying assumptions that $R = y$ and $f = 0.030$, determine the water surface profile upstream from the dam to a depth of 6 m. In your numerical calculations, let the first increment of depth change be y_c; use increments of depth change of 10 m until a depth of 10 m is reached; and then use 2-m increments until the desired limit is reached.



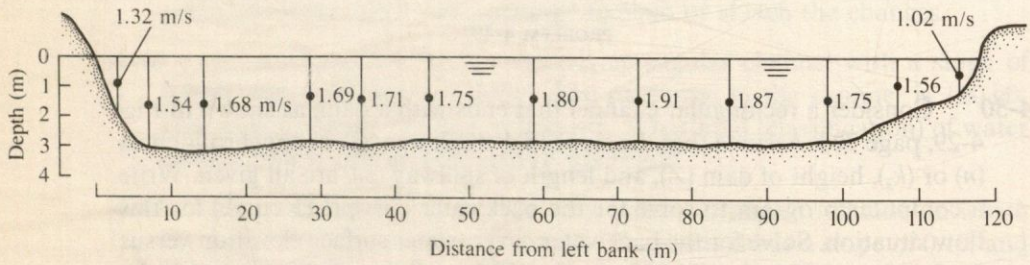
PROBLEM 4-51

4-52 Water flows at a steady rate of 12 cfs per foot of width ($q = 12$ cfs) in the wide rectangular concrete channel shown on the next page. Determine the water surface profile from section 1 to section 2.



PROBLEM 4-52

4-53 Theory and experimental verification indicate that the mean velocity along a vertical line in a wide stream is closely approximated by the velocity at 0.6 depth. If the indicated velocities at 0.6 depth in a river cross section are measured, what is the discharge in the river?



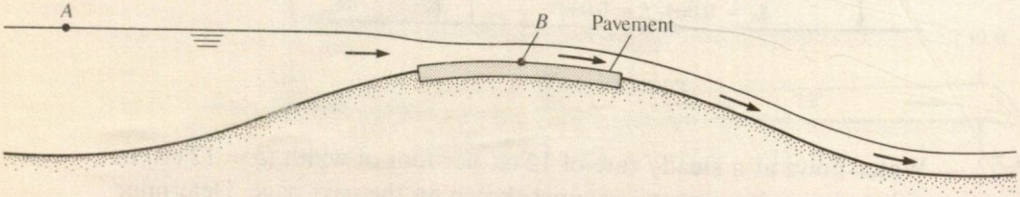
PROBLEM 4-53

4-54 Water flows over a rectangular weir that is 2 m wide and 30 cm high. If the head on the weir is 13 cm, what is the discharge in cubic meters per second?

4-55 What is the discharge over a rectangular weir 1 m high in a channel 2 m wide if the head on the weir is 25 cm?

4-56 What is the discharge over a rectangular weir 2 ft high in a channel 6 ft wide if the head on the weir is 1 ft?

4-57 A flood caused water to flow over a highway as shown below. The water surface elevation upstream of the highway (at *A*) was measured to be 101.00 ft. The elevation at the top of the crown of the pavement of the highway is 100.10 ft. Estimate the discharge over a stretch of highway with this elevation, which is 100 ft long. What was the depth of flow at the crown of the highway?



PROBLEM 4-57

4-58 The head on a 60° triangular weir is 1.5 ft. What is the discharge of water over the weir?

4-59 A rectangular irrigation canal 3 m wide carries water with a discharge of 6 m^3/s. What height of rectangular weir installed across the canal will raise the water surface to a level 2 m above the canal floor?

4-60 What discharge of water will occur over a high, broad-crested weir that is 10 m long if the head on the weir is 60 cm?

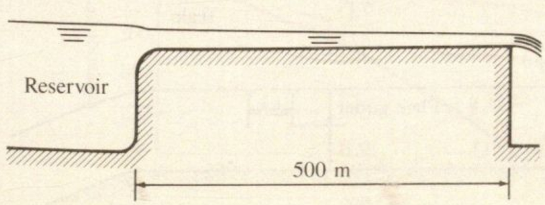
4-61 The crest of a high, broad-crested weir has an elevation of 100.00 m. If the weir is 20 m long and the discharge of water over the weir is 50 m^3/s, what is the water surface elevation in the reservoir upstream?

4-62 The crest of a high, broad-crested weir has an elevation of 300.00 ft. If the weir is 50 ft long and the discharge of water over the weir is 1500 cfs, what is the water surface elevation in the reservoir upstream?

4-63 Several discharge measuring devices are being considered for measurement of flow to a part of an irrigation project. For a discharge of 100 cfs, determine the head H and approximate minimum head loss across the device for the following:

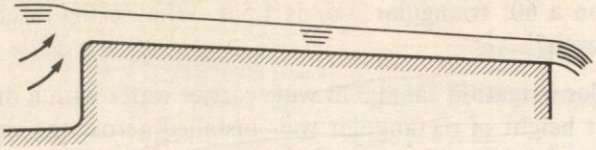
a. Rectangular weir, as shown in Fig. 4-33, page 211, with $L = 15$ ft and $P = 2$ ft
b. Broad-crested weir, as shown in Fig. 4-15, page 185, with $L = 20$ ft and $P = 2$ ft
c. Triangular weir with $\theta = 90°$
d. Parshall flume with $W = 6$ ft (free flow condition)
e. Ten foot wide sluice gate with $y = 1/2$ ft (free flow condition)
f. Ten foot wide Tainter gate with $y = 1/2$ ft (free flow condition); also for this gate, $a = 5$ ft and $r = 10$ ft

4-64 The steep rectangular concrete channel shown is 4 m wide and 500 m long. It conveys water from a reservoir and delivers it to a free outfall. The channel entrance is rounded and smooth (negligible head loss at the entrance). If the water surface elevation in the reservoir is 2 m above the channel bottom, what will the discharge in the channel be?



PROBLEM 4-64

4-65 The concrete rectangular channel shown on the next page is 3.5 m wide and has a bottom slope of 0.001. The channel entrance is rounded and



PROBLEM 4-65

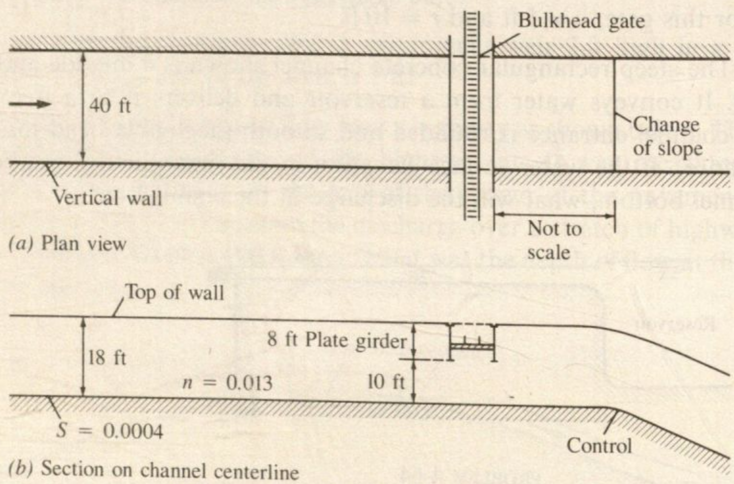
smooth (negligible head loss at the entrance), and the reservoir water surface is 2.5 m above the bed of the channel at the entrance.

a. If the channel is 3000 m long, estimate the discharge in it.
b. If the channel is only 100 m long, tell how you would solve for the discharge in it.

4-66 A straight rectangular concrete ($n = 0.013$) flood control channel flows through a city in a valley. Upstream of a control section, the slope is 0.0004, the width is 40 ft, and sidewall height is 18 ft. In the upstream reach, the uniform approach flow is subcritical, whereas in the reach downstream from the control, the flow is supercritical. The slope change isolates the upstream channel from any backwater effects of the downstream channel.

An industrial developer proposes to build a railroad bridge across the channel, upstream from the control section. To match track grades, his design calls for a bridge with 8 ft deep plate girders having a 10-ft clearance above the channel bottom. (Special bulkhead gates are to be provided to close the holes breached in the channel walls when water depth in the channel approaches the bottom of the plate girders.)

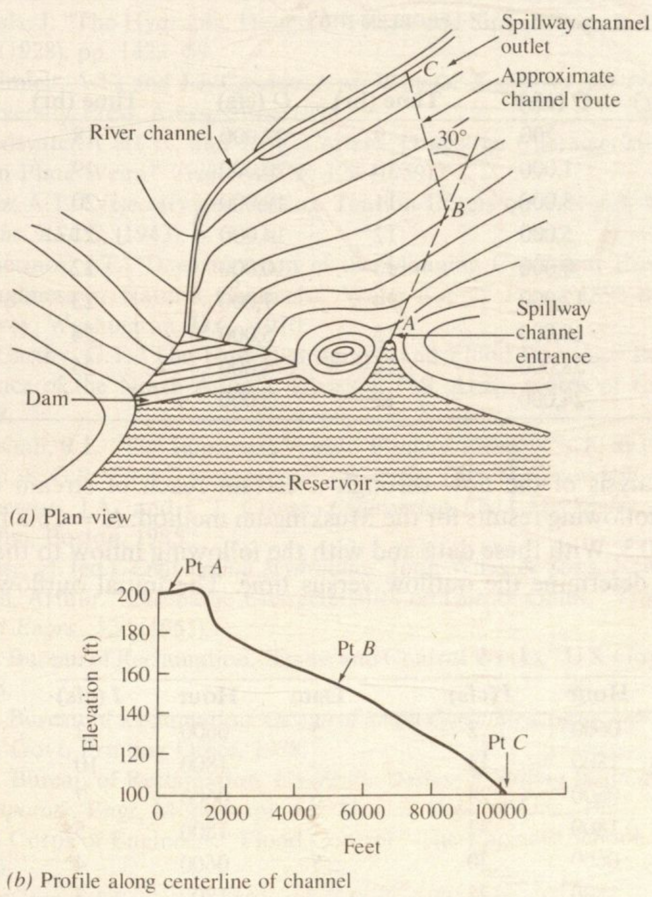
a. What is the maximum flow (cfs) in the channel before the water surface rises to the bottom of the plate girders?



PROBLEM 4-66

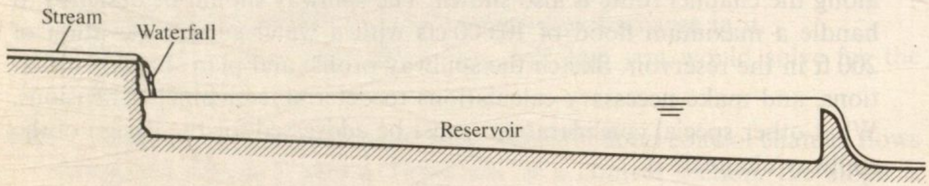
b. Estimate the flow at which the girder will first be overtopped.
c. What is the horizontal force on the bridge, due to the water, at this time?
d. What is the reduction caused by the bridge in the maximum discharge that will just overtop the sidewall at the bridge location?

4-67 The sketch for this problem is a plan view of an earth fill dam and proposed centerline of a spillway channel. Because of geology and other restrictions, assume that this location is fixed. The profile of the ground surface along the channel route is also shown. The spillway should be designed to handle a maximum flood of 10,000 cfs with a water surface elevation of 200 ft in the reservoir. Sketch the spillway profile and plan showing elevations, and make necessary calculations to determine channel dimensions. What other special considerations must be addressed for the design of the spillway channel?



PROBLEM 4-67

4-68 Consider the stream, reservoir, and dam shown below. Assume that the reservoir is enclosed by vertical rock walls except at the lower end where the dam is located. The reservoir pool area is 1000 acres. The length of the spillway is 200 ft. Before the flood hit the basin, assume that the steady flow rate was 200 cfs. A flood hit the basin, and the flood discharges for the incoming stream are given in the table. Route the flood through the reservoir. Plot the inflow and outflow hydrograph on the same scale. Assume the value of the spillway flow coefficient, K, is 0.50.



PROBLEM 4-68

Time (hr)	Q (cfs)	Time (hr)	Q (cfs)	Time (hr)	Q (cfs)
0	200	9	22,000	18	2,000
1	1,000	10	20,000	19	1,500
2	3,000	11	19,000	20	1,000
3	5,000	12	14,000	21	500
4	8,000	13	10,000	22	400
5	12,000	14	7,000	23	300
6	18,000	15	5,000	24	250
7	23,000	16	4,000	25	200
8	24,000	17	3,000		

4-69 The analysis of the flow through a certain reach of stream channel yielded the following results for the Muskingum method: $K = 25.2$ hr, $\Delta t = 12$ hr, $X = 0.3$. With these data and with the following inflow to that reach of channel, determine the outflow versus time. The initial outflow rate is 2 cfs.

Date	Hour	I (cfs)	Date	Hour	I (cfs)
1	0600	2	5	0600	13
	1800	15		1800	10
2	0600	28	6	0600	7
	1800	32		1800	5
3	0600	30	7	0600	4
	1800	25		1800	3
4	0600	20	8	0600	3
	1800	16		1800	2

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