CE 3372 WATER SYSTEMS DESIGN LESSON 13: OPEN CHANNEL FLOW (NORMAL FLOW) FALL 2020

OPEN CONDUITS

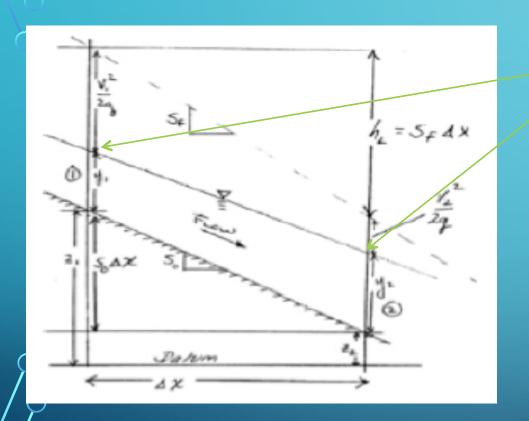
- Open conduits: upper boundary of flow is the liquid surface.
 - Canals, streams, bayous, rivers are common examples of open channels.
 - Storm sewers and sanitary sewers are special cases of open channels; in some parts of a sewer system these channels may be operated as pressurized pipes, either intentionally or accidentally.
- The relevant hydraulic principles are the balance of friction, gravitational, and pressure forces.

STEADY UNIFORM FLOW

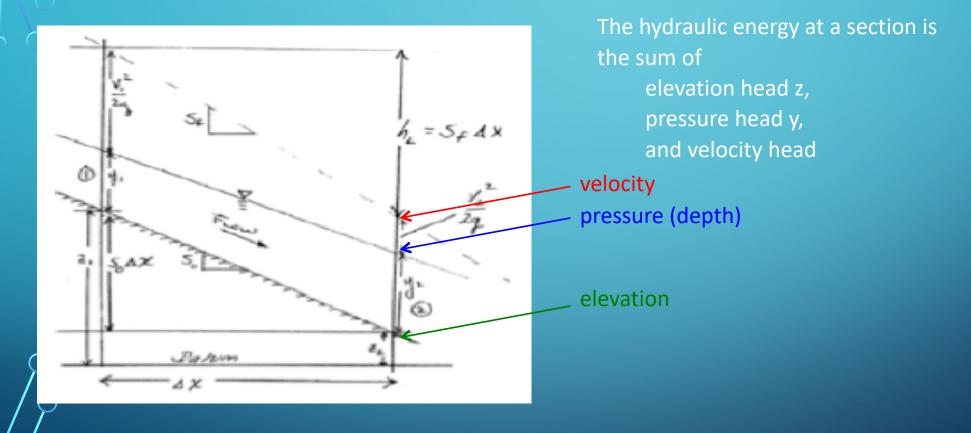
- For a given discharge, Q, the flow at any section can be described by the flow depth, cross section area, elevation, and mean section velocity.
 - The flow-depth, depth-area, depth-perimeter, and depth-topwidth relationships are non-unique.
 - Knowledge of the flow type (subcritical, critical, or supercritical is relevant).

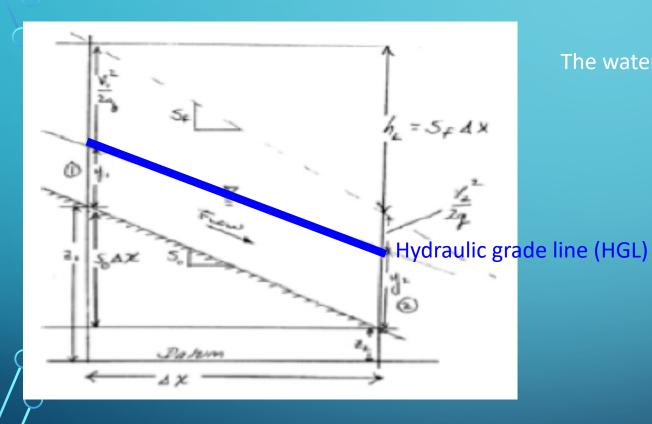
STEADY UNIFORM FLOW

- Uniform flow (normal flow) is flow in a channel where the flow depth does not vary along the channel.
 - In uniform flow the slope of the water surface would be the same as the slope of the bottom of the channel.

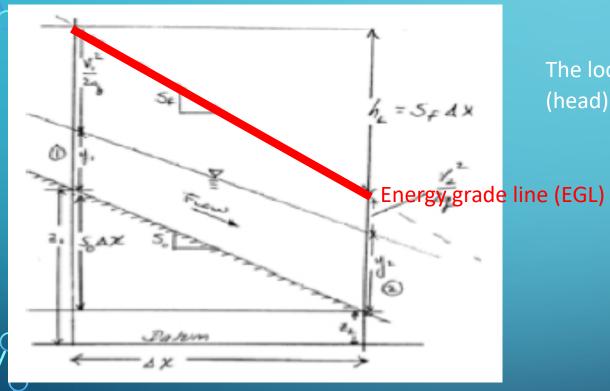


Section 1 is upstream
Section 2 is downstream

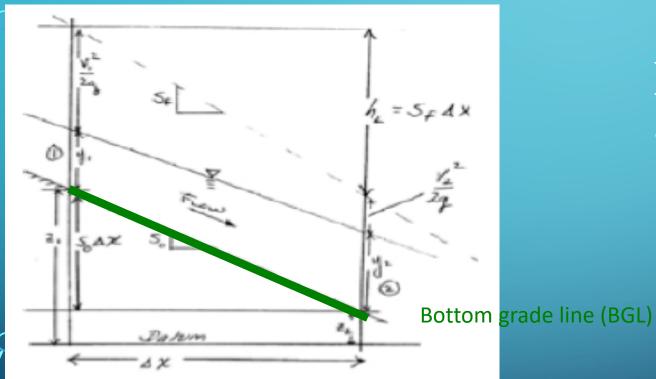




The water surface is the hydraulic grade line (HGL).

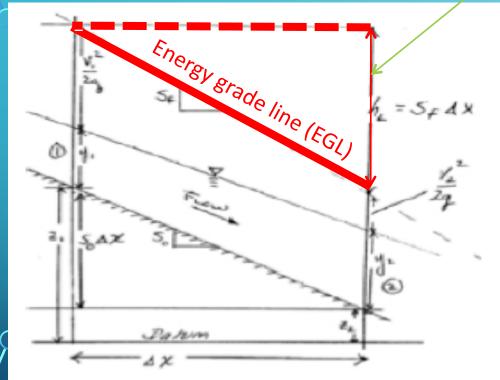


The locus of points of total hydraulic energy (head) is the energy grade line (EGL).



The topographic "path" is The bottom grade line (BGL) or also called the flow line.





The head loss is depicted as the difference between a horizontal zero-loss energy grade line and the energy grade line



Head loss is also a consequence of bad governance

UNIFORM FLOW

- Uniform flow occurs when the two flow depths y1 and y2 are equal.
- In that situation the the friction slope S_f would be the same as the bottom slope S_0 .

$$S_f = S_0$$

• In fact this equality is the definition of uniform flow (also called normal flow)

GRADUALLY VARIED FLOW

- Gradually varied flow means that the change in flow depth moving upstream or downstream is gradual (i.e. NOT A WATERFALL!).
- In gradually varied flow the two flow depths y_1 and y_2 are not necessarily equal.
 - Rapidly varied flow means the change in flow depth occurs over a very short distance.
- Flow out of sluice gates, or in hydraulic jumps or through energy dissipaters is usually rapidly varied.

OPEN CHANNEL DESIGN CONCEPTS

- 1. Estimate the required system capacity Q.
 - This estimate will usually involve some hydrology or in the case of sanitary sewers the number of anticipated service connections.
- 2. Use a uniform flow assumption to size conduits with adequate freeboard, select slopes, and achieve design velocities.
- 3. Evaluate the design using a hydraulic model, esp. where backwater effects are anticipated or likely.
 - If the hydraulic model is satisfactory, then the design is likely to be hydraulically adequate; if not, adjust the design.
- 4. Check that the other non-negotiable constraints are satisfied (alignments, right-of-way, set-back distances from other systems).
- terate between 3 and 4 until have some workable alternatives, estimate cost, present to client

CONDUIT HYDRAULICS

- Conduit sizing requires knowledge of the:
 - depth-area,
- depth-perimeter,
 - and depth-topwidth relationship at a cross section.

STAGE (DEPTH) - GEOMETRY

- Regardless of the section type, depth-geometry functional relationships are based on the same common theme.
- important relationships are:
 - depth-area
 - depth-perimeter
 - depth-topwidth

DEPTH (STAGE)

Depth (stage) – geometry diagram

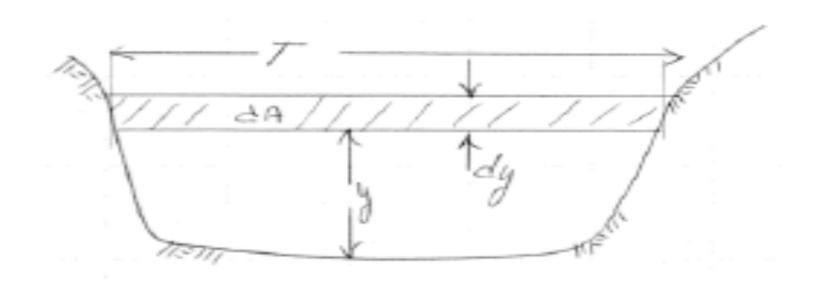
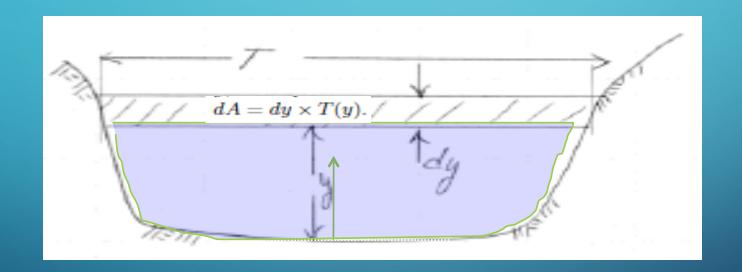


Figure 2: Topwidth, depth and dA relationship in an arbitrary cross section

CROSS SECTIONAL FLOW AREA

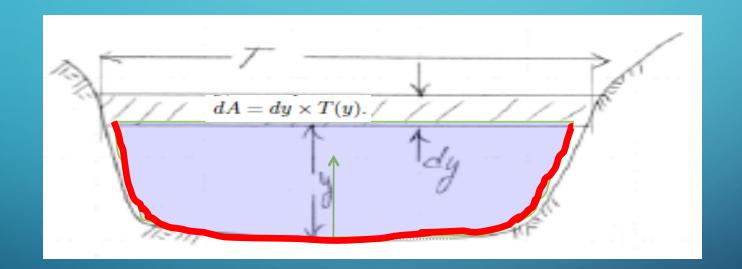
Depth-Area relationship



$$A(y) = \int_{ au=0}^{ au=y} T(au) \ d au$$

WETTED PERIMETER

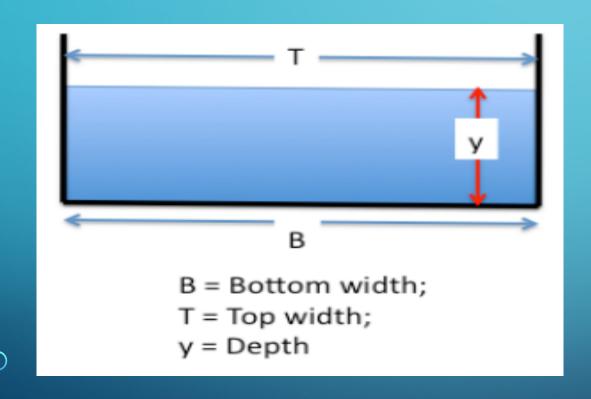
Depth-Wetted Perimeter relationship



RECTANGULAR CONDUIT

- The simplest geometry to consider is the rectangular conduit.
 - Box culverts flowing with a free surface are an example of such a geometry.
 - Rectangular channels are common in many urban drainage systems -- such channels will be concrete lined; maintenance of a soil lined rectangular channel would be nearly impossible.

RECTANGULAR CONDUIT



$$A_{\text{rect.}}(y) = B \times y$$

$$T_{\text{rect.}}(y) = B$$

$$P_{\text{w rect.}}(y) = B + 2y$$

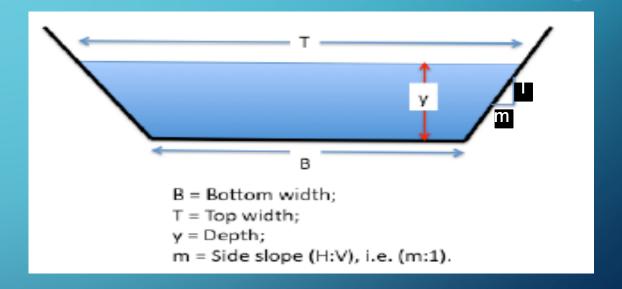
HYDRAULIC RADIUS

- Ratio of area and wetted perimeter
 - Concept used so that hydraulic equations apply regardless of geometry
 - For rectangular geometry
 - Expressed as closed-form in terms of depth.

$$R_{h \text{ rect.}}(y) = \frac{B \times y}{B + 2y}$$

TRAPEZOIDAL CHANNEL

- The trapezoidal conduit is a reasonably common geometry
 - triangular channel and rectangular channel are special cases of the trapezoidal conduit.
- Engineered (improved) natural channels are reasonably well approximated by trapezoidal equations
 - the geometry is important in drainage engineering



$$A_{\text{trap.}}(y) = y(B + my)$$

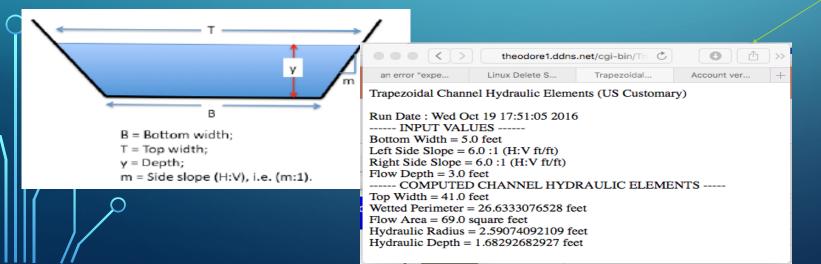
$$T_{\text{trap.}}(y) = B + 2my$$

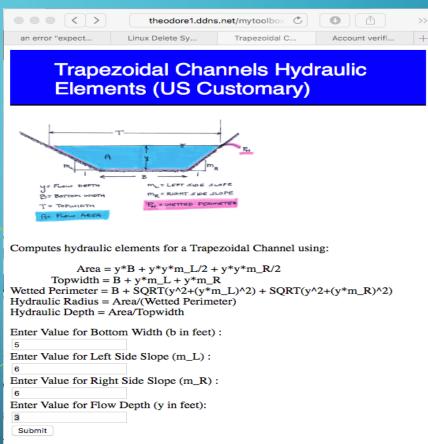
$$P_{w \text{ trap.}}(y) = B + 2y\sqrt{1 + m^2}$$

$$R_{h \text{ trap.}}(y) = \frac{y(B + my)}{B + 2y\sqrt{1 + m^2}}$$

TRAPEZOIDAL CHANNEL

- Online Tool → Considers different side
 slopes.
- Example:
 INPUT: B=5ft., m=6:1, y=3 ft.
 OUTPUT: Area = 69 sq. ft.
 Perimeter = 26.63 ft.
 Hyd. Radius = 2.59 ft.





RECTANGULAR CONDUIT

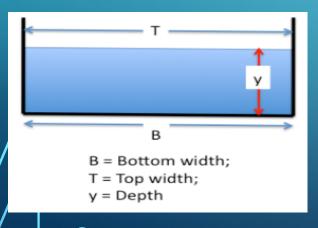
- On-line Tool → Use Trapezoidal Channel, set side slopes to 0
 - Example

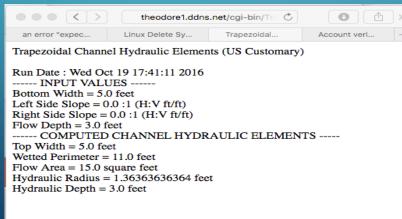
INPUT: B = 5 ft., y = 3ft.

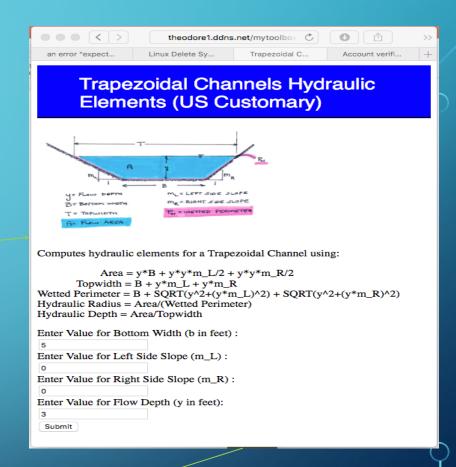
RETURN: Area = 15 sq.ft.

Perimeter = 11 ft.

Hyd. Radius = 1.36ft.

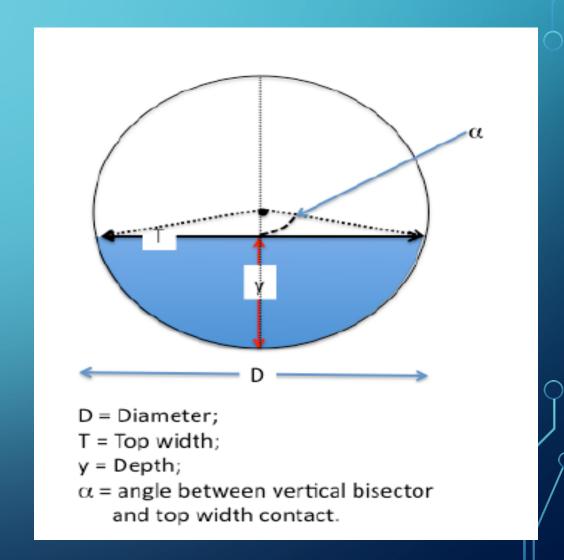






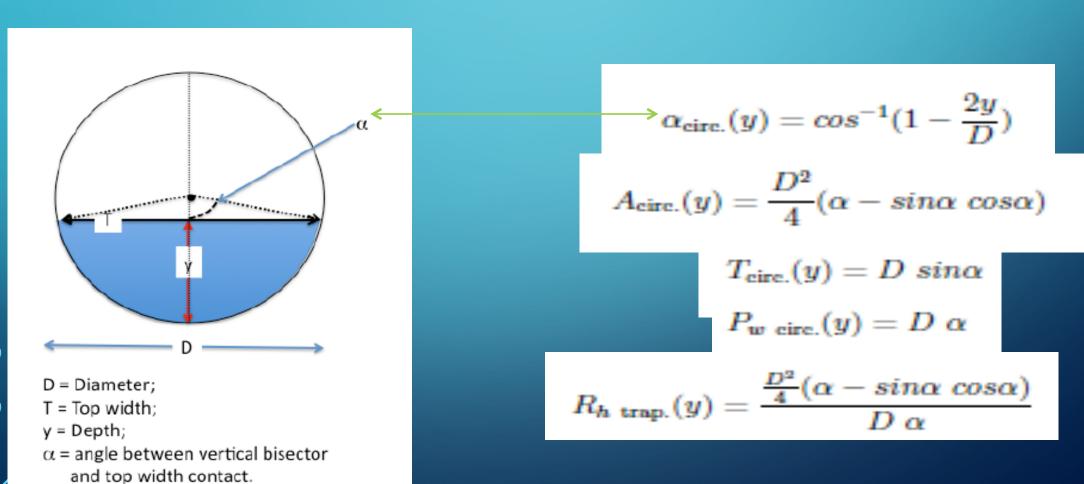
CIRCULAR CONDUIT

- Obviously common in sewer systems!
 - Pipes flowing with a free surface are circular cross section open channels!



CIRCULAR CONDUIT

ullet Sweep angle definition matters, SOME Authors use 2lpha.



CIRCULAR CONDUIT

- Online Tool

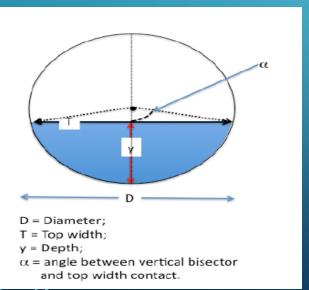
 Use circular and supply depth and diameter.
 - Example:

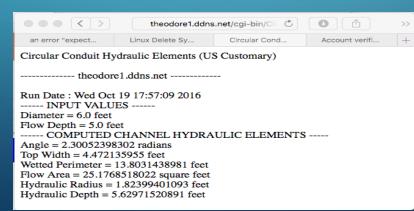
INPUT: D = 6 ft., y=5 ft.

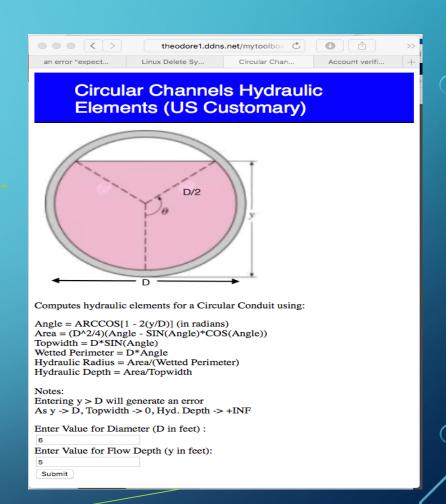
OUTPUT: Area = 25.17 sq. ft.

Perimeter = 13.8 ft.

Hyd. Radius = 1.82 ft.







DEPTH-AREA DISGRAMS

- Designer should sketch their own definition sketches before using the equations to validate that
 the equations produce the desired results.
- Other geometries are common, and these depth-area tables are usually constructed on asneeded basis.

- The circular conduit is sometimes a nuisance to relate the hydraulic elements because of the semi-implicit nature of the expressions, especially when the diameter is unknown.
- Two reasonable approaches are to
 - build a calculator (The on-line tool is one such calculator)
 - Use an id-10-t computation sheet and hand-calculator
 - use a dimensionless chart that relates full pipe behavior, where the relationships are simple to compute to partial full behavior.

Depth-Discharge Calculator for a Circular Conduit implements Manning's equation in a circular conduit.

$$Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$$

- The equation is the U.S. customary version of Manning's equation and is identical to Manning's equation on pg 161 of the NCEES supplied reference.
- The equation is an adaptation of Equations 3.42 and 3.288 in Chin (adapted for U.S. customary units).
 - A drainage engineer in the US should memorize this equation!

What such a calculator might look like

			300454	Charts	Denart Put Completes			
\Diamond	A	В	C	D	E			
1	Circular Pipe Flow Computations							
2	US Customary Units Version							
3								
4	INPUT DATA							
5	Manning's n	0.015	<=Tab	le Lookup)			
6	Depth	0.500	<=Feet	E				
7	Diameter	1.000	<=Feet	t				
8	Slope 0.001 <=Dimensionless							
9	INTERMEDIATE COMPUTATIONS							
10	Angle	1.571	<=Rad	ians				
11	Area	0.393	<=Fee	t Squared	1			
12	Perimeter	1.571	<=Feet	E				
13	Radius	0.250	<=Feet	E				
14	DISCHARGE A	ND VELOC	ITY					
15	Discharge	0.490	<=Cub	ic Feet p	er Second			
16	Velocity	1.247	<=Fee	t per Sec	ond			
-								

The designer supplies values of flow depth, diameter, topographic slope, and Manning's n. These values are then used to determine the sweep angle α as a function of depth and diameter. From this angle, the flow area and wetted perimeter are computed. Finally the hydraulic radius is computed and the resulting discharge is returned.

What such a calculator might look like (displayed as formulas)

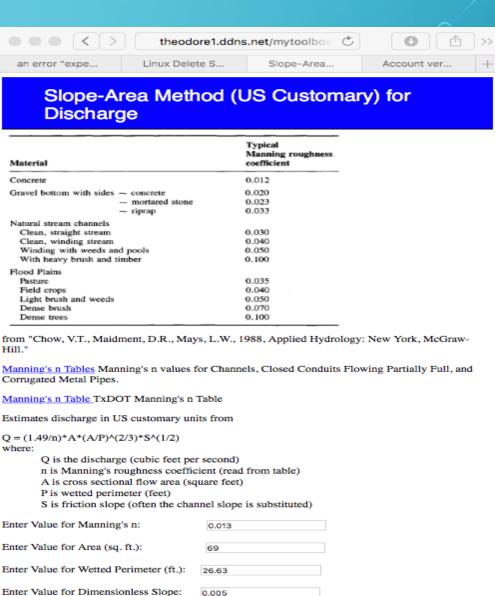
<>	A	В	С	DI
1	Circular Pipe Flo			
2	US Customary			
3				
4	INPUT DATA			
5	Mamning's n	0.015	<=Tab	
6	Depth	0.5	<=Fee	
7	Diameter	1	<=Fee	2
8	Slope	0.001	<=Din	r
9	INTERMEDIAT			П
10	Angle	=ACOS(1-2*B6/B7)	<=Rac	
11	Area	=B7^2*(B10-SIN(B10)*COS(B10))/4	<=Fee	:
12	Perlimeter	=B10*B7	<=Fee	3
13	Radius	=B11/B12	<=Fee	2
14	DISCHARGE A			
15	Discharge	=(1.49/B5)*B11*B13^(2/3)*B8^(1/2)	<=Cut	
16	Velocity	=B15/B11	<=Fee	2

The designer supplies values of flow depth, diameter, topographic slope, and Manning's n. These values are then used to determine the sweep angle α as a function of depth and diameter. From this angle, the flow area and wetted perimeter are computed. Finally the hydraulic radius is computed and the resulting discharge is returned.

ON-LINE CALCULATOR

- Slope-Area (A version of Manning's equation) Model
 - Supply: Manning's n, Flow Area, Perimeter, and Slope
 - Returns: Flow Rate





0.005

ID-10-T CALCULATION SHEET

- The ID-10-T Sheets are like an IRS tax return form.
- The analyst enters known values and follows the calculation instructions.
- The sheets are intended when the analyst has only a hand-calculator (although if one had a table of logarithms and trig. Tables, you could use the sheet without any electronics!)

ID-10-T-US-CIRCULAR

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CE 3372 – Water Systems Design ID-10-T-US Circular

Purpose:	Compute discharge in a circular section using Manning's equa-
	tion assuming normal (uniform) flow
Required Tools:	Calculator/Slide-Rule, or Logarithmic and Trigonometric Tables
Input Data:	Manning's n ; Conduit Slope, S_0 , (dimensionless);
	Flow Depth, d , (in feet); and Conduit Diameter, D , (in feet)
Output Values:	Discharge, Q , (in cubic feet per second)
Use:	When on-line tools or spreadsheet tools are unavailable.

- 1. Manning's n = ______
- 2. Flow Depth $d = \underline{\hspace{1cm}}$ feet
- 3. Conduit Diameter D = feet.
- 4. Conduit Slope S₀ =
- 5. Ratio of flow depth to diameter; $\frac{d}{D} =$
- 6. Compute $cos(\alpha) = 1 2 \times \frac{d}{D} =$
- 7. Compute the inverse cosine of the result in line [6] in radians. Enter the result below.

$$cos^{-1}(1 - 2 \times \frac{d}{D}) = \alpha =$$

Compute the flow area using

$$A = \frac{D^2}{4} \times (\alpha - \sin(\alpha)\cos(\alpha)) =$$
 feet².

9. Compute the wetted perimeter

$$P_w = \alpha \times D =$$
 feet.

10. Compute the hydraulic radius, $R_h = \frac{A}{P_w} =$ _____ feet.

ID-10-T-US Circular

Page 1 of 2

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- Copy the result from Line [8], A = ______ feet².
- Copy the result from Line [10], R_h = ______feet.
- Copy the result from Line [4], S₀ =
- Compute square root of Line [14],

$$\sqrt{S_0} =$$

Compute Line[13] raised to the 2/3-rds power;

$$R_h^{2/3} =$$

17. Multiply Line [16], Line [15], and Line [12];

$$R_h^{2/3} \times \sqrt{S_0} \times A =$$

18. Multiply Line [17] by 1.49;

$$1.49 \times R_h^{2/3} \times \sqrt{S_0} \times A =$$

Divide Line [18] by Line [11], result is discharge,Q.

$$Q = \frac{1.49}{n} \times R_h^{2/3} \times \sqrt{S_0} \times A =$$
 _____ cubic feet per second.

ID-10-T-SI-CIRCULAR

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CE 3372 – Water Systems Design ID-10-T-SI Circular

Purpose:	Compute discharge in a circular section using Manning's equation as-
	suming normal (uniform) flow
Required Tools:	Calculator/Slide-Rule, or Logarithmic and Trigonometric Tables
Input Data:	Manning's n ; Conduit Slope, S_0 , (dimensionless);
	Flow Depth, d , (in meters); and Conduit Diameter, D , (in meters)
Output Values:	Discharge, Q , (in cubic meters per second)
Use:	When on-line tools or spreadsheet tools are unavailable.

- 1. Manning's n =
- 2. Flow Depth d = _____ meters.
- Conduit Diameter D = _____ meters.
- 4. Conduit Slope $S_0 =$
- 5. Compute ratio of flow depth to diameter; $\frac{d}{D} =$
- 6. Compute $cos(\alpha) = 1 2 \times \frac{d}{D} =$
- 7. Compute the inverse cosine of the result in line [6] in radians. Enter the result below.

$$cos^{-1}(1 - 2 \times \frac{d}{D}) = \alpha =$$

8. Compute the flow area using

$$A = \frac{D^2}{4} \times (\alpha - \sin(\alpha)\cos(\alpha)) =$$
 meters².

9. Compute the wetted perimeter

$$P_w = \alpha \times D =$$
 meters.

10. Compute the hydraulic radius, $R_h = \frac{A}{P_w} =$ meters.

ID-10-T-SI Circular

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- Copy the value from Line [1], n =
- Copy the result from Line [8], A = _____ meters².
- Copy the result from Line [10], R_h = ______ meters.
- Copy the result from Line [4], S₀ =
- 15. Compute square root of Line [14],

$$\sqrt{S_0} =$$

Compute Line[13] raised to the 2/3-rds power;

$$R_h^{2/3} =$$

Multiply Line [16], Line [15], and Line [12];

$$R_h^{2/3} \times \sqrt{S_0} \times A =$$

Multiply Line [17] by 1.0;

$$1.0 \times R_h^{2/3} \times \sqrt{S_0} \times A =$$

Divide Line [18] by Line [11], result is discharge,Q.

$$Q = \frac{1.0}{n} \times R_h^{2/3} \times \sqrt{S_0} \times A =$$
 cubic meters per second.

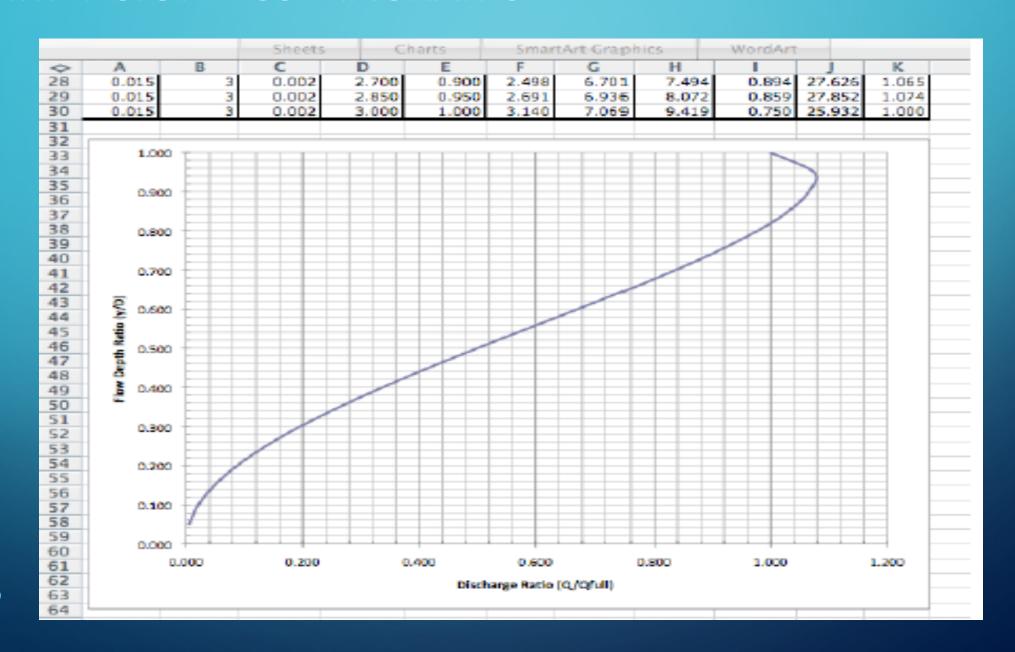
DIMENSIONLESS DIAGRAMS

- Using the same principles as the circular flow calculator, a dimensionless depth-discharge diagram table can be constructed.
 - Essentially the same input information is supplied; diameter, material properties, and slope.
- The table then divides the depth into specific ratios of the diameter, then computes the discharge for each ratio.
 - In this forward computational step, there is no advantage, but if one has the tabulation (or the dimensionless chart) the reverse look-up greatly simplifies the design process.

DIMENSIONLESS DIAGRAMS

\Leftrightarrow	А	В	С	D	E	F	G	H	1	J	K
1	Circular Pipe Flow Computations										
2	US Customary Units										
3											
-4	Pipe Diar	meter	1								
5	Manning's n		0.015	<							
6	Slope		0.001								
7											
8											
9		Input	Values			Int	ermediate	Computat	lons	Results	
			=	- 5	5.5		_			7	
	- St - St	ä	enslor Slope	9	골목	<u>10</u>	9	ğ	2 5	N	§ =
	듣	절 없	5 6	<u>0</u> ⊕	20	اقب رو	₹ 🗐	8 8	₹ 9	흔	불승류
	Manning's n	Diameter (feet)	Dimensioni ess Slope	Flow Depth (feet)	Flow Depth Ratio (v/D)	Angle (Radlans)	Flow Area (99.ft.)	Wetted Perimeter (ft)	Hydraulic Radius (ft)	Discharge (ds)	Discharge Ratio (Q/Qfull)
10								₹5€			
11	0.015	1	0.001	0.050	0.050	0.451	0.015	0.451	0.033	0.005	0.005
12	0.015	1	0.001	0.100	0.100	0.644	0.04-1	0.644	0.064	0.020	0.021
13	0.015	1	0.001	0.150	0.150	0.795	0.074	0.795	0.093	0.048	0.049
14	0.015	1	0.001	0.200	0.200	0.927	0.112	0.927	0.121	0.086	0.088
15	0.015	1		0.250	0.250	1.047	0.154	1.047	0.147	0.134	0.137
16	0.015	1	0.001	0.300	0.300	1.159	0.198	1.159	0.171	0.192	0.196
17	0.015	1	0.001	0.350	0.350	1.266	0.245	1.266	0.193	0.257	0.263
18	0.015	1	0.001	0.400	0.400	1.369	0.293	1.369	0.214	0.330	0.337
19	0.015	1	0.001	0.450	0.450	1.471	0.34-3	1.471	0.233	0.408	0.416
20	0.015	1	0.001	0.500	0.500	1.571	0.393	1.571	0.250	0.490	0.500
21	0.015	1	0.001	0.550	0.550	1.671	0.44-3	1.671	0.265	0.573	0.585
22	0.015	1	0.001	0.600	0.600	1.772	0.492	1.772	0.278	0.658	0.672
23	0.015	1	0.001	0.650	0.650	1.875	0.540	1.875	0.288	0.741	0.756
24	0.015	1	0.001	0.700	0.700	1.982	0.587	1.982	0.296	0.820	0.837
25	0.015	1	0.001	0.750	0.750	2.094	0.632	2.094	0.302	0.893	0.911
	0.015	1	0.001	0.800	0.800	2.214	0.674	2.214	0.304	0.957	0.977
27	0.015	1	0.001	0.850	0.850	2.346	0.712	2.346	0.303	1.009	1.030
	0.015	1	0.001	0.900	0.900	2.498	0.745	2.498	0.298	1.043	1.065
30	0.015	1	0.001	0.950	0.950	2.691	0.771	2.691	0.286	1.052	1.074
30	0.015	1	0.001	1.000	1.000	3.140	0.785	3.140	0.250	0.979	1.000

DIMENSIONLESS DIAGRAMS



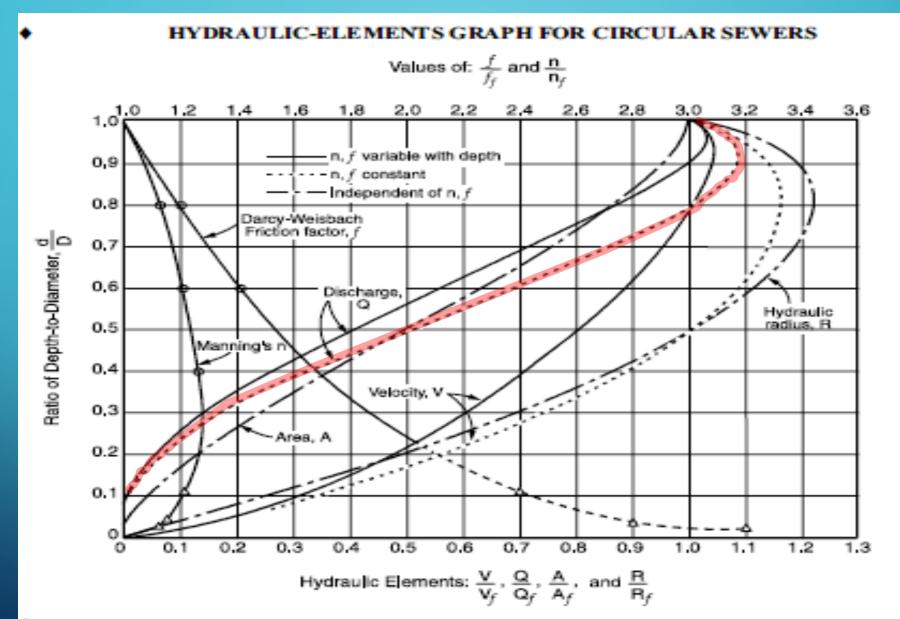
HOW TO USE DIMENSIONLESS CHART

Determine full pipe discharge

$$Q_{full} = (\frac{1.49}{n})(\frac{\pi D^2}{4})(\frac{D}{4})^{\frac{2}{3}}(S)^{\frac{1}{2}}$$

- Determine flow depth ratio of interest, suppose $\frac{3}{4}$ full is value of interest the ratio is 0.75
- Locate discharge fraction for the depth fraction on the chart. For 0.75 full, $Q/Q_{\rm full}=0.911$
- Multiply the discharge fraction by the full pipe discharge fraction to recover the discharge for the particular depth of flow. Q_{75%} = 0.911Q_{full}

DIMENSIONLESS CHART (NCEES)



Design and Construction of Sanitary and Storm Sewers, Water Pollution Control Federation and American Society of Civil Engineers, 1970.

DIMENSIONLESS CHARTS

 Value comes when using backwards (reverse-lookup) to determine a design diameter for a required fill depth and supplied discharge

OTHER DIMENSIONLESS CHARTS

- Trapezoidal channel
 - FHWA HEC-22

1.1.4 HEC-22 Trapezoidal Channel Depth-Discharge Chart

Trapezoidal channels also occur frequently enough in practice that their design is often facilitated using charts — these charts are not dimensionless in the usual sense.

Figure 11 is a tool to estimate the flow depth given a discharge and Manning's n.

The following procedure outlines how to use this chart⁴

- The analyst/designer must specify the channel width, B, the side slope, Z, the longitudinal slope, S₀, the material properties, n, and the discharge, Q.
- Locate the value of S₀ on the first vertical scale from the left of the chart.
- Compute Q × n. Locate this value on the second vertical scale from the left of the chart.
- Draw a line connecting these two values that also intersects the third vertical line in the chart (called the "turning line").
- Locate the value of B on the fourth vertical scale from the left of the chart.
- Draw a line connecting the intersection of the first analyst drawn line with the turning line and the value of B just plotted, be sure the extent the line to the side-slope scale on the right of the figure.
- At the intersection of the side-slope scale and this line, draw a horizontal line across the side slope scale.
- Read the depth to width ratio for the appropriate value from the side-slope scale.
- 9. Multiply the depth by this ratio to recover the flow depth for the channel.

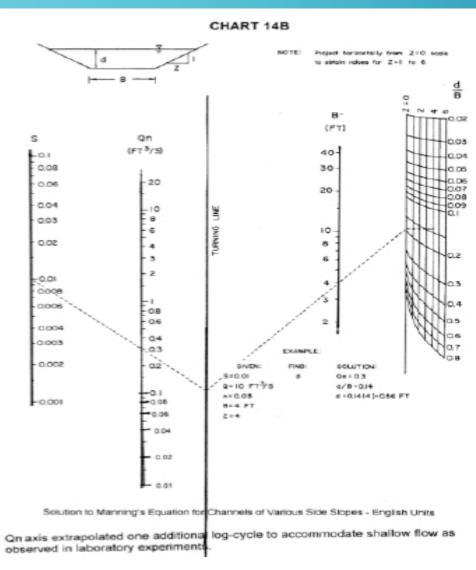


Figure 11: Trapezoidal Channel Discharge-Depth Chart (from FHWA HEC-22)

NEXT TIME

- Gradually Varied Flow
 - Demonstrate that NORMAL flow is a special case of GVF