CE 3372 WATER SYSTEMS DESIGN

PIPE HYDRAULICS PART 2 (FALL 2020)

ENERGY EQUATION

$$\frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_l$$

- The energy equation relates the total dynamic head at two points in a system, accounting for frictional losses and any added head from a pump.

  h_L = head lost to friction

  h_p = head supplied by a pump

  h_t = head recovered by a turbine

ENERGY EQUATION APPLICATION

- Estimate discharge between two reservoirs:
    - Head Loss is given as $h_L = 0.136 \cdot L \frac{Q^2}{\pi^2 g D^5}$

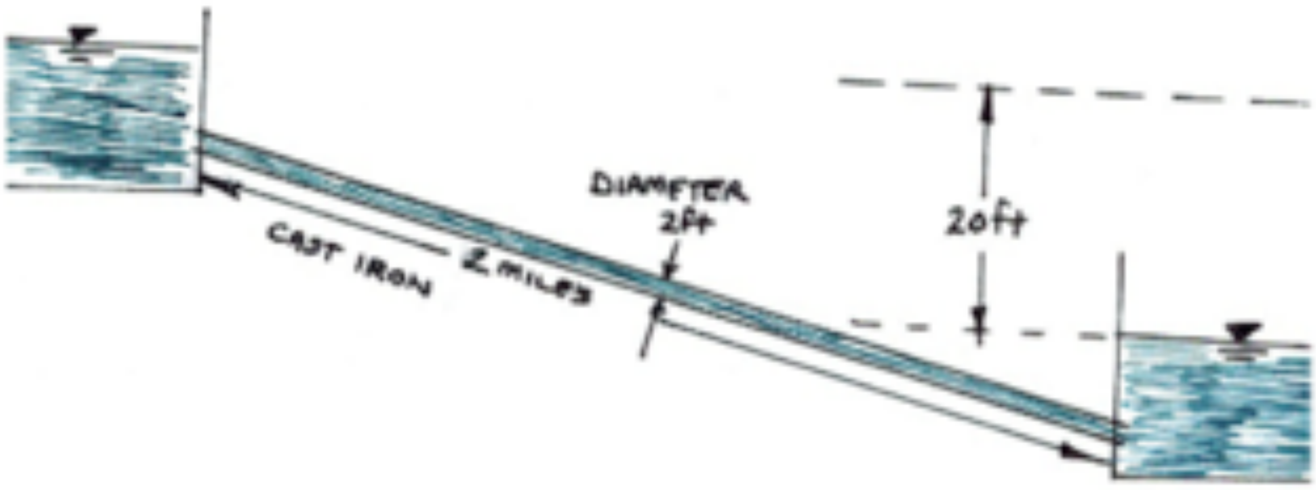


Figure 1: Two reservoirs connected by a cast iron pipe

HEAD LOSS MODELS

- Head Loss Models (for losses in pipes)
  - Darcy-Weisbach
    - Moody Chart
    - Jain Equations
  - Hazen-Williams
  - Chezy-Manning
- Fitting (Minor) Losses

PIPELINE SYSTEM

Head Loss Model

$$\frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_l$$

Pump Curve

DARCY-WEISBACH LOSS MODEL FOR PIPE LOSS

- Frictional loss proportional to
  - Length, Velocity**2
- Inversely proportional to
  - Cross sectional area
- Loss coefficient (f) depends on
  - Reynolds number (fluid and flow properties)
  - Roughness height (pipe material properties)

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

MOODY CHART

Material	e (ft)	e (mm)
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

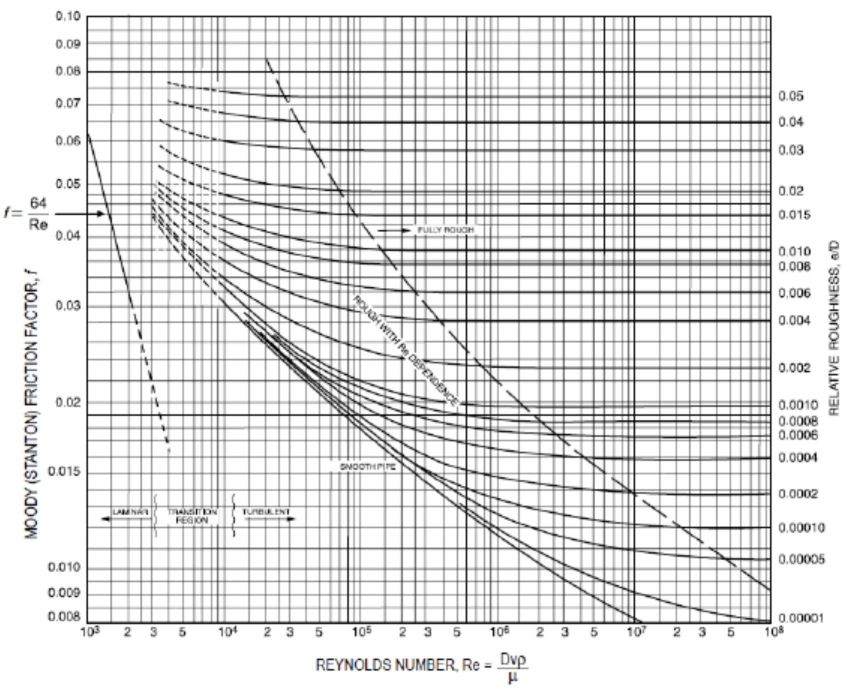


Figure 2: Moody-Stanton Diagram (from CITE NCEES).

- Moody-Stanton chart is a tool to estimate the friction factor in the DW head loss model
- Used for the pipe loss component of friction

EXAMPLES

- Three “classical” examples using Moody Chart
  - Head loss for given discharge, diameter, material
  - Discharge given head loss, diameter, material
  - Diameter given discharge, head loss, material

EXAMPLE 1

- Head loss for given discharge, diameter, material

Find h_f given Q, D, ϵ This kind of problem is relatively straightforward. The engineer computes Re_d from the discharge Q and the pipe diameter D. Then computes the roughness ratio from the tabulated ϵ for the pipe material. The friction factor, f, is then recovered directly from the Moody chart.

Example Oil with specific gravity 0.9, viscosity 0.00003 ft^2/sec flows in a 2000-foot long, 6-inch diameter, cast-iron pipe at a flow rate of 1.0 cubic-feet-per-second. The pipe slopes upward at an angle of 5^o in the direction of flow. Estimate the head loss in the pipe. Estimate the pressure drop in the pipe.

Solution Figure 15 is a sketch of the situation.

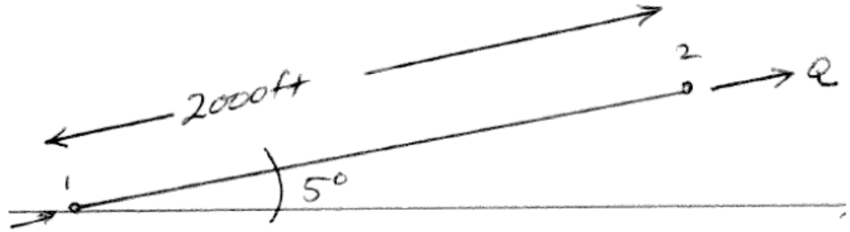


Figure 15: Sketch for Example

EXAMPLE 1

- Head loss for given discharge, diameter, material

Equation 11 is the energy equation for the situation. The velocity terms are absent because they are equal, the added pump head and removed turbine head are absent because these devices are absent. All that remains is the pressure, elevation, and head loss terms.

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2 + h_l \tag{10}$$

Rearranging equation to isolate the head loss will be of value when we try to find the pressure drop.

$$(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}) + (z_1 - z_2) = h_l \tag{11}$$

The first term in parenthesis is the pressure drop (rise), and the second term is the elevation drop (rise).

The head loss is evaluated using the Darcy-Weisbach head loss model. First we compute Re_d

$$Re_d = \frac{V\ D}{\nu} = \frac{4\ (1cfs)\ (0.5ft)}{\pi\ (0.5ft)^2\ (0.00003\ sq.ft/sec)} \approx 84,822 \tag{12}$$

The Reynolds number is greater than 10,000 therefore we conclude the flow is turbulent.

EXAMPLE 1

- Head loss for given discharge, diameter, material

The roughness height is ϵ=0.00085 from the table on the Moody chart (in this document), so the roughness ratio is

$$\frac{\epsilon}{D} = \frac{0.00085}{0.5} \approx 0.0017 \qquad (13)$$

Material	e (ft)	e (mm)
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

EXAMPLE 1

- Head loss for given discharge, diameter, material

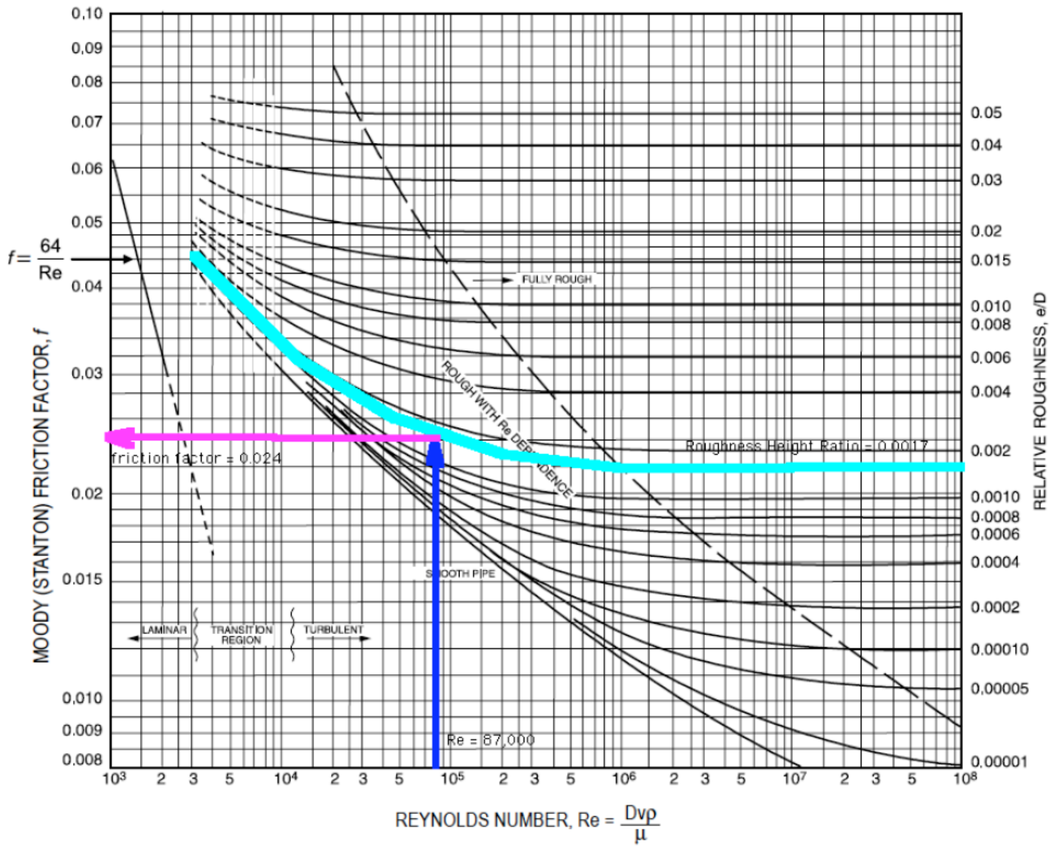


Figure 16: Moody-Stanton Diagram annotated with Example 1 components.

Figure 16 is the Moody chart with the roughness ratio shown as the light blue (cyan) curve,

EXAMPLE 1

- Head loss for given discharge, diameter, material

the reynolds number as the black line, and the recovered friction factor (f=0.024) from the magenta line.

To complete the analysis, we then use the Darcy-Weisbach equation for estimate the head loss as

$$h_l = 8fL\frac{Q^2}{\pi^2 g D^5} = 8(0.024)(2000ft)\frac{(1cfs)^2}{\pi^2(32.2ft/s^2)(0.5ft)^5} \approx 38.6ft \qquad (14)$$

Now to compute the pressure drop, we simply account for the elevation change and what remains must be pressure. First the change in elevation is about $2000sin5^o \approx 175ft$. The change in pressure is therefore

$$\Delta p = \rho g(h_l + (z_2 - z_1)) = (38.6ft + 175ft)(62.4)(0.9) \approx 11,999lb/ft^2 = 83psi \qquad (15)$$

Thus the oil pressure must be at least 83 psi greater at the lower elevation than the upper elevation for the oil to flow up the pipe.

EXAMPLE 2

- Discharge given head loss, diameter, material

Example An 80-foot horizontal, 1/2-inch diameter wrought iron pipe has an observed head loss of 40 feet. Estimate the discharge in the pipe.

Solution Apply Darcy-Weisbach directly — the pipe is horizontal so the energy equation is quite boring,

$$h_l = \frac{\Delta p}{\rho g} = 40ft \tag{16}$$

First compute the roughness height ratio — it will be needed to look up friction factors.

$$\frac{\epsilon}{D} = \frac{0.00015ft}{0.5i/12ft} \approx 0.0036 \tag{17}$$

Then construct a table of computations as shown in Table 2. Increase (decrease) the flow rate until the computed head loss is about the same as the required head loss. The moody chart is used the same way as in the previous example. The engineer will need to exercise some judgement of when to stop, because as one gets close to the specified head loss, the ability to read changes in f diminishes.

EXAMPLE 2

- Discharge given head loss, diameter, material

Table 1: Computation table for Estimating Q from head loss and material properties.

Q_{guess}	Re_d	f	$h_{l\ guess}$
0.001	2.83×10^3	0.036	≈ 0.57
0.005	1.41×10^4	0.032	≈ 12.7
0.008	2.26×10^4	0.031	≈ 31.6
0.009	2.25×10^4	0.030	≈ 37.9

The result in this example is that the pipe discharge is about 0.009 cfs.

EXAMPLE 3

- Diameter given discharge, head loss, material

Example An 600 foot wrought-iron pipe is to carry water at 20°C at a discharge of 3 CFS. The pipe drops 60 feet in the direction of flow and the desired pressure drop is 6 feet of head. What diameter pipe will function under these conditions?

Solution Figure 17 is a sketch of the situation.

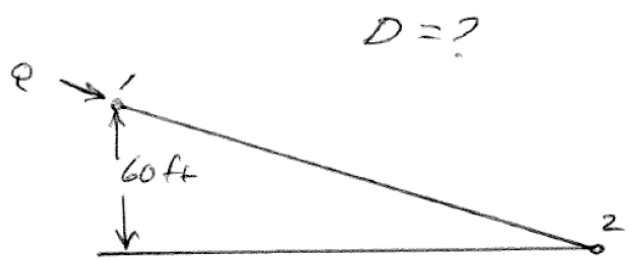


Figure 17: Sketch for Example

The energy equation for this situation is

$$(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}) + (z_1 - z_2) = h_l \tag{18}$$

The elevation change is given as 60 feet and the pressure drop is given as 6 feet (that is the pressure is greater at location 1 by 6 feet than location 2). Thus the Darcy-Weisbach head

EXAMPLE 3

- Diameter given discharge, head loss, material

loss equation is

$$h_l = 8fL\frac{Q^2}{\pi^2 g D^5} \approx 66ft \quad (19)$$

As in the prior example, a computation table is useful. The sixth column is a computational trick to make a hand calculations faster. The term $\frac{h_l}{f}$ is evaluated by taking the Darcy-Weisbach equation and dividing out the friction factor (i.e. $\frac{h_l}{f} = 8L\frac{Q^2}{\pi^2 g D^5}$)

Table 2: Computation table for Estimating D from head loss, discharge and material properties.

D_{guess}	Re_d	$\frac{\epsilon}{D}$	f	$\frac{h_l}{f}$	h_l
0.25	1.41×10^6	0.0006	0.018	1.39×10^5	≈ 2500
0.50	7.06×10^5	0.0003	0.016	4.35×10^3	≈ 69.6
0.51	6.92×10^5	0.00029	0.016	3.94×10^3	≈ 63.0

The result after three tries is that the diameter is between 6-7 inches, commercially available 7 inch iron pipe exists[8], so this size could be specified in such a situation.

DIRECT (JAIN) EQUATIONS

- An alternative to the Moody chart are regression equations that allow direct computation of discharge, diameter, or friction factor.

$$Q = -2.22D^{5/2} \times \sqrt{gh_f/L} \times [log_{10}(\frac{\epsilon}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_f/L}})]$$

$$D = 0.66[\epsilon^{1.25} \times (\frac{LQ^2}{gh_f})^{4.75} + \nu Q^{9.4} \times (\frac{L}{gh_f})^{5.2}]^{0.04}$$

JAIN EQUATIONS – COMPUTATIONAL THINKING/DATA SCIENCE APPLICATION

- Build a Computational Tool
  (e.g JupyterLab as in ENGR 1330)
  1. State the programming problem
  2. Known (Inputs)
  3. Unknown (Outputs)
  4. Governing Equation(s)
  5. Test the tool

JAIN EQUATIONS

- Link to On-Line Tool
    - A similar tool (to the Jupyter Notebook just developed) is available online at:
    - http://atomickitty.ddns.net/documents/mytoolbox-server/Hydraulics/QGivenHeadLoss/QGivenHeadLoss.html

HAZEN-WILLIAMS

- Frictional loss proportional to
  - Length, Velocity^(1.8)
- Inversely proportional to
  - Cross section area (as hydraulic radius)
- Loss coefficient (Ch) depends on
  - Pipe material and finish
  - Turbulent flow only (Re>4000)
    - WATER ONLY!

$$h_f = 3.02\ L\ D^{-1.167} (\frac{V}{C_h})^{1.85}$$

HAZEN-WILLIAMS

- HW Head Loss

$$h_f = 3.02\ L\ D^{-1.167}(\frac{V}{C_h})^{1.85}$$

- Discharge Form

$$h_f = 3.02\ L\ D^{-1.167}(\frac{4Q}{\pi D^2 C_h})^{1.85}$$

HAZEN-WILLIAMS

- Hazen-Williams C-factor

Table 3: Hazen-Williams Coefficients for Different Materials.

Material	C_h	Material	C_h
ABS - Acrylonite Butadiene Styrene	130	Aluminum	130 - 150
Asbestos Cement	140	Asphalt Lining	130 - 140
Brass	130 - 140	Brick sewer	90 - 100
Cast-Iron - new unlined (CIP)	130	Cast-Iron 10 years old	107 - 113
Cast-Iron 20 years old	89 - 100	Cast-Iron 30 years old	75 - 90
Cast-Iron 40 years old	64-83	Cast-Iron, asphalt coated	100
Cast-Iron, cement lined	140	Cast-Iron, bituminous lined	140
Cast-Iron, wrought plain	100	Cast-Iron, seal-coated	120
Cement lining	130 - 140	Concrete	100 - 140
Concrete lined, steel forms	140	Concrete lined, wooden forms	120
Concrete, old	100 - 110	Copper	130 - 140
Corrugated Metal	60	Ductile Iron Pipe (DIP)	140
Ductile Iron, cement lined	120	Fiber	140
Fiber Glass Pipe - FRP	150	Galvanized iron	120
Glass	130	Lead	130 - 140
Metal Pipes - Very to extremely smooth	130 - 140	Plastic	130 - 150
Polyethylene, PE, PEH	140	Polyvinyl chloride, PVC, CPVC	150
Smooth Pipes	140	Steel new unlined	140 - 150
Steel, corrugated	60	Steel, welded and seamless	100
Steel, interior riveted, no projecting rivets	110	Steel, projecting girth and horizontal rivets	100
Steel, vitrified, spiral-riveted	90 - 110	Steel, welded and seamless	100
Tin	130	Vitrified Clay	110
Wrought iron, plain	100	Wooden or Masonry Pipe - Smooth	120
Wood Stave	110 - 120		

Adapted from `http://www.engineeringtoolbox.com/hazen-williams-coefficients-d_798.html`.

EXAMPLE USING HAZEN-WILLIAMS FORMULA

Example Estimate the head loss in a 72-inch, 10,000-foot steel pipe carrying water at 200 CFS using the Hazen-Williams formula.

Solution Using Table 3 an estimate of the C_h is 100. Next substitute into the HW formula as

$$h_f = 3.02\ (10,000ft)\ (6ft)^{-1.167}(\frac{4(200cfs)}{\pi(6ft)^2 100})^{1.85} \approx 28ft \quad (23)$$

HYDRAULIC RADIUS

- HW is often presented as a velocity equation using the hydraulic radius

$$V = 1.381\ C_h\ R^{0.63}\ S^{0.54}$$

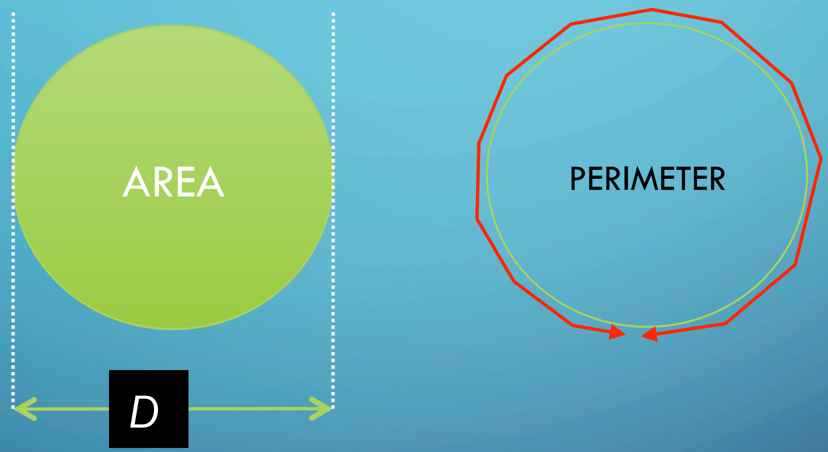
HYDRAULIC RADIUS

- The hydraulic radius is the ratio of cross section flow area to wetted perimeter

$$R_h = \frac{A}{P_w}$$

HYDRAULIC RADIUS

- For circular pipe, full flow (no free surface)



$$R_h = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$

CHEZY-MANNING

- Frictional loss proportional to
  - Length, Velocity^2
- Inversely proportional to
  - Cross section area (as hydraulic radius)
- Loss coefficient depends on
  - Material, finish

$$h_f = L\frac{n^2V^2}{2.22R^{4/3}}$$

FITTING (MINOR) LOSSES

- Fittings, joints, elbows, inlets, outlets cause additional head loss.
- Called "minor" loss not because of magnitude, but because they occur over short distances.
- Typical loss model is

$$h_{minor} = K\frac{V^2}{2g}$$

FITTING (MINOR) LOSSES

- The loss coefficients are tabulated for different kinds of fittings

Table 4: Minor Loss Coefficients for Different Fittings

Fitting Type	K
Tee, Flanged, Line Flow	0.2
Tee, Threaded, Line Flow	0.9
Tee, Flanged, Branched Flow	1.0
Tee, Threaded , Branch Flow	2.0
Union, Threaded	0.08
Elbow, Flanged Regular 90^o	0.3
Elbow, Threaded Regular 90^o	1.5
Elbow, Threaded Regular 45^o	0.4
Elbow, Flanged Long Radius 90^o	0.2
Elbow, Threaded Long Radius 90^o	0.7

EXAMPLE – FITTING (MINOR) LOSSES

Example What is the pressure drop across a valve with nominal diameter of 8 cm, a loss coefficient of 3.2, and a flow rate of 0.04 m^3/sec?

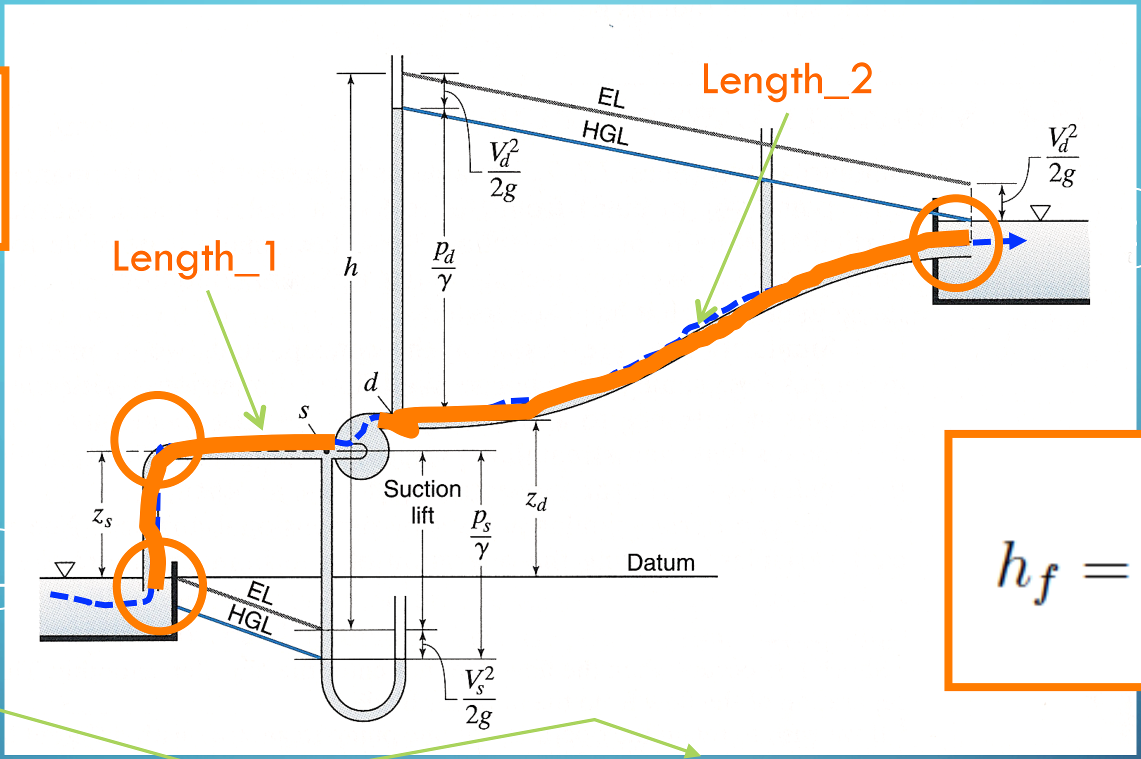
Solution First write the minor loss equation, solve for head loss.

$$h_l = K\frac{V^2}{2g} = (3.2)\frac{(\frac{4*0.04}{\pi(0.08)^2})^2}{2(9.8)} \approx 10.3m \qquad (29)$$

Then convert the head loss into a pressure drop from

$$\rho g * h_l = \Delta p = 9800N/m^3 * 10.3m = 101,321Pa \approx 101kPa \qquad (30)$$

$$h_{minor} = K\frac{V^2}{2g}$$



=Length_1 + Length_2

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$h_l = \sum h_{\text{minor}} + \sum h_{\text{pipelines}}$$

Head Loss Model

$$\frac{p_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_l$$