Problem #1

Figure 1 is a profile through an unconfined aquifer with two drains located left and right of the aquifer. Derive the equation that relates head in the aquifer to recharge (N) and position (x).

The water level in the left drain is fixed at elevation $h_L = 10.0$ m. The elevation of the land surface is $D = 18.5$ m. If the right drain is one kilometer ($L = 1000.0$ m) from the left drain and the desired minimum depth to water is, $d_{min} = 1.0$ m, find the required elevation h_R in the right drain. The recharge rate is $N = 0.001$ m/d, the hydraulic conductivity is $K = 1$ m/d.

$$\dot{I} - \dot{O} = \frac{\Delta S}{\Delta t}$$

$$N\Delta X - (Q_{OUT} - Q_{IN}) = 0$$

$$N = \frac{Q_{OUT} - Q_{IN}}{\Delta X}$$

$$\lim_{\Delta X \to 0} N = \lim_{\Delta X \to 0} \frac{Q_{OUT} - Q_{IN}}{\Delta X}$$

$$N = \frac{\partial Q}{\partial x}$$

$$Q = -Kh\frac{\partial h}{\partial x} \text{ (Darcy's Law)}$$

$$\therefore N = -\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)$$

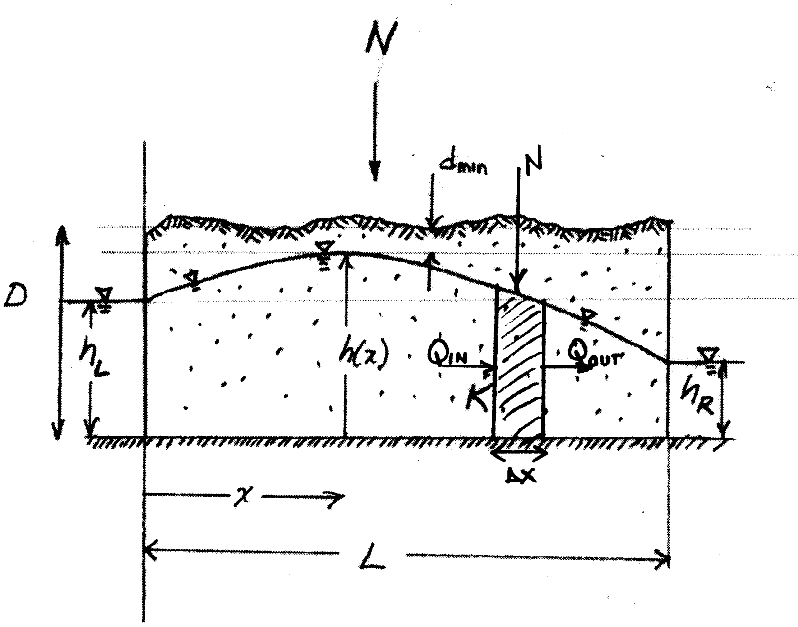


Figure 1. Diagram of unconfined aquifer

B.C.'s

$x = 0,\ h = h_L$

$x = L,\ h = h_R$

$$\int N\,dx = -\int \frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)dx$$

$$Nx + C_1 = -Kh\frac{\partial h}{\partial x}$$

$$\int Nx + C_1\,dx = \int -Kh\frac{\partial h}{\partial x}\,dx$$

$$\frac{Nx^2}{2} + C_1 x + C_2 = -\frac{K}{2}h^2$$

$x = 0,\ h = h_L$

$$C_2 = -\frac{Kh_L^2}{2}$$

$x = L,\ h = h_R$

$$\frac{NL^2}{2} + C_1 L - \frac{Kh_L^2}{2} = -\frac{Kh_R^2}{2}$$

$$C_1 = \frac{K}{2L}(h_L^2 - h_R^2) - \frac{NL}{2}$$

$$\therefore -\frac{Kh^2}{2} = -\frac{Kh_L^2}{2} + \frac{Nx^2}{2} + \frac{K}{2L}(h_L^2 - h_R^2)x - \frac{NLx}{2}$$

or

$$h^2(x) = h_L^2 + \frac{(h_R^2 - h_L^2)x}{L} + \frac{NLx}{K} - \frac{Nx^2}{K}$$

d_{min} @ h_{max}

$$h_{max} \text{ @ } \frac{\partial}{\partial x}(h^2) = 0 = \frac{h_R^2 - h_L^2}{L} + \frac{NL}{K} - \frac{2Nx^*}{K}$$

$$x^* = \frac{K}{2NL}(h_R^2 - h_L^2) + \frac{L}{2}$$

Substitute numbers & solve for $h_{max} = 18.5 - 1 = 17.5$

$h_L = 10$, $K = 1$ m/d, $L = 1000$ m, $N = 0.001$ m/d

$h_R = ?$

$$x^* = \frac{1\,m/d}{2(0.001\,m/d)(1000\,m)}\left(h_R^2 - 100\,m^2\right) + 500\,m = \frac{0.5}{m}(h_R^2) + 450\,m$$

$$17.5^2 = h^2 = 100\,m^2 + \left(\frac{h_R^2 - 100\,m^2}{1000\,m}\right)\left(\frac{0.5}{m}h_R^2 + 450\,m\right) + \frac{(0.001\,m/d)(1000\,m)}{1\,m/d}\left(0.5h_R^2 + 450\,m\right)$$

$$- \frac{0.001\,m/d}{1.0\,m/d}\left(\frac{0.5}{m}(h_R^2) + 450\,m\right)^2$$

$$h_{max}^2 - 100 = \left(\frac{h_R^2}{1000} - 0.1\,m\right)\left(0.5h_R^2 + 450\right) + \left(0.5h_R^2 + 450\right) - 0.001\left(0.5h_R^2 + 450\right.$$

$$h_{max}^2 - 100 = \left(\frac{A}{1000} - 0.1\right)\left(0.5A + 450\right) + \left(0.5A + 450\right) - 0.001\left(0.5A + 450\right)^2 = f(A)$$

$206.25 = f(A)$ guess values for A, evaluate $f(A)$ stop when $f(A) = 206.25$

h_R	$h_R^2 = A$	$f(A)$
10	100	250
8	64	232.32
6	36	219.02
4	16	209.76
2	4	204.3
3	9	206.57
2.82	8	206.11
2.91	8.5	206.34
2.88	8.3	206.2522

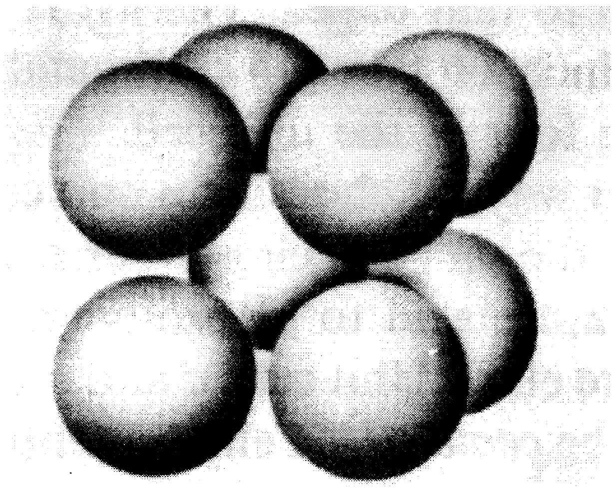
* close enough

$\therefore h_R = 2.88\,m$

$x^* = 0.5(8.3) + 450 = 454.15\,m$

Problem 2.

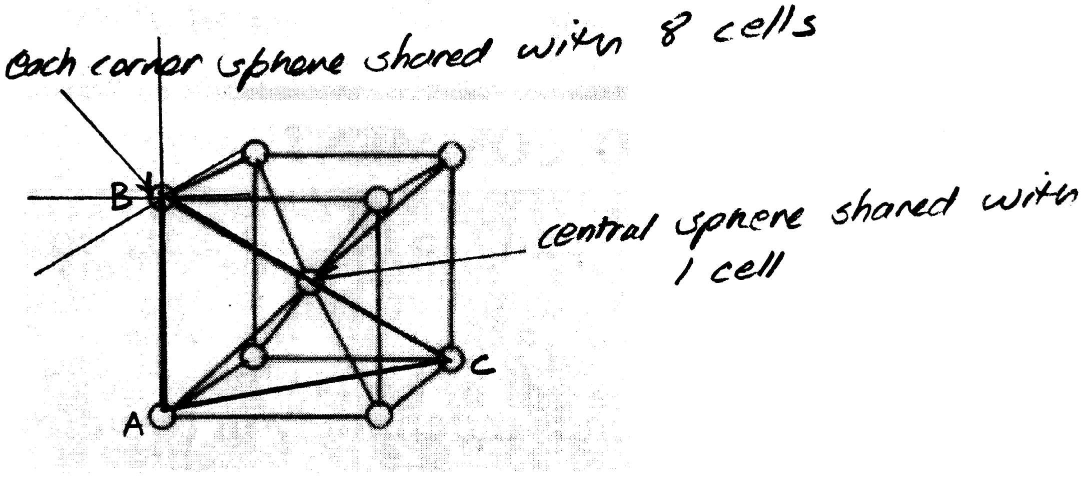
Calculate the porosity of uniform sized spheres packed in a body-centered cubic structure. Figure 2 depicts the structure.

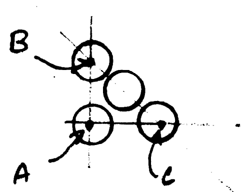


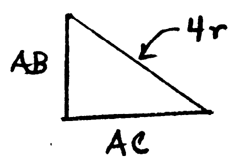
r = sphere radius

$$n = \frac{V_{void}}{V_{bulk}} = \frac{V_{void}}{V_{cell}}$$

$$V_{void} = V_{cell} - V_{spheres}$$







$AB^2 + AC^2 = 16r^2$

Also because cell is cubic

$AB^2 + AB^2 = AC^2$

$\therefore\ 3AB^2 = 16r^2$

$$AB = \frac{4}{\sqrt{3}} r$$

$$V_{CELL} = \frac{4^3 r^3}{(\sqrt{3})^3}$$

\# spheres = 1 central + $8\left(\frac{1}{8}\right)$ corners = 2 spheres

$$V_{spheres} = 2\left(\frac{4}{3}\pi r^3\right)$$

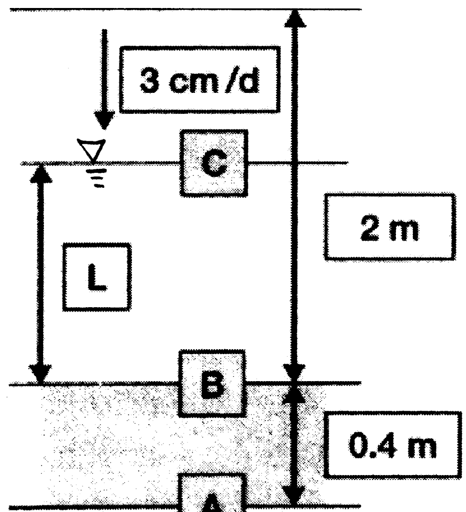
$$n = 1 - \frac{V_{spheres}}{V_{cell}} = 1 - \frac{2\left(\frac{4}{3}\pi \cancel{r^3}\right)}{\frac{4^3 \cancel{r^3}}{\sqrt{3}^3}}$$

$$n = 1 - \frac{\cancel{2}(\cancel{4}\pi)(\sqrt{3})^{\cancel{3}}}{\cancel{3}(\cancel{4})(\cancel{4})(4)2} = 1 - \frac{\pi\sqrt{3}}{8} = 0.319$$

$\therefore\ n \approx 32\%$

Problem 3.

Heavy infiltration (due to excess irrigation) of *3 cm/d* causes a perched water table to form above a low permeability, flow-restricting layer. The top of the flow-restricting layer is at a depth of *2 m*, it is *0.4 m* thick, and it has a K_v, of *0.01 m/d*. The material above the restricting layer is a silt loam with a K_v, of *0.12 m/d*. Coarse sand and gravel occurs below the restricting layer. After flowing through the restricting layer, the water moves as unsaturated flow through the sand and gravel to an unconfined aquifer. What is the height of the perched water table above the top of the restricting layer? The setting is shown below.



datum

$$h_A = \frac{\cancel{p_a}^{\,0}}{\gamma} + \cancel{z_a}^{\,0} = 0$$

$$h_B = \frac{p_b}{\gamma} + z_b = \frac{p_b}{\gamma} + 0.4\text{m}$$

$$h_c = \frac{\cancel{p_c}}{\gamma} + z_c = L + 0.4\text{m} \qquad (p_c = 0)$$

Darcy's law

$$q_{(\text{infiltration})} = K_{v_R}\frac{h_B - h_A}{0.4\text{ m}} \quad (1)$$

$$q_{(\text{infiltration})} = K_{v_s}\frac{h_c - h_B}{L} \quad (2)$$

Solve (1) for h_B

$$.03\text{ m/d} = (0.1\text{ m/d})\frac{h_B}{0.4\text{ m}} \qquad h_B = \frac{(0.4\text{m})(0.3\text{m/d})}{(0.1\text{m/d})} = \frac{0.12}{0.1}\text{ m} = 1.2\text{m}$$

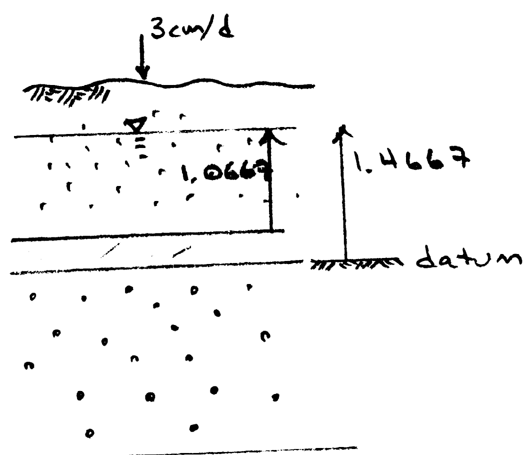
Solve (2) for L

$$.03\text{m/d} = (0.12\text{m/d})\frac{L+0.4-1.2}{L}$$

$$L(0.03) = (0.12)L - (0.12)(0.8)$$

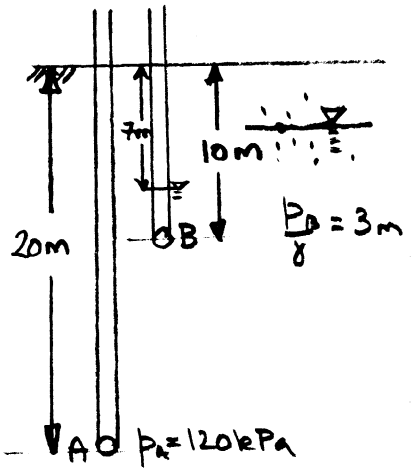
$$L(0.03 - 0.12) = -(0.12)(0.8)$$

$$L = 1.0667$$



Problem 4.

A piezometer is screened at a depth of *20 m* below land surface and records a pressure of *120 kPa* on a pressure transducer. An immediately adjacent piezometer is screened at a depth of *10 m* below land surface, and the depth to water in this piezometer is *7 m* below land surface. Is the vertical component of flow upward or downward at this location, and what is the depth of the water table below land surface? Assume the specific weight of water is *9810 N/m³*.



select "A" as datum

$$h_A = \frac{p_A}{\gamma} + z_a = \frac{120 \cdot 10^3 N/m^2}{9810 N/m^3} = 12.23m$$

$$h_B = \frac{p_B}{\gamma} + z_b = 3m + 10m = 13 \ m$$

Flow is <u>downward</u> (B to A)

because $h_B > h_A$

Water table at C

$$h_c = \cancel{\frac{p_c}{\gamma}}^{=0} + z_c$$

$$\frac{h_c - h_A}{z_c} = \frac{h_B - h_A}{10m} \quad \text{(same vertical gradient)}$$

$$\frac{z_c - 12.23}{z_c} = \frac{13 - 12.23}{10} = \frac{0.77}{10} = 0.077$$

$$z_c - 12.23 = z_c(0.077)$$

$$z_c(1 - 0.077) = 12.23$$

$$z_c = \frac{12.23}{(0.923)} = 13.249 \approx 13.25m$$

$\therefore$ Depth to water table is $20 - 13.25m = 6.75m$

Problem 5

During a drought period the following declines in the water table were recorded in an unconfined aquifer.

Area	Size (mi^2)	Decline (ft)
A	14	2.75
B	7	3.56
C	28	5.42
D	33	7.78

The total volume of water removed from storage in this aquifer during the time period was 5.7385 x 10^4 acre-feet. Estimate the specific yield of this aquifer.

$$S_y = \frac{V_{\text{water removed}}}{V_{\text{aquifer dewatered}}}$$

A $\quad 14 \cdot (5280)(5280)(2.75) = 1.073 \cdot 10^9 \qquad ft^3/43560 = 2.464 \cdot 10^4$ ac

B $\quad 7 \cdot (5280)(5280)(3.56) = 6.947 \cdot 10^9 \qquad ft^3/43560 = 1.595 \cdot 10^4$ ac

C $\quad 28 \cdot (5280)(5280)(5.42) = 4.231 \cdot 10^9 \qquad ft^3/43560 = 9.713 \cdot 10^4$ ac

D $\quad 33 \cdot (5280)(5280)(7.78) = 7.158 \cdot 10^9 \qquad ft^3/43560 = 1.643 \cdot 10^5$ ac

$\Sigma \qquad 3.020 \cdot 10^5$ ac

$$S_y = \frac{5.7385 \cdot 10^4}{3.020 \cdot 10^5} = 0.19$$

$$\therefore S_y \approx 19\%$$

Problem 6

Draw sufficient flow lines on the profile below to illustrate the regional flow pattern. Ignore that the aquifer is anisotropic and ignore the vertical/horizontal scale distortion. Determine whether the lake contributes water to the aquifer (recharge) or receives water from the aquifer (discharge).

The dashed line is the position of the water table; recall that the water table is a flow boundary.

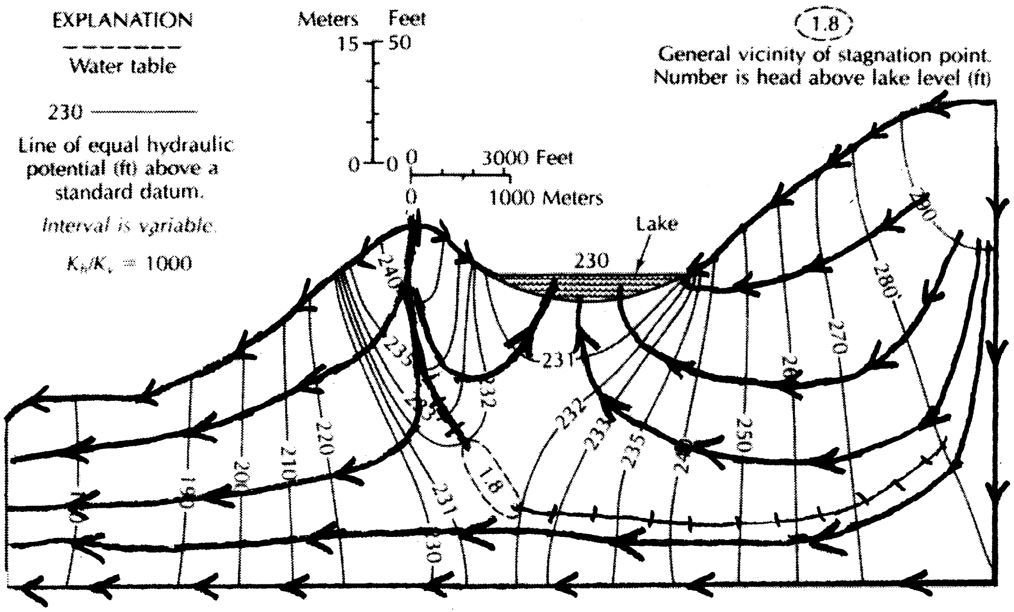


FIGURE 8.26 Hydrogeologic cross section showing head distribution in a one-lake system with a homogeneous, anisotropic aquifer system. Results are based on a two-dimensional, steady-state, numerical-simulation model. Source: T. C. Winter, U.S. Geological Survey Professional Paper 1001, 1976.

Lake receives water from aquifer – lake is a discharge area