Rainfall Intensity in Design

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Acknowledgements

- Texas Department of Transportation
  - Various projects since FY 2000.
  - Current: 0-6070 Use of Rational and Modified Rational Method for Drainage Design.
- William H. Asquith, USGS
  - Research colleague who provided much of the ideas and authored the R package that makes the simulations possible.

Introduction

- Result of a question:
  - “After all, how hard can it rain?”
- Intensity has variety of uses
  - BMP design
  - Rational method
- Examine use of recent tools:
  - Are estimated intensities consistent with observations?

Data Sources

- Texas specific:
  - Asquith and others (2004)
  - Williams-Sether and others (2004)
  - Asquith and others (2006)
- Global Maxima
  - Jennings (1950), Paulhus (1965), Barcelo and others (1997), Smith and others (2001)

Data Sources

- Asquith and others (2004).
  - 92 stations (up to 135).
  - 1600 paired events.

USGS
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In cooperation with the Texas Department of Transportation

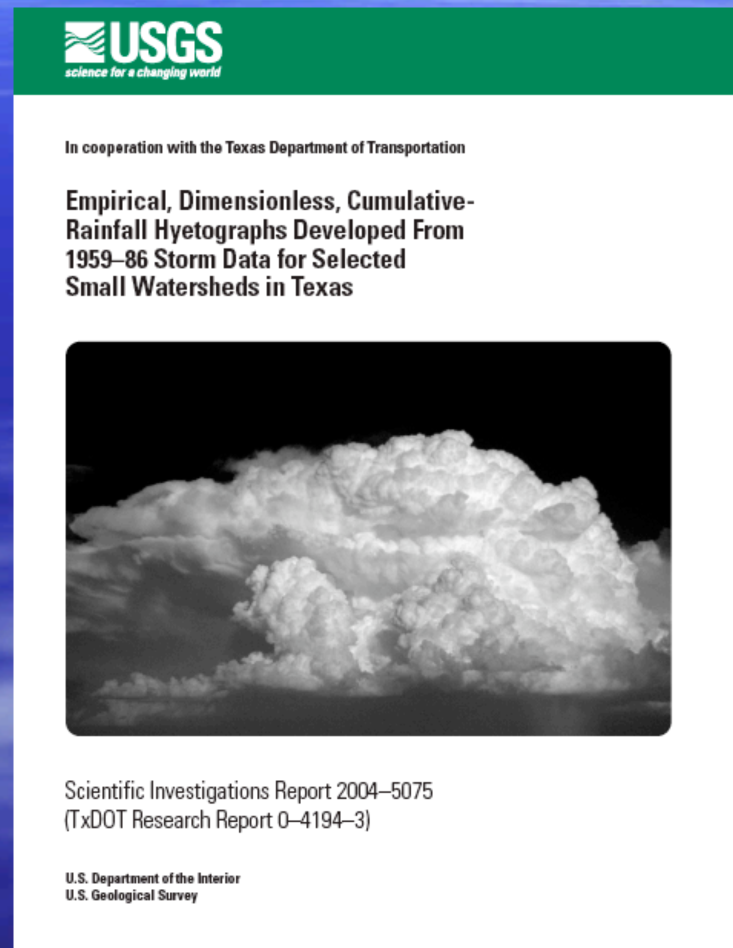
Synthesis of Rainfall and Runoff Data Used for Texas Department of Transportation Research Projects 0–4193 and 0–4194

Open-File Report 2004–1035
(TxDOT Research Reports 0–4193–2 and 0–4194–2)

U.S. Department of the Interior
U.S. Geological Survey

Empirical Hyetographs

- Williams-Sether and others (2004)
  - 92 stations, 1507 storms, known to have produced runoff.
  - Duration divided into 4-quartiles.
    - Quartile with largest accumulation of rainfall defines “storm quartile”
  - Observed rainfall collected into 2.5-percentile “bins”
    - Smoothing (to force monotonic dimensionless hyetographs).
  - Result is empirical-dimensionless-hyetograph


USGS
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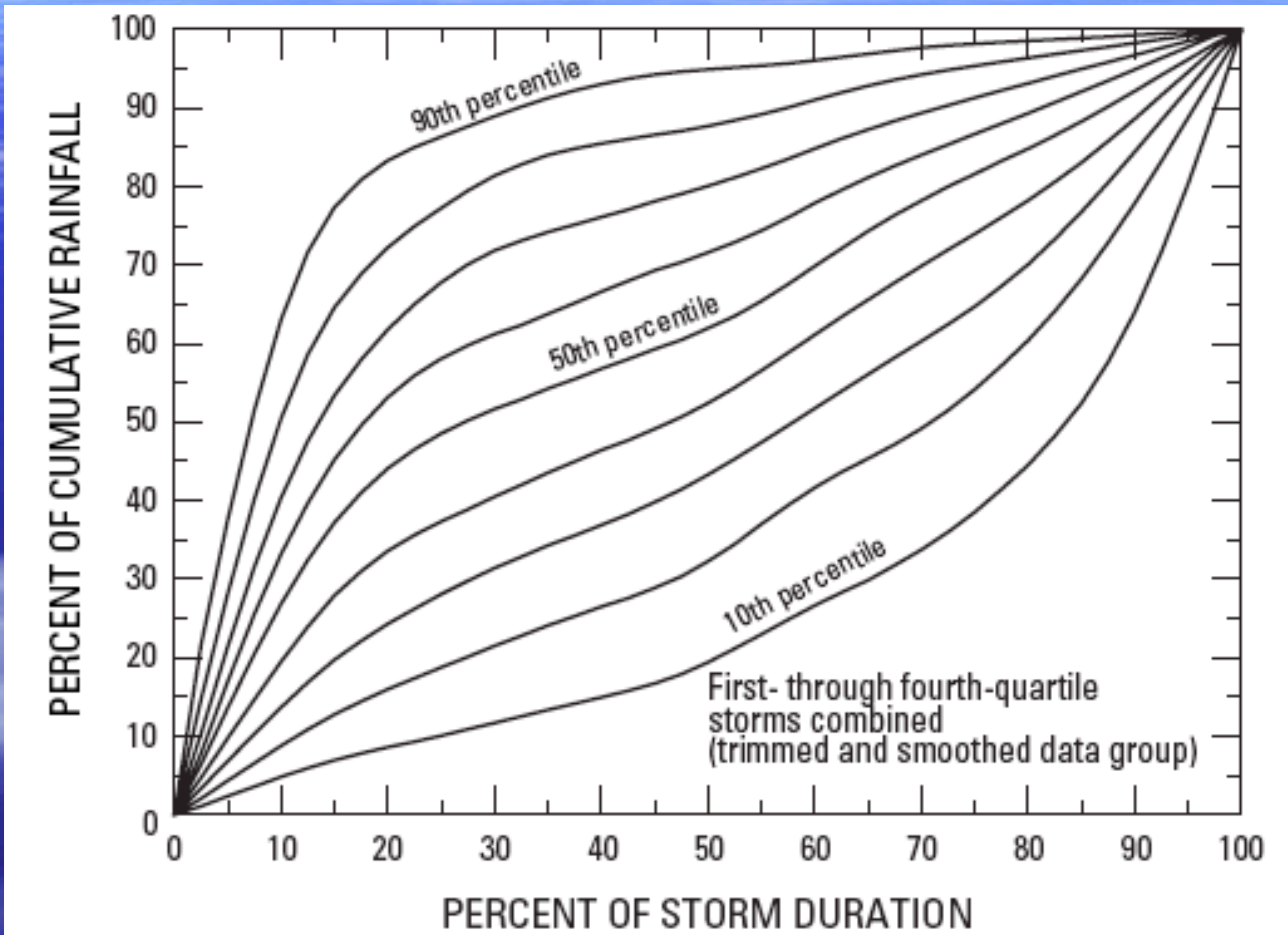
In cooperation with the Texas Department of Transportation

Empirical, Dimensionless, Cumulative-Rainfall Hyetographs Developed From 1959–86 Storm Data for Selected Small Watersheds in Texas

Scientific Investigations Report 2004–5075
(TxDOT Research Report 0–4194–3)

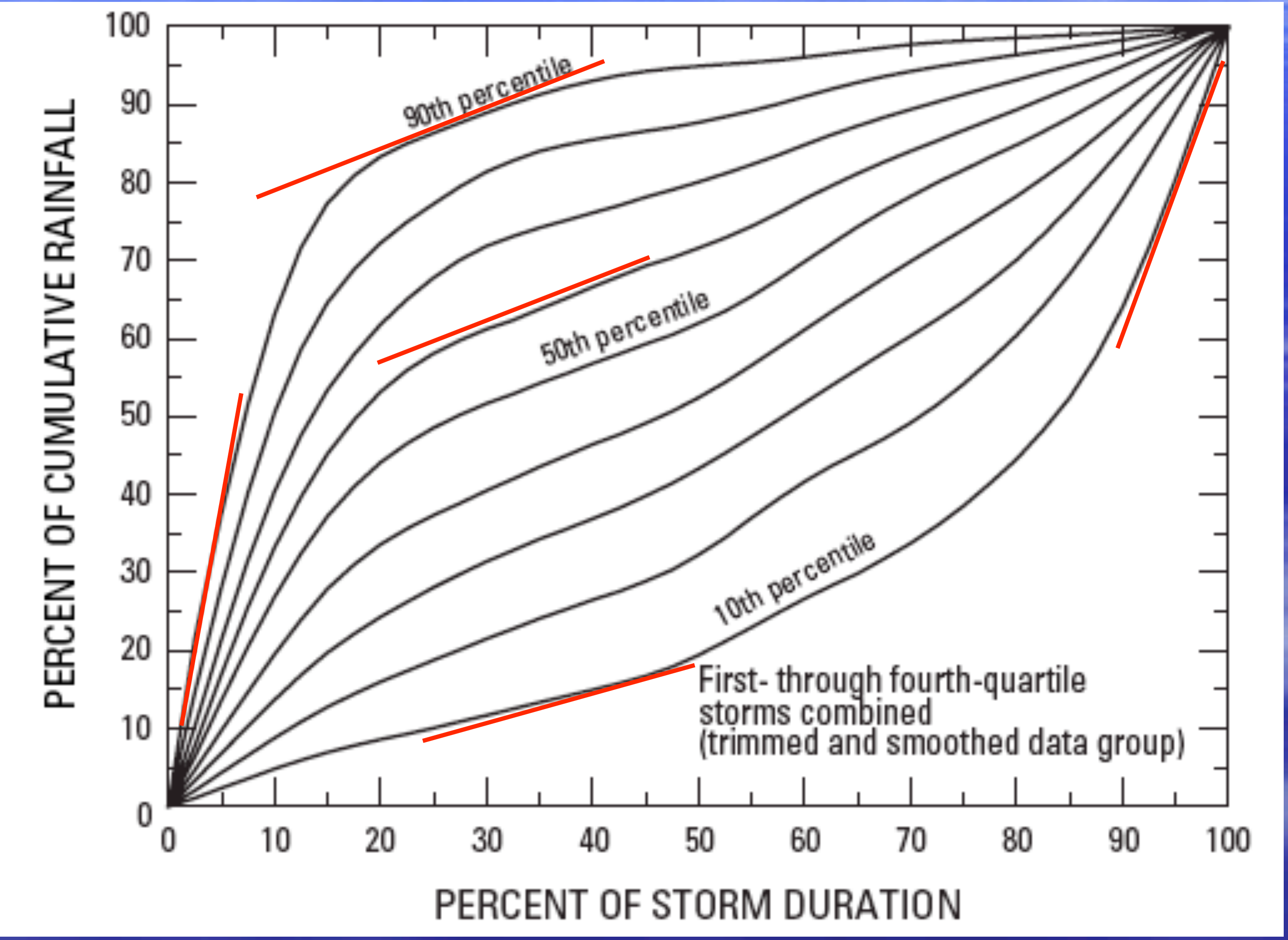
U.S. Department of the Interior
U.S. Geological Survey

Empirical Hyetographs



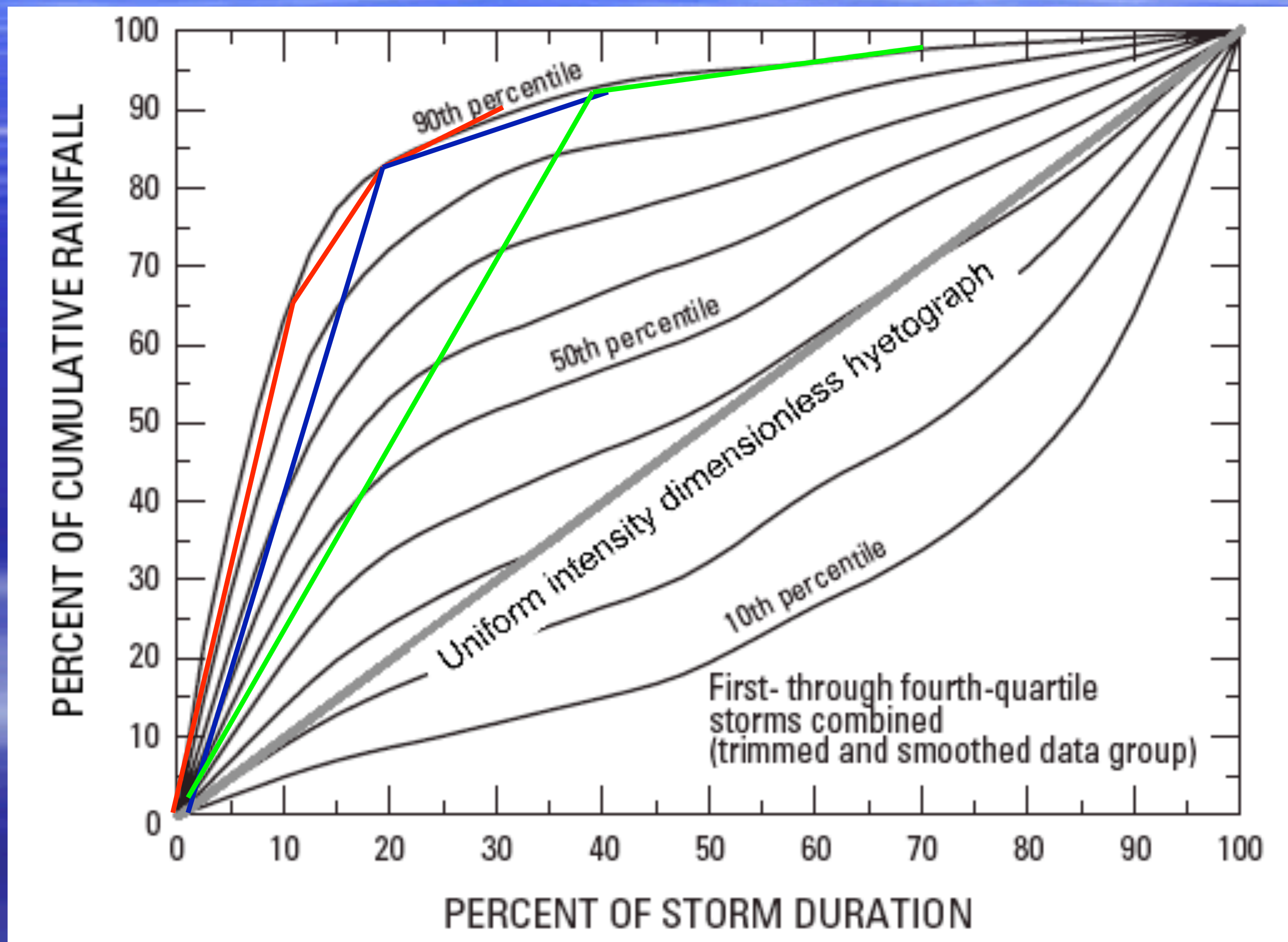
Empirical Hyetographs

- Slopes are dimensionless "intensity"

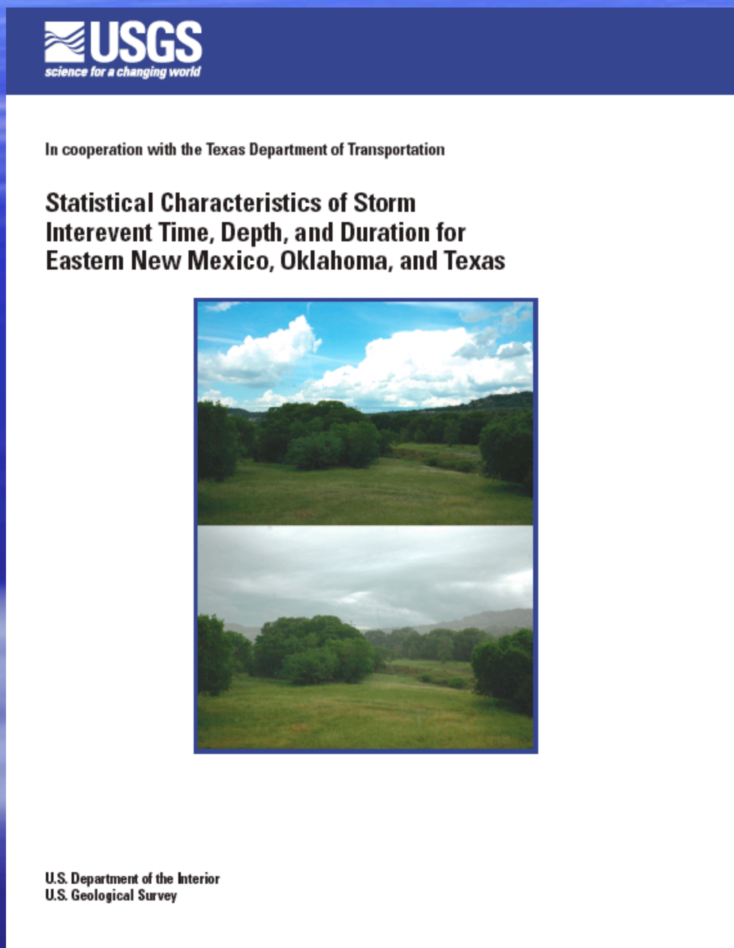


Intensity Simulations

- 10% steps
- 20% steps
- 25% steps
- 33% steps
- 50% steps
- Uniform (average) “intensity”



Intensity Simulations



- Asquith and others (2006)
  - 774 stations in New Mexico, Oklahoma, and Texas.
  - Quantiles for each “storm.” (Half-million in Texas).
  - L-moments computed for each station for duration and depth.
  - Kappa distribution recommended as most appropriate distribution for depth and duration.

Intensity Simulations

Papa, 2000, p. 74). The cumulative distribution of storm interevent times is

$$F(x) = 1 - e^{\frac{MIT - x}{\Lambda - MIT}} \text{ for } x \geq MIT \text{ and } n = 1, 2, \ldots \quad (10)$$

where F is the cumulative or nonexceedance probability for the x interevent time, and MIT is the minimum interevent time in days. The parameter Λ is the mean interevent time in days. The inclusion of the minimum interevent time adjusts the exponential distribution because interevent times less than the minimum interevent time are not possible. Equation 10 can be solved in terms of x. The resulting equation is the quantile function of interevent time and is

$$x(F) = MIT - (\Lambda - MIT)\ln(1 - F) \text{ for } x \geq MIT. \quad (11)$$

When random numbers between 0 and 1 are substituted for F in equation 11 with Λ equal to 7.60 days and MIT equal to 1 day (24 hours), a random sequence of interevent times is generated. Five simulations based on a random sequence of five interevent times are listed in table 21 (at end of report). The mean of the simulations is 7.19 days—the mean approaches 7.60 as the number of simulations becomes larger.

It is illustrative to compare the 7.60 days mean interevent time to the results of Asquith and Roussel (2003, fig. 4). Asquith and Roussel (2003, fig. 4) shows that the interoccurrence of daily rainfall (not hourly) of 0.05 inch or more is, on average about 8 days for the Amarillo area. The two interevent times are of the same order as expected, but the values should not be equal.

Example 3: Estimation of the Empirical Distribution of Storm Depth

PROBLEM: The 98th-percentile storm from the empirical distribution of storm depth for a site very close to station 4311 Houston Alief, Tex. (fig. 3C) (62 years of record), is required by an environmental consulting firm working on a project proposal in a watershed where BMPs are to have a 24-hour drawdown time. Hence, the statistics of storms with a 24-hour minimum interevent time are appropriate.

SOLUTION: The 98th percentile and other selected percentiles of storm depth are listed in appendix 4–4.5 and in column two of table 22 (at end of report). The 98th-percentile storm has a depth of 4.55 inches. (Column three of table 22 is a component of example 4.) The median storm depth is 0.44 inch and the interquartile range is 1.03 inches (1.18 minus 0.15) for station 4311.

Example 4: Estimation of the Continuous Distribution of Storm Depth

⋆PROBLEM: As part of a city ordinance, a BMP for a small urban watershed in the city is believed to accommodate 90 percent of all storms when 2 inches or less of runoff is captured. The temporal distribution of runoff (outflow rate) from the BMP is to be ignored. Engineering firm A is to design a BMP for a given watershed in which the ordinance applies. The ordinance states that the BMP is to have a 24-hour drawdown time; hence an analysis of storms with a 24-hour minimum interevent time is required. Engineering firm B is questioning whether a 2-inch design runoff would accommodate the 90th-percentile storm as reflected by the ordinance or instead would accommodate approximately the 95th-percentile storm. Thus, firm B believes that the ordinance might contribute to over-design of BMPs. The scientific credibility of the ordinance hence is in question; the results of this report can be used to evaluate the ordinance. Assume, for the purpose of illustration, that near the planned BMP is long-term station 4311 Houston Alief, Tex. (station considered in example 3).

SOLUTION: The first step toward the solution is to compute the depth of rainfall that produces 2 inches of runoff on the watershed. A simple runoff model (Adams and Papa, 2000, p. 121, eq. 6.28) used for illustration is

$$R = \phi(P - S_D), \quad (12)$$

where R is runoff in inches, ϕ is the runoff coefficient, P is rainfall in inches, and S_D is depression storage or an initial abstraction in inches. It is widely accepted that a typical initial abstraction for the watershed is 0.25 inch and the runoff coefficient is about 0.8. Upon variable substitution, the rainfall producing 2 inches of runoff is 2.75 inches.

The L-moments of storm depth for a 24-hour minimum interevent time for this station are 0.88849 inch, 0.52954 inch, 0.45778, and 0.23879 for the mean, L-scale, L-skew, and L-kurtosis, respectively (appendix 4–2.5). A four-parameter kappa distribution (see section "Quantile Functions of Storm Depth and Duration" in this report) can be fit by use of these L-moments using an algorithm such as in Hosking (1996) (data not shown in this report). The fitted kappa distribution corresponding to these L-moments is

$$P(F) = -0.4990 + \frac{1.028}{-0.1117}\left\{1 - \left[\frac{(1 - F^{1.650})}{1.650}\right]^{-0.1117}\right\}, \quad (13)$$

where P is storm depth and F is nonexceedance probability. Substituting 2.75 inches for the left side of the equation and solving the equation for F yields 0.932 or 93.2 percent. In other words, a rainfall depth of 2.75 inches is about the 93rd-percentile storm depth. Therefore, a statistical estimate of the storm percentage associated with 2 inches of runoff for the watershed is 3 percentage points larger than 90 percent. The 90th percentile for the distribution ($F = 0.90$) is 2.24 inches.

Thus, the ordinance reflects a depth of 2.75 inches; whereas, the statistical estimate of the 90th-percentile storm is 2.24 inches using the Hosking (1996) algorithm. Therefore, the claim of engineering firm B that a storm associated with 2 inches of runoff would accommodate approximately the 95th-percentile storm is questionable. The depth for the 95th-percentile storm is 3.18 inches by substituting $F = 0.95$ into

- Asquith and others (2006)
  - Examples provide "tools" to parameterize the empirical-dimensionless-hyetographs.
  - Page 42 explains how to use Kappa quantile function and L-moments to recover storm depth (vertical axis of dimensionless hyetograph).

Intensity Simulations

equation 13. The runoff from the 95th-percentile storm is about 2.34 inches from equation 12.

To further illustrate the application of this report, from equation 13 the quantiles for each of the selected percentiles or nonexceedance probabilities (0.01, 0.02, 0.10, 0.25, 0.50, 0.75, 0.90, 0.98, and 0.99) are listed in column three of table 22. As seen in the table, the empirical storm depth percentiles and storm depth percentiles from the kappa distribution are similar for each percentile as expected.

Example 5: Statistical Simulation of Rainfall Intensity

⋆PROBLEM: An analyst wants to construct synthetic temporal distributions of average rainfall intensity for station 4311 Houston Alief, Tex., to investigate the influence of rainfall rates on the spill volume of a numerical model of a particular BMP design.

SOLUTION: The kappa distribution of storm depth P for nonexceedance probability F is given as equation 13 in example 4. The L-moments of storm duration for the station are listed in appendix 4–3.5. The mean, L-scale, L-skew, and L-kurtosis are 13.434 hours, 8.1389 hours, 0.46763, and 0.20844, respectively. Fitting a kappa distribution to these L-moments using the Hosking (1996) algorithm (data not shown in this report) results in the following equation for the storm duration D in terms of nonexceedance probability, F:

$$D(F) = -23.466 + \left(\frac{28.137}{0.093897}\right)\left\{1 - \left[\frac{(1-F^{2.4775})}{2.4775}\right]^{0.093897}\right\}. \quad (14)$$

It is convenient to assume that storm depth and duration are independent random variables, which is supported by the scattered relation in figure 9. Under this assumption, storm depth is simulated by generating a random number between 0 and 1, substituting this value for F, and solving equation 13 for P. A similar process for storm duration is done with the generation of a new random number between 0 and 1, substituting this value for F, and solving equation 14 for D. This process is best illustrated by example. A random number of 0.78687 is generated for storm depth and results in a depth of 1.33 inches using equation 13. Another random number of 0.040703 is generated for storm duration and results in a duration of 1.01 hours using equation 14. The average rainfall intensity for this storm thus is 1.33 divided by 1.01 or 1.32 inches per hour.

Regional Approach by County

Example 6: Regional Estimation of Storm Occurrence

PROBLEM: The storm interevent time for storms defined by a 40-hour minimum interevent time in Randall County, Tex. (fig. 3A), is desired. The storm interevent time is a component of a design. The maps in this report can be used for estimation.

SOLUTION: The storm interevent time for a 40-hour minimum interevent time is not a statistic provided in this report. However, 24-hour and 48-hour minimum interevent times bracket 40 hours. At the approximate center of Randall County, the mean storm interevent time for the 24-hour minimum interevent time is about 10.5 days (table 18), and that for the 48-hour minimum interevent time is about 12.4 days (table 18). Linear interpolation can be used to estimate the mean storm interevent time for the 40-hour minimum interevent time; the result is about 11.8 days.

Example 7: Computation of the Storm-Captured Percentage

PROBLEM: A local ordinance for a county in Texas requires that a BMP capture a 1.5-inch storm and release this storm over a 24-hour period. The county has a mean storm depth of 0.750 inch (a randomly selected value from table 19). An estimate of the percentage of storms that will be captured under the ordinance is needed.

SOLUTION: The dimensionless storm depth frequency curve using the kappa distribution (eq. 6; table 7) for a 24-hour minimum interevent time in Texas is

$$x(F) = -0.5790 + \frac{1.115}{-0.1359}\left[1 - \left(\frac{1-F^{1.747}}{1.747}\right)^{-0.1359}\right]. \quad (15)$$

where $x(F)$ is the dimensionless multiplier (a frequency factor) for nonexceedance probability F. The storm depth distribution is the mean depth multiplied by the dimensionless distribution or

$$P(F) = 0.750 \times x(F), \quad (16)$$

where $P(F)$ is the storm depth for nonexceedance probability F. The left side of the equation is set to 1.5 inches, and the storm percentage can be estimated by solving the resulting equation for F. The equation is

$$\frac{1.5}{0.75} = -0.5790 + \frac{1.115}{-0.1359}\left[1 - \left(\frac{1-F^{1.747}}{1.747}\right)^{-0.1359}\right]. \quad (17)$$

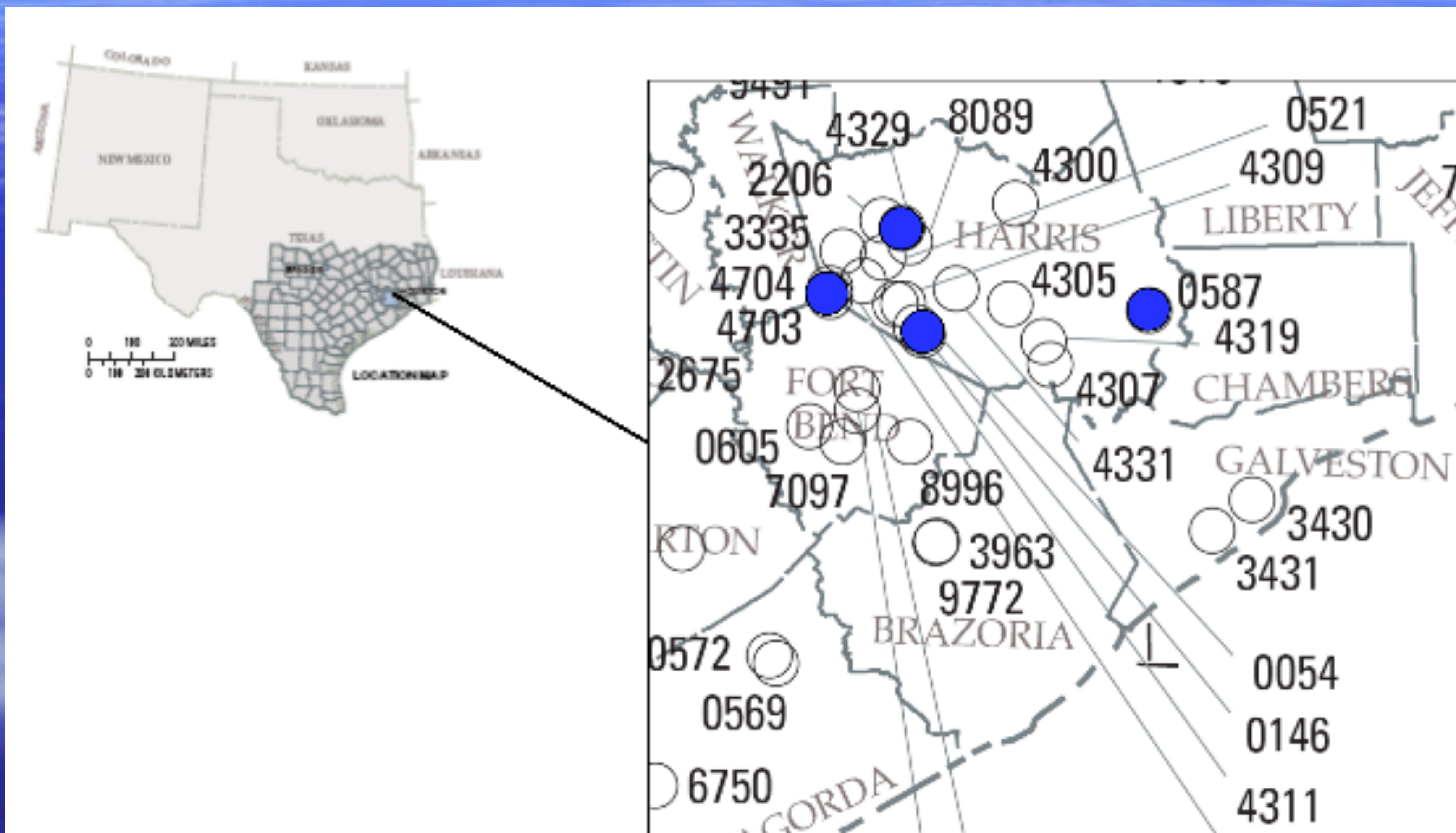
The F satisfying the equality is 0.859. Thus, under the ordinance, about 86 percent of all storms will be captured by the BMP.

Example 8: Regional Estimation of the Empirical Distribution of Storm Depth

PROBLEM: A BMP is to be built with a 36-hour drawdown time in Randall County, Tex. (fig. 3A). The empirical distribution, specifically the 50th, 75th, 90th, 98th, and 99th percentiles of storm depth, are needed as part of the design process.

- Asquith and others (2006)
  - Examples provide “tools” to parameterize the empirical-dimensionless-hyetographs.
  - Page 43 explains how to use Kappa quantile function and L-moments to recover duration (horizontal axis of the empirical hyetograph).
  - Did not provide ‘code.’

Intensity Simulations



Intensity Simulations

Appendix 4–2.1. L-moments of storm depth defined by 6-hour minimum interevent time for hourly rainfall stations in Texas.

[–, not available]

Station no.	Depth mean (inches)	Depth L-scale (inches)	Depth L-CV (dimensionless)	Depth L-skew (dimensionless)	Depth L-kurtosis (dimensionless)	Depth Tau5 (dimensionless)	Station no.	Depth mean (inches)	Depth L-scale (inches)	Depth L-CV (dimensionless)	Depth L-skew (dimensionless)	Depth L-kurtosis (dimensionless)	Depth Tau5 (dimensionless)
0015	0.10273	0.07309	0.71150	0.64511								[illegible]	0.19463
0016	.38511	.25319	.65743	.50821								[illegible]	.13521
0050	.50593	.29615	.58536	.43998								[illegible]	.11434
0054	.31767	.18918	.59551	.47295								[illegible]	.20723
0120	.60333	.34811	.57697	.41632								[illegible]	.22636
0145	.37637	.27471	.72989	.65386								[illegible]	.24622
0146	.35231	.20630	.58558	.38256								[illegible]	.18238
0174	.32717	.17482	.53434	.53045								[illegible]	.27911
0178	.26120	.17420	.66692	.59557								[illegible]	.21522
0179	.29057	.16757	.57667	.47202	.22912	.11305	1325	.57612	.36736	.63765	.52144	.28187	.17563
0202	.48328	.26817	.55490	.51610	.24559	.14331	1429	.51873	.31486	.60698	.50041	.27546	.17399
0206	.54830	.30667	.55931	.47264	.23593	.15685	1431	.55718	.34261	.61490	.49416	.25140	.14443
0208												.23891	.14393
0211												.27261	.17319
0244												.25455	.15369
0248												.23958	.13883
0262	.58471	.34641	.59244	.46963	.24353	.15408	1436	.57380	.34692	.60460	.48197	.25623	.16255
0271	.69897	.43081	.61636	.44289	.21929	.18320	1437	.45071	.30677	.68064	.51465	.19012	.04387
0380	.62676	.40903	.65261	.55519	.33996	.23983	1438	.55409	.33903	.61186	.48286	.24756	.14680
0394	.46000	.24564	.53399	.34123	.16605	.05181	1462	--	--	--	--	--	--
0408	.86676							.48235	.28420	.58919	.52638	.27591	.16216
0427	.43151							.53606	.30894	.57631	.35829	.04138	-.06899
0428	.40801							.47542	.29252	.61529	.55852	.31362	.18292
0429	.47570	.32658	.68653	.54493	.30187	.20464	1541	.65571	.37994	.57943	.47134	.18501	.11444
0463	.47370	.27083	.57173	.50067	.30770	.20084	1569	.50392	.34179	.67827	.57943	.37849	.29311
0493	.70842	.28749	.40581	.32264	.23370	.12487	1632	.47857	.26095	.54527	.12044	-.25730	.08212
0495	.32531	.18685	.57438	.46009	.25757	.17510	1641	.41386	.23517	.56824	.43397	.21010	.13376
										[illegible]	.53120	.28375	.15603
										[illegible]	.53790	.26175	.18446
										[illegible]	.51905	.27902	.16730
										[illegible]	.48079	.26018	.17040
										[illegible]	.47688	.16994	.09024
0556	.50496	[illegible]	[illegible]	.47864	[illegible]	[illegible]	1696	.42951	.24908	.57991	.44753	.22765	.14778
0569	.61755	.39031	.63203	.54491	.30312	[illegible]	1697	.43084	.25071	.58191	.46878	.23303	.12625
0572	.55580	.35213	.63355	.52475	.29810	.19635	1698	.40740	.23556	.57820	.51632	.28599	.16120
0576	[illegible]	[illegible]	.70234	[illegible]	[illegible]	.24110	1720	.45870	.27593	.60153	.59269	.28396	.13711
0580	[illegible]	[illegible]	.66393	[illegible]	[illegible]	.22153	1761	.27129	.16508	.60850	.43922	.19379	.12601
0587	.57882	.37118	.64128	.51392	.27750	.18293	1773	.63171	.36447	.57695	.47188	.23508	.15098
0605	.60080	.30497	.50760	.39297	.20415	.13276	1810	.40231	.24425	.60711	.52907	.33370	.23100

$$D(F) = -23.466 + \left(\frac{28.137}{0.093897}\right)\left\{1-\left[\frac{(1-F^{2.4775})}{2.4775}\right]^{0.093897}\right\}. \quad (14)$$

```
# generate depths and durations associated with probabilities
dep<-q_func(fdep,pardep$para[1],pardep$para[2],pardep$para[3],pardep$para[4])
```

```
# get Kappa parameters from L-moments
 pardep<-lmom2par(lmdep,type="kap")
```

```
# L-moments for each station from Appendix 4, Asquith and others, 2006
# Station 0587, 6-hour inter-event arrival time
 lmdep<-vec2lmom(c(0.57882, 0.37118, 0.51392, 0.2775 ))
```

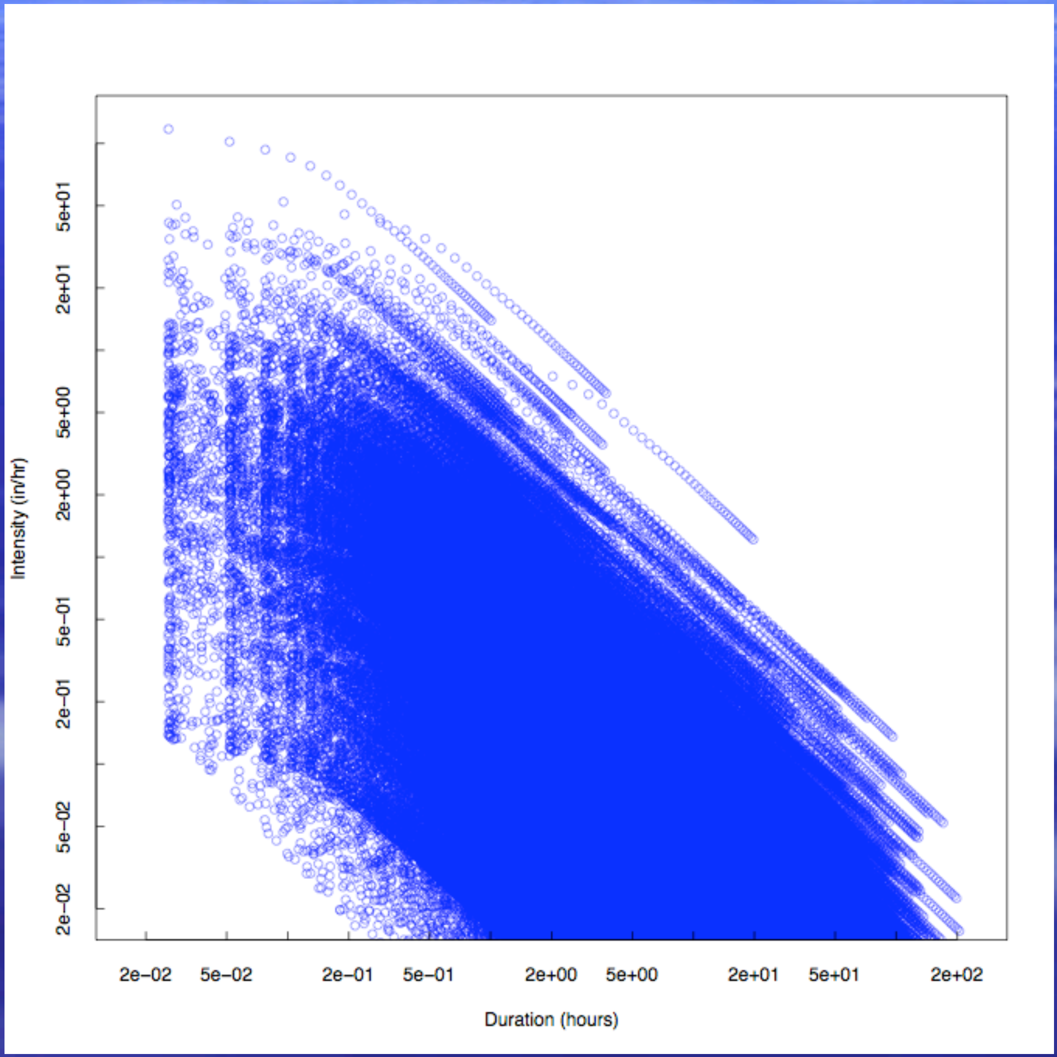
Intensity Simulations

- Asquith (2007)
  - LMOMCO package in R
  - Provides the necessary ‘code’ to make such computations.

```
# R Code to simulate Harris County Intensities (6-hour)
# Load the L-moment package from CRAN and attach as a library
 library(lmomco)
# Quantile Functions for Depth and Duration.
# Asquith and others, 2006, Eqns 13, and 14.
q_func<-function(f,p1,p2,p3,p4){(p1+(p2/p3)*(1-((1-f^p4)/p4)^p3))}
# L-moments for each station from Appendix 4, Asquith and others, 2006
# Station 0587, 6-hour inter-event arrival time
 lmdep<-vec2lmom(c(0.57882, 0.37118, 0.51392, 0.2775 ))
 lmdur<-vec2lmom(c(6.3865, 3.1849, 0.43733, 0.2504 ))
# get Kappa parameters from L-moments
 pardep<-lmom2par(lmdep,type="kap")
 pardur<-lmom2par(lmdur,type="kap")
 # generate 2500 random probabilities
 fdep<-runif(2500,0,1); fdur<-runif(2500,0,1)
# generate depths and durations associated with probabilities
dep<-q_func(fdep,pardep$para[1],pardep$para[2],pardep$para[3],pardep$para[4])
dur<-q_func(fdur,pardur$para[1],pardur$para[2],pardur$para[3],pardur$para[4])
# calculate intensities
avg_intensity<-dep/dur
```

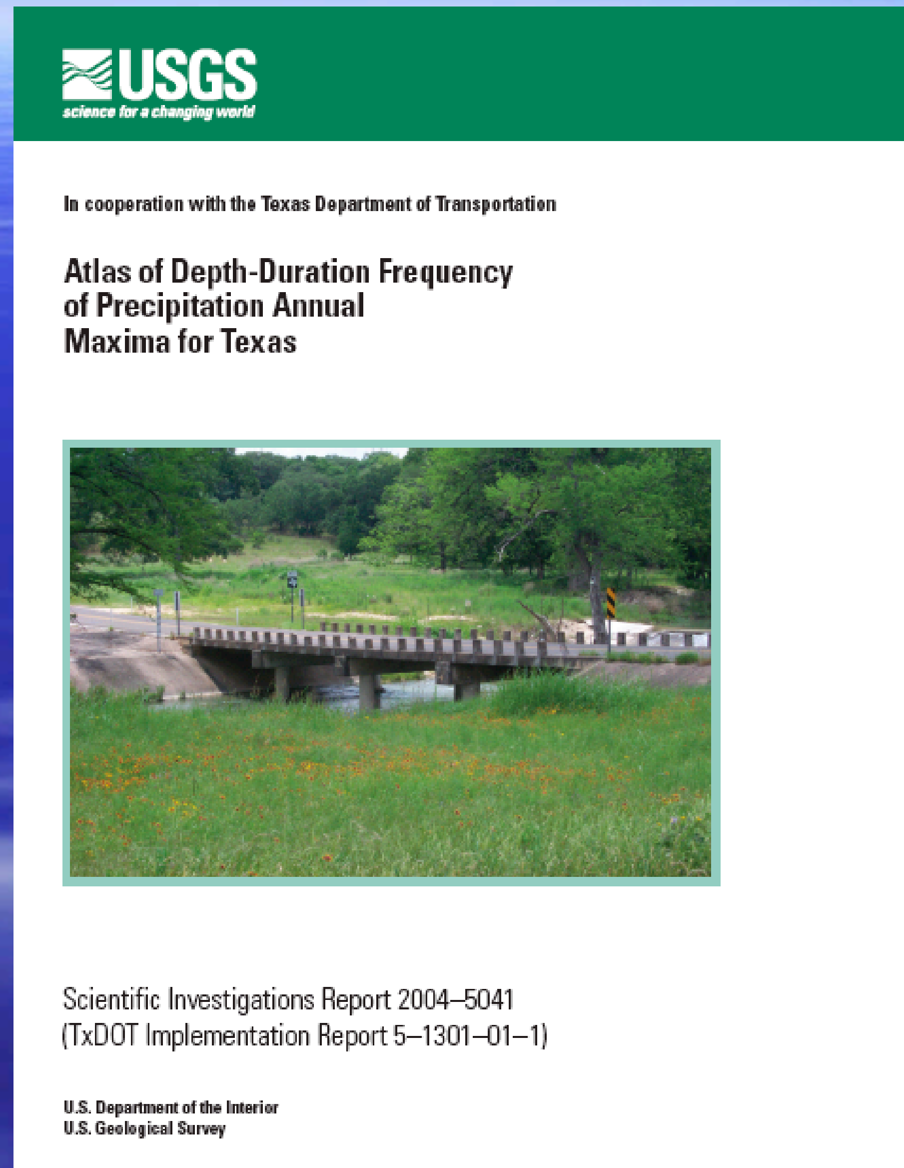
Intensity Simulations

- Resulting Plot,
  - 5000 ‘events’



Comparisons to Prior Work

- Asquith and Roussel (2004)
  - L-moments analysis.
  - Product similar to TP-40; HY-35


USGS
science for a changing world

In cooperation with the Texas Department of Transportation

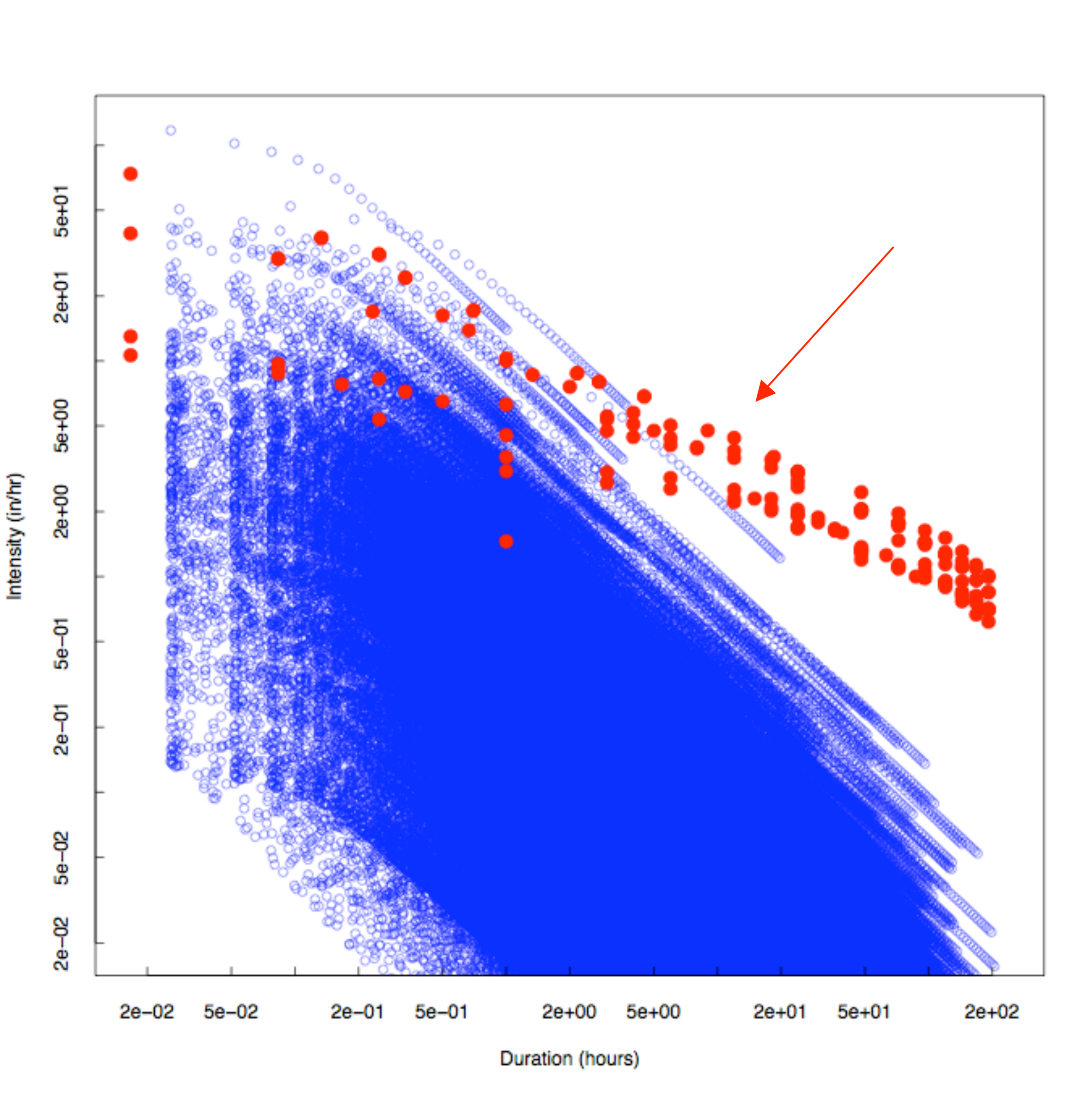
Atlas of Depth-Duration Frequency of Precipitation Annual Maxima for Texas

Scientific Investigations Report 2004–5041
(TxDOT Implementation Report 5–1301–01–1)

U.S. Department of the Interior
U.S. Geological Survey

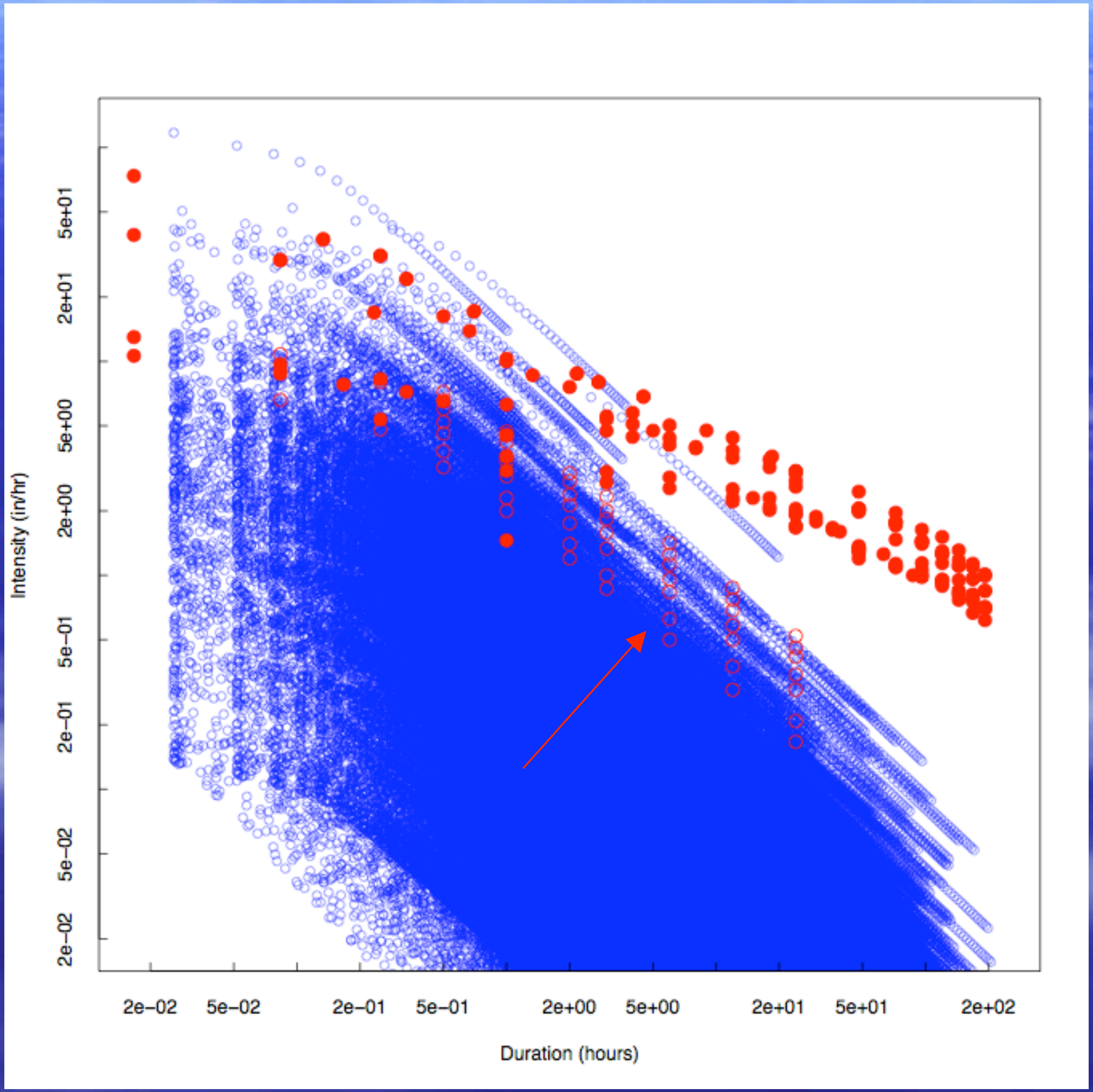
Comparison to Global Maxima

- Include Global Maxima
  - Avg. Intensities from Depth and Duration.



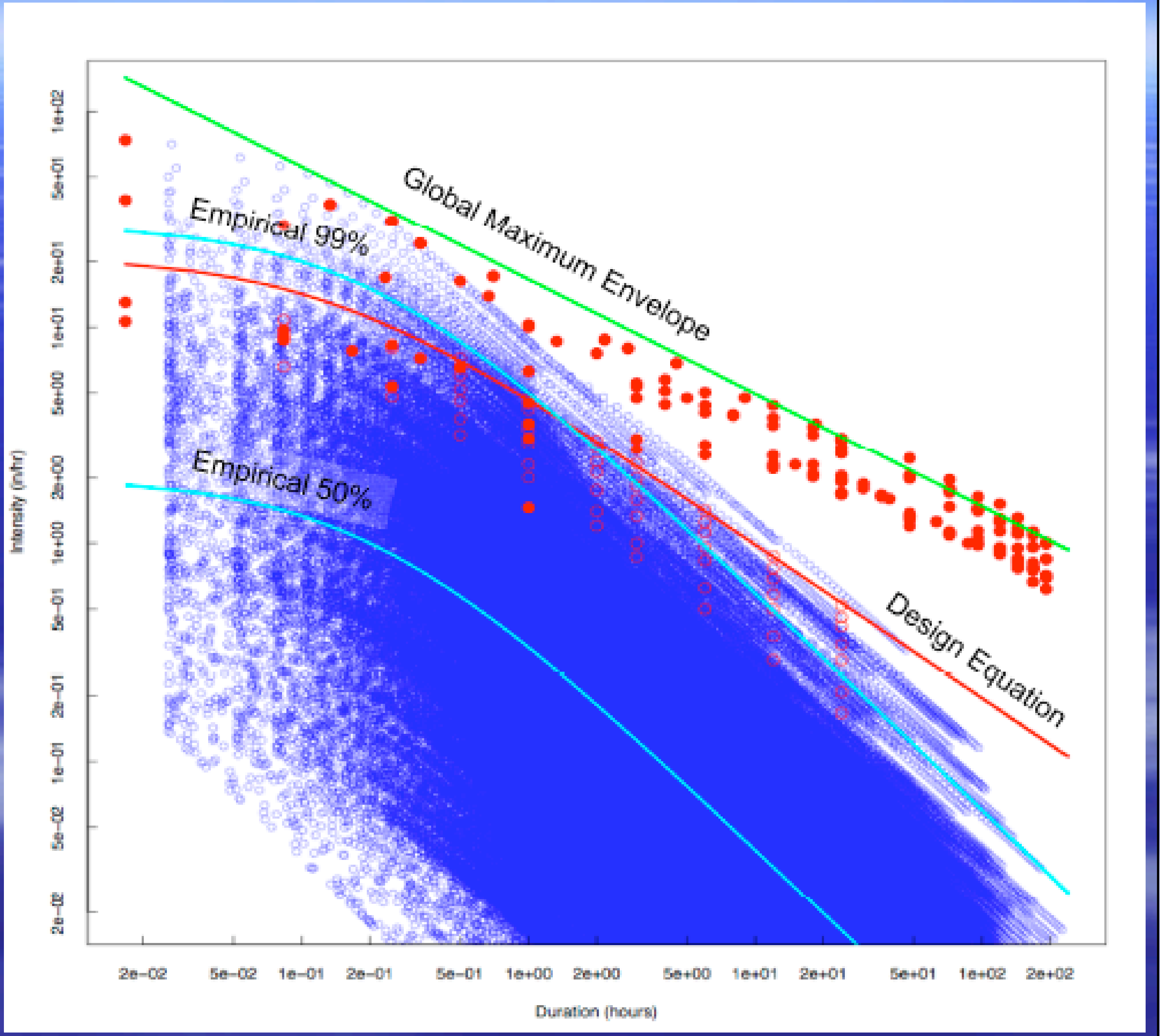
Comparison to TP-40/HY35

- Include Global Maxima
  - Avg. Intensities from Depth and Duration.
- Include TP-40 values.
- Include HY-35 values.



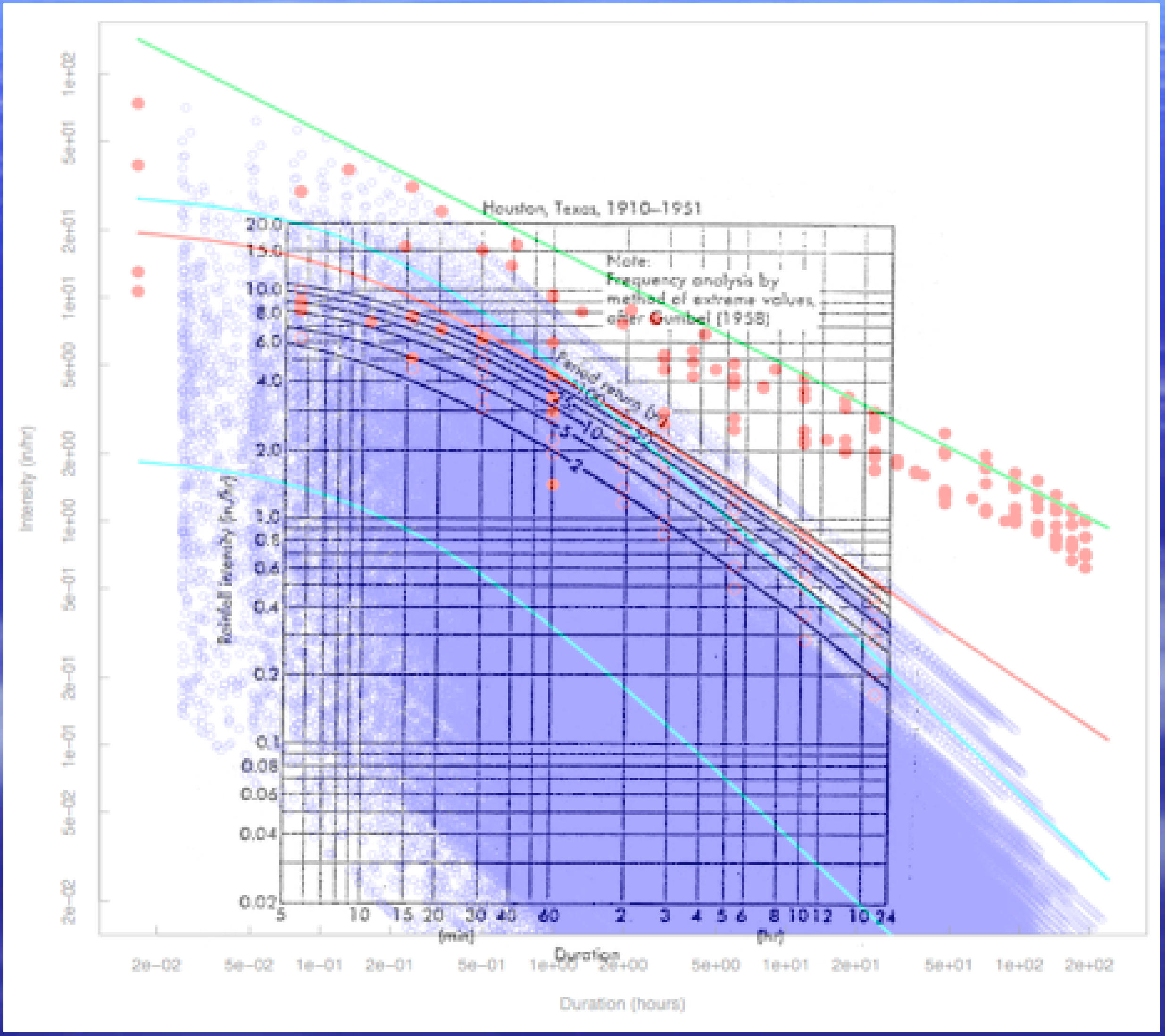
Empirical Percentiles

- Empirical 'Percentiles'
  - Count fraction above and below line.
  - Fraction establishes percentile.
  - Line is an ad-hoc model.
  - "Design" Equation is from TxDOT manual



Empirical Percentiles

- COH IDF Overlay.
  - 2-year line is about the 95% empirical percentile.



Conclusions

- Results are consistent with prior work.
- Results are within the global envelope.
- Differences at higher duration - Texas storms less intense if long.
- Rare (99th-percentile) estimates about the same.
- Median (50th-percentile) quite different.
  - Consequence of what simulations actually represent.

Future Directions

- Biggest assumption is independent depth and duration.
  - There is evidence that these variables are highly coupled, especially for longer durations.
- Conditional dependence should be examined.
  - Important for water quality issues.