CE 3354 Engineering Hydrology

Lecture 22: Groundwater Hydrology Concepts – Part 2

Outline

- Direct Application of Darcy's Law
- Steady flow solutions
  - Rectilinear flow
  - Flow to wells

Direct Application of Darcy’s Law

- A groundwater map is a topographic representation of the 3-D piezometric or water table surface.
- Darcy’s law implies that flow should be perpendicular to the lines of constant head (or potential)
  - unless the medium is anisotropic (permeability is direction dependent).

Direct Application of Darcy's Law

- Flowlines are thus usually plotted perpendicular to the water level distribution.
- Flowlines diverge away from a recharge area or source of water to the subsurface,
- Flowlines converge towards a discharge area.
  - The concept is similar to water flowing downhill in a surface water system.

Direct Application of Darcy's Law

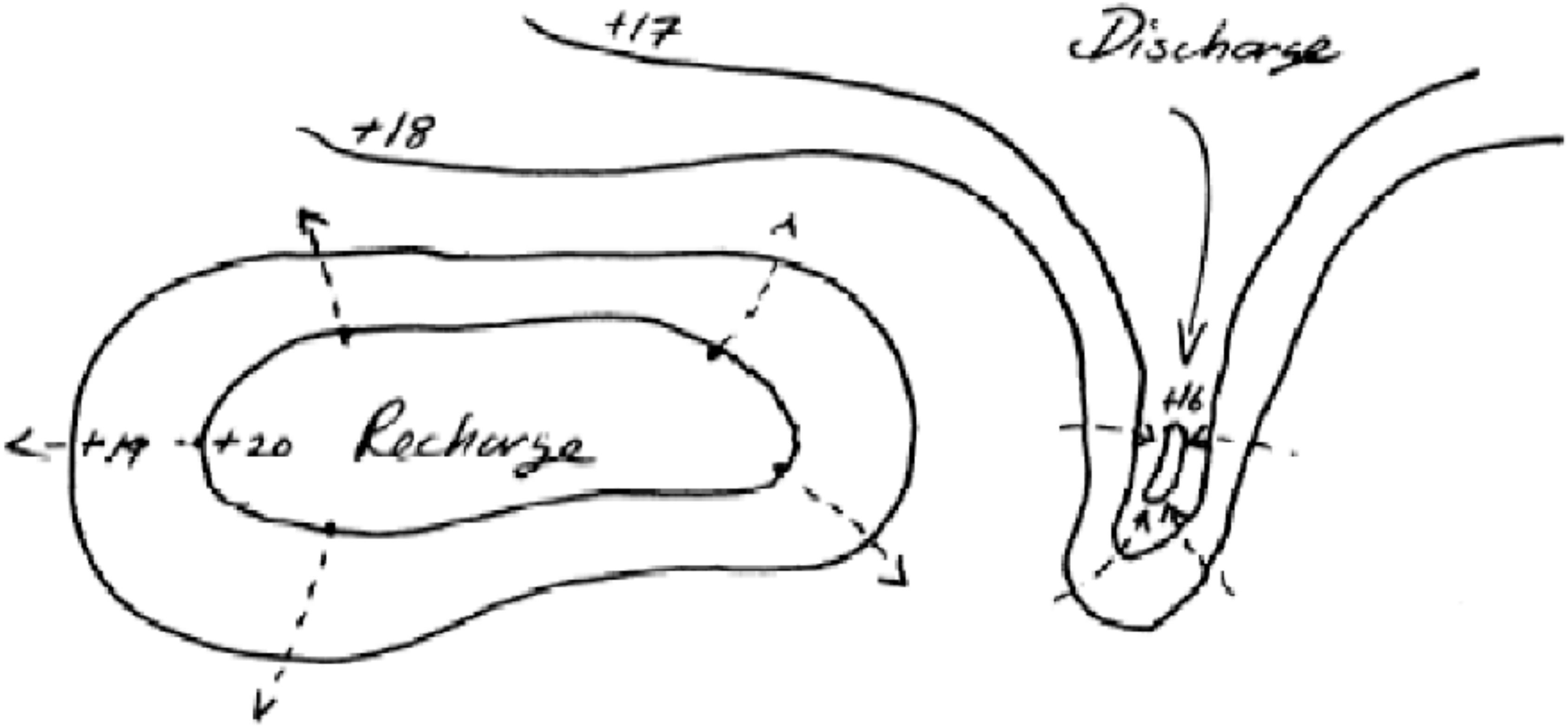


Figure 14: Some representative flownets

Direct Application of Darcy's Law

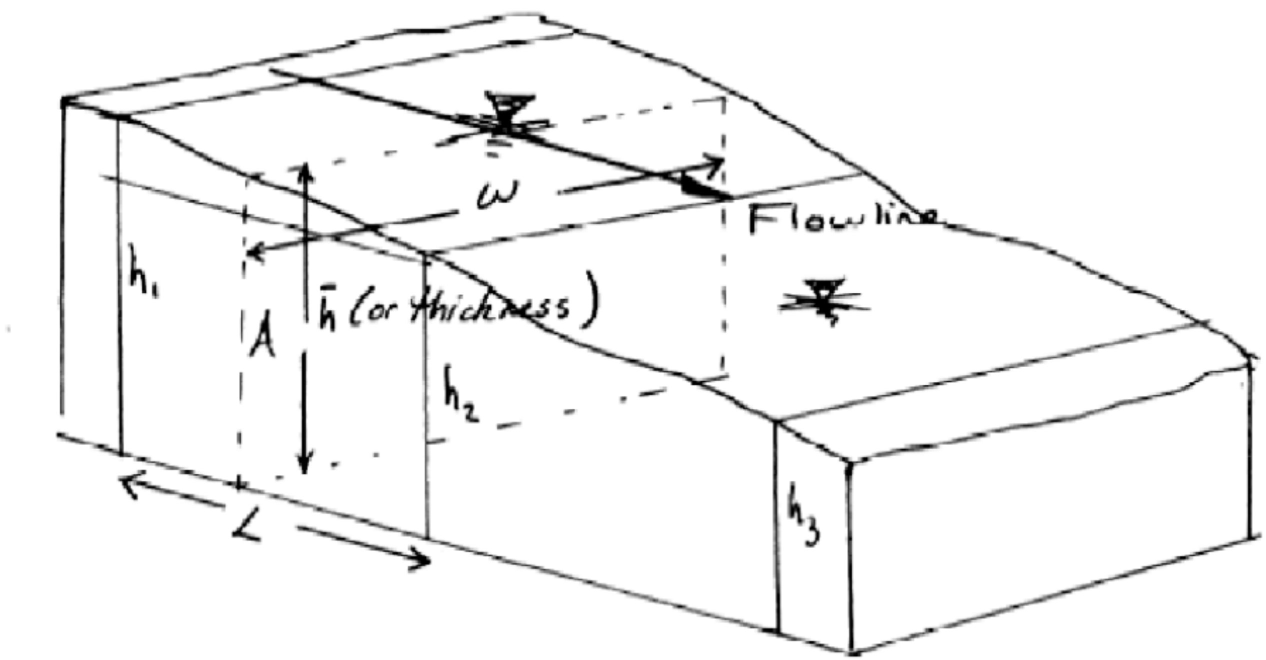


Figure 15: Aquifer 3D Solid

$$Q = Kw\bar{h}\frac{\Delta h}{\Delta L}$$

Direct Application of Darcy's Law

- Another example using a plan-view groundwater map is that the flow between any two flow lines is determined by the head change along the path of flow, the width between the flow lines, the thickness of the aquifer and the hydraulic conductivity

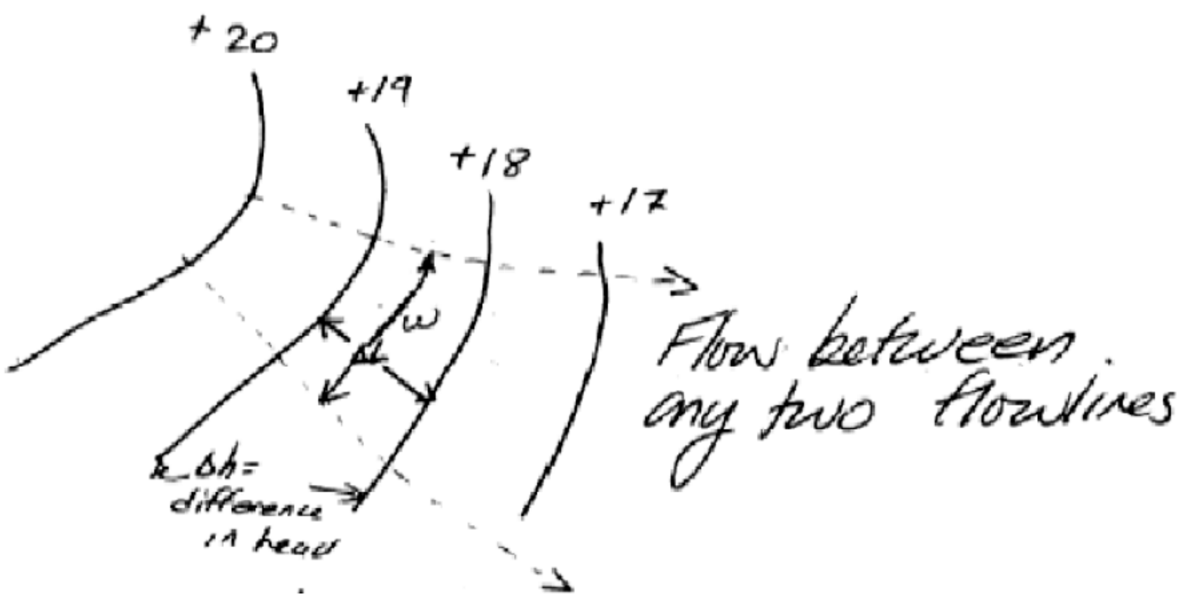


Figure 16: Aquifer Plan View

Confined-Rectilinear

- Darcy's Law

$$Q = -K\ b\ w\frac{\partial h}{\partial x}$$

- Rearrange into differential equation

$$\frac{\partial h}{\partial x} = -\frac{Q}{K\ b\ w}$$

- Integrate to recover the equation of head

$$h(x) = -\frac{Q}{K\ b\ w}x + h_0$$

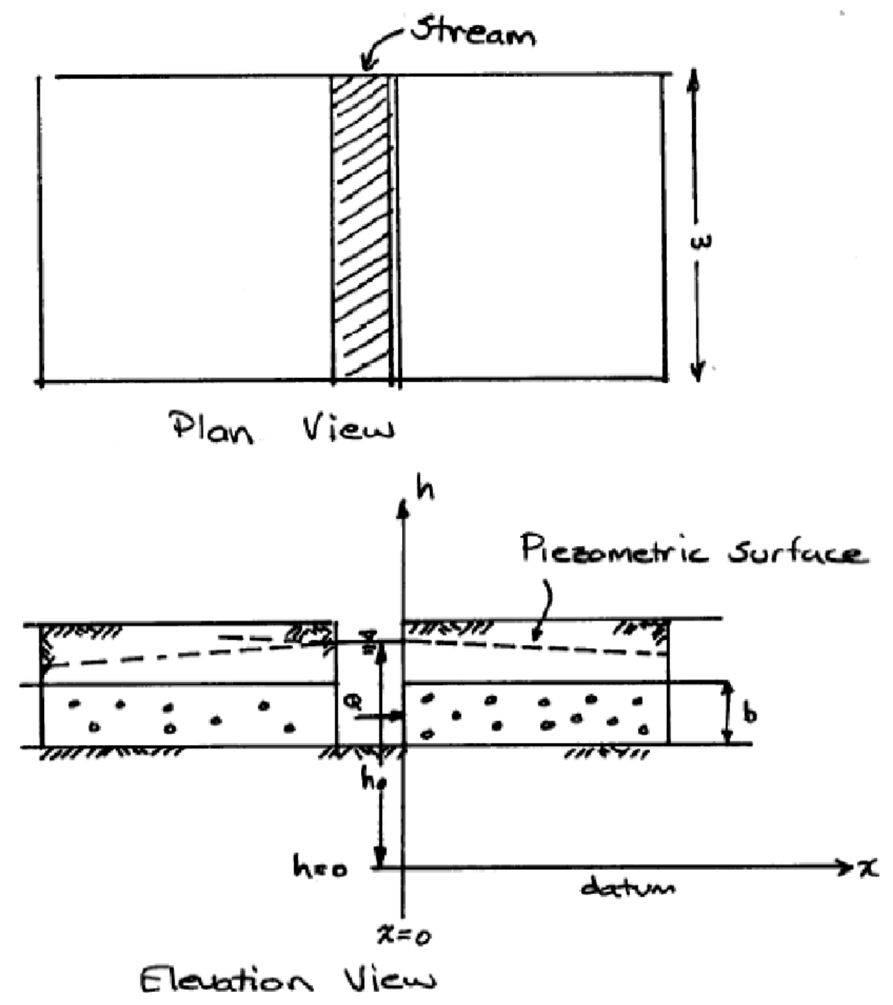


Figure 18: Steady flow in a confined aquifer

Unconfined – Rectilinear

- Darcy's Law

$$Q = -K\ h\ w\frac{\partial h}{\partial x}$$

- Rearrange into differential equation

$$h\frac{\partial h}{\partial x} = -\frac{Q}{K\ w}$$

- Recall some calculus

$$\frac{\partial h^2}{\partial x} = 2h\frac{\partial h}{\partial x}$$

$$2h\frac{\partial h}{\partial x} = -2\frac{Q}{K\ w} = \frac{\partial h^2}{\partial x}$$

$$h^2(x) = h_0^2 - \frac{2Q}{K\ w}x$$

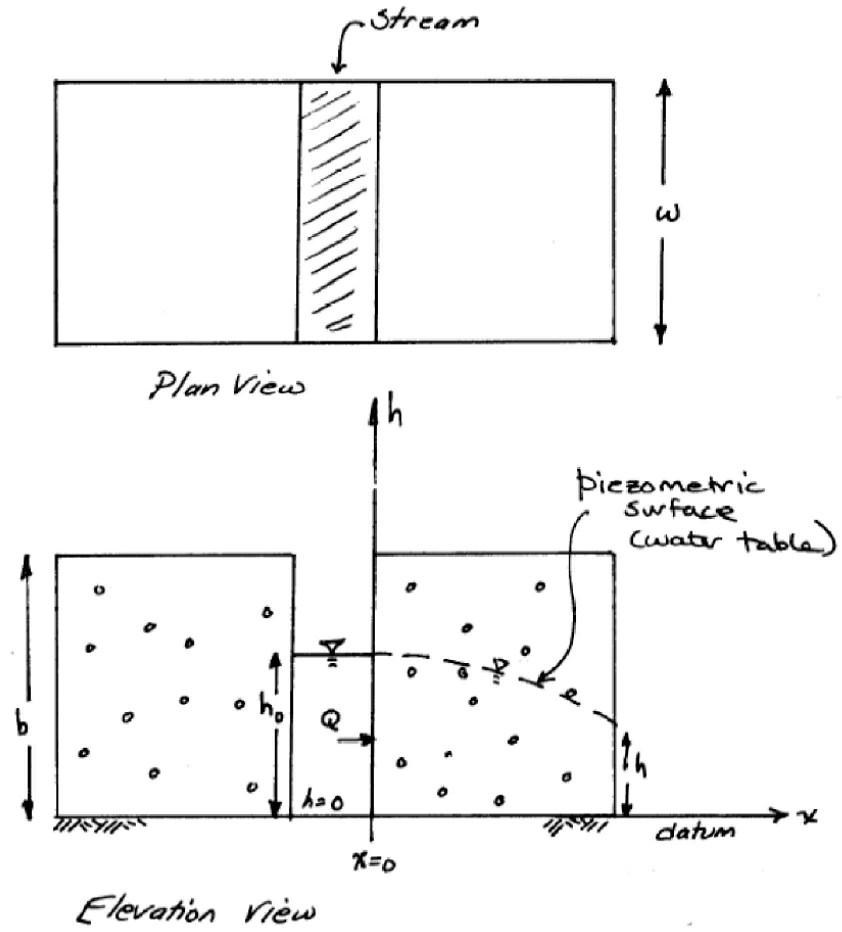
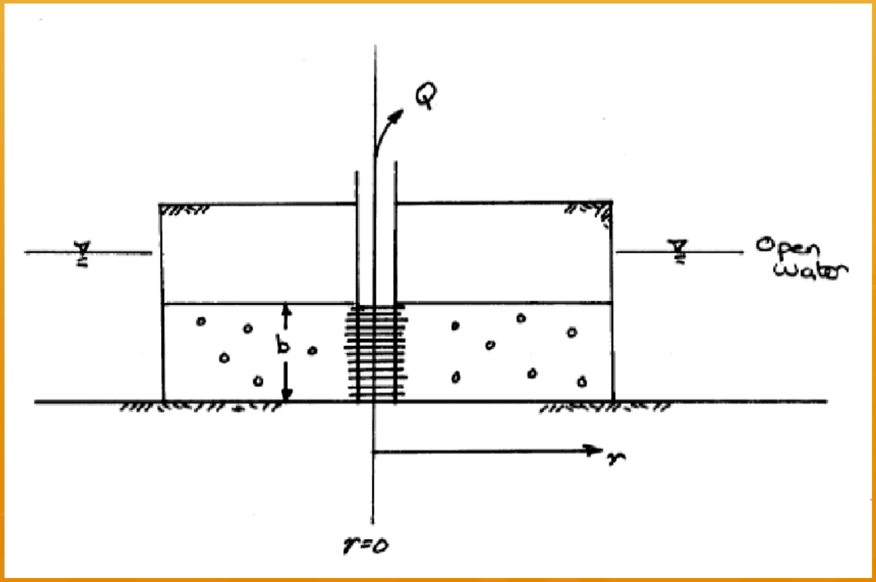
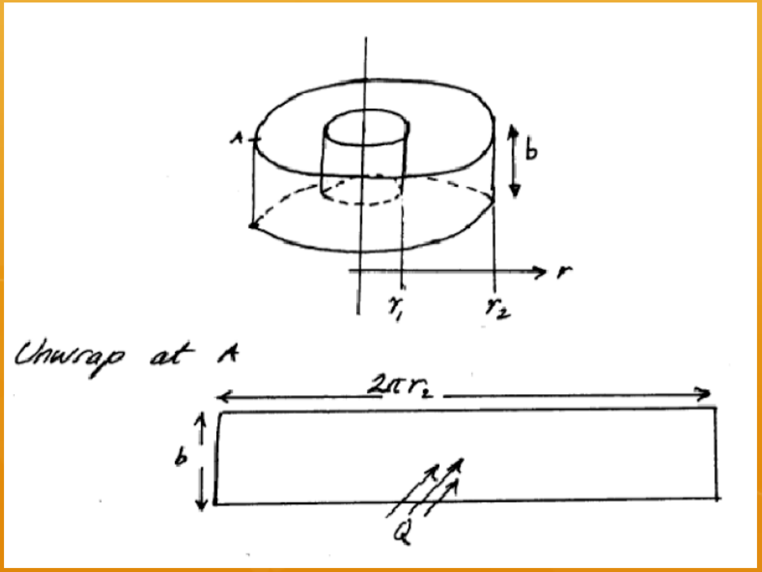


Figure 19: Steady flow in an un-confined aquifer

Confined – Cylindrical

- Now consider a circular, confined aquifer with a well in the center





Confined – Cylindrical

- Darcy's Law

$$-Q = -K2\pi rb\frac{\partial h}{\partial r}$$

- Calculus

$$\frac{\partial(ln(r))}{\partial r} = \frac{1}{r}$$

- Differential Equation

$$r\frac{\partial h}{\partial r} = \frac{Q}{K2\pi b}$$

$$\frac{\partial h}{\partial r} = \frac{\partial h}{\partial ln(r)}\frac{\partial ln(r)}{\partial r} = \frac{1}{r}\frac{\partial h}{\partial ln(r)}$$

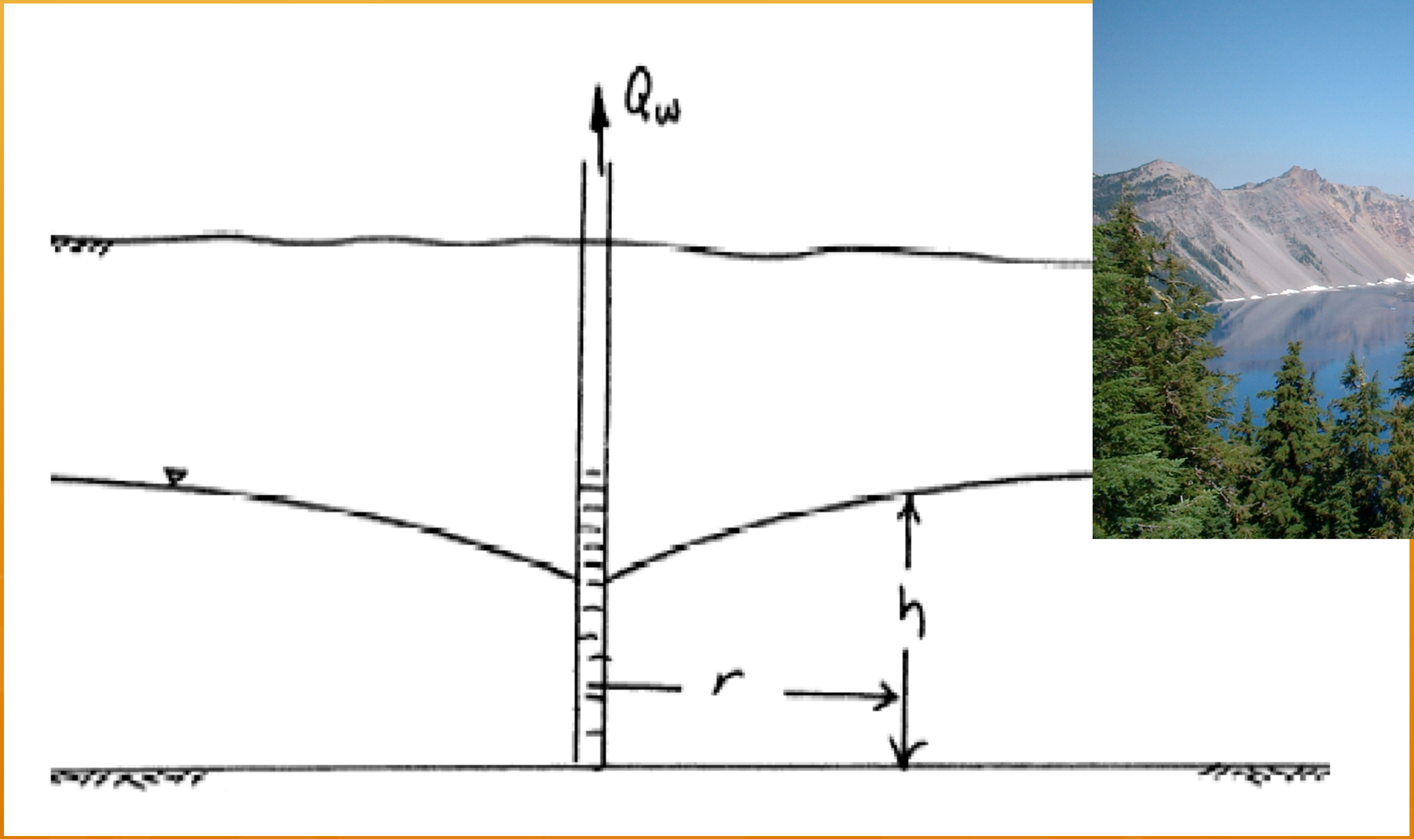
$$\frac{\partial h}{\partial ln(r)} = \frac{Q}{K2\pi b}$$

- Integrate

$$h_2 = h_1 + \frac{Q}{K2\pi b}(ln(r_2) - ln(r_1)) = h_1 + \frac{Q}{K2\pi b}ln(\frac{r_2}{r_1})$$

Unconfined – Cylindrical

- Unconfined aquifer on circular island



Unconfined – Cylindrical

- Unconfined aquifer on circular island

Darcys law for this cylinder is

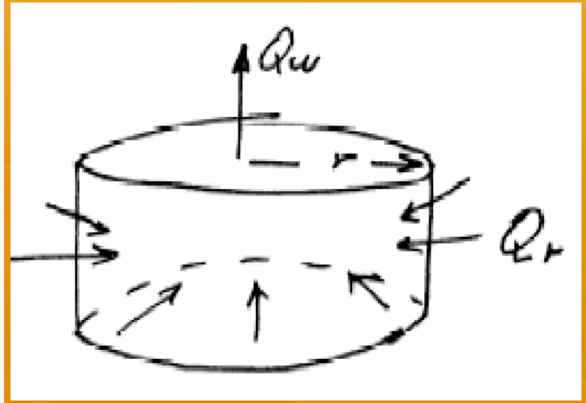
$$-Q = -K2\pi rh\frac{\partial h}{\partial r}$$

Rearrange into a differential equation

$$rh\frac{\partial h}{\partial r} = \frac{-Q}{-K2\pi}$$

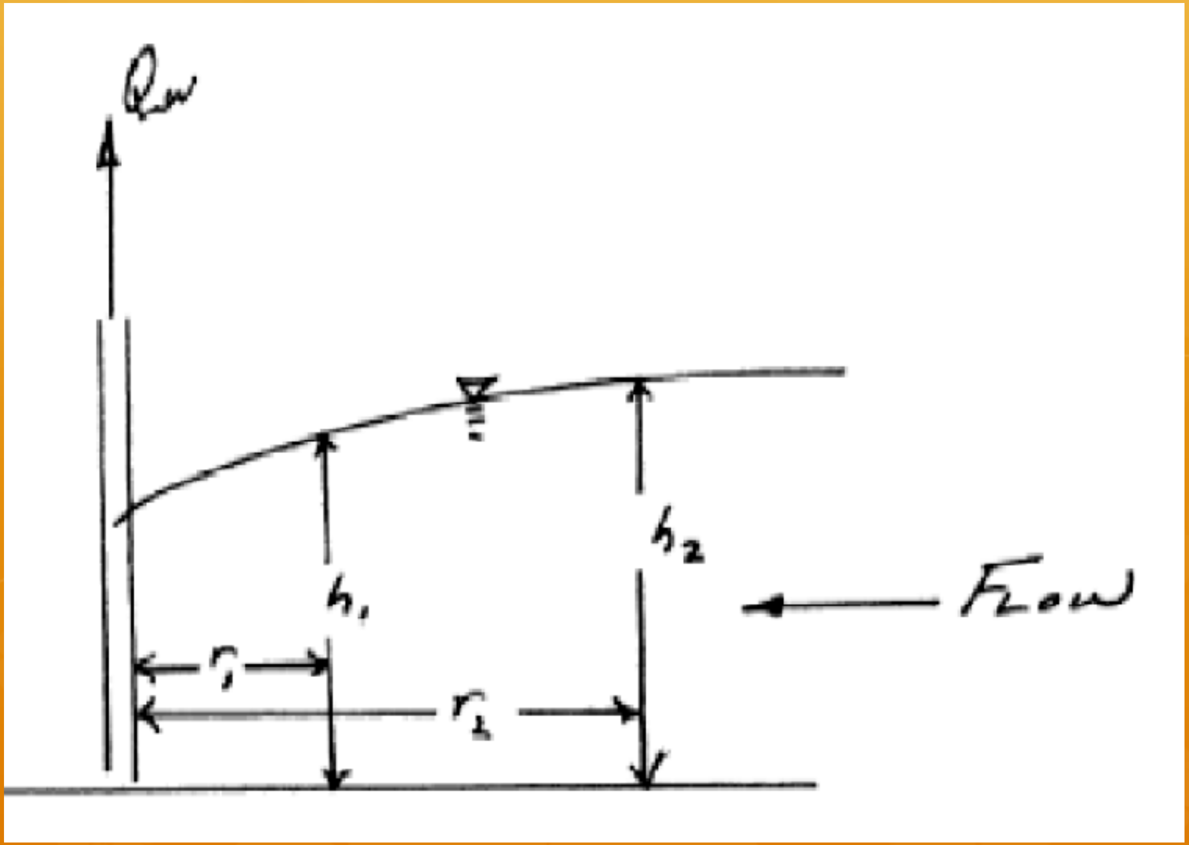
Make following substitutions

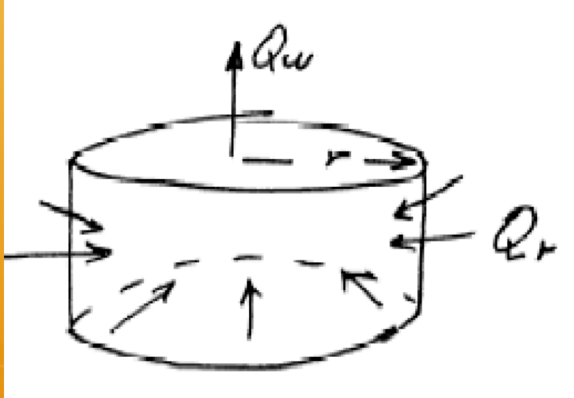
$$h\partial h = \frac{1}{2}\partial h^2$$



Unconfined – Cylindrical

- Unconfined aquifer on circular island





$$h_2^2 - h_1^2 = \frac{Q}{K\pi} ln(\frac{r_2}{r_1})$$

Transmissivity

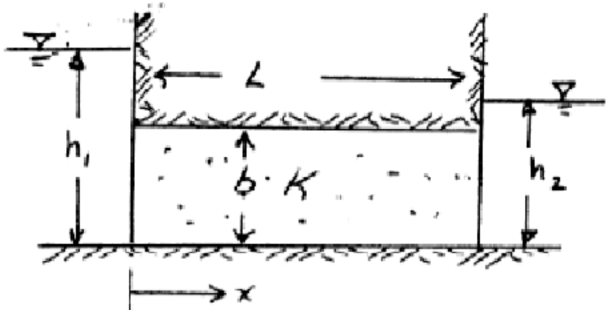
- Transmissivity is the term that refers to the amount of water that will flow through an entire thickness of aquifer

$$\frac{Q}{w} = -Kb\frac{\partial h}{\partial x}$$

- The term Kb is the transmissivity it represents the discharge per unit width under unit gradient through an aquifer

- The usual symbol is T

Flow Between Two Ditches



Steady flow, confined aquifer connecting two reservoirs

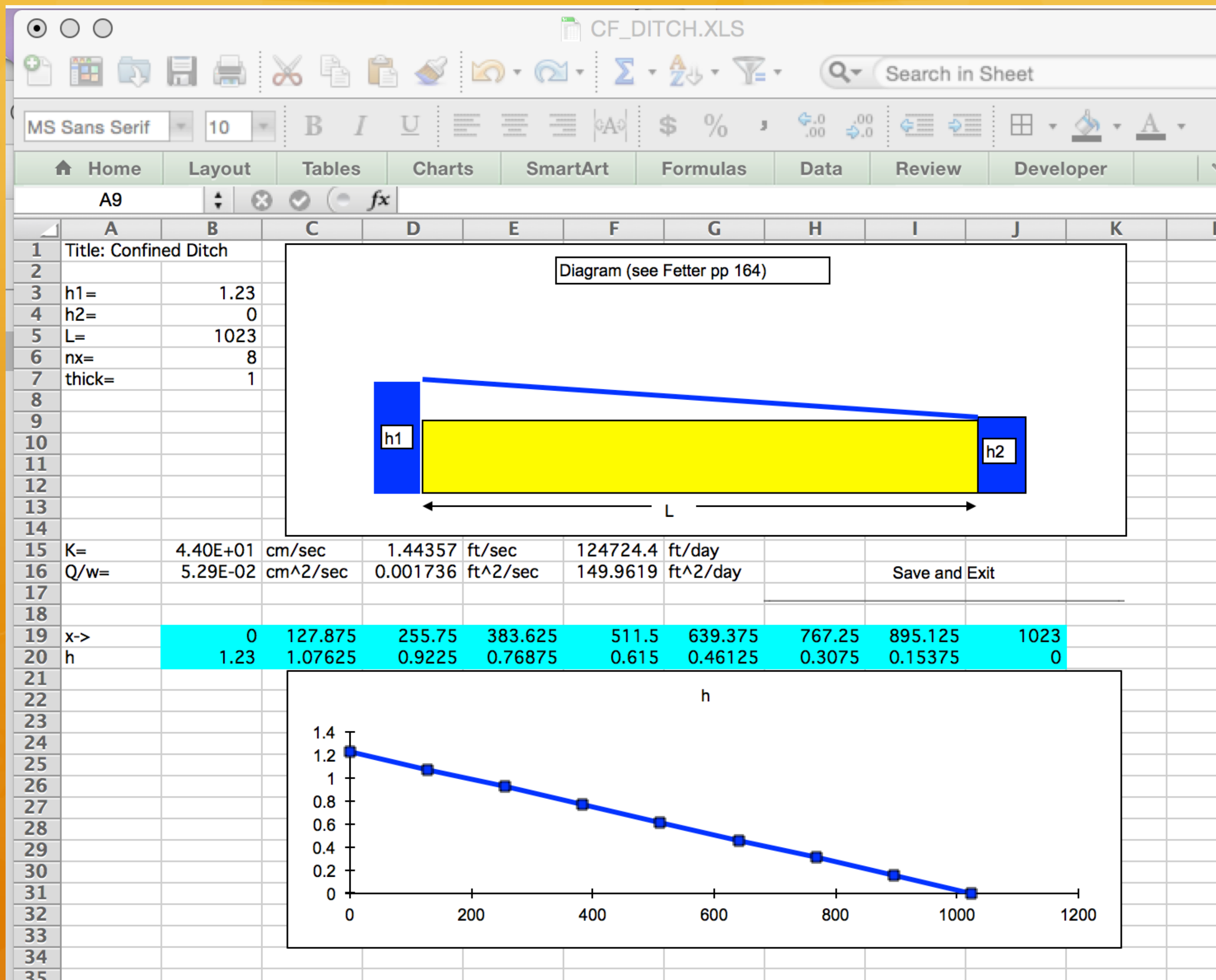
$$Q = KA\frac{\Delta h}{\Delta l} = -KA\frac{dh}{dx}$$

$$A = b \cdot w$$

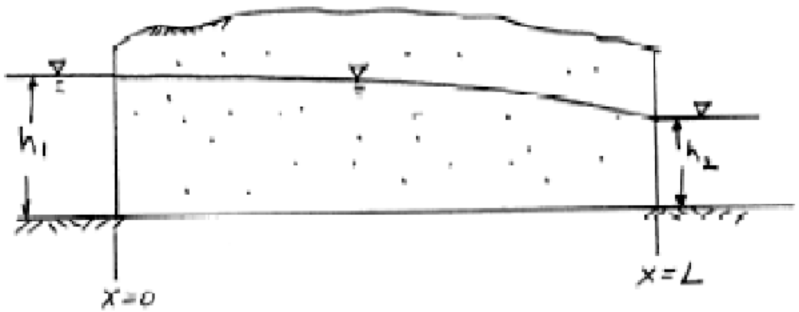
$$\frac{dh}{dx} = \frac{h_2 - h_1}{L}$$

$$Q = -K b w \frac{h_2 - h_1}{L}$$

Flow Between Two Ditches



Flow Between Two Ditches



$$Q = -KA\frac{\partial h}{\partial x} \quad \text{(Darcy's Law)}$$

$$Q = -Khw\frac{\partial h}{\partial x} \quad \text{(Dupuit assumptions)}$$

but

$$h\frac{\partial h}{\partial x} = \frac{1}{2}\frac{\partial h^2}{\partial x}$$

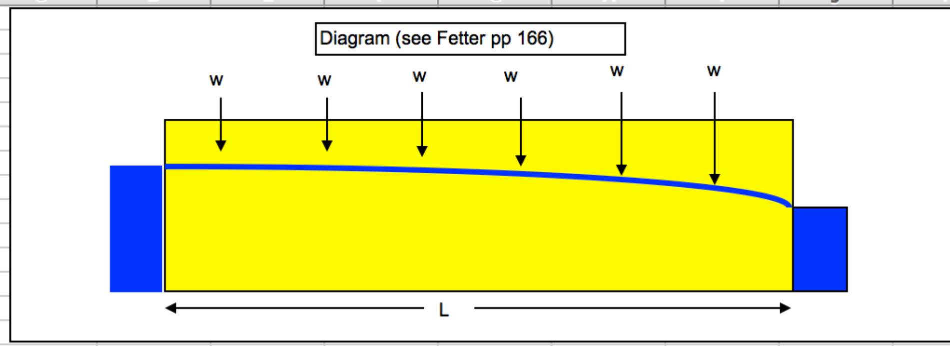
$$\therefore \quad Q = -\frac{Kw}{2}\frac{\partial h^2}{\partial x} \quad \text{or}$$

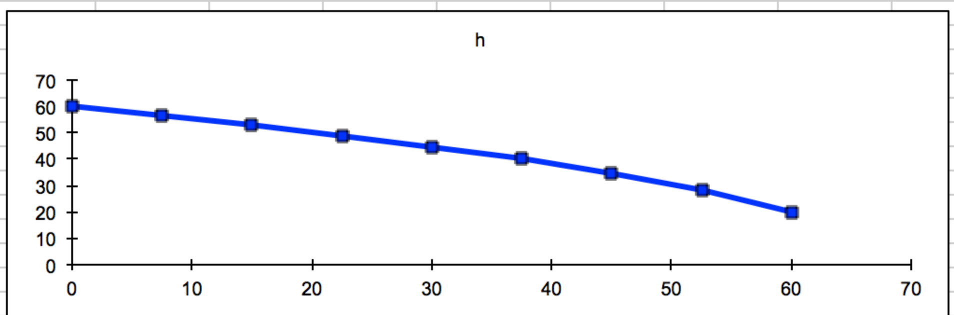
$$Q = \frac{Kw}{2}\frac{h_1^2 - h_2^2}{L}$$

Flow Between Two Ditches

UN_DITCH.xlsx

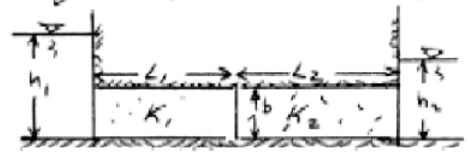
	A	B	C	D	E	F	G	H	I	J
1	Title: Unconfined Ditch									
3	h1=	60								
4	h2=	20								
5	L=	60								
6	nx=	8								
7	width=	100								
9	Flows valid when									
10	w=0									
11	Q(cfs)=	0.148731								
12	Q(ctd)=	12850.39								
15	K=	1.70E-03	cm/sec	5.58E-05	ft/sec	4.818898	ft/day		Save and Exit	
17	w	0	0	0	0	0	0	0	0	0
19	x->	0	7.5	15	22.5	30	37.5	45	52.5	60
20	h	60	56.56854	52.91503	48.98979	44.72136	40	34.64102	28.28427	20
22	h^2	3600	3200	2800	2400	2000	1600	1200	800	400



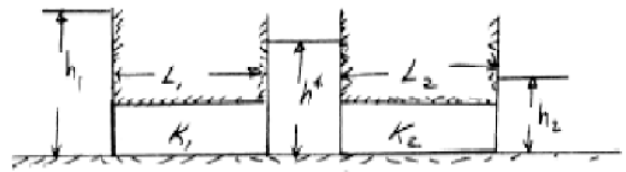


- Different material properties
- Use hydraulics principles (head loss) to find average K (or T)

Steady flow, confined aquifer comprised of two different geologic media.



Typically we want to know K_{mean}, the apparent mean hydraulic conductivity



$$Q_1 = K_1 A \frac{h_1 - h^*}{L_1} \qquad Q_2 = K_2 A \frac{h^* - h_2}{L_2} \qquad Q_T = \bar{K} A \frac{h_1 - h_2}{L_1 + L_2}$$

$$Q_1 = Q_2 = Q_T$$

$$\therefore \quad h_1 - h^* = \frac{Q_T L_1}{K_1 A} \qquad h^* - h_2 = \frac{Q_T L_2}{K_2 A} \qquad h_1 - h_2 = \frac{Q_T (L_1 + L_2)}{\bar{K} A}$$

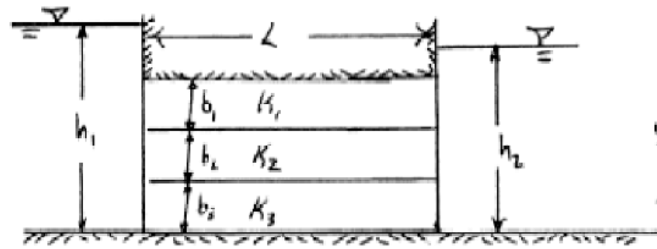
but $h_1 - h_2 = h_1 - h^* + h^* - h_2$

$$\therefore \frac{Q_T (L_1 + L_2)}{\bar{K} A} = \frac{Q_T L_1}{K_1 A} + \frac{Q_T L_2}{K_2 A}$$

$$\frac{\bar{K}}{(L_1 + L_2)} = \frac{1}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} \Rightarrow \bar{K} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

- Different material properties
- Use hydraulics principles (head loss) to find average K (or T)

Steady flow, confined aquifer, comprised of several layers of geologic media



Again we wish to know K_{mean}, the apparent hydraulic conductivity

Each layer is exposed to same gradient

$$\therefore \quad Q_1 = K_1 b_1 w \frac{h_1 - h_2}{L}$$

$$Q_2 = K_2 b_2 w \frac{h_1 - h_2}{L}$$

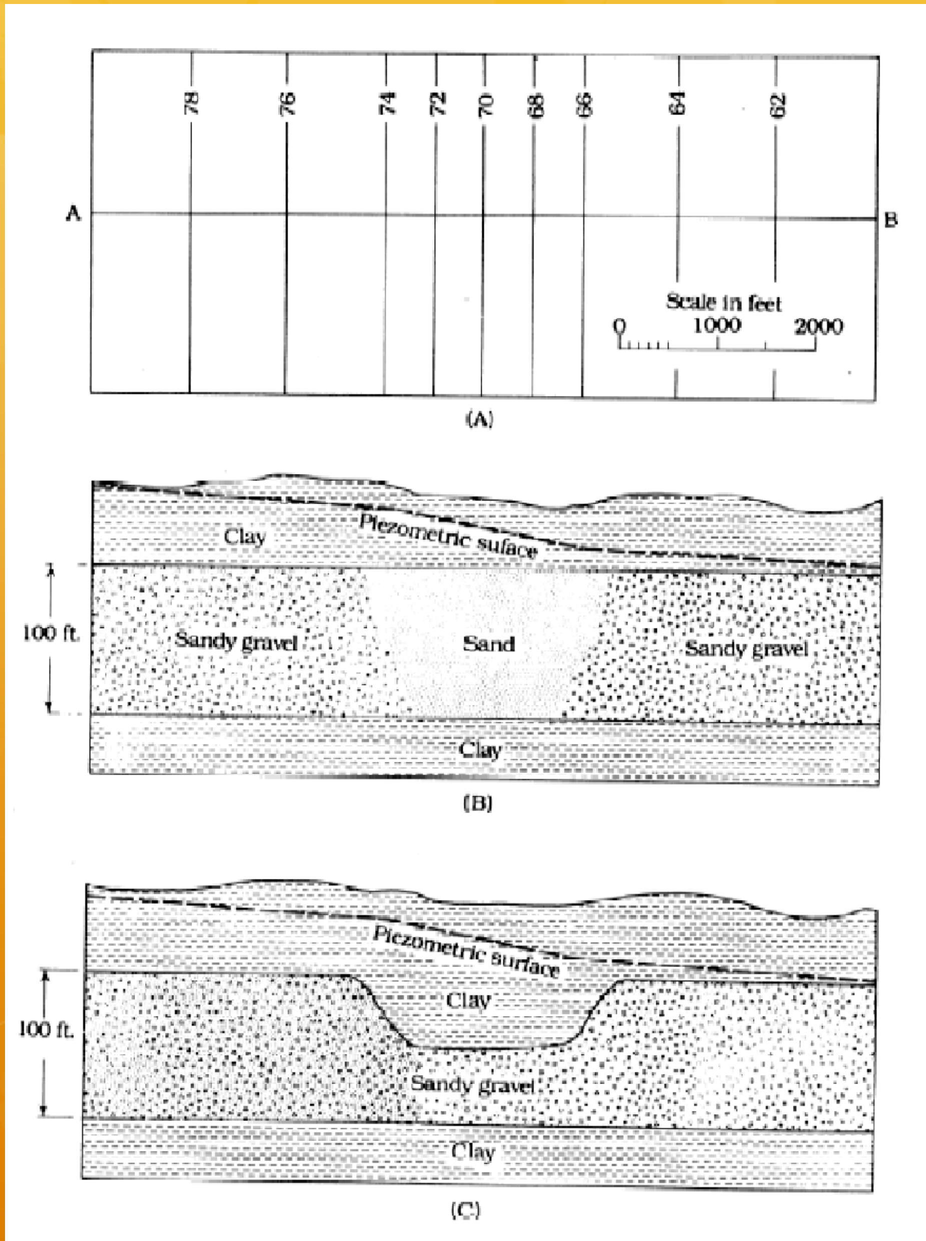
$$Q_3 = K_3 b_3 w \frac{h_1 - h_2}{L}$$

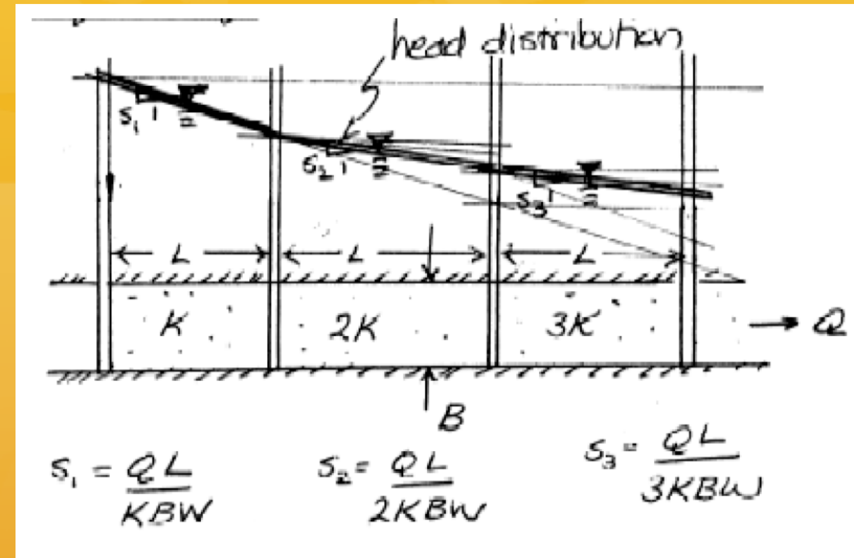
$$Q_T = \bar{K}(b_1 + b_2 + b_3) w \frac{h_1 - h_2}{L} = Q_1 + Q_2 + Q_3$$

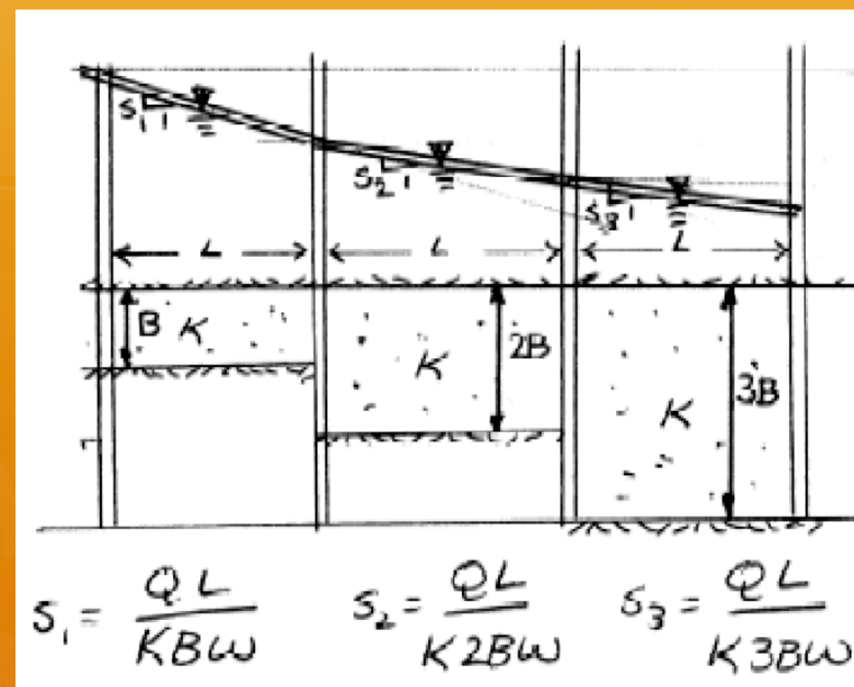
$$\therefore \bar{K}(b_1 + b_2 + b_3) w \frac{h_1 - h_2}{L} = (K_1 b_1 w + K_2 b_2 w + K_3 b_3 w) * \frac{h_1 - h_2}{L}$$

So

$$\bar{K} = \frac{K_1 b_1 + K_2 b_2 + K_3 b_3}{b_1 + b_2 + b_3}$$


78
76
74
72
70
68
66
64
62
A
B
Scale in feet
0
1000
2000
(A)
Clay
Piezometric suface
100 ft.
Sandy gravel
Sand
Sandy gravel
Clay
(B)
Piezometric surface
Clay
100 ft.
Sandy gravel
Clay
(C)


head distribution
L
L
L
K
2K
3K
Q
B
S1 = QL / KBW
S2 = QL / 2KBW
S3 = QL / 3KBW


L
L
L
B
K
2B
K
3B
K
S1 = QL / KBW
S2 = QL / K2BW
S3 = QL / K3BW

Summary

- Porosity, Specific Yield
- Storage
- Darcy’s Law – Hydraulic Conductivity
- Gradients and head
- Some steady flow solutions – use geometry, Darcy’s law, and calculus

Next Time

- Transient (time-varying) solutions
- Superposition of solutions
- Well hydraulics