CE 3354 Engineering Hydrology

Lecture 18: Channel Routing

Outline

- Level Pool Routing applied to a stream reach
  - Example
- Muskingum Routing Background
  - CMM pp. 257-260
- Muskingum-Cunge Routing applied to a stream reach
  - CMM pp. 302-304

Routing

- Routing simulates movement of a discharge signal (flood wave) through reaches
  - Accounts for storage in the reach and flow resistance.
  - Allows modeling of a basin comprised of interconnected sub-basins
  - Hydraulic routing – uses continuity and momentum (St. Venant Equations)
  - Hydrologic routing – uses continuity equation

Hydrologic Routing

- Hydrologic routing techniques use the equation of continuity and some linear or curvilinear relation between storage and discharge within the river.
- Methods include:
  - Lag Routing (no attenuation)
  - Modified Puls (level pool routing)
  - Muskingum-Cunge (almost a hydraulic model)

Level Pool Routing

- Technique to approximate the outflow hydrograph passing through a reach with the pool (water surface) assumed always level.
- Uses a reach (reservoir) mass balance equation, and

$$Q_{\mathrm{in}} - Q_{\mathrm{out}} = \frac{\Delta S}{\Delta t}$$

- a storage-outflow relationship.

$$Q_{\mathrm{out}} = f(S)$$

Level Pool Routing

- Variable names are typically changed:

$$Q_{\text{in}} \Rightarrow I_t$$

$$Q_{out} \Rightarrow O_t$$

- So the reach mass balance is

$$\bar{I} - \bar{O} = \frac{\Delta S}{\Delta t}$$

Level Pool Routing

- The time averaged values are taken at the beginning and end of the time interval, and the first-order difference quotient is used to approximate the rate of change in storage.
- The reach mass balance is then

$$\frac{I_t + I_{t-\Delta t}}{2} - \frac{O_t + O_{t-\Delta t}}{2} = \frac{S_t - S_{t-\Delta t}}{\Delta t}$$

Level Pool Routing

- Use stream-reach hydraulics, and depth-area-storage to build a storage-outflow function

$$O = f(S)$$

- Once we have that function, then build an auxiliary function (tabulation) called the storage-indication curve (function)

$$O = g(\frac{2S}{\Delta t} + O)$$

Level Pool Routing

- Once have the storage-indication curve then can use the reach mass balance to estimate the numerical value of :

$$\frac{2S_t}{\Delta t} + O_t$$

- Then use the storage-indication curve to find the value of outflow, subtract than from the result above, and now have both the end-of-interval outflow and storage.

Level Pool Routing

- Terminology: In Level Pool, X=0

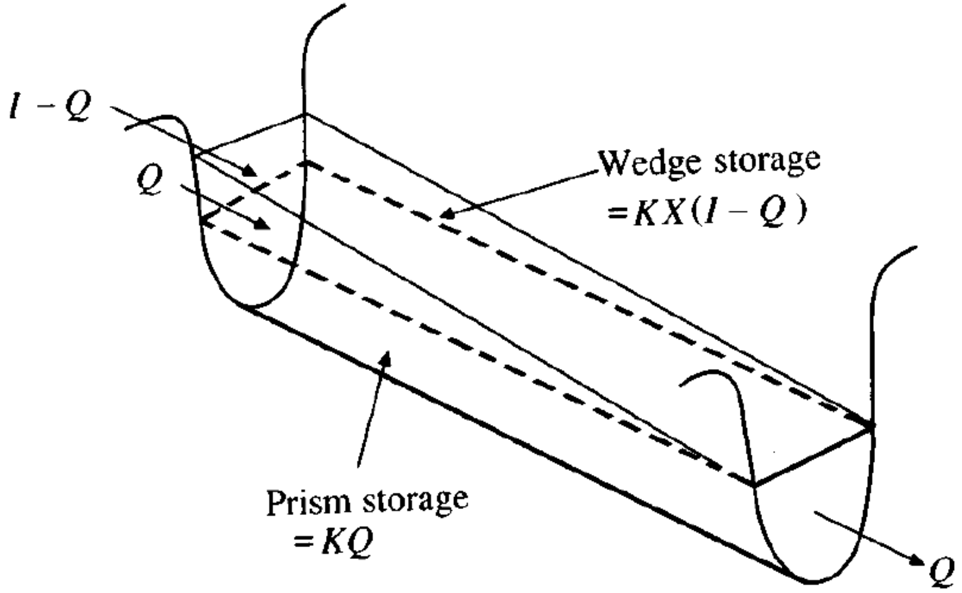
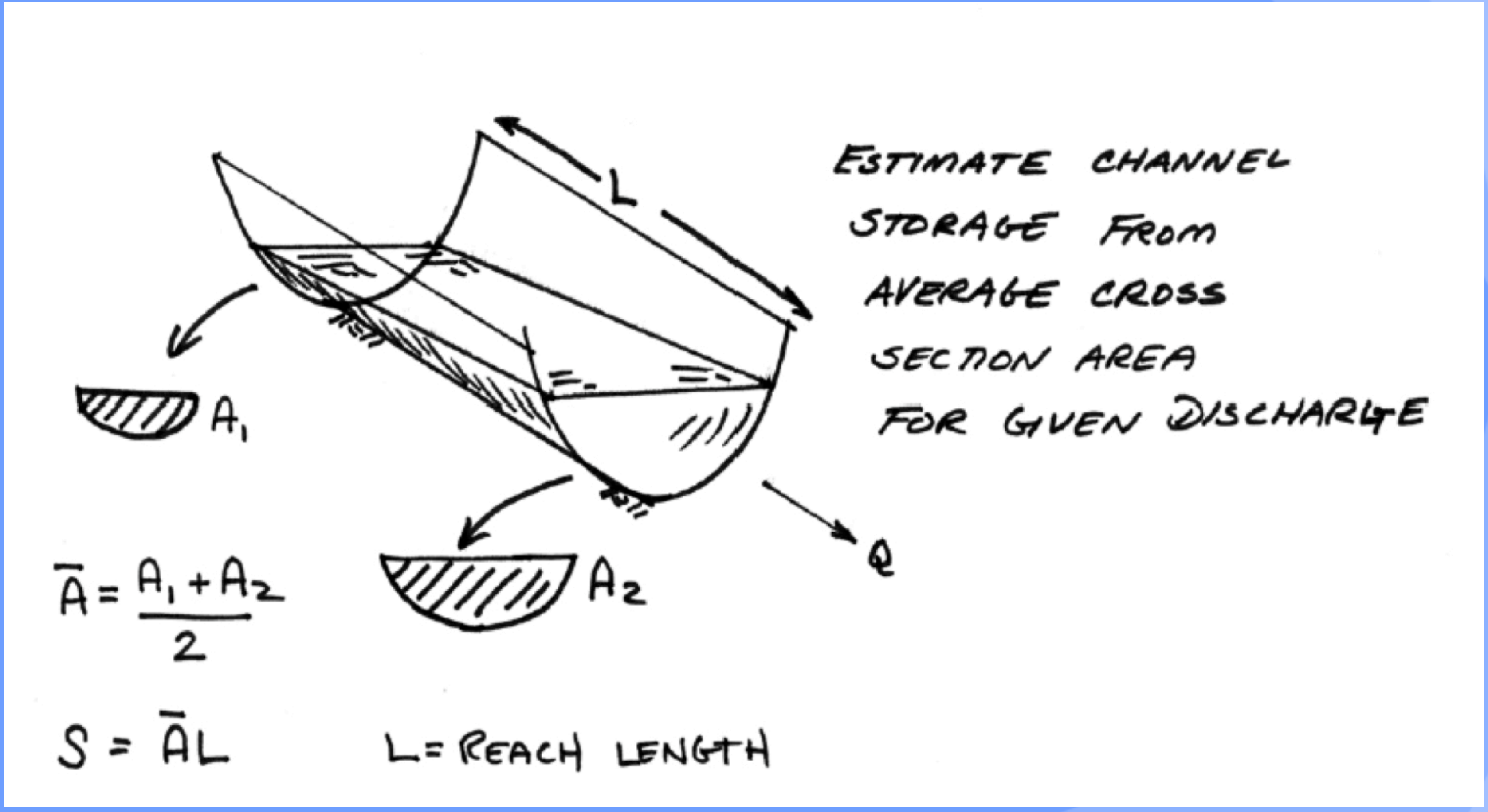


FIGURE 8.4.1
Prism and wedge storages in a channel reach.

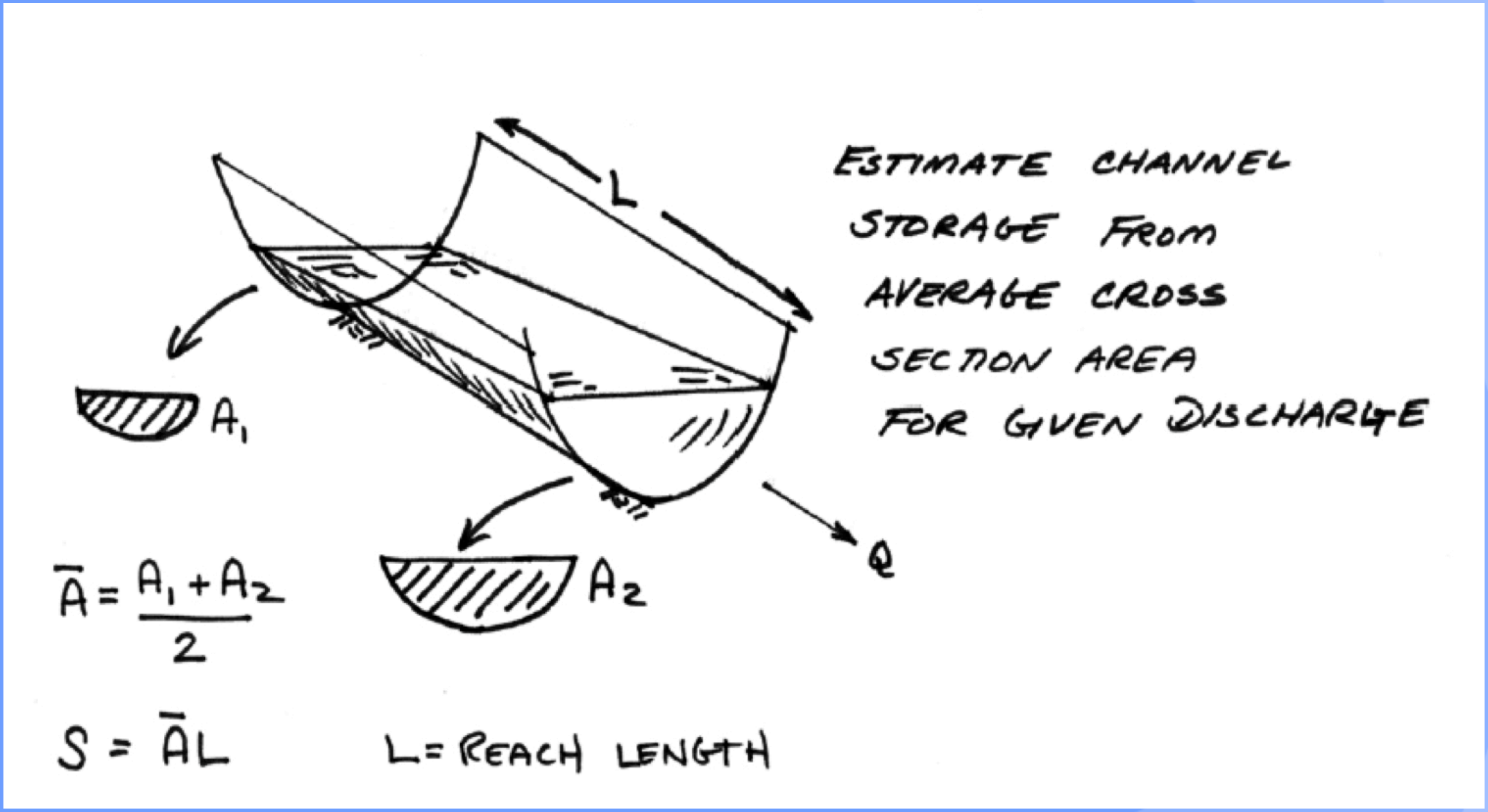
Channel Routing

- The storage in a reach can be estimated as the product of the average cross sectional area for a given discharge rate and the reach length.



Channel Routing

- A rating equation is used at each cross section to determine the cross section areas.



Approximating Ratings

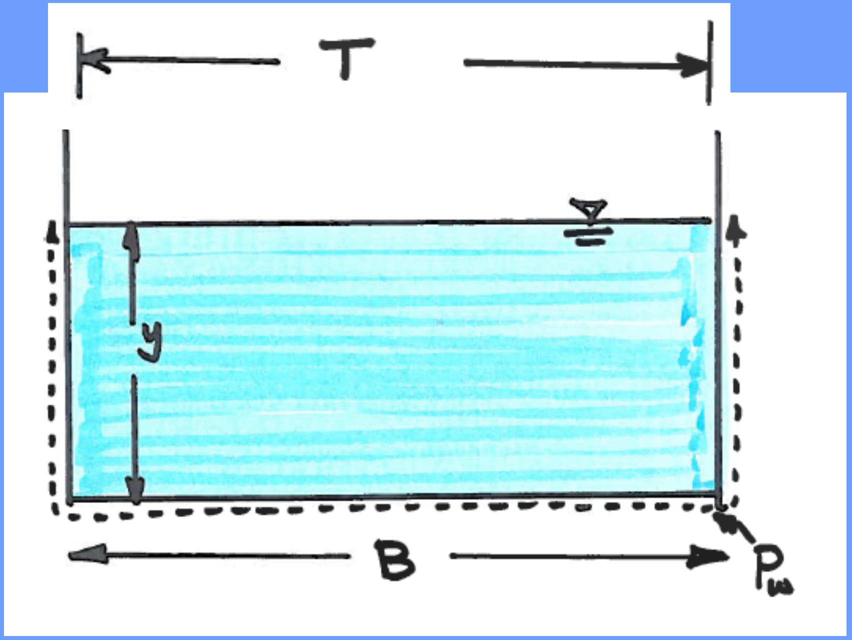
- Assume normal flow at each channel end section

$$Q = \frac{1.49}{n} A R^{2/3} S_0^{1/2}$$

- Use geometry to find values for A, and R.
- Engineered cross sections almost exclusively use just a handful of convenient geometry (rectangular, trapezoidal, triangular, and circular).
- Natural cross sections are handled in similar fashion as engineered, except numerical integration is used for the depth-area, topwidth-area, and perimeter-area computations.

Cross Section Geometry

- Rectangular Channel



- Depth-Area

$$A(y) = By$$

- Depth-Topwidth

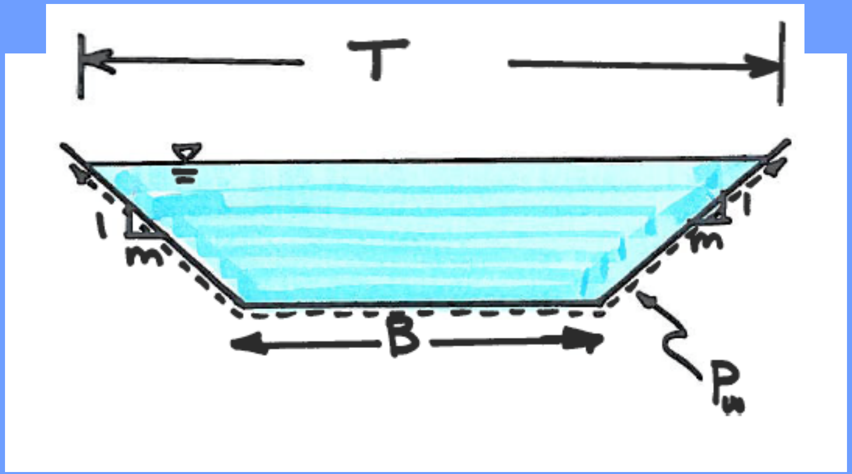
$$T(y) = B$$

- Depth-Perimeter

$$P_w(y) = B + 2y$$

Cross Section Geometry

- Trapezoidal Channel



- Depth-Area

$$A(y) = y(B + my)$$

- Depth-Topwidth

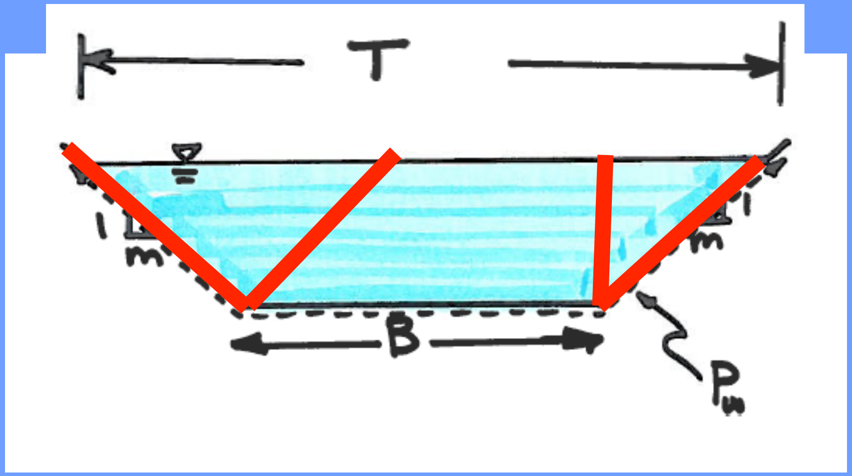
$$T(y) = B + 2my$$

- Depth-Perimeter

$$P_w(y) = B + 2y\sqrt{1 + m^2}$$

Cross Section Geometry

- Triangular Channels
  - Special cases of trapezoidal channel
- V-shape; set B=0
- J-shape; set B=0, use ½ area, topwidth, and perimeter



Cross Section Geometry

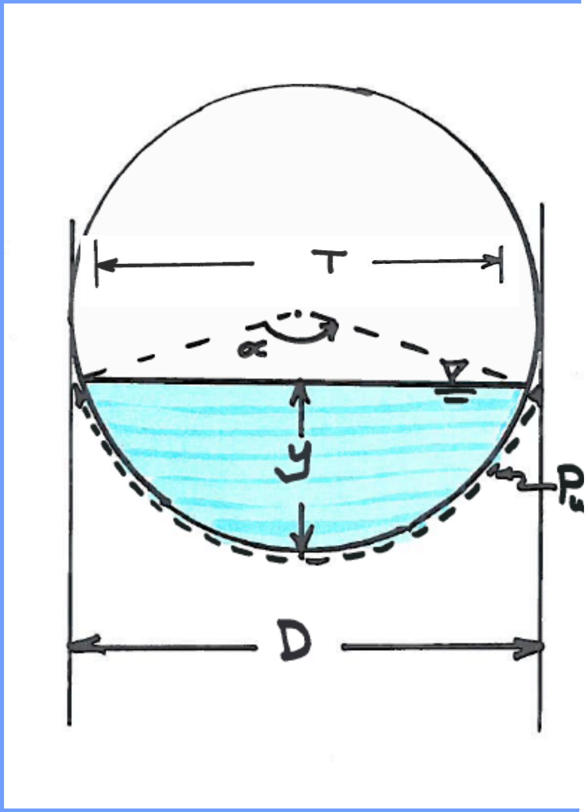
- Circular Channel (Conduit with Free-Surface)

- Contact Angle: $$\alpha(y) = 2\cos^{-1}(1 - \frac{2y}{D})$$

- Depth-Area: $$A(y) = \frac{D^2}{4}\left(\frac{\alpha}{2} - \sin(\frac{\alpha}{2})\cos(\frac{\alpha}{2})\right)$$

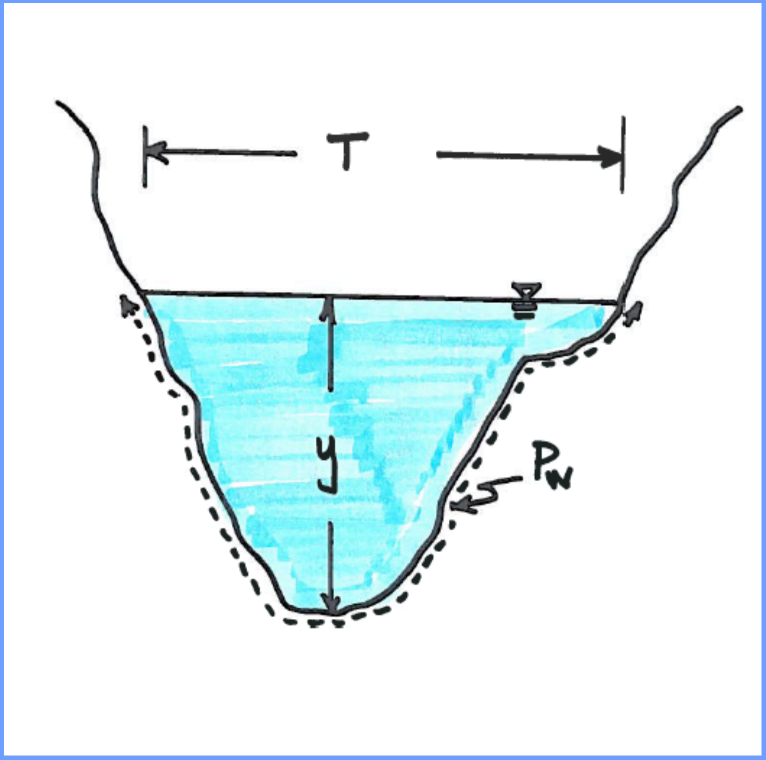
- Depth-Topwidth: $$T(y) = D\sin(\frac{\alpha}{2})$$

- Depth-Perimeter: $$P_w(y) = \frac{D\alpha}{2}$$



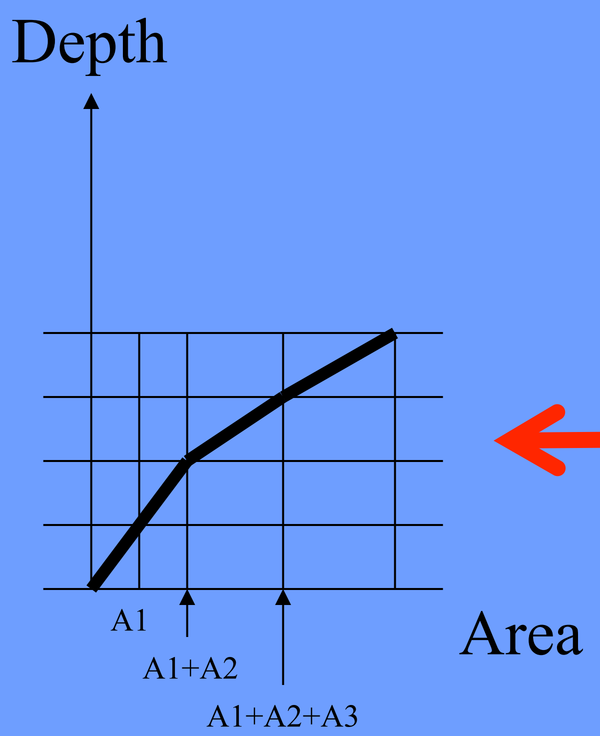
Cross Section Geometry

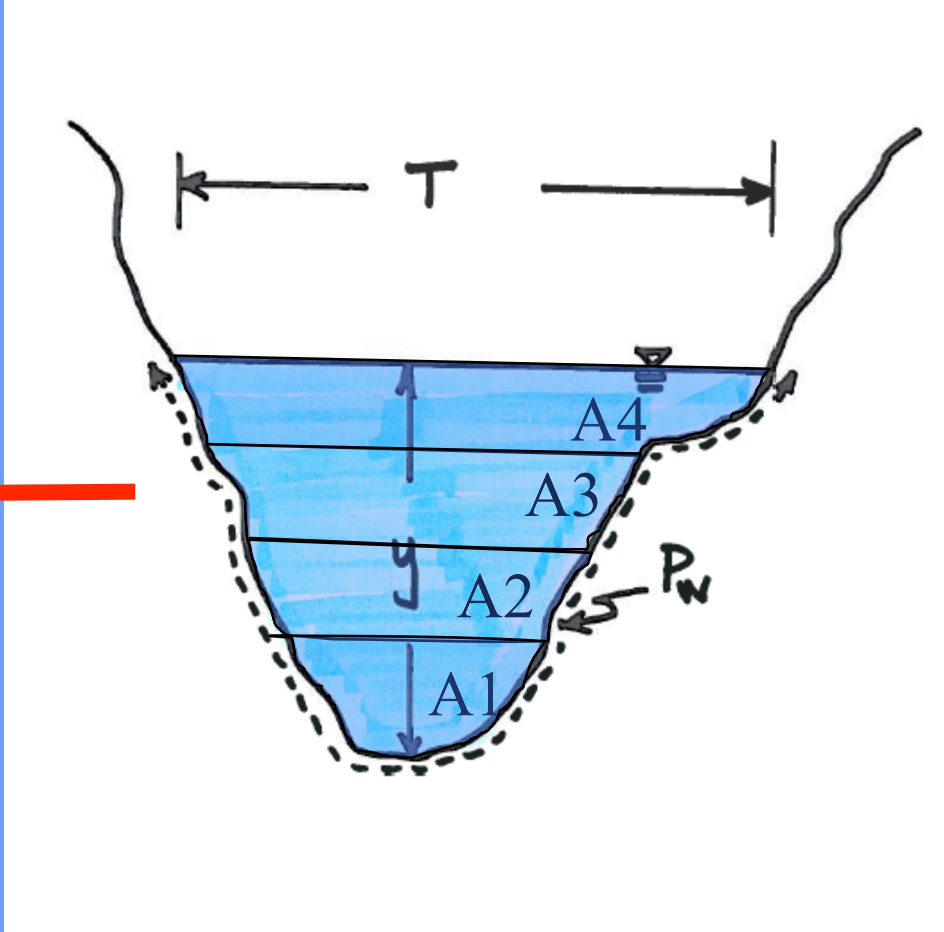
- Irregular Cross Section
  - Use tabulations for the hydraulic calculations



Cross Section Geometry

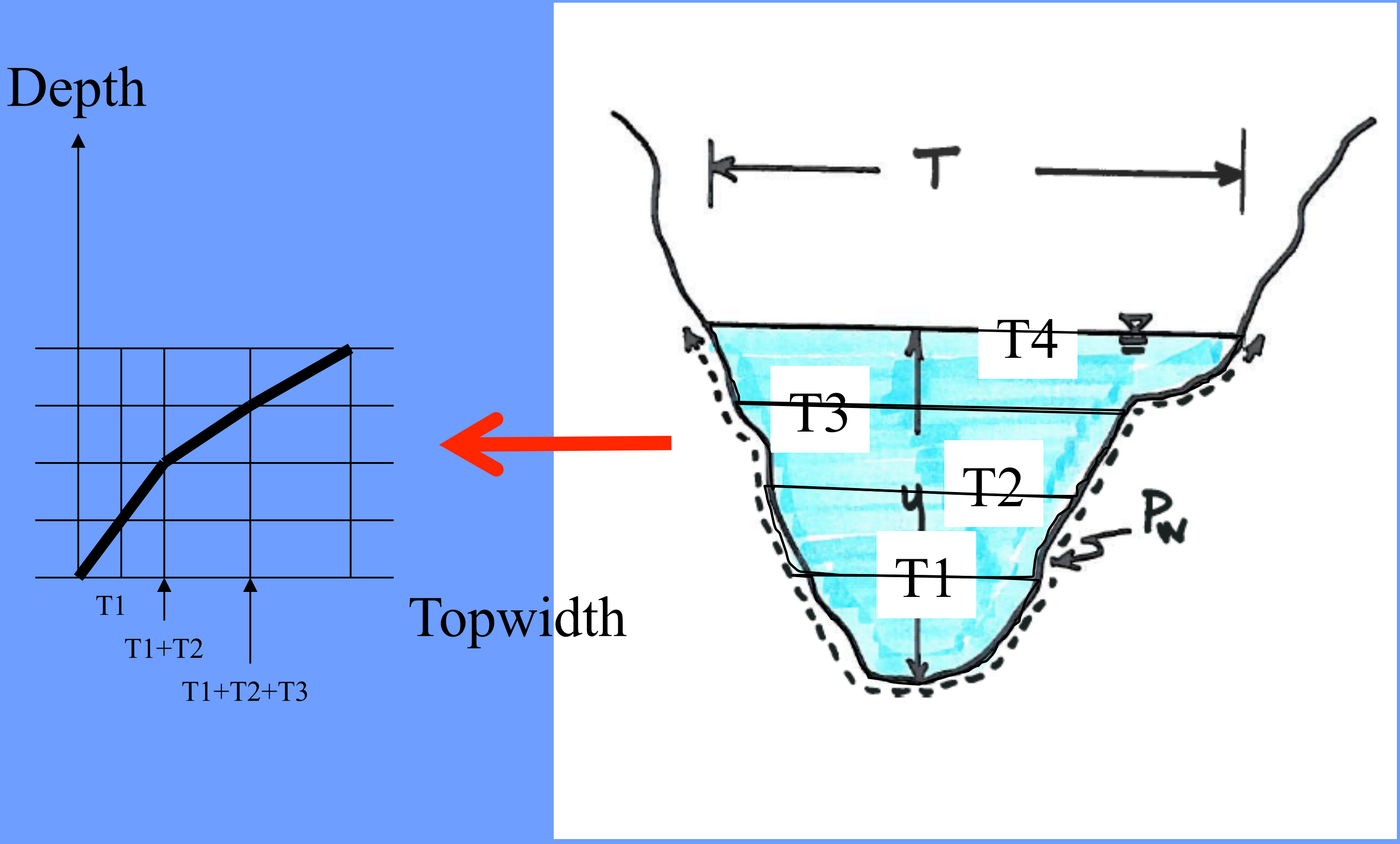
- Irregular Cross Section – Depth-Area





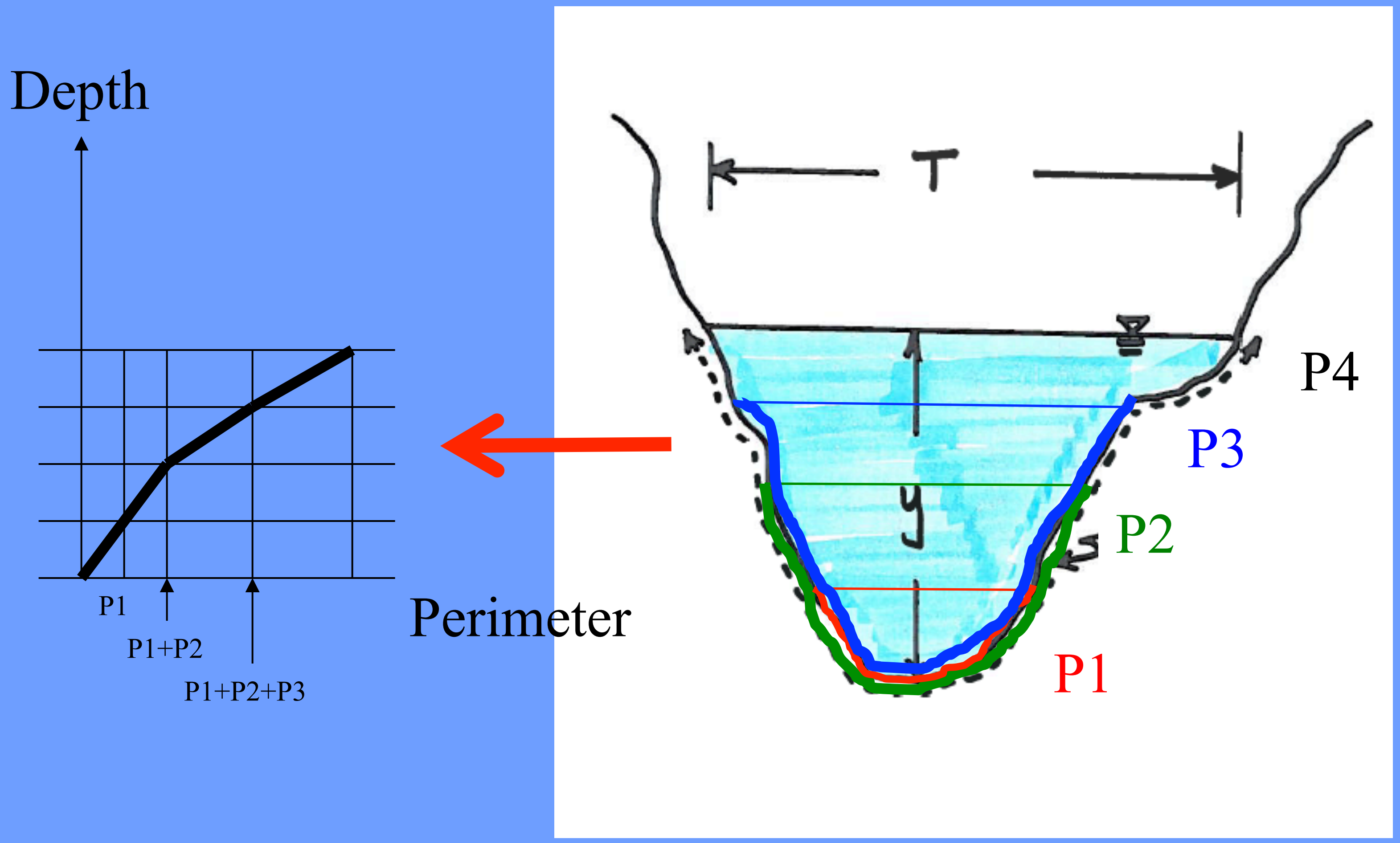
Cross Section Geometry

- Irregular Cross Section – Depth-Area



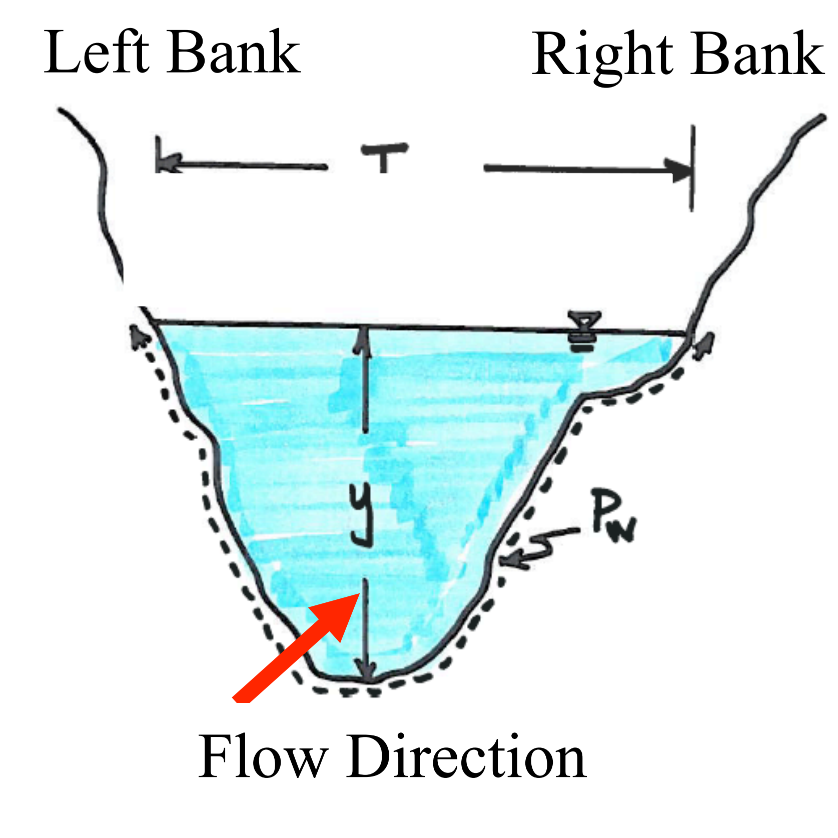
Cross Section Geometry

- Irregular Cross Section – Depth-Perimeter



Flow Direction/ Cross Section Geometry

- Convention is to express station along a section with respect to “looking downstream”
  - Left bank is left side of stream looking downstream (into the diagram)
  - Right bank is right side of stream looking downstream (into the diagram)

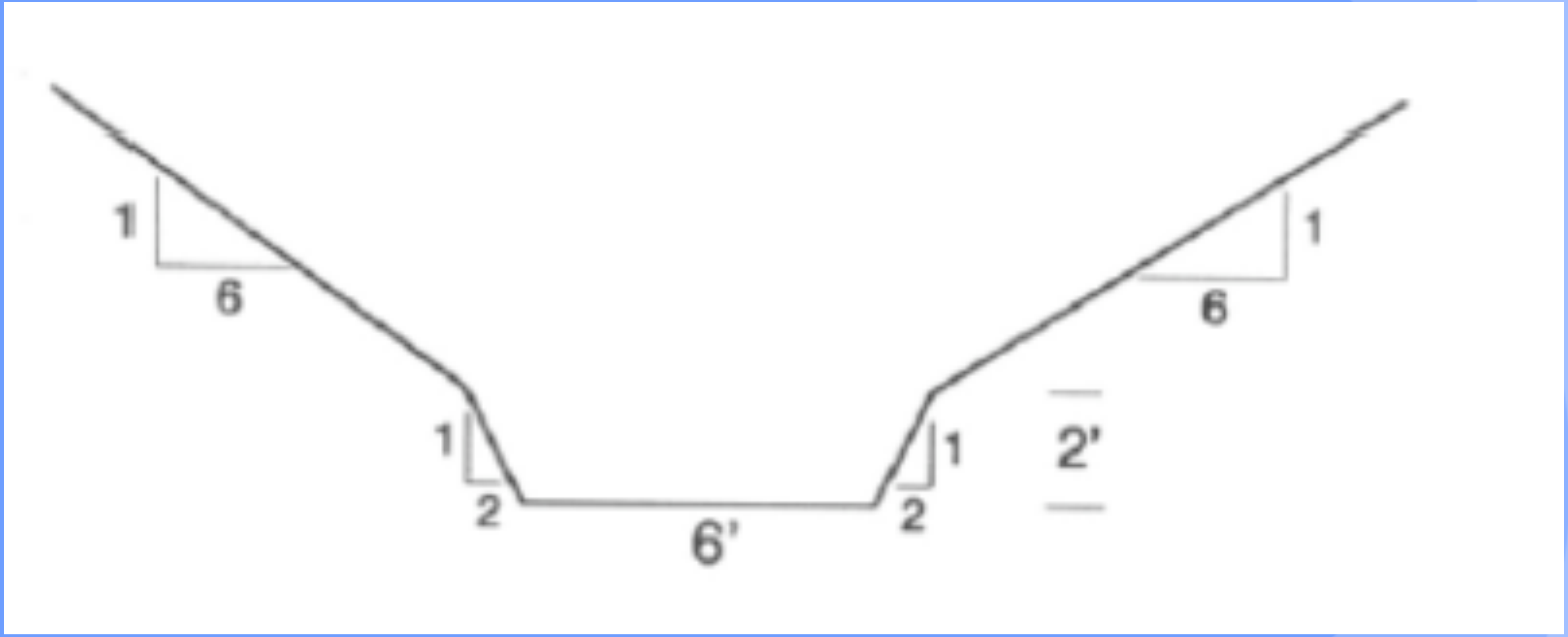


Channel Routing

- A known inflow hydrograph and initial storage condition can be propagated forward in time to estimate the outflow hydrograph.
- The choice of Dt value should be made so that it is smaller than the travel time in the reach at the largest likely flow and smaller than about 1/5 the time to peak of the inflow hydrograph
- HMS is supposed to manage this issue internally, if we roll-our-own, need to be cognizant of this important issue

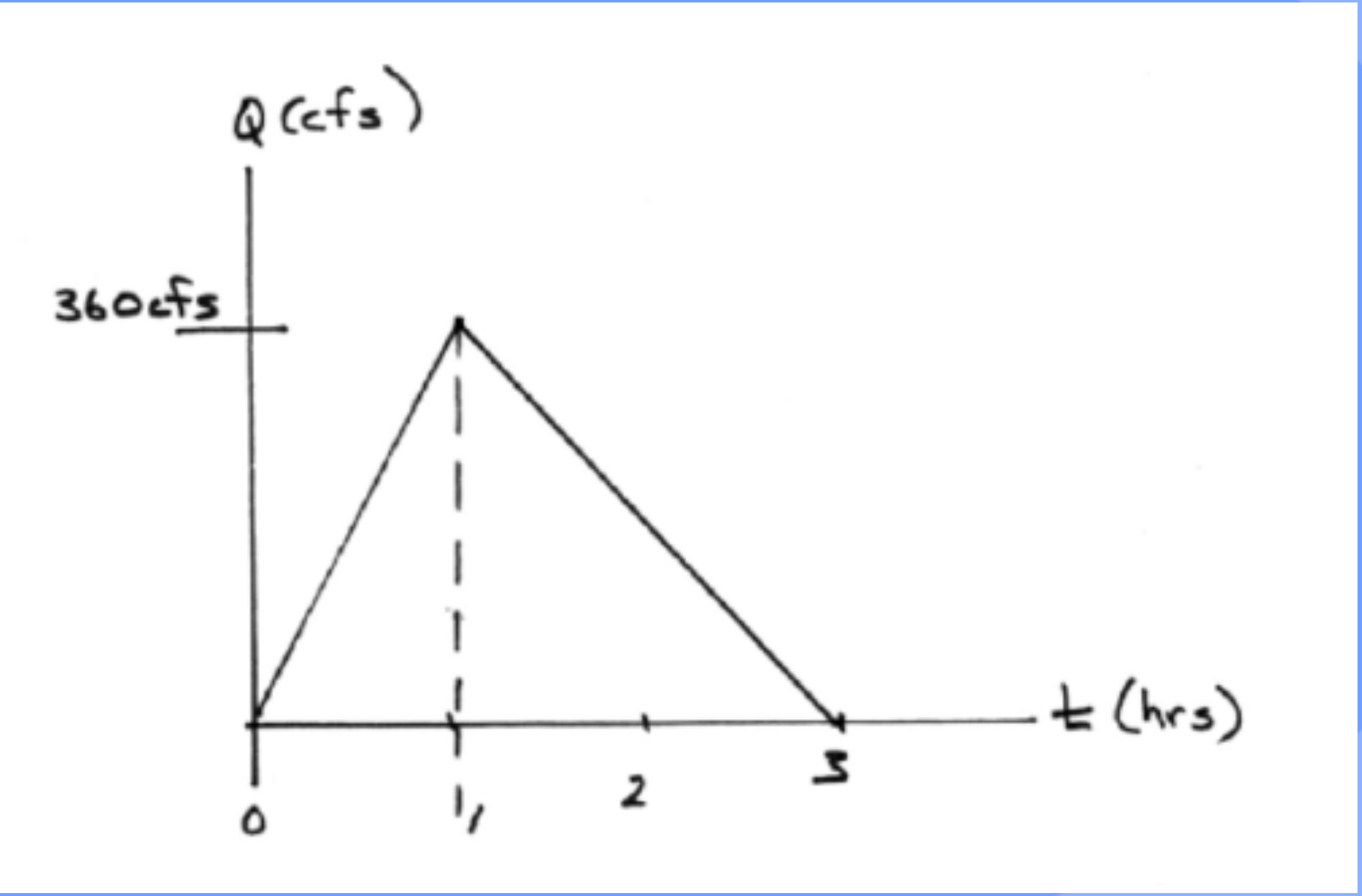
Channel Routing Example

- Consider a channel that is 2500 feet long, with slope of 0.09%, clean sides with straight banks and no rifts or deep pools. Manning's n is 0.030.



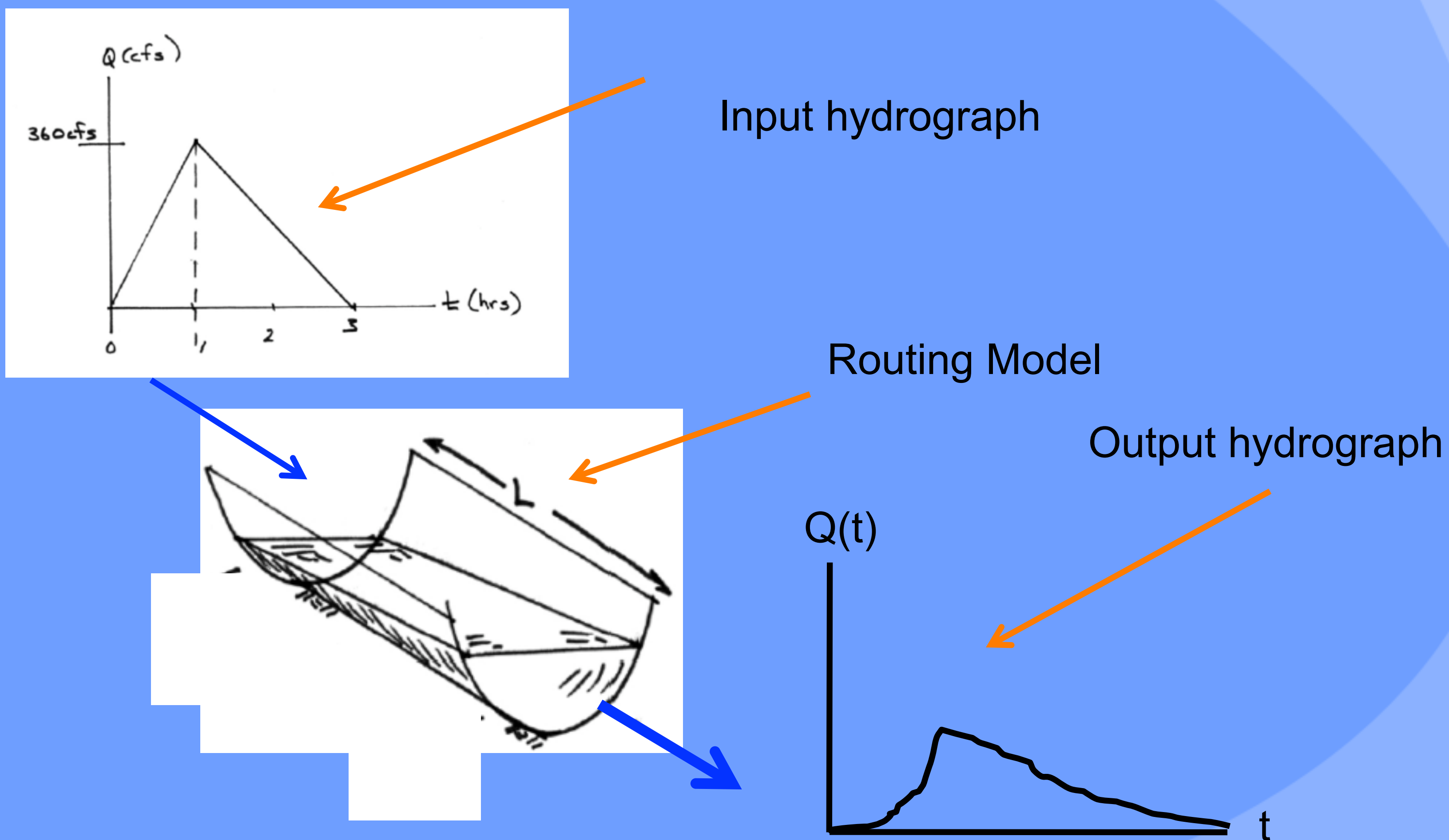
Channel Routing Example

- The inflow hydrograph is triangular with a time base of 3 hours, and time-to-peak of 1 hour. The peak inflow rate is 360 cfs.



Channel Routing Example

- Configuration:



Level Pool Routing

- Tasks:
  - Build a depth-storage table
  - Build a depth-outflow table
    - From 0 -6 feet deep use Manning's equation in variable-geometry conduit
  - Build the input hydrograph (make the picture into numb3rs).
  - Build the routing table (apply the reach mass balance)

Level Pool Routing

- Manning's Equation Calculations

$$Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$$

	A	B	C	D	E	F	G	H	I	J
1	Table 1: Depth-Discharge-Storage Table									
2	Dt	600								
3	n	0.03								
4										
5	y(ft)	A(ft^2)	Pw	R(=A/Pw)	u (ft/sec)	slope	Q(cfs)	S(ft^3)	S-Odt/2	S+Odt/2
6	0	0	0	0	0	0.0009	0	0	0	0
7	1	8	10.47214	0.763932	1.245153	0.0009	9.961225	20000	17011.63	22988.37
8	2	20	14.94427	1.338305	1.809491	0.0009	36.18982	50000	39143.05	60856.95
9	3	40	27.1098	1.475481	1.931118	0.0009	77.24471	100000	76826.59	123173.4
10	4	72	39.27532	1.832212	2.231832	0.0009	160.6919	130000	131792.4	228207.6
11	5	116	51.44085	2.255017	2.562238	0.0009	297.2196	290000	200834.1	379165.9
12	6	172	63.60637	2.704132	2.892043	0.0009	497.4315	430000	280770.6	579229.4
13										

Depth-Area

Depth-Perimeter

Depth-Radius

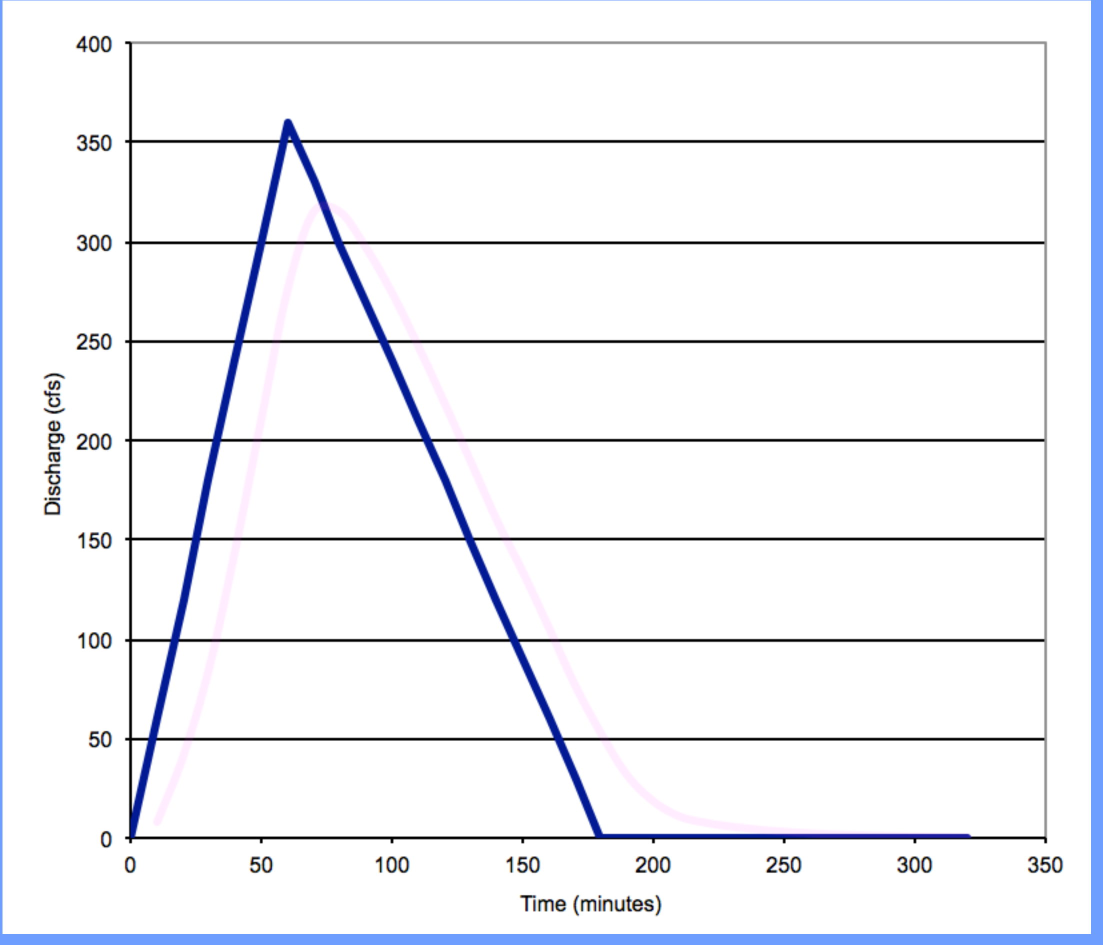
Use for decent Δt

Depth-Storage

- In this example, this table is also the storage-indication table

Level Pool Routing

- Input hydrograph



t(min)	I (cfs)
0	0
10	60
20	120
30	180
40	240
50	300
60	360
70	330
80	300
90	270
100	240
110	210
120	180
130	150
140	120
150	90
160	60
170	30
180	0
190	0

Level Pool Routing

- DEPTH-STORAGE-OUTFLOW

H18

	A	B	C	D	E	F	G
3	1-acre, vertical walls						
4	5-foot RCP outlet (assume short)						
5	10-foot max depth						
6							
7	Methods:						
8	Use Manning's equation in a circular channel for estimate Q vs Depth for 0 to 5 feet)						
9	Use Orifice equation (e.g. FHWA, TxDOT) for estimate Q vs Depth for 5 to 10 feet						
10	Use Depth*Area to estimate storage in cubic feet						
11					DELTA T	10	MIN
12							
13							
14	DEPTH(FT)	OUTFLOW(CFS)	DONE-HOW	STORAGE(FT^3)	2S/Dt + O (CFS)	REMARKS	
15	0	0	Mannings	0	0		
16	0.5	2.986243	Mannings	21780	75.586243	= 2*D16/(F11*60)+B16	
17	1	12.5257	Mannings	43560	157.7257		
18	1.5	28.01057	Mannings	65340	245.81057		
19	2	48.2008	Mannings	87120	338.6008		
20	2.5	71.51714	Mannings	108900	434.51714		
21	3	96.09617	Mannings	130680	531.69617		
22	3.5	119.7537	Mannings	152460	627.95368		
23	4	139.8113	Mannings	174240	720.61126		
24	4.5	152.4455	Mannings	196020	805.84555	Recall max flow in circular is at 85-95% fill depth	
25	5	143.0343	Mannings	217800	869.03427		
26	5	143.0343	Orifice	217800	869.03427	Adjust Cd to match flow at 5ft deep	
27	5.5	156.6862	Orifice	239580	955.28619		
28	6	169.2404	Orifice	261360	1040.4404		
29	6.5	180.9256	Orifice	283140	1124.7256		
30	7	191.9006	Orifice	304920	1208.3006		
31	7.5	202.281	Orifice	326700	1291.281		

Level Pool Routing – Routing Table

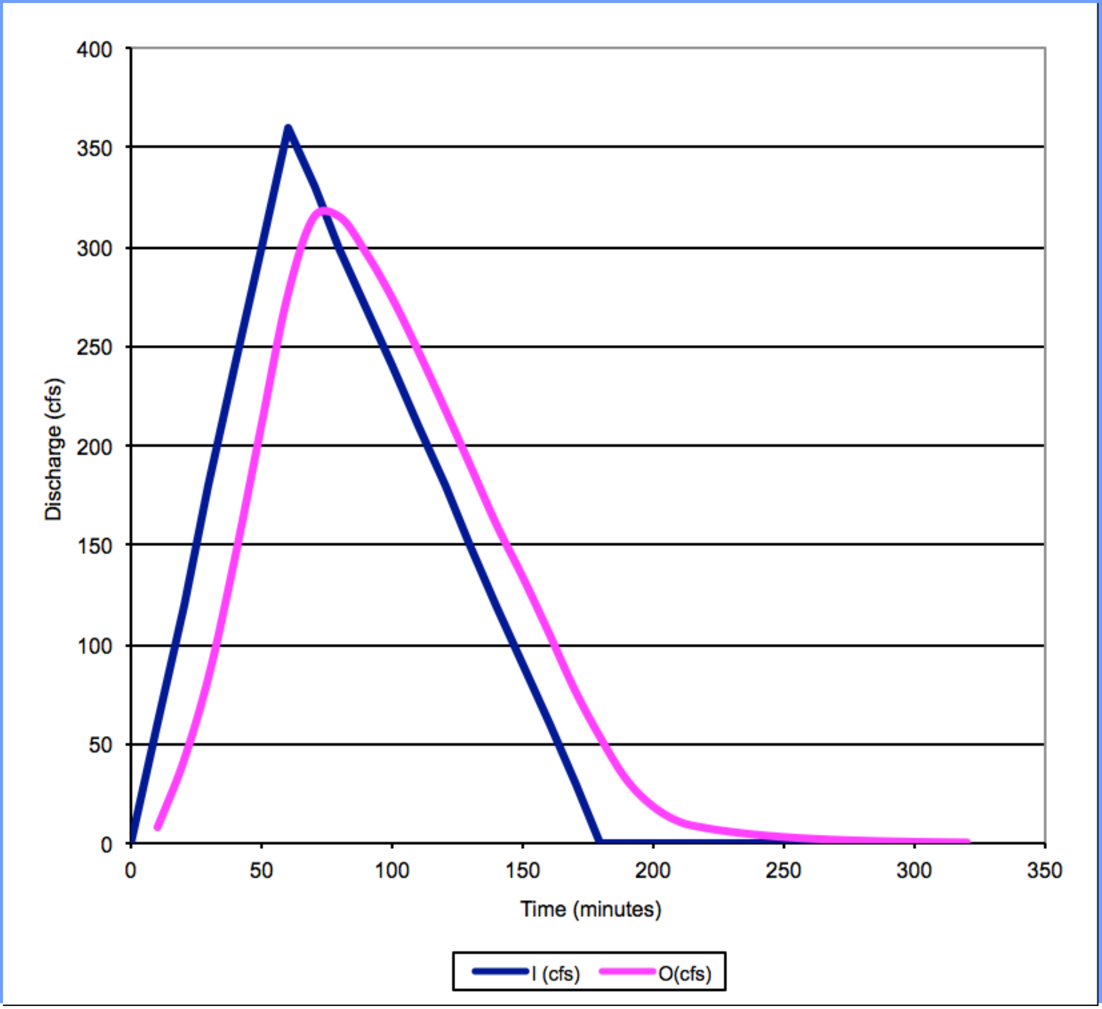
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
16				travel time	865.052													
17				tp/5	720													
18																		
19	Table 2: Routing Table																	
20					Table_Lookup				Interpolate	Mass Balance	Table_Lookup				Interpolate	Table Lookup		Interpolate
21	t(min)	I (cfs)	Dt*(I1+I2)/2	y1 (ft)	y_lo	y_hi	S-Odt/2_lo	S-Odt/2_hi	S-Odt/2	S+Odt/2	S+Odt/2_lo	S+Odt/2_hi	y_lo	y_hi	y2(ft)	O_low	O_high	O(cfs)
22	0	0	0	0														
23	10	60	18000	0	0	1	0	17011.6	0	18000	0	22988.4	0	1	0.783	0	9.96122	7.79969
24	20	120	54000	0.783	0	1	0	17011.6	13320.2	67320.2	60856.9	123173	2	3	2.10372	36.1898	77.2447	40.4479
25	30	180	90000	2.10372	2	3	39143.1	76826.6	43051.5	133051	123173	228208	3	4	3.09405	77.2447	160.692	85.0926
26	40	240	126000	3.09405	3	4	76826.6	131792	81995.9	207996	123173	228208	3	4	3.80757	77.2447	160.692	144.634
27	50	300	162000	3.80757	3	4	76826.6	131792	121215	283215	228208	379166	4	5	4.36439	160.692	297.22	210.441
28	60	360	198000	4.36439	4	5	131792	200834	156951	354951	228208	379166	4	5	4.83959	160.692	297.22	275.319
29	70	330	207000	4.83959	4	5	131792	200834	189759	396759	379166	579229	5	6	5.08794	297.22	497.431	314.826
30	80	300	189000	5.08794	5	6	200834	280771	207864	396864	379166	579229	5	6	5.08846	297.22	497.431	314.93
31	90	270	171000	5.08846	5	6	200834	280771	207905	378905	228208	379166	4	5	4.99827	160.692	297.22	296.984
32	100	240	153000	4.99827	4	5	131792	200834	200715	353715	228208	379166	4	5	4.8314	160.692	297.22	274.202
33	110	210	135000	4.8314	4	5	131792	200834	189194	324194	228208	379166	4	5	4.63585	160.692	297.22	247.503
34	120	180	117000	4.63585	4	5	131792	200834	175692	292692	228208	379166	4	5	4.42717	160.692	297.22	219.012
35	130	150	99000	4.42717	4	5	131792	200834	161285	260285	228208	379166	4	5	4.21249	160.692	297.22	189.703
36	140	120	81000	4.21249	4	5	131792	200834	146463	227463	123173	228208	3	4	3.99291	77.2447	160.692	160.101
37	150	90	63000	3.99291	3	4	76826.6	131792	131403	194403	123173	228208	3	4	3.67816	77.2447	160.692	133.835
38	160	60	45000	3.67816	3	4	76826.6	131792	114102	159102	123173	228208	3	4	3.34207	77.2447	160.692	105.789
39	170	30	27000	3.34207	3	4	76826.6	131792	95628.5	122629	60856.9	123173	2	3	2.99126	36.1898	77.2447	76.8857
40	180	0	9000	2.99126	2	3	39143.1	76826.6	76497.1	85497.1	60856.9	123173	2	3	2.3954	36.1898	77.2447	52.4231
41	190	0	0	2.3954	2	3	39143.1	76826.6	54043.2	54043.2	22988.4	60856.9	1	2	1.82007	9.96122	36.1898	31.4705
42	200	0	0	1.82007	1	2	17011.6	39143.1	35160.9	35160.9	22988.4	60856.9	1	2	1.32144	9.96122	36.1898	18.3922
43	210	0	0	1.32144	1	2	17011.6	39143.1	24125.6	24125.6	22988.4	60856.9	1	2	1.03003	9.96122	36.1898	10.7489
44	220	0	0	1.03003	1	2	17011.6	39143.1	17676.3	17676.3	0	22988.4	0	1	0.76892	0	9.96122	7.65941
45	230	0	0	0.76892	0	1	0	17011.6	13080.6	13080.6	0	22988.4	0	1	0.56901	0	9.96122	5.66804
46	240	0	0	0.56901	0	1	0	17011.6	9679.8	9679.8	0	22988.4	0	1	0.42107	0	9.96122	4.19441
47	250	0	0	0.42107	0	1	0	17011.6	7163.15	7163.15	0	22988.4	0	1	0.3116	0	9.96122	3.10391
48	260	0	0	0.3116	0	1	0	17011.6	5300.81	5300.81	0	22988.4	0	1	0.23059	0	9.96122	2.29692
49	270	0	0	0.23059	0	1	0	17011.6	3922.65	3922.65	0	22988.4	0	1	0.17064	0	9.96122	1.69975
50	280	0	0	0.17064	0	1	0	17011.6	2902.8	2902.8	0	22988.4	0	1	0.12627	0	9.96122	1.25783
51	290	0	0	0.12627	0	1	0	17011.6	2148.11	2148.11	0	22988.4	0	1	0.09344	0	9.96122	0.93081
52	300	0	0	0.09344	0	1	0	17011.6	1589.62	1589.62	0	22988.4	0	1	0.06915	0	9.96122	0.68881
53	310	0	0	0.06915	0	1	0	17011.6	1176.34	1176.34	0	22988.4	0	1	0.05117	0	9.96122	0.50973
54	320	0	0	0.05117	0	1	0	17011.6	870.501	870.501	0	22988.4	0	1	0.03787	0	9.96122	0.3772
55																		
56		3240																3238.93

Check Sums: should be nearly equal

Level Pool Routing

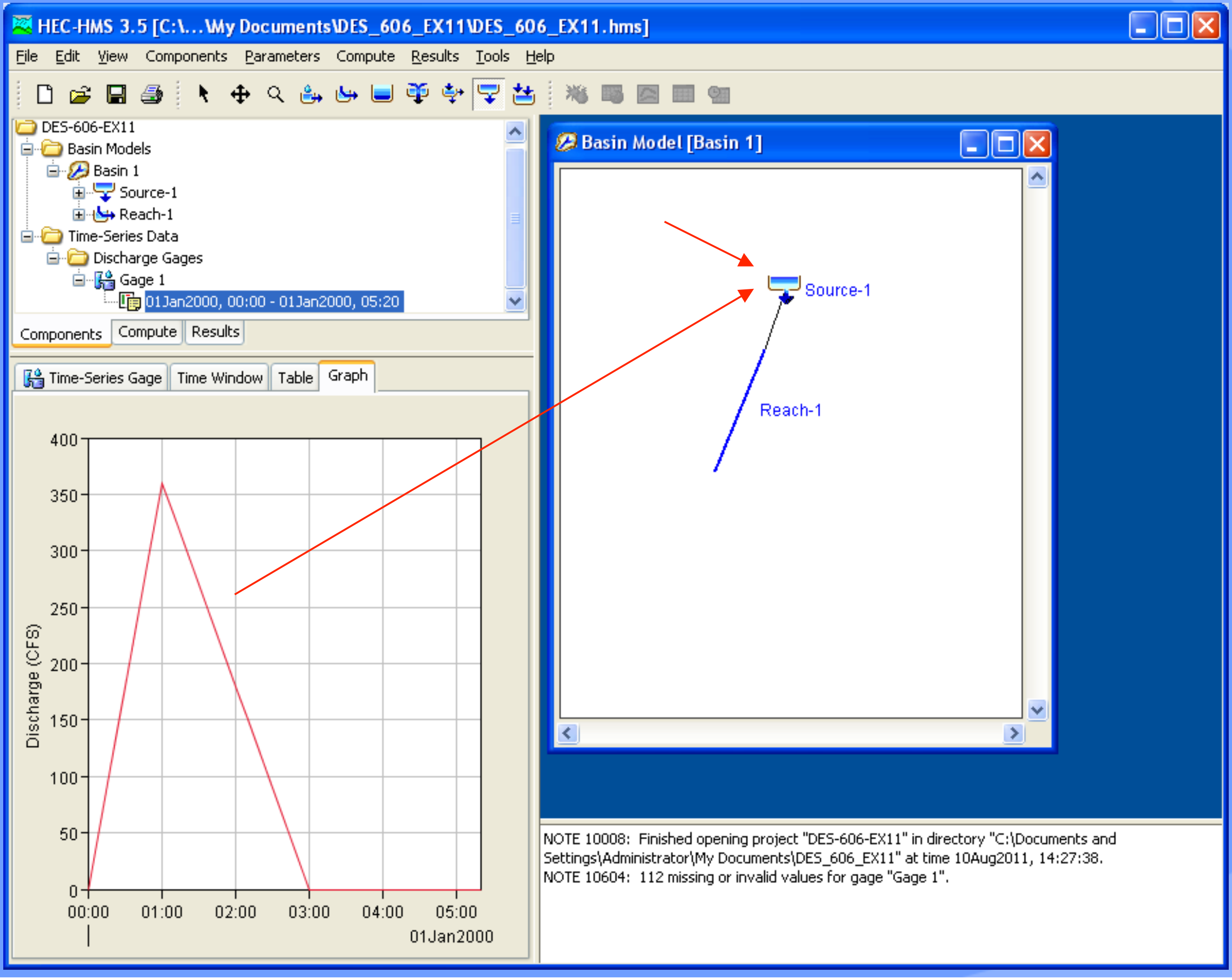
- Plot Results and Examine
  - Notice the reduction in peak, and the lag – is the lag time sensible?

- Lag ~ 20 min
- 2500 ft/20min =125 ft/min =2.08 ft/sec
- Check against depth-discharge, Velocities in 0-3 ft/sec – thus reasonable result



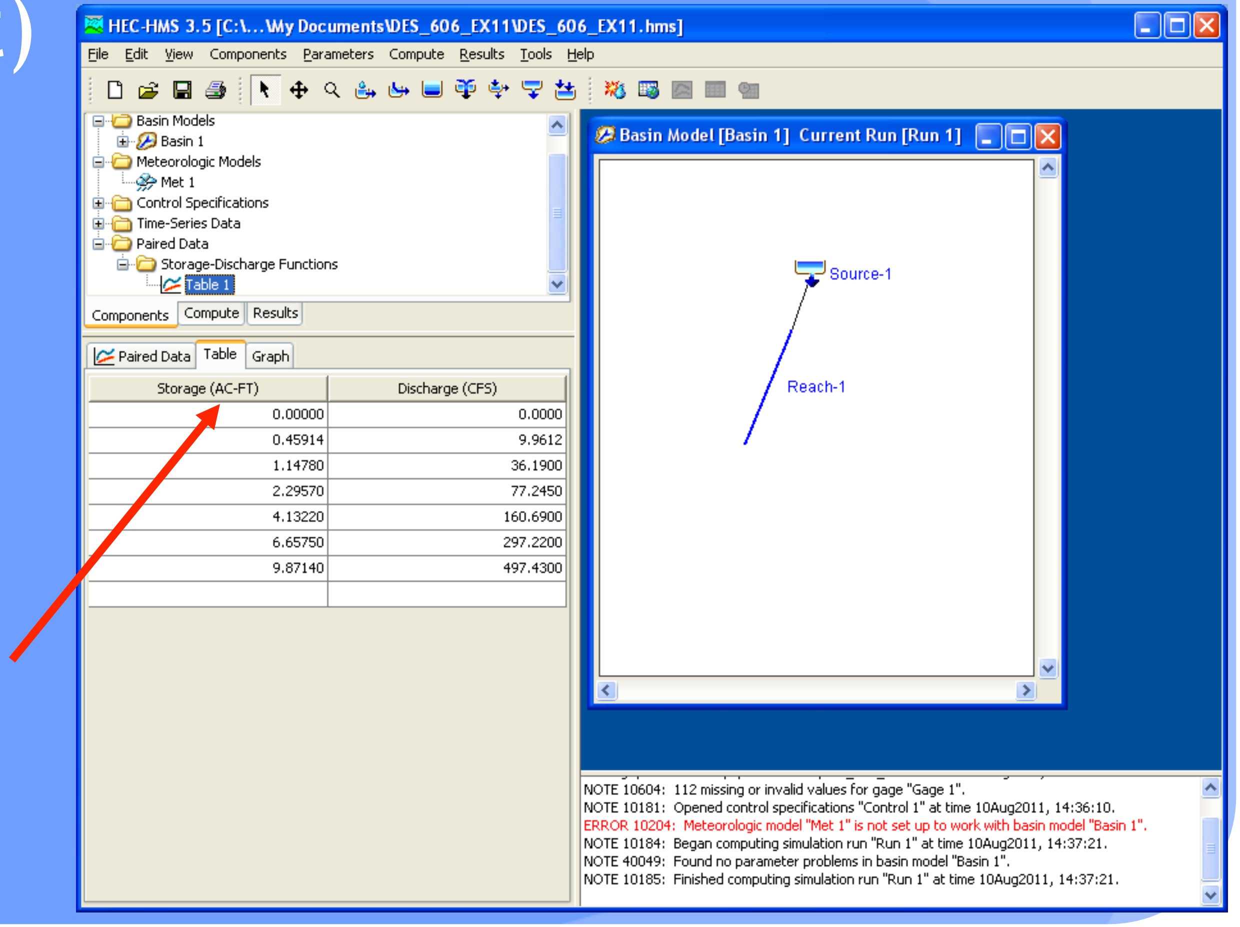
Same example, HEC-HMS

- Create a generic model, use as many null elements as practical (to isolate the routing component)



HEC-HMS

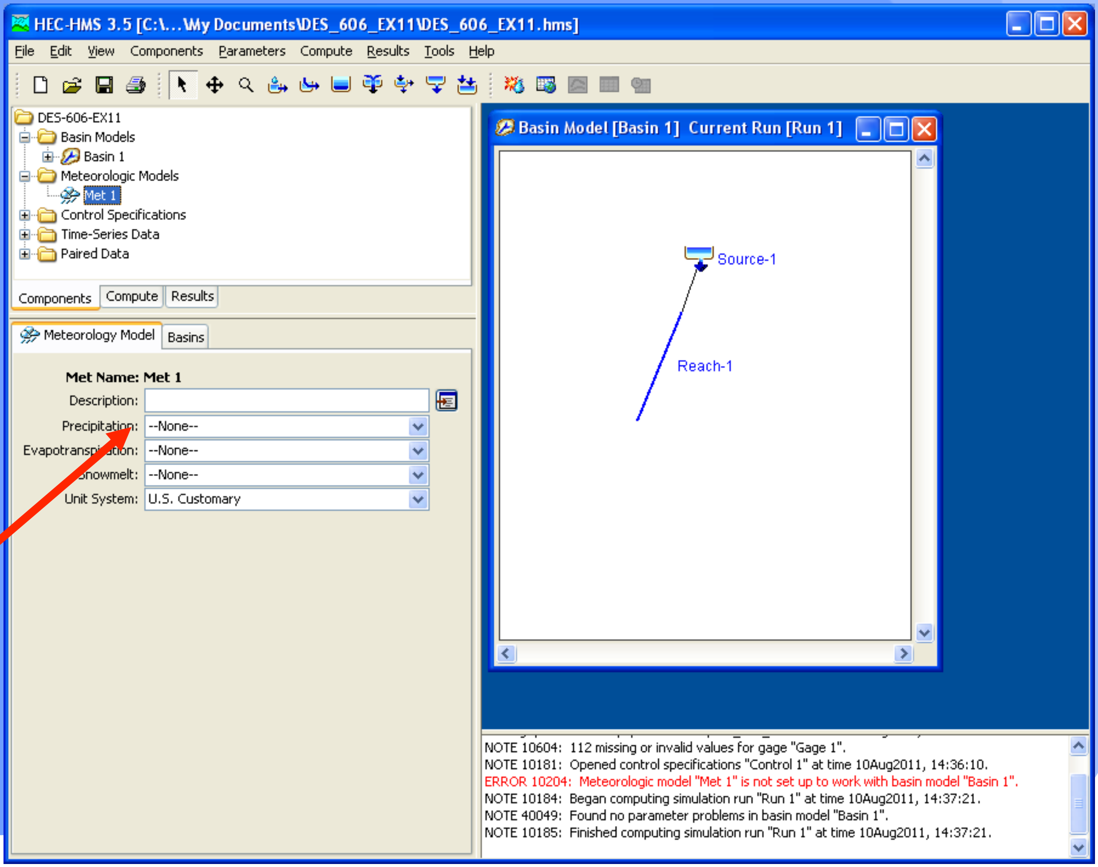
- Storage-Discharge Table (from the spreadsheet)



Note the units of storage

HEC-HMS

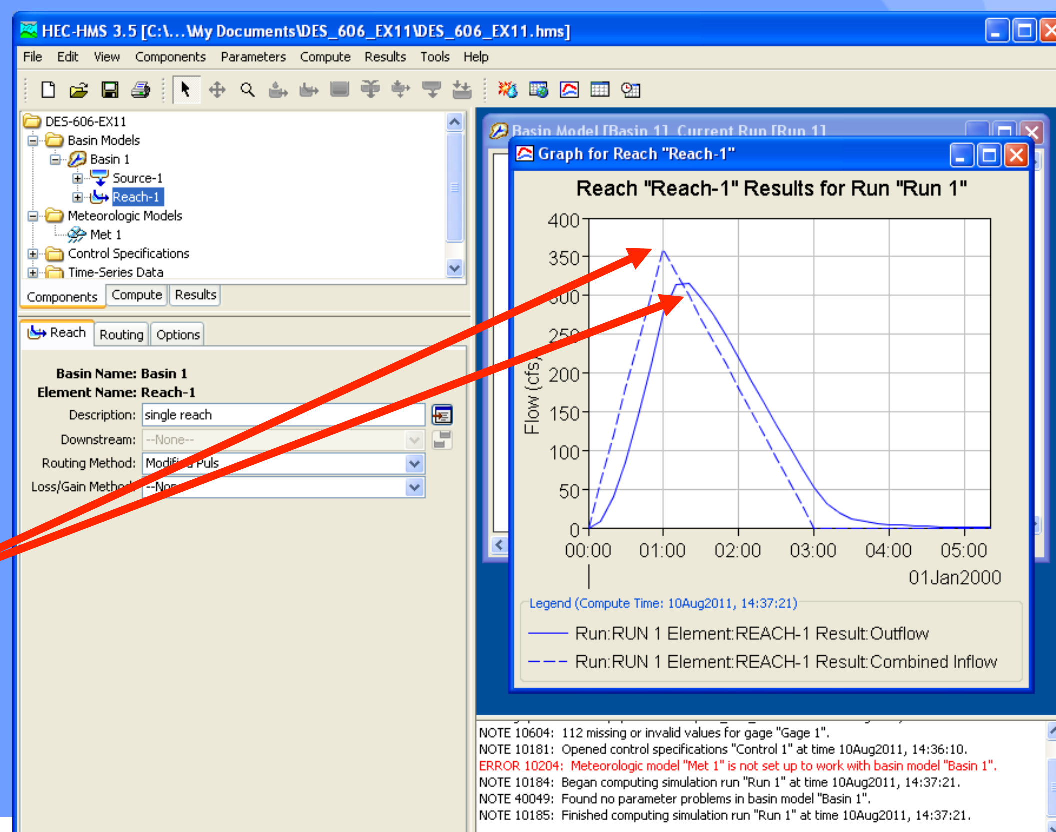
- Meterological Model (HMS needs, but won’t use this module)



Null meterological model

HEC-HMS

- Set control specifications, time windows, run manager – simulate response



Observe the lag from input to output and the attenuated peak from in-channel storage

HEC-HMS

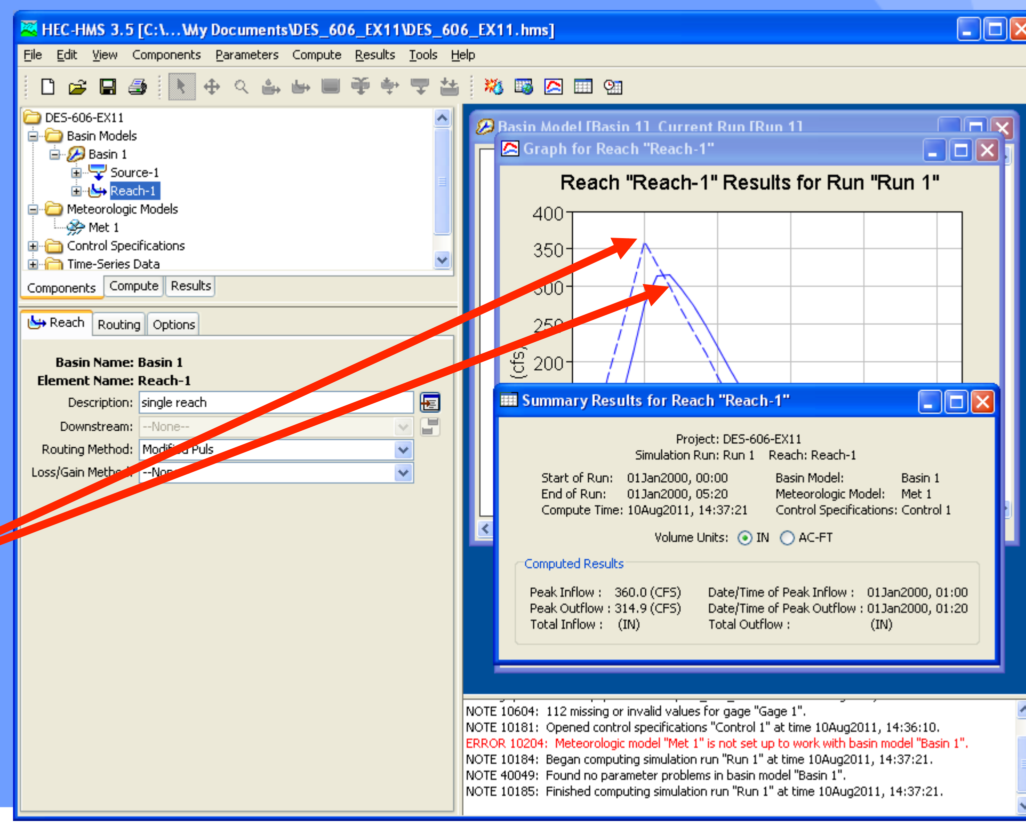
- Set control specifications, time windows, run manager – simulate response

Lag about 20 minutes

Attenuation (of the peak) is about 45 cfs

Average speed of flow about 2 ft/sec

Observe the lag from input to output and the attenuated peak from in-channel storage



Muskingum Routing

- Muskingum routing is a storage-routing technique that is used to:
  - translate and attenuate hydrographs in natural and engineered channels
  - avoids the added complexity of hydraulic routing.
- The method is appropriate for a stream reach that has approximately constant geometric properties.

Muskingum Routing

- At the upstream end, the inflow and storage are assumed to be related to depth by power-law models

$$I = a y_u^n$$

$$S_I = b y_u^m$$

Muskingum Routing

- At the downstream end, the outflow and storage are also assumed to be related to depth by power-law models

$$O = a y_d^n$$

$$S_O = b y_d^m$$

Muskingum Routing

- Next the depths at each end are rewritten in terms of the power law constants and the inflows

$$S_I = \frac{bI^{m/n}}{a^{m/n}}$$

$$S_O = \frac{bO^{m/n}}{a^{m/n}}$$

Muskingum Routing

- Then one conjectures that the storage within the reach is some weighted combination of the section storage at each end (weighted average)

$$S = wS_I + (1 - w)S_O$$

- The weight, w, ranges between 0 and 0.5.
  - When w = 0, the storage in the reach is entirely explained at the outlet end (like a level pool)
  - When w = 0.5, the storage is an arithmetic mean of the section storage at each end.

Muskingum Routing

- Generally the variables from the power law models are substituted

$$K = \frac{b}{a^{m/n}} \text{ and } z = m/n$$

- And the routing model is expressed as

$$S = K[wI^z + (1 - w)O^z]$$

- z is usually assumed to be unity resulting in the usual form

$$S = K[wI + (1 - w)O]$$

Muskingum Routing

- For most natural channels w ranges between 0.1 and 0.3 and are usually determined by calibration studies
- Muskingum-Cunge further refines the model to account to relate the values of the weights to channel geometry, slope, and resistance features
- At this level of abstraction (M-C) the model is nearly a hydraulic model (Kinematic wave)

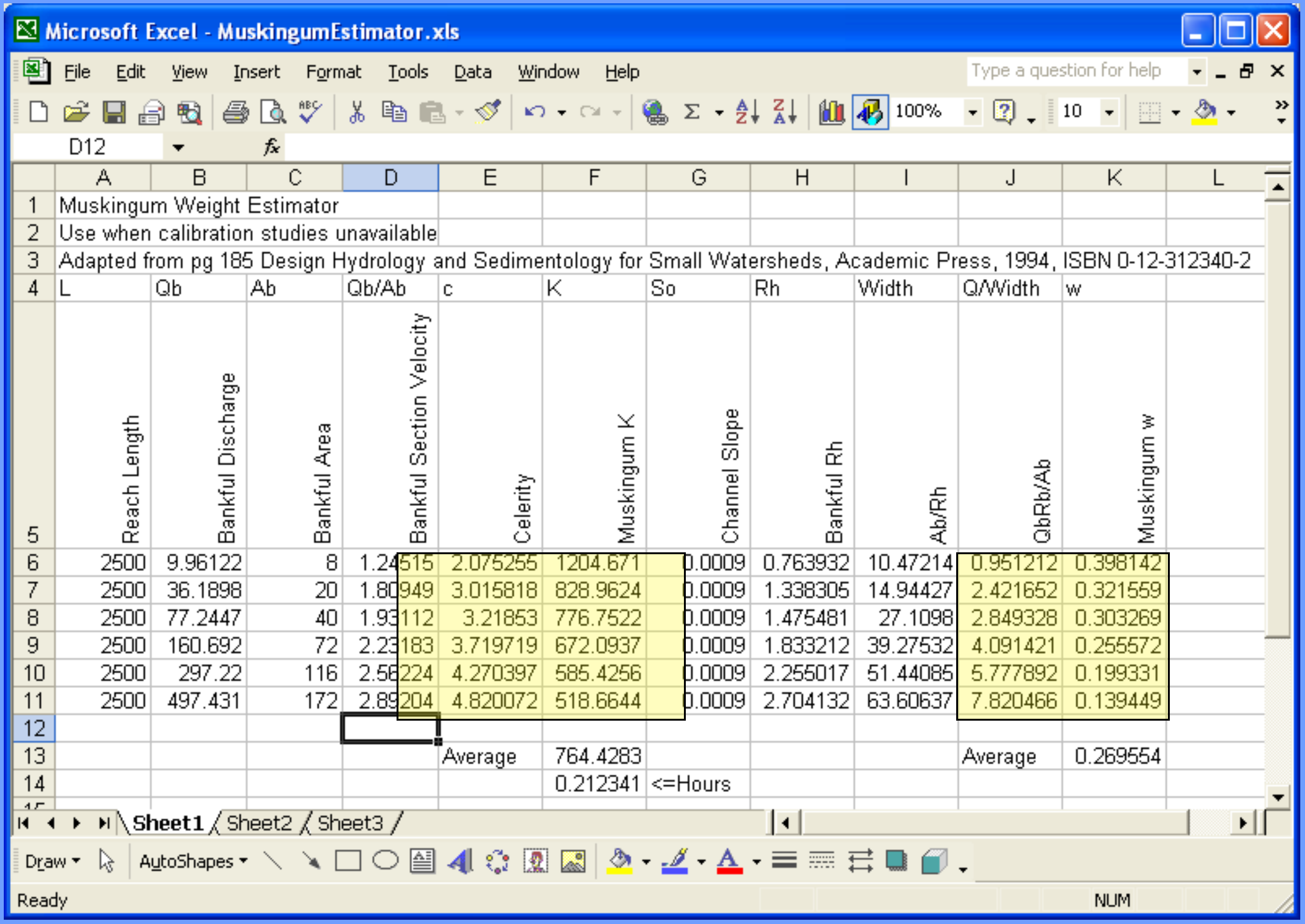
In HEC-HMS

- Use same example conditions
- From hydrologic literature (Haan, Barfield, Hayes) a rule of thumb for estimating w and K is
  - Estimate celerity from bankful discharge (or deepest discharge value)
  - Estimate K as ratio of reach length to celerity (units of time, essentially a reach travel time)
  - Estimate weight (w) as

$$w = \frac{1}{2}(1 - \frac{q_0}{S_0 c L})$$

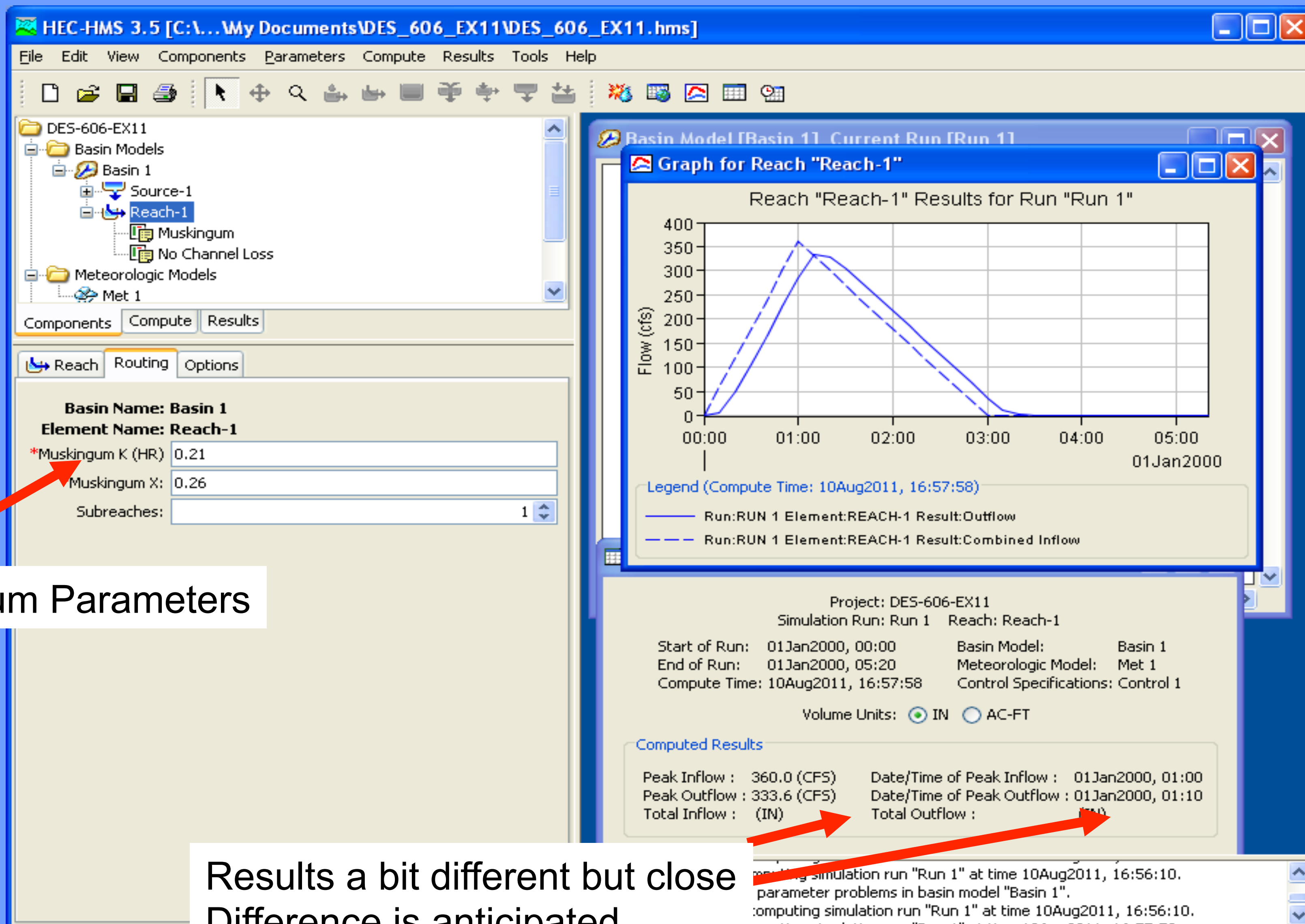
In HEC-HMS

- Use same example conditions



	A	B	C	D	E	F	G	H	I	J	K	L
1	Muskingum Weight Estimator											
2	Use when calibration studies unavailable											
3	Adapted from pg 185 Design Hydrology and Sedimentology for Small Watersheds, Academic Press, 1994, ISBN 0-12-312340-2											
4	L	Qb	Ab	Qb/Ab	c	K	So	Rh	Width	Q/Width	w	
5	Reach Length	Bankful Discharge	Bankful Area	Bankful Section Velocity	Celerity	Muskingum K	Channel Slope	Bankful Rh	Ab/Rh	QbRb/Ab	Muskingum w	
6	2500	9.96122	8	1.24515	2.075255	1204.671	0.0009	0.763932	10.47214	0.951212	0.398142	
7	2500	36.1898	20	1.80949	3.015818	828.9624	0.0009	1.338305	14.94427	2.421652	0.321559	
8	2500	77.2447	40	1.93112	3.21853	776.7522	0.0009	1.475481	27.1098	2.849328	0.303269	
9	2500	160.692	72	2.23183	3.719719	672.0937	0.0009	1.833212	39.27532	4.091421	0.255572	
10	2500	297.22	116	2.56224	4.270397	585.4256	0.0009	2.255017	51.44085	5.777892	0.199331	
11	2500	497.431	172	2.89204	4.820072	518.6644	0.0009	2.704132	63.60637	7.820466	0.139449	
12												
13					Average	764.4283				Average	0.269554	
14						0.212341	<=Hours					

In HEC-HMS

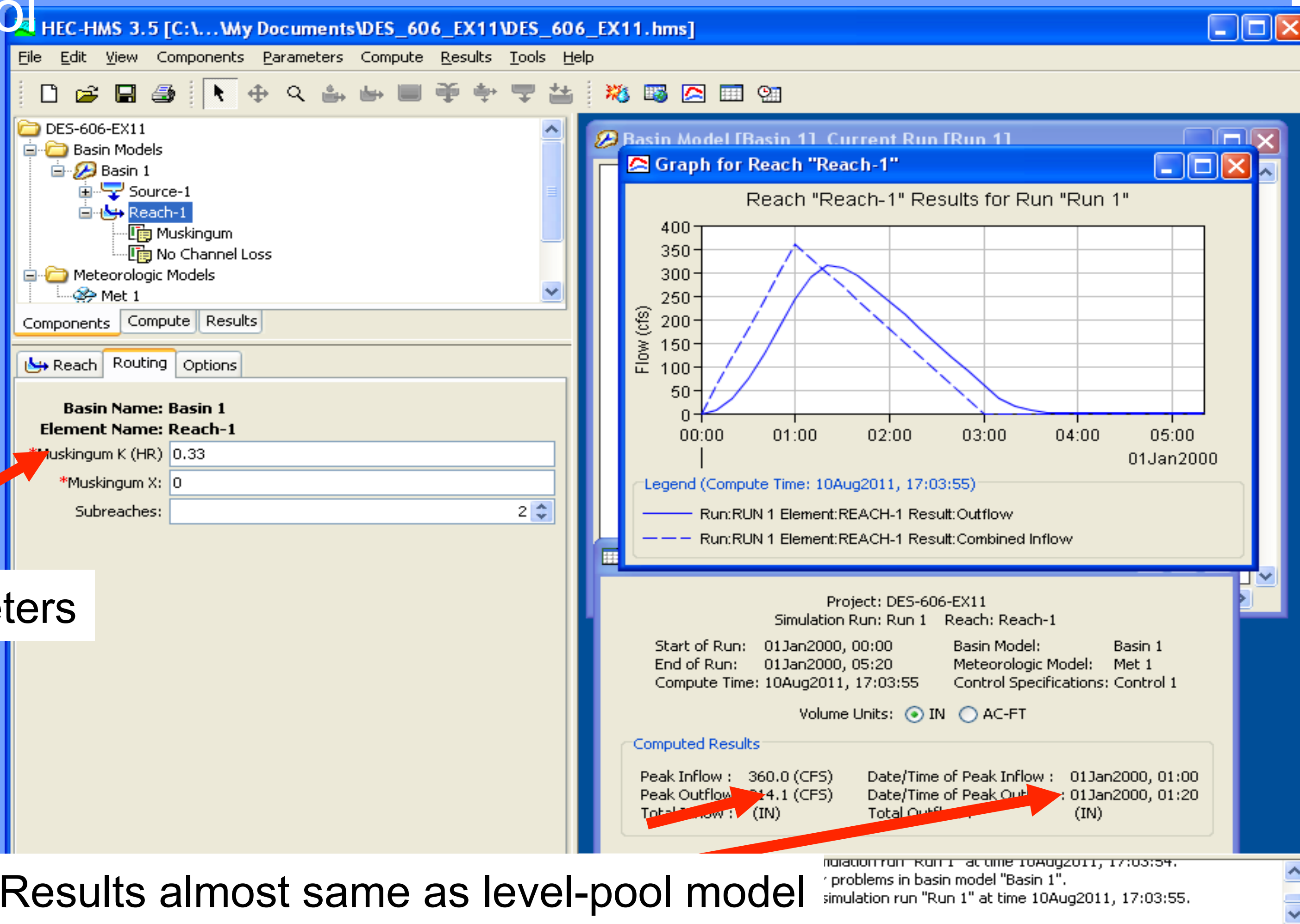


Muskingum Parameters

Results a bit different but close
Difference is anticipated

In HEC-HMS

- Change w to 0.0, K=20 minutes, NReach=2
  - Level Pool

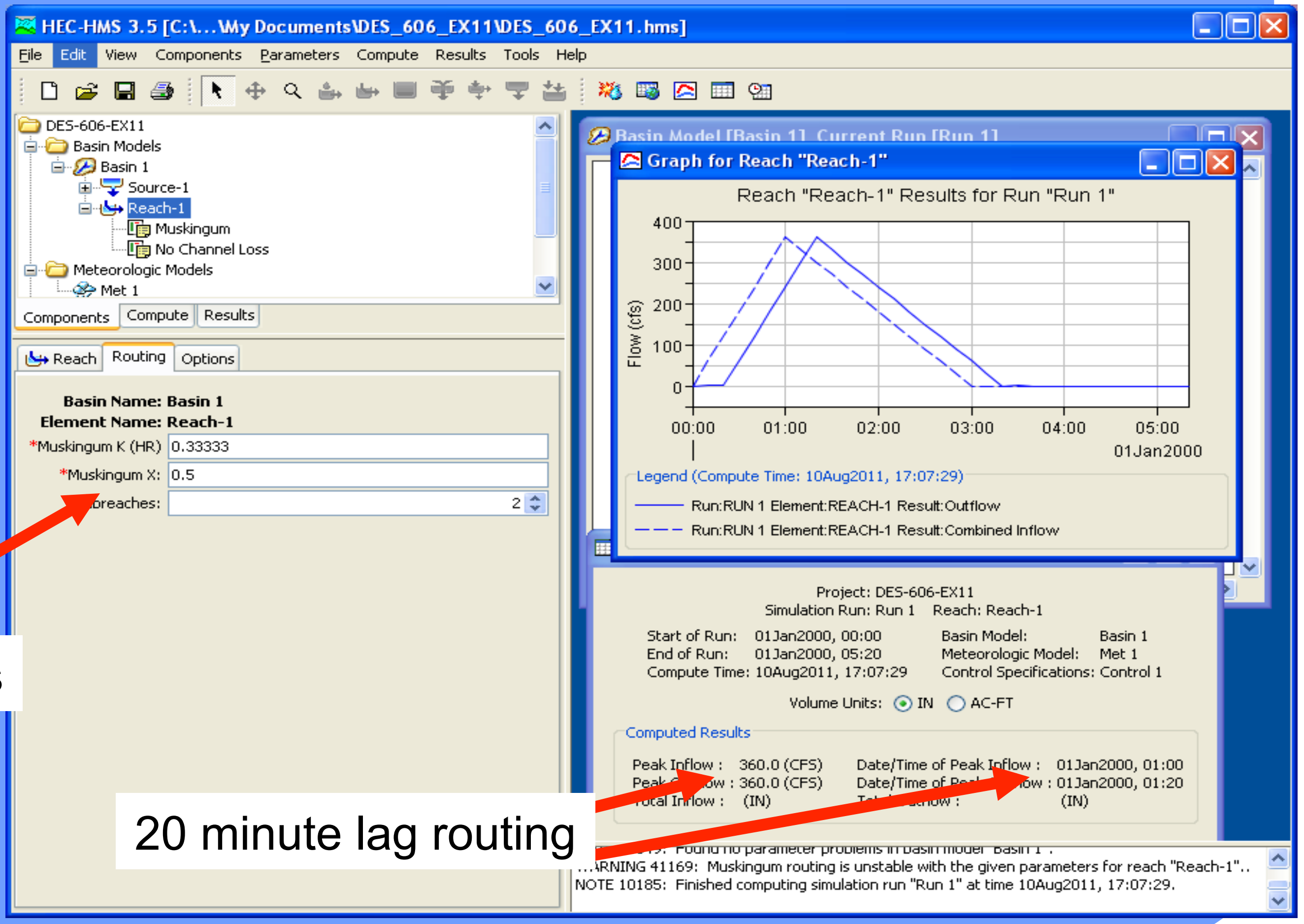


Muskingum Parameters

Results almost same as level-pool model

In HEC-HMS

- Change w to 0.5, K=20 minutes, NReach=2
  - Lag Routing



Muskingum Parameters

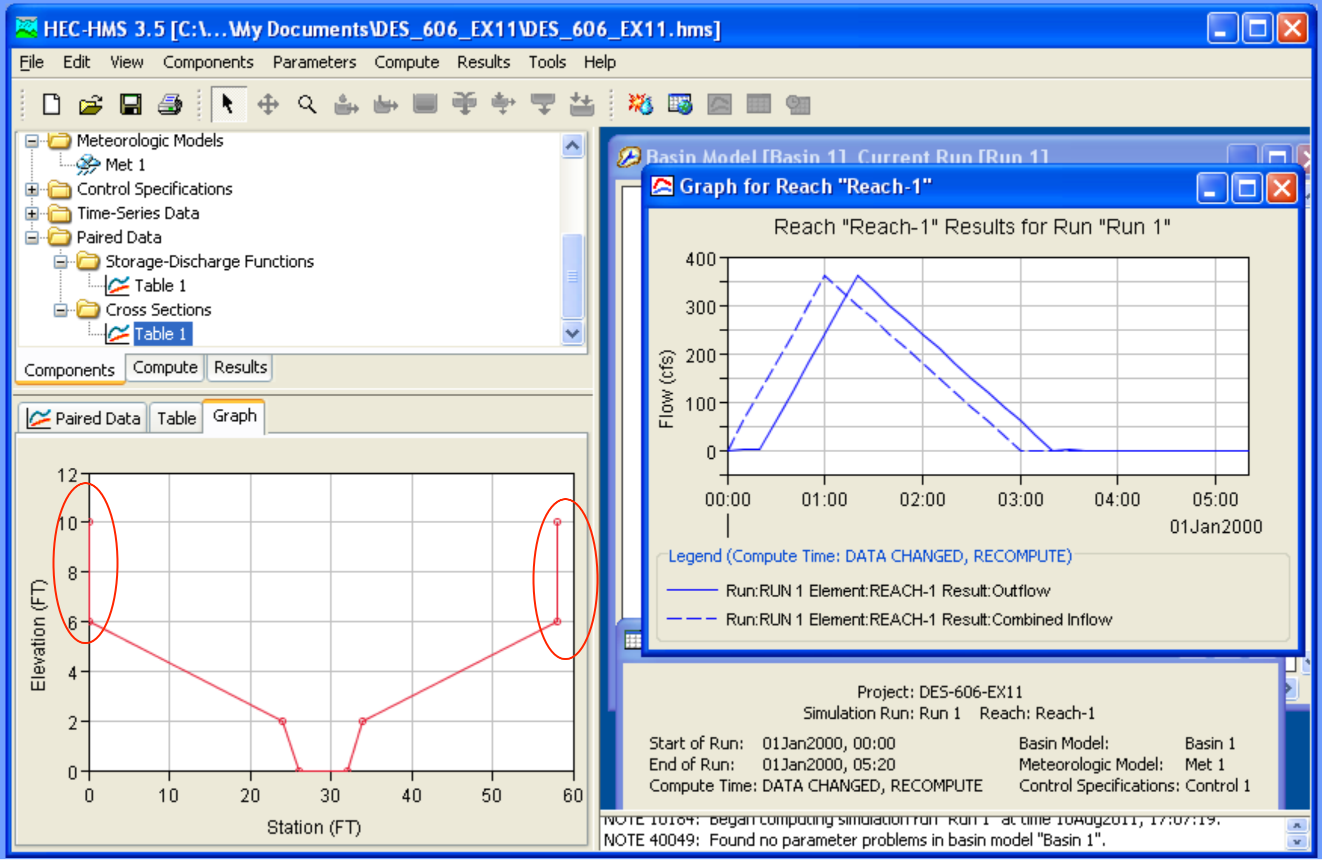
20 minute lag routing

Muskingum-Cunge

- CMM pp. 302-304
- Data needs are
  - Cross section geometry (as paired-data)
  - Manning's n in channel, left and right overbank
  - Slope
  - Reach length
  - Number of reaches (the program divides the reaches into sub-reaches for computation)

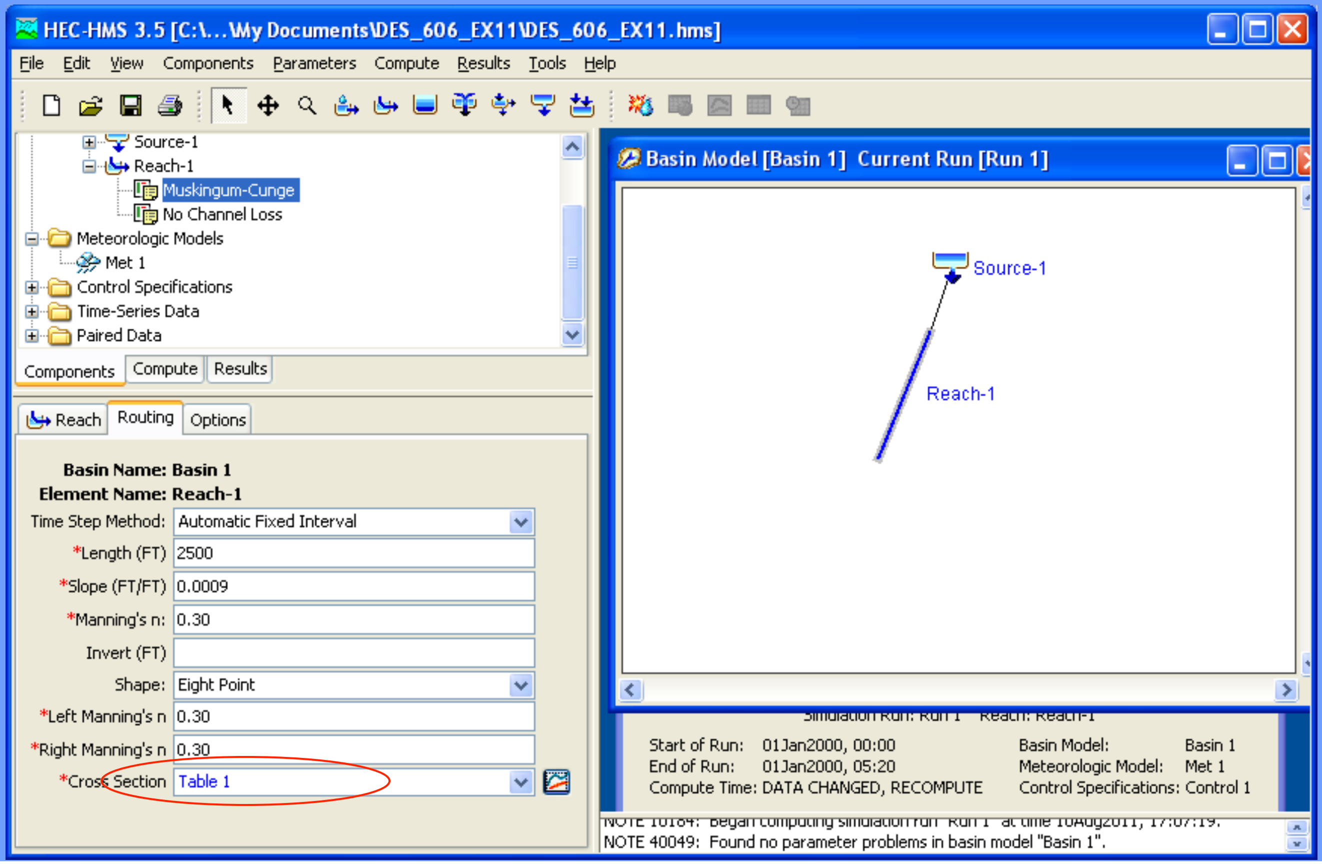
Muskingum-Cunge

- Cross section geometry
  - “Glass walls”



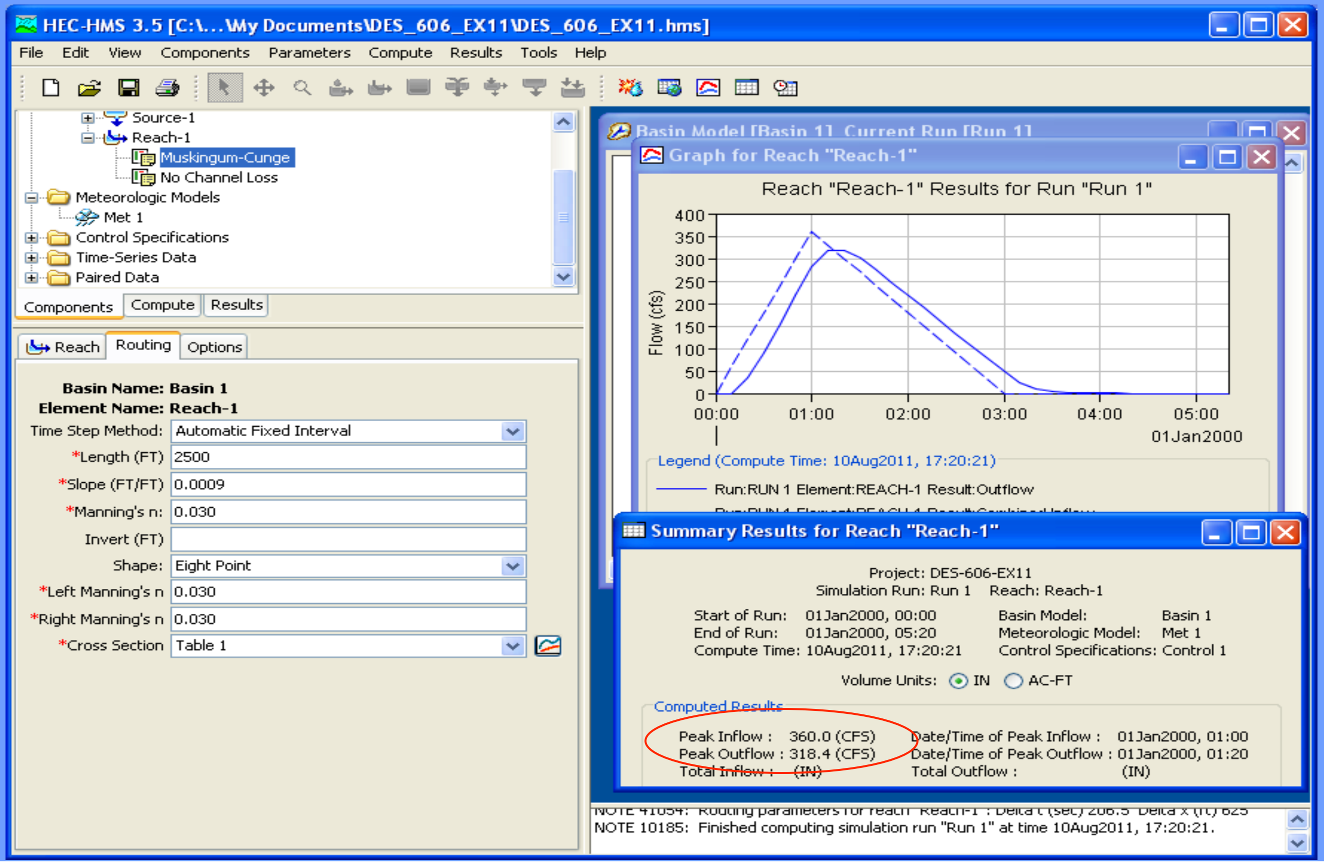
Muskingum-Cunge

- Associate the section with the routing element
  - Other data included



Muskingum-Cunge

- Run the simulation
  - Result comparable to level-pool.



Summary

- Examined routing using:
  - Lag
  - Level pool routing (Puls)
  - Muskingum
  - Muskingum-Cunge
- All require external data preparation

Summary

- Which method to consider
  - Lag – use to get connectivity to agree with the conceptualization of the system. Also can use in practice for short elements that don't overflow and don't have much storage capacity relative to the discharges.
  - Level pool routing (Puls) – use for reservoirs, detention basins, LID/GI practices where the flow out of the practice is weir or underdrain controlled.

Summary

- Which method to consider
  - Muskingum – use for streams
  - Muskingum-Cunge – use for streams and engineered channels
- None will work well in backwater conditions – that's the realm of HEC-RAS or SWMM.

Next Time

- HEC HMS workshop
- Lecture YBX
  - Review for Exam 2
- You will be expected to be able to build and run a HEC-HMS model.