CE 3354 ENGINEERING HYDROLOGY

LECTURE 13: UNIT HYDROGRAPHS

OUTLINE

- ES-5 Solution
- Project Status
  - Technology you already possess
- Unit Hydrographs
  - Theory
  - Data Analysis to construct a UH
    - Chapter 11 CMM

ES 5 SOLUTION SKETCH

1) Figure 1 is a Google-Earth image of a watershed. Assume the watershed is located near College Station, Texas. The distance on the image from Rain Gage R-1 to the Rocky Run Branch Gage is 1,500 feet.

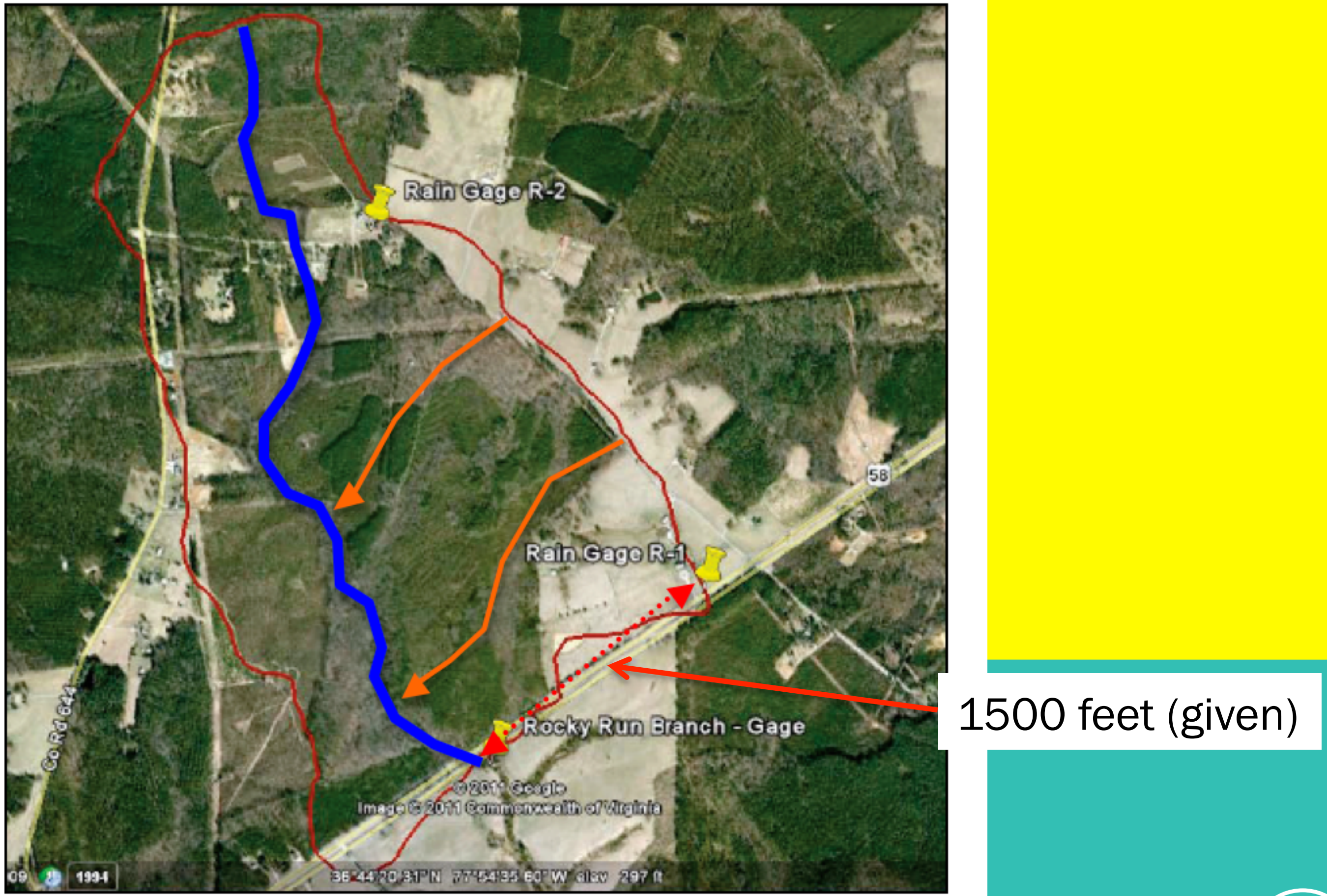


Figure 1. Rocky Run Branch Watershed

ES 5 SOLUTION SKETCH

1. **Identify the channel, measure its length relative to given distance**
2. **Identify some overland flow paths, measure(estimate) their length relative to given distance**
3. **Use Kerby-Kirpich to estimate Tc**
4. **Use Upland to estimate Tc**

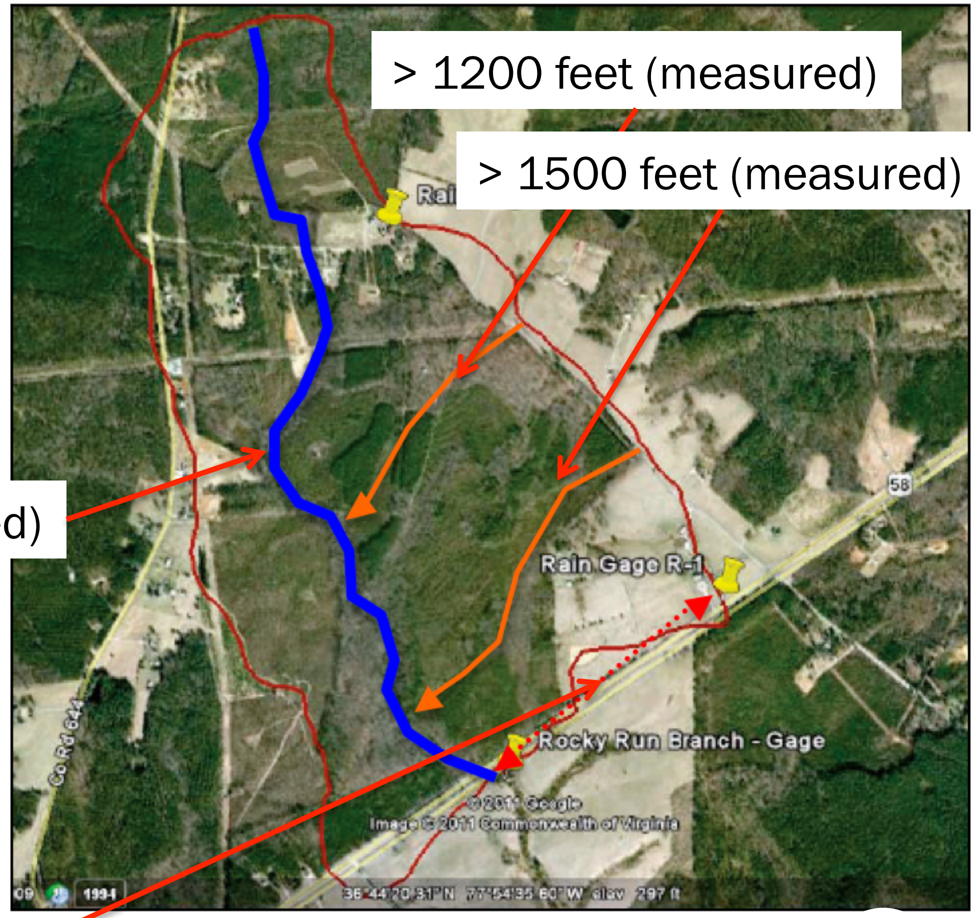


Figure 1. Rocky Run Branch Watershed

ES 5 SOLUTION SKETCH

- Blue line is main channel, it is approximately 4300 feet long using the R1 to Outlet distance as a reference distance.
- Orange lines are a couple overland flow paths. The flow path near R1 to outlet is at least 1500 feet, so use the 1,200 foot maximum length in Kerby-Kirpich method.
- Overland slope would be at least equal to channel slope (otherwise incised channel would not form) so use overland slope of 0.006
- Retardance somewhere between Poor Grass and Pasture (N=0.3)

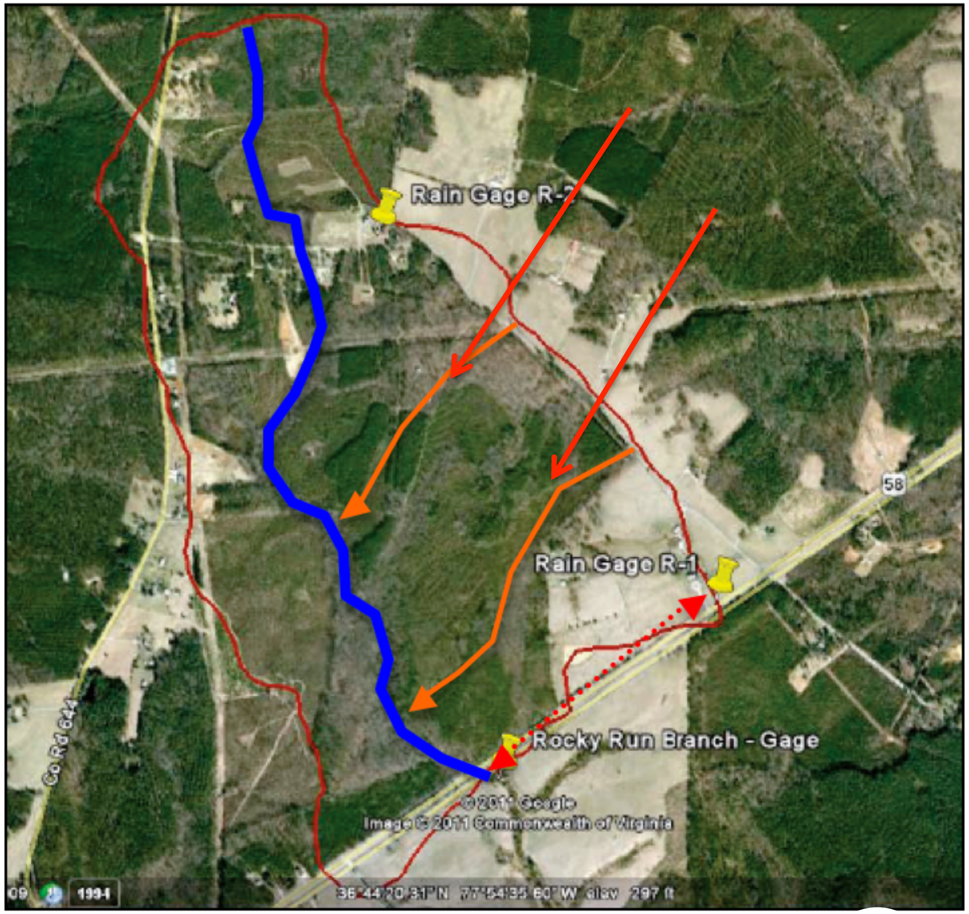


Figure 1. Rocky Run Branch Watershed

ES 5 SOLUTION SKETCH

- Use equations as presented, or simply use the spreadsheet

	A	B	C
1	Kerby-Kirpich Tc Estimator		
2			
3	--- Overland Flow Portion ---		
4	Unit Conversion, K (US)	0.828	
5	Retardance Coefficient, N	0.3	Table Look Up
6	Overland Length, L_{ov}	1200	Feet
7	Slope, S	0.006	Feet/Feet
8	Tc-overland	*43.0*	Minutes
9	--- Channel Flow Portion ---		
10	Unit Conversion, K (US)	0.0078	
11	Channel Length, L_{ch}	4300	Feet
12	Channel Slope, S	0.006	Feet/Feet
13	Tc-channel	*35.1*	Minutes
14	Tc (overland+channel)	***78.1***	Minutes

Generalized Terrain Condition	N
Pavement	0.02
Smooth, bare, packed soil	0.1
Poor grass, cultivated row crops, or moderately rough packed surfaces	0.2
Pasture, average grass	0.4
Deciduous forest	0.6
Dense grass, coniferous forest, or deciduous forest with deep litter	0.8

KerbyKirpich-US

ES 5 SOLUTION SKETCH

- Upland method only uses a path length (and assumption of cover)

NRCS Upland Method Tc Estimator

Input Values

0.6	<= Watershed Slope (%)
Grassed Waterway	<= Select Watershed Surface Type (Pull Down Menu)
4300	<= Path Length (ft)

Reference: NEH 630 Chapter 15
http://directives.sc.egov.usda.gov/OpenNonWebContent.aspx?content=27002.wba

Computed Values

1.161895004	<= Flow Velocity (ft/s)
3701	<= Time (seconds)
61.7	<= Time (minutes)
1.03	<= Time (hours)

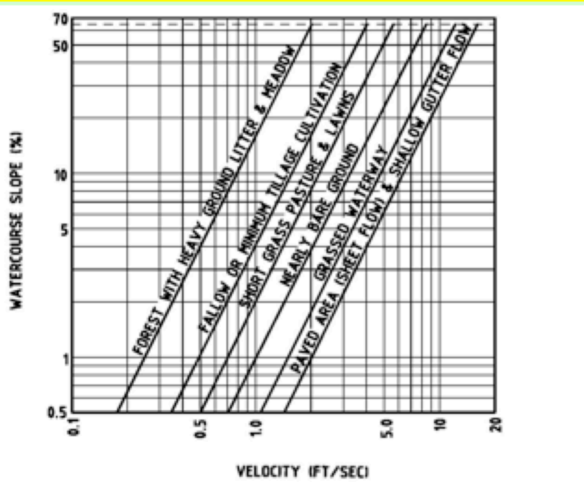
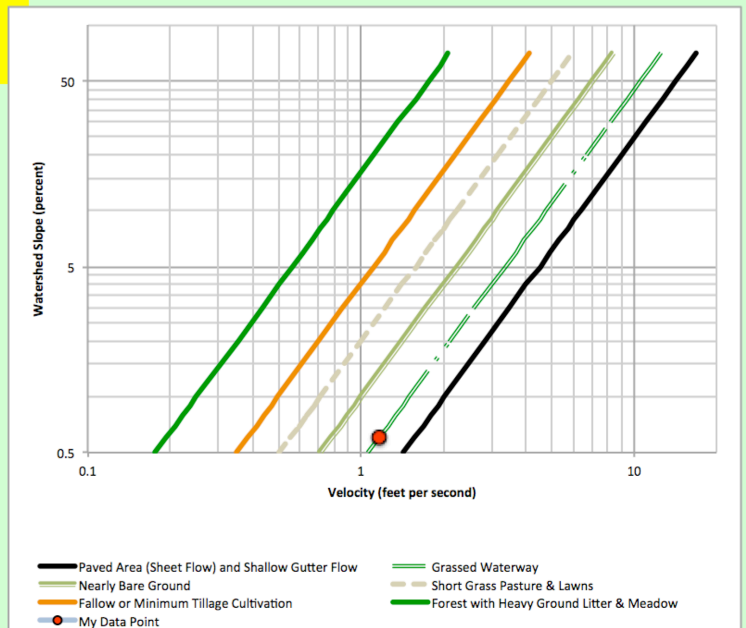


Figure 5-4. Velocities for Upland Method of Estimating Time of Concentration--English (Adapted from the National Engineering Handbook Volume 4)



ES 5 SOLUTION SKETCH

- Discussion:
  - Using these two methods we see that the estimated time is about 70 minutes
  - Falls within observed range for Texas watersheds

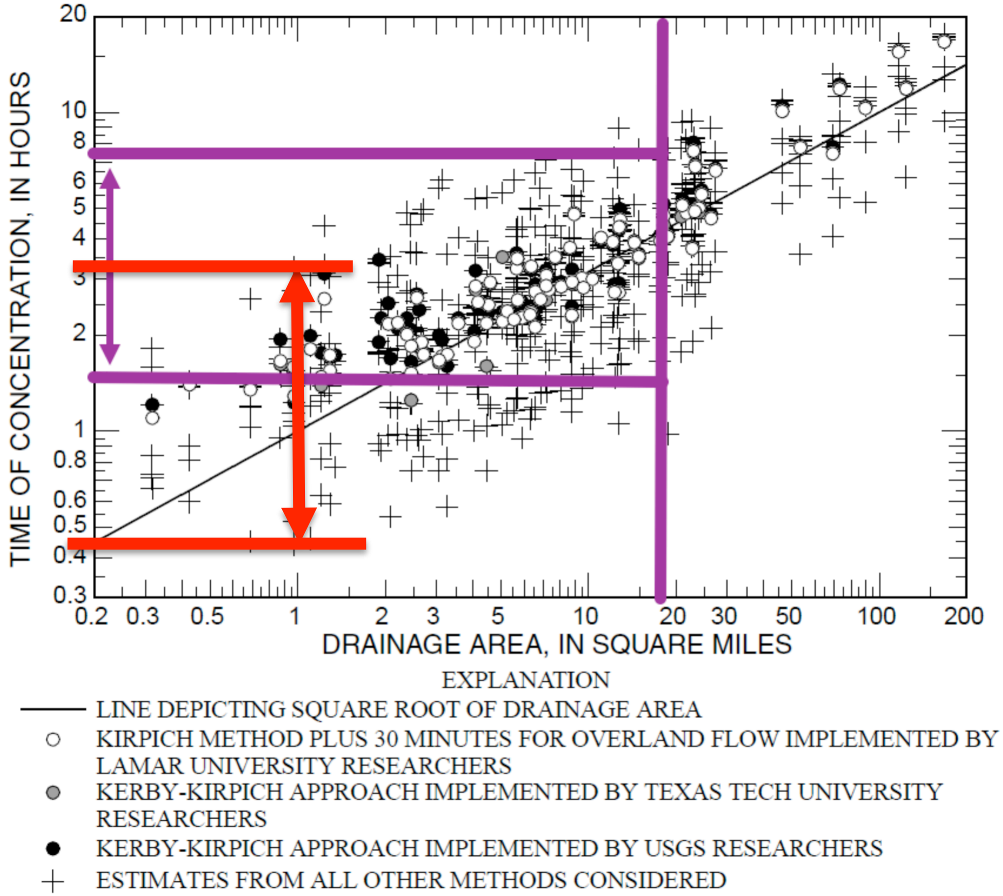
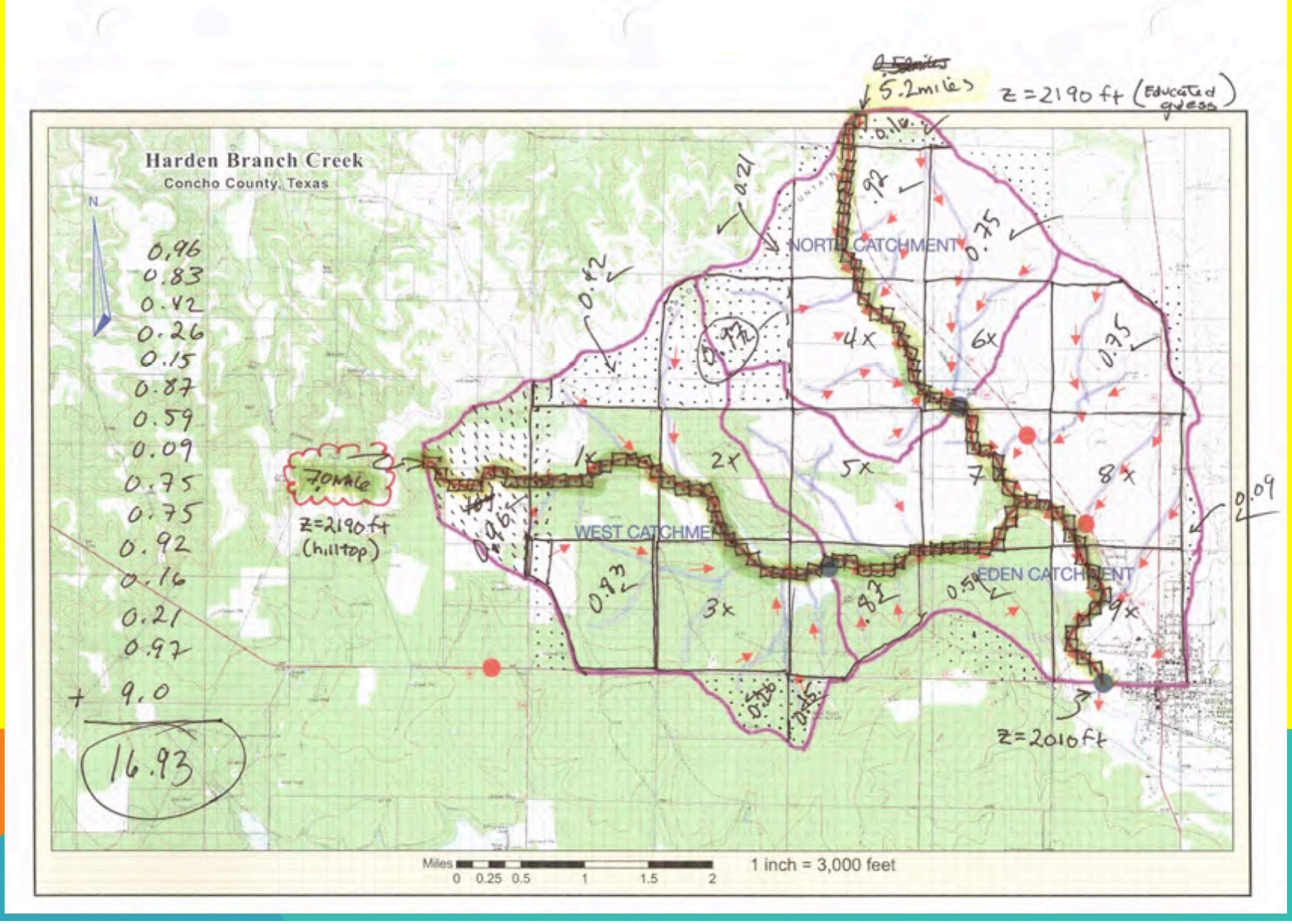


Figure 4. Relation between drainage area and time of concentration from evaluated approaches with distinction of Kirpich-inclusive approaches.

ES 5 SOLUTION SKETCH

2) Estimate the time of concentration for the Harden Creek watersheds (sub-basin to each reservoir, plus the portion directly to the outlet) using the Kerby-Kirpich method.



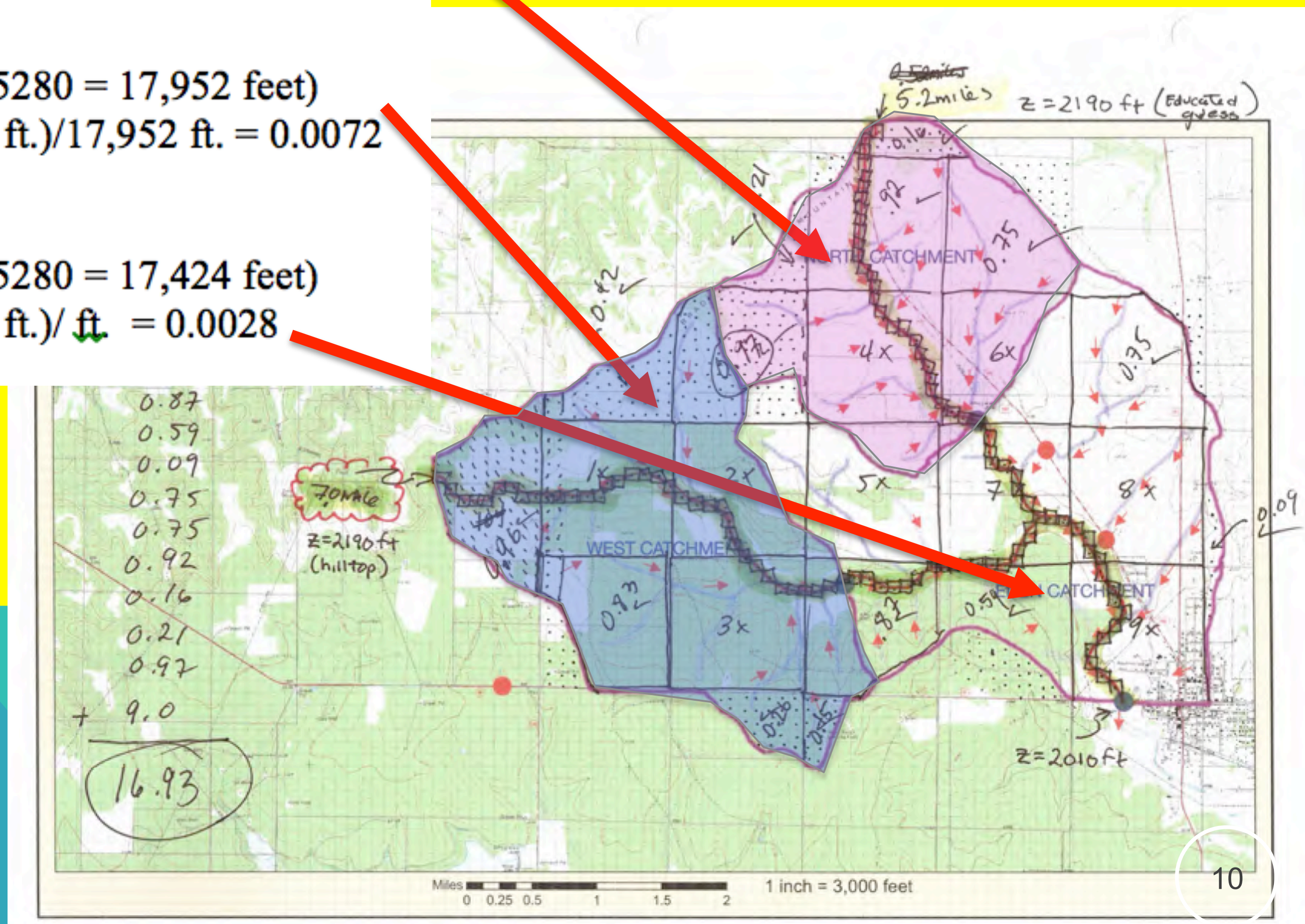
First identify the overland and channel flow path lengths. The channels are each well over a mile, so assume entire 1200 feet of overland is required. Next use the spreadsheet tool that implements the calculations

ES5 SOLUTION SKETCH

North Catchment
MCL = 2.7 miles (2.7*5280 = 14,256 feet)
Slope = (2190 ft. – 2065 ft.)/14,256 ft. = 0.0088

West Catchment
MCL = 3.4 miles (3.4 * 5280 = 17,952 feet)
Slope = (2190 ft. – 2060 ft.)/17,952 ft. = 0.0072

Eden Catchment
MCL = 3.3 miles (3.3 * 5280 = 17,424 feet)
Slope = (2060 ft. – 2010 ft.)/ ft. = 0.0028



ES5 SOLUTION SKETCH

North Catchment

	A	B	C
4	Unit Conversion, K (US)	0.828	
5	Retardance Coefficient, N	0.4	Table Look Up
6	Overland Length, L_{ov}	1200	Feet
7	Slope, S	0.009	Feet/Feet
8	Tc-overland	*45.0*	Minutes
9	--- Channel Flow Portion ---		
10	Unit Conversion, K (US)	0.0078	
11	Channel Length, L_{ch}	14256	Feet
12	Channel Slope, S	0.009	Feet/Feet
13	Tc-channel	*76.2*	Minutes
14	Tc (overland+channel)	***121.2***	Minutes

Generalized Terrain Condition	N
Pavement	0.02
Smooth, bare, packed soil	0.1
Poor grass, cultivated row crops, or moderately rough packed surfaces	0.2
Pasture, average grass	0.4
Deciduous forest	0.6
Dense grass, coniferous forest, or deciduous forest with deep litter	0.8

Figure 3. North Catchment

ES5 SOLUTION SKETCH

West Catchment

B8   `=(B7^(-0.235))*B4*(B6*B5)^(0.467)`

	A	B	C
4	Unit Conversion, K (US)	0.828	
5	Retardance Coefficient, N	0.4	Table Look Up
6	Overland Length, L_{ov}	1200	Feet
7	Slope, S	0.007	Feet/Feet
8	Tc-overland	*47.2*	Minutes
9	--- Channel Flow Portion ---		
10	Unit Conversion, K (US)	0.0078	
11	Channel Length, L_{ch}	17952	Feet
12	Channel Slope, S	0.007	Feet/Feet
13	Tc-channel	*98.3*	Minutes
14	Tc (overland+channel)	***145.5***	Minutes

Generalized Terrain Condition	N
Pavement	0.02
Smooth, bare, packed soil	0.1
Poor grass, cultivated row crops, or moderately rough packed surfaces	0.2
Pasture, average grass	0.4
Deciduous forest	0.6
Dense grass, coniferous forest, or deciduous forest with deep litter	0.8

ES5 SOLUTION SKETCH

Eden Catchment

Paste | Align | Conditional Formatting | Styles | Actions | Themes | Aa

B6 | fx | 1200

	A	B	C
4	Unit Conversion, K (US)	0.828	
5	Retardance Coefficient, N	0.2	Table Look Up
6	Overland Length, L_{ov}	1200	Feet
7	Slope, S	0.003	Feet/Feet
8	Tc-overland	*42.6*	Minutes
9	--- Channel Flow Portion ---		
10	Unit Conversion, K (US)	0.0078	
11	Channel Length, L_{ch}	17424	Feet
12	Channel Slope, S	0.003	Feet/Feet
13	Tc-channel	*138.2*	Minutes
14	Tc (overland+channel)	***180.8***	Minutes
15			

Generalized Terrain Condition	N
Pavement	0.02
Smooth, bare, packed soil	0.1
Poor grass, cultivated row crops, or moderately rough packed surfaces	0.2
Pasture, average grass	0.4
Deciduous forest	0.6
Dense grass, coniferous forest, or deciduous forest with deep litter	0.8

ES5 SOLUTION SKETCH

- The time of concentration for the entire watershed if the reservoirs do not store water would be the sum of the two longest times
    - 180 min. + 145 min. = 325 min. (about 5:24 hours)

ES5 SOLUTION SKETCH

3) Estimate the time of concentration for the Harden Creek watersheds using the Upland method.

For upland method use two longest path lengths and reasonable guess of cover, then add these two values

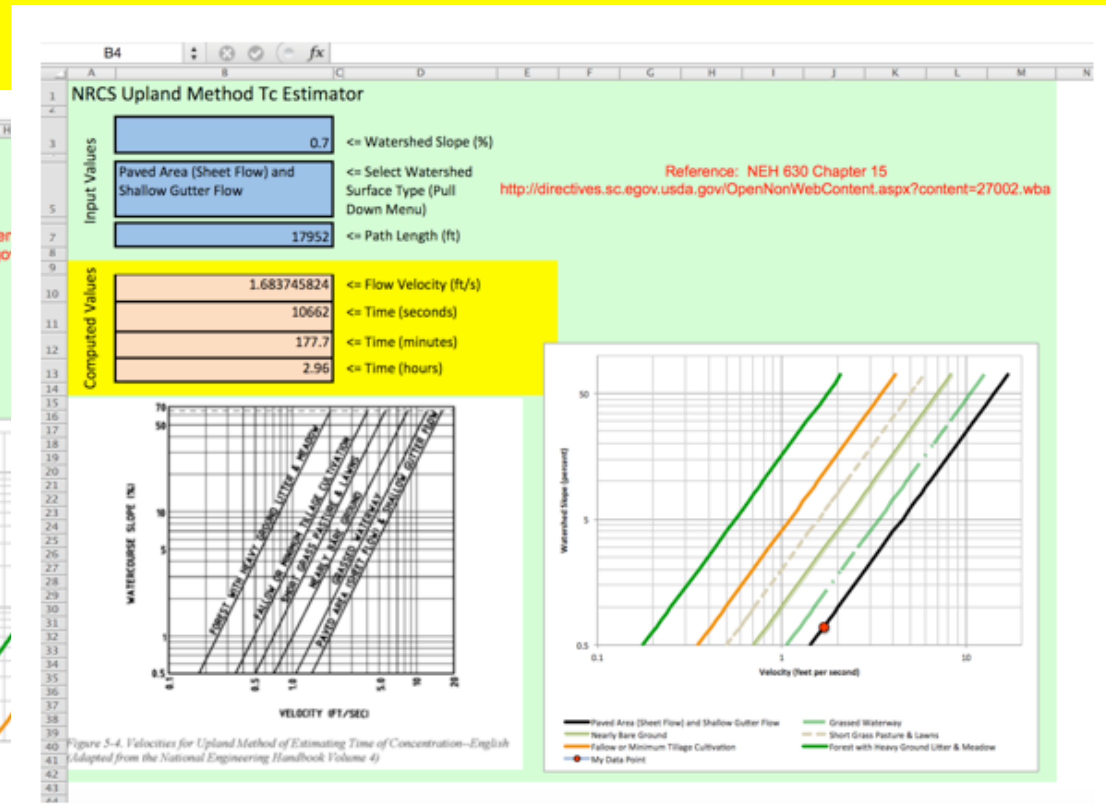


Figure 7. West Catchment Upland Estimate

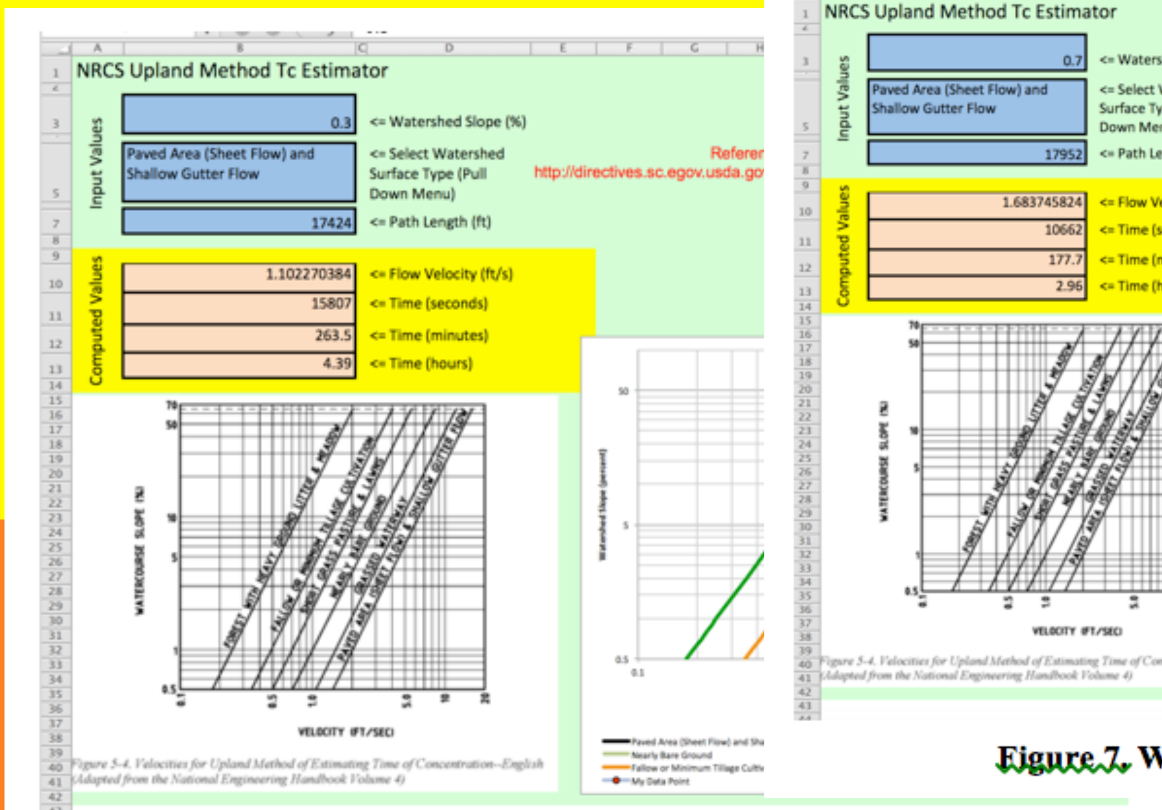
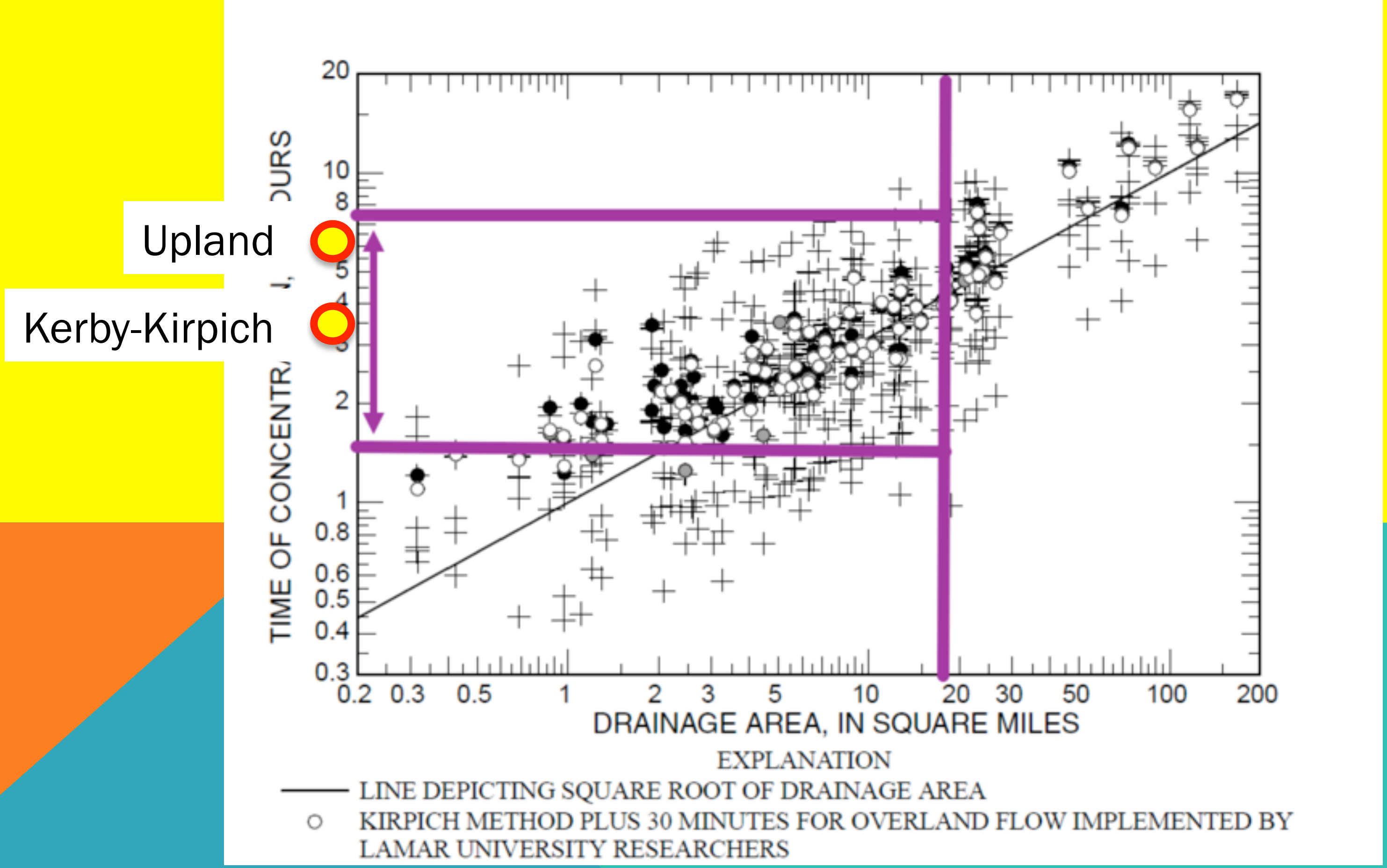


Figure 6. Eden Catchment Upland Estimate

ES5 SOLUTION SKETCH

- 177 min. + 263 min. = 440 min. (about 7:18 hours)
- Notice the effect of the reduced slope on the Eden catchment has pretty substantial impact on the estimate. Still both are about same order of magnitude and an estimate of 5-7 hours is about right.



PROJECT STATUS

Things you should already have completed

- Study area description, scope, and boundaries
- Watershed delineation
  - Sub-basin areas
  - Main channel lengths
  - Slopes
  - Connectivity diagram
- Time of concentration estimates
  - Sub-basin times
  - Combined times

PROJECT STATUS

Things you should already have completed

- Sketches of the 3-barrel and 4-barrel system
  - Elevations of toe (bottom) of embankment at the crossing
  - Elevations of the inlet and outlet at the crossing
  - Elevation of the road surface (and cross section sketch)
- Sketches of the SCS dams
  - Elevation of the spillway crests
  - Elevation of the riser pipes (outlets)
- Soil Properties
  - Infiltration rates (Green-Ampt values from textural description and WSS)
  - CN estimates (from WSS soils description)

PROJECT STATUS

Things you should already have completed

- Risk Levels
  - XX-year for the road-type from Table 4.1
  - 100-year check-storm
- Design storms
  - XX-Year for the watershed time (research how long as function of Tc)
  - 100-Year for the watershed time
  - SCS XX-Year and 100-Year, 24 hour (because its easy)
- Qxx and Q100 from Regional Regression Equations
  - because its easy
  - provides an order-of-magnitude estimate

HYDROGRAPHS

- A hydrograph is a plot (or paired time-discharge values) of discharge versus time for a location (on a stream)
- The ideal hydrograph has
  - Rise portion
  - Peak portion
  - Recession portion
  - Inflection point
- The hydrograph pictured also has a baseflow component
  - flow in absence of a storm

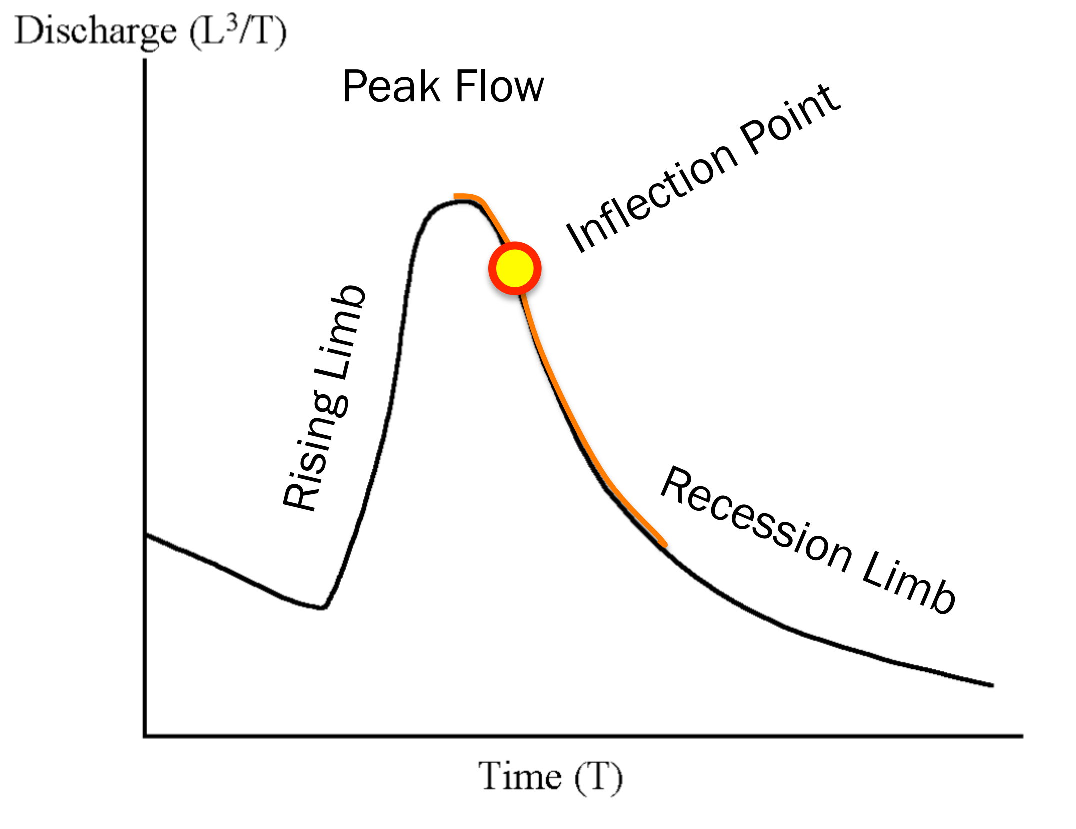


Figure 1: Idealized hydrograph

HYDROGRAPHS

- Baseflow separation is a first step in analysis – several methods
- Constant discharge method
- When rising limb starts – declare that value to constant rate during the event, rejoin as recession limb.
    - All flow above the value is declared storm flow

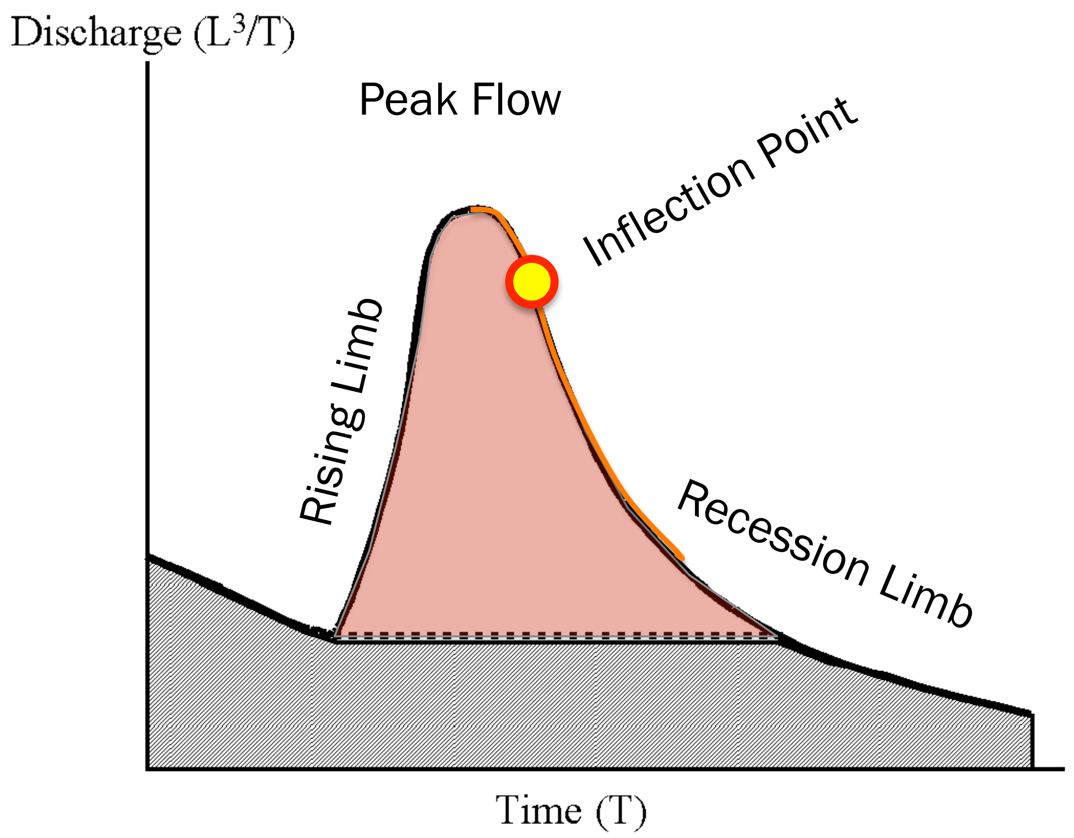


Figure 2: Baseflow Separation - Constant Discharge

HYDROGRAPHS

- Baseflow separation is a first step in analysis – several methods
- Constant slope method
- When rising limb starts – draw a segment from that value to the inflection point on the recession limb
    - All flow above the value is declared storm flow
    - Hard for multiple peak hydrographs (real hydrographs may exhibit many peaks)

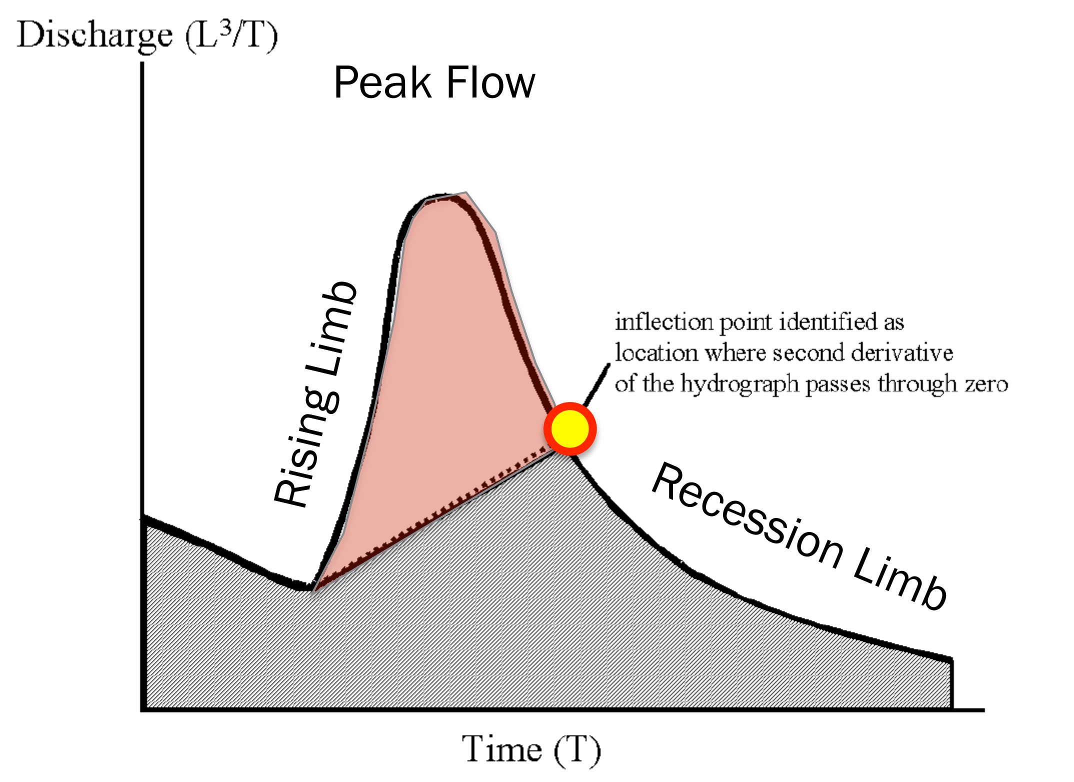


Figure 3: Baseflow Separation - Constant Slope

HYDROGRAPHS

- Baseflow separation is a first step in analysis – several methods
- Concave method
  - When rising limb starts – draw a segment from that value following the recession curve to a point beneath the peak flow.
  - Then draw a segment from the point above to the inflection point
- All flow above the segments are declared storm flow
  - Hard for multiple peak hydrographs (real hydrographs may exhibit many peaks)

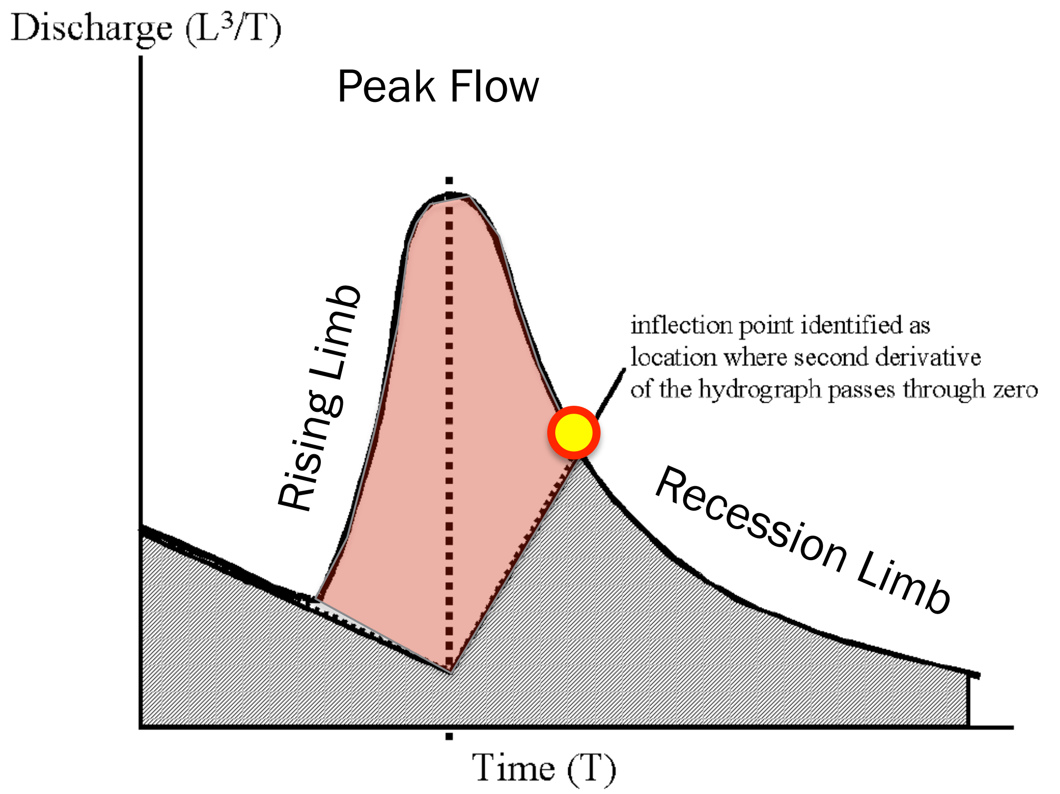


Figure 4: Baseflow Separation - Concave

HYDROGRAPHS

- **Baseflow separation is a first step in analysis – several methods**
- Concave method
  - When rising limb starts – draw a segment from that value following the recession curve to a point beneath the peak flow.
  - Then draw a segment from the point above to the inflection point
- **All flow above the segments are declared storm flow**
  - Hard for multiple peak hydrographs (real hydrographs may exhibit many peaks)

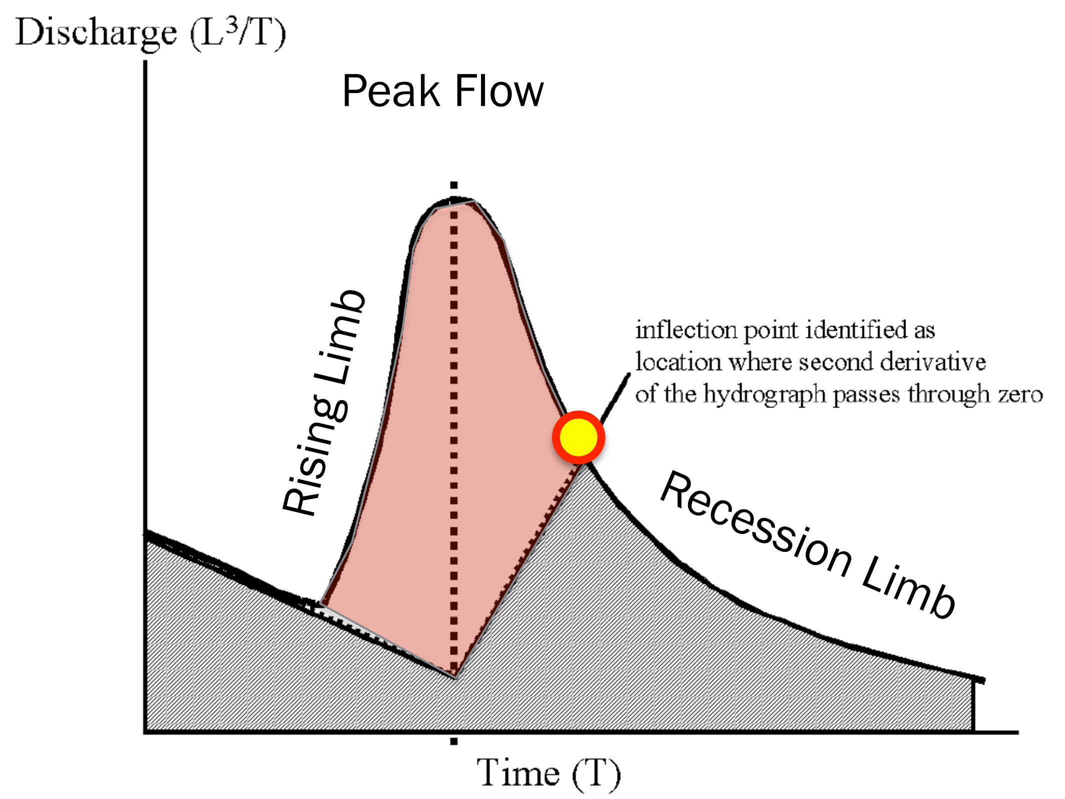


Figure 4: Baseflow Separation - Concave

HYDROGRAPHS

- There are a few more ways to accomplish baseflow separation
  - In Additional Readings the master-depletion curve method is outlined
- For many practical cases with multiple peaked hydrographs the constant discharge method is probably the most straightforward to apply (or use continuous simulation techniques – outside scope this course)

UNIT HYDROGRAPHS

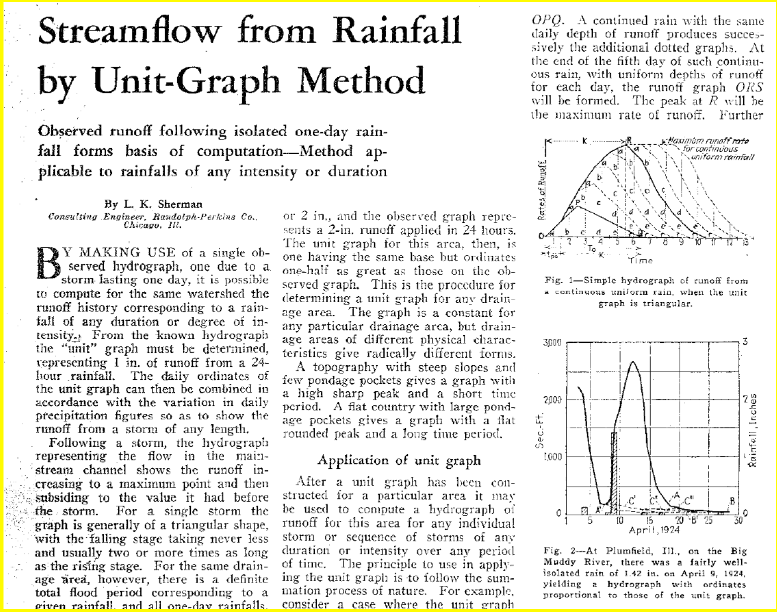
What is a unit hydrograph?

How are they used?

How are they built from data (analysis)?

How are they built when data do not exist (synthesis)?

WHAT IS A UNIT HYDROGRAPH?



Streamflow from Rainfall by Unit-Graph Method

Observed runoff following isolated one-day rainfall forms basis of computation—Method applicable to rainfalls of any intensity or duration

By L. K. Sherman
Consulting Engineer, Randolph-Perkins Co., Chicago, Ill.

BY MAKING USE of a single observed hydrograph, one due to a storm lasting one day, it is possible to compute for the same watershed the runoff history corresponding to a rainfall of any duration or degree of intensity. From the known hydrograph the "unit" graph must be determined, representing 1 in. of runoff from a 24-hour rainfall. The daily ordinates of the unit graph can then be combined in accordance with the variation in daily precipitation figures so as to show the runoff from a storm of any length.

Following a storm, the hydrograph representing the flow in the main-stream channel shows the runoff increasing to a maximum point and then subsiding to the value it had before the storm. For a single storm the graph is generally of a triangular shape, with the falling stage taking never less and usually two or more times as long as the rising stage. For the same drainage area, however, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls

or 2 in., and the observed graph represents a 2-in. runoff applied in 24 hours. The unit graph for this area, then, is one having the same base but ordinates one-half as great as those on the observed graph. This is the procedure for determining a unit graph for any drainage area. The graph is a constant for any particular drainage area, but drainage areas of different physical characteristics give radically different forms.

A topography with steep slopes and few pondage pockets gives a graph with a high sharp peak and a short time period. A flat country with large pondage pockets gives a graph with a flat rounded peak and a long time period.

Application of unit graph

After a unit graph has been constructed for a particular area it may be used to compute a hydrograph of runoff for this area for any individual storm or sequence of storms of any duration or intensity over any period of time. The principle to use in applying the unit graph is to follow the summation process of nature. For example, consider a case where the unit graph

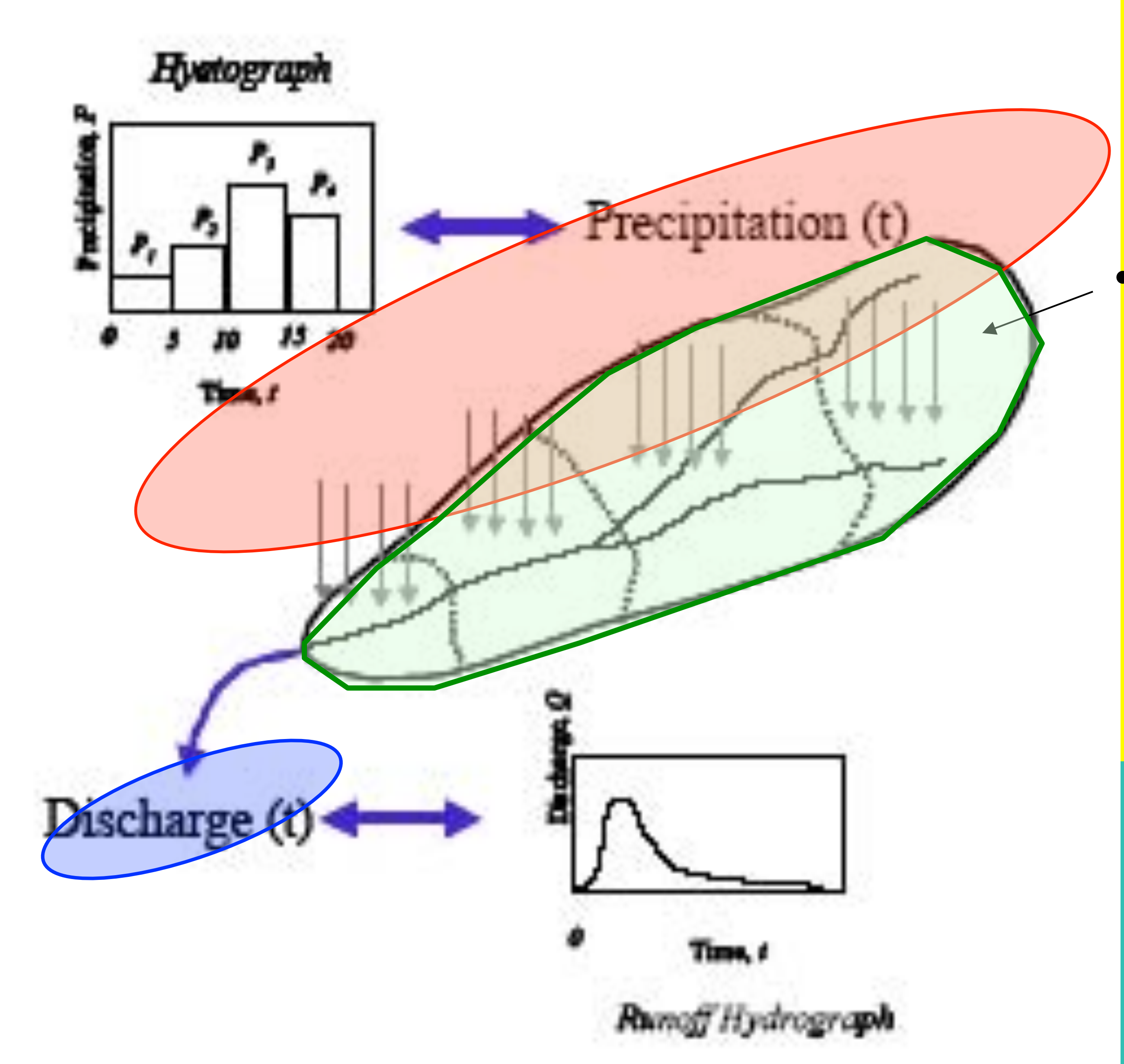
OPQ. A continued rain with the same daily depth of runoff produces successively the additional dotted graphs. At the end of the fifth day of such continuous rain, with uniform depths of runoff for each day, the runoff graph *ORS* will be formed. The peak at *R* will be the maximum rate of runoff. Further

Fig. 1—Simple hydrograph of runoff from a continuous uniform rain, when the unit graph is triangular.

Fig. 2—At Plumfield, Ill., on the Big Muddy River, there was a fairly well-isolated rain of 1.42 in. on April 9, 1924, yielding a hydrograph with ordinates proportional to those of the unit graph.

- Used to explain the time re-distribution of excess precipitation on a watershed
- Represents the response of the watershed at the outlet to a unit depth of EXCESS precipitation
  - EXCESS implies some kind of loss model is applied to the raw precipitation
  - Time re-distribution implies some kind of transfer behavior is applied
- L. K. Sherman 1932 is credited with seminal publication of the concept
  - Read the document in AdditionalReadings

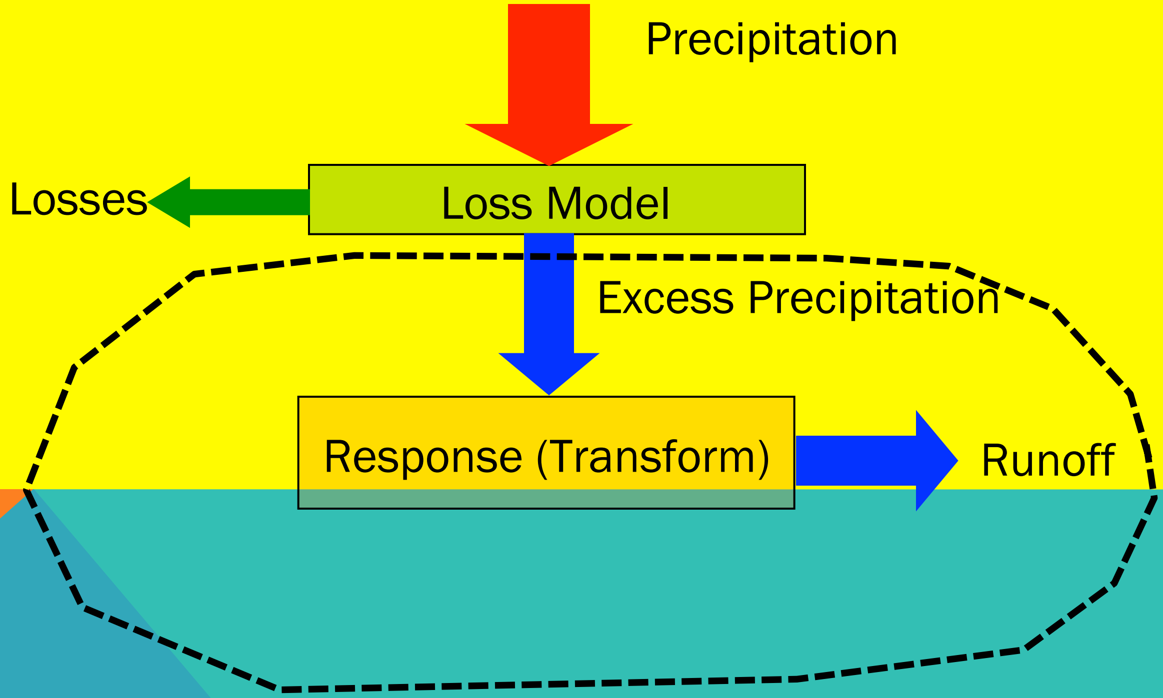
RESPONSE MODEL



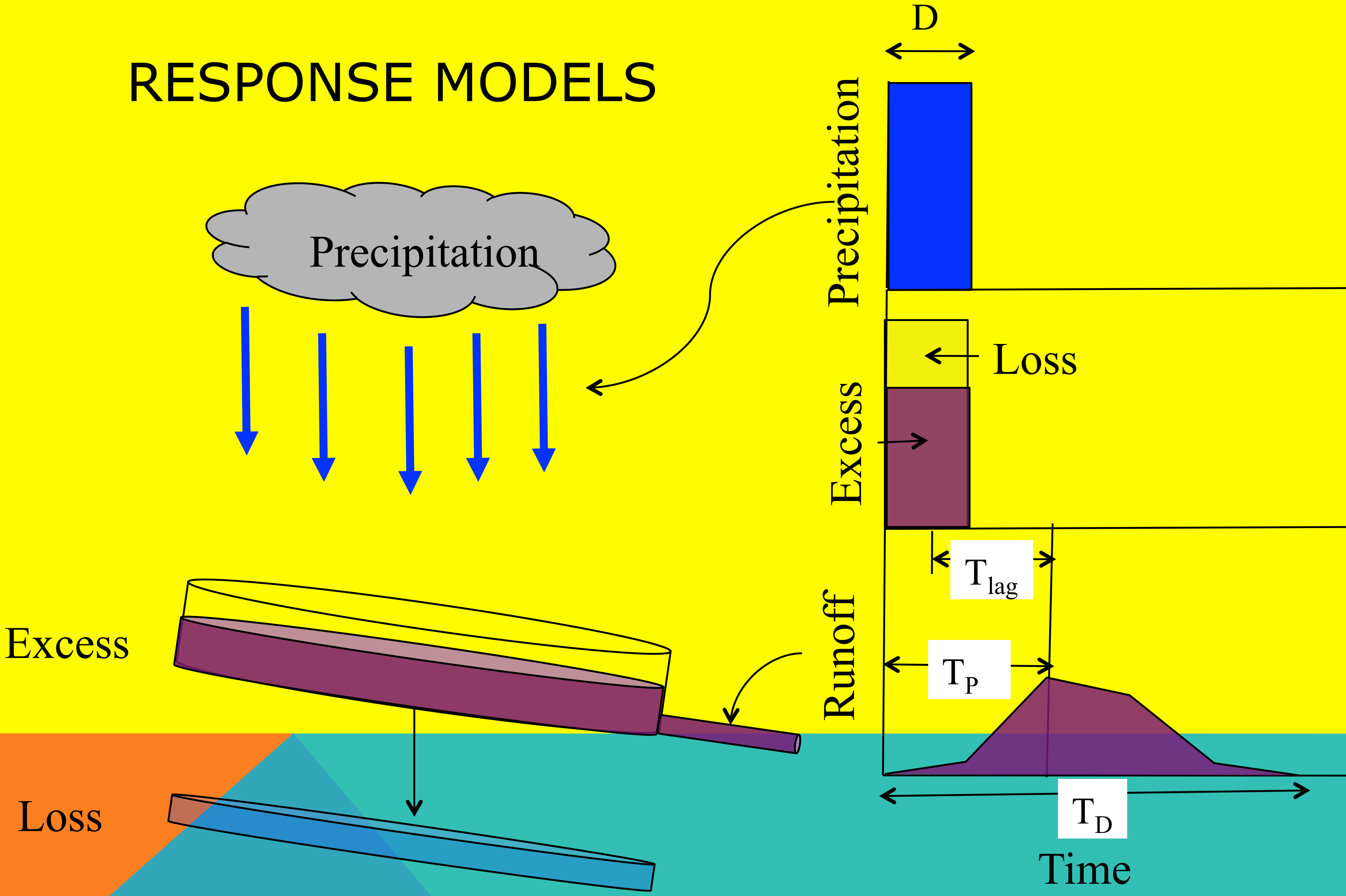
- Watershed
  - Losses
  - Transformation
  - Storage
  - Routing

RESPONSE MODEL

Response models convert the excess precipitation signal into a direct runoff hydrograph at the point of interest



RESPONSE MODELS



HYETOGRAPHS

Typically divided into three components

- Initial abstraction
- Loss
- Excess
    - This component becomes direct runoff

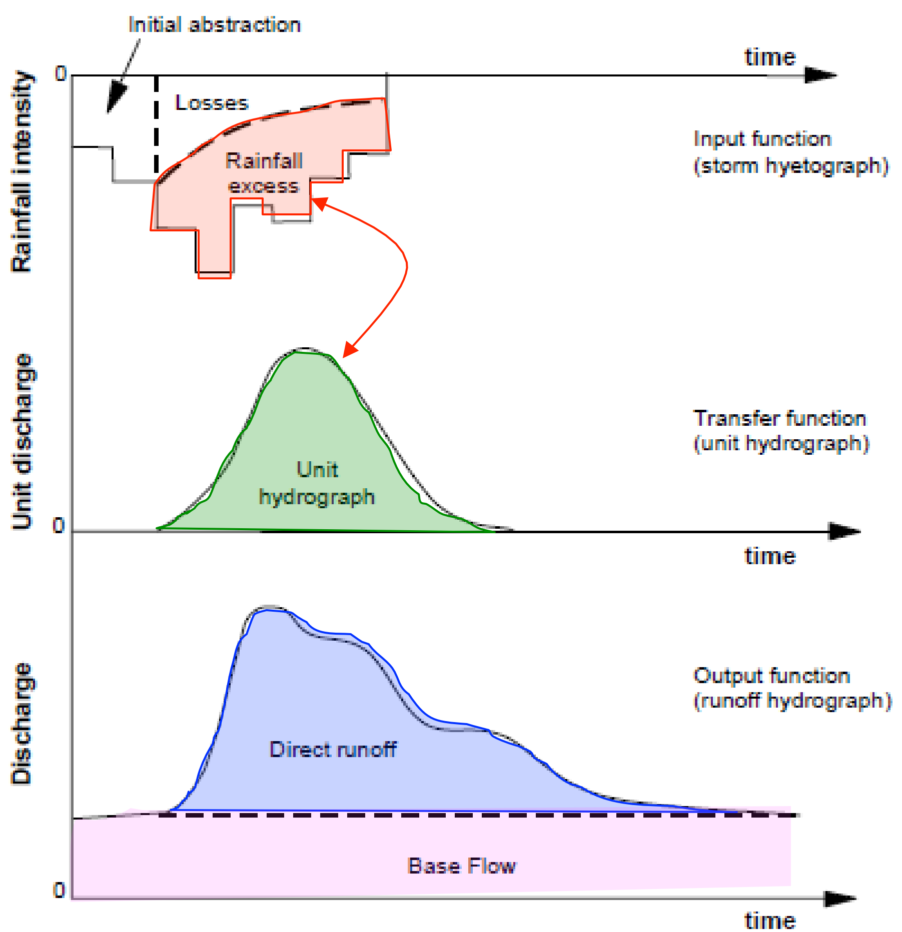
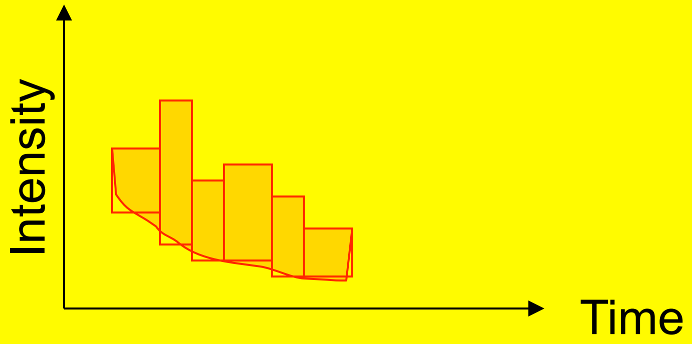


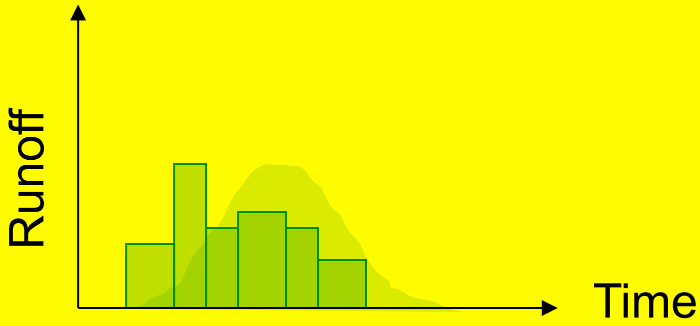
Figure 6.1. Rainfall/runoff as the system process

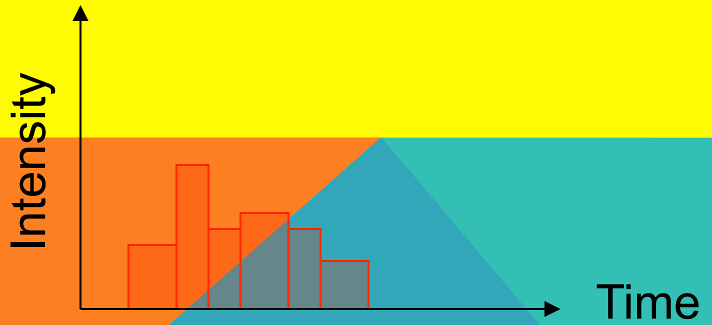
EXCESS HYETOGRAPHS

Excess as series of pulses



- If pulses went immediately to the outlet, would expect direct hydrograph to have same shape. Q(t) = i(i)A



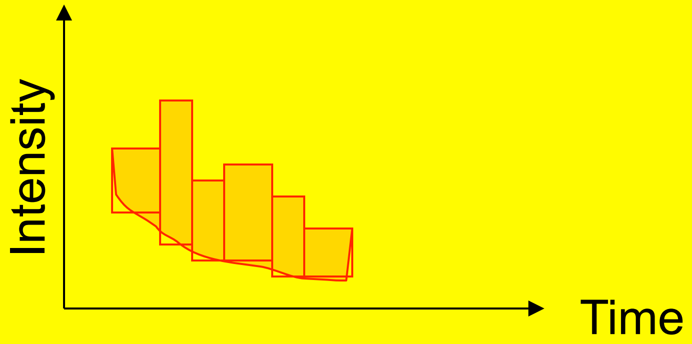


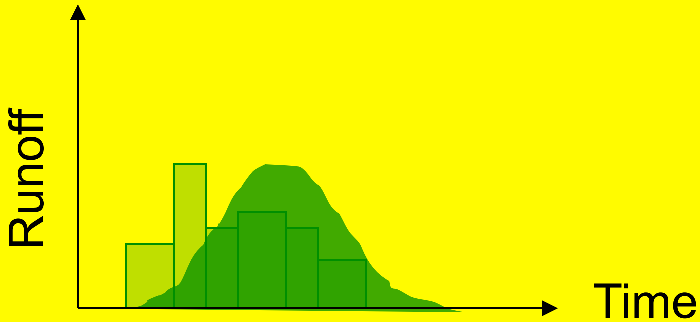
- But pulses are assumed uniform over whole area – close to outlet arrive sooner than far from outlet
  - Hence there is time re-distribution

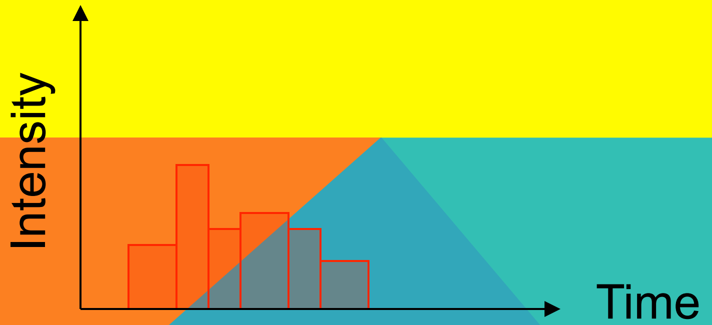
EXCESS HYETOGRAPHS

Excess as series of pulses

- If pulses went immediately to the outlet, would expect direct hydrograph to have same shape. Q(t) = i(i)A



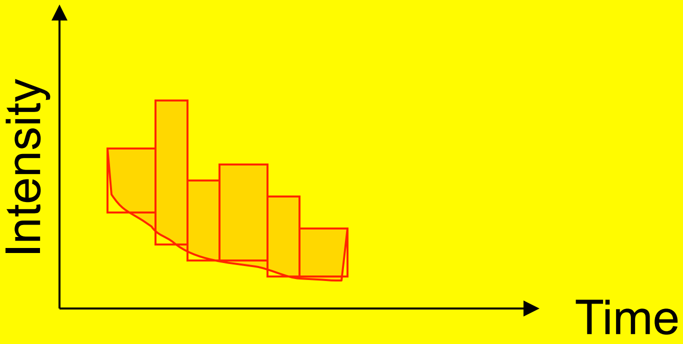




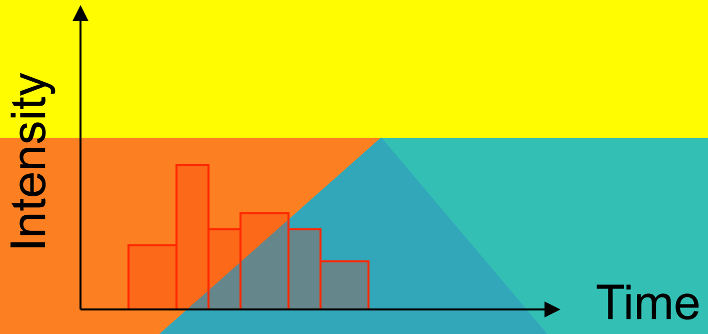
- But pulses are assumed uniform over whole area – close to outlet arrive sooner than far from outlet
  - Hence there is time re-distribution

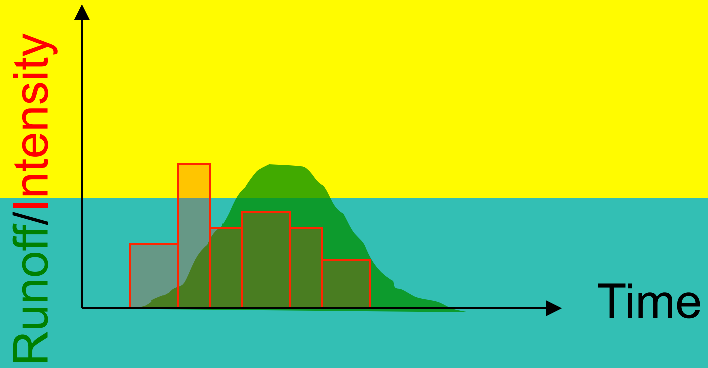
UNIT HYDROGRAPH

Excess as series of pulses



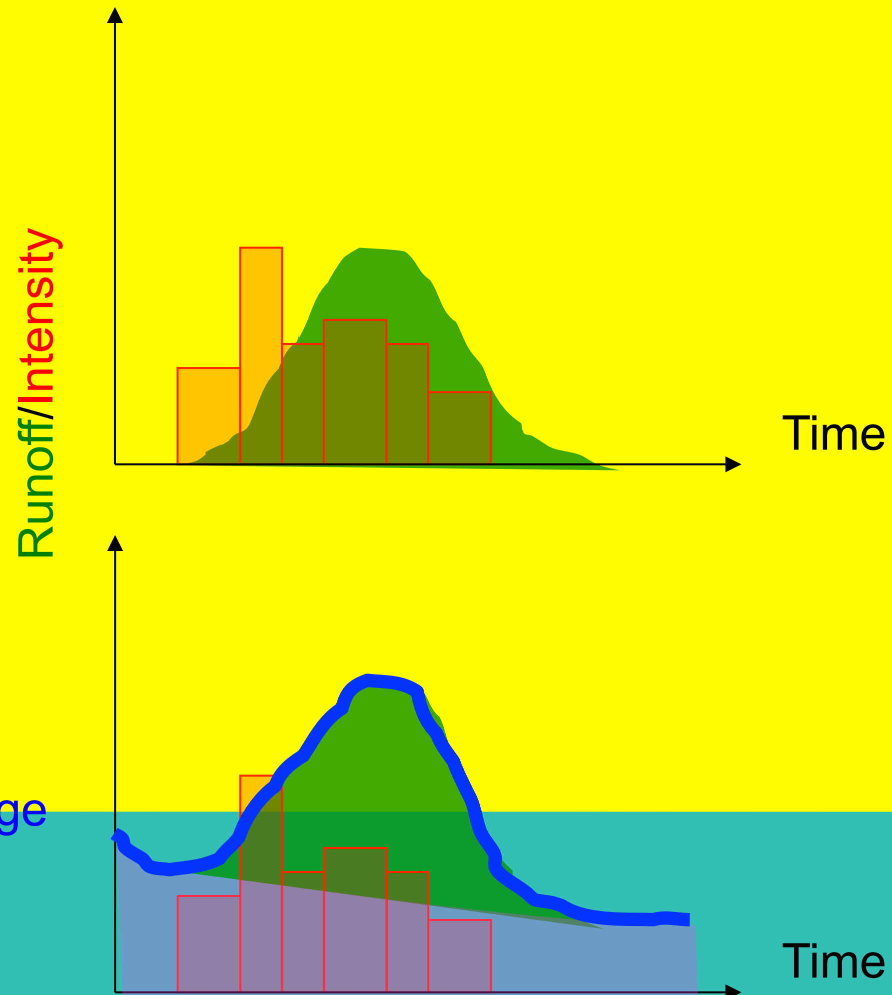
- The unit hydrograph is the “function” that maps the time-distribution of pulses of excess precipitation to the time-distribution of direct runoff.





TOTAL HYDROGRAPH

Total hydrograph is the algebraic combination (in time) of the direct runoff hydrograph and the baseflow hydrograph

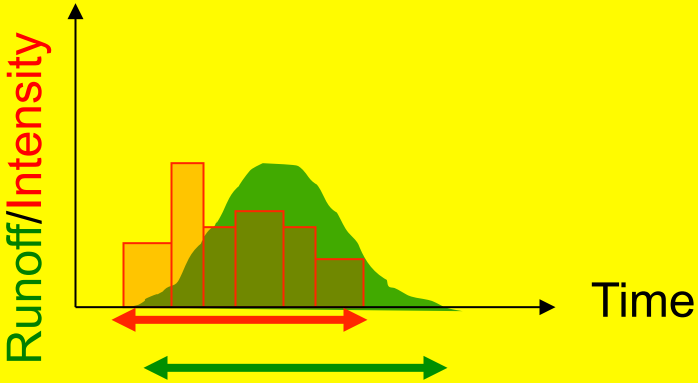


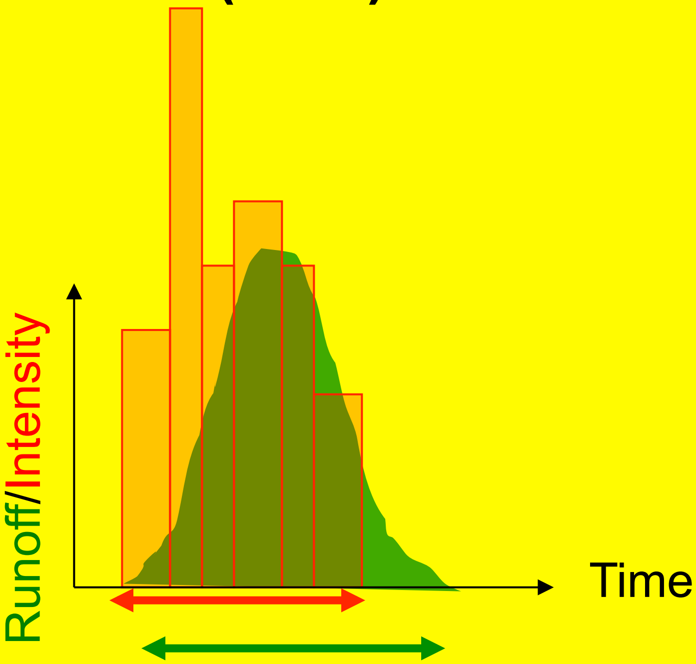
UNIT HYDROGRAPH ASSUMPTIONS

- Direct runoff duration (time) is same for all uniform-intensity storms of same duration (time)
- Two excess hyetographs of the same duration (time) will produce direct runoff hydrographs of the same duration (time) but with runoff rates proportional to the volumes (depth) of the excess hyetographs
- The time distribution of direct runoff from a given storm duration is independent of concurrent runoff from prior storms (no memory)

UNIT HYDROGRAPH

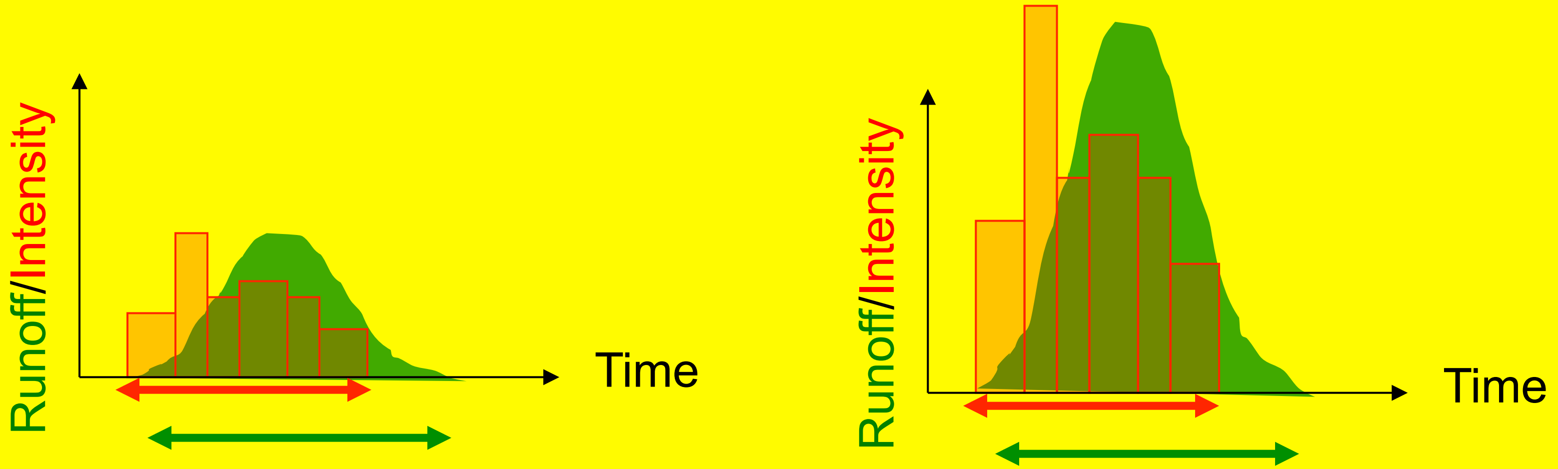
- **Direct runoff duration (time) is same for all uniform-intensity storms of same duration (time).**





UNIT HYDROGRAPH

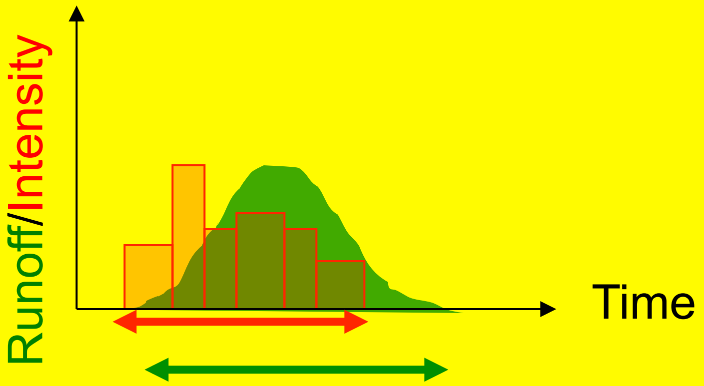
- Two excess hyetographs of the same duration (time) will produce direct runoff hydrographs of the same duration (time) but with runoff rates proportional to the volumes (depth) of the excess hyetographs.

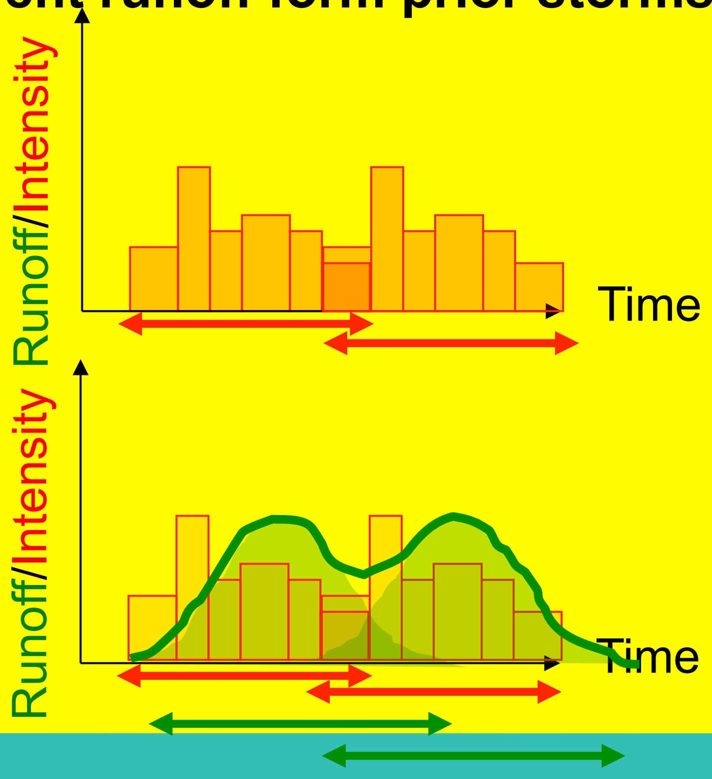


$$\int p(t)dt = \frac{1}{A}\int q(t)dt$$

UNIT HYDROGRAPH

- **The time distribution of direct runoff from a given storm duration is independent of concurrent runoff form prior storms.**





TIMING

Strictly speaking, each unit hydrograph has a particular duration associated with it, D in the diagram

- That duration must coincide with the time step size used in discrete aggregation

Thus a D-hour unit hydrograph is a response to a D-hour “pulse” of excess precipitation.

The flow associated with that response is reported every D-hours until there is no further response (T_D in the diagram)

TIMING

Each watershed has a characteristic response time, T_{lag} and T_P in the diagram

The characteristic time of the watershed is related to physical characteristics of the watershed- contributing area, slope, etc.

The time step size for aggregation must the same as the duration, and the time-to-peak for the watershed must be an integer multiple of that value.

UNIT HYDROGRAPHS

$$q(t) = \int_0^T r(\tau) f(t-\tau) d\tau$$

$$Q_n = \sum_{m=1}^{n<=M} P_m U_{n-m}$$

UNIT HYDROGRAPHS – EXAMPLE

- Continuous convolution

$$q(t) = \int_0^T r(\tau) f(t-\tau) d\tau$$

In watershed inches/time – multiply by area to get into flow units

- Discrete convolution

$$Q_n = \sum_{m=1}^{n<=M} P_m U_{n-m}$$

UNIT HYDROGRAPH - EXAMPLE

- The example is from CMM pp 216-223
- Method is called "back-substitution"

Unit Hydrograph (Classical Example)				
Time		Depth	Flow	
0.5	1	1.06	428	
1	2	1.93	1923	
1.5	3	1.81	5297	
2	4		9131	
2.5	5		10625	
3	6		7834	
3.5	7		3921	
4	8		1846	
4.5	9		1402	
5	10		830	
5.5	11		313	

Figure 7: Data for UH application example

UNIT HYDROGRAPH - EXAMPLE

- Build an equation array using the unknown unit weights and the known EXCESS precipitation depths, and the known discharge values

$$
\begin{array}{lllllllllll}
Q_1 = & P_1U_1 & & & & & & & & & \\
Q_2 = & P_2U_1 & +P_1U_2 & & & & & & & & \\
Q_3 = & P_3U_1 & +P_2U_2 & +P_1U_3 & & & & & & & \\
Q_4 = & & +P_3U_2 & +P_2U_3 & +P_1U_4 & & & & & & \\
Q_5 = & & & +P_3U_3 & +P_2U_4 & +P_1U_5 & & & & & \\
Q_6 = & & & & +P_3U_4 & +P_2U_5 & +P_1U_6 & & & & \\
Q_7 = & & & & & +P_3U_5 & +P_2U_6 & +P_1U_7 & & & \\
Q_8 = & & & & & & +P_3U_6 & +P_2U_7 & +P_1U_8 & & \\
Q_9 = & & & & & & & +P_3U_7 & +P_2U_8 & +P_1U_9 & \\
Q_{10} = & & & & & & & & +P_3U_8 & +P_2U_9 & +P_1U_{10} \\
Q_{11} = & & & & & & & & & +P_3U_9 & +P_2U_{10} & +P_1U_{11}
\end{array}
$$

Figure 8: UH Equation Array (Discrete Convolution)

UNIT HYDROGRAPH - EXAMPLE

- Express in vector-matric form (makes spreadsheet a little easier to interpret)

$$\begin{bmatrix} P_1 & & & \\ P_2 & P_1 & & \\ & & \ddots & \\ & & & P_M \end{bmatrix} \bullet \begin{bmatrix} U_1 \\ U_2 \\ \\ U_{N-M+1} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ \vdots \\ Q_N \end{bmatrix}$$

Figure 9: UH Equation Array (Vector-Matrix)

UNIT HYDROGRAPH – EXAMPLE

5.1.1 Back-substitution

Straightforward. Solve each equation successively (back substitute) for U .
$U_1 = Q_1/P_1 = 428/1.06 = 404 cfs/in.$
$U2 = (Q2 - P_2 U_1)/P1 = (1923 - 1.93 * 404)/1.06 = 1079 cfs/in.$
And so on . . . .

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Unit Hydrograph (Back-Substitute)																					
2	Observations						[P]												[U]	[Q*]	[Q]-[Q*]	
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11					
4	0.5	1	1.06	428		1	1.06	0	0	0	0	0	0	0	0	0	0		403.77	428	-0.000228	403.774
5	1	2	1.93	1923		2	1.93	1.06	0	0	0	0	0	0	0	0	0		1079	1923.02	-0.023434	1078.98
6	1.5	3	1.81	5297		3	1.81	1.93	1.06	0	0	0	0	0	0	0	0		2343.1	5297	0	2343.11
7	2	4	0	9131		4	0	1.81	1.93	1.06	0	0	0	0	0	0	0		2505	9130.5	0.5025618	2505.5
8	2.5	5	0	10625		5	0	0	1.81	1.93	1.06	0	0	0	0	0	0		1461	10624.3	0.6560813	1461.66
9	3	6	0	7834		6	0	0	0	1.81	1.93	1.06	0	0	0	0	0		453	7833.96	0.04	453.04
10	3.5	7	0	3921		7	0	0	0	0	1.81	1.93	1.06	0	0	0	0		379.5	3920.97	0.03	379.53
11	4	8	0	1846		8	0	0	0	0	0	1.81	1.93	1.06	0	0	0		276.9	1845.88	0.121	277.021
12	4.5	9	0	1402		9	0	0	0	0	0	0	1.81	1.93	1.06	0	0		170.5	1402.04	-0.042	170.458
13	5	10	0	830		10	0	0	0	0	0	0	0	1.81	1.93	1.06	0		-0.47	829.756	0.2442	-0.2258
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1.81	1.93	1.06		5.32	313.337	-0.3371	4.9829
15																						
16									Back substitute (lazy way)													
17									(1) Guess U													
18									(2) Compute Q*													
19									(3) Compute difference													
20									(4) Adjust U one-by-one to get difference small													
21									(5) Stop when difference is small percent of actual Q													
22									(6) If SOLVER works on machine - can automate - crashes on Mac (Microsoft hates Mac!)													

Figure 10: Backsubstitution in a spreadsheet

UNIT HYDROGRAPHS – EXAMPLE

- Observe that if the linear system has full ranked matrix (rows=columns) and non-zero diagonal, one could just solve the resulting linear equation for the unitgraph weights
- Probably better than manual back-substitution which is error prone
    - Many instances the system is over-determined – more equations than unknowns and an optimization technique is usually applied

UNIT HYDROGRAPHS – EXAMPLE

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Unit Hydrograph (Solve Linear System)																					
2	Observations						[P]												[U]	[Q*]	[Q]-[Q*]	
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs			1	2	3	4	5	6	7	8	9	10	11					
4	0.5	1	1.06	428		1	1.06	0	0	0	0	0	0	0	0	0	0		403.77	428	-0.00023	403.77
5	1	2	1.93	1923		2	1.93	1.06	0	0	0	0	0	0	0	0	0		1079	1923.02	-0.02343	1079
6	1.5	3	1.81	5297		3	1.81	1.93	1.06	0	0	0	0	0	0	0	0		2343.1	5297	0	2343.1
7	2	4	0	9131		4	0	1.81	1.93	1.06	0	0	0	0	0	0	0		2505	9130.5	0.502562	2505.5
8	2.5	5	0	10625		5	0	0	1.81	1.93	1.06	0	0	0	0	0	0		1461	10624.3	0.656081	1461.7
9	3	6	0	7834		6	0	0	0	1.81	1.93	1.06	0	0	0	0	0		453	7833.96	0.04	453.04
10	3.5	7	0	3921		7	0	0	0	0	1.81	1.93	1.06	0	0	0	0		379.5	3920.97	0.03	379.53
11	4	8	0	1846		8	0	0	0	0	0	1.81	1.93	1.06	0	0	0		276.9	1845.88	0.121	277.02
12	4.5	9	0	1402		9	0	0	0	0	0	0	1.81	1.93	1.06	0	0		170.5	1402.04	-0.042	170.46
13	5	10	0	830		10	0	0	0	0	0	0	0	1.81	1.93	1.06	0		-0.47	829.756	0.2442	-0.226
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1.81	1.93	1.06		5.32	313.337	-0.3371	4.9829

Manual Back-substitution
I must have made an arithmetic mistake here

	[A]											[x]	[b]	[Q*]
17	1.06	0	0	0	0	0	0	0	0	0	0	403.77	428	428
18	1.93	1.06	0	0	0	0	0	0	0	0	0	1079	1923	1923
19	1.81	1.93	1.06	0	0	0	0	0	0	0	0	2343.2	5297	5297
20	0	1.81	1.93	1.06	0	0	0	0	0	0	0	2505.4	9131	9131
21	0	0	1.81	1.93	1.06	0	0	0	0	0	0	1460.8	10625	10625
22	0	0	0	1.81	1.93	1.06	0	0	0	0	0	452.74	7834	7834
23	0	0	0	0	1.81	1.93	1.06	0	0	0	0	380.42	3921	3921
24	0	0	0	0	0	1.81	1.93	1.06	0	0	0	275.77	1846	1846
25	0	0	0	0	0	0	1.81	1.93	1.06	0	0	170.93	1402	1402
26	0	0	0	0	0	0	0	1.81	1.93	1.06	0	0.8985	830	830
27	0	0	0	0	0	0	0	0	1.81	1.93	1.06	1.7752	313	313

=MMULT(MINVERSE(G17:Q27),T17:T27)

Solution of [A][x]=[b]
Where [A]=[P]
[x]=[U]
[b]=[Q_obs]

SUMMARY

- Unit hydrographs map the excess precipitation signal to the outlet\
- Base-flow separation isolates the total discharge from the storm-induced discharge
- Loss models are implicit; the unit hydrograph maps excess to the outlet
- Back-substitution (linear equation) method illustrated.

NEXT TIME

Unit Hydrographs (continued)

- CMM pp. 201-223
- Analysis
  - Least-Squares Method
- How to Use the UH
- Synthesis
  - NRCS DUH
  - Clark UH
- HEC-HMS Transform Model