SCRIPT

FM PROBLEMS RELY UPON EXPERIMENTAL DATA. MOST CASES EMPIRICAL RESULTS APPLY TO GENERAL SITUATIONS, SUCH THAT ENGINEERS NEED THE DATA IN NORMAL DESIGN PRACTICE.

DATA REPORTED IN HANDBOOKS, JOURNALS, ON-LINE AUTHORATIVE SOURCES.

EXAMPLES ARE FRICTION LOSS COEFFICIENTS FOR PIPES, VALVES, AND OTHER FITTINGS; DRAG COEFFICIENTS; FLUID PROPERTIES

BOARD

DIMENSIONAL ANALYSIS & SIMILITUDE

TOOL(S) TO USE EXPERIMENTS (DIMENSIONAL ANALYSIS) AND TO CHANGE SCALES (SIMILITUDE)

EXPERIMENT ≡ DIMENSIONAL ANALYSIS

SCALING ≡ SIMILITUDE

SCRIPT

PHYSICAL MODELING COMMON IN FM.

SCALE "MODELS" USED TO STUDY AND ESTIMATE BEHAVIOR AT FULL SCALE "PROTOTYPE"

TESTBEDS COMMON IN AEROSPACE ENGINEERING USUALLY SEVERAL SCALE MODELS, THEN FIRST PROTOTYPE.

THE FIRST ONE IS CALLED A TESTBED AND USES PROVEN ENGINES AND OTHER FEATURES TO GUARENTEE FLIGHT. *

BOARD

MANY PRACTICAL PROBLEMS HAVE UNUSUAL ENOUGH GEOMETRY THAT STUDIES ON A REPLICA AT SMALL SCALE ARE USED TO PREDICT FLOW PATTERNS, PRESSURE VARIATION, FRICTIONAL LOSS

WHEN SUCH TESTS PERFORMED

REPLICA ≡ MODEL

FULL SIZE ≡ PROTOTYPE

AEROSPACE USES A SLIGHTLY DIFFERENT APPROACH, OFTEN TESTING FULL SCALE WITH A "TESTBED"

SCRIPT

* BOEING'S ENTRY IN THE JSF COMPETITION A DECADE AGO WAS FIRST EVER ALL-COMPUTER DESIGN; IT EVENTUALLY WORKED, BUT WAS NOT SELECTED FOR PRODUCTION

BOEING 757 WAS ALSO ALL COMPUTER.
- HAD ENGINE ISSUES DISCOVERED DURING UNSCHEDULED "TESTBED"

CONSIDER USUAL ORIFICE PLATE MOUNTED "BACKWARDS": CONSTRICTS LIKE A NOZZLE BUT IN OPPOSITE DIRECTION USUALLY ASSOCIATED WITH A NOZZLE.

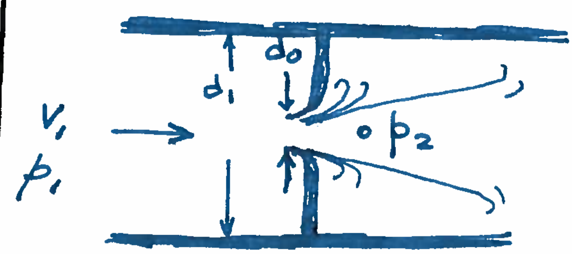
BOARD

EXACT SOLUTIONS FOR ALL HYDROSTATIC AND MANY LAMINAR FLOW EXIST.

GENERAL CASES (RANS) ON SUPERCOMPUTERS YIELD ONLY FAIR RESULTS.

OLDER RESULTS ARE ONLY AVAILABLE IN DIMENSIONLESS FORM

CONSIDER



SCRIPT

TEST PROCEDURE WOULD INVOLVE SEVERAL ORIFICES WITH DIFFERENT d_0

WE WOULD MEASURE $p_1 - p_2$ FOR DIFFERENT VELOCITIES V_1 AND LIQUID DENSITIES, ρ.

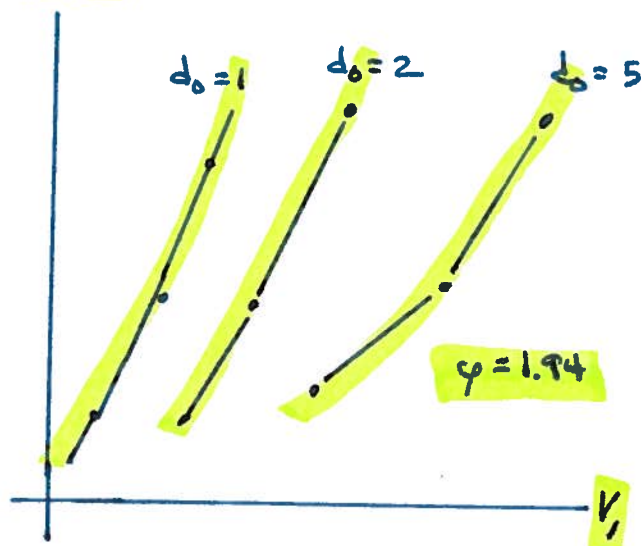
REASONABLE APPROACH WOULD BE TO MAKE MEASUREMENTS AT DIFFERENT VALUES V_1, d_0, ρ

THEN PLOT RESULTS ON A DESIGN CHART

SUCH AN APPROACH WOULD TAKE TREMENDOUS AMOUNT OF EFFORT (\$)

BOARD

$\Delta p = p_1 - p_2$



SCRIPT

EACH d_0 NEED 10–20 RUNS IN TRIPLICATE (30–60 INDIVIDUAL EXPERIMENTS), FOR EACH FLUID (ρ).

SUPPOSE WANTED TO EXAMINE

HEXANE
WATER
GLYCERINE

OVER 90 EXPERIMENTS, FOR EACH ORIFICE PLATE.

BOARD

A MORE TYPICAL SCHEME IS TO CONJECTURE BERNOULLI/ENERGY MIGHT APPLY

$$p_1 + \frac{\rho V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

WE WOULD THEN ISOLATE Δp

$$\Delta p = p_1 - p_2 = \rho \frac{V_2^2}{2} - \rho \frac{V_1^2}{2}$$

THEN DIVIDE BOTH SIDES BY UPSTREAM VELOCITY HEAD

$$\frac{\Delta p}{\rho \frac{V_1^2}{2}} = \frac{\frac{\rho V_2^2}{2}}{\rho \frac{V_1^2}{2}} - \frac{\rho \frac{V_1^2}{2}}{\rho \frac{V_1^2}{2}}$$

SCRIPT

WE KNOW FROM CONTINUITY THAT

$$\frac{V_2}{V_1} = \frac{A_1}{A_2},$$

BUT WE DONT REALLY KNOW A_2, HOWEVER WE ARE INTERESTED IN EQUATION IN TERMS OF A_0 ANYWAY, BECAUSE WE CAN CONTROL SIZE OF THE ORIFICE

BOARD

$$\frac{\Delta p}{\rho \frac{V_1^2}{2}} = \frac{V_2^2}{V_1^2} - 1$$

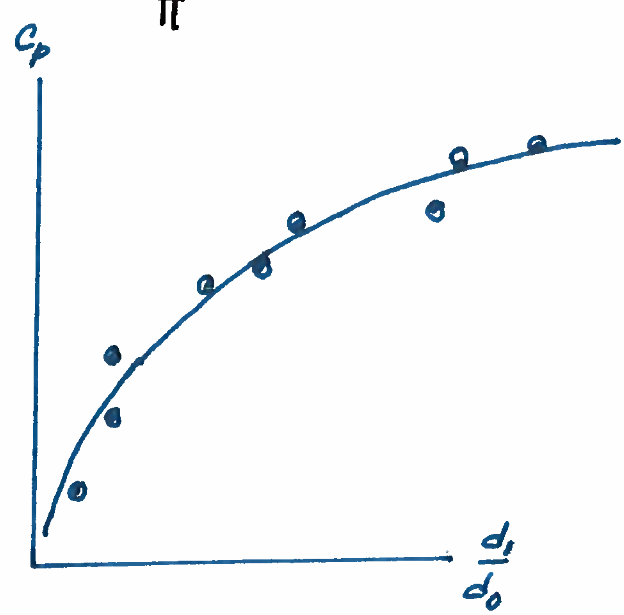
"INVENT" A COEFFICIENT

$$\frac{V_2}{V_1} = f\left(\frac{A_1}{A_0}\right) = f_1\left(\left(\frac{d_1}{d_0}\right)^2\right)$$

IN WORDS WE ARE STATING:

"THE VELOCITY RATIO IS EQUAL TO SOME FUNCTION OF DIAMETER RATIO"

SCRIPT	BOARD
SO NOW REWRITE THE EXPRESSION	$C_p = \dfrac{\Delta p}{\frac{\rho V_1^2}{2}} = \left[f_1\left[\left(\frac{d_1}{d_0}\right)^2\right]\right]^2 - 1$
AT THIS POINT IN THE ANALYSIS, WE HAVE NO IDEA OF THE EXACT FORM OF THE FUNCTION BUT WE HAVE RELATED THE DIAMETER RATIO (MEASUREABLE) TO THE PRESSURE → COEFFICIENT, ALSO MEASURABLE.	THE RHS IS SOLELY A FUNCTION OF THE DIAMETER RATIO, ABSORB THE "1" INTO C_p AND CALL IT (C_p) THE PRESSURE COEFFICIENT
NOW IF WE PLOT C_p VS d_1/d_0	$C_p = \left[f_2\left(\frac{d_1}{d_0}\right)\right]$

SCRIPT	BOARD
EVERYTHING COLLAPSES ONTO A SINGLE CURVE (WE HOPE!)	
TO COLLECT DATA FOR THIS CURVE MIGHT ONLY REQUIRE 20 EXPERIMENTS – TOTAL	
– WE WOULD WANT TO VARY APPROACH VELOCITIES TO TAKE FULL ADVANTAGE OF VERTICAL RANGE OF C_p GRAPH	THIS PROCESS (NON-DIMENSIONALIZATION) REDUCES THE NUMBER OF INDEPENDENT PARAMETERS FROM 5 TO 2. $(\Delta p, \rho, V_1, d_1, d_0)$ $\left(\dfrac{\Delta p}{\frac{\rho V_1^2}{2}}, \dfrac{d_1}{d_0}\right)$

SCRIPT

IN THE EXAMPLE, HAD AN IDEA OF GOVERNING EQUATION.

IN MANY PRACTICAL CASES WE HAVE NO A-PRIORI IDEA, AND SEEK STRUCTURE BY MEANS OF DIMENSIONAL ANALYSIS.

ALL EQUATIONS MUST BALANCE IN MAGNITUDE AND BE DIMENSIONALLY

BOARD

DIMENSIONS & EQUATIONS

ENGINEERING VARIABLES EXPRESSED IN A LIMITED NUMBER OF BASIC DIMENSIONS

$$[F] = \frac{ML}{T^2}$$

BRACKETS MEAN "DIMENSION OF..."

$$[\mu] = \frac{FT}{L^2} = \frac{M}{LT}$$

$$[p] = \frac{F}{A} = \frac{ML}{L^2T^2} = \frac{M}{LT^2}$$

SCRIPT

HOMOGENEOUS —

THE LEFT HAND SIDE MUST HAVE SAME DIMENSIONS AS RIGHT HAND SIDE.

BOARD

BUCKINGHAM π THEOREM

TOOL TO CONSTRUCT DIMENSIONLESS CORRELATIONS TO GUIDE EXPERIMENTS.

NUMBER OF DIMENSIONLESS GROUPS IS

$n - m$

$n \equiv$ NUMBER OF VARIABLES INVOLVED

$m \equiv$ NUMBER OF FUNDAMENTAL DIMENSIONS

THE DIMENSIONLESS PARAMETERS ARE CALLED Π; THEY ARE COLLECTED INTO GROUPS CALLED π GROUPS

SCRIPT

IF EQUATION DESCRIBING A SYSTEM HAS n DIMENSIONAL VARIABLES

IT CAN BE REARRANGED AND EXPRESSED IN TERMS OF $n-m$ DIMENSIONLESS GROUPS

BOARD

$$y_1 = f(y_2, y_3, \dots, y_n)$$

FUNCTIONAL STRUCTURE CHANGES!

$$\pi_1 = \Phi(\pi_2, \pi_3, \dots, \pi_{n-m})$$

THUS, IF DRAG FORCE F OF A LIQUID FLOWING PAST A SPHERE IS KNOWN TO BE A FUNCTION OF V, ρ, μ, d

THESE 5 VARIABLES HAVE

SCRIPT

BOARD

3 FUNDAMENTAL DIMENSIONS (M, L, T)

THUS WE WILL HAVE $5-3=2$ BASIC GROUPINGS THAT CAN BE USED TO COORELATE EXPERIMENTAL RESULTS IN THE FORM OF

$$\pi_1 = \Phi(\pi_2)$$

WE FIND Φ BY EXPERIMENT!

- BUT REQUIRE FAR FEWER EXPERIMENTS THAN WITH THE 5 ORIGINAL ~~EX~~ VARIABLES

SCRIPT

TWO METHODS TO FIND THE π-GROUPS ARE STEP-BY-STEP AND EXPONENT. BOTH PRODUCE SAME RESULT; STEP-BY-STEP APPROPRIATE FOR SIMPLE CASES.

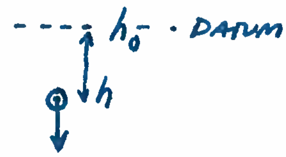
SUPPOSE WE DON'T KNOW HOW TO DERIVE RELATIONSHIP

BOARD

METHODS
1. STEP-BY-STEP
2. EXPONENT

EXAMPLE 1

OBJECT FALLING IN A VACUUM. ITS VELOCITY V IS A FUNCTION OF g AND DISTANCE TRAVELED h.



SCRIPT

FOR FALL VELOCITY. AND WILL DETERMINE BY DIMENSIONAL ANALYSIS AND EXPERIMENTATION

NEXT COMBINE THE DIMENSIONAL VARIABLES IN SUCH A WAY AS TO ELIMINATE BASIC DIMENSIONS AND DETERMINE π GROUPS

BOARD

STATE OUTCOME: $V = f(g, h)$

EXAMINE COMPONENTS:

$$[V] = \frac{L}{T}$$

$$[g] = \frac{L}{T^2}$$

$$[h] = L$$

SCRIPT

BUILD A TABLE.

OBSERVE h[L] IS COMMON TO ALL VARIABLES, HENCE A GOOD CANDIDATE FOR DIMENSIONAL ELIMINATION

BOARD

VARIABLE	[ ]	VARIABLE	[ ]	VARIABLE	[ ]
$\frac{V}{1}$	$\frac{L}{T}$	$\frac{V}{h}$	$\frac{L}{L \cdot T} = \frac{1}{T}$	$\frac{V}{h\sqrt{\frac{g}{h}}} = \frac{V}{\sqrt{gh}}$	$\frac{1}{1}$
$\frac{g}{1}$	$\frac{L}{T^2}$	$\frac{g}{h}$	$\frac{L}{T^2}\frac{1}{L} = \frac{1}{T^2}$		
$\frac{h}{1}$	$\frac{L}{1}$	$\frac{h}{h}$	1		

ONLY T REMAINS

NOW DIVIDE BY $\sqrt{T^2}$

SCRIPT

IN THIS EXAMPLE THE REMAINING DIMENSIONLESS PARAMETER WAS CONSISTENT WITH THE EXPECTED NUMBER OF π GROUPS.

WE HAD 3 VARIABLES AND 2 FUNDAMENTAL DIMENSIONS, THUS ONLY A SINGLE (3-2=1) PI GROUP.

BOARD

NOW THE RATIO $\frac{V}{\sqrt{gh}}$ IS A NON-DIMENSIONAL GROUP

THE FUNCTIONAL EQUATION IS WRITTEN DIMENSIONLESS AS

$$C = \frac{V}{\sqrt{gh}}$$

SUBTLETY: START WITH

$$V = f(g, h)$$

SCRIPT

BOARD

BUCKINGHAM'S THEORY STATES THAT THE EQUATION CAN BE REARRANGED AS

$$\pi_1 = \varphi(\pi_2, \pi_3 \ldots)$$

IF RHS HAS NO π GROUPS, THE FUNCTION IS A CONSTANT.

EXAMPLE 2

DRAG FORCE ON A SPHERE

$$F_d = f(V, \rho, \mu, D)$$

SCRIPT

BUILD TABLE

ALL VARIABLES INVOLVE LENGTH;
~~DIVIDE BY D~~
FACTOR BY D

NEXT 3/4 HAVE MASS,
TRY TO FACTOR M

NOW ALL REMAIN IS TIME, T

BOARD

5 VARIABLES
3 DIMENSIONS (M, L, T)

V	[ ]	V	[ ]	V	[ ]	V	[ ]
$\frac{F_d}{1}$	$\frac{ML}{T^2}$	$\frac{F_d}{D}$	$\frac{M}{T^2}$	$\frac{F_d}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F_d}{\rho V^2 D^2}$	$\frac{1}{1}$
$\frac{V}{1}$	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{VD}{DV}$	$\frac{1}{1}$
$\frac{\rho}{1}$	$\frac{M}{L^3}$	$\frac{\rho D^3}{1}$	$\frac{M}{1}$	$\frac{\rho}{\rho}$	$\frac{1}{1}$		
$\frac{\mu}{1}$	$\frac{ML}{T}$	$\frac{\mu}{D}$	$\frac{M}{T}$	$\frac{\mu D}{\rho D^3}$	$\frac{1}{T}$	$\frac{\mu}{\rho V D}$	$\frac{1}{1}$
$\frac{D}{1}$	$\frac{L}{1}$	$\frac{D}{D}$	$\frac{1}{1}$				

FACTOR L FACTOR M FACTOR T

SCRIPT

BOARD

THUS THE TWO π-GROUPS ARE

$$\pi_1 = \frac{F_d}{\rho V^2 D^2}$$

$$\pi_2 = \frac{\mu}{\rho V D}$$

AND THE "CORRELATION" WOULD BE

$$\frac{F_d}{\rho V^2 D^2} = \Phi\left(\frac{\mu}{\rho V D}\right)$$

SOME FUNCTION.

FIND Φ BY EXPERIMENTS!

SCRIPT

BOARD

UPON INSPECTION WE SEE THAT THE RIGHT-HAND SIDE IS THE RECIPROCAL OF REYNOLDS NUMBER - A COMMON DIMENSIONLESS GROUP IN ALL FLOWS

RATIO OF VISCOSITY TO INERTIA)

RELATES FRICTION AND MOMENTUM

SCRIPT

BEAWARE – THE FUNCTIONAL FORM IS STILL UNKNOWN – IT MUST BE DETERMINED BY EXPERIMENTATION.

BOARD

SUMMARY STEP-BY-STEP.

1) IDENTIFY ALL SIGNIFICANT VARIABLES EXPRESS AS FUNCTIONAL RELATION

$$Z = f(U, V, W, X, Y)$$

2) USE BUCKINGHAMS THEOREM TO PREDICT NUMBER EXPECTED DIMENSIONLESS GROUPS

3) MAKE A TABLE, COMBINE VARIABLES WITH OTHERS TO ELIMINATE A DIMENSION. THEN CHOOSE SECOND DIMENSION TO ELIMINATE

REPEAT UNTIL HAVE REMAINING DIMENSIONLESS π- GROUPS.

4) WRITE THE FINAL FORM OF

SCRIPT

BOARD

EXPRESSION IN TERMS OF π- GROUPS $\pi_1 = \psi(\pi_2 \dots)$

SCRIPT

BOARD

EXPONENT METHOD

USEFUL FOR CASES WHERE HARD TO FIGURE WHICH DIMENSIONS TO ELIMINATE

REPEAT THE SPHERE CASE

$$F_d = f(V, \rho, \mu, D)$$

5 = VARIABLES

3 = DIMENSIONS

5 − 3 = 2 Π-GROUPS

APPLY PRINCIPLE DIMENSIONAL HOMOGENITY

SCRIPT

BOARD

$$[F_d] = [V^a][\rho^b][\mu^c][D^d]$$

a, b, c, d ARE EXPONENTS TO BE SELECTED SO RHS HAS DIMENSION OF FORCE

NOW INSERT "DIMENSIONS"

$$\frac{ML}{T^2} = \left(\frac{L}{T}\right)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{LT}\right)^c \left(\frac{L}{1}\right)^d = \frac{L^{a-3b-c+d}\, M^{b+c}}{T^{a+c}}$$

NOW EXAMINE REQUIRED EXPONENTS

M: $b + c = 1$

L: $a - 3b - c + d = 1$

T: $a + c = 2$

SCRIPT

BOARD

THE ALGEBRAIC SYSTEM CONTAINS THE REQUIRED RELATIONSHIPS, BUT 3 EQN, 4 UNK — OVERDETERMINED SYSTEM.

USUAL APPROACH IS TO SELECT A VALUE FOR ONE OF EXPONENTS; THEN SOLVE FOR REMAINDER,

RESULTING SYSTEM NEEDS NON-ZERO DETERMINANT.

USEFUL RULE OF THUMB IS TO SELECT THE EXPONENT THAT OCCURS MOST FREQUENTLY (IN THIS CASE c)

SCRIPT

BOARD

$a = 2 - c$

$b = 1 - c$

$d = 2 - c$

NOW SUBSTITUDE BACK INTO "CORRELATION" EQUATION

$$V^{2-c}\rho^{1-c}\mu^{c}D^{2-c} = \rho V^2 D^2 \left(\frac{\mu}{\rho V D}\right)^{c}$$

AT THIS POINT WE DON'T REALLY NEED TO KNOW c (AND WONT KNOW) UNTIL EXPERIMENTS ARE COMPLETED

SCRIPT

$$F = \frac{mL}{T^2}$$

$$\rho V^2 D^2 = \frac{m}{\cancel{L^3}} \frac{\cancel{L^2}}{T^2} \cdot \cancel{L^2}$$

$$= \frac{mL}{T^2}$$

BOARD

BY CONVENTION SET $c = 1$ AND RESULT IS

$$F_d = \underbrace{\rho V^2 D^2 \left(\frac{\mu}{\rho V D}\right)^c}_{\text{NOT DIMENSIONLESS YET}}$$

$$\underbrace{\frac{F_d}{\rho V^2 D^2}}_{\pi_1} = \Phi\left(\underbrace{\frac{\mu}{\rho V D}}_{\pi_2}\right)$$

ACTUAL FUNCTIONAL FORM FOR Φ IS FOUND BY EXPERIMENT

SCRIPT

BOARD

IN ACTUAL EXPERIMENT PROBABLY EASIEST TO ADJUST V FIVE OR SIX VALUES, ADJUST D AND REPEAT, ADJUST D AGAIN.

REPEAT AT DIFFERENT T (TO IMPACT ρ & μ).

TOTAL 30 EXPERIMENTS, 3 MODELS, 5 FLOW SETTINGS.

USE 3 D TO DETECT CURVATURE

SCRIPT

GOOD EXERCISES
8.4; 8.6; 8.12

BOARD

COMMON Π-GROUPS (DIMENSIONLESS VARIABLES) ARE LISTED IN TABLE 8.3.

Reynolds Number is really important in pipelines

Froude Number is really important in open flows

Other groups are:

Peclet, Lewis, Schmidt, Fourier important in heat transfer & contaminant transport.

SCRIPT

BOARD

SIMILITUDE

THEORY & ART OF PREDICTING PROTOTYPE PERFORMANCE FROM MODEL OBSERVATIONS

THEORY INVOLVES RELATING DIMENSIONLESS NUMBERS TO MODEL & PROTOTYPE SCALES

BEST PRACTICAL EXAMPLE ARE PUMPS. PUMP CURVES RARELY REFLECT ACTUAL MEASUREMENTS, BUT

SCRIPT

*MANY MODEL STUDIES HAVE THIS AS A BASIC REQUIREMENT, IN SOME CASES

BOARD

INSTEAD ARE REPRESENTATIVE OF TYPES OF MANUFACTURERS PUMPS AT A HANDFUL OF OPERATING POINTS, ENGINEER USES SIMILARITY LAWS TO ADJUST TO ACTUAL APPLICATIONS

DIFFERENT KINDS OF SIMILARITY

GEOMETRIC SIMILARITY*

$$\frac{L_m}{L_p} = L_r$$

L_r IS CALLED THE SCALE RATIO.

$$A_r = L_r^2 \quad , \quad \forall_r = L_r^3$$

SCRIPT

NON-UNITY ASPECT RATIOS ARE NECESSARY (VERTICAL DISTORTION)

BOARD

KINEMATIC SIMILARITY

REQUIREMENT THAT MODEL HAVE EXACT RELATIVE VELOCITIES OF MOVING PARTS.

$$\frac{T_m}{T_p} = T_r \quad \text{(CHARACTERISTIC TIME)}$$

$$\frac{V_m}{V_p} = \frac{\frac{L_m}{T_m}}{\frac{L_p}{T_p}} = V_r = \frac{L_r}{T_r} \quad \text{(CHARACTERISTIC VELOCITY)}$$

SCRIPT

BOARD

CHARACTERISTIC ACCELERATIONS

$$\frac{a_m}{a_p} = \frac{L_r}{T_r^2}$$

CHARACTERISTIC DISCHARGE

$$\frac{Q_m}{Q_p} = \frac{L_r^3}{T_r}$$

KINEMATIC SIMILARITY IMPORTANT IN HYDRAULIC MACHINERY; ANGULAR VELOCITY WHICH HAS DIMENSIONS $1/T$ IS AN IMPORTANT PROPERTY

SCRIPT

BOARD

DYNAMIC SIMILARITY

MODEL AND PROTOTYPE HAVE EXACT RELATIVE FORCES APPLIED TO MASSES IN THE SYSTEM

$$\left(\frac{F_m}{F_p} = C \right)$$

Pg 303-305 USES SPILLWAY AS AN EXAMPLE.

START WITH (GRAVITY)

$$\frac{m_m \cdot a_m}{m_p \, a_p} = \frac{F_{gm}}{F_{gp}}$$

SCRIPT

BOARD

$Fr_m^2 = Fr_p^2$ (PRESERVATION OF FROUDE NUMBER)

FRICTION

$$\frac{m_m \cdot a_m}{m_p \cdot a_p} = \frac{F_{\nu m}}{F_{\nu p}} \qquad F \propto \mu V L$$

$\vdots$

$Re_m = Re_p$ (PRESERVATION OF REYNOLDS NUMBER)

PRESSURE

$$\frac{m_m \cdot a_m}{m_p \cdot a_p} = \frac{F_{P_m}}{F_{P_p}} \quad \cdots \quad C_{P_m} = C_{P_p}$$

SCRIPT

DYNAMIC SIMILARITY NOT ALWAYS POSSIBLE
- FOR INSTANCE IN A RIVER MODEL WE MAY FIND WE NEED A LIQUID WITH VISCOSITY OF AIR AND DENSITY OF MERCURY.

SUCH LIQUID DOES NOT EXIST ON EARTH, AND A COMPROMISE WOULD BE USED.

BOARD

SO DYNAMIC SIMILARITY IS ACHIEVED IF GEOMETRIC SIMILARITY EXISTS; Fr & Re NUMBERS ARE PRESERVED, AND PRESSURE COEFFICIENT (EULER NUMBER) IS PRESERVED.

DYNAMIC SIMILARITY MEANS THAT VALUES OF Π-GROUPS ARE EQUAL IN MODEL & PROTOTYPE

SCRIPT

BOARD

EXAMPLE 1

Open channel model with geometric ratio 5:1 is operated at $Q = 6.86\ ft^3/s$

What is prototype discharge?

$$\frac{L_M}{L_P} = \frac{1}{5}$$

$$\frac{\forall_m}{\forall_p} = \frac{L_M^3}{L_P^3} = \frac{(1)^3}{(5)^3} = \frac{1}{125}$$

$$Q_M = \frac{\forall_m}{T_m}$$

$$Q_p = \frac{\forall_p}{T_p}$$

SCRIPT

BOARD

$$\frac{Q_m}{Q_p} = \frac{\frac{\forall_m}{T_m}}{\frac{\forall_p}{T_p}} = \frac{\forall_m}{\forall_p} \cdot \frac{T_p}{T_m} = \frac{1}{125}$$

ASSUME NO ATTEMPT TO SCALE TIME

$$Q_p = 125\left(6.86\ ft^3/s\right) = 858\ ft^3/s$$

SCRIPT

BOARD

A 10:1 scale model is constructed to study flow patterns in a stormwater detention pond.

If prototype discharge is 200 ft³/s and model can produce a maximum flow of 2 ft³/s. What is time ratio?

$$\frac{L_m}{L_p} = \frac{1}{10} \qquad \frac{Q_m}{Q_p} = \frac{2}{200} = \frac{1}{100}$$

$$\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^3 \left(\frac{T_p}{T_m}\right)$$

$$\therefore \frac{T_p}{T_m} = \frac{Q_m}{Q_p}\left(\frac{L_p}{L_m}\right)^3 = \frac{1}{100}\left(\frac{10}{1}\right)^3 = \frac{1000}{100} = 10$$

SCRIPT

BOARD

$$\therefore T_m = 0.1\, T_p$$

SO EVERY 10 REAL SECONDS IS 1 MODEL SECOND. SCALING TIME OFTEN FORGOTTON, BUT USEFUL EXPERIMENTAL TRICK.

SCRIPT

BOARD

80 hp Pump to supply a prototype water system.

A model is built at 8:1 scale.

A velocity ratio of 2:1 is anticipated, how much pump power is needed in the model?

$$\frac{L_m}{L_p} = \frac{1}{8} \qquad \frac{V_m}{V_p} = \frac{1}{2} = \frac{L_m}{T_m} \cdot \frac{T_p}{L_p} = \frac{1}{8} \cdot \frac{4}{1} = \frac{1}{2}$$

$$\therefore \frac{T_m}{T_p} = \frac{1}{4}$$

$$\frac{F_m}{F_p} = \frac{\rho V_m \frac{L_m}{T_m^2}}{\rho V_p \frac{L_p}{T_p^2}}$$

SCRIPT

GOOD EXERCISES
8.44; 8.66; 8.77

BOARD

ASSUME SAME WORKING LIQUID IN MODEL & PROTOTYPE

THEN:

$$\frac{F_m}{F_p} = \frac{\cancel{\rho} V_m}{\cancel{\rho} V_p} \cdot \frac{L_m}{T_m^2} \cdot \frac{T_p^2}{L_p} = \frac{L_m^4 T_p^2}{L_p^4 T_m^2} = \left(\frac{1}{8}\right)^4 \left(\frac{4}{1}\right)^2$$

$$= \frac{16}{8^4} = \frac{\cancel{4} \cdot \cancel{4}}{8 \cdot 8 \cdot \cancel{4} \cdot 2 \cdot \cancel{4} \cdot 2} = \frac{1}{16^2} = \frac{1}{256}$$

$$\frac{P_m}{P_p} = \frac{F_m L_m}{T_m} \cdot \frac{T_p}{F_p L_p} = \left(\frac{F_m}{F_p}\right)\left(\frac{T_p}{T_m}\right)\left(\frac{L_m}{L_p}\right)$$

$$= \left(\frac{1}{256}\right)\left(\frac{4}{1}\right)\left(\frac{1}{8}\right) = \frac{1}{256} \cdot \frac{1}{2} = \frac{1}{512}$$

$$\therefore P_m = \frac{1}{512} P_p = \frac{80\,hp}{512} = 0.156\,hp.$$