AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852
TEXAS TECH UNIVERSITY
J.H. MURDOUGH
ASCE STUDENT CHAPTER
ENGINEERING TEXAS TECH

NAME CLEVELAND DATE 27FEB14
COURSE CE3305 SHEET 1 OF 6

SCRIPT

LINEAR MOMENTUM RELATES FORCES TO CHANGE IN VELOCITY.

ESSENTIALLY NEWTON'S SECOND LAW

USE REYNOLD'S THEOREM TO RELATE FORCES & SYSTEM q TO INTEGRAL MODEL

BOARD

MOMENTUM

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM IS

$$m\frac{d\underline{V}}{dt}\bigg|_{sys} = \Sigma \underline{F}$$

IF WE APPLY REYNOLD'S TRANSPORT THEOREM

EXTENSIVE PROPERTY

$$m\frac{d\underline{V}}{dt}\bigg|_{sys} = \frac{d}{dt}(M\cdot\underline{V})$$

SO $\beta = \dfrac{M\underline{V}}{M}$ INTENSIVE PER UNIT MASS

SCRIPT

BOARD

$\rho\underline{V}$ INTENSIVE PER UNIT VOLUME

RECALL REYNOLD'S TRANSPORT THEOREM

$$\frac{dB}{dt}\bigg|_{sys} = \frac{d}{dt}\int_{CV}\beta\rho\, d\forall + \int_{C.S.}\beta\rho(\underline{V}\cdot d\underline{A}) = \Sigma\underline{F}$$

FOR LINEAR MOMENTUM

$$B = \cancel{m\forall}\; m\underline{V}$$

$$\beta = \underline{V}$$

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NAME CLEVELAND DATE 27FEB14
COURSE CE3305 SHEET 2 OF 6

SCRIPT	BOARD
THE RESULT IS THE SUM OF EXTERNAL FORCES IS EQUAL TO THE RATE OF CHANGE OF LINEAR MOMENTUM ~~IN~~ C.V. PLUS THE NET MOMENTUM LEAVING ACROSS THE C.S. VECTOR EQUATION	$\underbrace{\Sigma \underline{F}}_{\text{NET FORCE ON MATL. INSIDE C.V.}} = \underbrace{\frac{d}{dt}\int_{CV} \rho \underline{V}\, d\forall}_{\text{RATE OF CHANGE OF LINEAR MOMENTUM IN CV}} + \underbrace{\int_{C.S.} \rho \underline{V}(\underline{V}\cdot d\underline{A})}_{\text{NET MOMENTUM LEAVING C.V. ACROSS C.S.}}$ NET FORCE ON MATL. INSIDE C.V. → BODY + SURFACE FORCES ON C.V.

SCRIPT	BOARD
APPLICATION USES 3 PRINCIPLES	1) SELECT AN INERTIAL REFERENCE FRAME (NON-ACCELERATING) 2) INDICATE POSITIVE & NEGATIVE COORDINATE DIRECTIONS 3) DRAW C.S./C.∀. i) INDICATE FORCES ii) VELOCITIES iii) $d\underline{A}$ VECTORS RELEVANT TO THE PROBLEM TWO EXAMPLES FOLLOW

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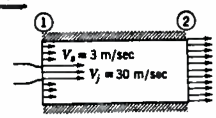
COURSE CE3305 SHEET 3 OF 6

A water jet pump has jet area 0.01 m² and jet speed 30 m/sec. The jet is within a secondary stream of water having speed $V_s = 3$ m/sec. The total area of the duct (the sum of the jet and secondary stream areas) is 0.075 m². The water is thoroughly mixed and leaves the jet pump in a uniform stream. The pressures of the jet and secondary stream are the same at the pump inlet. Determine the speed at the pump exit and the pressure rise, $p_2 - p_1$.



KNOWN + SKETCH

GOVERNING EQUATION(S)

FIND

V_2

$p_2 - p_1$

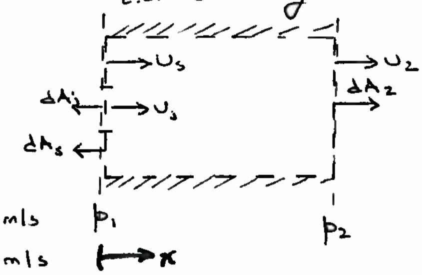
Solution

Continunity

$$0 = \frac{d}{dt}\int_{c.v.} \rho\, d\forall + \int_{c.s.} \rho \vec{V}\cdot d\vec{A}$$

0

C.V. = Water in pump at any time
C.S. = bounding surface



$$\int_{c.s.} \rho(\vec{V}\cdot d\vec{A}) =$$

$$-\rho U_j A_j - \rho U_s A_s + \rho U_2 A_2$$

$A_2 = 0.075\,m^2$ $\quad U_j = 30\,m/s$

$A_j = 0.01\,m^2$ $\quad U_s = 3\,m/s$

$A_s = 0.065\,m^2$

$$\cancel{\rho} U_j A_j + \cancel{\rho} U_s A_s = \cancel{\rho} U_2 A_2$$

$$\frac{(30)(0.01) + (3)(0.065)}{0.075} = U_2 = 6.6\ m/sec$$

Momentum

$$\Sigma F = \frac{d}{dt}\int_{c.v.} \vec{V}\rho\, d\forall + \int_{c.s.} \vec{V}\rho(\vec{V}\cdot d\vec{A})$$

NOTE C.V. DIAGRAM:

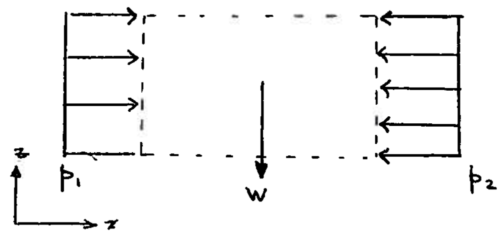
INDICATE +, − DIRECTIONS $\longrightarrow x$

DRAW C.∀ & C.S.

DRAW dA VECTORS

DRAW V VECTORS

THEN APPLY CONTINUNITY & MOMENTUM



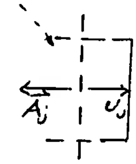
FORCE DIAGRAM

$$\sum F_x = p_1 A_1 \hat{i} - p_2 A_2 \hat{i} = p_1 - p_2 (A)\, \hat{i}$$

$$\frac{d}{dt}\int_{c.v.} \vec{V} \rho \, d\forall = 0$$ (steady flow; momentum in c.v. is not changing in time)

$$\int_{c.s.} \vec{V} \rho (\vec{V}\cdot d\vec{A}) = u_j(-\rho u_j A_j)\hat{i} + u_s(-\rho u_s A_s)\hat{i} + u_2(\rho u_2 A_2)\hat{i}$$

DEAL WITH $\underline{V\cdot A}$ CONCEPT(S)



$\vec{A_j} = -A_j \hat{i}$

$\vec{u_j} = u_j \hat{i}$

$u_j \hat{i}(-\rho u_j \hat{i} \cdot A_j \hat{i})$

$= -u_j(\rho u_j A_j)\hat{i}$

$\therefore$

$$(p_1 - p_2) A \hat{i} = \rho u_2^2 A_2 \hat{i} - \rho u_j^2 A_j \hat{i} - \rho u_s^2 A_s \hat{i}$$

(Drop vector notation for clarity)

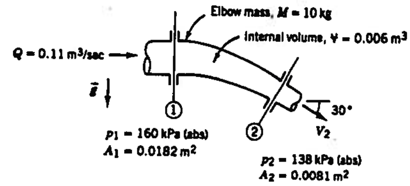
$$(p_1 - p_2) A = \rho (u_2^2 A_2 - u_j^2 A_j - u_s^2 A_s)$$

$$p_1 - p_2 = \frac{\rho}{A}(u_2^2 A_2 - u_j^2 A_j - u_s^2 A_s)$$

$$\Delta p = \frac{1000\,kg/m^3}{0.075\,m^2}\left((6.6)^2(0.075) - (30)^2(0.01) - (3)^2(0.065)\right)$$

$$\Delta p = -84240\ Pa \quad \left(\frac{kg\cdot m}{s^2}\cdot\frac{m^3}{m^3}\cdot\frac{1}{m^2}\right)$$

A 30° reducing elbow is shown. The fluid is water. Evaluate the components of force that must be provided by the adjacent pipes to keep the elbow from moving.



KNOWN + SKETCH

Solution

Continuity

$$0 = \frac{d}{dt}\int_{CV} \rho\, d\forall + \int_{C.S.} \rho \vec{V}\cdot d\vec{A}$$

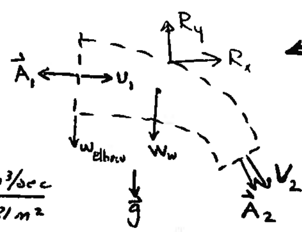
$$-\rho u_1 A_1 + \rho V_2 A_2 = 0$$

Incompressible

$$V_2 = \frac{u_1 A_1}{A_2} = \frac{Q_1}{A_2} = \frac{0.11 m^3/sec}{0.0081 m^2}$$

$$V_2 = 13.5 m/sec$$

$$u_1 = 6.04 m/sec$$



FBD (C.V.)

SHOW FORCES, COORDINATES, dA VECTORS, V VECTORS

Momentum

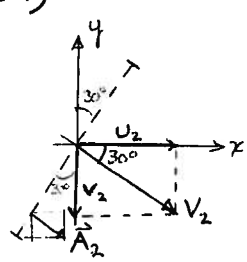
$$\Sigma F_x = \frac{d}{dt}\int u\rho\, d\forall + \int u \rho (u \cdot dA)$$

$$\Sigma F_y = \frac{d}{dt}\int v\rho\, d\forall + \int v \rho (v \cdot dA)$$

$$u_2 = V_2 \cos 30° \hat{i}$$

$$v_2 = -V_2 \sin 30° \hat{j}$$

$$\vec{A}_2 = A_2 \cos 30° \hat{i} - A_2 \sin 30° \hat{j}$$



V·dA CONCEPT

$$\frac{d}{dt}\int u\rho\, d\forall = \frac{d}{dt}\int v \rho\, d\forall = 0$$

$$\int u\varphi(u\cdot dA) = -u_1\varphi(u_1 A_1) + u_2\varphi\left(u_2\hat{i}\cdot(A_2\cos 30\hat{i} - A_2\sin 30\hat{j})\right)$$

$$= -u_1^2\varphi A_1 + u_2^2\varphi A_2\cos 30$$

$$\int v\varphi(v\cdot dA) = v_2\varphi\left(-v_2\hat{j}\cdot(A_2\cos 30\hat{i} - A_2\sin 30\hat{j})\right)$$

$$= v_2^2\varphi A_2\sin 30$$

$$\Sigma F_x = p_1 A_1 - p_2 A_2\cos 30 + R_x$$

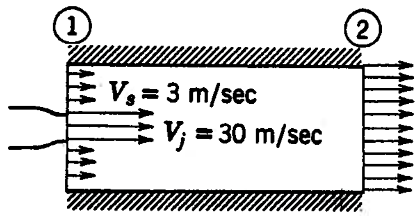
$$\therefore\ p_1 A_1 - p_2 A_2\cos 30 + R_x = u_2^2\varphi A_2\cos 30 - u_1^2\varphi A_1$$

$$\Sigma F_y = p_2 A_2\sin 30 + R_y - W_w - W_{elbow}$$

$$\therefore\ p_2 A_2\sin 30 + R_y - W_w - W_{elbow} = v_2^2\varphi A_2\sin 30$$

Substitute numerical values and solve for R_y & R_x

A water jet pump has jet area 0.01 m^2 and jet speed 30 m/sec. The jet is within a secondary stream of water having speed $V_s = 3$ m/sec. The total area of the duct (the sum of the jet and secondary stream areas) is 0.075 m^2. The water is thoroughly mixed and leaves the jet pump in a uniform stream. The pressures of the jet and secondary stream are the same at the pump inlet. Determine the speed at the pump exit and the pressure rise, $p_2 - p_1$.



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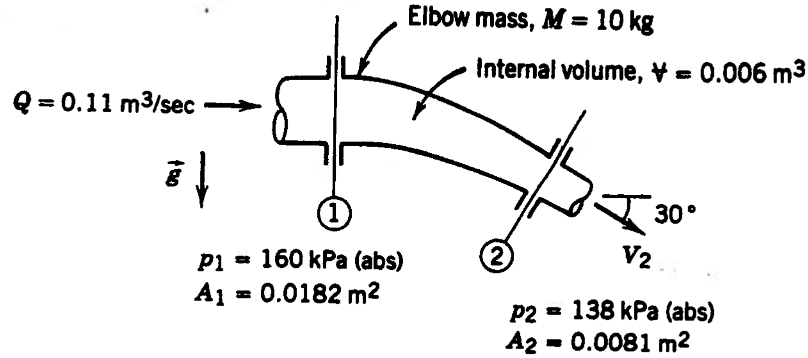


SCRIPT

BOARD

MOMENTUM PRINCIPLE

MOMENTUM IS USED TO FIND FORCES

FORCES KIND OF IMPORTANT
- BRIDGE PIERS
- RETAINING WALLS
- DAMS

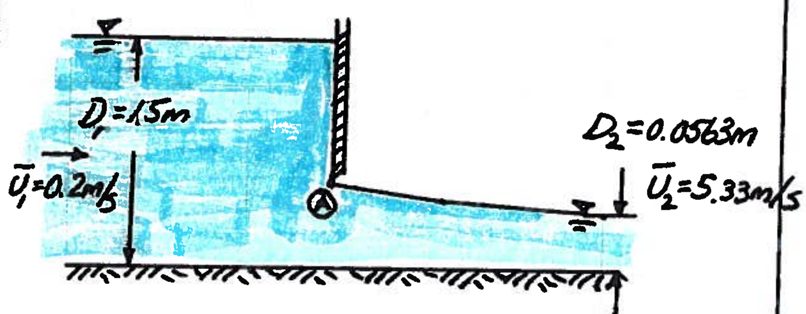
AS AN EXAMPLE, CONSIDER FORCE ON A SLUICE GATE (UNDERFLOW FROM A POWER-HOUSE)

SCRIPT

EXAMPLE OF USING MOMENTUM

ONE WOULD INITIALLY BE TEMPED TO TREAT GATE AS SUBMERGED PLATE AND USE HYDROSTATIC CALCULATIONS, EXCEPT AT POINT (A) THE PRESSURE IS ATMOSPHERIC - THE PRESSURE DIST. IS NOT HYDROSTATIC AT THE GATE!

BOARD

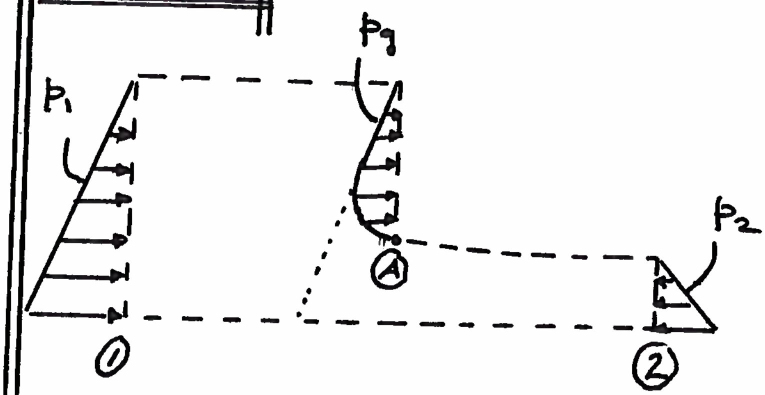


FIND FORCE OF WATER ON THE GATE

SCRIPT

SO INSTEAD OF TRYING TO FIND PRESSURE ON THE GATE, FIND FORCE OF GATE ON WATER - THEN BY EQUAL-OPPOSITE ACTION-REACTION WE FIND FORCE OF WATER ON GATE.

BOARD



PRESSURE AT GATE (A) IS NOT HYDROSTATIC - NEED A DIFFERENT APPROACH

-MOMENTUM!

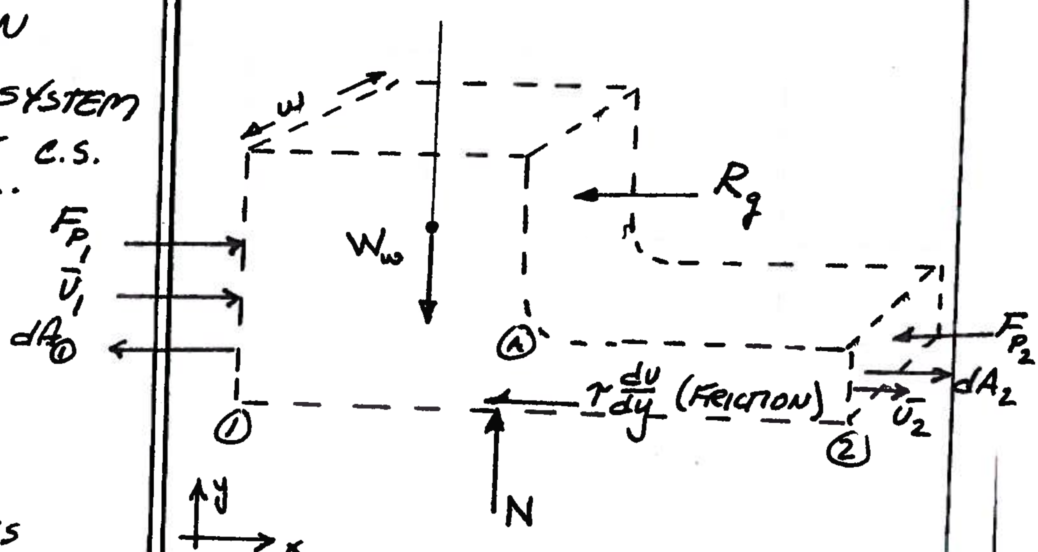
FIRST TAKE ABOVE SKETCH AS A CONTROL VOLUME

SCRIPT

DRAW THE C.V.
PUT FORCES IN COORDINATE SYSTEM
SHOW $\underline{dA}$ AT C.S.
SHOW $\bar{U}_{IN}, \bar{U}_{OUT}$ ...
AT C.S.

NOW READY FOR ANALYSIS

BOARD



SCRIPT

ASSUMPTIONS

DISTANCE UPSTREAM & DOWNSTREAM WILL BE RELATIVELY SMALL (100's OF FEET)

- NEGLECT FRICTION

- USE THE VELOCITY AND AREA DIRECTIONS TO GET ± SIGN FOR THE FLUX INTEGRALS

BOARD

MOMENTUM

$$\Sigma \underline{F} = \frac{d}{dt}\int_{C.V.} \rho \underline{v}\, d\forall + \int_{C.S.} \rho \underline{v} (\underline{v}\cdot d\underline{A})$$

0 STEADY FLOW
NON-DEFORMING C.V.
ρ = CONSTANT

$$\Sigma F_x = F_{P_1} - F_{P_2} - R_g = \int_{C.S.} \rho u (\underline{v}\cdot d\underline{A})$$

KNOWN $\bar{U}_1$ →
A_1 ←
①
$A_1 = D_1 \cdot W$

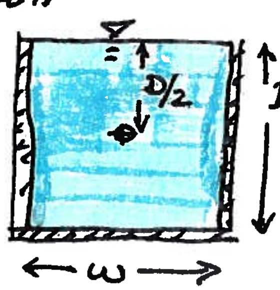
KNOWN $\bar{U}_2$ →
→ A_2
②
$A_2 = D_2 \cdot W$

SCRIPT

THE TWO PRESSURE FORCES ARE HYDROSTATIC SO FIND THE MAGNITUDE FROM

$$h = \frac{p}{\gamma}$$

RECALL GO TO CENTROID OF PROJECTED AREA



BOARD

$\therefore$

$$F_{P_1} - F_{P_2} - R_g = -\rho v_1^2 D_1 W + \rho v_2^2 D_2 W$$

→ THESE ARE HYDROSTATIC — STRAIGHT & PARALLEL STREAMLINES

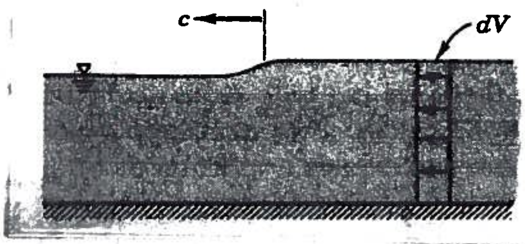
$$F_{P_1} = \underbrace{\frac{\rho g D_1}{2}}_{p = \gamma \bar{h}} \cdot w \cdot D_1$$

$$F_{P_2} = \frac{\rho g D_2}{2} \cdot w \cdot D_2$$

SCRIPT	BOARD
NOT GIVEN ω, SOLVE AS A FORCE PER WIDTH - $\frac{F}{w}$	NOW SOLVE FOR $\frac{R_g}{w}$ (FORCE PER UNIT WIDTH, ω,) $$\frac{R_g}{w} = \frac{\rho g}{2}\left[D_1^2 - D_2^2\right] + \rho\left[D_1 v_1^2 - D_2 v_2^2\right]$$
INSERT #'S WE HAVE FOUND R_g	NUMERICAL VALUES $\frac{R_g}{w} = 9.47\,kN/m$
USE NEWTONS 3RD LAW $F_{fluid-g} = -R_g$	SO FORCE OF GATE ON FLUID IS $9.47\,kN/m \leftarrow$ HENCE FORCE OF FLUID ON GATE IS $9.47\,kN/m \rightarrow$ $(9.47\,kN/m\ \hat{i})$

SCRIPT	BOARD

Propagation of small waves on a liquid free surface may be analyzed using a differential control volume. Consider a small solitary wave moving with speed c from right to left. Assume a small change in water surface elevation across the wave. To make the flow appear steady, choose a differential control volume that encloses the wave and moves with it. Apply conservation of mass and the momentum equation to derive an expression for wave speed. Be sure to include in your analysis the hydrostatic pressure forces on the control surface. Neglect friction on the channel bed.



SCRIPT

BOARD

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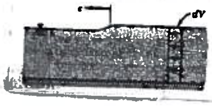
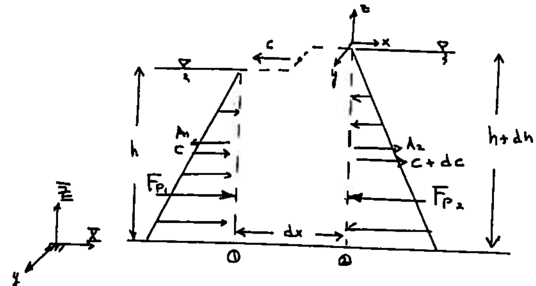




Steady flow, moving C.V.
ρ = const.
Uniform velocity at ① & ②
Neglect bottom friction
Assume hydrostatic pressure

SCRIPT

BOARD

Continuity

$$\frac{d}{dt}\underbrace{\int_{C.V.} \rho \, d\forall}_{0} + \int_{C.S.} \rho (\vec{V} \cdot d\vec{A}) = 0$$

$$\int_{C.S.} \rho (\vec{V} \cdot d\vec{A}) = -\cancel{\rho} c h \cancel{dy} + \cancel{\rho}(c + dc)(h + dh)\cancel{dy} = 0$$

$$ch = (c + dc)(h + dh)$$

$$ch = ch + c\,dh + h\,dc + \underbrace{\cancel{dc\,dh}}_{\text{Negligible}}$$

$$\therefore \quad c\,dh + h\,dc = 0$$

or

$$dc = -\frac{c}{h}\,dh$$

Momentum

$$\Sigma F_x = \frac{d}{dt}\int_{C.V.} \vec{V} \rho \, d\forall + \int_{C.S.} \vec{V} \rho (\vec{V} \cdot d\vec{A})$$

$$F_{P_1} - F_{P_2} = -\rho c^2 h\,dy + \rho (c + dc)^2 (h + dh)\,dy$$

$$F_{P_1} = \rho g \frac{h^2}{2} dy \qquad F_{P_2} = \frac{\rho g (h + dh)^2 dy}{2}$$

$$\therefore$$

$$\frac{\cancel{\rho} g}{2}\left[h^2 \cancel{dy} - (h + dh)^2 \cancel{dy}\right] = -\cancel{\rho} c^2 h \cancel{dy} + \cancel{\rho}(c + dc)(c + dc)(h + dh)\cancel{dy}$$

SCRIPT

BOARD

$$\frac{gh^2}{2} - \frac{gh^2}{2} - \frac{\cancel{2}ghdh}{\cancel{2}} - \cancel{\frac{g\,dh\,dh}{2}}^{\text{Negligible}} = -c^2h + (c+dc)\underbrace{(c+dc)(h+dh)}_{=ch \text{ (from continuity)}}$$

$$-ghdh = \underbrace{-c^2h + c^2h}_{=0} + ch\,dc$$

$$-ghdh = ch\,dc$$

Also from continuity $dc = -\frac{c}{h}dh$

$$\therefore -gh\cancel{dh} = c\cancel{h}\left(-\frac{c}{\cancel{h}}\right)\cancel{dh}$$

$$-gh = -c^2$$

$$\therefore c = \sqrt{gh}$$

SCRIPT

BOARD

SCRIPT

CONSERVATION OF ANGULAR MOMENTUM (ALSO CALLED MOMENT OF MOMENTUM) IS USED TO EXPLAIN HOW FORCES CAUSE ROTATIONS.

FUNDAMENTAL IN PUMPS AND SIMILAR MACHINES.

BOARD

ANGULAR MOMENTUM IS USED WITH ROTATING THINGS

IN TERMS OF REYNOLD'S TRANSPORT THEOREM, THE EXTENSIVE ANGULAR MOMENTUM IS

$$m(\vec{r} \times \vec{V})$$

THE INTENSIVE ANGULAR MOMENTUM IS (PER UNIT MASS)

$$(\vec{r} \times \vec{V})$$

THE INTENSIVE ANGULAR MOMENTUM IS (PER UNIT VOLUME)

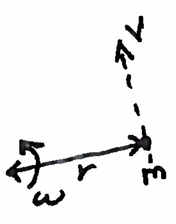
$$\rho(\vec{r} \times \vec{V})$$

SCRIPT

START WITH SYSTEM EQUATION

MOMENTUM $= m \cdot v$

ANGULAR:



$r \cdot \omega = v$

$\therefore mv = m r \omega$

angular speed

momentum

distance

$\frac{d}{dt}(m r \omega) = m \frac{dr}{dt} \omega + m r \frac{d\omega}{dt}$

~~CONSTANT SPEED~~

BOARD

APPLY REYNOLDS TRANSPORT THEOREM

$$\underbrace{\frac{d}{dt}\left(m(\vec{r} \times \vec{V})\right) = \vec{r} \times \vec{F}}_{\text{From system mechanics}}$$

(e.g. $\Sigma M = \vec{r} \times \vec{F}$)

APPLY RTT TO OBTAIN THE INTEGRAL FORM AS

$$\Sigma(\vec{r} \times \vec{F}) = \frac{d}{dt}\int_{C.V.} \rho(\vec{r} \times \vec{V})\, d\forall + \int_{C.S.} \rho(\vec{r} \times \vec{V})(\vec{V} \cdot \vec{dA})$$

SCRIPT

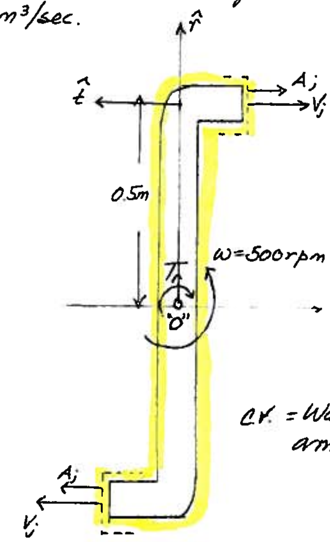
APPLICATION OF ANGULAR MOMENTUM OFTEN INVOLVES USE OF CONTINUITY & BERNOULLI'S EQ

SELECTION OF C.V. IS IMPORTANT TO MAKE ANALYSIS STRAIGHTFORWARD

– EASIEST ILLUSTRATED BY EXAMPLE

BOARD

Determine the power produced by a simple reaction turbine that rotates in a horizontal plane at 500 rpm. Water enters turbine from a vertical pipe, coaxial with axis of rotation, and exits through short nozzles each with cross section area $10\,cm^2$. Total discharge through turbine is $0.1\,m^3/sec$.



C.V. = Water in turbine arms

SCRIPT

CONSIDER A REACTION TURBINE (LIKE A RAIN-BIRD™)

SKETCH C.V.

- LET C.V. ROTATE WITH THE TURBINE

(OK. FOR C.V. TO MOVE; JUST CHOOSE CAREFULLY)

USE CONTINUITY TO FIND V_{jets}

APPLY MOMENTUM; RELATIVE TO POINT O

BOARD

Continuity

$$0 = \frac{d}{dt}\int_{CV} \underbrace{\rho\, d\forall}_{0} + \int_{CS} \rho(\vec{V}\cdot d\vec{A})$$

$$0 = -\rho Q_{in} + \rho V_j A_j + \rho V_j A_j$$

V_j is velocity relative to the control volume

$$\frac{Q_{in}}{2} = V_j A_j$$

$$\therefore V_j = \frac{Q_{in}}{2A_j} = \frac{0.1\,m^3/s}{2(10\,cm^2)\left(\frac{1}{100\,cm}\right)^2} = 50\,m/s$$

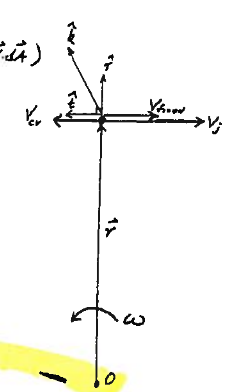
Angular Momentum

$$\Sigma \vec{r}\times\vec{F} = \frac{d}{dt}\int_{CV} \vec{r}\times\vec{V}\rho\, d\forall + \int_{CS} \vec{r}\times\vec{V}\rho(\vec{V}\cdot d\vec{A})$$

$$\vec{V}_{fixed} = \vec{V}_j + \vec{V}_{cv} = (\omega R - V_j)\,\hat{t}$$

$$\vec{r} = R\,\hat{r}$$

$$\therefore \vec{r}\times\vec{V} = (\omega R^2 - R V_j)\,\hat{k}$$



SCRIPT

APPLY REYNOLDS TRANS.

BREAK

WRITE AS

$$\frac{d}{dt}\int_0^R \bar{r}\times\bar{V}\rho\, d\forall$$

$$+\ \frac{d}{dt}\int_0^{-R} \bar{r}\times\bar{V}\rho\, dV \rightarrow 0$$

$$= \underbrace{\frac{d}{dt}\int_0^R - \int_0^{-R}}_{\text{CANCEL}}$$

ALL LEFT IS FLUX INTEGRAL

BOARD

$$\Sigma\vec{r}\times\vec{F} = \frac{d}{dt}\int \vec{r}\times\vec{V}\rho\, d\forall + \int_{c.s.} \vec{r}\times\vec{V}\rho(\vec{V}\cdot d\vec{A})$$

$$= \Big[\underbrace{\frac{d}{dt}\Big(\frac{\rho\omega R^3}{3}A_j - \frac{\rho R^2}{2}V_jA_j\Big)}_{\text{Upper arm volume } \int_0^R} - \underbrace{\frac{d}{dt}\Big(\frac{\rho\omega R^3}{3}A_j - \frac{\rho R^2}{2}V_jA_j\Big)}_{\text{lower arm } \int_0^{-R}}\Big]\hat{k}$$

$$\underbrace{-\frac{\rho Q}{2}R(V_j-\omega R)\hat{k}}_{\text{Upper arm flux}} \quad \underbrace{-\frac{\rho Q}{2}R(V_j-\omega R)\hat{k}}_{\text{lower arm}}$$

$$\therefore\ \Sigma\vec{r}\times\vec{F} = -\rho QR(V_j-\omega R)\hat{k}$$

But $\Sigma\vec{r}\times\vec{F} = T_r$

$$\therefore\ T_r = -\rho QR(V_j-\omega R)\hat{k}$$

Reaction torque is torque of generator on arms $\therefore$ torque of arms on generator is $T_g = -T_r = \rho QR(V_j-\omega R)\hat{k}$

SCRIPT

BOARD

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force}\cdot\text{distance}}{\text{Time}} = \text{Force}\cdot\text{Velocity}$$

$$T_g = \underbrace{(\rho QV_j - \rho Q\omega R)}_{\text{Force}}\underbrace{R}_{\text{distance}}$$

$$R\omega = \frac{\text{distance}}{\text{time}}$$

$$\therefore\ \text{Power} = T_g\omega$$

$$= \rho QV_jR\omega - \rho Q\omega^2R^2$$

Substitute numerical values

$$\text{Power} = (1000\,\text{kg/m}^3)(0.1\,\text{m}^3/\text{s})(50\,\text{m/s})(0.5\,\text{m})\left(\frac{500\cdot 2\pi}{60}\right)$$

$$-(1000\,\text{kg/m}^3)(0.1\,\text{m}^3/\text{s})(0.5)^2\left(\frac{500\cdot 2\pi}{60}\right)^2$$

$$= 62.36\,\text{kN}\cdot\text{m/sec} \quad = 62.4\,\text{kW}$$

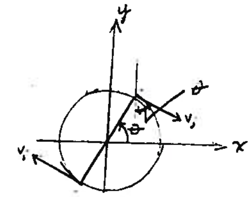
SCRIPT

IF WE DO SAME PROBLEM IN INERTIAL REF. FRAME

Consider same problem in an inertial reference frame.

$V_{j-rel} = 50 m/sec$ (unchanged)

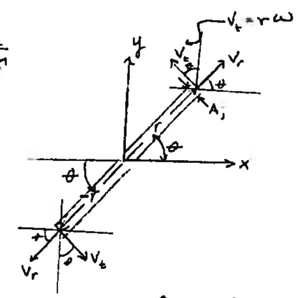
$$\Sigma \vec{r} \times \vec{F} = \frac{d}{dt}\int_{cv} \vec{r} \times \vec{V} \rho + \int_{c.s.} \vec{r} \times \vec{V} \rho (\vec{V} \cdot d\vec{A})$$



$$\underline{\frac{d}{dt}\int_{cv} \vec{r} \times \vec{V} \rho d\forall}$$

$$\vec{r} = r\cos\theta \hat{i} + r\sin\theta \hat{j}$$

$$\vec{V} = V_r\cos\theta \hat{i} - r\omega\sin\theta \hat{i} + V_r\sin\theta \hat{j} + r\omega\cos\theta \hat{j}$$



$$\frac{d}{dt}\int_{cv} \vec{r} \times \vec{V} \rho d\forall =$$

$$\frac{d}{dt}\left[\int_{c.v\text{-upper arm}} \vec{r} \times \vec{V} \rho d\forall + \int_{c.v\text{-lower arm}} \vec{r} \times \vec{V} \rho d\forall\right]$$

$$= \frac{d}{dt}\left[\int_0^R r^2\omega\rho A_j dr + \int_0^{-R} r^2\omega\rho A_j dr\right]$$

$$= \underline{0}$$

$$\vec{r} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r\cos\theta & r\sin\theta & 0 \\ V_r\cos\theta - r\omega\sin\theta & V_r\sin\theta + r\omega\cos\theta & 0 \end{vmatrix}$$

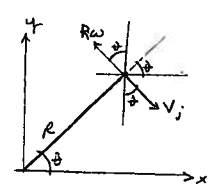
$$= r^2\omega \hat{k}$$

SCRIPT

$$\underline{\int_{c.s.} \vec{r} \times \vec{V} \rho (\vec{V} \cdot d\vec{A})}$$

$$\vec{r}_j = R\cos\theta \hat{i} + R\sin\theta \hat{j}$$

$$\vec{V}_j = V_j\sin\theta \hat{i} - R\omega\sin\theta \hat{i} + R\omega\cos\theta \hat{j} - V_j\cos\theta \hat{j}$$



$$\vec{r}_j \times \vec{V}_j = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R\cos\theta & R\sin\theta & 0 \\ V_j\sin\theta - R\omega\sin\theta & R\omega\cos\theta - V_j\cos\theta & 0 \end{vmatrix}$$

$$= R\cos\theta(R\omega\cos\theta - V_j\cos\theta) - R\sin\theta(V_j\sin\theta - R\omega\sin\theta)\ \hat{k}$$

$$= \cos^2\theta(R^2\omega - RV_j) + \sin^2\theta(R^2\omega - RV_j)\ \hat{k}$$

$$= \underbrace{(\cos^2\theta + \sin^2\theta)}_{=1}(R^2\omega - RV_j)\ \hat{k}$$

$$\vec{r} \times \vec{V} = (R^2\omega - RV_j)\ \hat{k}$$

SAME RESULT FROM THIS POINT FORWARD

$$\therefore \int_{c.s.} \vec{r} \times \vec{V} \rho (\vec{V} \cdot d\vec{A}) = \underbrace{-\frac{\rho Q}{2} R(V_j - \omega R)}_{\text{upper arm}} - \underbrace{\frac{\rho Q}{2} R(V_j - \omega R)}_{\text{lower arm}} \hat{k}$$