## SCRIPT

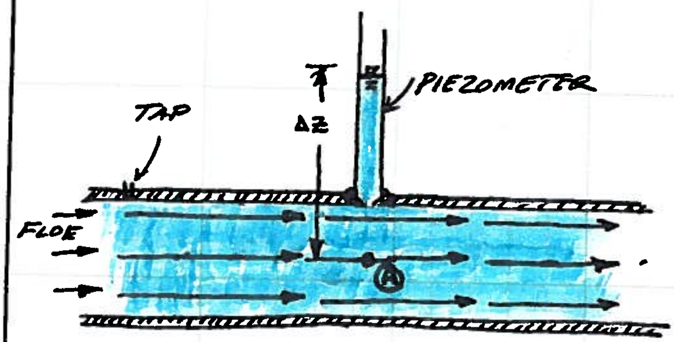
PRESSURE & VELOCITY CAN BE MEASURED USING SIMPLE TOOLS BASED ON BERNOULLI'S EQUATION

WHILE SOMEWHAT ARCHAIC, THESE OLD SCHOOL DEVICES WORK WITHOUT POWER; WORK AT NIGHT (NEED A FLASHLIGHT) AND ARE A DESIRABLE BACK-UP TO MODERN TRANSDUCERS

## BOARD

MEASURING VELOCITY & PRESSURE

PRESSURE IS OFTEN MEASURED USING PIEZOMETERS, PITOT TUBES & SIMILAR TECHNOLOGY.




STATIC PRESSURE (HEAD) IS THE HEIGHT LIQUID WILL RISE IN A TUBE (PIEZOMETER) ORIENTED NORMAL (PERPINDICULAR) TO FLOW

## SCRIPT

STAGNATION (PITOT) TUBE ORIENTS INTO FLOW AND CAPTURES VELOCITY HEAD PLUS STATIC HEAD

WRITE BERNOULLI'S FROM 0 TO 1

LOCATION 0 IS A FREE-STREAM LOCATION.

$d$ IS DISTANCE FROM PITOT TUBE CENTERLINE TO INSIDE WALL OF PIPE OR DEPTH IN AN OPEN FLOW

$\ell_0$ IS THE HEIGHT OF RISE IN A STATIC TUBE AT THE ~~FREE~~ STREAM

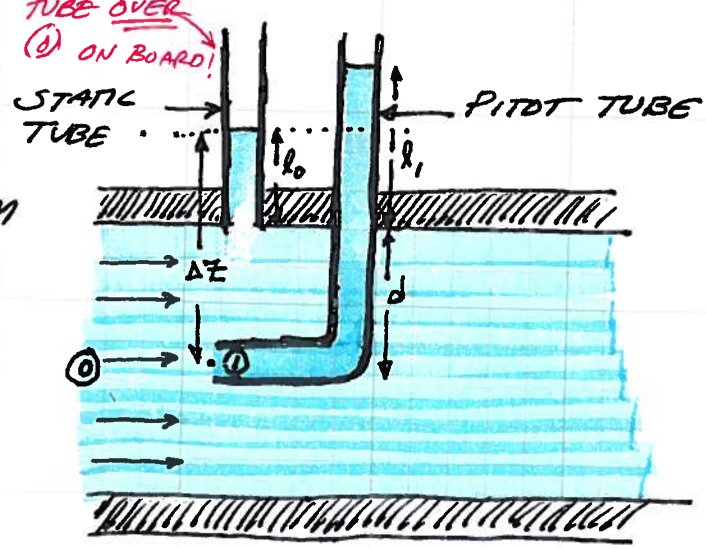
## BOARD

IN CASE OF DRAWING

$$\Delta z = \frac{p_A}{\gamma}$$

BY CONVENTION MEASURED FROM CENTERLINE OF CONDUIT.

DRAW THIS TUBE OVER 0 ON BOARD!



**SCRIPT**

$\ell_1$ IS HEIGHT OF RISE ABOVE $d$ IN THE PITOT TUBE.

APPLY BERNOULLI'S EQUATION (IN HEAD FORM) AND OBSERVE:

$V_1$ IS ZERO (STAGNATION)

$z_0 = z_1$ = ELEVATION OF FLOWLINE.

$p_0$ = STATIC PRESSURE

$p_1$ = KINETIC PRESSURE

$\Delta p \propto V_0^2$

**BOARD**

$$\frac{p_0}{\gamma} + \cancel{z_0} + \frac{V_0^2}{2g} = \frac{p_1}{\gamma} + \cancel{z_1} + \cancel{\frac{V_1^2}{2g}}$$

$z_0 = z_1$

$V_1 = 0$ (STAGNATION POINT)

$$\therefore V_0^2 = \left(\frac{p_1 - p_0}{\gamma}\right) 2g$$

$$V_0 = \sqrt{\frac{2(p_1 - p_0)\cancel{g}}{\rho g}}$$

$p_0 = \gamma(d + \ell_0)$

$p_1 = \gamma(d + \ell_1)$

$$V_0 = \sqrt{2\,\frac{\cancel{\gamma}(d+\ell_1) - \cancel{\gamma}(d+\ell_0)}{\cancel{\gamma}}\,g}$$

**SCRIPT**

TEXTBOOK PG 139-141 DISCUSSES FOR OPEN FLOW.

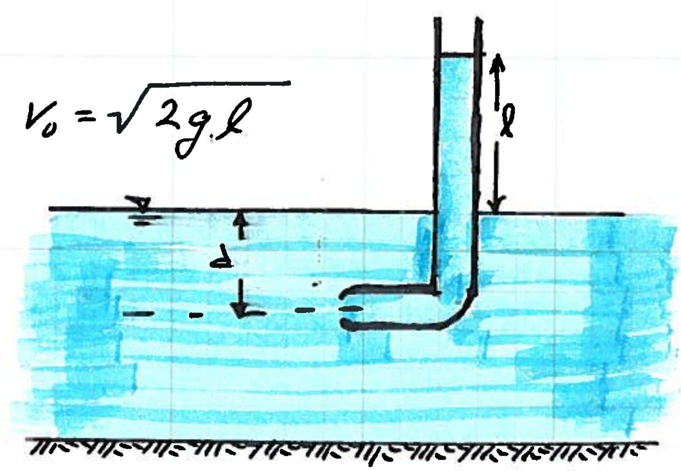
IN PRACTICE THE STATIC TUBE AND PITOT TUBE ARE BUILT INTO A SINGLE INSTRUMENT.

**BOARD**

$$V_0 = \sqrt{2(\ell_1 - \ell_0)g}$$

NOW IF IN AN OPEN FLOW (FREE SURFACE)

$$V_0 = \sqrt{2g\ell}$$



PITOT-STATIC TUBE ON LIGHT AIRCRAFT

[14] discusses the use of titanium tetrachloride for low-speed flow measurements. One of the best fuels for producing nontoxic, noncorrosive, dense smoke is a product called Type-1964 Fog Juice. This fuel has a boiling temperature of approximately 530°F (276°C), contains petroleum hydrocarbon, and may be obtained from most theatrical supply houses. Some smoke-generation techniques are discussed in Ref. [28].

## 7-15 PRESSURE PROBES

A majority of fluid dynamic applications involve measuring the total flow rate by one or more of the methods discussed in the previous sections. These measurements ignore the local variations of velocity and pressure in the flow channel and permit an indication of only the total flow through a particular cross section. In applications involving external flow situations, such as aircraft or wind-tunnel tests, an entirely different type of measurement is required. In these instances probes must be inserted in the flow to measure the local static and stagnation pressures. From these measurements the local flow velocity may be calculated. Several probes are available for such measurements, and summaries of characteristic behaviors are given in Refs. [4], [7], [13], and [18]. We shall discuss some of the basic probe types in this section.

The total pressure for isentropic stagnation of an ideal gas is given by

COMPRESSIBLE FLOW

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} \tag{7-47}$$

where $p_0$ is the stagnation pressure, $p_\infty$ is the free-stream static pressure, and $M_\infty$ is the free-stream Mach number given by

$$M_\infty = \frac{u_\infty}{a} \tag{7-48}$$

$a$ is the acoustic velocity and may be calculated with

$$a = \sqrt{\gamma g_c RT} \tag{7-49}$$

for an ideal gas. It is convenient to express Eq. (7-47) in terms of dynamic pressure $q$ defined by

$$q = \tfrac{1}{2}\rho u_\infty^2 = \tfrac{1}{2}\gamma p M_\infty^2 \tag{7-50}$$

Equation (7-47) thus becomes

$$p_0 - p_\infty = \frac{2q}{\gamma M_\infty^2}\left[\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} - 1\right] \tag{7-51}$$

This relation may be simplified to

$$p_0 - p_\infty = \frac{2q}{\gamma M_\infty^2}\left(1 + \frac{M_\infty^2}{4} + \frac{2-\gamma}{24} M_\infty^4 + \cdots\right) \tag{7-52}$$

when

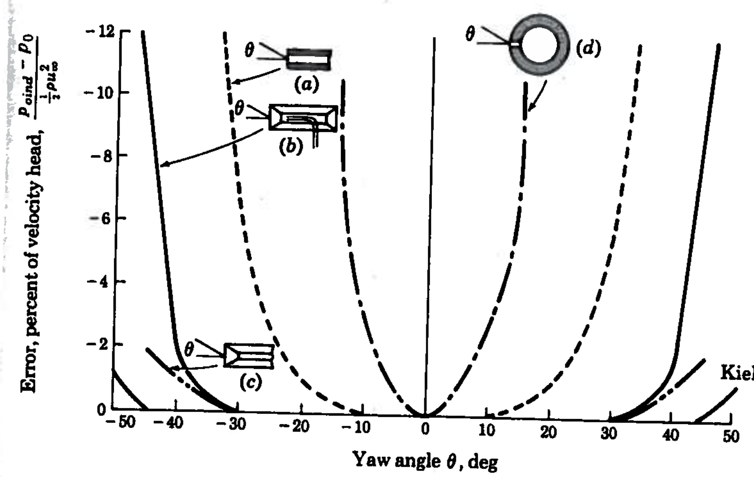
$$M_\infty^2\left(\frac{\gamma-1}{2}\right)<1.$$

For very small Mach numbers Eq. (7-52) reduces to the familiar incompressible flow formula

$$p_0 - p_\infty = \tfrac{1}{2}\rho u_\infty^2 \tag{7-53}$$

We thus observe that a measurement of static and stagnation pressures permits a determination of the flow velocity, by either Eq. (7-53) or Eq. (7-51), depending on the fluid medium.

A basic total-pressure probe may be constructed in several different ways, as shown in Fig. 7-40. In each instance the opening in the probe is oriented in a direction exactly parallel to the flow when a measurement of the total stream pressure is desired. If the probe is inclined at an angle $\theta$ to the free-stream velocity, a somewhat lower pressure will be observed. This reduction in pressure is indicated in Fig. 7-40 according to Ref. [7]. Configuration *a* represents an open-ended tube placed in the flow. Configuration *b* is called a shielded probe and consists of a venturi-shaped tube placed in the flow with an open-ended tube at the throat of the section to sense the stagnation pressure. It may be noted that this probe is rather insensitive to flow direction. Configuration *c* represents an open-ended tube with a chamfered opening. The chamfer is about 15°, and the ratio of OD to ID of the tube is about 5. Configuration *d* represents a tube having a small hole drilled in its side, which is placed normal to the flow direction. This type of probe, as might be expected, is the most sensitive to changes in yaw angle. Also indicated in Fig. 7-40 is a portion of the curve for a Kiel probe, which is similar in construction to configuration *b*



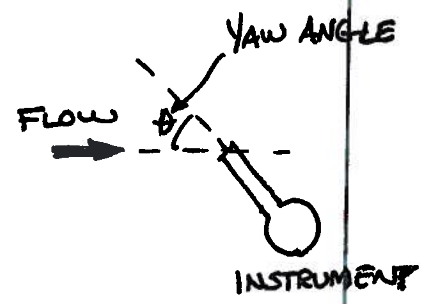




**FIGURE 7-40**
Stagnation pressure response of various probes to changes in yaw angle. (*a*) Open-ended tube; (*b*) channel tube; (*c*) chamfered tube; (*d*) tube with orifice in side. (*From Ref.* [7].)

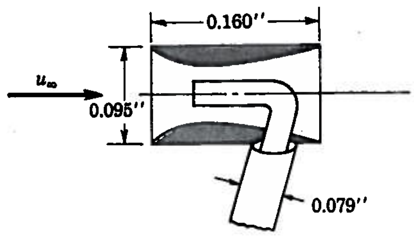
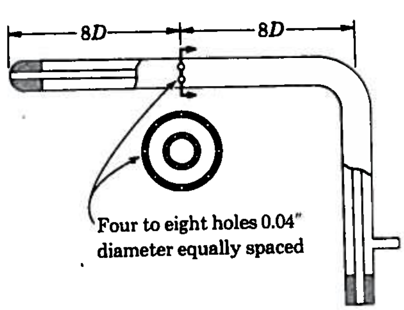



**FIGURE 7-41**
Kiel probe (Model 3696) for measurement of stagnation pressure. (*Courtesy of Airflo Instrument Co., Glastonbury, Conn.*)

except that a smoother venturi shape is used, as shown in Fig. 7-41. The Kiel probe is the least sensitive to yaw angle.

The measurement of static pressure in a flowstream is considerably more difficult than the measurement of stagnation pressure. A typical probe used for the measurement of both static and stagnation pressures is the *Pitot tube* shown in Fig. 7-42. The opening in the front of the probe senses the stagnation pressure, while the small holes around the outer periphery of the tube sense the static pressure. The static-pressure measurement with such a device is strongly dependent on the distance of the peripheral openings from the front opening as well as on the yaw angle. Figure 7-43 indicates the dependence of the static-pressure indication on the distance from the leading edge of the probe for both blunt subsonic and conical supersinic configurations. To alleviate this condition, the static-pressure holes are normally placed at least eight diameters downstream from the front of the probe. The dependence of the static and stagnation pressures on yaw angle for a conventional Pitot tube is indicated in Fig. 7-44. This device is quite sensitive to flow direction. The probe in Fig. 7-42 is sometimes called a *Pitot static tube* because it measures both static and stagnation pressure.

The static-pressure characteristics of three types of probes are shown in Figs. 7-45 and 7-46 as functions of Mach number and yaw angle. It may be noted that both the wedge and Prandtl tube indicate static-pressure values that are too low, while the cone indicates a value that is too high. The wedge is least sensitive to yaw angle. All three probes have two static-pressure holes located 180° apart.




TYP. PITOT-STATIC TUBE DESIGN; SUBSONIC FLOW

**FIGURE 7-42**
Schematic drawing of Pitot tube.

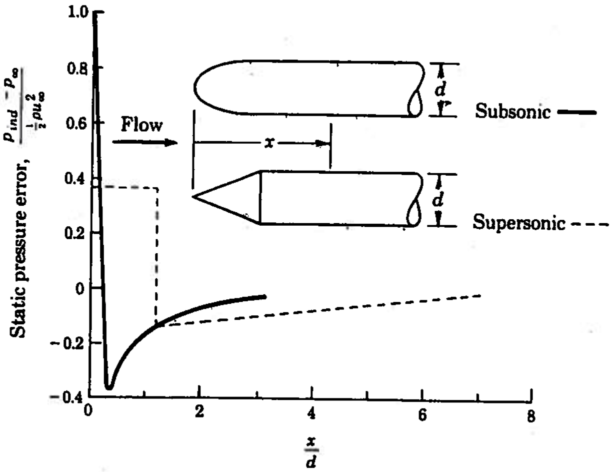



**FIGURE 7-43**
Variation of static pressure along standard subsonic and supersonic probe types. (*From Ref.* [4].)

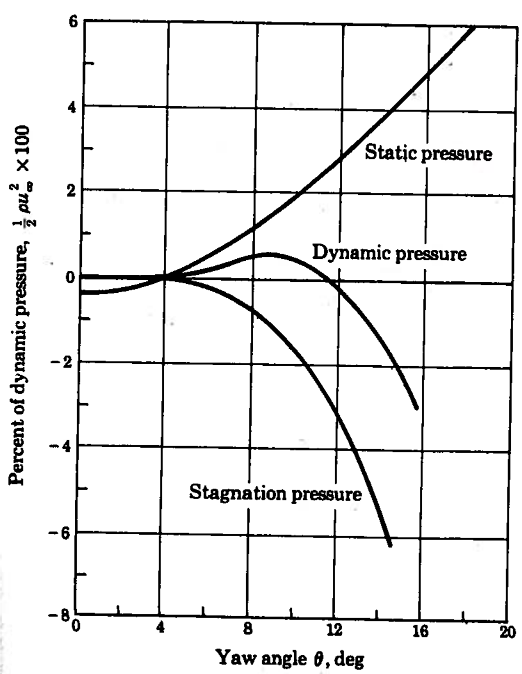



**FIGURE 7-44**
Variation of static, stagnation, and dynamic pressure with yaw angle for Pitot tube. (*Courtesy of Airflo Instrument Co., Glastonbury, Conn.*)

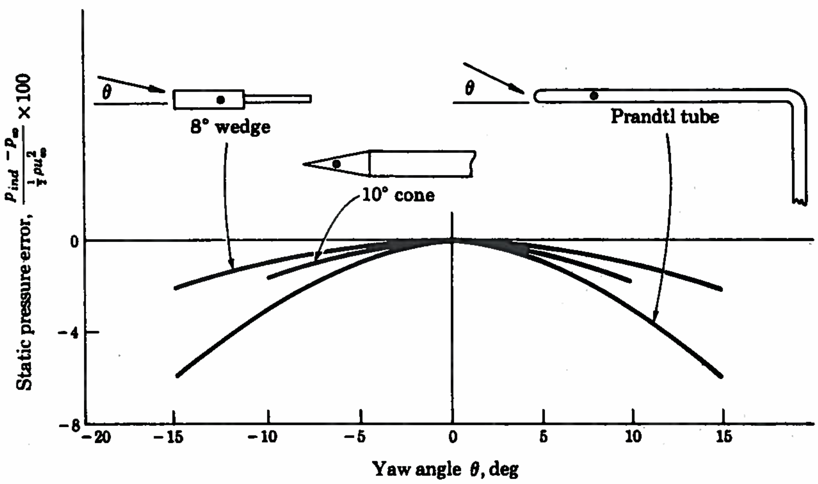



**FIGURE 7-45**
Yaw-angle characteristics of various static-pressure probes. (*From Ref.* [7].)

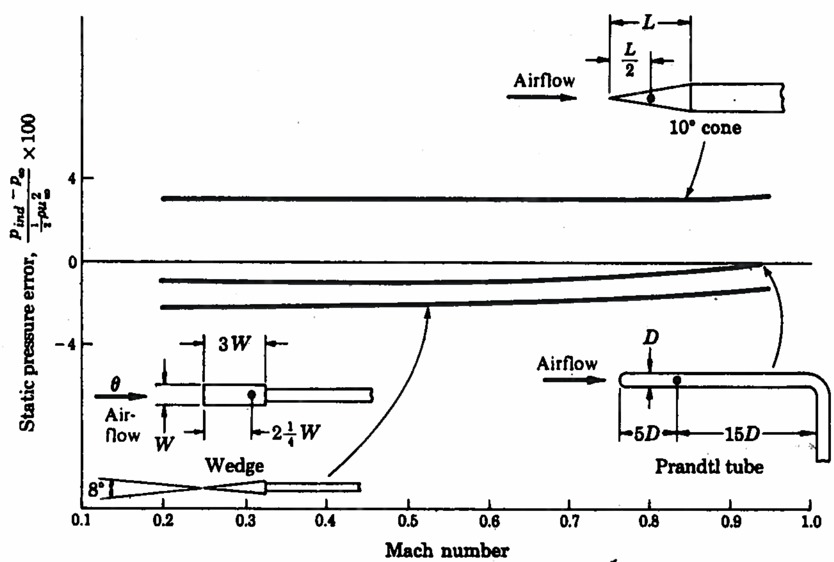



**FIGURE 7-46**
Mach-number characteristics of various static-pressure probes. (*From Ref.* [7].)

**Example 7-7.** A Pitot tube is inserted in a flowstream of air at 30°C and 1.0 atm. The dynamic pressure is measured as 1.12 in water when the tube is oriented parallel to the flow. Calculate the flow velocity at that point.

***Solution.*** We use Eq. (7-53) for this calculation. The air density is calculated as

$$\rho = \frac{p_\infty}{RT_\infty} = \frac{1.0132 \times 10^5}{(287)(303)} = 1.165 \text{ kg/m}^3$$

We also have

$$p_0 - p_\infty = 1.12 \text{ in water} = 5.82 \text{ psf} = 278.7 \text{ pa}$$

so that the velocity is

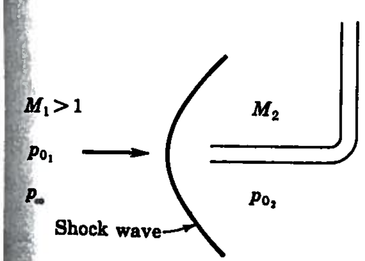
$$u_\infty = \sqrt{\frac{2(p_0 - p_\infty)}{\rho}} = \left[\frac{(2)(278.7)}{1.165}\right]^{1/2}$$
$$= 21.9 \text{ m/s } (71.8 \text{ ft/s})$$

## 7-16 IMPACT PRESSURE IN SUPERSONIC FLOW

Consider the impact probe shown in Fig. 7-47 which is exposed to a free stream with supersonic flow; that is, $M_1 > 1$. A shock wave will be formed in front of the probe as shown, and the total pressure measured by the probe will not be the free-stream total pressure before the shock wave. It is possible, however, to express the impact pressure at the probe in terms of the free-stream static pressure and the free-stream Mach number. The resulting expression as given in Ref. [10] is

$$\frac{p_\infty}{p_{0_2}} = \frac{\{[2\gamma/(\gamma+1)]M_1^2 - (\gamma-1)/(\gamma+1)\}^{1/(\gamma-1)}}{\{[(\gamma+1)/2]M_1^2\}^{\gamma/(\gamma-1)}} \tag{7-54}$$

where $p_\infty$ is the free-stream static pressure and $p_{0_2}$ is the measured impact pressure behind the normal shock wave. This equation is valid for Reynolds numbers based on probe diameter greater than 400. Equation (7-54) is called the *Rayleigh supersonic Pitot formula*. We see that in order to determine the value of the Mach number it is necessary to have a measurement of the free-stream static pressure. It is possible to make this measurement with special calibrated probes.




SUPERSONIC FLOW
NEEDS SPECIAL
TOOLS

**FIGURE 7-47**
Impact tube in supersonic flow.

**SCRIPT**

SEGMENTS AB & AC ARE INITIALLY ORTHOGONAL;

A SHORT TIME LATER, ELEMENT HAS MOVED AS SHOWN.

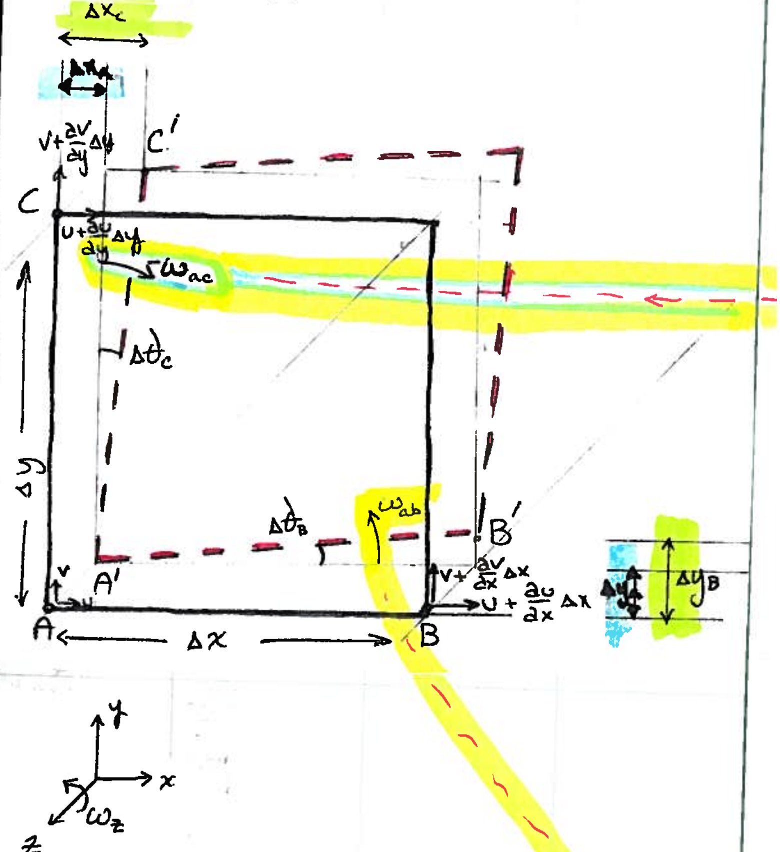
ANTICIPATED TRANSLATION OF ALL VERTICES IS $\Delta x_A, \Delta y_A$

SUPPOSE ~~[illegible]~~ POINTS B' & C' SHOW A LITTLE EXTRA TRANSLATION, $\Delta x_c, \Delta y_B$

BECAUSE OF SLIGHT DEFORMATION (ROTATION) OF THE ELEMENT

**BOARD**

ROTATION OF A FLUID




**SCRIPT**

RATE OF ROTATION OF SEGMENT AB IS

TANGENT OF SMALL ANGLE IS EXTRA TRANSLATION LESS ANTICIPATED TRANSLATION DIVIDED BY ELEMENT LENGTH $\Delta x$.

WE ARE CONSIDERING SMALL TIME; HENCE SMALL ANGLES SO $\tan(\alpha) \approx \alpha$.

SUBSTITUTE VELOCITY AND APPLY ALGEBRA

**BOARD**

$$\omega_{ab} = \lim_{\Delta t \to 0} \frac{\Delta\theta_B}{\Delta t}$$

$$\tan(\Delta\theta_B) = \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

FOR SMALL ANGLES $\tan(\alpha) \approx \alpha$

$$\therefore \Delta\theta_B \approx \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

$$\Delta y_B - \Delta y_A = \left(v + \frac{\partial v}{\partial x}\Delta x - v\right)\Delta t$$

$$\text{SO} \quad \Delta\theta_B = \frac{\frac{\partial v}{\partial x}\cancel{\Delta x}\Delta t}{\cancel{\Delta x}} = \frac{\partial v}{\partial x}\Delta t$$

$$\therefore \omega_{ab} = \lim_{\Delta t \to 0} \frac{\Delta\theta_B}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\partial v}{\partial x}\Delta t}{\Delta t} = \frac{\partial v}{\partial x}$$

| SCRIPT | BOARD |
|---|---|
| RATE OF ROTATION OF SEGMENT AC IS | $\omega_{ac} = \lim_{\Delta t \to 0} \frac{-\Delta\theta_c}{\Delta t}$ |
| USING SIMILAR ANALYSIS THE RESULT IS | $\Delta\theta_c \simeq -\left(\frac{\Delta X_c - \Delta X_A}{\Delta y}\right) = -\frac{\partial u}{\partial y}\Delta t$ |
| | $\boxed{\omega_{ac}} = \lim_{\Delta t \to 0} \frac{-\frac{\partial u}{\partial y}\Delta t}{\Delta t} = \boxed{-\frac{\partial u}{\partial y}}$ |
| THE AVERAGE RATE OF ROTATION (OF ELEMENT) IS | $\omega_z = \frac{\omega_{ab} + \omega_{ac}}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ |

| SCRIPT | BOARD |
|---|---|
| EXTENSION OF SUCH ANALYSIS INTO 3 DIMENSIONS PRODUCES AN ANGULAR VELOCITY VECTOR | $\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$ <br> WHERE |
| NOTICE FOR EACH DIRECTION, THE ROTATION INVOLVES THE OTHER TWO DIMENSIONS | $\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$ |
| E.G. $\omega_x$ INVOLVES $\frac{\partial w}{\partial y}$ & $\frac{\partial v}{\partial z}$ | $\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$ |
| ALSO NOTICE THE VELOCITY VARIATIONS ARE CROSS TERMS: <br> $\underline{V} = u\underline{i} + v\underline{j} + w\underline{k}$ <br> $\omega_x$ INVOLVES $\frac{\partial w}{\partial y}$ & $\frac{\partial v}{\partial z}$ <br> "z" "y" <br> "y" "z" | $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ <br> THE VORTICITY VECTOR IS DEFINED AS TWICE THE AVERAGE ANGULAR VELOCITY VECTOR |

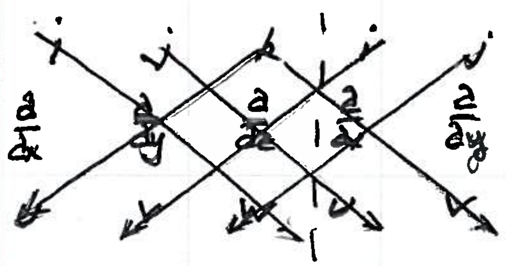
**SCRIPT**

THE VORTICITY VECTOR IS TWICE THE ANGULAR VELOCITY VECTOR.

IT IS COMPUTED AS THE CURL OF VELOCITY

NOTICE THE NOTATION

$\nabla \times \underline{V}$



$$\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)i+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)j+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\underline{k}$$

**BOARD**

VORTICITY VECTOR

$$\underline{\Omega} = \left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\underline{i}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\underline{j}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\underline{k}$$

THE VECTOR IS EQUAL TO THE CURL OF THE VELOCITY FIELD

$$\left.\begin{array}{l}\underline{\Omega} = curl(\underline{V}) \\ \quad\text{OR} \\ \underline{\Omega} = \nabla \times \underline{V}\end{array}\right\} \text{GENERAL DIFFERENTIAL FORMS OF VORTICITY}$$

**SCRIPT**

VORTICITY – STREAMFUNCTION IS A WAY OF MODELING REAL FLOWS.

MANY SITUATIONS ARE WELL MODELED AS IRROTATIONAL

- FLOW IN AN AQUIFER
- FLOW IN AN OIL RESERVOIR
- FLOW IN A REACTOR (DEPENDS ON KIND OF REACTOR)
- FLOW IN A LARGE STREAM
- FLOW IN CERTAIN CONDUITS
- FLOW OVER AIRFOIL

TEXTBOOK SAVES FOR LATER
NOT SURE WHY?

**BOARD**

MANY REAL FLOW SITUATIONS ARE WELL MODELED AS IRROTATIONAL FLOW

$$\underline{\Omega} = \underline{0}$$

$$\Rightarrow \quad \frac{\partial w}{\partial y}=\frac{\partial v}{\partial z}\ ;\ \frac{\partial u}{\partial z}=\frac{\partial w}{\partial x}\ ;\ \frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}$$

IMPORTANT FLUID CONCEPTS SO FAR:

$\rho \underline{a} = \rho \underline{g} - \nabla p$ EULER'S EQN

$\frac{\partial \rho}{\partial t} = -\nabla \rho \underline{V}$ (CONTINUITY –COMING SOON!)

$\underline{\Omega} = \nabla \times \underline{V}$ VORTICITY DEFINED

plus

$\frac{p}{\gamma}+z+\frac{V^2}{2g}=C$ BERNOULLI'S EQN