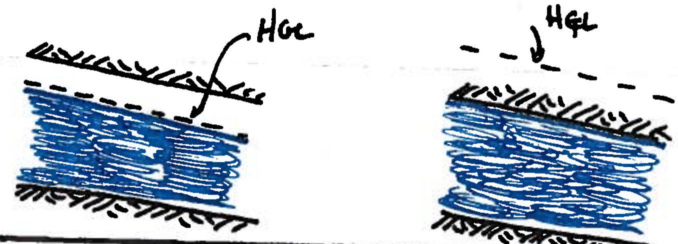
Open Channel Flow

- DEFINE AN OPEN CHANNEL
- DEFINE UNIFORM FLOW
- DEFINE NON-UNIFORM FLOW
- DEFINE CRITICAL FLOW
- COMPUTE Q FROM SLOPE AREA

OPEN CHANNEL

A CONDUIT WHERE THE UPPER BOUNDARY OF FLOW IS THE LIQUID SURFACE



EXAMPLES: CREEKS, DITCHES, RIVERS, CANALS

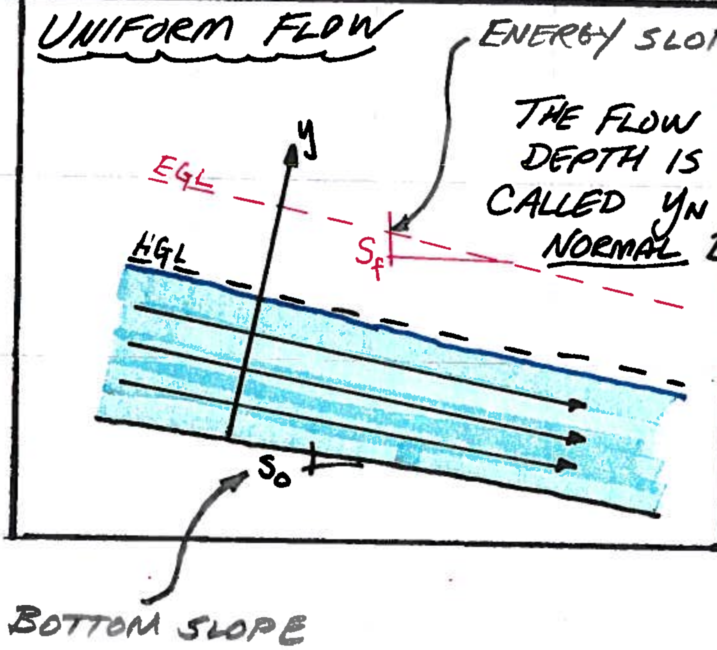
- SEWER PIPES PARTIAL FLOW

UNIFORM FLOW

VELOCITY CONSTANT ALONG A STREAMLINE

⇒ DEPTH & CROSS SECTION ARE CONSTANT ALONG THE PORTION OF THE CHANNEL

UNIFORM FLOW



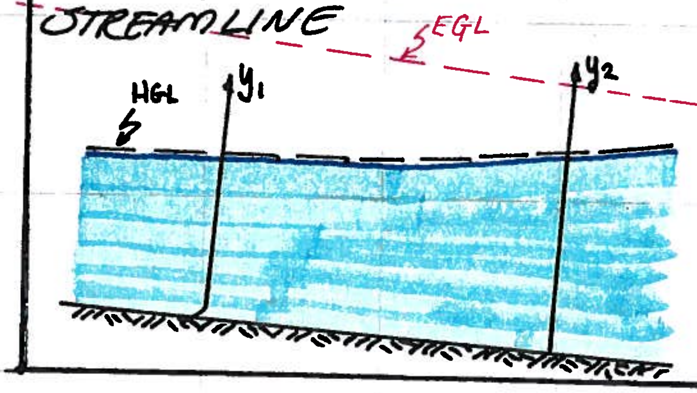
THE FLOW DEPTH IS CALLED y_N NORMAL DEPTH

NON-UNIFORM FLOW

VELOCITY VARIES ALONG A STREAMLINE

$y_1 \neq y_2$

$\bar{V}_1 \neq \bar{V}_2$



SUPPOSE STEADY FLOW $(Q_1 = Q_2)$; $V_2 \neq V_1$; WHAT KIND OF ACCELERATION IS OCCURING?

DIMENSIONLESS RELATIONS

$$Fr^2 = \frac{\text{kinetic forces}}{\text{gravity forces}} = \frac{V^2}{gL}$$

RELATED TO WAVE CELERITY

V IS MEAN SECTION VELOCITY

g IS GRAVITATIONAL CONSTANT

L IS CHARACTERISTIC LENGTH

REYNOLDS NUMBER MATTERS TOO!

DIMENSIONLESS RELATIONS

$$Re = \frac{VR_h}{\nu}$$

R_h IS THE HYDRAULIC RADIUS

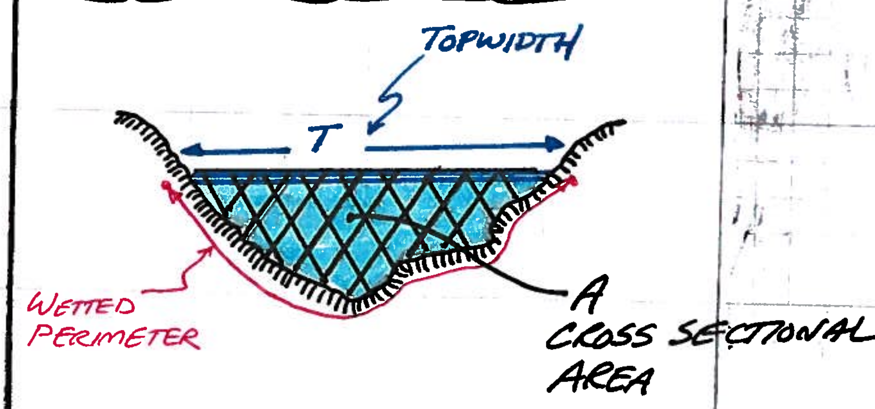
$$R_h = \frac{A}{P}$$

P IS THE WETTED PERIMETER

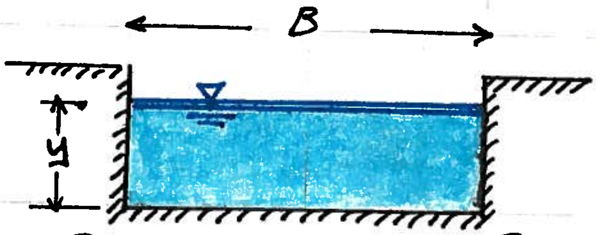
$Re_{R_h} < 500 \Rightarrow$ LAMINAR

$Re_{R_h} > 750 \Rightarrow$ TURBULENT

HYDRAULIC VARIABLES



HYDRAULIC VARIABLES IN RECTANGULAR CHANNEL



DEPTH-AREA

$A = By$

$P = B + 2y$

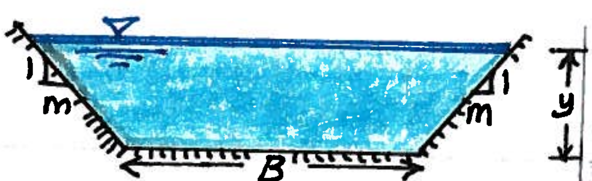
$R_h = \dfrac{By}{B+2y}$

RADIUS-AREA

OBSERVE FOR WIDE CHANNEL $B >> 2y$

$R_h \approx \dfrac{By}{B} = y$

HYDRAULIC VARIABLES IN TRAPEZOIDAL CHANNEL

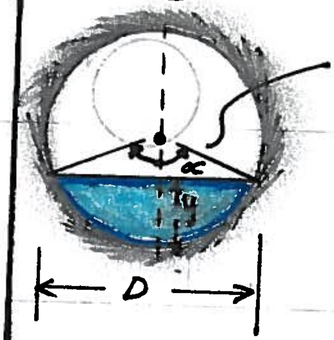


$A = y(B + my)$

$P = B + 2y\sqrt{1+m^2}$

$T = B + 2my$

CIRCULAR CHANNEL



$$\alpha = 2\cos^{-1}\left(1 - \frac{2y}{D}\right)$$

$$A = \frac{D^2}{4}\left(\frac{\alpha}{2} - \sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)$$

$$T = D\sin\left(\frac{\alpha}{2}\right)$$

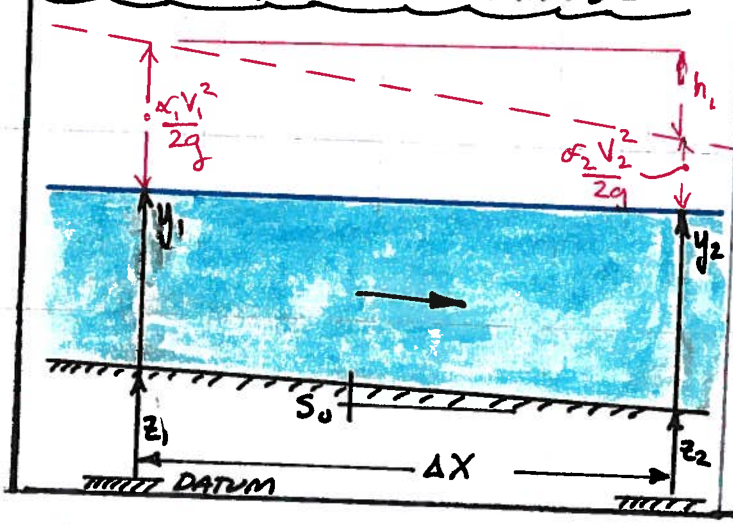
$$P = \frac{D\alpha}{2}$$

! CHECK THE SKETCH; SOME CONTEXTS USE A HALF-ANGLE (LIKE OUR BOOK p567)

!! α (HERE) IS IN RADIANS

p557

ENERGY EQ, OPEN CHANNEL



DEFINITION SKETCH — DIFFERENT FROM BOOK; BY CONVENTION USUALLY MEASURE TO BOTTOM OF CHANNEL

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_L$$

WHEN TURBULENT (COMMON) AND NOT TOO MUCH CURVATURE

$$\alpha_1 = \alpha_2 \approx 1.0$$

$$\frac{p_1}{\gamma} = y_1 \qquad \frac{p_2}{\gamma} = y_2$$

HYDROSTATIC APPROXIMATION. AND FLOW DEPTH IS EQUIVALENT TO PRESSURE HEAD

$$z_1 - z_2 = S_0 \Delta x$$

$$\therefore$$

$$y_1 + \frac{V_1^2}{2g} + S_0 \Delta x = y_2 + \frac{V_2^2}{2g} + h_L$$

$$h_L = S_f \Delta x$$

ENERGY SLOPE
FRICTION SLOPE

UNIFORM (NORMAL) FLOW

UNIFORM FLOW IS DEFINED AS

$$S_o = S_f$$

BUT

$$S_f = \frac{h_L}{\Delta x}$$

RECALL DARCY-WEISBACH HEAD LOSS MODEL

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

D IN PIPE IS EQUIVALENT TO $4R_h$

$$\therefore h_L = f \frac{L}{4R_h} \frac{V^2}{2g}$$

SO $\frac{h_L}{L} = \frac{h_L}{\Delta X} = S_f$

$\therefore S_f = \frac{f}{4R_h}\frac{V^2}{2g}$

UNIFORM

$S_f = S_0$

$\therefore V = \sqrt{\frac{8g}{f} R_h S_0}$

USING D-W MODEL IN THIS WAY
USE MOODY CHART: SUBSTITUTE
$4R_h$ FOR THE DIAMETER

$\frac{k_s}{4R_h}$ FROM MATERIAL PROPERTIES

ROCK CHANNEL ALTERNATIVE

$f = \frac{1}{\left[1.2 + 2.03\log\left(\frac{R_h}{d_{84}}\right)\right]^2}$

ALTERNATIVE HEAD LOSS MODELS

CHEZY

$$V = C\sqrt{R_h S_0}$$

MANNING

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad (SI)$$

$$V = \frac{1.49}{n} R^{2/3} S_0^{1/2} \quad (U.S)$$

MULTIPLY BY AREA TO RECOVER DISCHARGE Q

$$Q = \frac{1.49}{n} A R^{2/3} S_0^{1/2}$$

U.S 1.49

S.I. 1.0

MOST COMMON MODEL IN OPEN CHANNEL HYDRAULICS

NOTE: S_0 IS FRICTION SLOPE IN UNIFORM FLOW

THE "n" IS CALLED A RESISTANCE COEFFICIENT.

REFERRED TO AS "MANNING'S n"

LOOK UP IN TABLES (LIKE IN TEXT; INTERNET; OTHER SOURCES)

BOOK DESCRIBES "BEST" HYDRAULIC SECTION
SELF-STUDY (P563)

NON-UNIFORM FLOW

NON-UNIFORM $S_f \neq S_o$

SPECIFIC ENERGY

$$E = y + \frac{V^2}{2g}$$

E IS SUM OF PRESSURE & VELOCITY HEAD.

EXPRESS IN DISCHARGE FORM

$$E = y + \frac{Q^2}{2gA^2}$$

ENERGY EQUATION IS

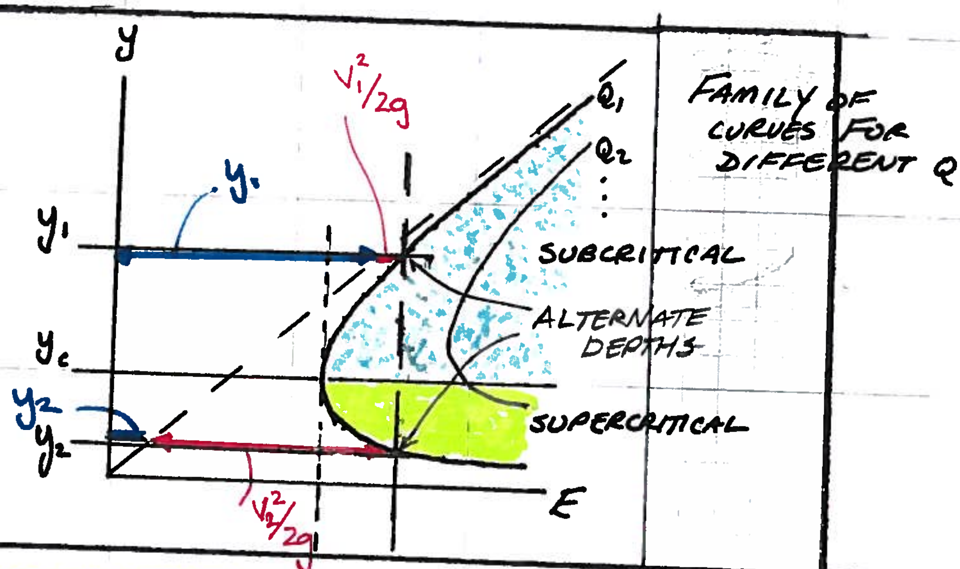
$$y_1 + \frac{Q^2}{2gA_1^2} + S_0 \Delta X = y_2 + \frac{Q^2}{2gA_2^2} + S_f \Delta X$$

NOW CONSIDER ZERO SLOPE AND FRICTIONLESS SYSTEM

$$S_0 = 0 \; ; \; S_f = 0$$

$$y_1 + \frac{Q}{2gA_1^2} = y_2 + \frac{Q}{2gA_2^2}$$

A PLOT OF y VS E IS CALLED SPECIFIC ENERGY PLOT



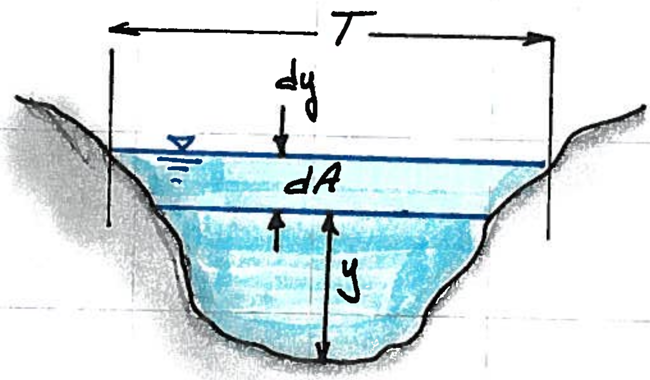
THE MINIMUM ENERGY POINT, FOR ANY GIVEN Q IS CALLED CRITICAL FLOW
THE DEPTH IS CALLED CRITICAL DEPTH

CRITICAL FLOW CHARACTERISTICS

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy}$$

AT MINIMUM

$$\frac{dE}{dy} = 0 \quad \therefore \quad \frac{Q^2}{gA^3}\frac{dA}{dy} = 1$$



$$dA = T\,dy \quad \therefore \quad \frac{dA}{dy} = T$$

dA

THUS

$$\frac{Q^2 T}{g A^3} = 1$$

OR

$$\frac{V^2 T}{g A} = 1$$

OBSERVE T/A = A CHARACTERISTIC LENGTH

SO

$$\underbrace{\frac{V^2}{gL}}_{Fr^2} = 1$$

AT CRITICAL FLOW

$$Fr^2 = 1$$

THE CRITICAL CRITERION IS MORE USEFUL IN DISCHARGE-AREA FORM

$$\underbrace{\frac{Q^2 T}{g A^3}}_{} = 1$$

ALSO HOW TO COMPUTE Fr^2 FOR ANY GIVEN Q & y

CRITICAL FLOW OCCURS AT CRITICAL (CONTROL) SECTIONS

BROAD WEIR

PARSHALL FLUME
CHANGE IN SLOPE
FREE OVERFALL

SLOPE-AREA

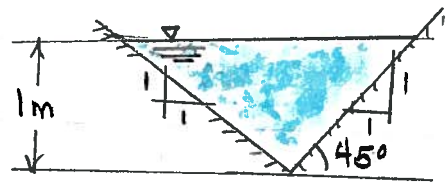
15.5
15.7 } IN-CLASS EXAMPLES

SCRIPT

BOARD

15-5

TRIANGLE CHANNEL, LONG. SLOPE $S_0 = 0.0019$



PLANED WOOD
WHAT IS Q?

SLOPE-AREA (MANNING'S)

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

$n = 0.012$ (TABLE 15.1 pg 561)

SCRIPT

BOARD

(TRAPEZOID; $B=0$ $m=1$)

$$A = y(B+my) = y^2 \qquad y=1$$

$$P = B + 2y\sqrt{1+m^2}$$

$$= 2y\sqrt{2}$$

$$R_H = \frac{y^2}{2y\sqrt{2}} = \frac{y}{2\sqrt{2}}$$

$$\therefore Q = \frac{1}{0.012}(1^2)\left(\frac{1}{2\sqrt{2}}\right)^{2/3}(0.0019)^{1/2}$$

$$= 1.82\,m^3/s$$

SCRIPT

BOARD

15.7

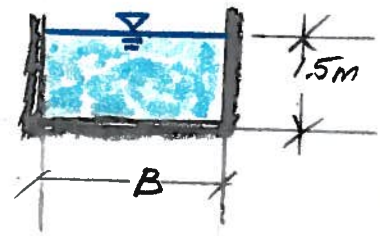
RECTANGULAR CHANNEL

$S_0 = 0.001$

$B = 3m$

$y = 1.5m$

FIND Q



SLOPE-AREA

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

SCRIPT

BOARD

$n = 0.012$

$$Q = \frac{1}{0.012}(3)(1.5)(6)^{2/3}(0.001)^{1/2}$$

$$= 9.78\, m^3/s$$

IF CHOOSE D-W

$A = 4.5m^2$

$P = 6m$

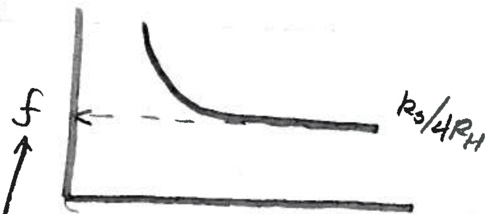
$R_h = A/P = 0.75m$

$$\frac{k_s}{4R_h} = 0.333 \cdot 10^{-8}$$

SCRIPT

BOARD

USE MOODY CHART



USE THIS VALUE AS 1st GUESS

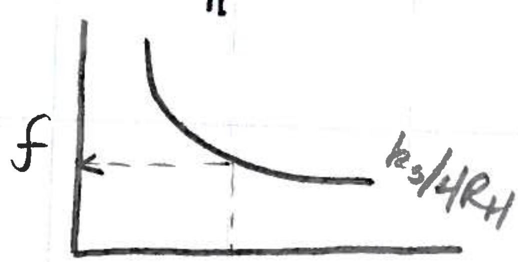
$f = 0.016$

$V = \sqrt{\frac{8g}{f} R_h S_o} = 1.92\,m/s$

$Re_D = \frac{1.92 \cdot 4(0.75)}{1.31 \cdot 10^{-6}} = 4.4 \cdot 10^6$

SCRIPT

BOARD



$f = 0.015$

$V = \sqrt{\left(\frac{8g}{f}\right) R_h S_o} = 1.98\,m/s$

$Q = (1.95\,m/s)(4.5\,m^2)$
$= 8.91\,m^3/s$

ABOUT SAME RESULT