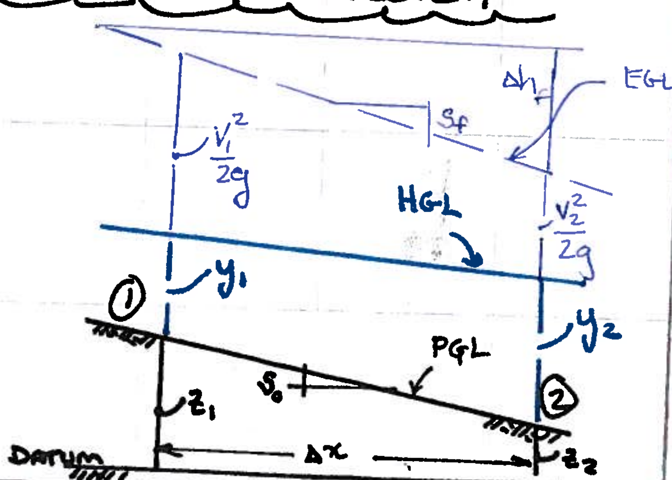
Open Channel Flow

- Gradually varied flow
- Rapidly varied flow

Definition Sketch



Energy from ① to ②

$$\underbrace{y_1 + \frac{V_1^2}{2g}}_{E_1} + \overset{\text{Channel Slope}}{S_0} \Delta X = \underbrace{y_2 + \frac{V_2^2}{2g}}_{E_2} + \overset{\text{EGL Slope}}{S_f} \Delta X$$

$$E_1 + S_0 \Delta X = E_2 + S_f \Delta X$$

Algebra & Calculus

$$S_0 - S_f = \frac{E_2 - E_1}{\Delta X}$$

SLOPE OF HGL

$$\lim_{\Delta X \to 0} \frac{E_2 - E_1}{\Delta X} = \frac{dE}{dx} = \frac{dE}{dy} \cdot \frac{dy}{dx}$$

__Calculus__

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{V^2}{2g}\right)$$

$$= \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right)$$

RECALL $A = A(y)$

$$= 1 + \underbrace{\frac{Q^2}{2gA^3}\frac{dA}{dy}}_{\text{Also is } Fr^2 \text{ (from last class)}}$$

__Combine__

$$S_0 - S_f = (1 - Fr^2)\frac{dy}{dx}$$

__Rearrange__

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

} Integrating this is used to recover $y(x)$ (the water surface profile)

Gravity

Friction

$$y(x) = \int \frac{S_0 - S_f}{1 - Fr^2} \, dx$$

Geometry

To perform the integration, need to know type profile; determined from Fr.

Slope Designations

Slope	Depths	Remarks
S-Steep	$y_n < y_c$	
C-Critical	$y_n = y_c$	
M-mild	$y_n > y_c$	
H-Horizontal	$S_0 \to 0$	$y_n \to \infty$
A-Adverse	$S_0 < 0$	

FIG 15.30
pg 585

Profile Types

Type	y, y_n, y_c	LOGIC	y, y_n, y_c
Type 1	$y > y_c$	.AND.	$y > y_n$
Type 2	$y_c < y < y_n$	.OR.	$y_n < y < y_c$
Type 3	$y < y_c$	.AND.	$y < y_n$

FIG 15.31 pg 586

Slope + Profile Type Tells How to Integrate

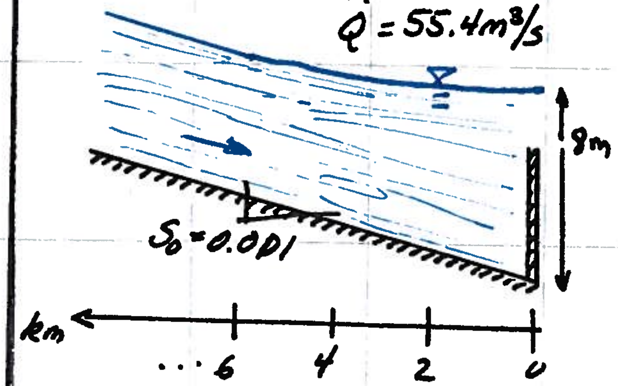
Type 1 & 2 Usually downstream control so integrate in $-x$ direction

Type 3 is upstream control so integrate in $+x$ direction

Example 15.12 pg 587

Example

RECTANGULAR
$B = 5m$
$n = 0.02$
$Q = 55.4 m^3/s$



FIND SHAPE HGL $(y(x))$ FROM WEIR TO POINT UPSTREAM WHERE NORMAL DEPTH

1) Start at location with known depth. The pool at the weir.
$y = 8m$, $x = 0$

2) Slope & Profile Type

- $y_n = 5m$ ← Slope-Area Manning's Eqn.

$y_c = 2.3m$ ← Defn. Fr in rectangular channel. (pg 571, middle of page)

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$55.4 = \frac{1}{0.02}(5 \cdot y)(2y+5)^{2/3}(0.001)^{1/2}$$

Solve for y

$$y_c = \left[\frac{55.4^2}{(5^2 \cdot 9.8)}\right]^{1/3}$$

$y_n = 5$, $y_c = 2.3$, $y_o = 8$

∴ MILD

∴ TYPE-1 (Downstream control)

Integrate in -X direction

BUILD A WORKSHEET

- Calculate E_1 at starting section
- Calculate S_{f_1} (Use Manning's)
- Change depth a little bit

$$y_2 = y_1 - 0.05y_1 \leftarrow 5\% \text{ offset}$$

- Calculate E_2
- Calculate S_{f_2}
- Compute $\bar{S}_f$

- Compute $\bar{S}_0$
- Solve

$$\Delta x = \frac{E_2 - E_1}{S_0 - S_f}$$

- Move to next section and repeat

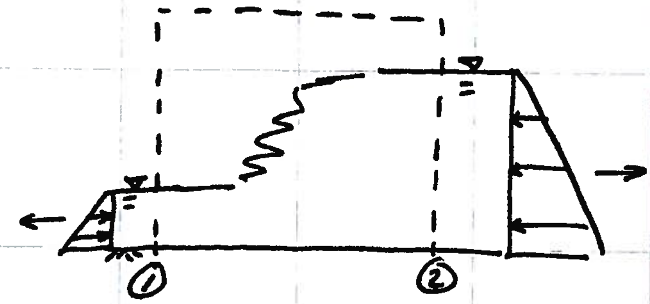
- Go over handout.

Rapidly Varied Flow

- Flow transitions from supercritical subcritical over short distance.
- Hydraulic jump is common RVF phenomenon
  - Mixing chemicals
  - Energy dissipation (erosion control)

- Energy is not conserved in RVF, but distance short so friction alone cannot explain Δh_L
- Momentum is nearly conserved

Conservation Linear Momentum



$$\Sigma F = \frac{d}{dt}\int_{CV} \rho V \cdot d\forall + \int_{c.s.} \rho V (V \cdot dA)$$

0 fixed C.V.

$$\Sigma F = -\rho V_1 A_1 V_1 + \rho V_2 A_2 V_2$$

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho \underbrace{V_2^2 A_2}_{V_2 Q} - \rho \underbrace{V_1^2 A_1}_{V_1 Q}$$

$$\therefore \bar{p}A_1 + \rho Q V_1 = \bar{p}A_2 + \rho Q V_2$$

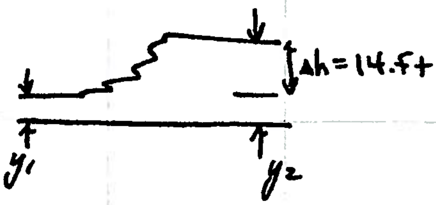
Typical problems are to find depth given upstream depth and Q.

IN CE 3105 (Fluids lab) will have to do such experiment in rectangular channel

Textbook pg 579 has derivation of rectangular channel formula

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

Example 15.53



$Q = 65 cfs$

what is y_1

$$(\Delta h + y_1) = \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

$$Fr_1^2 = \frac{Q^2 T}{g A^3}$$

$T = 1$ (UNIT WIDTH RECTANGULAR CHANNEL)

$A = Ty$

$Q = 65 ft^3/s$

$$= \frac{(65)^2}{(32.2)(y_1)^3}$$

$$\Delta h + y_1 = \frac{y_1}{2}\left(\sqrt{1 + 8\left(\frac{(65)^2}{32.2 \cdot y_1^3}\right)} - 1\right)$$

$$\Delta h = \frac{y_1}{2}\left(\sqrt{1 + \frac{8(65)^2}{32.2 y_1^3}} - 1\right) - y_1$$

PUT INTO SOLVER;
FIND y_1

y_1	$\frac{8(65)^2}{32.2 y_1^3}$	$\frac{y_1}{2}\left(\sqrt{\quad} - 1\right) - y_1$	Δh_{TARGET}
0.4	16401	25.01	14
0.8	2050	16.91	
1.2	607	13.	14
1.1	788	13.8	14
1	1049	14.7	14
1.05	906	14.2	14
1.077	840	14.003	14