SCRIPT

BOARD

CLOSED CONDUITS

ENERGY EQUATION

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\underbrace{h_L}_{\text{HEAD LOSS.}}$$

PIPE LOSSES

$$h=f\frac{L}{D}\frac{V^2}{2g}$$

FITTING LOSSES
(MINOR LOSSES)

SCRIPT

BOARD

TO PRESERVE "STRUCTURE" OF EQUATION FITTING LOSSES ARE MODELED AS QUADRATIC DRAG LAW

$$h=K\frac{V^2}{2g}$$

USUALLY THE VELOCITY IN SMALLEST DIAMETER COMPONENT

LOSS COEFFICIENT
TABULATED

SCRIPT

BOARD

KINDS OF COMPONENTS

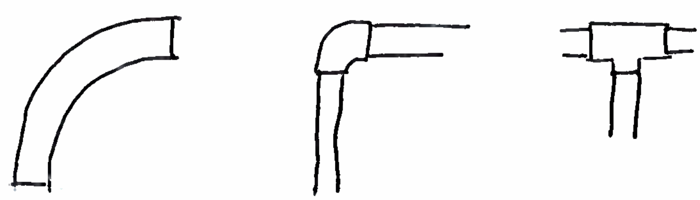
CONTRACTIONS & EXPANSIONS

RESERVOIR ENTRANCE & EXIT

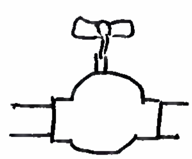
SCRIPT

BOARD

BENDS, ELBOWS, TEES



VALVES



- ANOTHER SOURCE OF FRICTIONAL LOSSES IN PIPELINES ARE CALLED MINOR LOSSES.

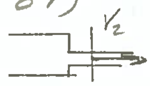
MINOR LOSSES ARE SIGNIFICANT DEPENDING ON THE SCALE OF THE SYSTEM THAT YOU ARE ANALYZING

LIFT STATION - MINOR LOSSES MATTER

REGIONAL DISTRIBUTION NETWORK - PROBABLY CAN NEGLECT

LOSSES - CONTRACTIONS

① CONTRACTION (pg 81)



$$h_{L_c} = K_c \frac{V_2^2}{2g}$$

EXIT VELOCITY

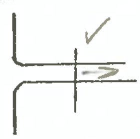
TABLE LOOK-UP

② TAPERED CONTRACTION (pg 82)

$$h'_{L_c} = K_c' \frac{V_2^2}{2g}$$

EXIT VELOCITY

③ ENTRANCES (pg 83)



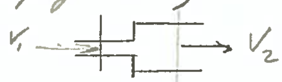
$$h_e = K_e \frac{V^2}{2g}$$

"EXIT" VELOCITY

ALL VELOCITIES ARE TAKEN "AFTER" THE LOSS PRODUCING STRUCTURE

LOSSES - EXPANSIONS

① EXPANSION (pg 83)



$$h_{L_E} = \frac{(V_1 - V_2)^2}{2g}$$

② DIFFUSER (pg 84)

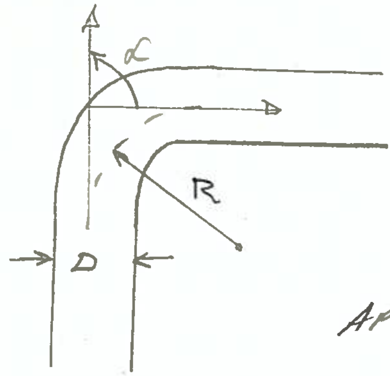
$$h_{L_E'} = K_e' \frac{V_1^2 - V_2^2}{2g}$$

(EXIT INTO RESERVOIR $K_e' = 1.0$; $V_2 = 0$)

BOTH APPROACH & EXIT VELOCITY ARE USED

AMPAD 22-141 50 SHEETS 22-142 100 SHEETS 22-144 200 SHEETS

LOSSES — BENDS



$$h_b = K_b \frac{V^2}{2g}$$

APPROXIMATION:

$$K_b = (\sin\alpha) K_b(90°)$$

REMEMBER TO INCLUDE LENGTH OF BEND AS PART OF PIPE LOSS, IF LENGTH OF BEND IS SIGNIFICANT. ($R \gg D/2$)

LOSSES — VALVES

- NEARLY ALWAYS EMPIRICAL (TABLES)

$$h_v = K_v \frac{V^2}{2g}$$

FROM TABLE — DEPENDS ON DEGREE OF OPENING.

CAN OBTAIN TABLES FROM MANUFACTURER'S.

ENERGY EQUATION WITH MINOR LOSSES

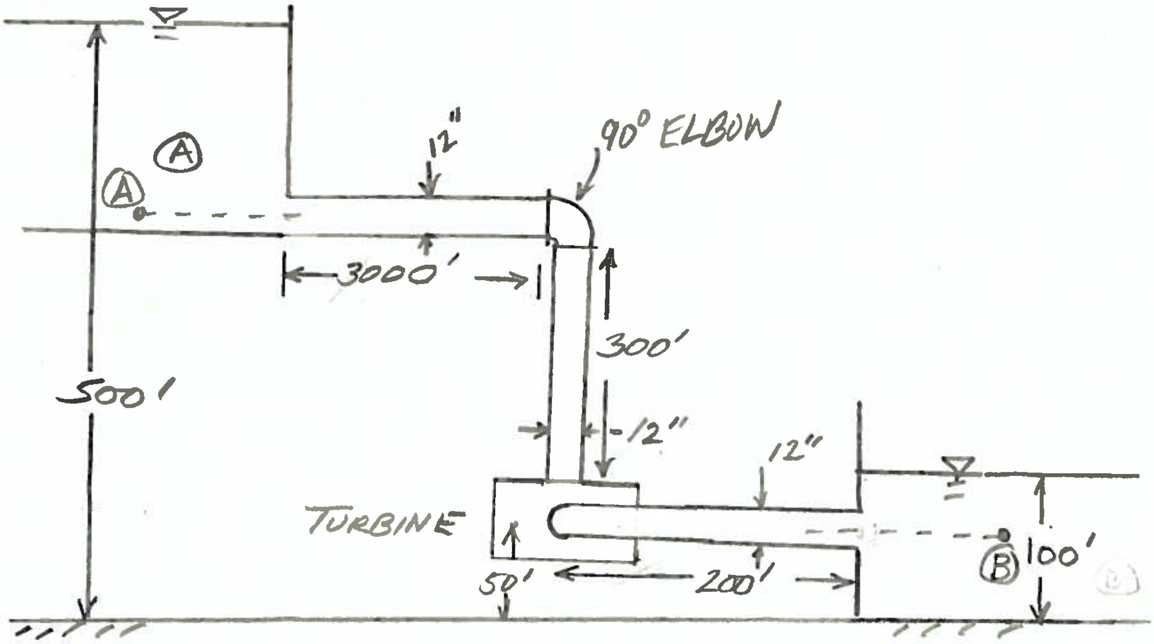
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L + \underbrace{\Sigma h_m}_{\text{SUM OF ALL MINOR LOSSES}} + h_T$$

TURBINE LOSS

PIPE LOSS

"DISSIPATION" TERMS IN INTEGRAL EQUATIONS

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD



WATER IS RELEASED FROM FOREBAY Ⓐ AT Q=2000gpm. ALL POWER PIPING IS 12" SMOOTH CONCRETE. HOW MUCH HEAD LOSS DOES THE TURBINE PRODUCE? IF THE TURBINE IS 80% EFFICIENT HOW MUCH POWER WILL THIS SYSTEM PRODUCE?

ENERGY

$$\underbrace{\frac{p_A}{\gamma} + z_A}_{500} + \cancel{\frac{V_A^2}{2g}}^{\text{small}} + \cancel{h_p}^{0} = \underbrace{\frac{p_B}{\gamma} + z_B}_{100} + \cancel{\frac{V_B^2}{2g}}^{\text{small}} + h_L + \Sigma h_m + h_T$$

$$500' = 100' + h_L + \Sigma h_m + h_T$$

Page 211

MINOR LOSSES

① POWER TUNNEL ENTRANCE $\quad K_e = 0.5 \quad h_e = K_e \frac{V^2}{2g}$

② 90° ELBOW $\quad K_b = 0.35 \quad h_e = K_b \frac{V^2}{2g}$

③ POWER TUNNEL EXIT $\quad K_e = 1 \quad h_e = K_e \frac{V^2}{2g}$

$$Q = 2000\,gpm \cdot \frac{ft^3}{7.48\,gal} \cdot \frac{1\,min}{60\,sec} = 4.45\,ft^3/s$$

$$V = \frac{Q}{A} = \frac{4.45\,ft^3/s}{\frac{\pi (1ft)^2}{4}} = 5.674\,ft/s$$

PIPE LOSSES

$$Re_d = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(5.674\,ft/s)(1\,ft)}{1.082 \cdot 10^{-5}\,\frac{ft^2}{s}}$$

$$= 524{,}399 > 2000 \therefore \text{TURBULENT}$$

$$\frac{\varepsilon}{D} = \frac{0.0006}{1.0} = 0.0006$$

$$\left.\begin{array}{l} Re = 5.24 \cdot 10^5 \\ \frac{\varepsilon}{D} = 0.0006 \end{array}\right) f \simeq 0.02 \text{ (FROM MOODY CHART)}$$

$$L = 3000' + 300' + 200' = 3500'$$

LOSS TERMS

$$h_L = f \frac{L}{D} \frac{V^2}{2g} = \frac{(0.02)(3500)(5.674)^2}{(1.0)(2)(32.2)} = 34.99'$$

$$\Sigma h_m = (1.85)\frac{V^2}{2g} = (1.85)\frac{(5.674)^2}{(2)(32.2)} = 0.924'$$

ENERGY

$$500' = 100' + 35' + 1' + h_T$$

$$500' - 136' = h_T = \underline{\underline{364'}}$$ ← HEAD EXTRACTED BY TURBINE

IF TURBINE IS 80% EFFICIENT

$(364')(0.8) = 291.2'$ OF ENERGY IS CONVERTED

POWER = WORK/TIME

= FORCE · DISTANCE / TIME

$$= Q\gamma h = 4.45\,\frac{ft^3}{s} \cdot \frac{62.4\,lbs}{ft^3} \cdot 291.2'$$

$$= 80{,}860\ \frac{ft\cdot lbs}{s}$$

$$= 80{,}860\,\frac{ft\cdot lbs}{sec} \cdot \frac{1.3558\,W}{1\,\frac{ft\cdot lbs}{sec}}$$

$$= \underline{\underline{109\,kW}}$$ ← USEFUL POWER PRODUCED

QUIZ QUESTION:

DERIVE THIS POWER RELATIONSHIP FROM FIRST PRINCIPLES – IGNORE FRICTION

Minor Losses

Losses at inlets, fittings, elbows, values, etc. are called minor losses

Minor refers to the actual length of the fitting and does not mean the losses are negligible

Energy Equation

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T+\overbrace{h_f}^{\text{pipe loss}}+\underbrace{\Sigma h_L}_{\text{other losses}}$$

Observe:

$$h_f=\underbrace{f\frac{L}{D}}\frac{V^2}{2g}$$

K - Convegence factor.

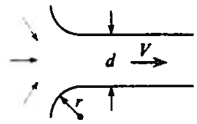
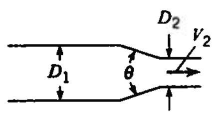
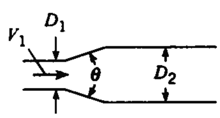
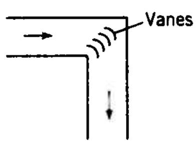
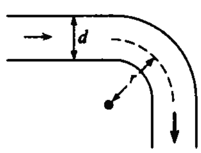
$$h_L=\underbrace{K}\frac{V^2}{2g}$$

loss coefficient (for the particular fitting)

Note both loss equations have the same structure.

Typical Table of Loss Coefficients

TABLE 10.2 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

Description	Sketch	Additional Data		K	Source
		r/d		K_e	(2)*
Pipe entrance		0.0		0.50	
		0.1		0.12	
$h_L = K_e V^2/2g$		>0.2		0.03	
			K_C	K_C	
Contraction		D_2/D_1	$\theta = 60°$	$\theta = 180°$	(2)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
$h_L = K_C V_2^2/2g$		0.90	0.06	0.10	
			K_E	K_E	
Expansion		D_1/D_2	$\theta = 20°$	$\theta = 180°$	(2)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend		Without vanes		$K_b = 1.1$	(37)
		With vanes		$K_b = 0.2$	(37)
90° smooth bend		r/d			(5) and (19)
		1		$K_b = 0.35$	
		2		0.19	
		4		0.16	
		6		0.21	
		8		0.28	
		10		0.32	
Threaded pipe fittings	Globe valve—wide open	$K_v = 10.0$			(37)
	Angle valve—wide open	$K_v = 5.0$			
	Gate valve—wide open	$K_v = 0.2$			
	Gate valve—half open	$K_v = 5.6$			
	Return bend	$K_b = 2.2$			
	Tee				
	straight-through flow	$K_t = 0.4$			
	side-outlet flow	$K_t = 1.8$			
	90° elbow	$K_b = 0.9$			
	45° elbow	$K_b = 0.4$			

*Reprinted by permission of the American Society of Heating, Refrigerating and Air Conditioning Engineers, Atlanta, Georgia, from the 1981 ASHRAE Handbook-Fundamentals.

Some tables use both approach & exit velocities

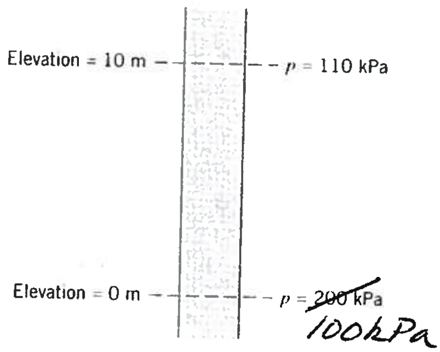
$$h_L = K\frac{V_1^2 - V_2^2}{2g}$$

be sure to read the material that accompanies a particular table

CIVE 3434 FALL 2005 EXERCISES WK7

EX 07-01

Liquid in the pipe shown in the figure has a specific weight 10.0 kN/m^3. The acceleration of the liquid is zero. Is the liquid ›nary, moving upward, or moving downward in the pipe? If pipe diameter is 1 cm and the liquid viscosity is 3.0 x 10^{-3} N · s/m^2, what is the magnitude of the mean velocity : pipe?



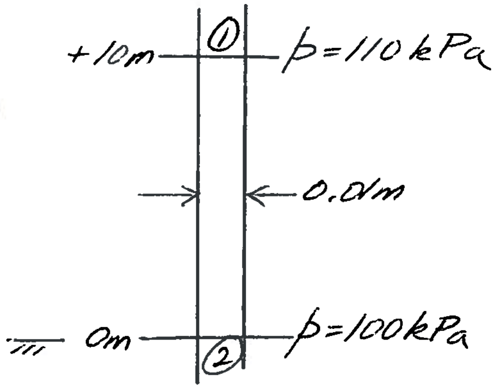
PROBLEM 10.2

$\gamma = 10\ kN/m^3$

$\mu = 3 \cdot 10^{-3} N \cdot s/m^2$

Solutions

10.2



← Adjust to 90kPa to match text

$\rho g = 10 \cdot 10^3 N/m^3$ ← Adjust to $8 \cdot 10^3 N/m^3$ to match text

$\vec{a} = 0$

$\mu = 3 \cdot 10^{-3} N \cdot s/m^2$

Is liquid moving?

$$\frac{p_1}{\gamma} + z_1 + \cancelto{}{\frac{V_1^2}{2g}} + \cancelto{=0}{h_P} = \frac{p_2}{\gamma} + \cancelto{=0}{z_2} + \cancelto{}{\frac{V_2^2}{2g}} + \cancelto{=0}{h_T} + h_L$$

$V_1 = V_2$ (continuity)

$$\frac{p_1}{\gamma} + 10m = \frac{p_2}{\gamma} + h_L$$

$$\frac{p_1}{\gamma} = \frac{110 \cdot 10^3 N/m^2}{10 \cdot 10^3 N/m^3} = 11m$$

$$\frac{p_2}{\gamma} = \frac{100 \cdot 10^3 N/m^2}{10 \cdot 10^3 N/m^3} = 10m$$

$\therefore$

$$11m + 10\cancel{m} = 10\cancel{m} + h_L$$

$11m = h_L$ Because $h_L \neq 0 \Rightarrow$ fluid is moving

How fast is liquid moving?

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \qquad Re = \frac{\rho V D}{\mu}$$

$$\frac{L}{D 2g} = \frac{(10m)}{(0.01m)(2)(9.8)} = 51.02$$

$$\frac{\rho D}{\mu} = \frac{(10 \cdot 10^3)(0.01)}{(9.8)(3 \cdot 10^{-3})} = 3401.4$$

$\therefore \quad h_L = f \; 51.02 \, V^2 \qquad Re = 3401.4\, V$

$h_L = 11m = f \; 51.02 \, V^2$

$\frac{11m}{51.02} = f V^2 = 0.2156$

Assume turbulent flow, apply Prandtl's formula

	A	B	C	D	E	F
1	Prandtl's Formula					
2						
3	$\frac{1}{\sqrt{f}} = 2\log[\text{Re}\sqrt{f}] - 0.8$					
4						
5						
6	ρ	1020.4082	(kg/m³)			
7	μ	3.00E-03	(N s/m)			
8	D	0.01	(meters)			
9				A	B	C $= fV^2$
10	V(m/s)	Re	fg	1/sqrt(fg)	1/sqrt(fg) Prandtl	f(V)V
11	1	3.40E+03	0.042	4.8795	4.89E+00	0.042
12	1.5	5.10E+03	0.037	5.198752	5.18E+00	0.08325
13	2	6.80E+03	0.034	5.423261	5.40E+00	0.136
14	2.5	8.50E+03	0.032	5.59017	5.56E+00	0.2
15	2.6	8.84E+03	0.032	5.59017	5.60E+00	0.21632
16	2.55	8.67E+03	0.032	5.59017	5.58E+00	0.20808
17	2.59	8.81E+03	0.0319	5.598925	5.59E+00	0.21398839

$V \approx 2.6 \, m/sec$

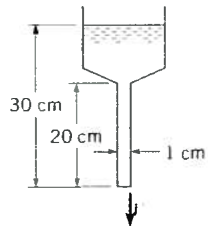
$\frac{1}{\sqrt{f}}$ $\qquad 2\log(Re\sqrt{f}) - 0.8$

① Enter different Values for V

② Adjust f Until A & B match

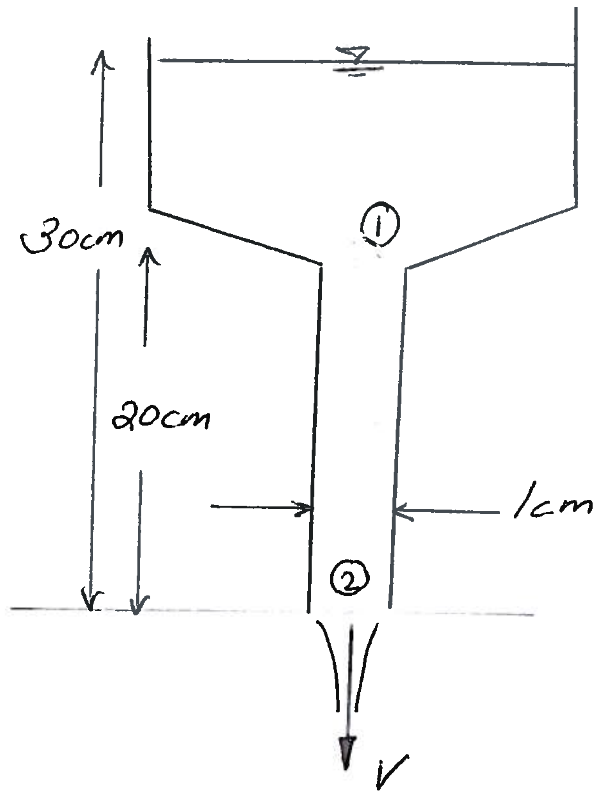
CIVE 3434 FALL 2005 EXERCISES WK 7
EX 07-02

10.16 Glycerine ($T = 20°C$) flows through a funnel as shown. Calculate the mean velocity of the glycerine exiting the tube. Assume the only head loss is due to friction in the tube.



PROBLEM 10.16

10.16 Glycerine @20°C Calculate V in tube



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2} + h_L$$

$=0$ $=0$

$\underbrace{\frac{p_1}{\gamma} + \frac{V_1^2}{2g}}_{10cm}$

$$10cm + 20cm = \frac{V_2^2}{2g} + h_L$$

$$30cm = f\frac{L}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\rho = 1260\ kg/m^3$$

$$\mu = 6.2 \cdot 10^{-1}\ N \cdot s/m^2$$

$$Re = \frac{\rho D}{\mu} V$$

$$= \frac{(1260)(0.01)}{6.2 \cdot 10^{-1}} V$$

$$= 20.3\ V$$

(V > 100 m/s flow will still be laminar)

$$\therefore f = \frac{64}{Re}$$

$$30cm = \left(1 + \frac{64}{Re}\frac{L}{D}\right)\frac{V^2}{2g}$$

$$= \left(1 + \frac{64}{20.3V}\frac{L}{D}\right)\frac{V^2}{2g}$$

$$2g(30cm) = 2(9.8)(0.3) = 65.3$$

$$65.3 = V^2 + \frac{64L}{20.3VD}V^2$$

$$\frac{64(0.2)}{(0.01)20.3} = 63.05$$

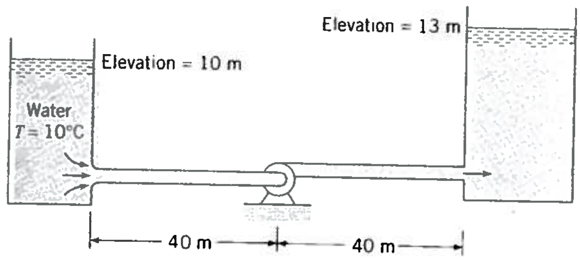
$$\therefore 65.3 = V^2 + 63.05V$$

V	V^2	$63.05V$	$V^2 + 63.05V$
1	1	63.05	64.05
1.2	1.44	75.66	77.1
1.1	1.21	69.35	70.56
1.05	1.1025	66.25	67.3
1.025	1.0506	64.55	65.6

CIVE 3434 FALL 2005 EXERCISES WK7

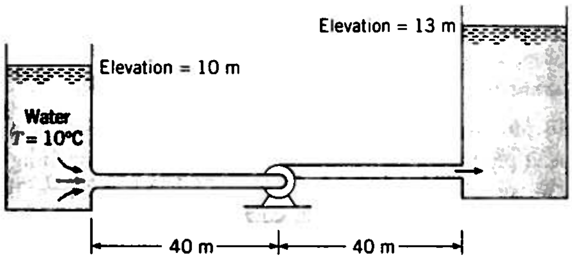
EX 07-04

• **10.61** If the flow of $0.10\ m^3/s$ of water is to be maintained in the system shown, what power must be added to the water by the pump? The pipe is made of steel and is 15 cm in diameter. Draw the EGL and the HGL for the system.



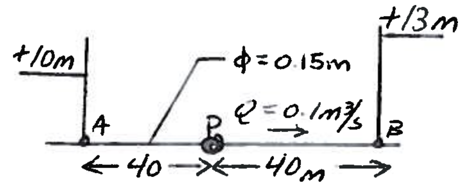
PROBLEM 10.61

10.61 If a flow of 0.10 m^3/s of water is to be maintained in the system shown, what power must be added to the water by the pump? The pipe is made of steel and is 15 cm in diameter. Draw the EGL and the HGL for the system.



PROBLEM 10.61

Sketch



Energy from Ⓐ → Ⓑ

$$\underbrace{\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_a}_{=10m} + h_p = \underbrace{\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_b}_{=13m} + h_{f_{A\to B}}$$

$$h_p + 10m = h_f + 13m$$

$$h_p = h_f + 3m \qquad h_f = \frac{fL}{D}\frac{V^2}{2g} = \frac{8fL}{\pi^2 g D^5} Q^2$$

$L = 80m$

$D = 0.15m$

$$\therefore h_f \approx 0.0205(871.36)$$

$$= 17.8 \text{ m}$$

$$h_f = f \cdot 871.36$$

$$\therefore h_p = 20.8 \text{ m}$$

$$Re_d = \frac{4\rho Q}{\pi \mu D} = 8.5 \cdot 10^5$$

$$\text{Power} = Q\gamma h$$

$$= (0.1 m^3/s)(9800 N/m^3)(20.8)$$

$$= \underline{\underline{20.4 \text{ kW}}}$$

$$\frac{\varepsilon}{D} = \frac{0.000046}{0.15} = 0.0003$$

$$f \approx 0.0205 \qquad (*)$$

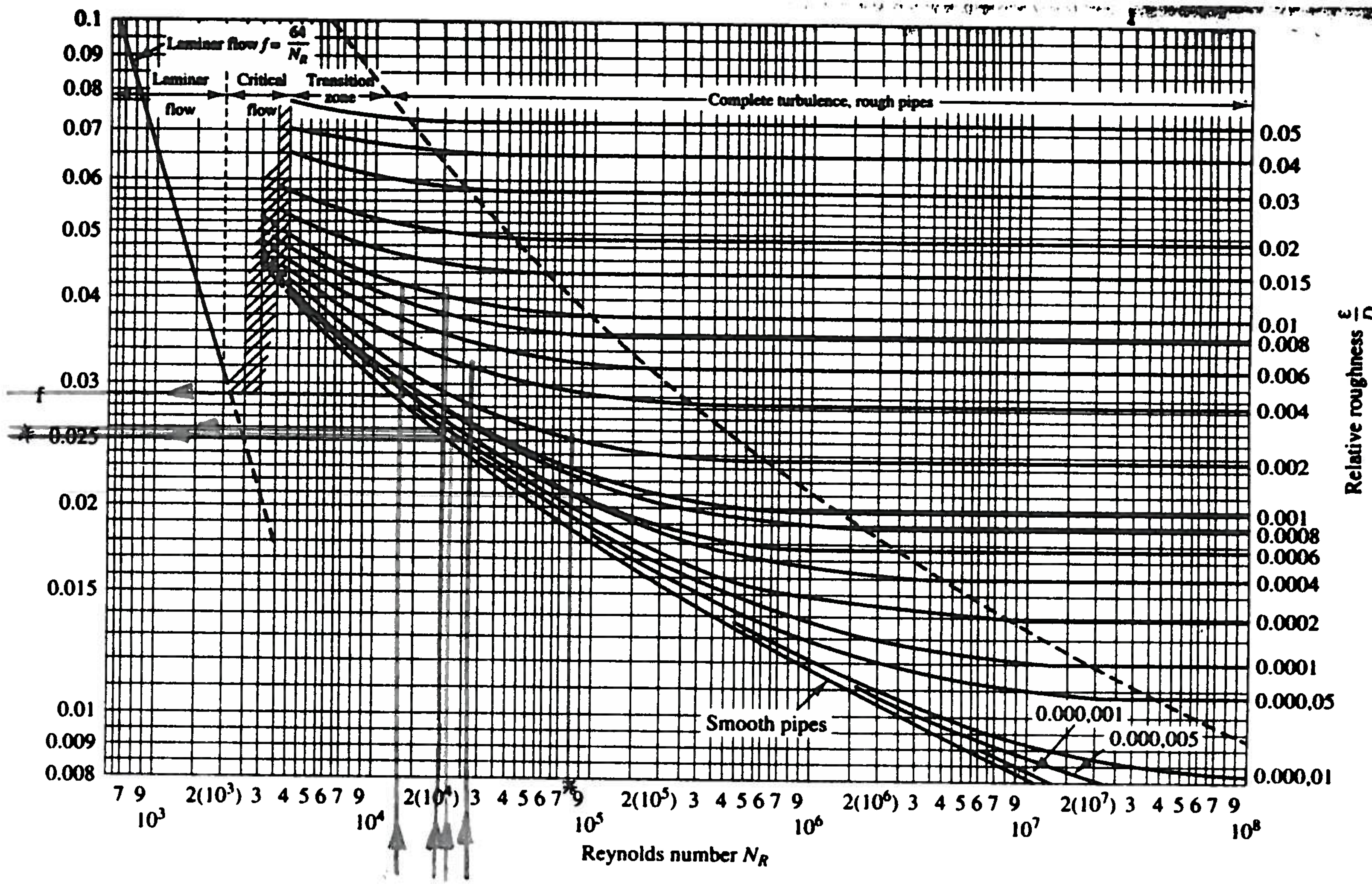
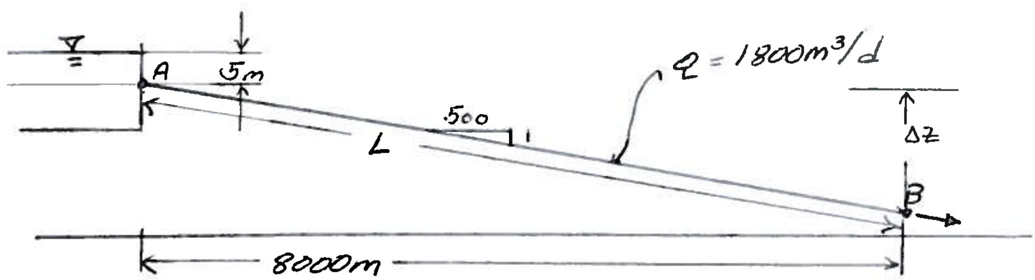


Figure 3.8 Friction factors for flow in pipes, the Moody diagram (From L. F. Moody, "Friction factors for pipe flow," *Trans. ASME*, vol. 66, 1944.)

Determine the minimum diameter concrete pipe that may be used to transport 1800 m^3/d of water from a reservoir to a water treatment plant. The reservoir is 8 km from the plant, and the water surface in the reservoir is 5m above the pipe entrance. The pipeline is to be laid on a 1/500 slope. Assume the pipe discharges to open air.



Solution

Find elevation of A.

Let $z_B = 0$. $\quad z_A = \frac{1}{500}(8000) = 16m$

Energy Equation from Ⓐ → Ⓑ

$$\frac{p_A}{\gamma} + z_a + \frac{V_a^2}{2g} = \frac{p_B}{\gamma} + z_b + \frac{V_b^2}{2g} + h_f$$

$\frac{p_a}{\gamma} + \frac{V_a^2}{2g} = 5m$ (given), $z_a = 16m$

$$\therefore \quad 21m = \underbrace{\frac{p_B}{\gamma}}_{=0 \text{ (discharge to open air)}} + \overbrace{z_b}^{=0 \text{ (by selection of datum}} + \frac{V_b^2}{2g} + h_f$$

$$\therefore \quad 21m = h_f + \frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\frac{V^2}{2g} = \frac{8Q^2}{g\pi^2 D^4} \quad \therefore \quad 21m = \frac{8}{g\pi^2}\left(\frac{1}{D^4} + \frac{fL}{D^5}\right)Q^2$$

$$Re_d = \frac{\rho V D}{\mu} = \frac{4\rho Q}{\mu \pi D} \qquad \rho = 1000\,kg/m^3$$

$$\mu = 1\cdot 10^{-3}\,N\cdot s/m^2$$

$L = 8000m$

$Q = 1800 m^3/d \approx 0.021 m^3/s$

$$\therefore \quad 21m = \underbrace{\frac{8}{g\pi^2}\left(\frac{1}{D^4} + \frac{8000f}{D^5}\right)Q^2}_{h_f}$$

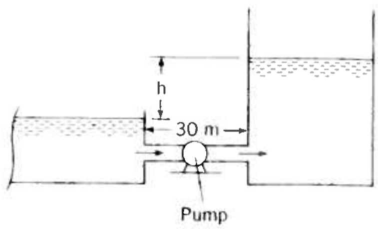
D (m)	ε/D	Re	f	h_f
1.0m	0.0003	$2.7\cdot 10^4$	0.025	57.4m
2.0m	0.00015	$1.3\cdot 10^4$	0.029	2.1m
1.25m	0.00024	$2.1\cdot 10^4$	0.026	19.5m
1.2m	0.00025	$2.2\cdot 10^4$	0.025	23.0m
1.22m	0.000246	$2.2\cdot 10^4$	0.025	21.2m

$$1.22m \cdot \frac{3.25ft}{m} \cdot \frac{12in}{1ft} \approx 47.58\,in$$

Specify a 48" (4 foot) I.D. RCP.
(48" Reinforced Concrete Pipe)

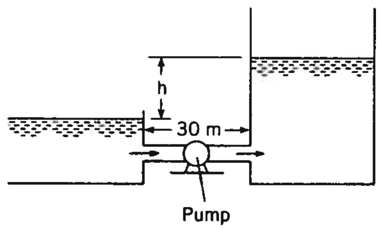
EX 07-03

10.54 A pump is used to fill a tank from a reservoir as shown. The head provided by the pump is given by $h_p = h_0(1 - (Q^2/Q^2_{max}))$ where h_0 is 50 meters, Q is the discharge through the pump, and Q_{max} is 2 m^3/s. Assume $f = 0.018$ and the pipe diameter is 90 cm. Initially the water level in the tank is the same as the level in the reservoir. The cross-sectional area of the tank is 100 m^2. How long will it take to fill the tank to a height, h, of 40 m?



PROBLEM 10.54

10.54 A pump is used to fill a tank from a reservoir as shown. The head provided by the pump is given by $h_p = h_0(1 - (Q^2/Q_{max}^2))$ where h_0 is 50 meters, Q is the discharge through the pump and Q_{max} is 2 m^3/s. Assume $f = 0.015$ and the pipe diameter is 90 cm. Initially the water level in the tank is the same as the level in the reservoir. The cross-sectional area of the tank is 100 m^2. How long will it take to fill the tank to a height, h, of 40 m?



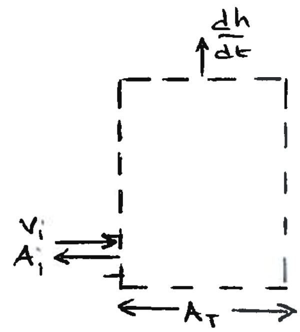
PROBLEM 10.54

Continuity for tank

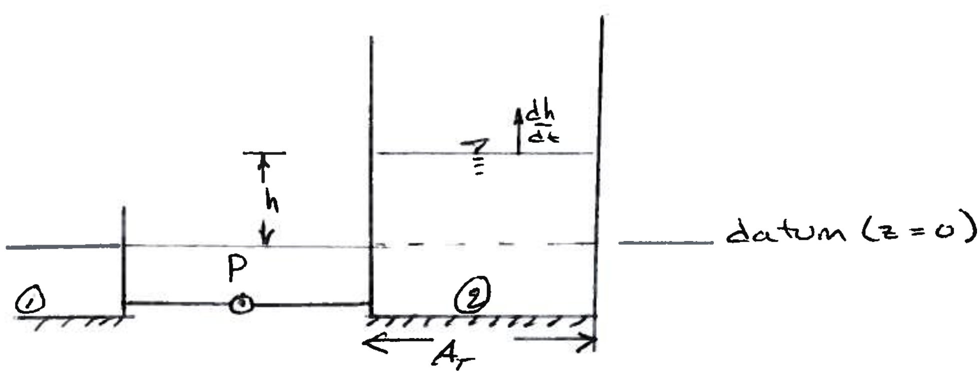
$$0 = \frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{c.s.} \rho V \cdot dA$$

$$0 = \rho A_T \frac{dh}{dt} - \rho \underbrace{V_i A_i}_{Q}$$

$$\therefore \frac{dh}{dt} = \frac{Q}{A_T}$$



Sketch



$$\underbrace{\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1}_{=0} + h_p = \underbrace{\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2}_{=h} + h_L$$

$$\therefore h_p = h + h_L$$

$$h_p = h_0\left(1 - \frac{Q^2}{Q_m^2}\right)$$

$$h_L = \frac{8fL}{\pi^2 g D^5} Q^2$$

<u>Given</u>

$h_0 = 50\,m$

$Q_m = 2\,m^3/s$

$f = 0.015$

$L = 30m$

$D = 0.9m$

$$\therefore h_L = \frac{8(0.015)(30)Q^2}{\pi^2(9.8)(0.9)^5}$$

$$= 0.063Q^2$$

$$h_p = 50\left(1 - \frac{Q^2}{4}\right) = 50 - 12.5Q^2$$

$$\therefore 50 - 12.5Q^2 = h + 0.063Q^2$$

$$h = 50 - 12.563Q^2$$

$$\frac{dh}{dt} = -12.563\left(2Q\frac{dQ}{dt}\right) = \frac{Q}{A_T}$$

$$-25.126\,A_T\,dQ = dt$$

$$\int -25.126\,A_T\,dQ = \int dt$$

$$-25.126\,A_T\,Q = t + C$$

$=100m$

$$-2512.6\,Q = t + C$$

Evaluate constant of integration

@ $t = 0 \qquad Q = 1.99\ m^3/s$

$\therefore \quad C = -5012.5$

So $\quad Q = \dfrac{t - 5012.5}{-2512.6}$

Now substitute back into related rate equation

$$\frac{dh}{dt} = \frac{\dfrac{t - 5012.5}{-2512.6}}{100} = \frac{t - 5012.5}{-251{,}260}$$

$$dh = \frac{t - 5012.5}{-251{,}260}\, dt$$

$$\int_0^H dh = \int_0^T \frac{t - 5012.5}{-251{,}260}\, dt$$

$$H = \frac{1}{-251{,}260}\left[\frac{t^2}{2} - 5012.5\, t\right]_0^T$$

$$H = \frac{1}{-251{,}260}\left[\frac{T^2}{2} - 5012.5\, T\right]$$

Substitute $H = 40m$, Solve for T

$$40m = \frac{1}{-251{,}260}\left[\frac{T^2}{2} - 5012.5\,T\right]$$

$$-10{,}050{,}400 = \frac{T^2}{2} - 5012.5\,T$$

$$-20{,}100{,}800 = T^2 - 10{,}025\,T$$

$$T^2 - 10{,}025\,T + 20{,}100{,}800 = 0$$

$$T = \frac{10{,}025 \pm \sqrt{10{,}025^2 - 4(20{,}100{,}800)}}{2}$$

$$= \frac{10{,}025 \pm 4483}{2} \qquad = 2770.9\text{ sec}$$

$$7254.0\text{ sec}$$

Now which time is correct?

V_T @ $40m = (40)(100) = 4000m^3$

Average flow rate $\sim 1.5\,m^3/sec$

So 2770.9 sec $\sim 4155\,m^3$ in tank

While 7250 sec $\sim 10500\,m^3$ in tank

∴ Smaller time is correct

$$T = 2770.9\text{ sec} = \underline{\underline{46.2\text{ min}}} \Longleftarrow$$

Alternate approach

h	V_T	Q	ΔV_T	time to add ΔV_t
0	0	~2 m³/s	0	0
5	500 m³	1.89 m³/s	500 m³	264 sec
10	1000 m³	1.78 m³/s	500 m³	280 sec
15	1500 m³	1.66 m³/s	500 m³	301 sec
20	2000 m³	1.54 m³/s	500 m³	324 sec
25	2500 m³	1.41 m³/s	500 m³	355 sec
30	3000 m³	1.26 m³/s	500 m³	397 sec
35	3500 m³	1.09 m³/s	500 m³	459 sec
40	4000 m³	0.89 m³/s	500 m³	562 sec

$\Sigma T = \boxed{2942 \text{ sec}} \approx 49 \text{ min}$

$$h = 50 - 12.563 Q^2$$

$$\therefore \quad \frac{h-50}{-12.563} = Q^2$$

$$\sqrt{\frac{h-50}{-12.563}} = Q$$