CLEVELAND 4FEB14
CE3305 1/2
EXTRA NOTES

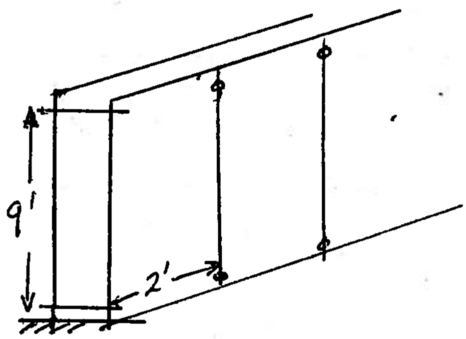
Exercise 02_02_04

Assuming that concrete behaves as a liquid (γ = 150 lbf/cu.ft.) just after it is placed, determine the force per foot of length exerted on a form by the concrete if it is poured into forms for a wall that is to be 9 feet high. If the forms are held in place as shown, with ties between vertical braces spaced every 2 feet, what force is exerted on the bottom tie?

Figure 4. Concrete retaining wall.

3.68 Force per unit length on concrete form 9' tall. Find force on bottom tie if vertical braces are every 2'

$\gamma_{conc.} = 150\ lbf/ft^3$



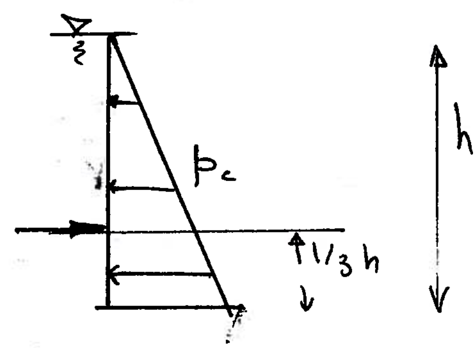
Force per unit length

$\Sigma F = 0$

$F_R = \int -p_c \, dA$

$\quad = (\tfrac{1}{2})(9')^2(150\ lbf/ft^3)$

$F_R = 6075\ lbf/ft$



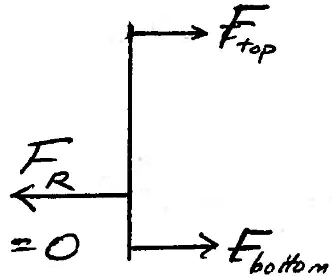
line of action is 3ft from bottom

Force on ties.

1 tie / two feet

$\Sigma F = F_{TOP} + F_{BOT} - F_R$

$\Sigma M_{bottom} = 0$



$F_R(3ft) - F_{TOP}(9ft) = 0$

$\dfrac{F_R(3)}{9} = F_{TOP} = 2025\ lbf/ft$

$F_{BOT} = 4050\ lbf/ft * 2ft = \underline{\underline{8100\ lbf}}$

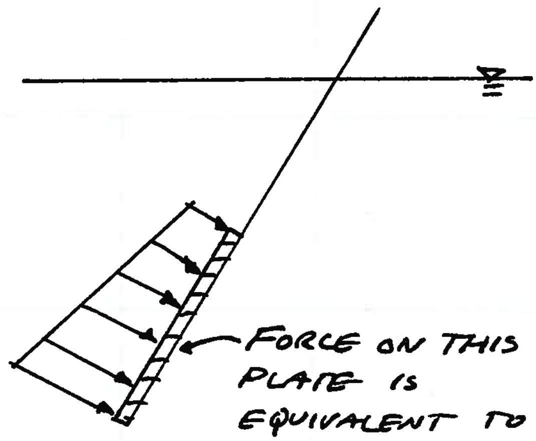
NAME CLEVELAND DATE 4FEB14
COURSE CE3305 SHEET 1 OF 5

SCRIPT

BOARD

THE "INTEGRAL" METHODS ARE FINE AND WORK ALWAYS

A PRACTICAL APPROACH USES DISPLACEMENT VOLUMES TO ACHIEVE RESULTANT FORCE



FORCE ON THIS PLATE IS EQUIVALENT TO

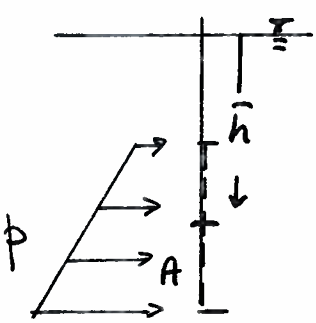
SCRIPT

F_z VERTICAL FORCE EQUAL TO WEIGHT OF LIQUID DISPLACED ABOVE SURFACE OF PLATE

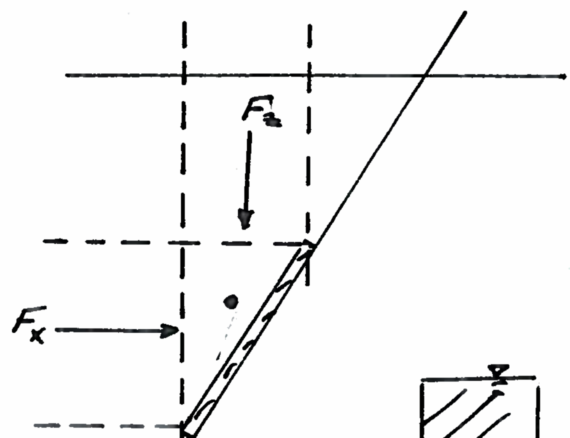
$F_z = \gamma \forall$

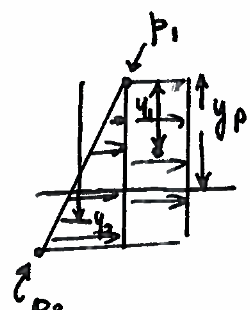
F_x MAGNITUDE IS

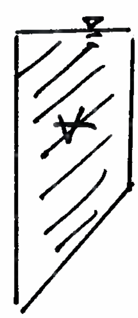
$F_x = \gamma \bar{h} A$



BOARD







$F_z = \gamma \forall$

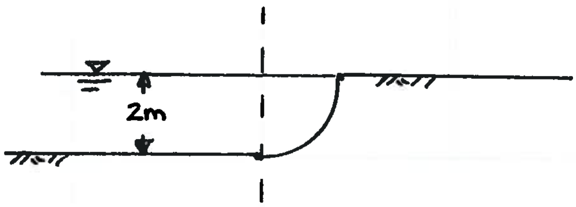
SCRIPT	BOARD
	$\bar{h}$ IS DISTANCE FROM FREE SURFACE TO CENTROID OF PROJECTED AREA LINE OF ACTION THROUGH CENTROID OF PRESSURE PRISM $y_p = \dfrac{y_1 p_1 A + y_2 p_2 A/2}{(p_1 + \frac{p_2}{2})A}$

SCRIPT	BOARD
EXAMPLE NEXT PAGE	F_z LINE OF ACTION THROUGH CENTROID OF COLUMN PROJECTION

EXAMPLE

FIND RESULTANT FORCE ON CURVED SURFACE SHOWN. WIDTH IS 1 METER INTO DIAGRAM

SKETCH



GIVEN

2m WATER.
CURVED SURFACE
1m INTO BOARD

GOVERNING EQUATIONS

$p = \gamma h$

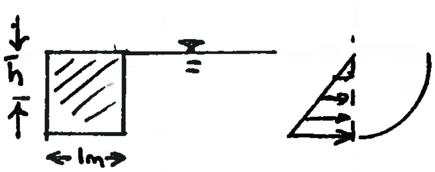
$F = pA$

FIND

RESULTANT FORCES ON CURVED PLATE

SOLUTION

HORIZONTAL FORCES



PROJECTION

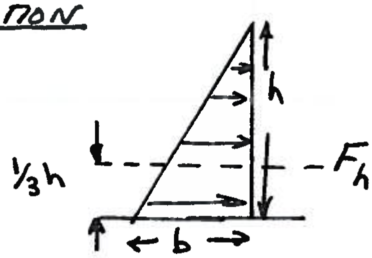
$A = 1m \times 2m = 2m^2$

$\bar{h} = 1m$

$F_h = \gamma \bar{h} A = (9800 N/m^3)(1m)(2m^2)$

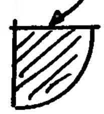
$= 19600 N$

LINE OF ACTION



THROUGH CENTROID OF PRESSURE PRISM

VERTICAL FORCES



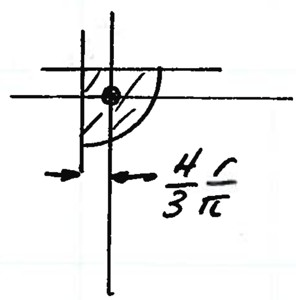
$$\forall = \frac{1}{4}\pi r^2 L$$

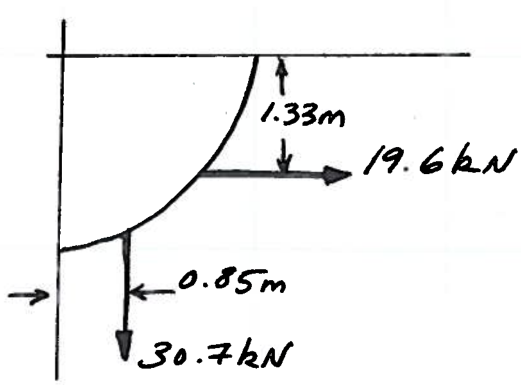
$$= \frac{1}{4}\pi(2m)^2(1m) = \pi m^3 \approx 3.142 m^3$$

$$F_v = \gamma \forall = (9806 N/m^3)(3.142 m^3) = 30,787 N$$

LINE OF ACTION

THROUGH CENTROID OF PROJECTED VOLUME





REMARKS

① GET SAME RESULTS IF WENT FOR INTEGRAL

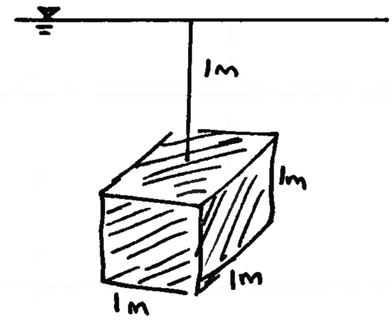
SCRIPT

SUBMERGED BODIES ARE SUBJECT TO A BOUYANT FORCE CREATED BY THE VERTICAL VARIATION OF PRESSURE IN A STATIC FLUID

IMAGINE A BLOCK AS SHOWN

BOARD

BOUYANCY



SCRIPT

IN THIS CASE

$$p_{BOT} - p_{TOP} = \gamma(2m - 1m)$$
$$= \gamma(1m)$$

$$\therefore F_B = \gamma(1m)(A)$$
$$= \gamma(1m)(1m)^2$$
$$= \gamma \forall_{BLOCK}$$

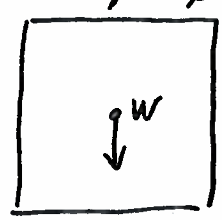
FOR ANY OBJECT, THE BOUYANT FORCE WILL ALWAYS EQUAL THE WEIGHT OF THE WATER DISPLACED

$$F_B = \gamma \forall_{OBJECT}$$

BOARD

FORCE BALANCE ON BLOCK

$p_{top} A_{top}$



$p_{Bot} A_{BOT}$

$$\Sigma F_y = \underbrace{p_{Bot} A_{BOT} - p_{TOP} A_{TOP}}_{\text{BOUYANT FORCE}} - W$$

THE RESULT IS CALLED ARCHIMEDE'S PRINCIPLE

LINE OF ACTION VERTICAL UP THROUGH CENTROID OF DISPLACED VOLUME

SCRIPT

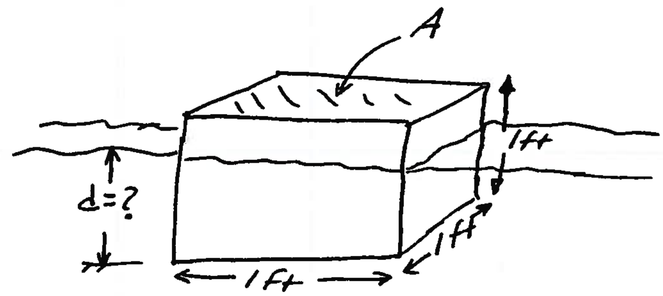
A BODY FLOATS IF THE BOUYANT FORCE EQUALS THE WEIGHT OF THE BODY.

CONSIDER THE CONCRETE BLOCK, WITH DENSITY SHOWN.

ANALYZE THE VERTICAL FORCE BALANCE

BOARD

FLOATING BODIES AND STABILITY



BLOCK $\rho g = 40\,lbf/ft^3$

HOW DEEP WILL BLOCK FLOAT?

KNOWN

$\rho g = 40\,lbf/ft^3$

$\forall = 1\,ft^3$

SCRIPT

BOARD

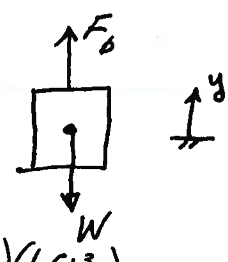
GOVERNING EQUATIONS

$\Sigma F = m\cancel{a}^{=0}$; $F_b = \gamma \forall_{displaced}$

FIND

d

SOLUTION



$\Sigma F_y = 0 = F_b - W$

$\therefore F_b = W$

$= \rho g \forall = (40\,lbf/ft^3)(1\,ft^3)$

$= 40\,lbf$

$F_b = \rho_w g \forall_{displaced} = 40\,lbf$

$\forall_{displaced} = \dfrac{40\,lbf}{\rho_w g} = \dfrac{40\,lbf}{62.4\,lbf} \cdot ft^3$

SCRIPT

THE DEPTH d IS CALLED THE DRAFT OF THE VESSEL.

IMPORTANT IN SHALLOW WATER OPERATION (NEAR SHORE) OF VESSELS AND OFFSHORE OIL PLATFORMS.

BOARD

$$\forall_{displaced} = 0.641\ ft^3$$

$$\forall_{displaced} = Ad = (1ft^2)d$$

$$d = \frac{\forall_{displaced}}{Area} = \frac{0.641 ft^3}{1 ft^2}$$

$$= 0.641\ ft$$

$\therefore$ Object floats with $d = 0.64\ ft$

SCRIPT

DETERMINATION OF WHETHER AN OBJECT WILL REMAIN ORIENTED AS PLACED OR NOT IS STABILITY ANALYSIS.

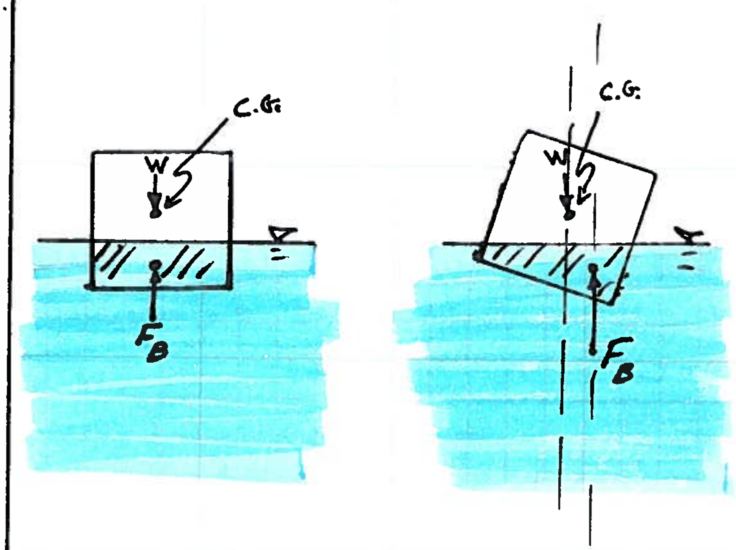
KIND OF IMPORTANT FOR VESSELS, SUBMARINES, PLATFORMS.

ALSO IMPORTANT FOR DUCKS; IF A DUCK IS UNSTABLE IN FLIGHT AND TURNS UPSIDE DOWN; THEN IT QUACKS UP!

BOARD

STABILITY

IMMERSED OR FLOATING OBJECTS



CALLED A RIGHTING MOMENT OR OVERTURNING MOMENT

W & F_B FORM A COUPLE. IF THE MOMENT-COUPLE RETURNS STABLE

SCRIPT

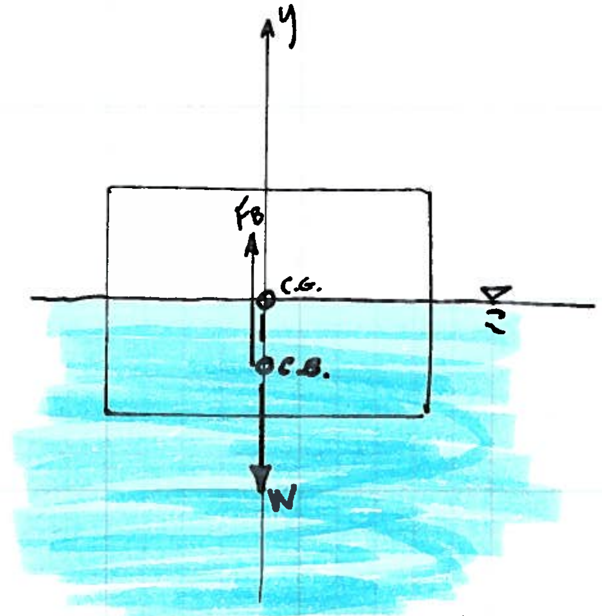
THE WEIGHT W ACTS THROUGH THE GRAVITY CENTER OF THE BODY.

THIS C.G. DOES NOT CHANGE LOCATION RELATIVE TO THE BODY.

THE BOUYANT FORCE F_B ACTS THROUGH THE CENTROID OF THE SUBMERGED SECTION.

THE FORCES CREATE A MOMENT-COUPLE (b) CALLED A RIGHTING MOMENT (STABLE) OR AN OVERTURNING MOMENT (UNSTABLE)

BOARD



(a)

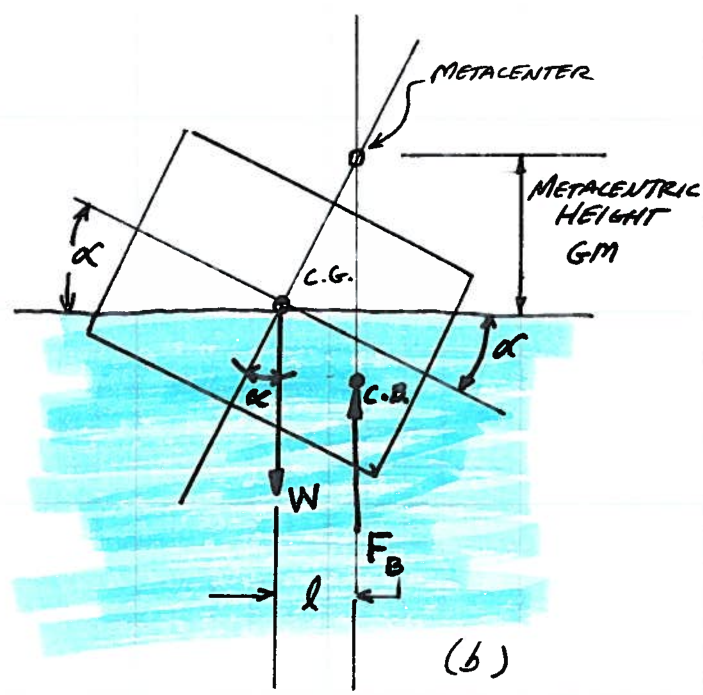
SCRIPT

THE INTERSECTION OF A BISECTOR RUNNING UP THROUGH THE VESSEL IN CONDITION (a) THROUGH THE C.G. AND A LINE OF ACTION THROUGH THE BOUYANT FORCE VECTOR IS CALLED THE VESSEL METACENTER.

IF METACENTER IS ABOVE THE C.G., THE BODY IS STABLE, OTHERWISE UNSTABLE

IN (b) THE RIGHTING MOMENT IS

BOARD



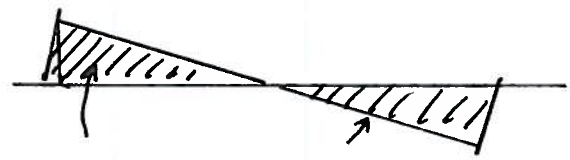
(b)

$M^{\curvearrowright} = W \cdot GM \cdot \sin\alpha$

SCRIPT

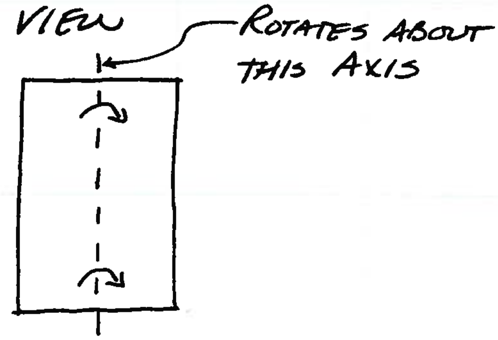
THE METACENTRIC HEIGHT GM IS OBTAINED FROM THE MOMENT OF INERTIA OF THE WEDGE THAT LIFTS AND SINKS.

BOARD



I_o OF THESE WEDGES

PLAN VIEW



SCRIPT

BOARD

EXAMPLE NEXT PAGE

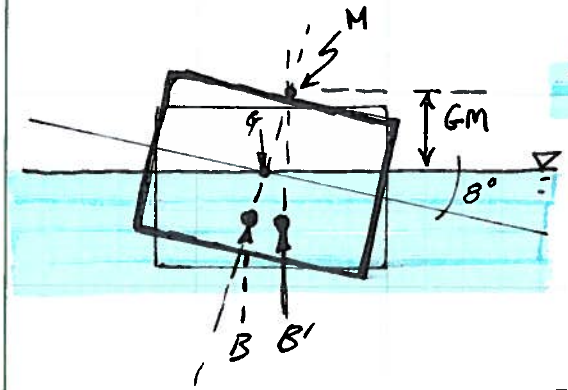
SUMMARY CHAPTER 3

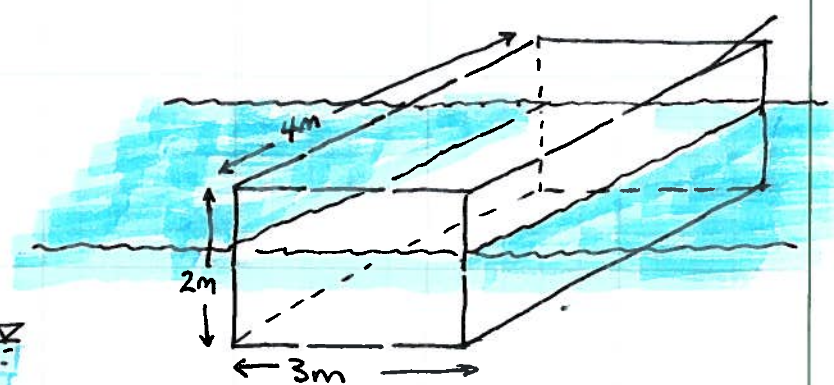
- PRESSURE, HYDROSTATIC EQUILIBRIUM
  HYDROSTATIC EQUATION
- PRESSURE DISTRIBUTIONS & FORCES
- PLATE PRESSURE
  + CURVED SURFACE AP
- BOUYANT FORCE & STABILITY

EXAMPLE

A 3m x 4m RECTANGULAR BOX PONTOON IS 2m DEEP. IT DRAWS 1.2m WHEN FLOATING UPRIGHT. COMPUTE THE METACENTRIC HEIGHT AND RIGHTING MOMENT IN SEA WATER (S.G. = 1.03) WHEN THE ANGLE OF HEEL IS 8°.

SKETCH





$$GM = \frac{I_o}{\forall_{displaced}} - CG$$

Eq. 3.47

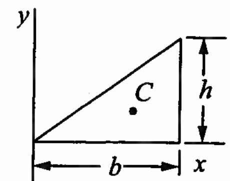
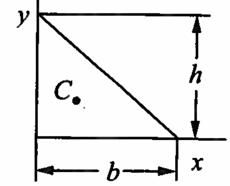
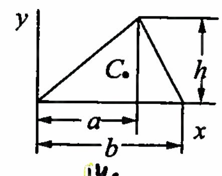
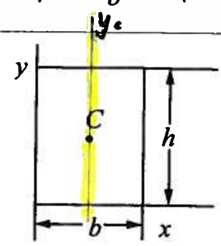
CG: DISTANCE FROM C.G. TO C.B. IN UPRIGHT CONDITION

$$\forall_{displaced} = (1.2m)(3m)(4m) = 14.4m^3$$

$$I_o = \frac{1}{12} b^3 l = \frac{1}{12}(3^3)(4) = 9\,m^4$$

$$CG = \left(\frac{2}{2}m\right) - \left(\frac{1.2}{2}m\right) = 1 - 0.6 = 0.4m$$

$$GM = \frac{9m^4}{14.4m^3} - 0.4m = 0.225m$$

Figure	Area & Centroid	Area Moment of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ ← CENTROIDAL AXIS $I_x = bh^3/3$ $I_y = b^3h/3$ $J = [bh(b^2 + h^2)]/12$

NEXT APPLY RIGHTING MOMENT EQUATION

$$M\!\circlearrowleft = W \cdot GM \cdot \sin\alpha$$

$$= (14.4\,m^3)(1.03 \cdot 9800\,N/m^3)\sin\left(\frac{8^\circ}{180^\circ}\right)$$

$$\approx 4560\ N\cdot m$$