SCRIPT

BODY FORCES ARE DEVELOPED WITHOUT CONTACT AND ARE DISTRIBUTED OVER THE VOLUME OF A FLUID

THE WEIGHT OF A QUANTITY OF LIQUID IS A BODY FORCE

SURFACE FORCES ACT AT BOUNDARIES OF A MEDIUM THROUGH CONTACT

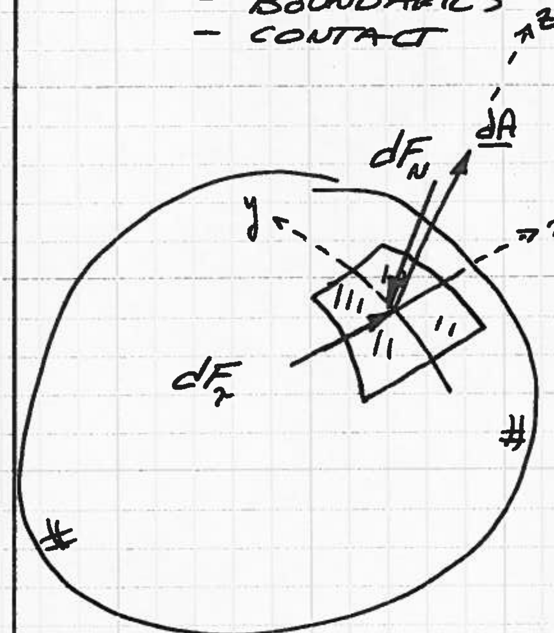
CONSIDER SURFACE OF A BUBBLE, WITH SOME SMALL AREA DEFINED ON THE SURFACE

BOARD

FLUID STATICS

BODY FORCES
- VOLUME

SURFACE FORCES
- BOUNDARIES
- CONTACT



STRESS IS LIMITING VALUE OF dF/dA

TWO KINDS OF STRESS ARE DEFINED

NORMAL
SHEAR (TANGENTIAL)

BOARD

~~STRESS IS THE LIMITING VALUE OF~~

$$\frac{dF}{dA}$$

NORMAL STRESS (PRESSURE)

SHEAR STRESS (FRICTION)

SCRIPT

USUAL NOTATION IS

σ FOR NORMAL STRESS

AND

τ FOR SHEAR STRESS

WHEN APPLIED IN 3-DIMENSIONS

first subscript is direction of normal vector to the plane ($\underline{n}$)

second subscript is direction of stress application

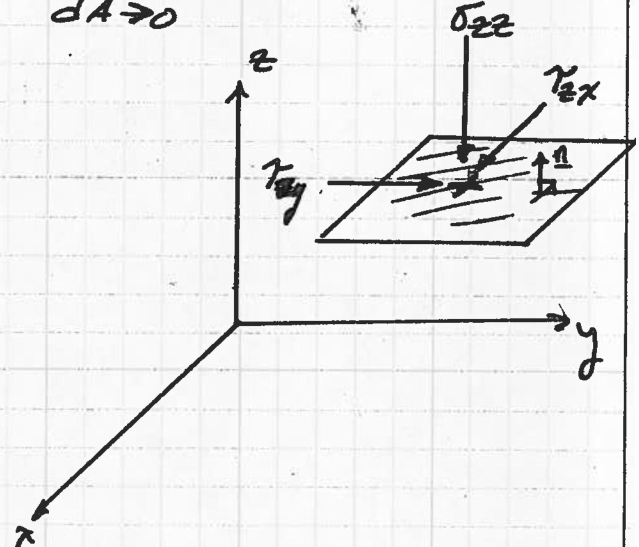
(normal stress is + into plane by convention)

BOARD

USUAL NOTATION IS

$$\sigma = \lim_{dA \to 0} \frac{dF_N}{dA} \quad \text{(NORMAL)}$$

$$\tau = \lim_{dA \to 0} \frac{dF_T}{dA} \quad \text{(SHEAR)}$$



SCRIPT

BECAUSE PRESSURE IS A NORMAL FORCE PER UNIT AREA, AT ANY POINT IN A FLUID, THE PRESSURE IS A SCALAR QUANTITY

BOARD

FLUID PRESSURE IS THE NORMAL STRESS APPLIED TO A FLUID ELEMENT

UNITS OF PRESSURE ARE

FORCE/AREA (psi, Pa)

LIQUID COLUMN HEIGHT (in_{H_2O}, in_{Hg})

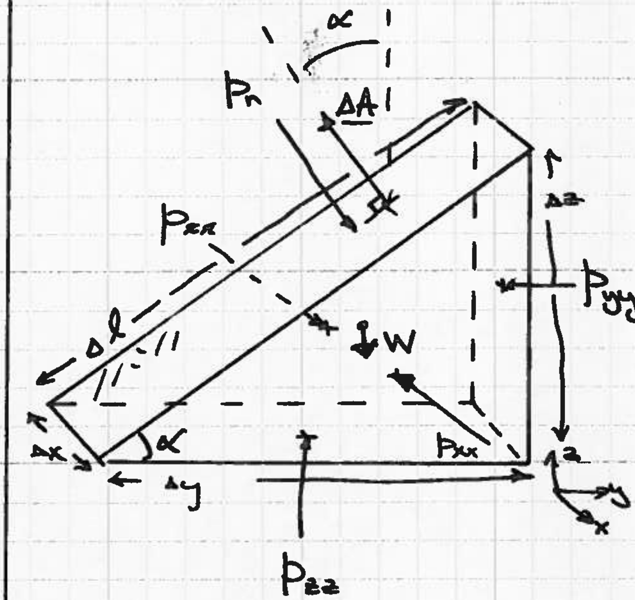
ATMOSPHERES

SCRIPT

CONSIDER A WEDGE OF FLUID

- PERFORM A FORCE BALANCE IN EACH DIRECTION
- INVOKE "STATIC" SYSTEM, NO ACCELERATION

BOARD



$\Delta z = \Delta l \sin\alpha$
$\Delta y = \Delta l \cos\alpha$

SCRIPT

ANALYZE THE ELEMENT EACH DIRECTION.

- OBSERVE

$F = p \cdot A$

$A = \Delta y \Delta z$
$= \Delta x \Delta y$
= AND SUCH.

ALSO NOTE USE TRIGONOMETRY FOR THE WEDGE

BOARD

$\Sigma F_x = 0$

$\frac{1}{2} p_{xx} \Delta y \Delta z - \frac{1}{2} p_{xx} \Delta y \Delta z = 0$
$\Rightarrow p_{xx} = p_{xx}$

$\Sigma F_y = 0$

$-p_{yy} \Delta x \Delta z + p_n \Delta l \sin\alpha \Delta x = 0$
$p_{yy} \cancel{\Delta x} \cancel{\Delta l} \sin\alpha = p_n \cancel{\Delta l} \cancel{\sin\alpha} \cancel{\Delta x}$
$\therefore p_{yy} = p_n$

$\Sigma F_z = 0$

$p_{zz} \Delta x \Delta y - p_N \Delta x \underbrace{\Delta l \cos\alpha}_{\Delta y}$
$- \underbrace{\frac{1}{2} \rho g \Delta y \Delta z \Delta x}_{\text{weight}} = 0$

SCRIPT

CONTINUE WITH Z

FROM THIS ANALYSIS, ONE CONCLUDES THAT PRESSURE IN A STATIC FLUID AT A POINT IS A SINGLE VALUE, INDEPENDENT OF DIRECTION.

IN OTHER WORDS, PRESSURE IS A SCALAR.

FROM NOW ON; DROP DOUBLE SUBSCRIPT FOR PRESSURE.

BOARD

$$\therefore p_{zz} - p_N - \frac{1}{2}\rho g \Delta z = 0$$

$$\lim_{\Delta z \to 0} p_{zz} - p_N - \frac{1}{2}\rho g \Delta z$$

$$= p_{zz} - p_N = 0$$

$$\therefore \quad p_{zz} = p_N$$

FINALLY

$$p_N = p_{yy} = p_{zz}$$

BECAUSE ORIENTATION IS ARBITRAY;

$$p_N = p_{xx}$$

SCRIPT

IN A CLOSED SYSTEM, A CHANGE IN PRESSURE IS TRANSMITTED THROUGHOUT THE ENTIRE SYSTEM, UNDIMINISHED

BOARD

PRESSURE TRANSMISSION

CLOSED SYSTEM

- ΔP ~~SAME~~ TRANSMITTED EVERYWHERE

PASCAL'S LAW;

BASIS OF HYDRAULIC MACHINERY

SCRIPT

PRESSURE USUALLY MEASURED AGAINST SOME REFERENCE VALUE

IF THE REFERENCE VALUE IS ONE "STANDARD" ATMOSPHERE THE MEASURE IS CALLED "GAGE PRESSURE"

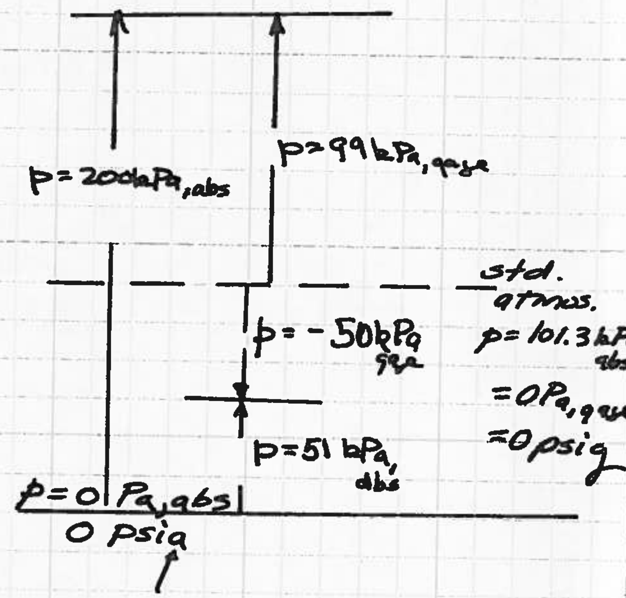
IF THE REFERENCE VALUE IS A VACCUM (ZERO PRESSURE), THE MEASURE IS CALLED "ABSOLUTE PRESSURE"

BOARD

REFERENCE VALUE

1 ATM ⇒ GAGE

0 ATM ⇒ ABSOLUTE



SCRIPT

FUNDAMENTAL EQUATION OF FLUID STATICS

FLUID STATICS MEANS THAT A FLUID IS FREE OF RELATIVE MOTION — ENTIRE FLUID BEHAVES AS A RIGID BODY

THE ABSENCE OF ANGULAR DEFORMATION IMPLIES ABSENCE OF SHEAR STRESS

STATIC FLUIDS ONLY SUSTAIN NORMAL STRESS (PRESSURE)

BOARD

FLUID STATICS

— NO RELATIVE MOTION

$$\frac{dV}{dy} = 0 \Rightarrow \tau = 0$$

OFTEN ALSO MEANS (BUT NOT ALWAYS)

$$\frac{dV}{dt} = 0$$

SCRIPT

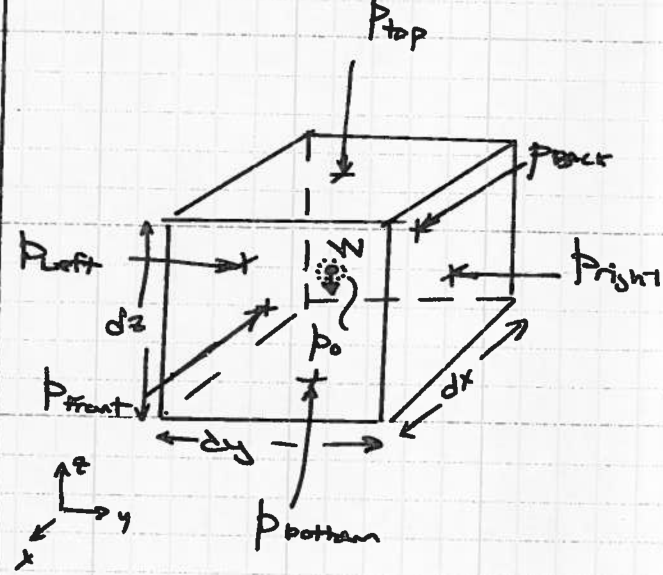
CONSIDER A FLUID ELEMENT;

ANALYZE FORCES.

USE SMALL ELEMENT;

WILL TAKE LIMITS
TO RECOVER DIFFERENTIAL
EQUATION OF FLUID

BOARD



SCRIPT

THEN USE TAYLOR-SERIES
EXPANSION ABOUT THE
CENTER OF THE ELEMENT
TO EXPRESS PRESSURES
AT THE ELEMENT FACES IN
TERMS OF p_0

BOARD

$$\Sigma d\underline{F} = d\underline{F}_{BODY} + d\underline{F}_{SURF} = dm\,\underline{a}$$

$$dm = \rho\, dx\, dy\, dz$$

$$d\underline{F}_{BODY} = \rho g\, dx\, dy\, dz$$

$$d\underline{F}_{SURF} = (p_{back} - p_{front})\, dz\, dy\, \underline{i}$$
$$(p_{left} - p_{right})\, dx\, dz\, \underline{j}$$
$$(p_{bot.} - p_{top})\, dx\, dy\, \underline{k}$$

SCRIPT	BOARD
THE INDIVIDUAL TERMS EXPANDED $\pm \frac{dx,y,z}{2}$ NOTE PARTIALS $\frac{\partial p}{\partial z} \rightarrow$ VARIATION IN PRESSURE AS MOVE IN +Z DIRECTION SAME FOR X & Y	$p_{back} = p_0 - \frac{\partial p}{\partial x}\frac{dx}{2}$ $p_{front} = p_0 + \frac{\partial p}{\partial x}\frac{dx}{2}$ $p_{left} = p_0 - \frac{\partial p}{\partial y}\frac{dy}{2}$ $p_{right} = p_0 + \frac{\partial p}{\partial y}\frac{dy}{2}$ $p_{\cancel{bot}} = p_0 - \frac{\partial p}{\partial z}\frac{dz}{2}$ $p_{top} = p_0 + \frac{\partial p}{\partial z}\frac{dz}{2}$

SCRIPT	BOARD
RECALL FROM CALCULUS; TERM IN PARENTHESIS IS THE GRADIENT OF PRESSURE (ALREADY KNOW IS A SCALAR)	see below

$$d\underline{F}_{SURF} = \left\{ \begin{array}{l} \left(p_0 - \frac{\partial p}{\partial x}\frac{dx}{2}\right) - \left(p_0 + \frac{\partial p}{\partial x}\frac{dx}{2}\right)\underline{i}\, dy\,dz \\ \left(p_0 - \frac{\partial p}{\partial y}\frac{dy}{2}\right) - \left(p_0 + \frac{\partial p}{\partial y}\frac{dy}{2}\right)\underline{j}\, dx\,dz \\ \left(p_0 - \frac{\partial p}{\partial z}\frac{dz}{2}\right) - \left(p_0 + \frac{\partial p}{\partial z}\frac{dz}{2}\right)\underline{k}\, dx\,dy \end{array} \right.$$

$$= -\left(\frac{\partial p}{\partial x}\underline{i} + \frac{\partial p}{\partial y}\underline{j} + \frac{\partial p}{\partial z}\underline{k}\right)dx\,dy\,dz$$

$$\therefore d\underline{F}_{SURF} = -\mathrm{grad}(p)\,dx\,dy\,dz$$

or

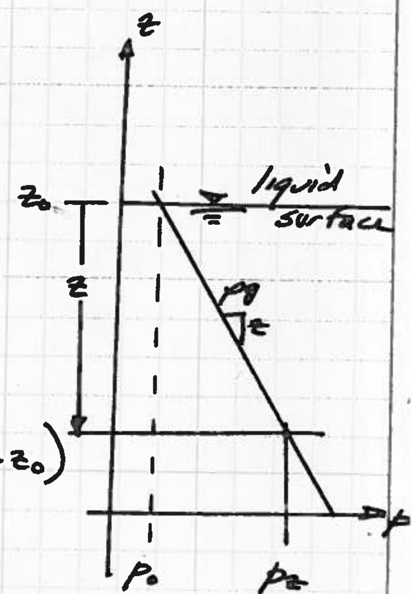
$$-\nabla p\; dx\,dy\,dz$$

SCRIPT	BOARD
SUBSTITUTE INTO THE FORCE BALANCE	$\Sigma \underline{F} = \rho \underline{g}\, dx\,dy\,dz - \nabla p\, dx\,dy\,dz = \rho \underline{a}\, dx\,dy$
NEXT CONSIDER JUST THE POINT "0" — THE FORCE PER UNIT VOLUME (SPECIFIC FORCE) IS	$\frac{\Sigma d\underline{F}}{d\forall} = \rho g - \nabla p = \rho \underline{a}$ THIS LAST EXPRESSION IS FUNDAMENTAL EQUATION OF FLUID STATICS
ONE VALUE OF ACCELERATION THAT IS COMMON IN <u>FLUID</u> <u>STATICS</u> PROBLEMS IS $\underline{a} = \underline{0}$	$\rho g - \nabla p = \rho \underline{a}$

SCRIPT	BOARD
IN THE SPECIAL CASE OF $\underline{a} = \underline{0}$ THE BALANCE EQUATION RELATES WEIGHT AND PRESSURE	SPECIAL CASE: $\underline{a} = \underline{0}$ $\underbrace{\rho g}_{\text{BODY FORCE PER UNIT VOLUME}} - \underbrace{\nabla p}_{\text{PRESSURE FORCE PER UNIT VOLUME}} = 0$

SCRIPT	BOARD
THE EQUATION IS A VECTOR EQUATION	$\rho g_x - \frac{\partial p}{\partial x} = 0$
USEFUL TO EXAMINE COMPONENTS	$\rho g_y - \frac{\partial p}{\partial y} = 0$
	$\rho g_z - \frac{\partial p}{\partial z} = 0$
OFTEN CHOOSE COORDINATE SYSTEMS SO THAT $\underset{\sim}{g}$ COINCIDES WITH $-z$ DIRECTION; THIS SPECIAL CASE REDUCES TO: $\longrightarrow$	IF $\underset{\sim}{g} = -g\underset{\sim}{k}$ $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$ $\frac{\partial p}{\partial z} = -\rho g = -\gamma$

SCRIPT	BOARD
THE LAST EXPRESSION IS THE FUNDAMENTAL RELATIONSHIP BETWEEN PRESSURE VARIATION AND DEPTH IN A STATIC FLUID AT ZERO ACCELERATION	$\frac{\partial p}{\partial z} = -\rho g$ IF ρ = CONST. $\partial p = -\rho g\, \partial z$
THE RELATIONSHIP SHOWS THAT PRESSURE VARIATION IS A FUNCTION OF DEPTH	$\int_{p_0}^{p_z} \partial p = -\rho g \int_{z_0}^{z}$ $p_z = p_0 - \rho g(z - z_0)$
IF ρ IS CONSTANT WHAT KIND OF FLUID? (INCOMPRESSIBLE)	



SCRIPT

IF $\rho \neq$ CONSTANT, COMPRESSIBLE.

NEED AN EQUATION OF STATE

IDEAL GAS LAW IS AN EXAMPLE

BOARD

$\rho \neq$ constant

$$p = \rho R T$$

$$dp = -\frac{p}{RT} g \, dz$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{z_0}^{z} \frac{g}{RT} dz$$

$$\ln|p| = -\frac{g}{RT}(z - z_0)$$

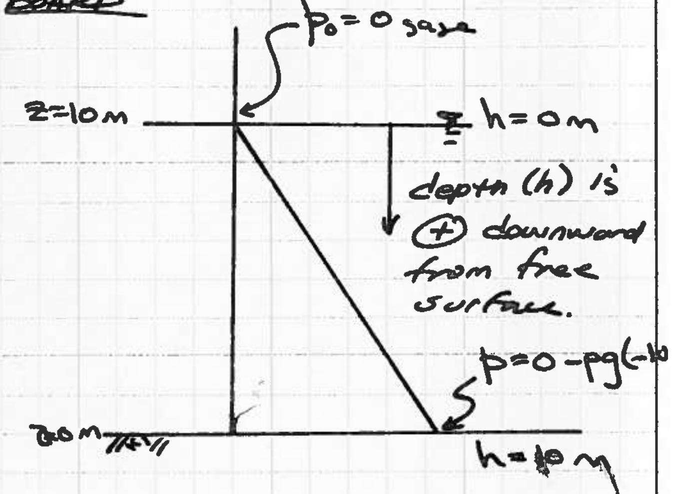
$$p_z = p_0 e^{-\frac{g}{RT}(z - z_0)}$$

SCRIPT

IN HYDRAULICS, WATER AND OTHER LIQUIDS ARE USUALLY TREATED AS INCOMPRESSIBLE FLUIDS

IN LIQUID SYSTEMS THE FREE SURFACE IS USUALLY TAKEN AS THE DATUM, AND DISTANCE (DEPTH) IS MEASURED DOWNWARD FROM THIS REFERENCE

BOARD



SCRIPT

(SAME RESULT!

BOARD

GENERAL

$p = p_0 - \rho g (z - z_0)$

$= p_0 - \rho g (0 - 10m)$

$= \rho g (10m)$

SP. CASE LIQUIDS

$p = p_0 + \rho g h$

$= \cancel{p_0} + \rho g (10m)$

$= \rho g (10m)$

SCRIPT

BOARD

SCRIPT

WRITE: CE3305
: QUIZ # 4
: ROLL
R : ES# 4 DUE NEXT...

AGENDA IS CONTINUATION OF FLUID STATICS.

LAST TIME WE DERIVED THE INVISCID (NO VISCOSITY, NO SHEAR) FLOW EQUATION

ALSO APPLIED TO PRESSURE VARIATION IN A LIQUID

NOW EXAMINE SOME PRESSURE MEASURING DEVICES

BOARD

FLUID STATICS — CONTINUES

MEASURING PRESSURE

- BAROMETER
- BOURDON-TUBE
- PIEZOMETER
- MANOMETER
- ABSOLUTE & DIFFERENTIAL PRESSURE TRANSDUCERS

(SCRIPT)

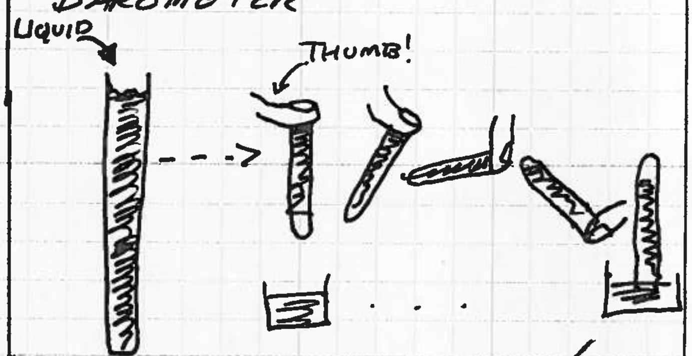
BAROMETERS ARE TUBES OF LIQUID; OIL, Hg, H_2O ← RARELY!

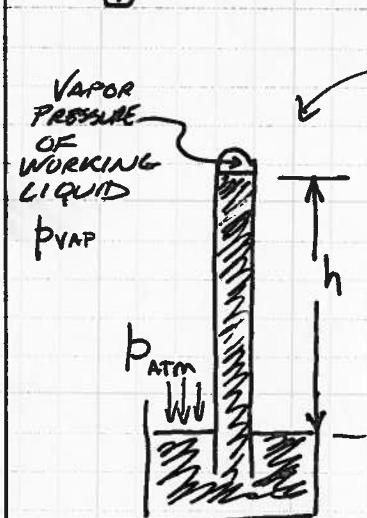
TUBES ARE FILLED; INVERTED INTO A BASIN; ALLOWED TO EQUILIBRATE

THE HEIGHT OF RISE IS OBTAINED FROM A FORCE BALANCE BETWEEN THE VAPOR PRESSURE OF THE WORKING FLUID AND THE ATMOSPHERIC PRESSURE

(BOARD)

BAROMETER





$$p_{ATM} = \gamma h + p_{VAP}$$

IF p_{VAP} PRETTY SMALL

$$p_{ATM} \approx \gamma h$$

SCRIPT

ANEROID BAROMETERS WORK BY MECHANICAL FORCE BALANCE.

BOURDON-TUBES ARE SIMILAR; ENTIRE TUBE DEFLECTS TO DETECT THE FORCE DIFFERENTIAL BY CHANGE IN PRESSURE

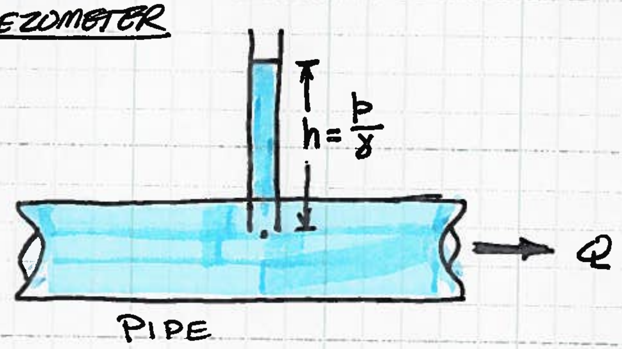
A PIEZOMETER IS A TUBE INTRODUCED INTO A FLUID (USUALLY LIQUID)

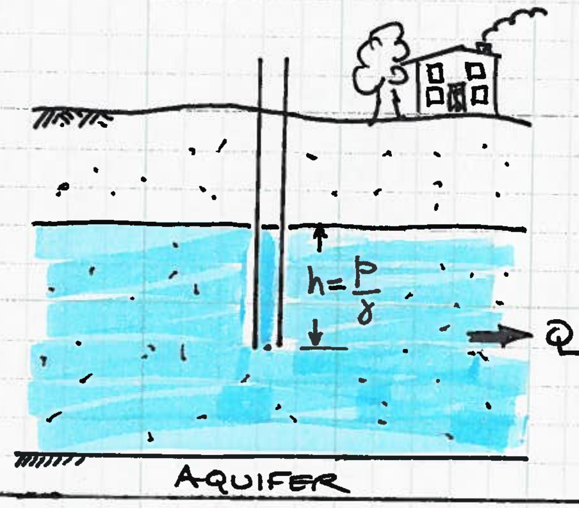
HEIGHT OF RISE IS PROPORTIONAL TO STATIC PRESSURE IN LIQUID

WELLS (NON-PUMPING) ARE PIEZOMETERS

BOARD

PIEZOMETER





SCRIPT

MANOMETERS ARE LIKE A HYBRID OF BAROMETER AND A PIEZOMETER

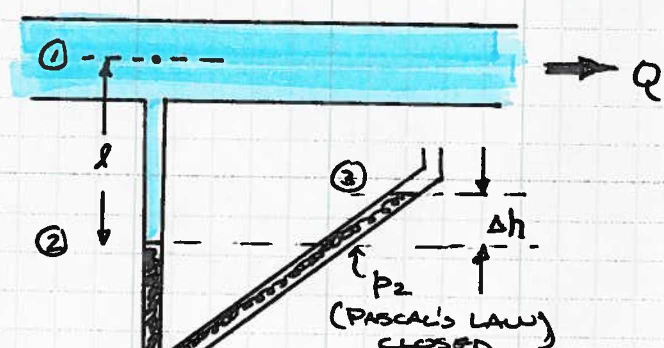
MANOMETERS ARE CONNECTED TO FLUID; HAVE A WORKING FLUID; MEASURE FORCE DIFFERENTIAL BETWEEN THE FLUID AND (TYP.) THE ATMOSPHERE.

USE THE DEVICES TO MEASURE "CENTERLINE" PRESSURE.

AS WITH OTHER DEVICES FORCE/AREA IS EQUATED TO COLUMN HEIGHT THROUGH $\frac{p}{\rho g} = h$

BOARD

MANOMETER



MOVED DOWN

$p_2 = p_1 + \gamma_A \ell$

MOVED UP

$p_3 = p_2 - \gamma_B \Delta h$

SCRIPT

SUMMARIZE MANOMETERS

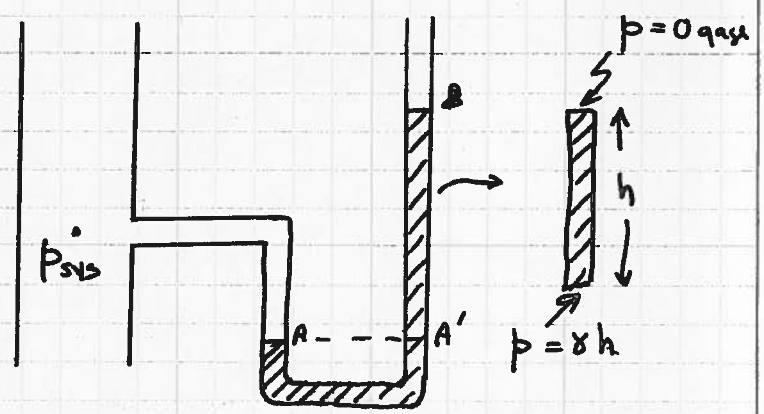
MULTIPLE TUBE/FLUID MANOMETERS FOLLOW 2 RULES

1. ANY TWO POINTS AT SAME ELEVATION IN A CONTINUOUS LENGTH OF THE SAME FLUID ARE AT SAME PRESSURE (PASCAL'S LAW)
2. PRESSURE INCREASES WITH DEPTH IN A LIQUID COLUMN.

BOARD

MANOMETERS

- DEVICES FOR MEASURING PRESSURE IN FLUID SYSTEM
- FORCE BALANCE BETWEEN MANOMETER FLUID AND FLUID SYSTEM BEING MEASURED
- PASCAL'S LAW
- UP/DOWN IN EACH FLUID MATTER



SCRIPT

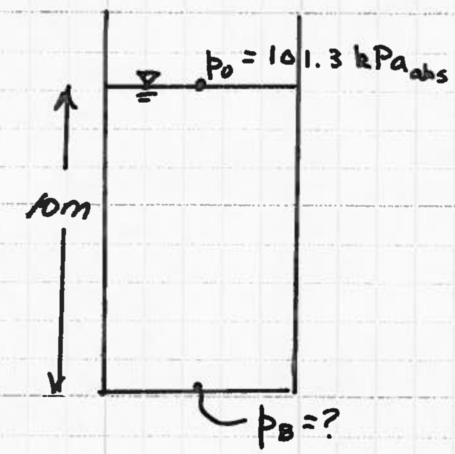
WORK EXAMPLES (ATTACHED)

BOARD

- PRESSURE IN BOTTOM OF WATER TANK
- PRESSURE IN API OIL/WATER SEPERATOR
- MANOMETER SYSTEM
- PRESSURE IN WATER DROP

EXAMPLES

WHAT IS THE ABSOLUTE WATER PRESSURE AT A DEPTH OF 10 METERS IN THE TANK SHOWN?



~~GIVEN~~ KNOWN

LIQUID IS WATER $\therefore \rho = 1000\ kg/m^3$
$\gamma = 9800\ N/m^3$

$h = 10\ m$

$p_0 = 101.325\ kPa_{abs}$ (use 101 kPa — close enough)

GOVERNING EQUATIONS

$$\frac{dp}{dz} = -\rho g$$

DOWNWARD FROM SURFACE!

OR $p_B = p_0 + \rho g h$

UNKNOWN

p_B

SOLUTION

APPLY GOVERNING EQUATION; INSERT NUMERICAL VALUES

$$p_B = 101 \cdot 10^3 Pa + 9800\ N/m^3\,(10m) = 101 \cdot 10^3 Pa + 98 \cdot 10^3 Pa$$
$$= 199 \cdot 10^3 Pa = 199\ kPa_{ABS}$$

DISCUSSION

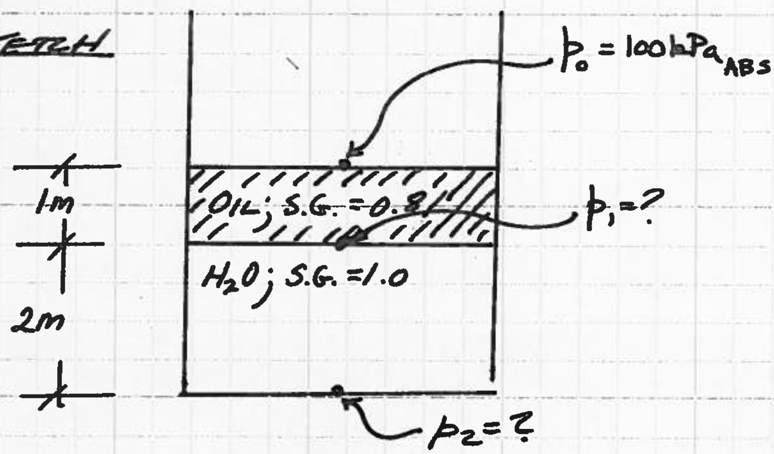
1. MEASURE DOWN FROM SURFACE
2. EXPRESS IN CONSISTENT UNITS; CONVERT TO USEFUL UNITS AT END

EXAMPLE

WHAT IS ABSOLUTE PRESSURE AT BOTTOM OF THE OIL-WATER SEPARATION TANK BELOW?

(SIMILAR TO 3.16, Pg 96)

SKETCH



KNOWN

H_2O; S.G. = 1.0; $\rho g = 9800\,N/m^3$

Oil; S.G. = 0.8; $\rho g = 9800(0.8)\,N/m^3$

$\Delta h_{OIL} = 1.0\,m$

$\Delta h_{H_2O} = 2.0\,m$

$p_0 = 100\,kPa_{abs}$

GOVERNING EQUATIONS

$p_B = p_A + \rho g \Delta h$ ← MEASURED DOWNWARD (PASCAL'S LAW)

UNKNOWN

p_2 (AND p_1 AT OIL/WATER INTERFACE)

SOLUTION

APPLY GOVERNING EQUATION IN OIL TO FIND p_1

APPLY GOVERNING EQUATION IN H_2O TO FIND p_2

$$p_1 = p_0 + \rho_{oil} g \Delta h$$

$$= p_0 + (9800\,N/m^3)(0.8)(1.0\,m)$$

$$= 100 \cdot 10^3\,N/m^2 + 7.84 \cdot 10^3\,N/m^2 = 107.84 \cdot 10^3\,N/m^2$$

$$= 107.84\,kPa_{ABS}$$

$$p_2 = p_1 + \rho_{H_2O} g \Delta h$$

$$= 107.84 \cdot 10^3\,N/m^2 + 9800\,N/m^3 (2.0\,m)$$

$$= 107.84 \cdot 10^3\,N/m^2 + 19.6 \cdot 10^3\,N/m^2$$

$$= 127.44 \cdot 10^3\,N/m^2 = 127.44\,kPa_{ABS}$$

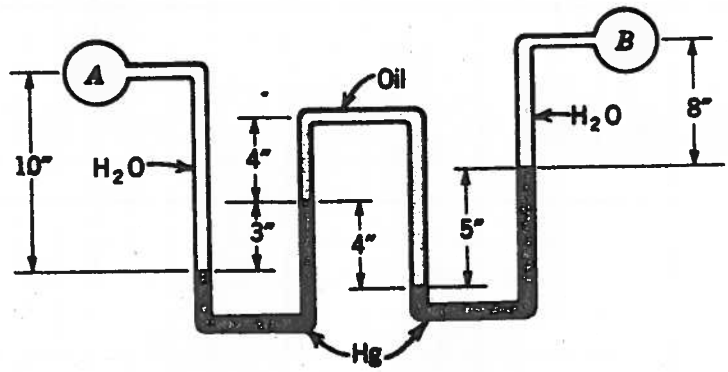
DISCUSSION

1. TWO LIQUIDS; START AT TOP WORK DOWN.
2. USE S.G. TO RELATE γ_{OIL} TO γ_{WATER} AS $\frac{\gamma_{OIL}}{\gamma_{WATER}} = S.G.$
3. REMEMBER TO USE CORRECT γ FOR PORTION OF FLUID EXAMINED
4. IF WE WERE INTERESTED IN GAGE PRESSURE

$$p_{GAGE} = p_{ABS} - p_{REF}$$

$$= 127.44\,kPa - 100\,kPa = 27.44\,kPa_{gage}$$

Water flows through pipes *A* and *B*. Oil, with specific gravity 0.8, is in the upper portion of the inverted U. Mercury (specific gravity 13.6) is in the bottom of the manometer bends. Determine the pressure difference, $p_A - p_B$, in units of lbf/in.^{2}



KNOWN

PIPE FLUIDS ARE H_2O $\rho g = 9800 N/m^3 = 62.4 \; lbf/ft^3$

MANOMETER FLUIDS ARE Hg $S.G. = 13.6$
AND
OIL $S.G. = 0.8$

GOVERNING EQUATIONS

(3.21) $p_2 = p_1 + \sum_{down} \gamma_i h_i - \sum_{up} \gamma_i h_i$

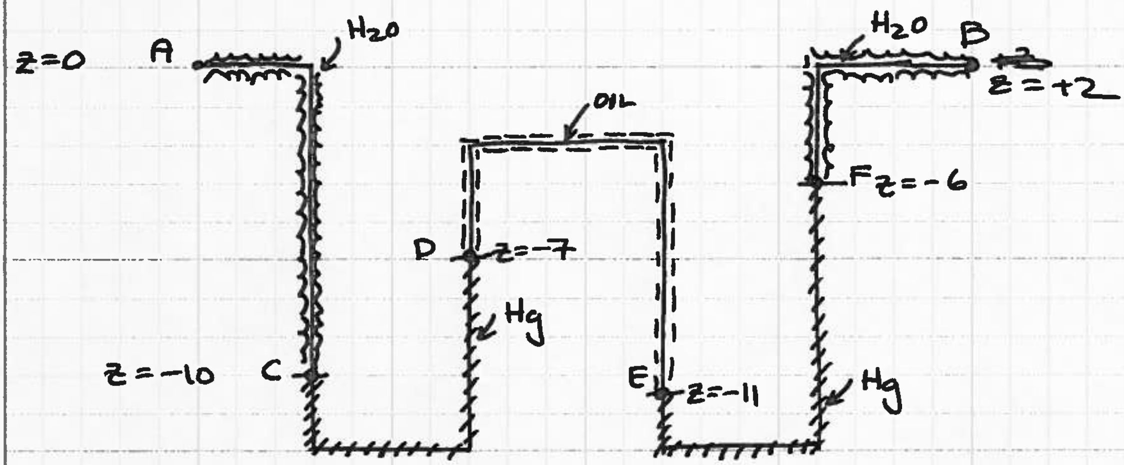
UNKNOWN

$\Delta p = p_A - p_B$

SOLUTION

APPLY EQN 3.21 FROM (A) TO (B)

SKETCH



$$p_A - p_C = -\rho_w g\,(z_A - z_C)$$
$$p_C - p_D = -\rho_{Hg} g\,(z_C - z_D)$$
$$p_D - p_E = -\rho_{OIL} g\,(z_D - z_E)$$
$$p_E - p_F = -\rho_{Hg} g\,(z_E - z_F)$$
$$p_F - p_B = -\rho_w g\,(z_F - z_B)$$

ADD ALL Δps FROM A TO B

$$(p_A - \cancel{p_C}) + (\cancel{p_C} - \cancel{p_D}) + (\cancel{p_D} - \cancel{p_E}) + (\cancel{p_E} - \cancel{p_F}) + (\cancel{p_F} - p_B)$$
$$= p_A - p_B = \Delta p$$

NOW SUBSTITUTE IN RHS

$$\Delta p = -\rho_w g(z_A - z_C) - \rho_{Hg} g\,(z_C - z_D) - \rho_{OIL} g\,(z_D - z_E)$$
$$-\rho_{Hg} g(z_E - z_F) - \rho_w g\,(z_F - z_B)$$

$z_A = 0;\ z_C = -10,\ z_D = -7,\ z_E = -11,\ z_F = -6,\ z_B = +2$

$$\Delta p = -\rho_w g(10) - \rho_{Hg} g(-3) - \rho_{oil} g(4) - \rho_{Hg} g(-5)$$
$$-\rho_w g(-8)$$

ALL IN INCHES

$$\Delta p = g\left(-10\rho_w + 3\rho_{Hg} - 4\rho_{oil} + 5\rho_{Hg} + 8\rho_w\right)$$

$$= g\left(-2\rho_w + 8\rho_{Hg} - 4\rho_{oil}\right)$$

S.G. Hg S.G. OIL

$$= g\left(-2\rho_w + 8(13.6)\rho_w - 4(0.8)\rho_w\right)$$

$$= g\left(-2\rho_w + 108.8\rho_w - 3.2\rho_w\right)$$

$$= g\left(103.6\,\rho_w\right)$$

$$= \gamma_w (103.6\,in)$$

$$= 62.4\,\frac{lbf}{\cancel{ft^3}}\,(103.6\,\cancel{in})\,\frac{1\,\cancel{ft}}{12\,\cancel{in}}\cdot\frac{1\,\cancel{ft^2}}{144\,in^2}$$

$$= 3.74\ lbf/in^2$$

DISCUSSION

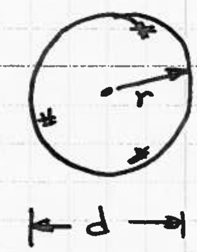
① ILLUSTRATED DIRECT USE OF STATIC FLUID AND MANOMETER RULES

② TOOK ADVANTAGE OF ALGEBRA, APPLIED S.G. TO MAKE A SINGLE CALCULATION AT END.

EXAMPLE

FIND A FORMULA FOR GAGE PRESSURE IN A SMALL SPHERICAL DROP OF WATER WITH SURFACE TENSION σ.

SKETCH



KNOWN

σ, d, r TOLD "SMALL"

GOVERNING EQUATION(S)

$\Sigma \underline{F} = m \underline{a}$

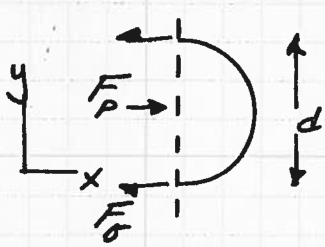
$F_\sigma = 2\pi r \sigma$ (2.24)

$\forall_{SPHERE} = 4/3 \pi r^3$

UNKNOWN

$p_{GAGE} = f(\sigma, d)$, FIND f

SOLUTION



CUT DROP IN HALF.
ANALYZE FORCES

NEGLECT VERTICAL VARIATION IN PRESSURE
$\left(\frac{dp}{dd} \approx \text{REALLY SMALL} \right)$

$$\Sigma F_x = m \cancel{a_x}^{=0}$$

$$F_p - F_\sigma = 0 \Rightarrow F_p = F_\sigma$$

$$F_p = p \cdot \frac{\pi d^2}{4} \quad \left(\text{or } p \cdot \pi r^2\right)$$

$$F_\sigma = \pi d \sigma \quad \left(\text{or } 2\pi r \sigma\right)$$

$\therefore$

$$p \frac{\cancel{\pi} d^2}{4} = \cancel{\pi} d \sigma$$

$$p d^{\cancel{2}} = 4 \cancel{d} \sigma$$

$$\underline{\underline{p = \frac{4\sigma}{d}}} \longleftarrow !$$

DISCUSSION

① PROBLEM IS A FORCE BALANCE

② NO NUMERICAL VALUES; ANALYSIS PRODUCES EQUATION

③ COULD USE $\frac{dp}{dd} = -\rho g$ TO DECIDE HOW SMALL IS SMALL (WELL OUTSIDE SCOPE THIS CLASS)

SCRIPT

CONSIDER AREA A SHOWN, WITH UNIFORM PRESSURE APPLIED AS SHOWN.

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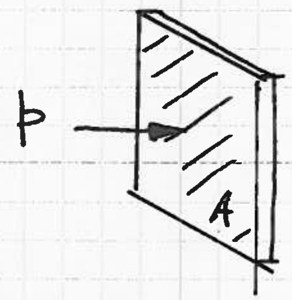
HYDROSTATIC FORCE OF SUBMERGED PLANE (PANEL) SURFACES



FORCE IS

$$F = pA$$

IF PRESSURE IS DISTRIBUTED OVER THE PLATE, THEN THE DIFFERENTIAL FORCE IS SIMILAR, BY APPLIED OVER A SMALL dA

FOR A SUBMERGED PLATE, THE PRESSURE p IS A FUNCTION OF DEPTH.

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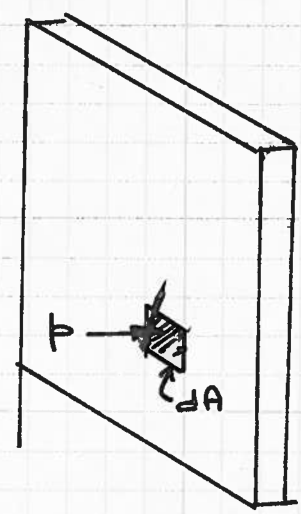
DISTRIBUTED PRESSURE



$$dF = p\,dA$$

TOTAL FORCE IS

$$F = \int_A p\,dA$$

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NOW EXAMINE A PLATE AT SOME ARBITRARY ANGLE

DEPTH TO "dy" PANEL IS h.

DISTANCE ALONG + y AXIS IS y.

h IS RELATED TO y BY ANGLE α.

NOW LOOK NORMAL TO PLATE

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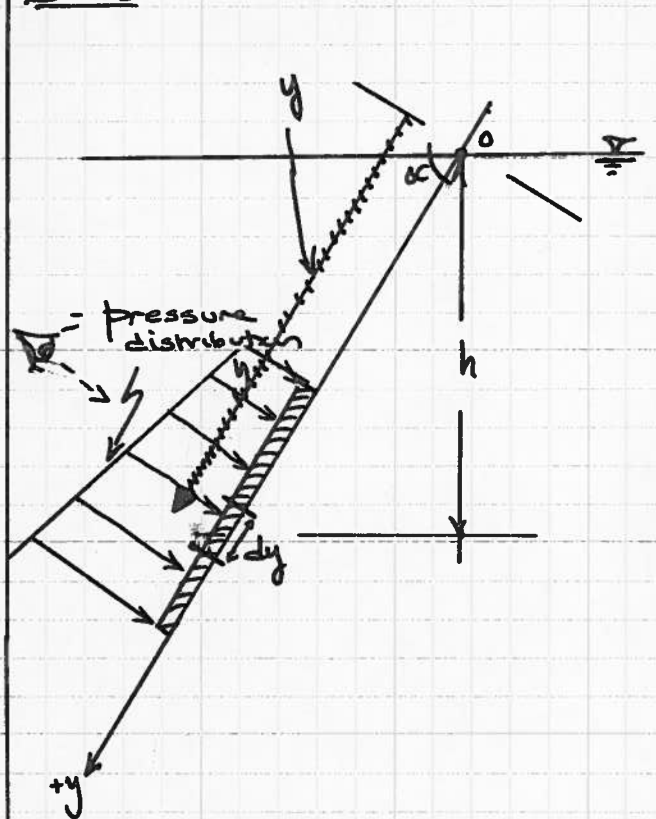

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PANEL WIDTH * dy IS dA.

LOOKING ⊥ TO ~~ω~~ Y AXIS.

WHAT IS RESULTANT FORCE ON SURFACE?

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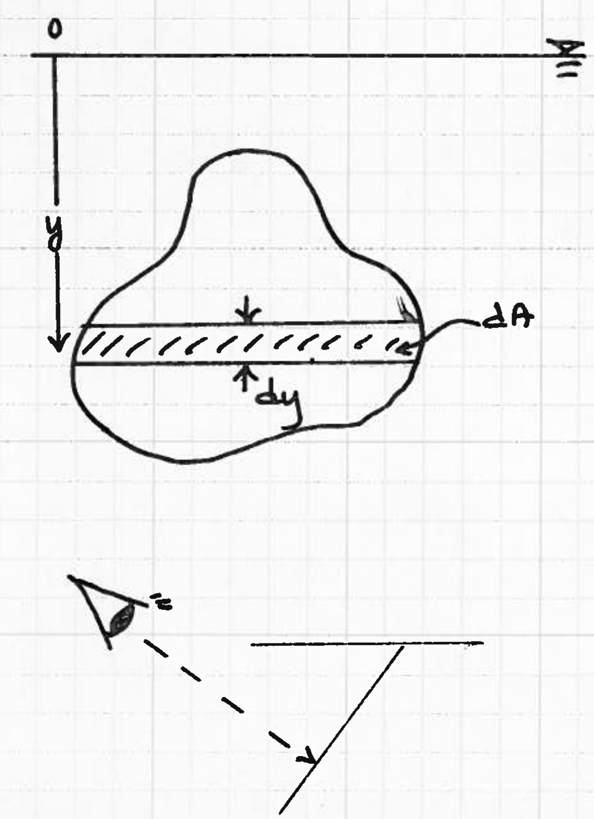

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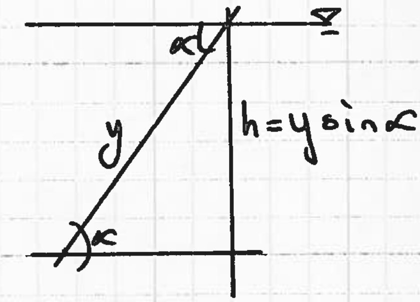
NEED TO EXPRESS PRESSURE IN TERMS OF y OR h $$p = \rho g h = \rho g y \sin\alpha$$ DOING ACTUAL INTEGRATION IS A PITA; INSTEAD WOULD LIKE TO BE ABLE TO DEAL IN TOTAL AREA AND DEPTH THE LAST ~ TERM IS FIRST MOMENT OF AREA. ~~NOTICE~~ NOTICE HOW MULTIPLY AND DIVIDE BY A $A \cdot \frac{1}{A} = 1$	 $$dF = p\,dA$$ $$p = \rho g h = \rho g y \sin\alpha$$ $$F = \int_A \rho g y \sin\alpha\, dA$$ $$= \rho g \sin\alpha A \cdot \underbrace{\frac{1}{A}\int_A y\, dA}_{\text{FIRST MOMENT OF AREA ABOUT AX}}$$
NOW RECALL FROM STATICS THAT THE FIRST MOMENT IS DISTANCE TO CENTROID FROM AXIS USE THE TRIG. RELATIONSHIP TO EQUATE LOCATION OF CENTROID OF OBJECT TO DEPTH	STATICS $$\frac{1}{A}\int_A y\, dA = \bar{y}$$ USE TRIGONOMETRY $$F = \rho g \sin\alpha A \bar{y}$$ $$= \rho g A \bar{h}$$ $\bar{h}$ IS DEPTH FROM FREE SURFACE TO CENTROID OF AREA. $$\boxed{F = \gamma \bar{h} A}$$ MAGNITUDE OF FORCE

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MAGNITUDE IS

$F = \gamma \bar{h} A$

NEXT WANT LINE OF ACTION.

y_P IS THE DISTANCE FROM EDGE OF PLATE (A) TO POINT OF APPLICATION OF RESULTANT FORCE THAT CAUSES SAME MOMENT ON THE PLATE AS THE ACTUAL DISTRIBUTED LOAD.

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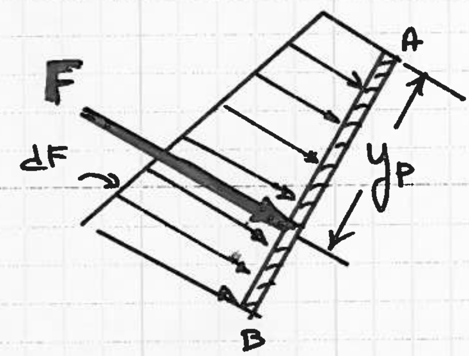

$$\sum M_A \circlearrowright = \int_A y\, dF = y_P F$$

$$\therefore y_P = \frac{\int_A y\, dF}{F} = \frac{\int_A \gamma y^2 \sin\alpha\, dA}{\int_A \gamma y \sin\alpha\, dA}$$

SCRIPT

I_x IS MOMENT OF INERTIA ABOUT X-AXIS.

APPLY PARALLEL AXIS THEOREM TO TRANSLATE I_x TO CENTROID OF THE PLATE.

THESE METHODS WORK FINE FOR ANY SURFACE; A MORE PRACTICAL APPROACH WILL BE ILLUSTRATED NEXT MEETING

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$$= \frac{\int y^2\, dA}{\int y\, dA} = \frac{I_x}{A\bar{y}}$$

PARALLEL AXIS THEOREM

$$y_P = \frac{I_o}{A\bar{y}} + \bar{y}$$

I_o IS MOMENT OF INERTIA ABOUT PLATE CENTROID.