SCRIPT

SEGMENTS AB & AC ARE INITIALLY ORTHOGONAL;

A SHORT TIME LATER, ELEMENT HAS MOVED AS SHOWN.

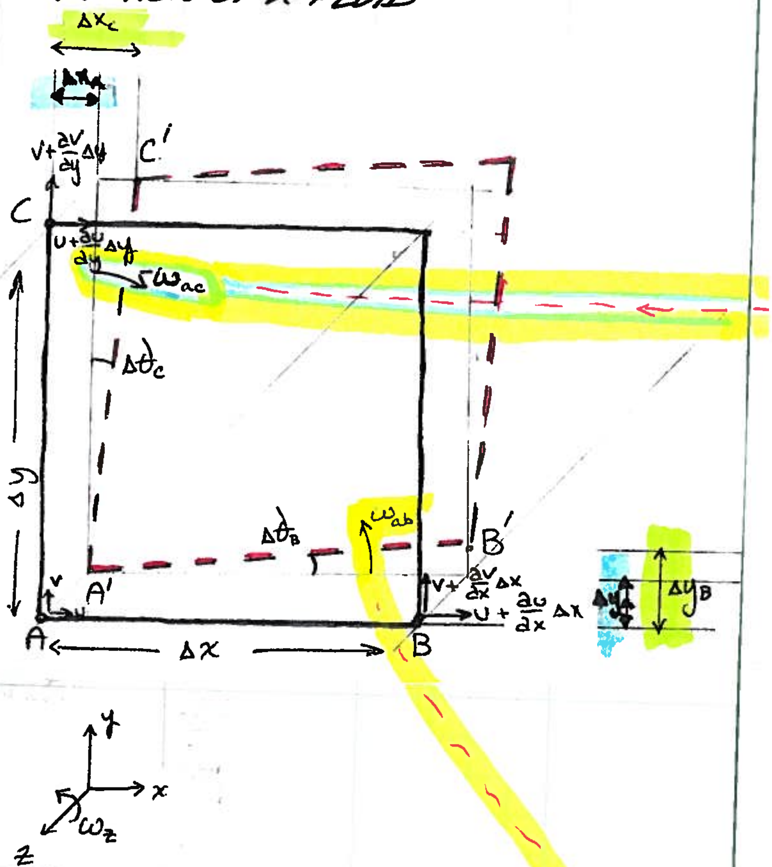
ANTICIPATED TRANSLATION OF ALL VERTICES IS $\Delta x_A, \Delta y_A$

SUPPOSE ~~POINTS~~ POINTS B' & C' SHOW A LITTLE EXTRA TRANSLATION, $\Delta x_c, \Delta y_B$

BECAUSE OF SLIGHT DEFORMATION (ROTATION) OF THE ELEMENT

BOARD

ROTATION OF A FLUID



SCRIPT

RATE OF ROTATION OF SEGMENT AB IS

TANGENT OF SMALL ANGLE IS EXTRA TRANSLATION LESS ANTICIPATED TRANSLATION DIVIDED BY ELEMENT LENGTH Δx.

WE ARE CONSIDERING SMALL TIME; HENCE SMALL ANGLES SO $\tan(\alpha) \approx \alpha$.

SUBSTITUTE VELOCITY AND APPLY ALGEBRA

BOARD

$$\omega_{ab} = \lim_{\Delta t \to 0} \frac{\Delta\theta_B}{\Delta t}$$

$$\tan(\Delta\theta_B) = \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

FOR SMALL ANGLES $\tan(\alpha) \approx \alpha$

$$\therefore \Delta\theta_B \approx \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

$$\Delta y_B - \Delta y_A = \left(v + \frac{\partial v}{\partial x}\Delta x - v\right)\Delta t$$

$$\text{SO} \quad \Delta\theta_B = \frac{\frac{\partial v}{\partial x}\cancel{\Delta x}\Delta t}{\cancel{\Delta x}} = \frac{\partial v}{\partial x}\Delta t$$

$$\therefore \omega_{ab} = \lim_{\Delta t \to 0} \frac{\Delta\theta_B}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\partial v}{\partial x}\Delta t}{\Delta t} = \frac{\partial v}{\partial x}$$

SCRIPT	BOARD
RATE OF ROTATION OF SEGMENT AC IS	$\omega_{ac} = \lim_{\Delta t \to 0} \dfrac{-\Delta\theta_c}{\Delta t}$
USING SIMILAR ANALYSIS THE RESULT IS	$\Delta\theta_c \approx -\left(\dfrac{\Delta X_c - \Delta X_A}{\Delta y}\right) = -\dfrac{\partial u}{\partial y}\Delta t$
	$\boxed{\omega_{ac}} = \lim_{\Delta t \to 0} \dfrac{-\frac{\partial u}{\partial y}\Delta t}{\Delta t} = \boxed{-\dfrac{\partial u}{\partial y}}$
THE AVERAGE RATE OF ROTATION (OF ELEMENT) IS	$\omega_z = \dfrac{\omega_{ab} + \omega_{ac}}{2} = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$

SCRIPT	BOARD
EXTENSION OF SUCH ANAYLSIS INTO 3 DIMENSIONS PRODUCES AN ANGULAR VELOCITY VECTOR	$\underline{\omega} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}$
	WHERE
NOTICE FOR EACH DIRECTION, THE ROTATION INVOLVES THE OTHER TWO DIMENSIONS	$\omega_x = \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} - \dfrac{\partial v}{\partial z}\right)$
E.G. ω_x INVOLVES $\dfrac{\partial w}{\partial y}$ & $\dfrac{\partial v}{\partial z}$	$\omega_y = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial x}\right)$
ALSO NOTICE THE VELOCITY VARIATIONS ARE CROSS TERMS: $\underline{V} = u\underline{i} + v\underline{j} + w\underline{k}$	$\omega_z = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$
ω_x INVOLVES $\dfrac{\partial w}{\partial y}$ & $\dfrac{\partial v}{\partial z}$ "z" "y" "y" "z"	THE VORTICITY VECTOR IS <u>DEFINED</u> AS TWICE THE AVERAGE ANGULAR VELOCITY VECTOR

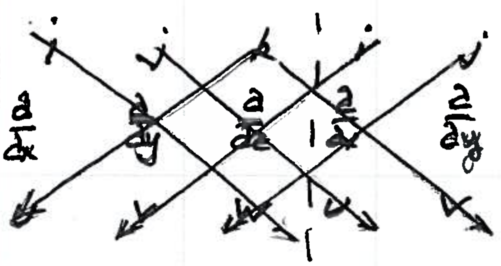
SCRIPT

THE VORTICITY VECTOR IS TWICE THE ANGULAR VELOCITY VECTOR.

IT IS COMPUTED AS THE CURL OF VELOCITY

NOTICE THE NOTATION

$\nabla \times \underline{V}$



$$\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)i+\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial x}\right)j+\left(\frac{\partial v}{\partial x}-\frac{\partial v}{\partial y}\right)\underline{k}$$

BOARD

VORTICITY VECTOR

$$\underline{\Omega} = \left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\underline{i}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\underline{j}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\underline{k}$$

THE VECTOR IS EQUAL TO THE CURL OF THE VELOCITY FIELD

$$\left.\begin{array}{l}\underline{\Omega} = curl(\underline{V}) \\ \quad\text{OR} \\ \underline{\Omega} = \nabla \times \underline{V}\end{array}\right\}\text{GENERAL DIFFERENTIAL FORMS OF VORTICITY}$$

SCRIPT

VORTICITY – STREAMFUNCTION IS A WAY OF MODELING REAL FLOWS.

MANY SITUATIONS ARE WELL MODELED AS IRROTATIONAL

- FLOW IN AN AQUIFER
- FLOW IN AN OIL RESERVOIR
- FLOW IN A REACTOR (DEPENDS ON KIND OF REACTOR)
- FLOW IN A LARGE STREAM
- FLOW IN CERTAIN CONDUITS
- FLOW OVER AIRFOIL

TEXTBOOK SAVES FOR LATER
NOT SURE WHY?

BOARD

MANY REAL FLOW SITUATIONS ARE WELL MODELED AS IRROTATIONAL FLOW

$$\underline{\Omega} = \underline{0}$$

$$\Rightarrow \quad \frac{\partial w}{\partial y}=\frac{\partial v}{\partial z}\ ;\ \frac{\partial u}{\partial z}=\frac{\partial w}{\partial x}\ ;\ \frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}$$

IMPORTANT FLUID CONCEPTS SO FAR:

$\rho a = \rho g - \nabla p$ EULER'S EQN

$\frac{\partial \rho}{\partial t} = -\nabla \rho \underline{V}$ (CONTINUITY – COMING SOON!)

$\underline{\Omega} = \nabla \times \underline{V}$ VORTICITY DEFINED

plus

$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C$ BERNOULLI'S EQN

SCRIPT

CONSIDER A CONDUIT WITH CROSS SECTION AREA, A.

VOLUME OF FLUID THAT PASSES THE AREA AT x IN TIME INTERVAL Δt IS

$$\Delta x A = \forall$$

FLOW RATE IS

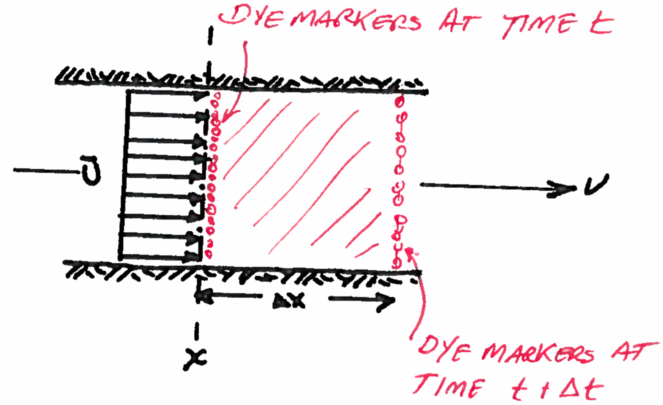
$$Q = \frac{\forall}{\Delta t} = \frac{\Delta x}{\Delta t} A$$

BOARD

CONTROL VOLUMES & CONTINUITY

VOLUMETRIC FLOW RATE

VOLUME OF FLUID CROSSING AN AREA PER UNIT OF TIME



SCRIPT

$$\frac{\Delta x}{\Delta t} = \bar{U} \text{ (IN DRAWING)}$$

$\bar{U}$ IS CALLED MEAN SECTION VELOCITY

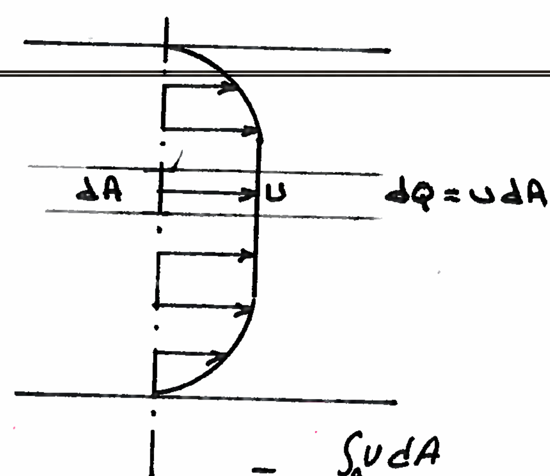
$$dQ = u\,dA$$

$$\int dQ = \int_A u\,dA$$

$$\bar{U} = \frac{\int_A u\,dA}{\int_A dA}$$

BOARD

IF VELOCITY VARIES ACROSS SECTION, THEN MEAN SECTION VELOCITY IS FOUND BY INTEGRATION



$$\bar{U} = \frac{\int_A u\,dA}{\int_A dA}$$

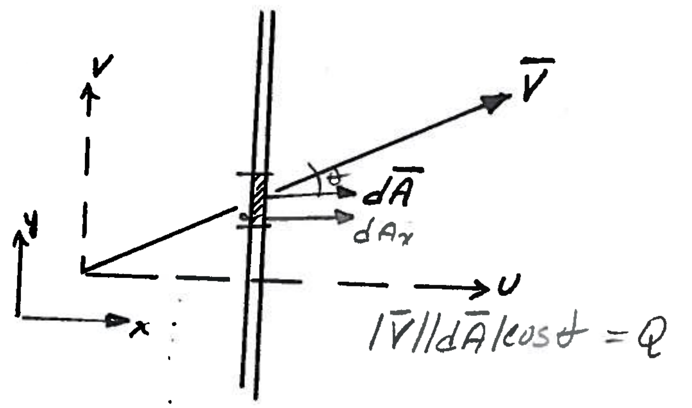
SCRIPT

FOR ARBITRARY ORIENTATION THE "INTEGRALS" ARE RESULT OF INNER PRODUCT OF VELOCITY VECTOR $\bar{V}$ AND AREA VECTOR $d\bar{A}$

SCALAR RESULT SHOWN IN PENCIL.

BOARD

OBSERVE THAT dA IS NORMAL TO V IN THIS DEFINITION



$$Q = \int_A \bar{V} \cdot d\bar{A} = \int_A u\,dA_x + \int_A v\,dA_y$$

SCRIPT

MASS OF FLUID THAT PASSES THE AREA AT x IN TIME INTERVAL Δt IS

$$\rho \Delta x A = \rho V$$

$$\frac{\rho V}{\Delta t} = \rho \frac{\Delta x}{\Delta t} A = \dot{m}$$

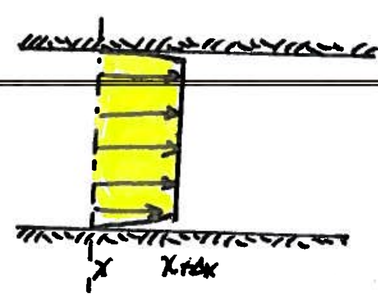
$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \to \bar{U}$$

$$\dot{m} = \rho \bar{U} A$$

BOARD

MASS FLOW RATE

MASS OF FLUID CROSSING AN AREA PER UNIT OF TIME



NEARLY SAME EQUATION; DECIDEDLY THE SAME CONCEPT.

SCRIPT

THE "INTEGRALS" WILL BE CALLED THE "FLUX" INTEGRALS

IF HAVE TO PERFORM INTEGRATIONS, NEED TO CONSIDER HOW VELOCITY VARIES ACROSS SECTION

$\bar{V}(d\bar{A}) \cdot d\bar{A}$

REALLY WHAT'S GOING ON!

BOARD

AS WITH VOLUMETRIC FLOW RATE, IF VELOCITY VARIES THEN

$$\dot{m} = \int_A \rho \bar{V} \cdot d\bar{A}$$

KEY CONCEPTS:

$$Q = \int_A \bar{V} \cdot d\bar{A}$$

VECTOR INNER PRODUCT OF $\bar{V}$ & $d\bar{A}$

$$\dot{m} = \int_A \rho \bar{V} \cdot d\bar{A}$$

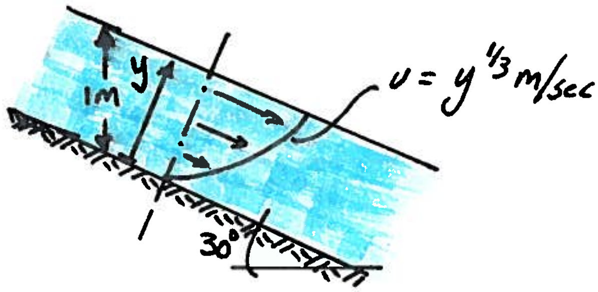
IF $\bar{V} \perp d\bar{A}$ THEN $\bar{V} \cdot d\bar{A} = V dA$

OTHERWISE NEED COMPONENTS

SCRIPT

BOARD

CHANNEL SHOWN IS 2M WIDE. WHAT IS VOLUMETRIC DISCHARGE?



KNOWN

$v(y) = y^{1/3}$ m/s

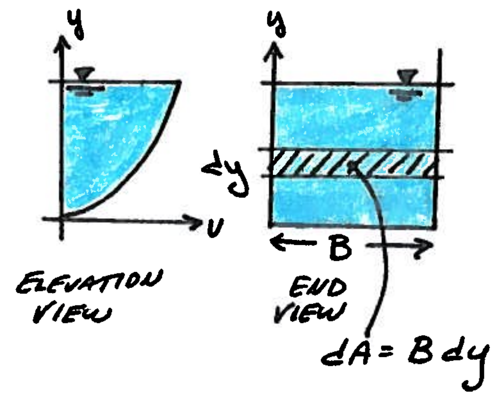
DISTANCE IN +Z AXIS 1M.

SLOPE 30°

UNKNOWN

Q

SOLUTION



1) DEPTH OF FLOW Y

$y = 1\,m \cos 30° = 0.866m$

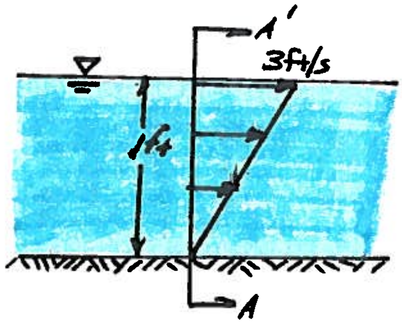
2) $Q = \int \vec{v}\, d\vec{A}$

$dA = B dy$

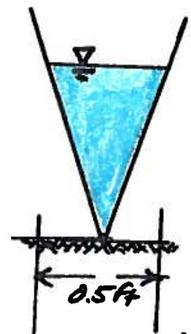
$$Q = \int_0^{0.866} y^{1/3} B\, dy = \frac{3}{4} y^{\frac{4}{3}} B \Big|_0^{0.866} = \left(\frac{3}{4}\right)(0.825)(2)$$

$$= 1.23\, m^3/sec \leftarrow Q$$

SECTIONAL WATER VELOCITY IN V-CHANNEL VARIES LINEARLY WITH DEPTH FROM ZERO AT THE BOTTOM TO A MAXIMUM AT THE WATER SURFACE AS SHOWN. DETERMINE THE DISCHARGE IN THE CHANNEL



ELEVATION VIEW



SECTION A-A' END VIEW

KNOWN

$v(y)$

GEOMETRY

UNKNOWN

Q

GOVERNING EQUATION(S)

$$Q = \int_A \bar{v}\, d\bar{A}$$

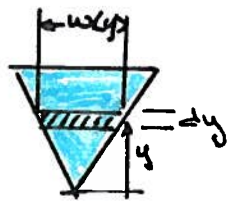
SOLUTION

① GEOMETRY

$@ y=0,\ w=0$

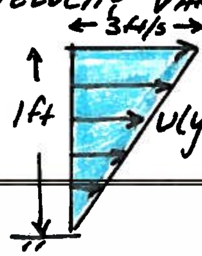
$@ y=1,\ w=0.5$

$\therefore\ w(y) = 0.5\,y$



$$dA = w\,dy = 0.5y \cdot dy = \frac{y}{2} dy$$

② VELOCITY VARIATION



$@ y=0,\ v=0$

$@ y=1,\ v=3\text{ft/s}$

$$v(y) = \frac{3\text{ft}}{\text{s}} \cdot \frac{y}{1\text{ft}} = 3y$$

③ $$Q = \int_A v\,dA = \int_0^1 3y \cdot \frac{y}{2} dy = \int_0^1 \frac{3}{2} y^2 dy = \frac{3}{2} \cdot \frac{y^3}{3}\Big|_0^1$$

$$= \underline{\underline{\frac{1}{2}\ \text{ft}^3/\text{sec}}} \longleftarrow Q$$