6 MOMENTUM EQUATION

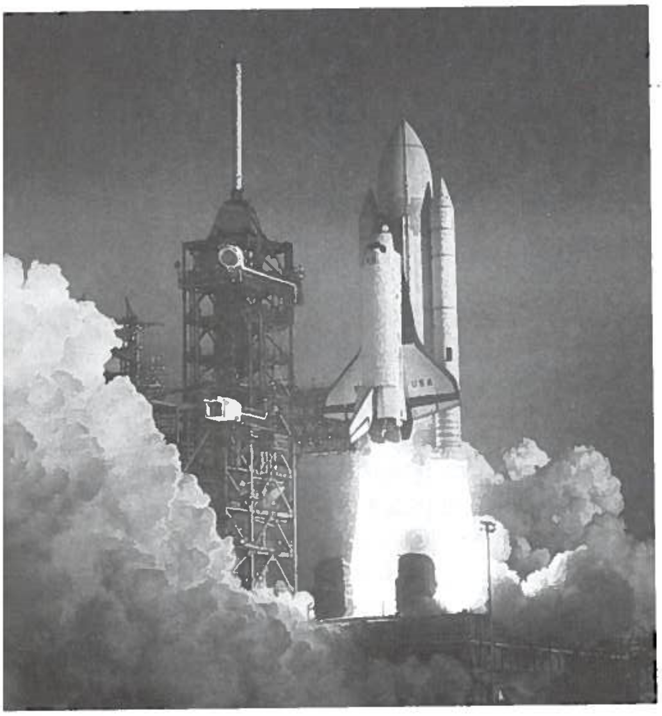


FIGURE 6.1
Engineers design systems by using a small set of fundamental equations such as the momentum equation. (Photo courtesy of NASA.)

Chapter Road Map

This chapter presents (a) the linear momentum equation and the (b) angular momentum equation. Both equations are derived from Newton's second law of motion.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define a force, a body force, and a surface force. (§6.
- Explain Newton's second law (particle or system of particles). (§6.1)
- Solve a vector equation with the VSM (Visual Solution Method). (§6.1)
- List the steps to derive the linear momentum equation. (§6.2)
- Describe or calculate (a) momentum flow and (b) momentum accumulation. (§6.2)
- Sketch a force diagram. Sketch a momentum diagram. (§6.3)
- Describe the physics of the momentum equation and the meaning of the variables that appear in the equation. (§6.2, §6.3)
- Describe the process for applying the momentum equati (§6.3)
- Apply the linear momentum equation to problems involv jets, vanes, pipe bends, nozzles, and other stationary objects. (§6.4)
- Apply the linear momentum equations to moving objects such as carts and rockets. (§6.5)
- Apply the angular momentum equation to analyze rotati machinery such as pumps and turbines. (§6.6)

6.1 Understanding Newton's Second Law of Motion

Because Newton's second law is the theoretical foundation of the momentum equation, this section reviews relevant concepts.

Body and Surface Forces

A **force** is an interaction between two bodies that can be idealized as a push or pull of one body on other body. A push/pull interaction is one that can cause acceleration.

Newton's third law tells us that forces must involve the interaction of *two bodies* and that *forces occur in pairs*. The two forces are equal in magnitude, opposite in direction, and colinear.

EXAMPLE. To give examples of force, consider an airplane that is flying in a straight path at constant speed (Fig. 6.2). Select the airplane as the *system* for analysis. Idealize the airplane as a *particle*. Newton's first law (i.e., force equilibrium) tells us that the sum of forces must balance. There are four forces on the airplane.

- The *lift force* is the net upward push of the air (body 1) on the airplane (body 2).
- The *weight* is the pull of the earth (body 1) on the airplane (body 2) through the action of gravity.
- The *drag force* is the net resistive force of the air (body 1) on the airplane (body 2).
- The *thrust force* is the net horizontal push of the air (body 1) on the surfaces of the propeller (body 2).

Notice that each of the four interactions just described can be classified as a force because: (a) they involve a push or pull, and (b) they involve the interaction of two bodies of matter.

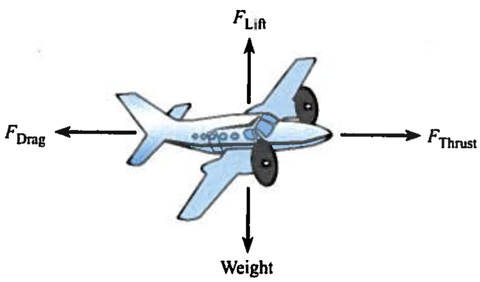


FIGURE 6.2
When an airplane is fly in straight and level flig the forces sum to zero.

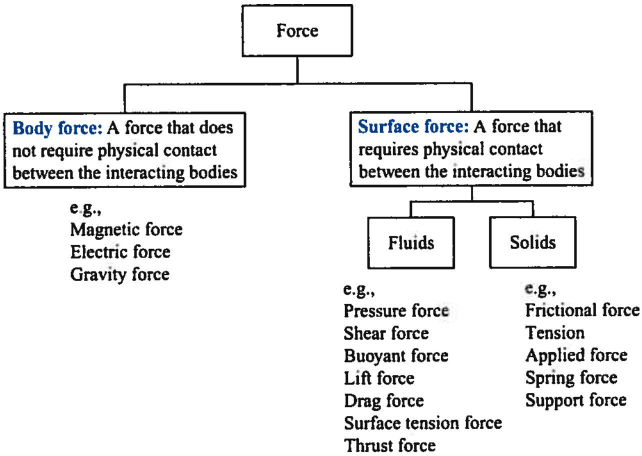
Forces can be classified into two categories: body force and surface force. A **surface force** (also known as a contact force) is a force that requires physical contact or touching between the two interacting bodies. The lift force (Fig. 6.2) is a surface force because the air (body 1) must touch the wing (body 2) to create the lift force. Similarly, the thrust and drag forces are surface forces.

A **body force** is a force that can act without physical contact. For example, the weight force is a body force because the airplane (body 1) does not need to touch the earth (body 2) for the weight force to act.

A body force acts on every particle within a system. In contrast, a surface force acts only on the particles that are in physical contact with the other interacting body. For example, consider a system comprised of a glass of water sitting on a table. The weight force is pulling on every particle within the system, and we represent this force as a vector that passes through the center of gravity of the system. In contrast, the normal force on the bottom of the cup acts on on the particles of glass that are touching the table.

Summary Forces can be classified in two categories: body forces and surface forces (s Fig. 6.3). Most forces are surface forces.

FIGURE 6.3
Forces can be classified as body forces or surface forces.



Newton's Second Law of Motion

In words, Newtons' second law is: *The sum of forces on a particle is proportional to the acceler tion, and the constant of proportionality is the mass of the particle.* Notice that this law appli only to a particle. The second law asserts that *acceleration and unbalanced forces are propc tional.* This means, for example, that

- If a particle is accelerating, then the sum of forces on the particle is nonzero.
- If the sum of forces on a particle is nonzero, then the particle will be accelerating.

Newton's second law can be written as an equation:

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = m\mathbf{a} \qquad (6.$$

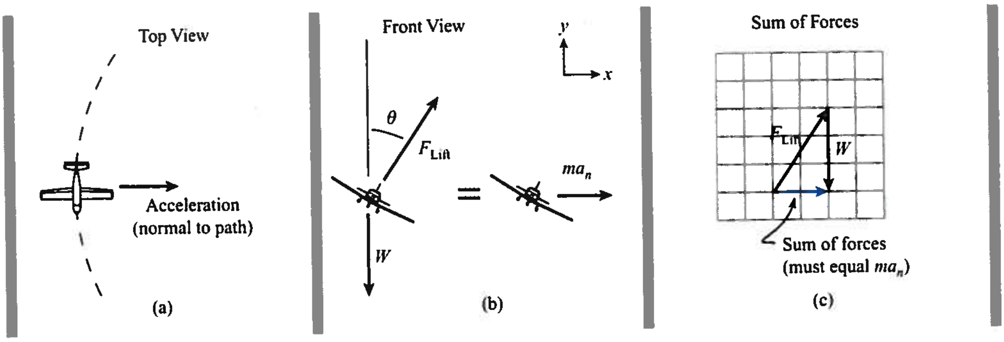
where the subscript "ext" is a reminder to sum only external forces.

EXAMPLE. To illustrate the relationship between unbalanced forces and acceleration, co sider an airplane that is turning left while flying at a constant speed in a horizontal pla (Fig. 6.4a). Select the airplane as a *system*. Idealize the airplane as a *particle*. Because t airplane is traveling in a circular path at constant speed, the acceleration vector must poi inward. Fig. 6.4b shows the vectors that appear in Newton's second law. For Newton's seco law of motion to be satisfied, the sum of the force vectors (Fig. 6.4c) must be equal to the r vector.

The airplane example illustrates a method for visualizing and solving a vector equati called the *Visual Solution Method* (VSM). This method was adapted from Hibbler (1). Tl method is presented in the next subsection. Checkpoint Problem 6.1 gives you a chance to t your understanding of Newton's second law.

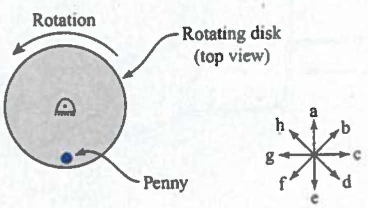
FIGURE 6.4

An airplane flying with a steady speed on curved path in a horizontal plane (a) Top view, (b) Front view, (c) Sketch showing how the $\Sigma\mathbf{F}$ vectors balance the **ma** vector.



✔CHECKPOINT PROBLEM 6.1

A disk in a horizontal plane is rotating in a counterclockwise direction and the speed of rotation is decreasing. A penny stays in place on the disk due to friction. Which letter (a to h) best represents the direction of acceleration of the penny? Which letter best represents the direction of the sum of forces vector?



Solving a Vector Equation with the Visual Solution Method (VSM)

The VSM is an approach for solving a vector equation that reveals the physics while also showing visually how the equation can be solved. Thus, the VSM simplifies problem solving. The VSM has three steps.

Step 1: Identify the vector equation in its general form.

Step 2: Draw a diagram that shows the vectors that appear in the left side of the equation. Then, draw a second diagram that shows the vectors that appear on the right side of the equation. Add the equal sign between the diagrams.

Step 3: From the diagrams, apply the general equation and simplify the results to create the reduced equation(s). The reduced equation(s) can be written as a vector equation or as one or more scaler equations.

EXAMPLE. This example shows how to apply the VSM to the airplane problem (see Fig. 6.4).

Step 1: The general equation is Newton's second law $(\sum \mathbf{F})_{ext} = m\mathbf{a}$.

Step 2: The two diagrams separated by an equal sign are shown in Fig. 6.4b.

Step 3: By looking at the diagrams, one can write the reduced equation using scalar equations:

$$(x\text{-direction}) \qquad F_{lift} \sin\theta = ma_n$$

$$(y\text{-direction}) \qquad -W + F_{lift}\cos\theta = 0$$

Alternatively, one can look at the diagrams and then write the reduced equation using a vector equation.

$$F_{Lift}(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) - W\mathbf{j} = (ma_n)\mathbf{i}$$

EXAMPLE. This example shows how to apply the VSM to a generic vector equation.

Step 1. Suppose the general equation is $\sum \mathbf{x} = \mathbf{y}_2 - \mathbf{y}_1$.

Step 2. Suppose the vectors are known. Then, one can sketch the diagrams (Fig. 6.5).

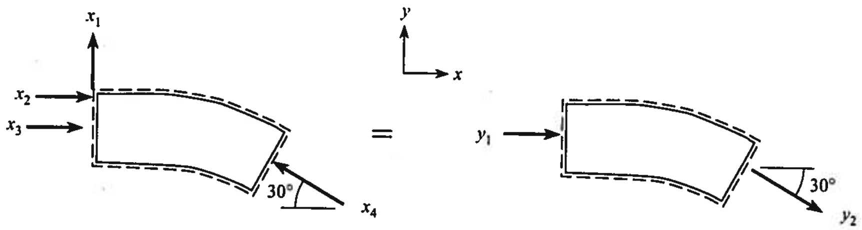
Step 3. By looking at the diagrams, write the reduced equations. To get the signs corr notice that the general equation shows that vector $\mathbf{y}_1$ is subtracted. The reduced equati are

$$\text{(x-direction)} \qquad x_2 + x_3 - x_4 \cos 30° = y_2 \cos 30° - y_1$$

$$\text{(y-direction)} \qquad x_1 + x_4 \sin 30° = -y_2 \sin 30°$$

FIGURE 6.5

Vectors used to illustrate how to solve a vector equation.



Newton's Second Law (System of Particles)

Newton's second law (Eq. 6.1) applies to one particle. Because a flowing fluid involves ma particles, the next step is to modify the second law so that it applies to a system of particles. begin the derivation, note that the mass of a particle must be constant. Then, modify Eq. (6. to give

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = \frac{d(m\mathbf{v})}{dt} \qquad (6.$$

Where $m\mathbf{v}$ is the momentum of one particle.

To extend Eq. (6.2) to multiple particles, apply Newton's second law to each particle, ai then add the equations together. Internal forces, which are defined as forces between tl particles of the system, cancel out, and the result is

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = \frac{d}{dt}\sum_{i=1}^{N}(m_i\mathbf{v}_i) \qquad (6.$$

where $m_i\mathbf{v}_i$ is the momentum of the ith particle, and $(\Sigma\mathbf{F})_{\text{ext}}$ are forces that are external to tl system. Next, let

$$\text{(Total momentum of the system)} \equiv \mathbf{M} = \sum_{i=1}^{N}(m_i\mathbf{v}_i) \qquad (6.$$

Combine Eqs. (6.3) and (6.4).

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = \left.\frac{d(\mathbf{M})}{dt}\right|_{\text{closed system}} \qquad (6.$$

The subscript "closed system" reminds us that Eq. (6.5) is for a closed system.

6.2 The Linear Momentum Equation: Theory

This section shows how to derive the linear momentum equation and explains the physics.

Derivation

Start with Newton's second law for a system of particles (Eq. 6.5). Next, apply the Reynolds transport theorem (Eq. 5.23) to the right side of the equation. The extensive property is momentum, and the corresponding intensive property is the momentum per unit mass which ends up being the velocity. Thus, Reynolds transport theorem gives

$$\left.\frac{d\mathbf{M}}{dt}\right|_{\text{closed system}} = \frac{d}{dt}\int_{\text{cv}} \mathbf{v}\rho\, d\mathcal{V} + \int_{\text{cs}} \mathbf{v}\rho \mathbf{V}\cdot d\mathbf{A} \tag{6.6}$$

Combining Eqs. (6.5) and (6.6) gives the *general form* of the *momentum equation*.

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = \frac{d}{dt}\int_{\text{cv}} \mathbf{v}\rho\, d\mathcal{V} + \int_{\text{cs}} \rho\mathbf{v}(\mathbf{V}\cdot d\mathbf{A}) \tag{6.7}$$

where $(\Sigma\mathbf{F})_{\text{ext}}$ is the sum of external forces acting on the matter in the control volume, $\mathbf{v}$ is fluid velocity relative to an inertial reference frame, and $\mathbf{V}$ is velocity relative to the control surface.

Eq. (6.7) can be simplified. To begin, assume that each particle inside the CV has the same velocity. Thus, the first term on the right side of Eq. (6.7) can be written as

$$\frac{d}{dt}\int_{\text{cv}} \mathbf{v}\rho\, d\mathcal{V} = \frac{d}{dt}\left[\mathbf{v}\int_{\text{cv}} \rho\, d\mathcal{V}\right] = \frac{d(m_{\text{cv}}\mathbf{v}_{\text{cv}})}{dt} \tag{6.8}$$

Next, assume that velocity is uniformly distributed as it crosses the control surface. Then, the last term in Eq. (6.7) can be written as

$$\int_{\text{cs}} \mathbf{v}\rho\mathbf{V}\cdot d\mathbf{A} = \mathbf{v}\int_{\text{cs}} \rho\mathbf{V}\cdot d\mathbf{A} = \sum_{\text{cs}} \dot{m}_o\mathbf{v}_o - \sum_{\text{cs}} \dot{m}_i\mathbf{v}_i \tag{6.9}$$

Combining Eqs. (6.7) to (6.9) gives the final result:

$$\left(\sum \mathbf{F}\right)_{\text{ext}} = \frac{d(m_{\text{cv}}\mathbf{v}_{\text{cv}})}{dt} + \sum_{\text{cs}} \dot{m}_o\mathbf{v}_o - \sum_{\text{cs}} \dot{m}_i\mathbf{v}_i \tag{6.10}$$

where m_{cv} is the mass of the matter that is inside the control volume. The subscripts o and i refer to the outlet and inlet ports, respectively. Eq. (6.10) is the *simplified form* of the momentum equation.

Physical Interpretation of the Momentum Equation

The momentum equation asserts that the sum of forces is exactly balanced by the momentum terms; see Fig. 6.6.

Momentum Flow (Physical Interpretation)

To understand what momentum flow means, select a cylindrical fluid particle passing across a CS (see Fig. 6.7). Let the particle be long enough so that it travels across the CS during a time interval Δt. Then, the particle's length is

$$L = (\text{length}) = \left(\frac{\text{length}}{\text{time}}\right)(\text{time}) = (\text{speed})(\text{time}) = v\Delta t$$

FIGURE 6.6
The conceptual meaning of the momentum equation.

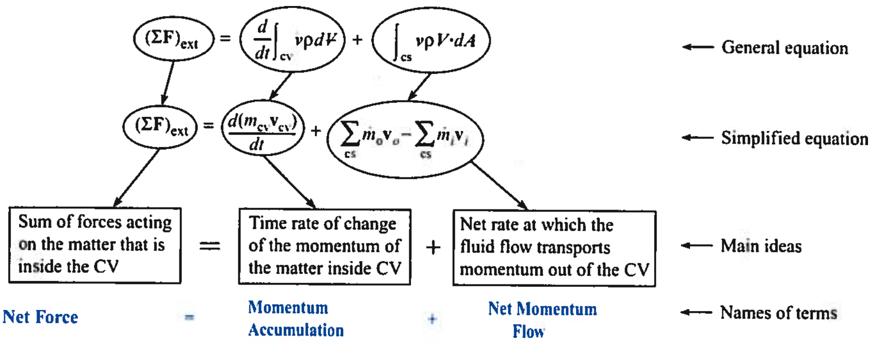
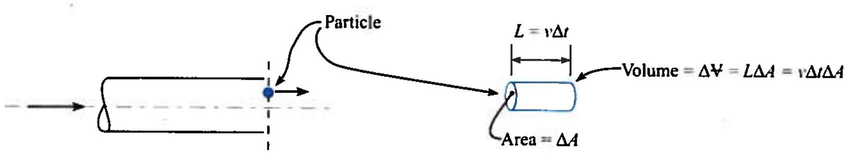


FIGURE 6.7
A fluid particle passing across the control surface during a time interval Δt.



and the particle's volume is $\Psi = (v\Delta t)\Delta A$. The momentum of the particle is

$$\text{momentum of one particle} = (\text{mass})(\text{velocity}) = (\rho\Delta\Psi)\mathbf{v} = (\rho v\Delta t\Delta A)\mathbf{v}$$

Next, add up the momentum of all particles that are crossing the control surface through given face.

$$\text{momentum of all particles} = \sum_{cs}(\rho v\Delta t\Delta A)\mathbf{v} \quad \text{(6.1}$$

Now, let the time interval Δt and the area ΔA approach zero and replace the sum with tl integral. Eq. (6.11) becomes

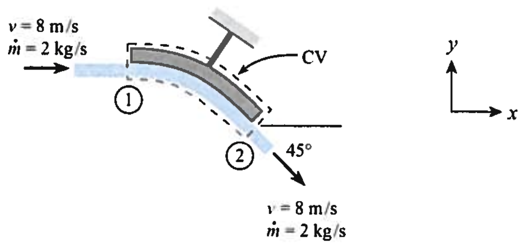
$$\left(\frac{\text{momentum of all particles crossing the CS}}{\text{interval of time}}\right)_{\text{instant in time}} = \int_{cs}(\rho v)\mathbf{v}\,dA$$

Summary Momentum flow describes the rate at which the flowing fluid transports momen tum across the control surface.

Momentum Flow (Calculations)

When fluid crosses the control surface, it transports momentum across the CS. At section (Fig. 6.8), momentum is transporting into the CV. At section 2, momentum is transported ou of the CV.

FIGURE 6.8
A fluid jet striking a flat vane.



When the velocity is uniformly distributed across the CS, Eq. (6.10) indicates that the

$$\begin{pmatrix}\text{magnitude of} \\ \text{momentum flow}\end{pmatrix} = \dot{m}v = \rho A v^2 \tag{6.12}$$

Thus, at section 1, the momentum flow has a magnitude of

$$\dot{m}v = (2\ \text{kg/s})(8\ \text{m/s}) = 16\ \text{kg} \cdot \text{m/s}^2 = 16\ \text{N}$$

and the direction of vector is to the right. Similarly at section 2, the momentum flow has a magnitude of 16 newtons and a direction of 45° below horizontal. From Eq. (6.10), the net momentum flow term is:

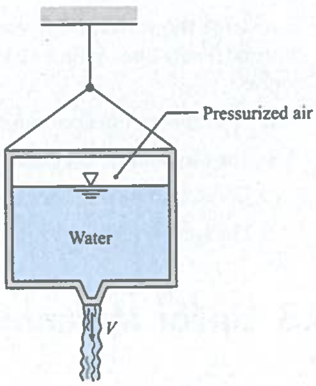
$$\dot{m}\mathbf{v}_2 - \dot{m}\mathbf{v}_1 = \{(16\ \text{N})\cos(45°\mathbf{i} - \sin 45°\mathbf{j})\} - \{(16\,\text{N})\mathbf{i}\}$$

Summary For uniform velocity, momentum flow terms have a magnitude $\dot{m}v = \rho A v^2$ and a direction parallel to the velocity vector. The net momentum flow is calculated by subtracting the inlet momentum flow vector(s) from the outlet momentum flow vector(s).

✔CHECKPOINT PROBLEM 6.2

Pressurized air forces water out of a tank. If the air pressure is increased so that the exit speed increases from V to $2V$, what happens to the rate of momentum flow out the bottom of the tank? The rate

a. Stays the same
b. Increases by 1x
c. Increases by 2x
d. Increases by 3x
e. Increases by 4x
f. Increases by 8x



Momentum Accumulation (Physical Interpretation)

To understand what accumulation means, consider a control volume around a nozzle (Fig. 6.9). Then, divide the control volume into many small volumes. Pick one of these small volumes, and note that the momentum inside this volume is $(\rho \Delta \mathcal{V})\mathbf{v}$.

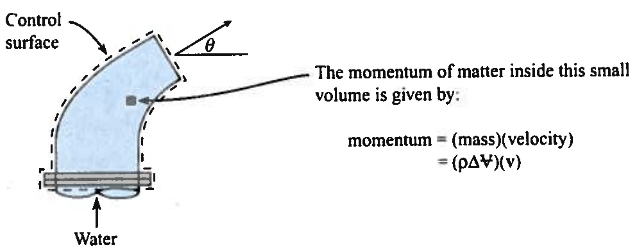


FIGURE 6.9
Water flowing through a nozzle.

To find the total momentum inside the CV add up the momentum for all the small volur that comprise the CV. Then, let $\Delta \mathcal{V} \to 0$, and use the fact that an integral is the sum of m small terms.

$$\begin{pmatrix}\text{Total momentum}\\ \text{inside the CV}\end{pmatrix} = \sum (\rho \Delta \mathcal{V})\mathbf{v} = \sum \mathbf{v}\rho\Delta\mathcal{V} = \int_{cv} \mathbf{v}\rho\, d\mathcal{V} \qquad (6.$$

Taking the time derivative of Eq. (6.13) gives the final result:

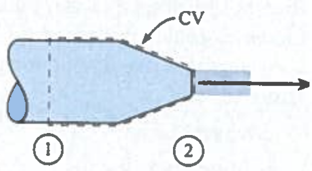
$$\begin{pmatrix}\text{Momentum}\\ \text{Accumulation}\end{pmatrix} = \begin{pmatrix}\text{Rate of change of the}\\ \text{total momentum}\\ \text{inside the CV}\end{pmatrix} = \frac{d}{dt}\int_{cv} \mathbf{v}\rho\, d\mathcal{V} \qquad (6.$$

Summary Momentum accumulation describes the time rate of change of the momentum ins the CV. For most problems, the accumulation term is zero or negligible. To analyze the momentu accumulation term, one can ask two questions. *Is the momentum of the matter inside the CV cha ing with time? Is this change significant?* If the answers to both questions are yes, then the mome tum accumulation term should be analyzed. Otherwise, the accumulation term can be set to ze

Checkpoint Problem 6.3 gives you a chance to test your understanding of the momentu equation.

✔CHECKPOINT PROBLEM 6.3

The sketch shows a liquid flowing through a stationary nozzle. Assume steady flow. Which statements are true? (select all that apply)



a. The momentum accumulation is zero.
b. The momentum accumulation is nonzero.
c. The sum of forces is zero.
d. The sum of forces is nonzero.

6.3 Linear Momentum Equation: Application

Working Equations

Table 6.1 summarizes the linear momentum equation.

TABLE 6.1 Summary of the Linear Momentum Equation

Description	Equation	Terms
General Equation	$\left(\sum \mathbf{F}\right)_{ext} = \frac{d}{dt}\int_{cv} \mathbf{v}\rho\, d\mathcal{V} + \int_{cs} \rho\mathbf{v}(\mathbf{V}\cdot d\mathbf{A})$ Eq. (6.7)	$(\Sigma\mathbf{F})_{ext}$ = sum of external forces (N) t = time (s) $\mathbf{v}$ = velocity measured from the selected ref. frame (m/s (must select a reference frame that is inertial) $\mathbf{v}_{cv}$ = velocity of CV from selected ref. frame (m/s) $\mathbf{V}$ = velocity measured from the control surface (m/s) ρ = density of fluid (kg/m^3) m_{cv} = mass of the matter inside the control volume (kg $\dot{m}_o$ = mass flow rate out of the control volume (kg/s) $\dot{m}_i$ = mass flow rate into the control volume (kg/s)
Simplified Equation Use this equation for most problems. Assumptions: (a) all particles inside the CV have the same velocity, and (b) when flow crosses the CS, the velocity is uniformly distributed.	$\left(\sum \mathbf{F}\right)_{ext} = \frac{d(m_{cv}\mathbf{v}_{cv})}{dt} + \sum_{cs} \dot{m}_o\mathbf{v}_o - \sum_{cs} \dot{m}_i\mathbf{v}_i$ Eq. (6.10)	

Force and Momentum Diagram

The recommended method for applying the momentum equation, the VSM (visual solution method), is illustrated in the next example.

EXAMPLE. This example explains how to apply the VSM for water flowing out a nozzle (Fig. 6.10a). The water enters at section 1 and jets out at section 2.

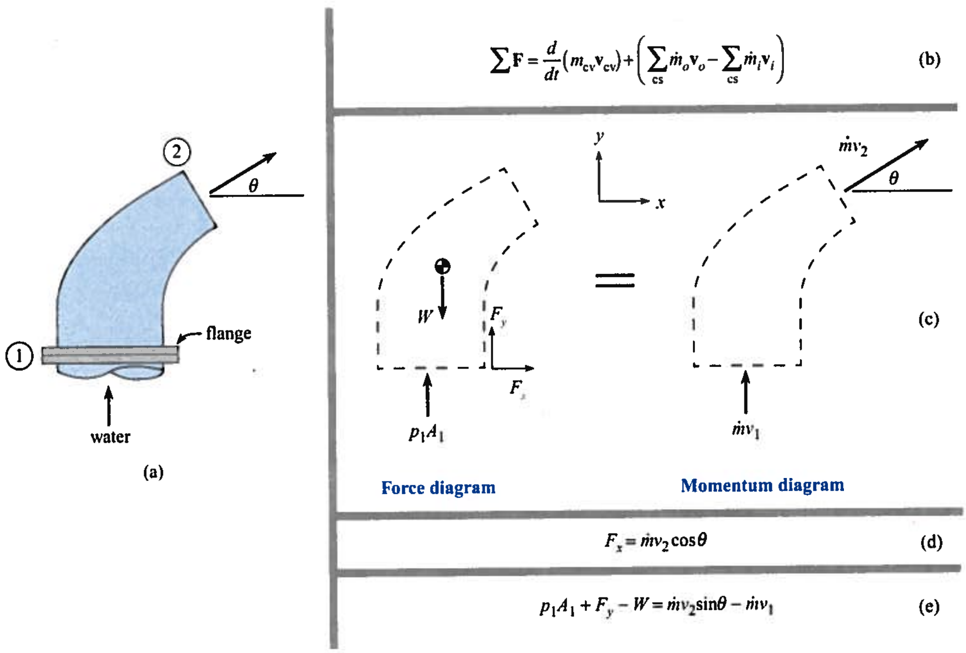


FIGURE 6.10

Step 1. Write the momentum equation (see Fig. 6.10b). Select a control volume that surrounds the nozzle.

Step 2a. To represent the force terms, sketch a *force diagram* (Fig. 6.10c). A **force diagram** illustrates the forces that are acting on the matter that is inside the CV. A force diagram is similar to a *free body diagram* in terms of how it is drawn and how it looks. However, a free-body diagram is an Lagrangian idea, whereas a force diagram is an Eulerian idea. This is why different names are used.

To draw the force diagram, sketch the CV, then sketch the external forces acting on the CV. In Fig. 6.10c, the weight vector, W, represents the weight of the water plus the weight of the nozzle material. The pressure vector, symbolized with p_1A_1, represents the water in the pipe pushing the water through the nozzle. The force vector, symbolized with F_x and F_y, represents the force of the support that is holding the nozzle stationary.

Step 2b. To represent the momentum terms, sketch a **momentum diagram** (Fig. 6.10c). This diagram shows the momentum terms from the right side of the momentum equation. The momentum outflow is represented with $\dot{m}v_2$ and momentum inflow is represented with $\dot{m}v_1$. The momentum accumulation term is zero because the total momentum inside the CV is constant with time.

Step 3. Using the diagrams, write the reduced equations (see Figs. 6.10d and 6.10e).

Summary The *force diagram* shows forces on the CV, and the *momentum diagram* shows momentum terms. We recommend drawing these diagrams and using the VSM.

A Process for Applying the Momentum Equation

Step 1. Selection. Select the linear momentum equation when the problem invol forces, accelerating fluid particles, and torque does not need to be considered.

Step 2. Sketching. Select a CV so that control surfaces cut through where (a) you know inf mation or (b) you want information. Then, sketch a force diagram and a momentum diagra

Step 3. Analysis. Write scalar or vector equations by using the VSM.

Step 4. Validation. Check that all forces are external force. Check the signs on vecto Check the physics. For example, if accumulation is zero, then the sum of forces shou balance the momentum flow out minus the momentum flow in.

A Road Map for Problem Solving

Fig. 6.11 shows a classification scheme for problems. Like a road map, the purpose of th diagram is to help navigate the terrain. The next two sections present the details of each catego of problems.

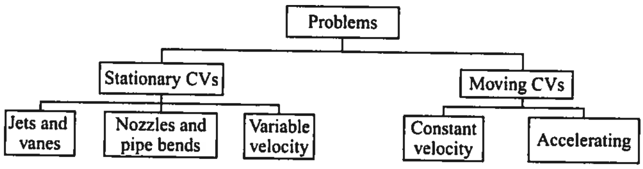


FIGURE 6.11
A classification scheme for problems that are solvable by application of the momentum equation.

6.4 The Linear Momentum Equation for a Stationary Control Volume

When a CV is stationary with respect to the earth, then the accumulation term is nearly alway zero or negligible. Thus, the momentum equations simplifies to

$$\text{(sum of forces)} = \text{(rate of momentum out)} - \text{(rate of momentum in)}$$

Fluid Jets

Problems in the category of **fluid jet** involve a free jet leaving a nozzle. However, analysis c the nozzle itself is not part of the problem. An example of a fluid jet problem is shown i Fig. 6.12. This problem shown involves a water cannon on a cart. The water leaves the nozzl with velocity V, and the goal is to find the tension in the cable.

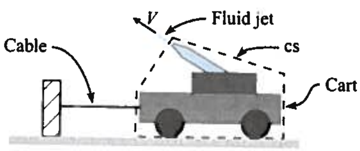


FIGURE 6.12
A problem involving a fluid jet.

Each category of problems has certain facts that make problem solving easier. These facts will be presented in the form of tips. **Tips** for fluid jet problems are

- When a free jet crosses the control surface, the jet does not exert a force. Thus, do not draw a force on the force diagram. The reason is that the pressure in the jet is ambient pressure, so there is no net force. This can be proven by applying Euler's equation.
- The momentum flow of the fluid jet is $\dot{m}v$.

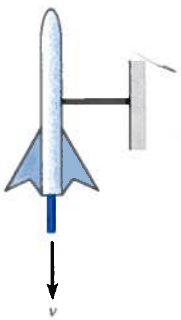
Example 6.1 shows a problem in the "fluid jet" category.

EXAMPLE 6.1

Momentum Equation Applied to a Stationary Rocket

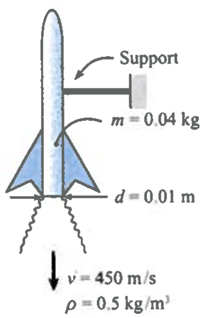
Problem Statement

The following sketch shows a 40 g rocket, of the type used for model rocketry, being fired on a test stand to evaluate thrust. The exhaust jet from the rocket motor has a diameter of $d = 1$ cm, a speed of $v = 450$ m/s, and a density of $\rho = 0.5$ kg/m^3. Assume the pressure in the exhaust jet equals ambient pressure. Find the force F_s acting on the support that holds the rocket stationary.



Define the Situation

A small rocket is fired on a test stand.



Assumptions. Pressure is 0.0 kPa gage at the nozzle exit plane.

State the Goal

F_s (N) ⬅ Force that acts on the support

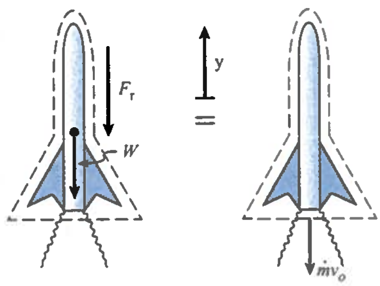
Generate Ideas and Make a Plan

Selection. Select the momentum equation because fluid particles are accelerating due to pressures generated by combustion and because force is the goal.

Sketching. Select a CV surrounding the rocket because the control surface cuts

- through the support (where we want information), and
- across the rocket nozzle (where information is known).

Then, sketch a *force diagram* and a *momentum diagram*. Notice that the diagrams include an arrow to indicate the positive *y*-direction. This is important because the moment equation is a vector equation.



In the force diagram, the body force is the weight (W). The force (F_r) represents the downward push of the support on the rocket. There is no pressure force at the nozzle exit plar because pressure is atmospheric.

Analysis. Apply the momentum equation in vertical direct by selecting terms off the diagrams.

$$F_r + W = \dot{m}v_o$$

In Eq. (a), the only unknown is F_r. Thus, the plan is

1. Calculate momentum flow: $\dot{m}v_o = \rho A v_o^2$.
2. Calculate weight.
3. Solve for force F_r. Then, apply Newton's third law.

Take Action (Execute the Plan)

1. Momentum flow.

$$\rho A v^2 = (0.5\ \text{kg/m}^3)(\pi \times 0.01^2\ \text{m}^2/4)(450^2\ \text{m}^2/\text{s}^2) = 7.952\ \text{N}$$

2. Weight

$$W = mg = (0.04\ \text{kg})(9.81\ \text{m/s}^2) = 0.3924\ \text{N}$$

3. Force on the rocket (from Eq. (a))

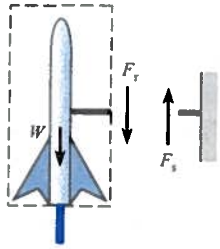
$$F_r = \rho A v_o^2 - W = (7.952\ \text{N}) - (0.3924\ \text{N}) = 7.56\ \text{N}$$

By Newton's third law, the force on the support is equal in magnitude to F_r and opposite in direction.

$$\boxed{F_s = 7.56\ \text{N (upward)}}$$

Review

1. *Knowledge.* Notice that forces acting on the rocket do not sum to zero. This is because the fluid is accelerating.
2. *Knowledge.* For a rocket, the term $\dot{m}v$ is sometimes called a "thrust force." For this example $\dot{m}v = 7.95$ N (1.79 lbf); this value is typical of a small motor used for model rocketry.
3. *Knowledge.* Newton's third law tells us that forces always occur in pairs, equal in magnitude and opposite in direction. In the sketch below, F_r and F_s are equal in magnitude and opposite in direction.



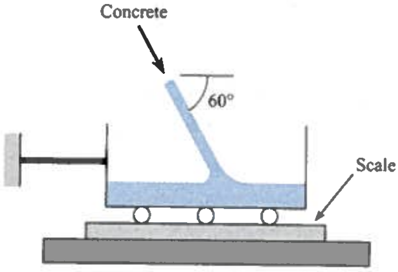
Example 6.2 gives another problem in the category of "fluid jet."

EXAMPLE 6.2

Momentum Equation Applied to a Fluid Jet

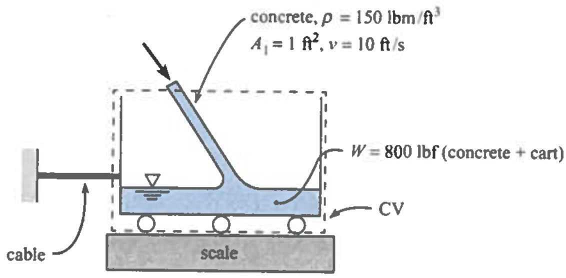
Problem Statement

As shown in the sketch, concrete flows into a cart sitting on a scale. The stream of concrete has a density of $\rho = 150\ \text{lbm/ft}^3$, an area of $A = 1\ \text{ft}^2$, and a speed of $v = 10$ ft/s. At the instant shown, the weight of the cart plus the concrete is 800 lbf. Determine the tension in the cable and the weight recorded by the scale. Assume steady flow.



Define the Situation

Concrete is flowing into a cart that is being weighed.

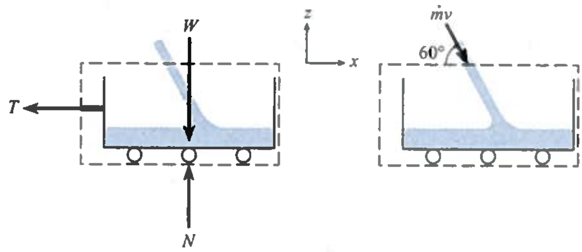


State the Goal

T(lbf) ⬅ Tension in cable
W_s(lbf) ⬅ Weight recorded by the scale

Generate Ideas and Make a Plan

Select the momentum equation. Then, select a CV and sketch this in the situation diagram. Next, sketch a force diagram and momentum diagram.



Notice in the force diagram that the liquid jet does not exert a force at the control surface. This is because the pressure in the jet equals atmospheric pressure.

To apply the momentum equation, use the force and momentum diagrams to visualize the vectors.

$$\sum \mathbf{F} = \dot{m}_o \mathbf{v}_o - \dot{m}_i \mathbf{v}_i$$

$$-T\mathbf{i} + (N - W)\mathbf{k} = -\dot{m}v((\cos 60°)\mathbf{i} - (\sin 60°)\mathbf{j})$$

Next, write scalar equations

$$-T = -\dot{m}v\cos 60° \quad \text{(a)}$$

$$(N - W) = \dot{m}v\sin 60° \quad \text{(b)}$$

Now, the goals can be solved for. The plan is to:

1. Calculate T using Eq. (a).
2. Calculate N using Eq. (b). Then let $W_s = -N$.

Take Action (Execute the Plan)

1. Momentum equation (horizontal direction)

$$T = \dot{m}v\cos 60° = \rho A v^2 \cos 60°$$

$$T = (150 \text{ lbm/ft}^3)\left(\frac{\text{slugs}}{32.2 \text{ lbm}}\right)(1 \text{ ft}^2)(10 \text{ ft/s})^2 \cos 60°$$

$$= \boxed{233 \text{ lbf}}$$

2. Momentum equation (vertical direction)

$$N - W = \dot{m}v\sin 60° = \rho A v^2 \sin 60°$$

$$N = W + \rho A v^2 \sin 60°$$

$$= 800 \text{ lbf} + 403 \text{ lbf} = \boxed{1200 \text{ lbf}}$$

Review

1. *Discussion.* The weight recorded by the scale is larger than the weight of the cart because of the momentum carried b the fluid jet.
2. *Discussion.* The momentum accumulation term in this problem is nonzero. However, it was assumed to be small and was neglected.

Vanes

A **vane** is a structural component, typically thin, that is used to turn a fluid jet (Fig. 6.13). A vane is used to idealize many components of engineering interest. Examples include a blade in a turbine, a sail on a ship, and a thrust reverser on an aircraft engine.

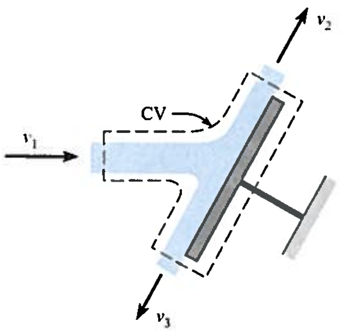


FIGURE 6.13
A fluid jet striking a flat vane.

To make solving of vane problems easier, we offer the following **Tips**.

- **Tip 1.** Assume that $v_1 = v_2 = v_3$. This assumption can be justified with the Bernoulli equation. In particular, assume inviscid flow and neglect elevation changes, and the Bernoulli equation can be used to prove that the velocity of the fluid jet is constant.

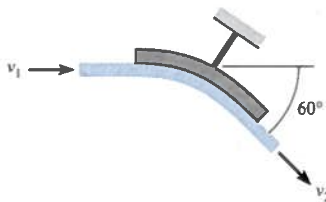
- **Tip 2.** Let each momentum flow equal $\dot{m}\mathbf{v}$. For example, in Fig. 6.13, the momentum inflow is $\dot{m}_1\mathbf{v}_1$. The momentum outflows are $\dot{m}_2\mathbf{v}_2$ and $\dot{m}_3\mathbf{v}_3$.
- **Tip 3.** If the vane is flat, as in Fig. 6.13, assume that the force to hold the vane stationary is normal to the vane because viscous stresses are small relative to pressure stresses. Thus, the load on the vane can assumed to be due to pressure, which acts normal to the vane.
- **Tip 4.** When the jet is a free jet, as in Fig. 6.13, recognize that the jet does not cause a net force at the control surface because the pressure in the jet is atmospheric. Only pressures different than atmospheric cause a net force.

EXAMPLE 6.3

Momentum Equation Applied to a Vane

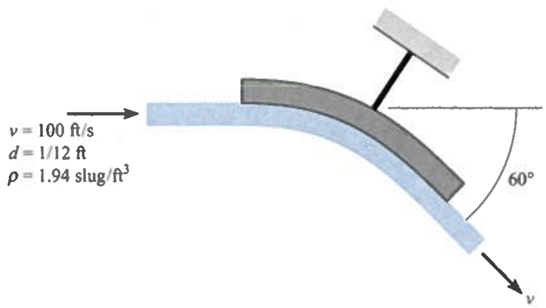
Problem Statement

A water jet ($\rho = 1.94$ slug/ft^3) is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Find the force exerted by the jet on the vane.



Define the Situation

A water jet is deflected by a vane.



Assumptions:

- Jet velocity is constant: $v_1 = v_2 = v$.
- Jet diameter is constant: $d_1 = d_2 = d$.
- Neglect gravitational effects.

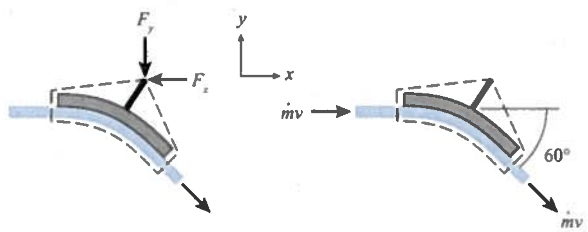
State the Goal

F_{jet}(N) ⬅ Force of the fluid jet on the vane

Generate Ideas and Make a Plan

Select. Because force is a parameter and fluid particles accelerate as the jet turns, select the linear momentum equation.

Sketch. Select a CV that cuts through support so that the force of the support can be found. Then, sketch a force diagram and a momentum diagram.



In the force and momentum diagrams, notice that

- Pressure forces are zero because pressures in the water jet at the control surface are zero gage.
- Each momentum flow is represented with $\dot{m}v$.

Analysis. To apply the momentum equation, use the force and momentum diagrams to write a vector equation.

$$\sum \mathbf{F} = \dot{m}_o\mathbf{v}_o - \dot{m}_i\mathbf{v}_i$$

$$(-F_x)\mathbf{i} + (-F_y)\mathbf{j} = \dot{m}v(\cos 60°\mathbf{i} - \sin 60°\mathbf{j}) - \dot{m}v\mathbf{i}$$

Now, write scalar equations

$$-F_x = \dot{m}v(\cos 60° - 1) \quad \text{(a)}$$

$$-F_y = -\dot{m}v(\sin 60°) \quad \text{(b)}$$

Because there is enough information to solve Eqs. (a) and (b), the problem is cracked. The plan is

1. Calculate $\dot{m}v$.
2. Apply Eq. (a) to calculate F_x.
3. Apply Eq. (b) to calculate F_y.
4. Apply Newton's third law to find the force of the jet.

Take Action (Execute the Plan)

1. Momentum flow rate.

$$\begin{aligned} \dot{m}v &= (\rho A v)v \\ &= (1.94\ \text{slug/ft}^3)(\pi \times 0.0417^2\ \text{ft}^2)(100\ \text{ft/s})^2 \\ &= 105.8\ \text{lbf} \end{aligned}$$

2. Linear momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}v(1 - \cos 60°) \\ &= (105.8\ \text{lbf})(1 - \cos 60°) \\ F_x &= 53.0\ \text{lbf} \end{aligned}$$

3. Linear momentum equation (y-direction)

$$\begin{aligned} F_y &= \dot{m}v \sin 60° \\ &= (105.8\ \text{lbf}) \sin 60° \\ F_y &= 91.8\ \text{lbf} \end{aligned}$$

4. Newton's third law

The force of the jet on the vane ($\mathbf{F}_{\text{jet}}$) is opposite in direction to the force required to hold the vane station (**F**). Therefore,

$$\mathbf{F}_{\text{jet}} = (53.0\ \text{lbf})\mathbf{i} + (91.8\ \text{lbf})\mathbf{j}$$

Review

1. *Discussion.* Notice that the problem goal was specified as a vector. Thus, the answer was given as a vector.
2. *Skill.* Notice how the common assumptions for a vane we applied in the "define the situation" portion.

Nozzles

Nozzles are flow devices used to accelerate a fluid stream by reducing the cross-sectional area of the flow (Fig. 6.14). Problems in this category involve analysis of the nozzle itself, not analysis of the free jet.

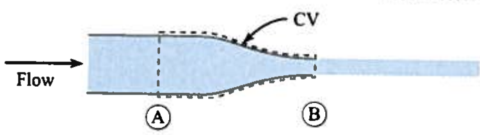


FIGURE 6.14
A fluid jet exiting a noz

To make solving of nozzle problems easier, we offer the following **Tips**.

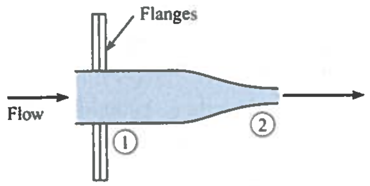
- **Tip 1.** Let each momentum flow equal $\dot{m}\mathbf{v}$. For the nozzle in Fig. 6.14, the momentum inflow is $\dot{m}\mathbf{v}_A$ and the outflow is $\dot{m}\mathbf{v}_B$.
- **Tip 2.** Include a pressure force where the nozzle connects to a pipe. For the nozzle in Fig. 6.14, include a pressure force of magnitude $p_A A_A$ on the force diagram. This pressure force, like all pressure forces, is compressive.
- **Tip 3.** To find p_A, apply the Bernoulli equation between A and B.
- **Tip 4.** To relate v_A and v_B, apply the continuity equation.
- **Tip 5.** When the CS cuts through a support structure (e.g., a pipe wall, a flange), represent the associated force on the force diagram. For the nozzle shown in Fig. 6.14, add a force F_{Ax} and F_{Ay} to the force diagram.

EXAMPLE 6.4

Momentum Equation Applied to a Nozzle

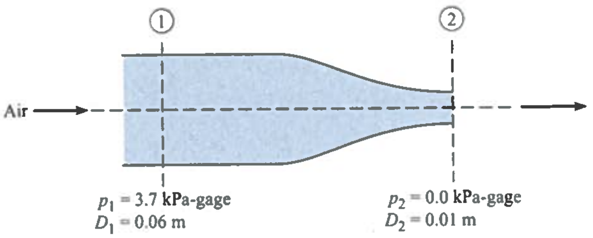
Problem Statement

The sketch shows air flowing through a nozzle. The inlet pressure is $p_1 = 105$ kPa abs, and the air exhausts into the atmosphere, where the pressure is 101.3 kPa abs. The nozzle has an inlet diameter of 60 mm and an exit diameter of 10 mm, and the nozzle is connected to the supply pipe by flanges. Find the force required to hold the nozzle stationary. Assume the air has a constant density of 1.22 kg/m³. Neglect the weight of the nozzle.



Define the Situation

Air flows through a nozzle



Properties. $\rho = 1.22$ kg/m³.

Assumptions

- Weight of nozzle is negligible.
- Steady flow, constant density flow, inviscid flow.

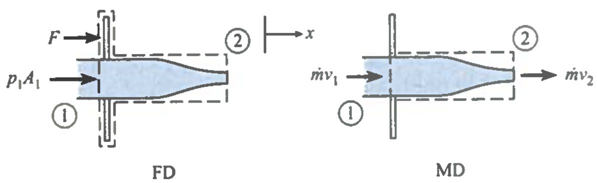
State the Goals

F(N) ⬅ Force required to hold nozzle stationary

Generate Ideas and Make a Plan

Select. Because force is a parameter and fluid particles are accelerating in the nozzle, select the momentum equation.

Sketch. Sketch a force and momentum diagram.



Write the momentum equation (x-direction)

$$F + p_1A_1 = \dot{m}(v_2 - v_1) \quad \text{(a)}$$

To solve for F, we need v_2 and v_1, which can be found using the Bernoulli equation. Thus, the plan is

1. Derive an equation for v_2 by applying the Bernoulli equation and the continuity equation.
2. Calculate v_2 and v_1.
3. Calculate F by applying Eq. (a).

Take Action (Execute the Plan)

1. *Bernoulli Equation* (apply between 1 and 2)

$$p_1 + \gamma z_1 + \frac{1}{2}\rho v_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho v_2^2$$

Term-by-term analysis

- $z_1 = z_2 = 0$
- $p_1 = 3.7$ kPa; $p_2 = 0.0$

The Bernoulli equation reduces to

$$p_1 + \rho v_1^2/2 = \rho v_2^2/2$$

Continuity Equation. Select a CV that cuts through sections 1 and 2. Neglect the mass accumulation terms. Continuity simplifies to

$$v_1A_1 = v_2A_2$$

$$v_1d_1^2 = v_2d_2^2$$

Substitute into the Bernoulli equation and solve for v_2:

$$v_2 = \sqrt{\frac{2p_1}{\rho(1 - (d_2/d_1)^4)}}$$

2. Calculate v_2 and v_1.

$$v_2 = \sqrt{\frac{2 \times 3.7 \times 1000 \text{ Pa}}{(1.22 \text{ kg/m}^3)(1 - (10/60)^4)}} = 77.9 \text{ m/s}$$

$$v_1 = v_2\left(\frac{d_2}{d_1}\right)^2$$

$$= 77.9 \text{ m/s} \times \left(\frac{1}{6}\right)^2 = 2.16 \text{ m/s}$$

3. Momentum equation

$$F + p_1A_1 = \dot{m}(v_2 - v_1)$$

$$F = \rho A_1 v_1(v_2 - v_1) - p_1A_1$$

$$= (1.22 \text{ kg/m}^3)\left(\frac{\pi}{4}\right)(0.06 \text{ m})^2(2.16 \text{ m/s})$$

$$\times (77.9 - 2.16)(\text{m/s})$$

$$-3.7 \times 1000 \text{ N/m}^2 \times \left(\frac{\pi}{4}\right)(0.06 \text{ m})^2$$

$$= 0.564 \text{ N} - 10.46 \text{ N} = -9.90 \text{ N}$$

Because F is negative, the direction is opposite to the direction assumed on the force diagram. Thus,

Force to hold nozzle = 9.90 N (← direction)

Review

1. *Knowledge*. The direction initially assumed for the force on a force diagram is arbitrary. If the answer for the force is negative, then the force acts in a direction opposite the chosen direction.
2. *Knowledge*. Pressures were changed to gage pressure in the "define the situation" operation because it is the pressures differences as compared to atmospheric pressure that caus net pressure forces.

Pipe Bends

A **pipe bend** is a structural component that is used to turn through an angle (Fig. 6.15). A pipe bend is often connected to straight runs of pipe by flanges. A flange is round disk with a hole in the center that slides over a pipe and is often welded in place. Flanges are bolted together to connect sections of pipe.

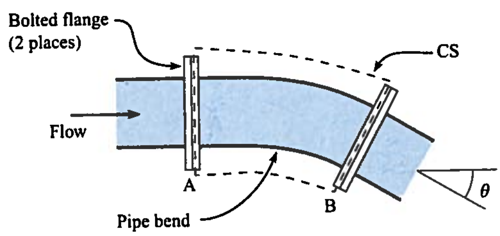


FIGURE 6.15
Pipe Bend

To make solving of nozzle problems easier, we offer the following **Tips**.

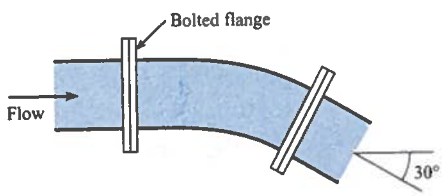
- **Tip 1.** Let each momentum flow equal $\dot{m}\mathbf{v}$. For the bend in Fig. 6.15, the momentum inflow is $\dot{m}\mathbf{v}_A$ and the outflow is $\dot{m}\mathbf{v}_B$.
- **Tip 2.** Include pressure forces where the CS cuts through a pipe. In Fig. 6.15, there is a pressure force at section A: $F_A = p_A A_A$ and at section B: $F_B = p_B A_B$. As always, both pressure forces are compressive.
- **Tip 3.** To relate p_A and p_B, it is most correct to apply the *energy equation* from Chapter 7 and include head loss. An alternative is to assume that pressure is constant or to assume inviscid flow and apply the Bernoulli equation.
- **Tip 4.** To relate v_A and v_B, apply the continuity equation.
- **Tip 5.** When the CS cuts through a support structure (pipe wall, flange), include the loads caused by the support on the force diagram.

EXAMPLE 6.5

Momentum Equation Applied to a Pipe Bend

Problem Statement

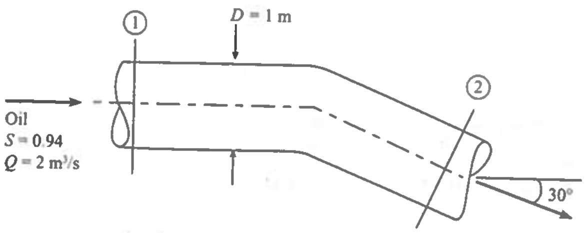
A 1-m-diameter pipe bend shown in the diagram is carrying crude oil ($S = 0.94$) with a steady flow rate of 2 m^3/s. The bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is 1.2 m^3, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the force requi to hold the bend in place.



Define the Situation

Crude oil flows through a pipe bend.

- Bend lies in a horizontal plane.
- $\mathcal{V}_{oil} = 1.2\ m^3$ = volume of oil in bend.
- $W_{bend} = 4000\ N$ = empty weight of bend.
- $p = 75$ kPa-gage = pressure along centerline.



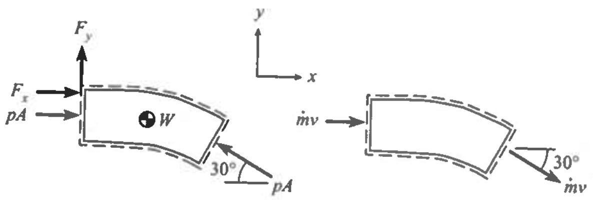
State the Goal

F(N) ⬅ Force to hold the bend stationary.

Generate Ideas and Make a Plan

Select. Because force is a parameter and fluid particles accelerate in the pipe bend, select the momentum equation.

Sketch. Select a CV that cuts through the support structure and through sections 1 and 2. Then, sketch the force and momentum diagrams.



Analysis. Using the diagrams as guides, write the momentum equation in each direction:

- x-direction

$$F_x + p_1A_1 - p_2A_2\cos 30° = \dot{m}v_2\cos 30° - \dot{m}v_1 \quad \text{(a)}$$

- y-direction

$$F_y - p_2A_2\sin 30° = -\dot{m}v_2\sin 30° \quad \text{(b)}$$

- z-direction

$$-F_z - W_{total} = 0 \quad \text{(c)}$$

Review these equations and notice that there is enough information to solve for the goals F_x, F_y, and F_z. Thus, create a plan

1. Calculate the momentum flux $\dot{m}v$.
2. Calculate the pressure force pA.
3. Solve Eq. (a) for F_x.
4. Solve Eq. (b) for F_y.
5. Solve Eq. (c) for F_z.

Take Action (Execute the Plan)

1. Momentum Flow

Example 6.6

- Apply the volume flow rate equation

$$v = Q/A = \frac{(2\ m^3/s)}{(\pi \times 0.5^2\ m^2)} = 2.55\ m/s$$

- Next, calculate the momentum flow

$$\dot{m}v = \rho Qv = (0.94 \times 1000\ kg/m^3)(2\ m^3/s)(2.55\ m/s)$$
$$= 4.79\ kN$$

2. Pressure Force

$$pA = (75\ kN/m^2)(\pi \times 0.5^2\ m^2) = 58.9\ kN$$

3. Momentum Equation (x-direction)

$$F_x + p_1A_1 - p_2A_2\cos 30° = \dot{m}v_2\cos 30° - \dot{m}v_1$$
$$F_x = -pA(1 - \cos 30°) - \dot{m}v(1 - \cos 30°)$$
$$= -(pA + \dot{m}v)(1 - \cos 30°)$$
$$= -(58.9 + 4.79)(kN)(1 - \cos 30°)$$
$$= -8.53\ kN$$

4. Momentum Equation (y-direction)

$$F_y + p_2A_2\sin 30° = -\dot{m}v_2\sin 30°$$
$$F_y = -(pA + \dot{m}v)\sin 30°$$
$$= -(58.9 + 4.79)(kN)(\sin 30°) = -31.8\ kN$$

Reaction force in z-direction. (The bend weight includes the oil plus the empty pipe).

$$-F_z - W_{total} = 0$$

$$W = \gamma\mathcal{V} + 4\ kN$$
$$= (0.94 \times 9.81\ kN/m^3)(1.2\ m^3) + 4\ kN = 15.1\ kN$$

Force to hold the bend

$$\boxed{\mathbf{F} = (-8.53\ kN)\mathbf{i} + (-31.8\ kN)\mathbf{j} + (15.1\ kN)\mathbf{k}}$$

Variable Velocity Distribution

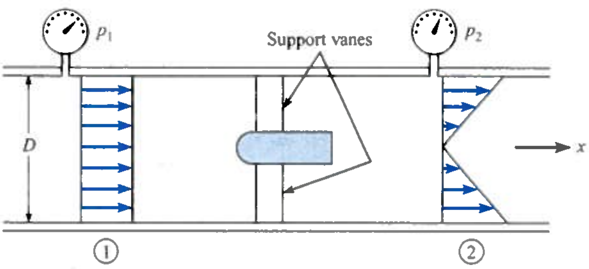
This subsection shows how to solve a problem when the momentum flow is evaluated by integration. This case is illustrated by Example 6.6.

EXAMPLE 6.6

Momentum Equation Applied with a Variable Velocity Distribution

Problem Statement

The drag force of a bullet-shaped device may be measured using a wind tunnel. The tunnel is round with a diameter of 1 m, the pressure at section 1 is 1.5 kPa gage, the pressure at section 2 is 1.0 kPa gage, and air density is 1.0 kg/m^3. At the inlet, the velocity is uniform with a magnitude of 30 m/s. At the exit, the velocity varies linearly as shown in the sketch. Determine the drag force on the device and support vanes. Neglect viscous resistance at the wall, and assume pressure is uniform across sections 1 and 2.



Define the Situation

Data is supplied for wind tunnel test (see above).
Assume. Steady flow.
Air: $\rho = 1.0$ kg/m^3.

State the Goal

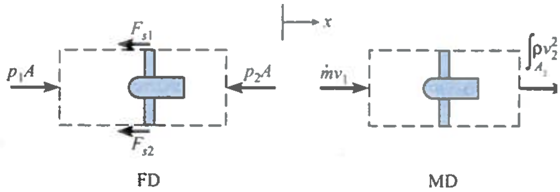
Find: Drag force (in newtons) on model

Make a Plan

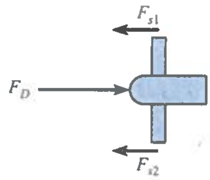
1. Select a control volume that encloses the model.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. The downstream velocity profile is not uniformly distributed. Apply the integral form of the momentum equation, Eq. (6.7).
5. Evaluate the sum of forces.
6. Determine velocity profile at section 2 by application of continuity equation.
7. Evaluate the momentum terms.
8. Calculate drag force on model.

Take Action (Execute the Plan)

1. The control volume selected is shown. The control volun is stationary.



2. The forces consist of the pressure forces and the force on the model support struts cut by the control surface. The drag force on the model is equal and opposite to the forc on the support struts: $F_D = F_{s1} + F_{s2}$.



3. There is inlet and outlet momentum flux.
4. Integral form of momentum equation in x-direction

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v_x d\forall + \int_{cs} \rho v_x (\mathbf{V}\cdot d\mathbf{A})$$

On cross section 1, $\mathbf{V}\cdot d\mathbf{A} = -v_x dA$, and on cross section 2, $\mathbf{V}\cdot d\mathbf{A} = v_x dA$, so

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v_x d\forall - \int_1 \rho v_x^2 dA + \int_2 \rho v_x^2 dA$$

5. Evaluation of force terms.

$$\sum F_x = p_1 A - p_2 A - (F_{s1} + F_{s2})$$
$$= p_1 A - p_2 A - F_D$$

6. Velocity profile at section 2.

Velocity is linear in radius, so choose $v_2 = v_1 K(r/r_o)$, whe r_o is the tunnel radius and K is a proportionality factor to be determined.

$$Q_1 = Q_2$$

$$A_1 v_1 = \int_{A_2} v_2(r)\, dA = \int_0^{r_o} v_1 K(r/r_o) 2\pi r\, dr$$

$$\pi r_o^2 v_1 = 2\pi v_1 K \frac{1}{3} r_o^2$$

$$K = \frac{3}{2}$$

7. Evaluation of momentum terms

- Accumulation term for steady flow is $\frac{d}{dt}\int_{cv} \rho v_x dV = 0$
- Momentum at cross section 1 with $v_x = v_1$ is

$$\int_1 \rho v_x^2 dA = \rho v_1^2 A = \dot{m}v_1$$

- Momentum at cross section 2 is

$$\int_2 \rho v_x^2 dA = \int_0^{r_o} \rho\left[\frac{3}{2}v_1\left(\frac{r}{r_o}\right)\right]^2 2\pi r dr = \frac{9}{8}\dot{m}v_1$$

8. Drag force

$$p_1A - p_2A - F_D = \dot{m}v_1\left(\frac{9}{8} - 1\right)$$

$$F_D = (p_1 - p_2)A - \frac{1}{8}\rho A v_1^2$$

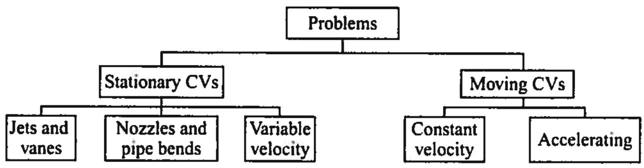
$$= (\pi \times 0.5^2\ \text{m}^2)(1.5 - 1.0)(10^3)\text{N/m}^2$$

$$-\frac{1}{8}(1\ \text{kg/m}^3)(\pi \times 0.5^2\ \text{m}^2)(30\ \text{m/s})^2$$

$$F_D = \boxed{304\ \text{N}}$$

6.5 Examples of the Linear Momentum Equation (Moving Objects)

This section describes how to apply the linear momentum equation to problems that invol moving objects such as carts in motion and rockets. When an object is moving, one le the CV move with the object. As shown below (repeated from Fig. 6.11), problems th involve moving CVs classify into two categories: objects moving with constant velocity an objects that are accelerating. Both categories involve selection of a reference frame, whic is the next topic.

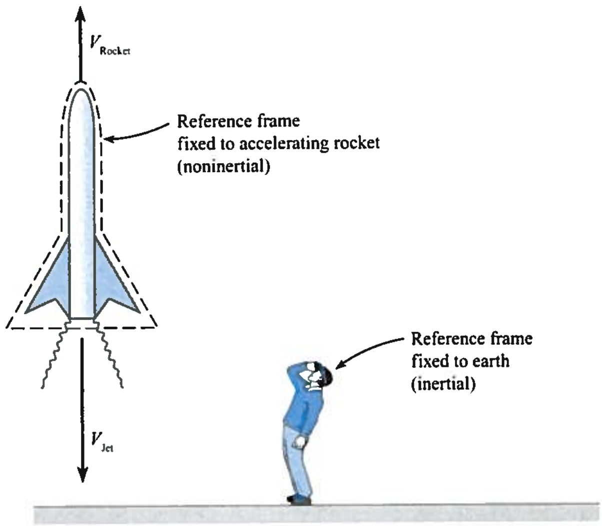


Reference Frame

When an object is moving, it is necessary to specify a reference frame. A **reference fram** is a three-dimensional framework from which an observer takes measurements. For ex ample, Fig. 6.16 shows a rocket in flight. For this situation, one possible reference frame i fixed to the earth. Another possible reference frame is fixed to the rocket. Observers i these two frames of reference would report different values of the rocket velocity V_{Rock} and the velocity of the fluid jet V_{jet}. The ground-based reference frame is inertial. An **in ertial reference frame** is any reference frame that is stationary or moving with constan velocity with respect to the earth. Thus, an inertial reference frame is a nonacceleratin reference frame. Alternatively, a **noninertial reference frame** is any reference frame tha is accelerating.

Regarding the linear momentum equation as presented in this text, this equation is onl valid for an inertial frame. Thus, when objects are moving, the engineer should specify a inertial reference frame.

FIGURE 6.16



Analyzing a Moving Body (Constant Velocity)

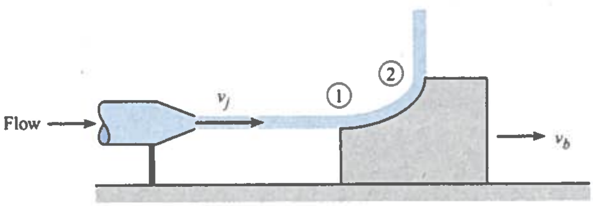
When an object is moving with constant velocity, then the reference frame can be placed on the moving object or fixed to the earth. However, most problems are simpler if the frame is fixed to the moving object. Example 6.7 shows how to solve a problem involving an object moving with constant velocity.

EXAMPLE 6.7

Momentum Equation Applied to a Moving CV

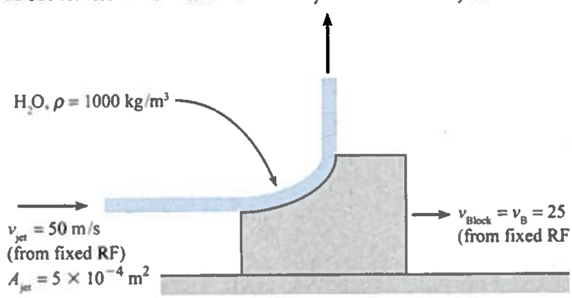
Problem Statement

A stationary nozzle produces a water jet with a speed of 50 m/s and a cross-sectional area of 5 cm^2. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed of 25 m/s on a surface with friction. The density of the water is 1000 kg/m^3. Find the frictional force F acting on the block.



Define the Situation

A block slides at constant velocity due to a fluid jet.



State the Goal

F_f(N) ⬅ The frictional force on the block

Solution Method I (Moving RF)

When a body is moving at constant velocity, the easiest way solve the problem is to put the RF on the moving body. This solution method is shown first.

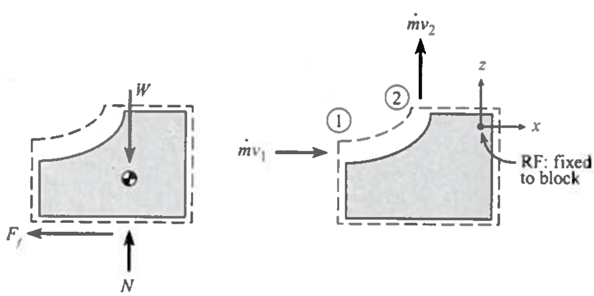
Generate Ideas and Make a Plan

Select the linear momentum equation because force is the goal and fluid particles accelerate as they interact with the block.

Select a moving CV that surrounds the block because this CV involves known parameters (i.e., the two fluid jets) and the goal (frictional force).

Because the CV is moving at a constant velocity, select a reference frame (RF) that is fixed to the moving block. This RF makes analysis of the problem simpler.

Sketch the force and momentum diagrams and the RF.



To apply the momentum equation, use the force and momentum diagrams to visualize the vectors. The momentum equation in the x-direction is

$$-F_f = -\dot{m}v_1 \quad \text{(a)}$$

In Eq. (a), the mass flow rate describes the rate at which mass is crossing the control surface. Because the CS is moving away from the fluid jet, the mass flow rate term becomes

$$\dot{m} = \rho AV = \rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}}) \quad \text{(b)}$$

In Eq. (a), the velocity v_1 is the velocity as measured from the selected reference frame. Thus,

$$v_1 = v_{\text{jet}} - v_{\text{block}} \quad \text{(c)}$$

Combining Eqs. (a), (b), and (c) gives

$$F_f = \dot{m}v_1 = \rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}})^2 \quad \text{(d)}$$

Because, all variables on the right side of Eq. (d) are known, we can find the problem goal. The plan is simple: plug numbers into Eq. (d).

Take Action (Execute the Plan)

$$F_f = \rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}})^2$$

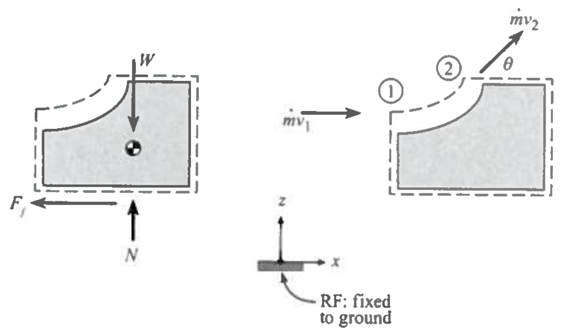
$$F_f = (1000\ \text{kg/m}^2)(5 \times 10^{-4}\ \text{m}^2)(50 - 25)^2(\text{m/s})^2$$

$$\boxed{F_f = 312\ \text{N}}$$

Solution Method II (Fixed RF)

Another way to solve this problem is to use a fixed reference frame. To implement this approach, sketch the force diagram, the momentum diagram, and the selected RF.

Notice that $\dot{m}v_2$ shows a vertical and horizontal component. This is because an observer in the selected RF would see these velocity components.



From the diagrams, one can write the momentum equation in the x-direction:

$$-F_f = \dot{m}v_2\cos\theta - \dot{m}v_1$$
$$F_f = \dot{m}(v_1 - v_2\cos\theta) \quad \text{(e)}$$

In the momentum equation, the mass flow rate is measured relative to the control surface. Thus, $\dot{m}$ is independent of the RF, and one can use Eq. (b), which is repeated below:

$$\dot{m} = \rho AV = \rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}}) \quad \text{(f)}$$

In Eq. (e), the velocity v_1 is the velocity as measured from the selected reference frame. Thus,

$$v_1 = v_{\text{jet}} \quad \text{(g)}$$

To analyze v_2, relate velocities by using a relative-velocity equation from a Dynamics Text:

$$\mathbf{v}_{\text{jet}} = \mathbf{v}_{\text{block}} + \mathbf{v}_{\text{jet/block}} \quad \text{(h)}$$

where

- $v_2 = v_{\text{jet}}$ is the velocity of the jet at section 2 as measured from the fixed RF.
- v_{block} is the velocity of the moving block as measured from the fixed RF.
- $v_{\text{jet/block}}$ is the velocity of the jet at section as measured from a RF fixed to the moving block.

Substitute numbers into Eq. (h) to give

$$\mathbf{v}_2 = (25\ \text{m/s})\mathbf{i} + (25\ \text{m/s})\mathbf{j} \quad \text{(i)}$$

Thus

$$v_2\cos\theta = v_{2x} = 25\ \text{m/s} = v_{\text{block}} \quad \text{(j)}$$

Substitute Eqs. (f), (g), and (j) into Eq. (e).

$$\begin{aligned} F_f &= \{\dot{m}\}(v_1 - v_2\cos\theta) \\ &= \{\rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}})\}(v_{\text{jet}} - v_{\text{block}}) \\ &= \rho A_{\text{jet}}(v_{\text{jet}} - v_{\text{block}})^2 \end{aligned} \qquad \text{(k)}$$

Eq. (k) is identical to Eq. (d). Thus, *Solution Method I* is equivalent to *Solution Method II*.

Review the Solution and the Process

1. *Knowledge.* When an object moves with constant velocity, select an RF fixed to the moving object because this is much easier than selecting an RF fixed to the earth.
2. *Knowledge.* Specifying the control volume and the reference frame are independent decisions.

Analyzing a Moving Body (Accelerating)

This section presents an example of an accelerating object, namely the analysis of a rocket (Fig. 6.17). To begin, sketch a control volume around the rocket. Note that the reference frame cannot be fixed to the rocket because the rocket is accelerating.

Assume the rocket is moving vertically upward with a speed v_r measured with respect to the ground. Exhaust gases leave the engine nozzle (area A_e) at a speed V_e relative to the rocket nozzle with a gage pressure of p_e. The goal is to obtain the equation of motion of the rocket.

The control volume is drawn around and accelerates with the rocket. The force and momentum diagrams are shown in Fig. 6.18. There is a drag force of D and a weight of W acting downward. There is a pressure force of p_eA_e on the nozzle exit plane because the pressure in a supersonic jet is greater than ambient pressure. The summation of the forces in the z-direction is

$$\sum F_z = p_eA_e - W - D \qquad (6.15)$$

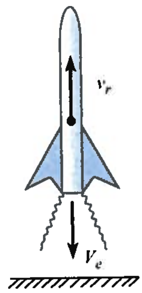


FIGURE 6.17
Vertical launch of rock[et]

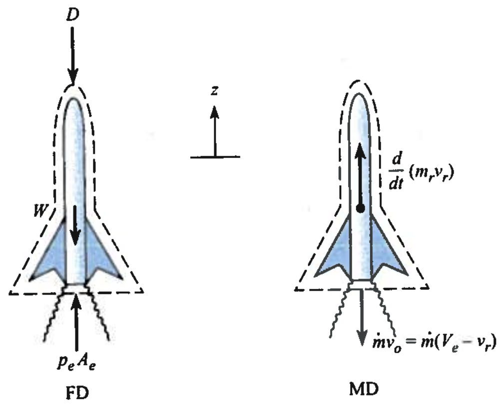


FIGURE 6.18
Force and momentum diagrams for rocket.

There is only one momentum flux out of the rocket nozzle, $\dot{m}v_o$. The speed v_o must be referenced to an inertial reference frame, which in this case is chosen as the ground. The speed of the exit gases with respect to the ground is

$$v_o = (V_e - v_r) \qquad (6.16)$$

because the rocket is moving upward with speed v_r with respect to the ground, and the exit gases are moving downward at speed V_e with respect to the rocket.

The momentum equation, in the z-direction is

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho d\forall + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz}$$

The velocity inside the control volume is the speed of the rocket, v_r, so the accumulation te
becomes

$$\frac{d}{dt}\left(\int_{cv} v_z \rho d\forall\right) = \frac{d}{dt}\left[v_r \int_{cv} \rho d\forall\right] = \frac{d}{dt}(m_r v_r)$$

Substituting the sum of the forces and momentum terms into the momentum equation give

$$p_e A_e - W - D = \frac{d}{dt}(m_r v_r) - \dot{m}(V_e - v_r) \quad (6.1$$

Next, apply the product rule to the accumulation term. This gives

$$p_e A_e - W - D = m_r \frac{dv_r}{dt} + v_r\left(\frac{dm_r}{dt} + \dot{m}\right) - \dot{m} V_e \quad (6.1$$

The continuity equation can now be used to eliminate the second term on the right. Applyi
the continuity equation to the control surface around the rocket leads to

$$\frac{d}{dt}\int_{cv} \rho d\forall + \sum \dot{m}_o - \sum \dot{m}_i = 0$$

$$\frac{dm_r}{dt} + \dot{m} = 0 \quad (6.1$$

Substituting Eq. (6.19) into Eq. (6.18) yields

$$\dot{m} V_e + p_e A_e - W - D = m_r \frac{dv_r}{dt} \quad (6.2$$

The sum of the momentum outflow and the pressure force at the nozzle exit is identified as th
thrust of the rocket

$$T = \dot{m} V_e + p_e A_e = \rho_e A_e V_e^2 + p_e A_e$$

so Eq. (6.20) simplifies to

$$m_r \frac{dv_r}{dt} = T - D - W \quad (6.21$$

which is the equation used to predict and analyze rocket performance.

Integration of Eq. (6.21) leads to one of the fundamental equations for rocketry: th
burnout velocity or the velocity achieved when all the fuel is burned. Neglecting the drag an
weight, the equation of motion reduces to

$$T = m_r \frac{dv_r}{dt} \quad (6.22$$

The instantaneous mass of the rocket is given by $m_r = m_i - \dot{m}t$, where m_i is the initial rocket mass and t is the time from ignition. Substituting the expression for mass into Eq. (6.22) and integrating with the initial condition $v_r(0) = 0$ results in

$$v_{bo} = \frac{T}{\dot{m}}\ln\frac{m_i}{m_f} \tag{6.23}$$

where v_{bo} is the burnout velocity and m_f is the final (or payload) mass. The ratio $T/\dot{m}$ is known as the specific impulse, I_{sp}, and has units of velocity.

6.6 The Angular Momentum Equation

This section presents the *angular momentum equation,* which is also called the *moment-of-momentum equation.* The angular momentum equation is very useful for situations that involve torques. Examples include analyses of rotating machinery such as pumps, turbines, fans, and blowers.

Derivation of the Equation

Newton's second law of motion can be used to derive an equation for the rotational motion of a system of particles:

$$\sum \mathbf{M} = \frac{d(\mathbf{H}_{sys})}{dt} \tag{6.24}$$

where $\mathbf{M}$ is a moment and $\mathbf{H}_{sys}$ is the total angular momentum of all mass forming the system.

To convert Eq. (6.24) to an Eulerian equation, apply the Reynolds transport theorem, Eq. (5.23). The extensive property B_{sys} becomes the angular momentum of the system: $B_{sys} = \mathbf{H}_{sys}$. The intensive property b becomes the angular momentum per unit mass. The angular momentum of an element is $\mathbf{r} \times m\mathbf{v}$, and so $b = \mathbf{r} \times \mathbf{v}$. Substituting for B_{sys} and b in Eq. (5.23) gives

$$\frac{d(\mathbf{H}_{sys})}{dt} = \frac{d}{dt}\int_{cv}(\mathbf{r}\times\mathbf{v})\rho\, d\forall + \int_{cs}(\mathbf{r}\times\mathbf{v})\rho\mathbf{V}\cdot d\mathbf{A} \tag{6.25}$$

Combining Eqs. (6.24) and (6.25) gives the integral form of the *moment-of-momentum equation*:

$$\sum \mathbf{M} = \frac{d}{dt}\int_{cv}(\mathbf{r}\times\mathbf{v})\rho\, d\forall + \int_{cs}(\mathbf{r}\times\mathbf{v})\rho\mathbf{V}\cdot d\mathbf{A} \tag{6.26}$$

where $\mathbf{r}$ is a position vector that extends from the moment center, $\mathbf{V}$ is flow velocity relative to the control surface, and $\mathbf{v}$ is flow velocity relative to the inertial reference frame selected.

If the mass crosses the control surface through a series of inlet and outlet ports with uniformly distributed properties across each port, the moment-of-momentum equation becomes

$$\sum \mathbf{M} = \frac{d}{dt}\int_{cv}(\mathbf{r}\times\mathbf{v})\rho\, d\forall + \sum_{cs}\mathbf{r}_o\times(\dot{m}_o\mathbf{v}_o) - \sum_{cs}\mathbf{r}_i\times(\dot{m}_i\mathbf{v}_i) \tag{6.27}$$

The moment-of-momentum equation has the following physical interpretation:

$$\begin{pmatrix}\text{sum of}\\ \text{moments}\end{pmatrix} = \begin{pmatrix}\text{angular momentum}\\ \text{accumulation}\end{pmatrix} + \begin{pmatrix}\text{angular momentum}\\ \text{outflow}\end{pmatrix} - \begin{pmatrix}\text{angular momentum}\\ \text{inflow}\end{pmatrix}$$

Application

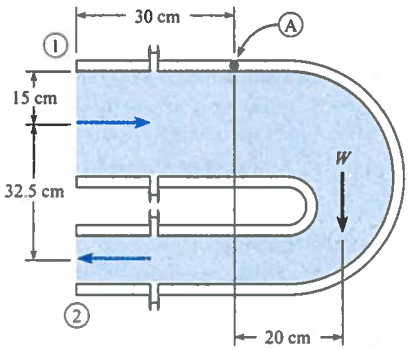
The process for applying the angular momentum equation is similar to the process for applyi the linear momentum equation. To illustrate this process, Example 6.8 shows how to apply t angular momentum equation to a pipe bend.

EXAMPLE 6.8

Applying the Angular Momentum Equation to Calculate the Moment on a Reducing Bend

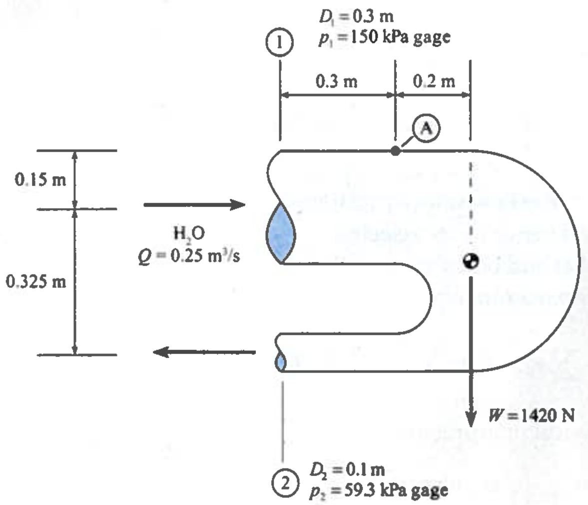
Problem Statement

The reducing bend shown in the figure is supported on a horizontal axis through point A. Water (20°C) flows through the bend at 0.25 m^3/s. The inlet pressure at cross section 1 is 150 kPa gage, and the outlet pressure at section 2 is 59.3 kPa gage. A weight of 1420 N acts 20 cm to the right of point A. Find the moment the support system must resist. The diameters of the inlet and outlet pipes are 30 cm and 10 cm, respectively.



Define the Situation

Water flows through a pipe bend.
Assume steady flow.
Water (Table A.5, 20°C, p = 1 atm): $\rho = 998\ kg/m^3$.



State the Goal

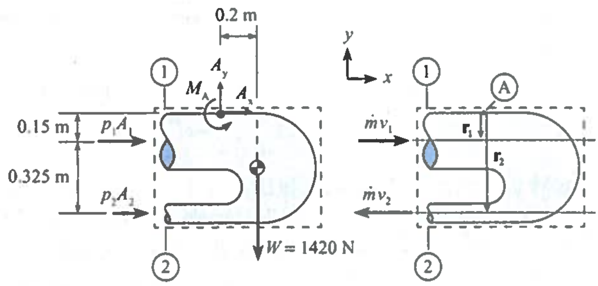
M_A(N) ⬅ Moment acting on the support structure

Generate Ideas and Make a Plan

Select the moment-of-momentum equation (Eq. 6.27) because (a) torque is a parameter and (b) fluid particles are accelerating as they pass through the pipe bend.

Select a control volume surrounding the reducing bend. The reason is that this CV cuts through point A (where we want to know the moment) and also cuts through sections 1 and 2 where information is known.

Sketch the force and momentum diagrams. Add dimensions to the sketches so that it is easier to evaluate cross products.



Select point "A" to sum moments about. Because the flow is steady, the accumulation of momentum term is zero. Also, there is one inflow of angular momentum and one outflow. Thus, the angular momentum equation (Eq. 6.27) simplifies to:

$$\sum \mathbf{M}_A = \{\mathbf{r}_2 \times (\dot{m}\mathbf{v}_2)\} - \{\mathbf{r}_1 \times (\dot{m}\mathbf{v}_1)\} \quad \text{(a)}$$

Sum moments in the z-direction

$$\sum M_{A,z} = (p_1A_1)(0.15\text{ m}) + (p_2A_2)(0.475\text{ m}) + M_A - W(0.2\text{ m}) \quad \text{(b)}$$

Next, analyze the momentum terms in Eq. (a).

$$\{\mathbf{r}_2 \times (\dot{m}\mathbf{v}_2)\} - \{\mathbf{r}_1 \times (\dot{m}\mathbf{v}_1)\}_z = \{-r_2\dot{m}v_2\} - \{r_1\dot{m}v_1\} \quad \text{(c)}$$

Substitute Eqs. (b) and (c) into Eq. (a)

$$(p_1A_1)(0.15\text{ m}) + (p_2A_2)(0.475\text{ m}) + M_A - W(0.2\text{ m}) = \{-r_2\dot{m}v_2\} - \{r_1\dot{m}v_1\} \quad \text{(d)}$$

All the terms in Eq. (d) are known, so M_A can be calculated. Thus, the plan is

1. Calculate torques to due to pressure: $r_1p_1A_1$ and $r_2p_2A_2$.
2. Calculate momentum flow terms: $r_2\dot{m}v_2 + r_1\dot{m}v_1$.
3. Calculate M_A.

Take Action (Execute the Plan)

1. Torques due to pressure

$$r_1 p_1 A_1 = (0.15\text{ m})(150 \times 1000\text{ N/m}^2)(\pi \times 0.3^2/4\text{ m}^2)$$
$$= 1590\text{ N}\cdot\text{m}$$
$$r_2 p_2 A_2 = (0.475\text{ m})(59.3 \times 1000\text{ N/m}^2)(\pi \times 0.15^2/4\text{ m}^2)$$
$$= 498\text{ N}\cdot\text{m}$$

2. Momentum flow terms

$$\dot{m} = \rho Q = (998\text{ kg/m}^3)(0.25\text{ m}^3/\text{s})$$
$$= 250\text{ kg/s}$$
$$v_1 = \frac{Q}{A_1} = \frac{0.25\text{ m}^3/\text{s}}{\pi \times 0.15^2\text{ m}^2} = 3.54\text{ m/s}$$
$$v_2 = \frac{Q}{A_2} = \frac{0.25\text{ m}^3/\text{s}}{\pi \times 0.075^2\text{ m}^2} = 14.15\text{ m/s}$$
$$\dot{m}(r_2 v_2 + r_1 v_1) = (250\text{ kg/s}) \times (0.475 \times 14.15 + 0.15 \times 3.54)(\text{m}^2/\text{s})$$
$$= 1813\text{ N}\cdot\text{m}$$

3. Moment exerted by support

$$M_A = -0.15 p_1 A_1 - 0.475 p_2 A_2 + 0.2W - \dot{m}(r_2 v_2 + r_1 v_1)$$
$$= -(1590\text{ N}\cdot\text{m}) - (498\text{ N}\cdot\text{m}) + (0.2\text{ m} \times 1420\text{ N}) - (1813\text{ N}\cdot\text{m})$$
$$M_A = -3.62\text{ kN}\cdot\text{m}$$

Thus, a moment of 3.62 kN · m acting in the clockwise, direction is needed to hold the bend stationary.

By Newton's third law, the moment acting on the support structure is $M_A = 3.62$ kN · m (counterclockwise).

Review the Solution and the Process

Tip. Use the "right-hand-rule" to find the correct direction of moments.

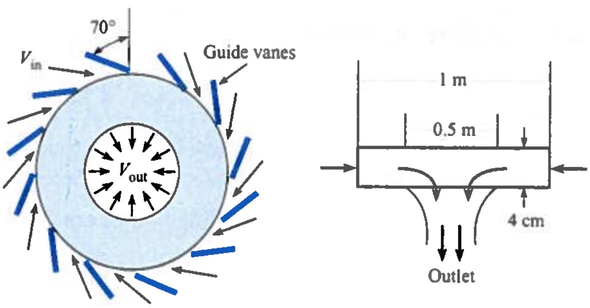
Example 6.9 illustrates how to apply the angular momentum equation to predict the power delivered by a turbine. This analysis can be applied to both power-producing machines (turbines) and power-absorbing machines (pumps and compressors). Additional information is presented in Chapter 14.

EXAMPLE 6.9

Applying the Angular Momentum Equation to Predict the Power Delivered by a Francis Turbine

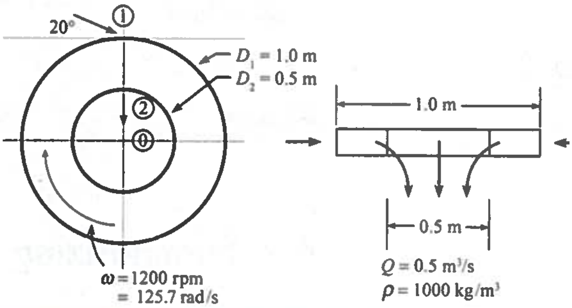
Problem Statement

A Francis turbine is shown in the diagram. Water is directed by guide vanes into the rotating wheel (runner) of the turbine. The guide vanes have a 70° angle from the radial direction. The water exits with only a radial component of velocity with respect to the environment. The outer diameter of the wheel is 1 m, and the inner diameter is 0.5 m. The distance across the runner is 4 cm. The discharge is 0.5 m³/s, and the rotational rate of the wheel is 1200 rpm. The water density is 1000 kg/m³. Find the power (kW) produced by the turbine.



Define the Situation

A Francis turbine generates power.



State the Goal

P(W) ⬅ Power generated by the turbine

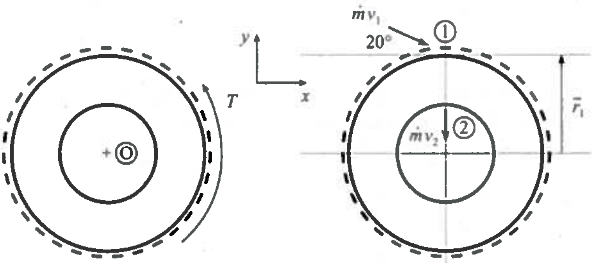
Generate Ideas and Make a Plan

Because power is the goal, select the *power equation*.

$$P = T\omega \qquad \text{(}$$

where T is torque acting on the turbine, and ω is turbine angular speed. In Eq. (a), torque is unknown, so it becomes the new goal. Torque can be found using the angular momentum equation.

Sketch. To apply the angular momentum equation, select a control volume surrounding the turbine. Then, sketch a force and momentum diagram



In the force diagram, the torque T is the external torque from the generator. Because this torque opposes angular acceleration, its direction is counterclockwise. The flow is idealized by using one inlet momentum flow at section 1 and one outlet momentum flow at section 2.

Select point "O" to sum moments about. Because the flow is steady, the accumulation of momentum is zero. Thus, the angular momentum equation (Eq. 6.26) simplifies to:

$$\sum \mathbf{M}_A = \{\mathbf{r}_2 \times (\dot{m}\mathbf{v}_2)\} - \{\mathbf{r}_1 \times (\dot{m}\mathbf{v}_1)\} \quad \text{(b)}$$

Apply Eq. (b) in the z-direction. Also, recognize that the flow at section 2 has no angular momentum. That is, $\{\mathbf{r}_2 \times (\dot{m}\mathbf{v}_2)\} = 0$. Thus, Eq. (b) simplifies to

$$T = 0 - \{-r_1 \dot{m} v_1 \cos 20°\}$$

which can be written as:

$$T = r_1 \dot{m} v_1 \cos 20° \quad \text{(c)}$$

In Eq. (c), the velocity v_1 can be calculated using the flow rate equation. Because velocity is not perpendicular to area, use the dot product.

$$Q_1 = \mathbf{V}_1 \cdot \mathbf{A}_1$$
$$Q = v_1 A_1 \sin 20°$$

which can be rewritten as

$$v_1 = \frac{Q}{A_1 \sin 20°} \quad \text{(d)}$$

Now, the number of equations equals the number of unknowns. Thus, the plan is to

1. Calculate inlet velocity v_1 using Eq. (d).
2. Calculate mass flow rate using $\dot{m} = \rho Q$.
3. Calculate torque using Eq. (c).
4. Calculate power using Eq. (a).

Take Action (Execute the Plan)

1. Volume flow rate equation:

$$v_1 = \frac{Q}{A_1 \sin 20°} = \frac{(0.5\ \text{m}^3/\text{s})}{\pi(1.0\ \text{m})(0.04\ \text{m})\sin 20°} = 11.63\ \text{m/s}$$

2. Mass flow rate equation:

$$\dot{m} = \rho Q = (1000\ \text{kg/m}^3)(0.5\ \text{m}^3/\text{s}) = 500\ \text{kg/s}$$

3. Angular momentum equation:

$$T = r_1 \dot{m} v_1 \cos 20°$$
$$= (0.5\ \text{m})(500\ \text{kg/s})(11.63\ \text{m/s})\cos 20°$$
$$= 2732\ \text{N} \cdot \text{m}$$

4. Power equation:

$$P = T\omega = (2732\ \text{N} \cdot \text{m})(125.7\ \text{rad/s})$$

$$\boxed{P = 343\ \text{kW}}$$

6.7 Summarizing Key Knowledge

Newton's Second Law of Motion

- A *force* is a push or pull of one body on another. A push/pull is an interaction that can cause a body to accelerate. A force always requires the interaction of two bodies.
- Forces can be classified into two categories:
  - *Body forces.* Forces in this category do not require that the interacting bodies be touchin Common body forces include weight, the magnetic force, and the electrostatic force.
  - *Surface forces.* Forces in this category require that the two interacting bodies are touching. Most forces are surface forces.
- Newton's second law $\sum \mathbf{F} = m\mathbf{a}$ applies to a fluid particle; other forms of this law are derived from this equation.

- Newton's second law asserts that forces are related to accelerations:
  - Thus, if $\sum \mathbf{F} > \mathbf{0}$, the particle must accelerate.
  - Thus, if $\mathbf{a} > \mathbf{0}$, the sum of forces must be nonzero.

Solving Vector Equations

- A vector equation is one whose terms are vectors.
- A vector equation can be written as one or more equivalent scalar equations.
- The Visual Solution Method (VSM) is an approach for solving a vector equation that makes problem solving easier. The process for the VSM is
  - **Step 1:** Identify the vector equation in its general form.
  - **Step 2:** Sketch a diagram that shows the vectors on the left side of the equation. Sketch an equal sign. Sketch a diagram that shows the vectors on the right side of the equation.
  - **Step 3:** From the diagrams, apply the general equation, write the final results, and simplify the results to create the reduced equation(s).

The Linear Momentum Equation

- The linear momentum equation is Newton's second law in a form that is useful for solving problems in fluid mechanics
- To derive the momentum equation
  - Begin with Newton's second law for a single particle.
  - Derive Newton's second law for a system of particles.
  - Apply the Reynolds transport theorem to give the final result.
- Physical Interpretation

$$\begin{pmatrix}\text{sum of}\\\text{forces}\end{pmatrix} = \begin{pmatrix}\text{momentum}\\\text{accumulation}\end{pmatrix} + \begin{pmatrix}\text{momentum}\\\text{outflow}\end{pmatrix} - \begin{pmatrix}\text{momentum}\\\text{inflow}\end{pmatrix}$$

- The *momentum accumulation* term gives the rate at which the momentum inside the control volume is changing with time.
- The *momentum flow* terms give the rate at which momentum is being transported across the control surfaces.

The Angular Momentum Equation

- The angular momentum equation is the rotational analog to the linear momentum equation.
  - This equation is useful for problems involving torques (i.e., moments)
  - This equation is commonly applied to rotating machinery such as pumps, fans, and turbines.
- The physics of the angular momentum equation are

$$\begin{pmatrix}\text{sum of}\\\text{moments}\end{pmatrix} = \begin{pmatrix}\text{angular momentum}\\\text{accumulation}\end{pmatrix} + \begin{pmatrix}\text{angular momentum}\\\text{outflow}\end{pmatrix} - \begin{pmatrix}\text{angular momentum}\\\text{inflow}\end{pmatrix}$$

- To apply the angular momentum equation, use the same process as that used for the linear momentum equation.

REFERENCES

1. Hibbeler, R.C. *Dynamics.* Englewood Cliffs, NJ: Prentice Hall, 1995.

PROBLEMS

Problem available in *WileyPLUS* at instructor's discretion.

Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Newton's Second Law of Motion (§6.1)

6.1 Identify the surface and body forces acting on a glider in flight. Also, sketch a free body diagram and explain how Newton's laws of motion apply.

6.2 Newton's second law can be stated that the force is equal to the rate of change of momentum, $F = d(mv)/dt$. Taking the derivative by parts yields $F = m(dv)/(dt) + v(dm)/(dt)$. This does not correspond to $F = ma$. What is the source of the discrepancy?

The Linear Momentum Equation: Theory (§6.2)

6.3 Which are the following are correct with respect to the derivation of the momentum equation? (Select all that apply.)

- **a.** Reynold's transport theorem is applied to Fick's law.
- **b.** The extensive property is momentum.
- **c.** The intensive property is mass.
- **d.** The velocity is assumed to be uniformly distributed across each inlet and outlet.
- **e.** The net momentum flow is the "ins" minus the "outs."
- **f.** The net force is the sum of forces acting on the matter inside the CV

The Linear Momentum Equation: Application (§6.3)

6.4 When making a force diagram (FD) and its partner momentum diagram (MD) to set up the equations for a momentum equation problem (see Fig. 6.10 on p. 217 in §6.3), which of the following elements should be in the FD, and which should be in the MD? (Classify all below as either FD or MD.)

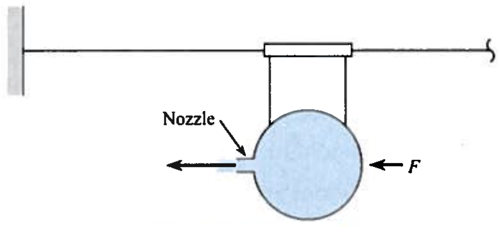
- **a.** Each mass stream with product $\dot{m}_o \mathbf{v}_o$ or product $\dot{m}_i \mathbf{v}_i$ crossing a control surface boundary.
- **b.** Reaction forces required to hold walls, vanes, or pipes in place.
- **c.** Weight of a solid body that contains or contacts the fluid.
- **d.** Weight of the fluid.
- **e.** Pressure force caused by a fluid flowing across a control surface boundary.

Applying the Momentum Equation to Fluid Jets (§6.4)

6.5 Give five examples of jets and how they are used in practice.

6.6 A "balloon rocket" is a balloon suspended from a taut wire by a hollow tube (drinking straw) and string. The nozzle is formed of a 0.8-cm-diameter tube, and an air jet exits the nozzl with a speed of 45 m/s and a density of 1.2 kg/m^3. Find the forc needed to hold the balloon stationary. Neglect friction.

6.7 The balloon rocket is held in place by a force *F*. The pressure inside the balloon is 8 in-H_2O, the nozzle diameter is 1.0 cm, and the air density is 1.2 kg/m^3. Find the exit velocity *v* and the force *F*. Neglect friction and assume the air flow is inviscid and irrotational.

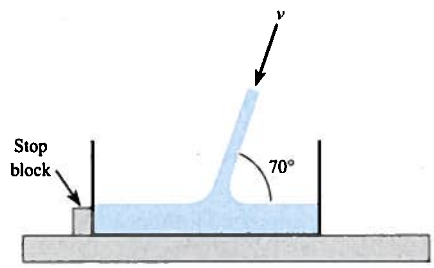


PROBLEMS 6.6, 6.7

6.8 For Example 6.2 in §6.4, the situation diagram shows concrete being "shot" at an angle into a cart that is tethered by a cable, and sitting on a scale. Determine whether the following two statements are "true" or "false."

- **a.** Mass is being accumulated in the cart.
- **b.** Momentum is being accumulated in the cart.

6.9 A water jet of diameter 30 mm and speed $v = 25$ m/s filling a tank. The tank has a mass of 25 kg and contains 25 liter of water at the instant shown. The water temperature is 15°C. Find the force acting on the bottom of the tank and the force acting on the stop block. Neglect friction.



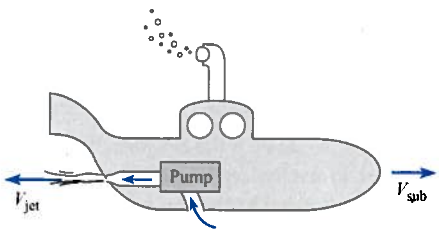
PROBLEMS 6.9, 6.10

6.10 A water jet of diameter 2 inches and speed $v = 60$ ft/ is filling a tank. The tank has a mass of 25 lbm and contains 6 gallons of water at the instant shown. The water temperature is 70°F. Find the minimum coefficient of friction such that the force acting on the stop block is zero.

6.11 A design contest features a submarine that will travel at a steady speed of $V_{sub} = 1$ m/s in 15°C water. The sub is powered by a water jet. This jet is created by drawing water from an inlet of diameter 25 mm, passing this water through a pump and then accelerating the water through a nozzle of diameter 5 mm to a speed of V_{jet}. The hydrodynamic drag force (F_D) can be calculated using

$$F_D = C_D\left(\frac{\rho V_{sub}^2}{2}\right)A_p$$

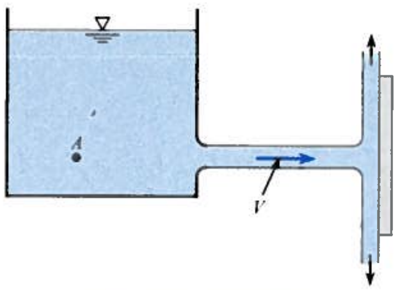
where the coefficient of drag is $C_D = 0.3$ and the projected area is $A_p = 0.28\ \text{m}^2$. Specify an acceptable value of V_{jet}.



PROBLEM 6.11

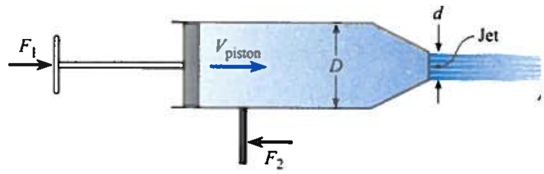
6.12 A horizontal water jet at 70°F impinges on a vertical-perpendicular plate. The discharge is 2 cfs. If the external force required to hold the plate in place is 200 lbf, what is the velocity of the water?

6.13 PLUS A horizontal water jet at 70°F issues from a circular orifice in a large tank. The jet strikes a vertical plate that is normal to the axis of the jet. A force of 600 lbf is needed to hold the plate in place against the action of the jet. If the pressure in the tank is 25 psig at point *A*, what is the diameter of the jet just downstream of the orifice?



PROBLEMS 6.12, 6.13

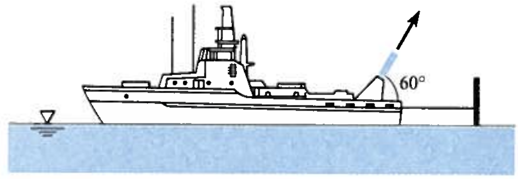
6.14 PLUS An engineer, who is designing a water toy, is making preliminary calculations. A user of the product will apply a force F_1 that moves a piston ($D = 80$ mm) at a speed of $V_{piston} = 300$ mm/s. Water at 20°C jets out of a converging nozzle of diameter $d = 15$ mm. To hold the toy stationary, the user applies a force F_2 to the handle. Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum principle. Then calculate F_1 and F_2. Neglect friction between piston and the walls.



PROBLEM 6.14

6.15 A firehose on a boat is producing a 4-in.-diameter water jet with a speed of $V = 60$ mph. The boat is held stationary by a cable attached to a pier, and the water temperature is 50°F. Calculate the tension in the cable.

6.16 PLUS A boat is held stationary by a cable attached to a pier. A firehose directs a spray of 5°C water at a speed of $V = 50$ m/s. If the allowable load on the cable is 5 kN, calculate the mass flow rate of the water jet. What is the corresponding diameter of the water jet?



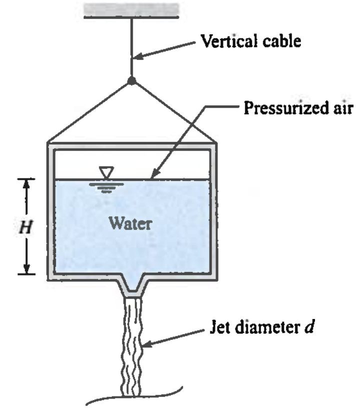
PROBLEMS 6.15, 6.16

6.17 GO A group of friends regularly enjoys white-water rafting, and they bring piston water guns to shoot water from one raft to another. One summer they notice that when on placid slack water (no current), after just a few volleys at each other they are drifting apart. They wonder whether the jet being ejected out of a piston gun has enough momentum to force the shooter and raft backward. To answer this question,

a. Sketch a CV, an FD, and an MD for this system.

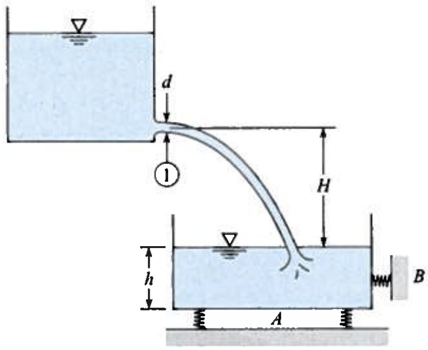
b. Calculate the momentum flux (N) generated by ejecting water with a flow rate of 1 gal/s from a cross section of 1.5 in.

6.18 GO A tank of water (15°C) with a total weight of 200 N (water plus the container) is suspended by a vertical cable. Pressurized air drives a water jet ($d = 12$ mm) out the bottom of the tank such that the tension in the vertical cable is 10 N. If $H = 425$ mm, find the required air pressure in units of atmospheres (gage). Assume the flow of water is irrotational.



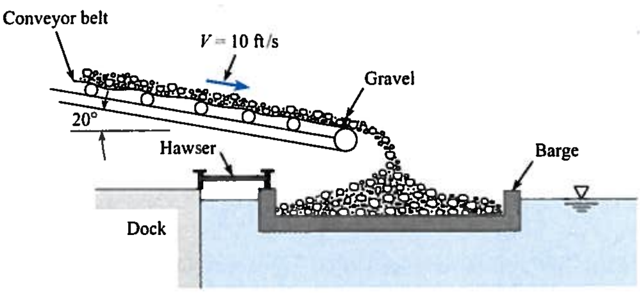
PROBLEM 6.18

6.19 WILEY PLUS A jet of water (60°F) is discharging at a constant rate of 2.0 cfs from the upper tank. If the jet diameter at section 1 is 4 in., what forces will be measured by scales A and B? Assume the empty tank weighs 300 lbf, the cross-sectional area of the tank is 4 ft², $h = 1$ ft, and $H = 9$ ft.



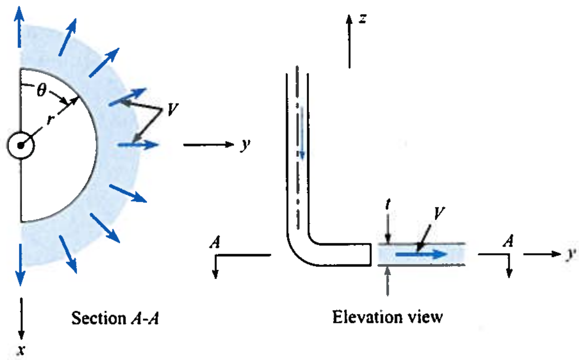
PROBLEM 6.19

6.20 A conveyor belt discharges gravel into a barge as shown at a rate of 50 yd³/min. If the gravel weighs 120 lbf/ft³, what is the tension in the hawser that secures the barge to the dock?



PROBLEM 6.20

6.21 The semicircular nozzle sprays a sheet of liquid through 180° of arc as shown. The velocity is V at the efflux section where the sheet thickness is t. Derive a formula for the external force I (in the y-direction) required to hold the nozzle system in place. This force should be a function of ρ, V, r, and t.

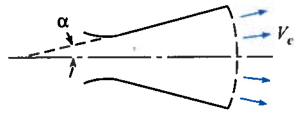


PROBLEM 6.21

6.22 The expansion section of a rocket nozzle is often conical i shape, and because the flow diverges, the thrust derived from tl nozzle is less than it would be if the exit velocity were everywh parallel to the nozzle axis. By considering the flow through the spherical section suspended by the cone and assuming that the exit pressure is equal to the atmospheric pressure, show that th thrust is given by

$$T = \dot{m}V_e\frac{(1 + \cos\alpha)}{2}$$

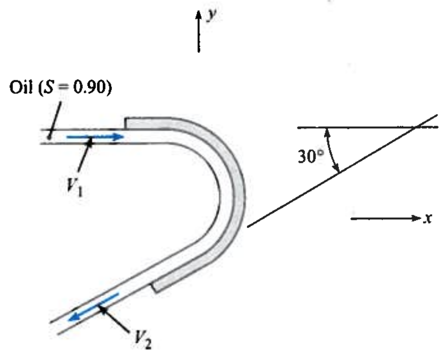
where $\dot{m}$ is the mass flow through the nozzle, V_e is the exit velocity, and α is the nozzle half-angle.



PROBLEM 6.22

Applying the Momentum Equation to Vanes (§6.4)

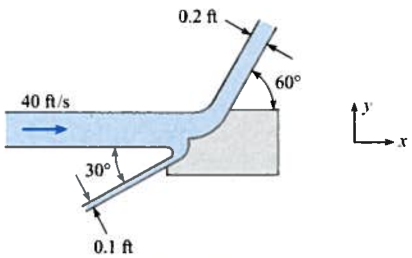
6.23 WILEY PLUS Determine the external reactions in the x- and y-directi needed to hold this fixed vane, which turns the oil jet ($S = 0.9$) in a horizontal plane. Here V_1 is 22 m/s, $V_2 = 21$ m/s, and $Q = 0.15$ m³.



PROBLEMS 6.23, 6.24

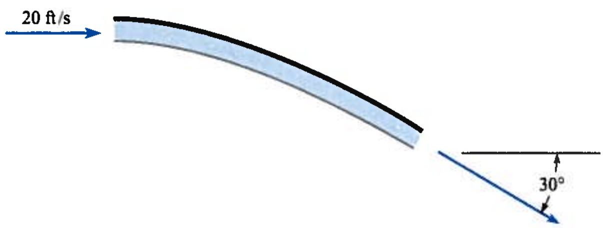
6.24 Solve Prob. 6.23 for $V_1 = 70$ ft/s, $V_2 = 65$ ft/s, and $Q = 1.5$ cfs.

6.25 WILEY PLUS This planar water jet (60°F) is deflected by a fixed vane. What are the x- and y-components of force per unit width needed to hold the vane stationary? Neglect gravity.



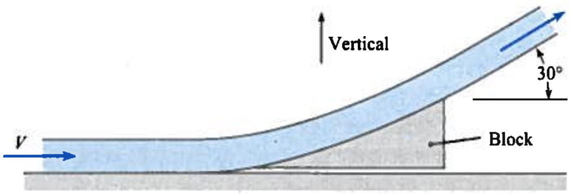
PROBLEM 6.25

6.26 WILEY PLUS A water jet with a speed of 30 ft/s and a mass flow rate of 35 lbm/s is turned 30° by a fixed vane. Find the force of the water jet on the vane. Neglect gravity.



PROBLEM 6.26

6.27 WILEY GO Water ($\rho = 1000$ kg/m^3) strikes a block as shown and is deflected 30°. The flow rate of the water is 1.5 kg/s, and the inlet velocity is $V = 10$ m/s. The mass of the block is 1 kg. The coefficient of static friction between the block and the surface is 0.1 (friction force/normal force). If the force parallel to the surface exceeds the frictional force, the block will move. Determine the force on the block and whether the block will move. Neglect the weight of the water.



PROBLEMS 6.27, 6.28

6.28 For the situation described in Prob. 6.27, find the maximum inlet velocity (V) such that the block will not slip

6.29 WILEY PLUS Plate A is 50 cm in diameter and has a sharp-edg orifice at its center. A water jet (at 10°C) strikes the plate concentrically with a speed of 90 m/s. What external force is needed to hold the plate in place if the jet issuing from the orifice also has a speed of 90 m/s? The diameters of the jets a $D = 10$ cm and $d = 3.5$ cm.

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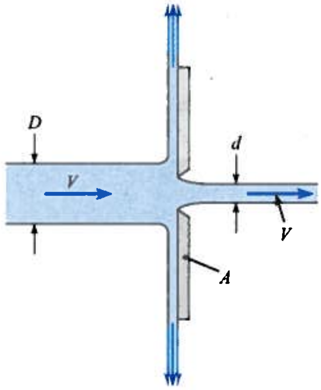
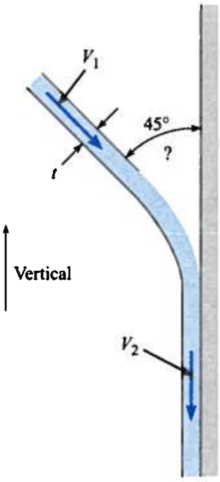


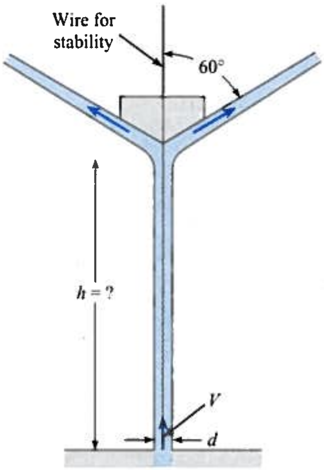
PROBLEM 6.29

6.30 A two-dimensional liquid jet impinges on a vertical wa Assuming that the incoming jet speed is the same as the exit jet speed ($V_1 = V_2$), derive an expression for the force per un width of jet exerted on the wall. What form do you think the upper liquid surface will take next to the wall? Sketch the sh you think it will take, and explain your reasons for drawing that way.



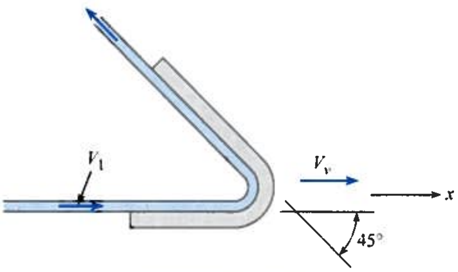
PROBLEM 6.30

6.31 WILEY PLUS A cone that is held stable by a wire is free to mov the vertical direction and has a jet of water (at 10°C) strikin it from below. The cone weighs 30 N. The initial speed of the jet as it comes from the orifice is 15 m/s, and the initial jet diameter is 2 cm. Find the height to which the cone will rise and remain stationary. *Note:* The wire is only for stability and should not enter into your calculations.



PROBLEM 6.31

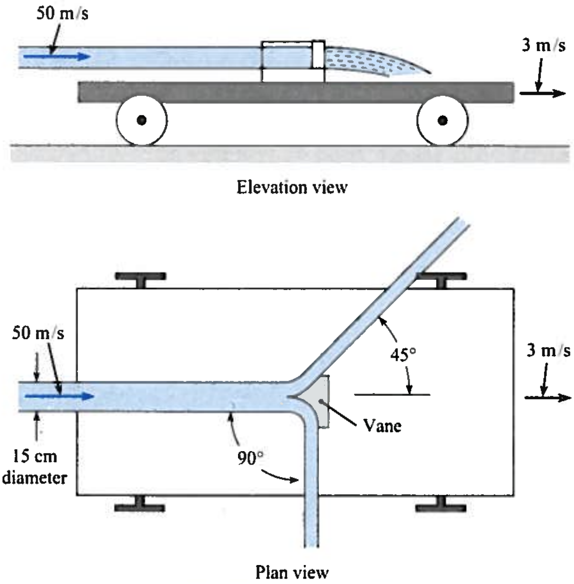
6.32 A horizontal jet of water (at 10°C) that is 6 cm in diameter and has a velocity of 20 m/s is deflected by the vane as shown. If the vane is moving at a rate of 7 m/s in the x-direction, what components of force are exerted on the vane by the water in the x- and y-directions? Assume negligible friction between the water and the vane.



PROBLEM 6.32

6.33 PLUS A vane on this moving cart deflects a 15-cm-diameter water ($\rho = 1000\ kg/m^3$) jet as shown. The initial speed of the water in the jet is 50 m/s, and the cart moves at a speed of 3 m/s. If the vane splits the jet so that half goes one way and half the other, what force is exerted on the vane by the water?

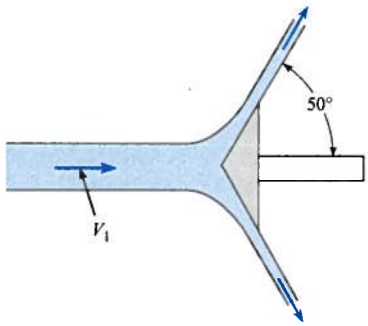
6.34 Refer to the cart of Prob. 6.33. If the cart speed is constant at 5 ft/s, and if the initial jet speed is 60 ft/s, and jet diameter = 0.15 ft, what is the rolling resistance of the cart? ($\rho = 62.4\ lbm/ft^3$)



PROBLEMS 6.33, 6.34

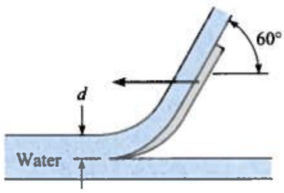
6.35 PLUS The water ($\rho = 1000\ kg/m^3$) in this jet has a speed of 60 m/s to the right and is deflected by a cone that is moving to the left with a speed of 5 m/s. The diameter of the jet is 10 cm. Determine the external horizontal force needed to move the cone. Assume negligible friction between the water and the vane.

6.36 This two-dimensional water (at 50°F) jet is deflected by th two-dimensional vane, which is moving to the right with a spee of 60 ft/s. The initial jet is 0.30 ft thick (vertical dimension), and its speed is 100 ft/s. What power per foot of the jet (normal to the page) is transmitted to the vane?



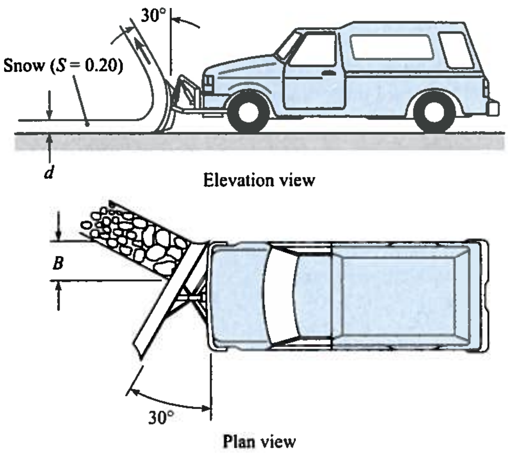
PROBLEMS 6.35, 6.36

6.37 PLUS Assume that the scoop shown, which is 20 cm wide, is used as a braking device for studying deceleration effects, such as those on space vehicles. If the scoop is attached to a 1000 kg sled that is initially traveling horizontally at the rate of 100 m/s, what will be the initial deceleration of the sled? The scoop dips into the water 8 cm ($d = 8$ cm). ($T = 10$°C.)



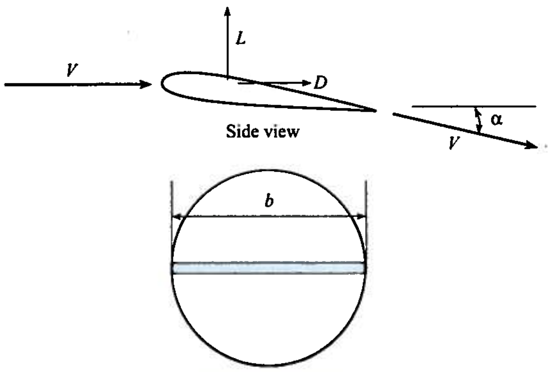
PROBLEM 6.37

6.38 This snowplow "cleans" a swath of snow that is 4 in. deep (d = 4 in.) and 2 ft wide (B = 2 ft). The snow leaves the blade in the direction indicated in the sketches. Neglecting friction between the snow and the blade, estimate the power required for just the snow removal if the speed of the snowplow is 40 ft/s.



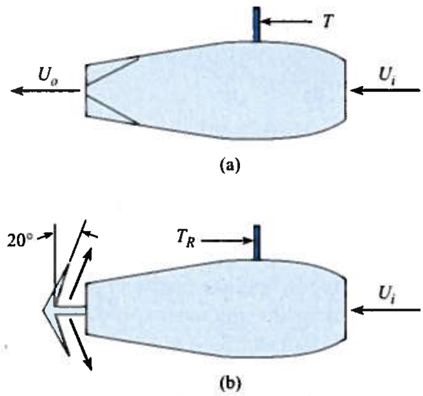
PROBLEM 6.38

6.39 GO A finite span airfoil can be regarded as a vane as shown in the figure. The cross section of air affected is equal to the circle with the diameter of the wing span, b. The wing deflects the air by an angle α and produces a force normal to the free-stream velocity, the lift L, and in the free-stream direction, the drag D. The airspeed is unchanged. Calculate the lift and drag for a 30 ft wing span in a 300 ft/s airstream at 14.7 psia and 60°F for flow deflection of 2°.



PROBLEM 6.39

6.40 The "clam shell" thrust reverser sketched in the figure is often used to decelerate aircraft on landing. The sketch shows normal operation (a) and when deployed (b). The vanes are oriented 20° with respect to the vertical. The mass flow throu the engine is 150 lbm/s, the inlet velocity is 300 ft/s, and the e velocity is 1400 ft/s. Assume that when the thrust reverser is deployed, the exit velocity of the exhaust is unchanged. Assu the engine is stationary. Calculate the thrust under normal operation (lbf) and when the thrust reverser is deployed.



PROBLEM 6.40

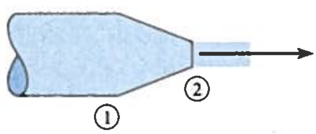
Applying the Momentum Equation to Nozzles (§6.4)

6.41 Firehoses are fitted with special nozzles. Use the Intern or contact your local fire department to find information on operational conditions and typical hose and nozzle sizes use

6.42 PLUS High-speed water jets are used for speciality cutti applications. The pressure in the chamber is approximately 60,000 psig. Using the Bernoulli equation, estimate the water speed exiting the nozzle exhausting to atmospheric pressure. Neglect compressibility effects and assume a water temperat of 60°F.

6.43 PLUS Water at 60°F flows through a nozzle that contrac from a diameter of 3 in. to 1 in. The pressure at section 1 is 2500 psfg, and atmospheric pressure prevails at the exit of the Calculate the speed of the flow at the nozzle exit and the forc required to hold the nozzle stationary. Neglect weight.

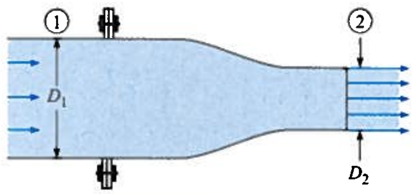
6.44 GO Water at 15°C flows through a nozzle that contrac from a diameter of 10 cm to 2 cm. The exit speed is v_2 = 25 r and atmospheric pressure prevails at the exit of the jet. Calcu the pressure at section 1 and the force required to hold the n stationary. Neglect weight.



PROBLEMS 6.43, 6.44

6.45 WILEY PLUS Water (at 50°F) flows through this nozzle at a rate of 20 cfs and discharges into the atmosphere. $D_1 = 26$ in., and $D_2 = 9$ in. Determine the force required at the flange to hold the nozzle in place. Assume irrotational flow. Neglect gravitational forces.

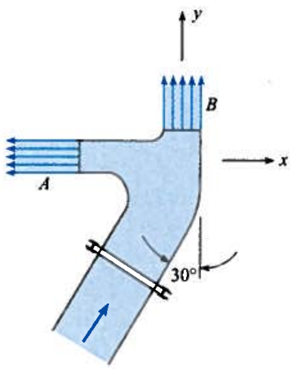
6.46 Solve Prob. 6.45 using the following values: $Q = 0.30\ \text{m}^3/\text{s}$, $D_1 = 30$ cm, and $D_2 = 10$ cm. ($\rho = 1000\ \text{kg/m}^3$.)



PROBLEMS 6.45, 6.46

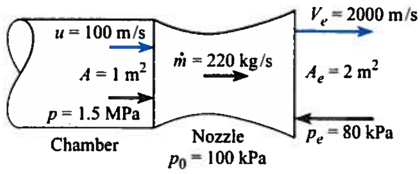
6.47 WILEY PLUS This "double" nozzle discharges water ($\rho = 62.4\ \text{lbm/ft}^3$) into the atmosphere at a rate of 16 cfs. If the nozzle is lying in a horizontal plane, what x-component of force acting through the flange bolts is required to hold the nozzle in place? *Note:* Assume irrotational flow, and assume the water speed in each jet to be the same. Jet A is 4 in. in diameter, jet B is 4.5 in. in diameter, and the pipe is 1 ft in diameter.

6.48 This "double" nozzle discharges water (at 10°C) into the atmosphere at a rate of $0.65\ \text{m}^3/\text{s}$. If the nozzle is lying in a horizontal plane, what x-component of force acting through the flange bolts is required to hold the nozzle in place? *Note:* Assume irrotational flow, and assume the water speed in each jet to be the same. Jet A is 8 cm in diameter, jet B is 9 cm in diameter, and the pipe is 30 cm in diameter.



PROBLEMS 6.47, 6.48

6.49 WILEY PLUS A rocket-nozzle designer is concerned about the force required to hold the nozzle section on the body of a rocket. The nozzle section is shaped as shown in the figure. The pressure and velocity at the entrance to the nozzle are 1.5 MPa and 100 m/s. The exit pressure and velocity are 80 kPa and 2000 m/s. The mass flow through the nozzle is 220 kg/s. The atmospheric pressure is 100 kPa. The rocket is not accelerating. Calculate the force on the nozzle-chamber connection. *Note:* The given pressures are absolute.

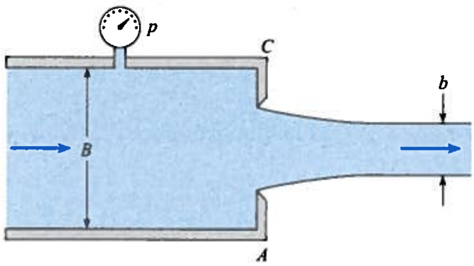


PROBLEM 6.49

6.50 A 15 cm nozzle is bolted with six bolts to the flange of a 30 cm pipe. If water ($\rho = 1000\ \text{kg/m}^3$) discharges from the nozzle into the atmosphere, calculate the tension load in each bolt when the pressure in the pipe is 200 kPa. Assume irrotational flow.

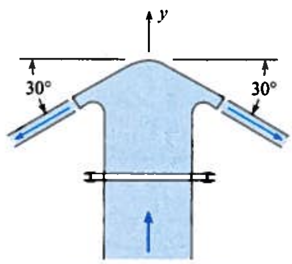
6.51 Water ($\rho = 62.4\ \text{lbm/ft}^3$) is discharged from the two-dimensional slot shown at the rate of 8 cfs per foot of slot. Determine the pressure p at the gage and the water force per fc on the vertical end plates A and C. The slot and jet dimensions and b are 8 in. and 4 in., respectively.

6.52 Water (at 10°C) is discharged from the two-dimensional slot shown at the rate of $0.40\ \text{m}^3/\text{s}$ per meter of slot. Determine the pressure p at the gage and the water force per meter on the vertical end plates A and C. The slot and jet dimensions B and are 20 cm and 7 cm, respectively.



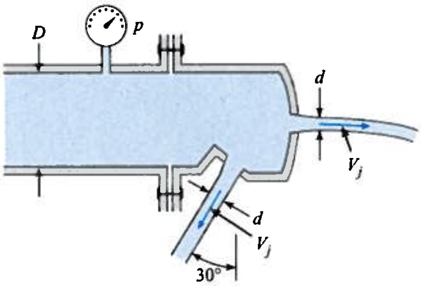
PROBLEMS 6.51, 6.52

6.53 This spray head discharges water ($\rho = 62.4\ \text{lbm/ft}^3$) at a rate of $4\ \text{ft}^3/\text{s}$. Assuming irrotational flow and an efflux speed o 65 ft/s in the free jet, determine what force acting through the bolts of the flange is needed to keep the spray head on the 6 in. pipe. Neglect gravitational forces.



PROBLEM 6.53

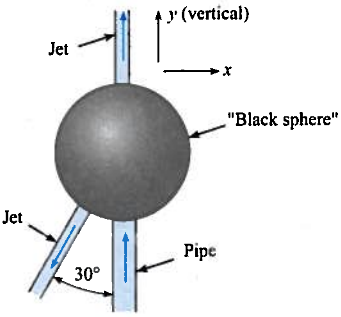
6.54 Two circular water ($\rho = 62.4$ lbm/ft^3) jets of 0.5 in. diameter ($d = 0.5$ in.) issue from this unusual nozzle. If the efflux speed is 80.2 ft/s, what force is required at the flange to hold the nozzle in place? The pressure in the 4 in. pipe ($D = 3.5$ in.) is 50 psig.



PROBLEM 6.54

6.55 Liquid (S = 1.2) enters the "black sphere" through a 2 in. pipe with velocity of 50 ft/s and a pressure of 60 psig. It leaves the sphere through two jets as shown. The velocity in the vertical jet is 100 ft/s, and its diameter is 1 in. The other jet's diameter is also 1 in. What force through the 2 in. pipe wall is required in the x- and y-directions to hold the sphere in place? Assume the sphere plus the liquid inside it weighs 200 lbf.

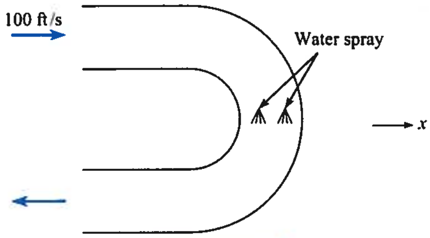
6.56 GO Liquid (S = 1.5) enters the "black sphere" through a 5 cm pipe with a velocity of 10 m/s and a pressure of 400 kPa. It leaves the sphere through two jets as shown. The velocity in the vertical jet is 30 m/s, and its diameter is 25 mm. The other jet's diameter is also 25 mm. What force through the 5 cm pipe wall is required in the x- and y-directions to hold the sphere in place? Assume the sphere plus the liquid inside it weighs 600 N.



PROBLEMS 6.55, 6.56

Applying the Momentum Equation to Pipe Bends (§6.4)

6.57 PLUS A hot gas stream enters a uniform-diameter return bend as shown. The entrance velocity is 100 ft/s, the gas density is 0.02 lbm/ft^3, and the mass flow rate is 1 lbm/s. Water is sprayed into the duct to cool the gas down. The gas exits with a density of 0.06 lbm/ft^3. The mass flow of water into the gas is negligible. The pressures at the entrance and exit are the same and equal to the atmospheric pressure. Find the force required to hold the bend.



PROBLEM 6.57

6.58 Assume that the gage pressure p is the same at sections and 2 in the horizontal bend shown in the figure. The fluid flowing in the bend has density ρ, discharge Q, and velocity The cross-sectional area of the pipe is A. Then the magnitud the force (neglecting gravity) required at the flanges to hold bend in place will be (a) pA, (b) $pA + \rho QV$, (c) $2pA + \rho QV$, (d) $2pA + 2\rho QV$.

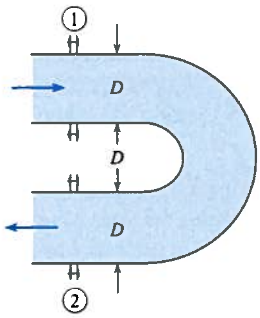
6.59 PLUS The pipe shown has a 180° vertical bend in it. Th diameter D is 1 ft, and the pressure at the center of the uppe pipe is 15 psig. If the flow in the bend is 20 cfs, what externa force will be required to hold the bend in place against the action of the water? The bend weighs 200 lbf, and the volum of the bend is 3 ft^3. Assume the Bernoulli equation applies. ($\rho = 62.4$ lbm/ft^3.)

6.60 The pipe shown has a 180° horizontal bend in it as sho and D is 20 cm. The discharge of water ($\rho = 1000$ kg/m^3) in pipe and bend is 0.35 m^3/s, and the pressure in the pipe and is 100 kPa gage. If the bend volume is 0.10 m^3, and the bend weighs 400 N, what force must be applied at the flanges to h the bend in place?

6.61 Set up the solution for Problem 6.60, and answer the following questions:

a. Do the two pressure forces from the inlet and exit a in the same direction, or in opposite directions?

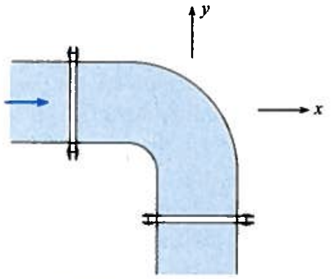
b. For the data given, which term has the larger magni (in N), the pressure force term, or the net momentu flux term?



PROBLEMS 6.58, 6.59, 6.60, 6.61

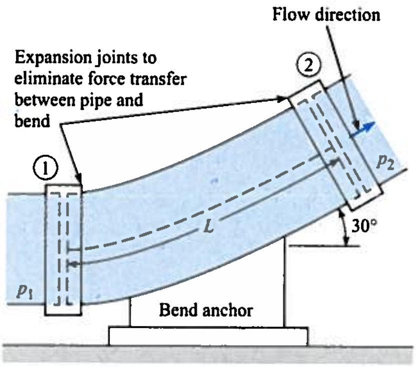
6.62 Water (at 50°F) flows in the 90° horizontal bend at a rate of 12 cfs and discharges into the atmosphere past the downstream flange. The pipe diameter is 1 ft. What force must be applied at the upstream flange to hold the bend in place? Assume that the volume of water downstream of the upstream flange is 4 ft³ and that the bend and pipe weigh 100 lbf. Assume the pressure at the inlet section is 4 psig.

6.63 The gage pressure throughout the horizontal 90° pipe bend is 300 kPa. If the pipe diameter is 1 m and the water (at 10°C) flow rate is 10 m³/s, what x-component of force must be applied to the bend to hold it in place against the water action?



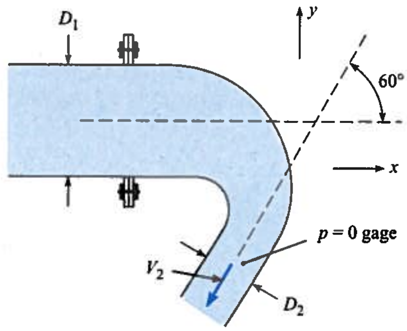
PROBLEMS 6.62, 6.63

6.64 This 30° vertical bend in a pipe with a 2 ft diameter carries water (ρ = 62.4 lbm/ft³) at a rate of 31.4 cfs. If the pressure p_1 is 10 psi at the lower end of the bend, where the elevation is 100 ft, and p_2 is 8.5 psi at the upper end, where the elevation is 103 ft, what will be the vertical component of force that must be exerted by the "anchor" on the bend to hold it in position? The bend itself weighs 300 lb, and the length L is 4 ft.



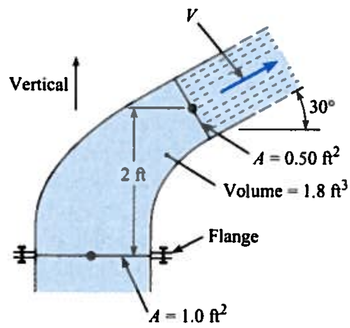
PROBLEM 6.64

6.65 This bend discharges water (ρ = 1000 kg/m³) into the atmosphere. Determine the force components at the flange required to hold the bend in place. The bend lies in a horizontal plane. Assume viscous forces are negligible. The interior volume of the bend is 0.25 m³, D_1 = 60 cm, D_2 = 30 cm, and V_2 = 10 m/s. The mass of the bend material is 250 kg.



PROBLEM 6.65

6.66 This nozzle bends the flow from vertically upward t 30° with the horizontal and discharges water (γ = 62.4 lbf/ft³) a speed of V = 130 ft/s. The volume within the nozzle itself is 1.8 ft³, and the weight of the nozzle is 100 lbf. For these conditioı what *vertical* force must be applied to the nozzle at the flange tı hold it in place?



PROBLEM 6.66

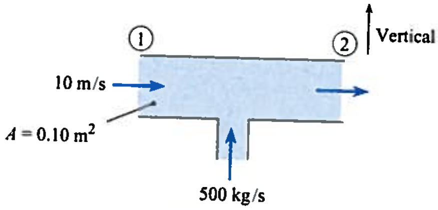
6.67 A pipe 1 ft in diameter bends through an angle of 135°. Tl velocity of flow of gasoline (S = 0.8) is 20 ft/s, and the pressure 10 psig in the bend. What external force is required to hold the bend against the action of the gasoline? Neglect the gravitation force.

6.68 A 6 in. horizontal pipe has a 180° bend in it. If the rate of flow of water (60°F) in the bend is 2 cfs and the pressure therein is 20 psig, what external force in the original direction c flow is required to hold the bend in place?

6.69 A pipe 15 cm in diameter bends through 135°. The velocit of flow of gasoline (S = 0.8) is 8 m/s, and the pressure is 100 kF gage throughout the bend. Neglecting gravitational force, determine the external force required to hold the bend against the action of the gasoline.

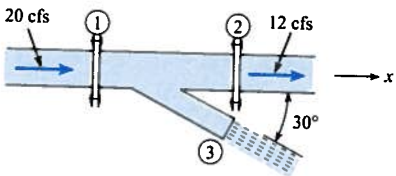
6.70 A horizontal reducing bend turns the flow of water (ρ = 1000 kg/m³) through 60°. The inlet area is 0.001 m², and th outlet area is 0.0001 m². The water from the outlet discharges int the atmosphere with a velocity of 50 m/s. What horizontal force (parallel to the initial flow direction) acting through the metal oı the bend at the inlet is required to hold the bend in place?

6.71 Water (at 10°C) flows in a duct as shown. The inlet water velocity is 10 m/s. The cross-sectional area of the duct is 0.1 m^2. Water is injected normal to the duct wall at the rate of 500 kg/s midway between stations 1 and 2. Neglect frictional forces on the duct wall. Calculate the pressure difference $(p_1 - p_2)$ between stations 1 and 2.



PROBLEM 6.71

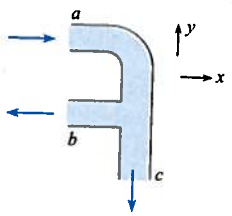
6.72 WILEY PLUS For this wye fitting, which lies in a horizontal plane, the cross-sectional areas at sections 1, 2, and 3 are 1 ft^2, 1 ft^2, and 0.25 ft^2, respectively. At these same respective sections the pressures are 1000 psfg, 900 psfg, and 0 psfg, and the water discharges are 20 cfs to the right, 12 cfs to the right, and exits to atmosphere at 8 cfs. What x-component of force would have to be applied to the wye to hold it in place?



PROBLEM 6.72

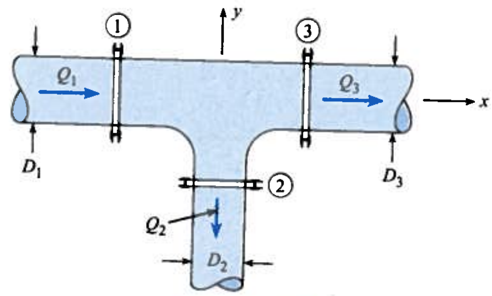
6.73 Water ($\rho = 62.4$ lbm/ft^3) flows through a horizontal bend and T section as shown. The mass flow rate entering at section a is 12 lbm/s, and those exiting at sections b and c are 6 lbm/s each. The pressure at section a is 5 psig. The pressure at the two outlets is atmospheric. The cross-sectional areas of the pipes are the same: 5 in^2. Find the x-component of force necessary to restrain the section.

6.74 Water ($\rho = 1000$ kg/m^3) flows through a horizontal bend and T section as shown. At section a the flow enters with a velocity of 6 m/s, and the pressure is 4.8 kPa. At both sections b and c the flow exits the device with a velocity of 3 m/s, and the pressure at these sections is atmospheric ($p = 0$). The cross-sectional areas at a, b, and c are all the same: 0.20 m^2. Find the x- and y-components of force necessary to restrain the section.



PROBLEMS 6.73, 6.74

6.75 For this horizontal T through which water ($\rho = 1000$ k is flowing, the following data are given: $Q_1 = 0.25$ m^3/s, $Q_2 = 0.10$ m^3/s, $p_1 = 100$ kPa, $p_2 = 70$ kPa, $p_3 = 80$ kPa, $D_1 = 15$ cm, $D_2 = 7$ cm, and $D_3 = 15$ cm. For these condit what external force in the x–y plane (through the bolts or supporting devices) is needed to hold the T in place?

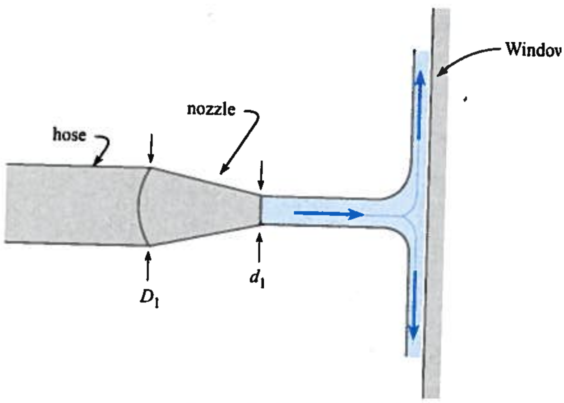


PROBLEM 6.75

Applying Momentum Equation: Other Situations (§6.4)

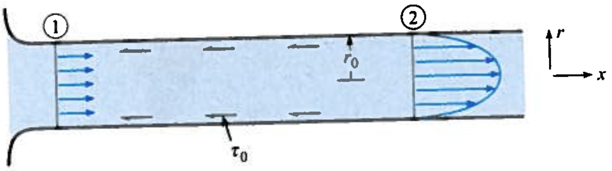
6.76 WILEY GO Firehoses can break windows. A 0.2-m diameter (firehose is attached to a nozzle with a 0.1 m diameter (d_2) ou The free jet from the nozzle is deflected by 90° when it hits t window as shown. Find the force the window must withstan due to the impact of the jet when water flows through the firehose at a rate of 0.15 m^3/s.

6.77 WILEY PLUS A fireman is soaking a home that is dangerously c to a burning building. To prevent water damage to the inside o neighboring home, he throttles down his flow rate so that it v not break windows. Assuming the typical window should be to withstand a force up to 25 lbf, what is the largest volumetri flow rate he should allow (gal/min.), given an 8-inch diamete (D_1) firehose discharging through a nozzle with 4-inch diame (d_2) outlet. The free jet from the nozzle is deflected by 90° wh it hits the window as shown.



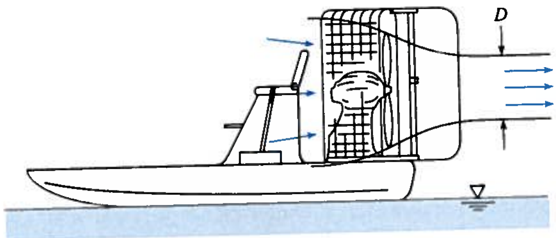
PROBLEMS 6.76, 6.77

6.78 For laminar flow in a pipe, wall shear stress (τ_0) causes the velocity distribution to change from uniform to parabolic as shown. At the fully developed section (section 2), the velocity is distributed as follows: $u = u_{max}[1 - (r/r_0)^2]$. Derive a formula for the force on the wall due to shear stress, F_τ, between 1 and 2 as a function of U (the mean velocity in the pipe), ρ, p_1, p_2, and D (the pipe diameter).



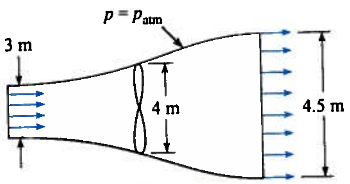
PROBLEM 6.78

6.79 WILEY PLUS The propeller on a swamp boat produces a slipstream 3 ft in diameter with a velocity relative to the boat of 100 ft/s. If the air temperature is 80°F, what is the propulsive force when the boat is not moving and also when its forward speed is 30 ft/s? *Hint:* Assume that the pressure, except in the immediate vicinity of the propeller, is atmospheric.



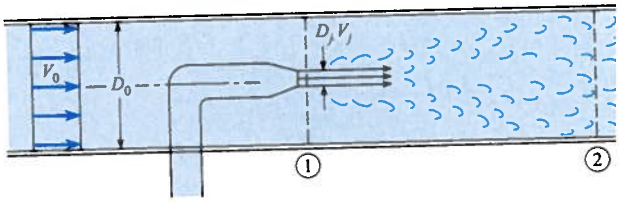
PROBLEM 6.79

6.80 WILEY PLUS A wind turbine is operating in a 12 m/s wind that has a density of 1.2 kg/m³. The diameter of the turbine silhouette is 4 m. The constant-pressure (atmospheric) streamline has a diameter of 3 m upstream of the windmill and 4.5 m downstream. Assume that the velocity distributions are uniform and the air is incompressible. Determine the thrust on the wind turbine.



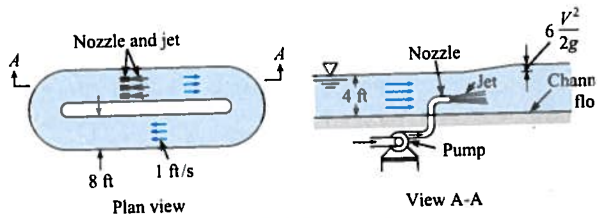
PROBLEM 6.80

6.81 WILEY PLUS The figure illustrates the principle of the jet pump. Derive a formula for $p_2 - p_1$ as a function of D_j, V_j, D_0, V_0, and ρ. Assume that the fluid from the jet and the fluid initially flowing in the pipe are the same, and assume that they are completely mixed at section 2, so that the velocity is uniform across that section. Also assume that the pressures are uniform across both sections 1 and 2. What is $p_2 - p_1$ if the fluid is water, $A_j/A_0 = 1/3$, $V_j = 15$ m/s, and $V_0 = 2$ m/s? Neglect shear stress.



PROBLEM 6.81

6.82 Jet-type pumps are sometimes used to circulate the flow in basins in which fish are being reared. The use of a jet-type pump eliminates the need for mechanical machinery that might be injurious to the fish. The accompanying figure shows the basic concept for this type of application. For this type of basin the jet would have to increase the water surface elevation by an amount equal to $6V^2/2g$, where V is the average velocity in the basin (1 ft/s as shown in this example). Propose a basic design for a je[t] system that would make such a recirculating system work for a channel 8 ft wide and 4 ft deep. That is, determine the speed, size, and number of jets.

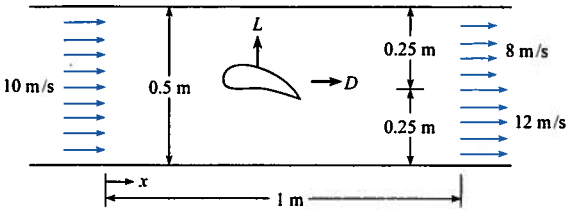


PROBLEM 6.82

6.83 An engineer is measuring the lift and drag on a wind turbine blade section mounted in a two-dimensional wind tunnel. The wind tunnel is 0.5 m high and 0.5 m deep (into the paper). The upstream wind velocity is uniform at 10 m/s, and t[he] downstream velocity is 12 m/s and 8 m/s as shown. The vertic[al] component of velocity is zero at both stations. The test section [is] 1 m long. The engineer measures the pressure distribution in t[he] tunnel along the upper and lower walls and finds

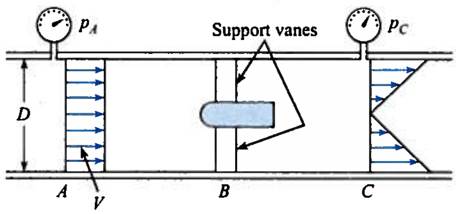
$$p_u = 100 - 10x - 20x(1 - x)(\text{Pa gage})$$
$$p_l = 100 - 10x + 20x(1 - x)(\text{Pa gage})$$

where x is the distance in meters measured from the beginnin[g] of the test section. The gas density is homogeneous througho[ut] and equal to 1.2 kg/m³. The lift and drag are the vectors indic[ated] on the figure. The forces acting on the fluid are in the opposit[e] direction to these vectors. Find the lift and drag forces acting on the wind turbine blade section.



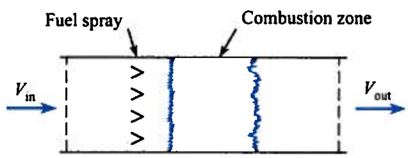
PROBLEM 6.83

6.84 WILEY PLUS A torpedolike device is tested in a wind tunnel with an air density of 0.0026 slugs/ft^3. The tunnel is 3 ft in diameter, the upstream pressure is 0.24 psig, and the downstream pressure is 0.10 psig. If the mean air velocity V is 120 ft/s, what are the mass rate of flow and the maximum velocity at the downstream section at C? If the pressure is assumed to be uniform across the sections at A and C, what is the drag of the device and support vanes? Assume viscous resistance at the walls is negligible.



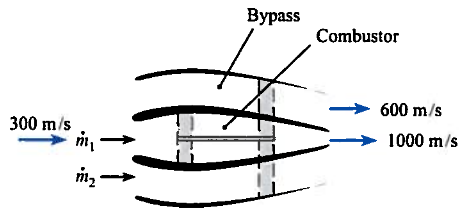
PROBLEM 6.84

6.85 A ramjet operates by taking in air at the inlet, providing fuel for combustion, and exhausting the hot air through the exit. The mass flow at the inlet and outlet of the ramjet is 60 kg/s (the mass flow rate of fuel is negligible). The inlet velocity is 225 m/s. The density of the gases at the exit is 0.25 kg/m^3, and the exit area is 0.5 m^2. Calculate the thrust delivered by the ramjet. The ramjet is not accelerating, and the flow within the ramjet is steady.



PROBLEM 6.85

6.86 WILEY PLUS A modern turbofan engine in a commercial jet takes in air, part of which passes through the compressors, combustion chambers, and turbine, and the rest of which bypasses the compressor and is accelerated by the fans. The mass flow rate of bypass air to the mass flow rate through the compressor-combustor-turbine path is called the "bypass ratio." The total flow rate of air entering a turbofan is 300 kg/s with a velocity of 300 m/s. The engine has a bypass ratio of 2.5. The bypass air exits at 600 m/s, whereas the air through the compressor–combustor–turbine path exits at 1000 m/s. What is thrust of the turbofan engine? Clearly show your control volum and application of momentum equation.



PROBLEM 6.86

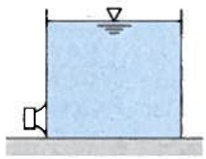
Applying Momentum Equation to Moving CVs (§6.5)

6.87 Using the Internet or some other source as reference, de in your own words the meaning of "inertial reference frame."

6.88 The surface of the earth is not a true inertial reference frame because there is a centripetal acceleration due to the earth's rotation. The earth rotates once every 24 hours and has a diameter of 8000 miles. What is the centripetal acceleration on the surface of the earth, and how does it compare to the gravitational acceleration?

6.89 A large tank of liquid is resting on a frictionless plane as shown. Explain in a qualitative way what will happen after the cap is removed from the short pipe.

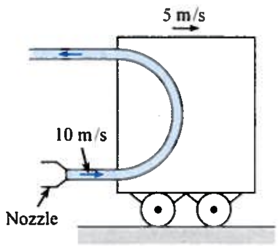
6.90 WILEY PLUS The open water tank shown is resting on a friction plane. The capped orifice on the side has a 4-cm diameter exit pipe that is located 3 m below the surface of the water. Ignore friction effects, and determine the force necessary to keep the tank from moving when the cap is removed.



PROBLEMS 6.89, 6.90

6.91 Consider a tank of water ($\rho = 1000$ kg/m^3) in a containe that rests on a sled. A high pressure is maintained by a compressor so that a jet of water leaving the tank horizontally from an orifice does so at a constant speed of 25 m/s relative t the tank. If there is 0.10 m^3 of water in the tank at time t and t diameter of the jet is 15 mm, what will be the acceleration of t sled at time t if the empty tank and compressor have a weight 350 N and the coefficient of friction between the sled and the is 0.05?

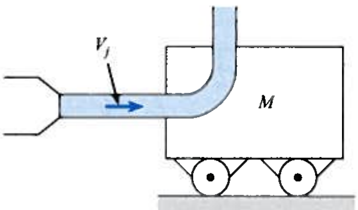
6.92 WILEY PLUS A cart is moving along a railroad track at a constan velocity of 5 m/s as shown. Water ($\rho = 1000$ kg/m^3) issues fro nozzle at 10 m/s and is deflected through 180° by a vane on th cart. The cross-sectional area of the nozzle is 0.002 m^2. Calcul the resistive force on the cart.



PROBLEM 6.92

6.93 A water jet is used to accelerate a cart as shown. The discharge (Q) from the jet is 0.1 m^3/s, and the velocity of the jet (V_j) is 10 m/s. When the water hits the cart, it is deflected normally as shown. The mass of the cart (M) is 10 kg. The density of water (ρ) is 1000 kg/m^3. There is no resistance on the cart, and the initial velocity of the cart is zero. The mass of the water in the jet is much less than the mass of the cart. Derive an equation for the acceleration of the cart as a function of Q, ρ, V_c, M, and V_j. Evaluate the acceleration of the cart when the velocity is 5 m/s.

6.94 WILEY PLUS A water jet strikes a cart as shown. After striking the cart, the water is deflected vertically with respect to the cart. The cart is initially at rest and is accelerated by the water jet. The mass in the water jet is much less than that of the cart. There is no resistance on the cart. The mass flow rate from the jet is 45 kg/s. The mass of the cart is 100 kg. Find the time required for the cart to achieve a speed one-half of the jet speed.



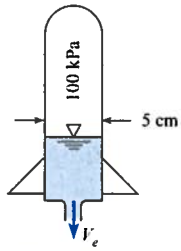
PROBLEMS 6.93, 6.94

6.95 It is common practice in rocket trajectory analyses to neglect the body-force term and drag, so the velocity at burnout is given by

$$v_{bo} = \frac{T}{\lambda}\ln\frac{M_0}{M_f}$$

Assuming a thrust-to-mass-flow ratio of 3000 N · s/kg and a final mass of 50 kg, calculate the initial mass needed to establish the rocket in an earth orbit at a velocity of 7200 m/s.

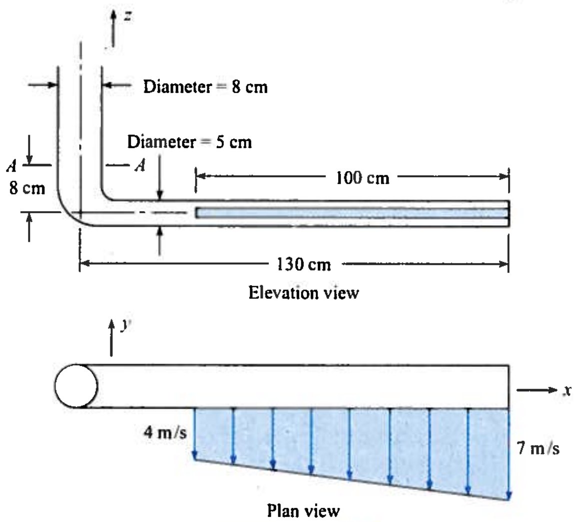
6.96 A very popular toy on the market several years ago was the water rocket. Water (at 10°C) was loaded into a plastic rocket and pressurized with a hand pump. The rocket was released and would travel a considerable distance in the air. Assume that a water rocket has a mass of 50 g and is charged with 100 g of water. The pressure inside the rocket is 100 kPa gage. The exit area is one-tenth of the chamber cross-sectional area. The inside diameter of the rocket is 5 cm. Assume that Bernoulli's equation is valid for the water flow inside the rocket. Neglecting air friction, calculate the maximum velocity it will attain.



PROBLEM 6.96

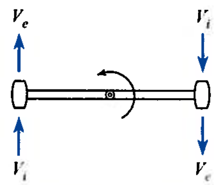
The Angular Momentum Equation (§6.6)

6.97 WILEY PLUS Water ($\rho = 1000$ kg/m^3) is discharged from the slot i the pipe as shown. If the resulting two-dimensional jet is 100 cn long and 15 mm thick, and if the pressure at section A-A is 30 kP what is the reaction at section A-A? In this calculation, do not consider the weight of the pipe.



PROBLEM 6.97

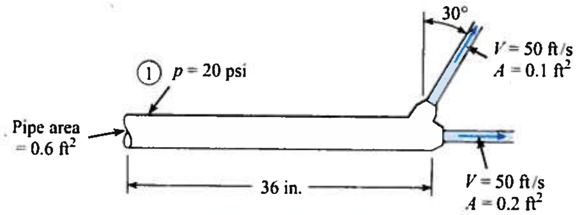
6.98 Two small liquid-propellant rocket motors are mounted at the tips of a helicopter rotor to augment power under emergenc conditions. The diameter of the helicopter rotor is 7 m, and it rotates at 1 rev/s. The air enters at the tip speed of the rotor, and exhaust gases exit at 500 m/s with respect to the rocket motor. The intake area of each motor is 20 cm^2, and the air density is 1.2 kg/m^3. Calculate the power provided by the rocket motors. Neglect the mass rate of flow of fuel in this calculation.



PROBLEM 6.98

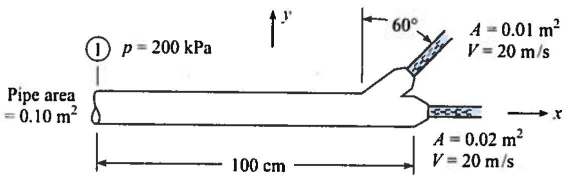
6.99 Design a rotating lawn sprinkler to deliver 0.25 in. of water per hour over a circle of 50 ft radius. Make the simplifying assumptions that the pressure to the sprinkler is 50 psig and that frictional effects involving the flow of water through the sprinkler flow passages are negligible (the Bernoulli equation is applicable). However, do not neglect the friction between the rotating element and the fixed base of the sprinkler.

6.100 PLUS What is the force and moment reaction at section 1? Water (at 50°F) is flowing in the system. Neglect gravitational forces.



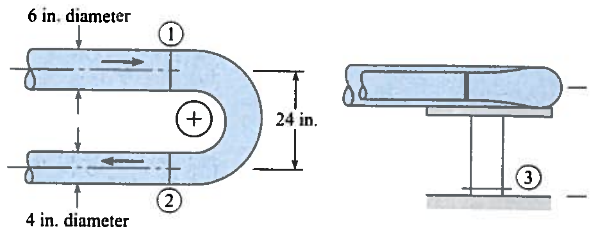
PROBLEM 6.100

6.101 What is the reaction at section 1? Water ($\rho = 1000\ \text{kg/m}^3$) is flowing, and the axes of the two jets lie in a vertical plane. The pipe and nozzle system weighs 90 N.



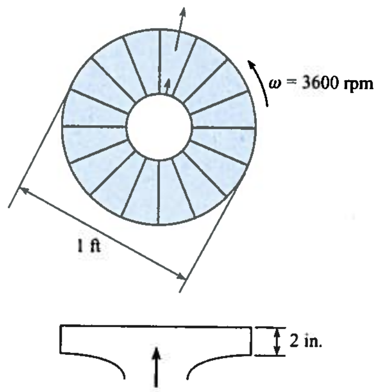
PROBLEM 6.101

6.102 A reducing pipe bend is held in place by a pedestal as shown. There are expansion joints at sections 1 and 2, so no force is transmitted through the pipe past these sections. The press at section 1 is 20 psig, and the rate of flow of water ($\rho = 62.4$ lbr is 2 cfs. Find the force and moment that must be applied at section 3 to hold the bend stationary. Assume the flow is irrotational, and neglect the influence of gravity.



PROBLEM 6.102

6.103 A centrifugal fan is used to pump air. The fan rotor is in diameter, and the blade spacing is 2 in. The air enters with angular momentum and exits radially with respect to the fan rotor. The discharge is 1500 cfm. The rotor spins at 3600 rev/ The air is at atmospheric pressure and a temperature of 60°F. Neglect the compressibility of the air. Calculate the power (h required to operate the fan.



PROBLEM 6.103

Hw