CONTROL VOLUME APPROACH AND CONTINUITY EQUATION

5

FIGURE 5.1

The photo shows an evacuated-tube solar collector that is being tested to measure the efficiency. This project was done by undergraduate engineering students. The team applied the control volume concept, the continuity equation, the flow rate equations as well as knowledge from thermodynamics and heat transfer. (Photo by Donald Elger.)

Chapter Road Map

This chapter describes how conservation of mass c be applied to a flowing fluid. The resulting equatic is called the *continuity equation*. The continuity equation is applied to a spatial region called a control volume, which is also introduced.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define mass flow rate and volume flow rate. (§5.1)
- Apply the flow rate equations. Describe how the flow rate equations are derived. (§5.1)
- Define and calculate the mean velocity. (§5.1)
- Describe the types of systems that engineers use for analysis. List the key differences between a CV and a closed system. (§5.2)
- Describe the purpose, application, and derivation of the Reynolds transport theorem. (§5.2)
- Describe and apply the continuity equation. Describe how the equation is derived. (§5.3, §5.4)
- Explain what cavitation means, describe why it is important, and list guidelines for designing to avoid cavitation. (§5.5)

5.1 Characterizing the Rate of Flow

Engineers characterize the rate of flow using the (a) mass flow rate, $\dot{m}$, and (b) the volume flc rate Q. Thus, these concepts and associated equations are introduced in this section.

Volume Flow Rate (Discharge)

Volume flow rate Q is the *ratio of volume to time at an instant in time.* In equation form,

$$Q = \left(\frac{\text{volume of fluid passing through a cross sectional area}}{\text{interval of time}}\right)_{\substack{\text{instant}\\ \text{in time}}} = \lim_{\Delta t \to 0} \frac{\Delta V\!\!\!/}{\Delta t} \quad (5.$$

EXAMPLE. To describe volume flow rate (Q) for a gas pump (Fig. 5.2a), select a cros sectional area. Then, Q is the volume of gasoline that flowed across the specified secti during a specified time interval (say one second) divided by the time interval. The un could be gallons per minute or liters per second.

EXAMPLE. To describe volume flow rate (Q) for a person inhaling while doing yo (Fig. 5.2b), select a cross-sectional area as shown. Then, Q is the volume of air that flow across the specified section during a specified time interval (say $\Delta t = 0.01$ s) divided by t time interval. Notice that the time interval should be short because the flow rate is contin ously varying during breathing. The idea is to let $\Delta t \to 0$ so that the flow rate is characte ized at an instant in time.

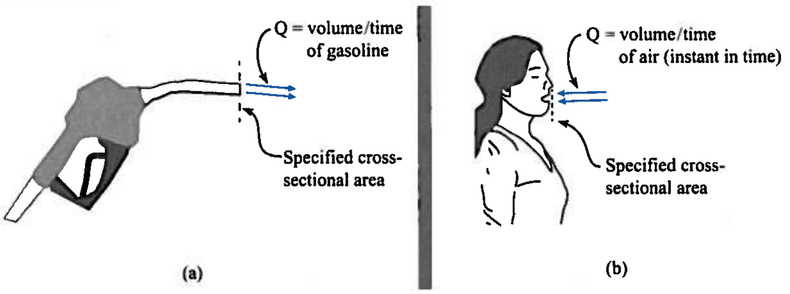


FIGURE 5.2
Sketches used to define volume flow rate (a) gasoline flowing out of a valve at a filling station, (b) air flowing inward to a person during inhalation.

Volume flow rate is often called *discharge.* Because these two terms are synonyms, this te uses both terms interchangeably.

The SI units of discharge are cubic meters of volume per second (m^3/s). In traditio units, the consistent unit is cubic feet of volume per second (ft^3/s). Often this unit is writt as cfs, which stands for cubic feet per second.

Deriving Equations for Volume Flow Rate (Discharge)

This subsection shows how to derive useful equations for discharge Q in terms of fluid veloc and section area A.

To relate Q to velocity V, select a flow of fluid (Fig. 5.3) in which velocity is assumed to constant across the pipe cross section. Suppose a marker is injected over the cross section section A-A for a period of time Δt. The fluid that passes A-A in time Δt is represented by t marked volume. The length of the marked volume is $V\Delta t$ so the volume is $\Delta \forall = AV\Delta t$. Apply the definition of Q:

$$Q = \lim_{\Delta t \to 0} \frac{\Delta \forall}{\Delta t} = \lim_{\Delta t \to 0} \frac{AV\Delta t}{\Delta t} = VA \quad (5.2)$$

In Eq. (5.2), notice how the units work out:

$$Q = VA$$

$$\text{Flow Rate } (\text{m}^3/\text{s}) = \text{Velocity } (\text{m/s}) \times \text{Area } (\text{m}^2)$$

FIGURE 5.3
Volume of fluid in flow with uniform velocity distribution that passes section *A-A* in time Δt.

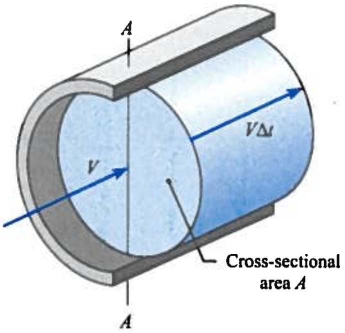
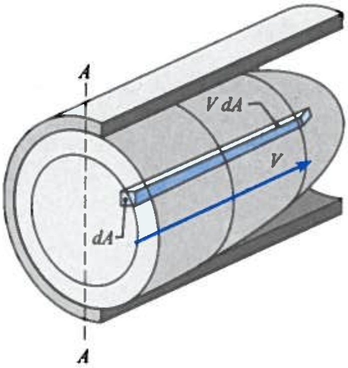


FIGURE 5.4
Volume of fluid that passes section *A-A* in time Δt.



Because Eq. (5.2) is based on a uniform velocity distribution, consider a flow in which the velocity varies across the section (see Fig. 5.4). The blue shaded region shows the volume of fluid that passes across a differential area of the section. Using the idea of Eq. (5.2), let $dQ = V\,dA$. To obtain the total flow rate, add up the volume flow rate through each differential element and then apply the definition of the integral:

$$Q = \sum_{\text{section}} V_i\, dA_i = \int_A V\,dA \quad (5.3)$$

Eq. (5.3) means that *velocity integrated over section area gives discharge.* To develop another useful result, divide Eq. (5.3) by area A to give

$$\overline{V} = \frac{Q}{A} = \frac{1}{A}\int_A V\,dA \quad (5.4)$$

Eq. (5.4) provides a definition of $\overline{V}$, which is called the **mean velocity**. As shown, the mean velocity is an area-weighted average velocity. For this reason, mean velocity is sometimes called *area-averaged velocity.* This label is useful for distinguishing an area-averaged velocity from a *time-averaged velocity,* which is used for characterizing turbulent flow (see Section 4.3). Some useful values of mean velocity are summarized in Table 5.1.

TABLE 5.1 Values of Mean Velocity

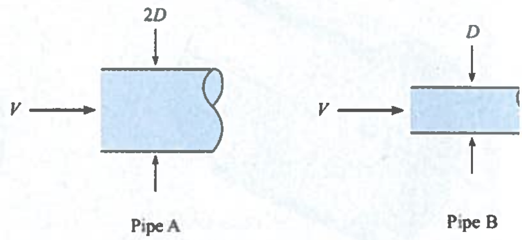
Situation	Equation for Mean Velocity
Fully developed laminar flow in a round pipe. For more information, see Section 10.5.	$\bar{V}/V_{max} = 0.5$, where V_{max} is the value of the maximum velocity in the pipe. Note that V_{max} is the value of the velocity at the center of the pipe
Fully developed laminar flow in a rectangular channel (channel has infinite width).	$\bar{V}/V_{max} = 2/3 = 0.667$
Fully developed turbulent flow in a round pipe. For more information, see Section 10.6.	$\bar{V}/V_{max} \approx 0.79$ to 0.86, where the ratio depends on Reynolds number.

The following checkpoint problems gives you a chance to test your understanding flow rate.

✔CHECKPOINT PROBLEM 5.1

Consider flow through two round pipes. Pipe A has twice the diameter of pipe B. The mean velocity in each pipe is the same. What is Q_A/Q_B?

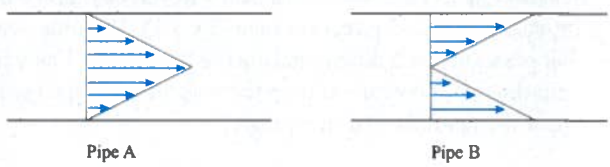
a. 1
b. 2
c. 4
d. 8



✔CHECKPOINT PROBLEM 5.2

Consider flow through two round pipes. The maximum velocity in each pipe is the same. The only difference is the velocity distribution. Which pipe has the larger value of mean velocity? Why?

a. Pipe A
b. Pipe B
c. They both have the same mean velocity



Eq. (5.4) can be generalized by using the concept of the dot product. The dot product is useful when the velocity vector is aligned at an angle with respect to the section area (Fig. 5.5). The only component of velocity that contributes to the flow through the differential area dA is the component normal to the area, V_n. The differential discharge through area dA is

$$dQ = V_n dA$$

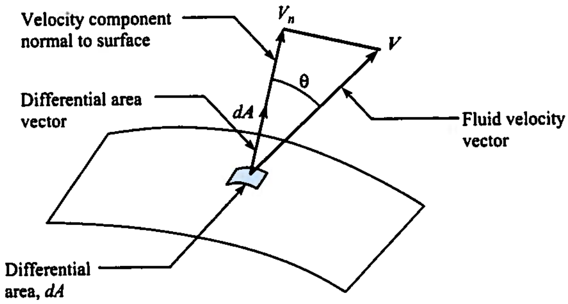


FIGURE 5.5
Velocity vector oriented at angle θ with respect to normal.

If the vector, **dA**, is defined with magnitude equal to the differential area, dA, and direction normal to the surface, then $V_n dA = |\mathbf{V}| \cos\theta\, dA = \mathbf{V} \cdot \mathbf{dA}$ where $\mathbf{V} \cdot \mathbf{dA}$ is the dot product of the two vectors. Thus a more general equation for the discharge or volume flow rate through a surface A is

$$Q = \int_A \mathbf{V} \cdot \mathbf{dA} \tag{5.5}$$

If the velocity is constant over the area and the area is a planar surface, then the discharge is

$$Q = \mathbf{V} \cdot \mathbf{A}$$

If, in addition, the velocity and area vectors are aligned, then

$$Q = VA$$

which reverts to the original equation developed for discharge, Eq. (5.2).

Mass Flow Rate

Mass flow rate $\dot{m}$ is the *ratio of mass to time at an instant in time.* In equation form,

$$\dot{m} = \left(\frac{\text{mass of fluid passing through a cross sectional area}}{\text{interval of time}}\right)_{\substack{\text{instant}\\\text{in time}}} = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} \tag{5.6}$$

The common units for mass flow rate are kg/s, lbm/s, and slugs/s.

Using the same approach as for volume flow rate, the mass of the fluid in the marked volume in Fig. 5.3 is $\Delta m = \rho \Delta \forall$, where ρ is the average density. Thus, one can derive several useful equations:

$$\begin{aligned}\dot{m} &= \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} = \rho \lim_{\Delta t \to 0} \frac{\Delta \forall}{\Delta t} = \rho Q \\ &= \rho A V\end{aligned} \tag{5.7}$$

The generalized form of the mass flow equation corresponding to Eq. (5.5) is

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \mathbf{dA} \tag{5.8}$$

where both the velocity and fluid density can vary over the cross-sectional area. If the density is constant, then Eq. (5.7) is recovered. Also if the velocity vector is aligned with the area vector, such as integrating over the cross-sectional area of a pipe, Eq. (5.8) reduces to

$$\dot{m} = \int_A \rho V \, dA \tag{5.9}$$

Working Equations

Table 5.2 summarizes the flow rate equations. Notice that multiplying Eq. (5.10) by density gives Eq. (5.11).

TABLE 5.2 Summary of the Flow Rate Equations

Description	Equation	Terms
Volume flow rate equation	$Q = \overline{V}A = \frac{\dot{m}}{\rho} = \int_A V\,dA = \int_A \mathbf{V}\cdot d\mathbf{A}$ (5.10)	Q = volume flow rate = discharge (m^3/s) $\overline{V}$ = mean velocity = area averaged velocity (m/s A = cross section area (m^2) $\dot{m}$ = mass flow rate (kg/s) V = speed of a fluid particle (m/s) dA = differential area (m^2) $\mathbf{V}$ = velocity of a fluid particle (m/s) $d\mathbf{A}$ = differential area vector (m^2) (points outward from control surface)
Mass flow rate equation	$\dot{m} = \rho A\overline{V} = \rho Q = \int_A \rho V\,dA = \int_A \rho\mathbf{V}\cdot d\mathbf{A}$ (5.11)	$\dot{m}$ = mass flow rate (kg/s) ρ = mass density (kg/m^3)

Example Problems

For most problems, application of the flow rate equation involves substituting numbers into t appropriate equation; see Example 5.1 for this case.

EXAMPLE 5.1

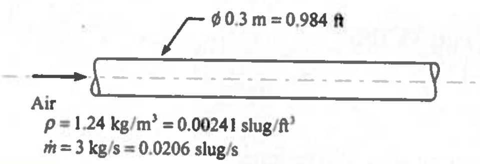
Applying the Flow Rate Equations to a Flow of Air in a Pipe

Problem Statement

Air that has a mass density of 1.24 kg/m^3 (0.00241 $slugs/ft^3$) flows in a pipe with a diameter of 30 cm (0.984 ft) at a mass rate of flow of 3 kg/s (0.206 slugs/s). What are the mean velocity and discharge in this pipe for both systems of units?

Define the Situation

Air flows in a pipe.



State the Goal

Q(m^3/s and ft^3/s) ⬅ Volume flow rate (discharge)
$\overline{V}$(m/s and ft/s) ⬅ Mean velocity

Generate Ideas and Make a Plan

Because Q is the goal and $\dot{m}$ and ρ are known, apply the mass flow rate equation (Eq. 5.11):

$$\dot{m} = \rho Q \qquad \text{(a)}$$

To find the last goal ($\overline{V}$), apply the volume flow rate equation (Eq. 5.10):

$$Q = \overline{V}A \qquad \text{(b)}$$

The plan is

1. Calculate Q using Eq. (a).
2. Calculate $\overline{V}$ using Eq. (b).

Take Action (Execute the Plan)

1. Mass flow rate equation:

$$Q = \frac{\dot{m}}{\rho} = \frac{3\text{ kg/s}}{1.24\text{ kg/m}^3} = \boxed{2.42\text{ m}^3/\text{s}}$$

$$Q = 2.42\text{ m}^3/\text{s} \times \left(\frac{35.31\text{ ft}^3}{1\text{ m}^3}\right) = \boxed{85.5\text{ cfs}}$$

2. Volume flow rate equation:

$$\overline{V} = \frac{Q}{A} = \frac{2.42\text{ m}^3/\text{s}}{(\frac{1}{4}\pi)\times(0.30\text{ m})^2} = \boxed{34.2\text{ m/s}}$$

$$\overline{V} = 34.2\text{ m/s} \times \left(\frac{1\text{ ft}}{0.3048\text{ m}}\right) = \boxed{112\text{ ft/s}}$$

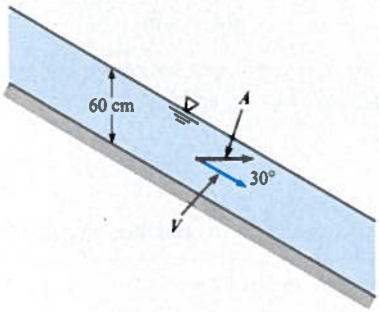
When fluid passes across a control surface and the velocity vector is at an angle with respect to the surface normal vector, then one uses the dot product. This case is illustrated by Example 5.2.

EXAMPLE 5.2

Calculating the Volume Flow Rate by Applying the Dot Product

Problem Statement

Water flows in a channel that has a slope of 30°. If the velocity is assumed to be constant, 12 m/s, and if a depth of 60 cm is measured along a vertical line, what is the discharge per meter of width of the channel?



Define the Situation

Water flows in an open channel.

State the Goal

$Q(m^3/s)$ ⬅ discharge per meter of width of the channel

Generate Ideas and Make a Plan

Because V and A are not at right angles, apply $Q = \mathbf{V} \cdot \mathbf{A} = VA\cos\theta$. Because all variables are known except Q, the plan is to substitute in values.

Take Action (Execute the Plan)

$$Q = \mathbf{V} \cdot \mathbf{A} = V(\cos 30°)A$$
$$= (12\ \text{m/s})(\cos 30°)(0.6\ \text{m})$$
$$= \boxed{6.24\ \text{m}^3/\text{s per meter}}$$

Review the Solution and the Process

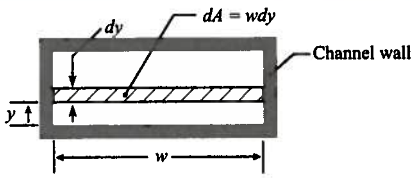
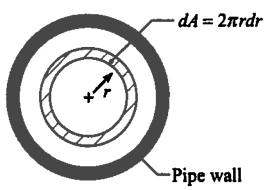
1. *Knowledge.* This example involves a channel flow. A flow is a *channel flow* when a liquid (usually water) flows with open surface exposed to air under the action of gravity.
2. *Knowledge.* The discharge per unit width is usually designated as q.

Another important case is when velocity varies at different points on the control surface. In this case, one uses an integral to determine flow rate as specified by Eq. (5.10):

$$Q = \int_A V\,dA.$$

In this integral, the differential area dA depends on the physics of the problem. Two common cases are shown in Table 5.3. Analyzing a variable velocity is illustrated by Example 5.3.

TABLE 5.3 Differential Areas for Determining Flow Rate

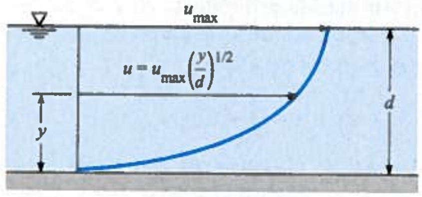
Label	Sketch	Description
Channel Flow		When velocity varies as $V = V(y)$ in a rectangular channel, then use a differential area dA given by $dA = wdy$ where w is the width of the channel and dy is a differential height.
Pipe Flow		When velocity varies as $V = V(r)$ in a round pipe, then use a differential area dA given by $dA = 2\pi r dr$ where r is the radius of th[e] differential area and dr is a differential radius.

EXAMPLE 5.3

Determining Flow Rate by Integration

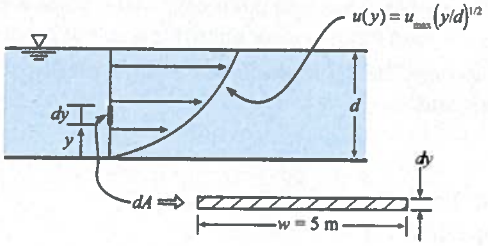
Problem Statement

The water velocity in the channel shown in the accompanying figure has a velocity distribution across the vertical section equal to $u/u_{max} = (y/d)^{1/2}$. What is the discharge in the channel if the water is 2 m deep ($d = 2$ m), the channel is 5 m wide, and the maximum velocity is 3 m/s?



Define the Situation

Water flows in a channel.



State the Goal

Q(m³/s) ⬅ Discharge (Volume Flow Rate)

Generate Ideas and Make a Plan

Because velocity is varying over the cross-sectional area, apply Eq. (5.10):

$$Q = \int_A V\,dA \qquad \text{(a)}$$

Because Eq. (a) has two unknowns (V and dA), find equations for these unknowns. The velocity is given:

$$V = u(y) = u_{max}(y/d)^{1/2} \qquad \text{(b)}$$

From Table 5.3, the differential area is

$$dA = w\,dy \qquad \text{(c)}$$

Notice that the differential area is sketched in the situation diagram. Substitute Eqs. (b) and (c) into Eq. (a):

$$Q = \int_0^d u_{max}(y/d)^{1/2}\,w\,dy \qquad \text{(d)}$$

The plan is to integrate Eq. (d) and then plug numbers in.

Take Action (Execute the Plan)

$$Q = \int_0^d u_{max}(y/d)^{1/2}\,w\,dy$$

$$= \frac{wu_{max}}{d^{1/2}}\int_0^d y^{1/2}\,dy$$

$$= \frac{wu_{max}}{d^{1/2}}\frac{2}{3}y^{3/2}\Big|_0^d = \frac{wu_{max}}{d^{1/2}}\frac{2}{3}d^{3/2}$$

$$= \frac{(5\text{ m})(3\text{ m/s})}{(2\text{ m})^{1/2}} \times \frac{2}{3} \times (2\text{ m})^{3/2} = \boxed{20\text{ m}^3/\text{s}}$$

5.2 The Control Volume Approach

Engineers solve problems in fluid mechanics using the *control volume approach*. Equations f this approach are derived using *Reynolds transport theorem*. These topics are presented in th section.

The Closed System and the Control Volume

As introduced in Section 2.1, a *system* is whatever the engineer selects for study. The *surroun ings* are everything that is external to the system, and the *boundary* is the interface between t system and the surroundings. Systems can be classified into two categories: the closed syste and the open system (also known as a control volume).

The closed system (also known as a *control mass*) is a fixed collection of matter that t engineer selects for analysis. By definition, mass cannot cross the boundary of a closed syste

The boundary of a closed system can move and deform.

EXAMPLE. Consider air inside a cylinder (see Fig. 5.6). If the goal is to calculate the pressure and temperature of the air during compression, engineers select a closed system comprised of the air inside the cylinder. The system boundaries would deform as the piston moves so that the closed system always contains the same matter. This is an example of a closed system because the mass within the system is always the same.

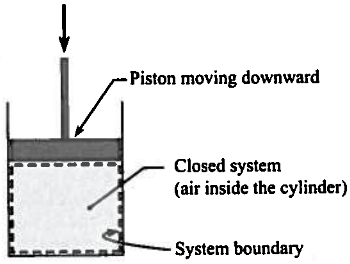


FIGURE 5.6
Example of a closed system.

Because the closed system involves selection and analysis of a specific collection of matter, the closed system is a Lagrangian concept.

The control volume (CV or cv; also known as an open system) is a specified volumetric region in space that the engineer selects for analysis. The matter inside a control volume is usually changing with time because mass is flowing across the boundaries. Because the control volume involves selection and analysis of a region in space, the CV is an Eulerian concept.

EXAMPLE. Suppose water is flowing through a tank (Fig. 5.7) and the goal is to calculate the depth of water h as a function of time. A key to solving this problem is to select a system, and the best choice of a system is a CV surrounding the tank. Note that the CV is always three dimensional because it is a volumetric region. However, CVs are usually drawn in two dimensions. The boundary surfaces of a CV are called the control surface. This is abbreviated as CS or cs.

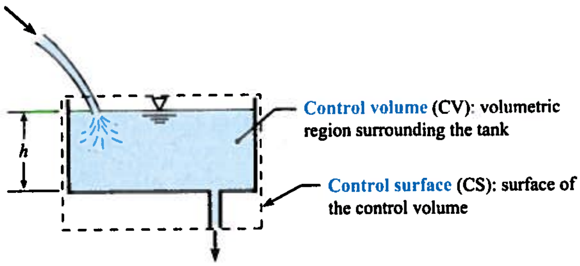


FIGURE 5.7
Water entering a tank through the top and exiti through the bottom

A control volume can be defined so that it is deforming or fixed. When a fixed CV is defined, this means that the shape of the CV and its volume are constant with time. When a deforming CV is defined, the shape of the CV and its volume change with time, typically to mimic the volume of a region of fluid.

EXAMPLE. To model a rocket made from a balloon suspended on a string, one can define a deforming CV that surrounds the deflating balloon and follow the shape of the balloon during the process of deflation.

Summary When engineers analyze a problem, they select the type of system that is m useful (see Fig. 5.8). There are two approaches. Using the *control volume approach* the engin selects a region in space and analyzes flow through this region. Using the *closed system appro* the engineer selects a body of matter of fixed identity and analyzes this matter.

Table 5.4 compares the *Control Volume Approach* and *Closed System Approach*.

FIGURE 5.8

When engineers select a system, they choose either the *control volume approach* or the *closed system approach*. Then, they select the specific type of system from a choice of six possibilities.

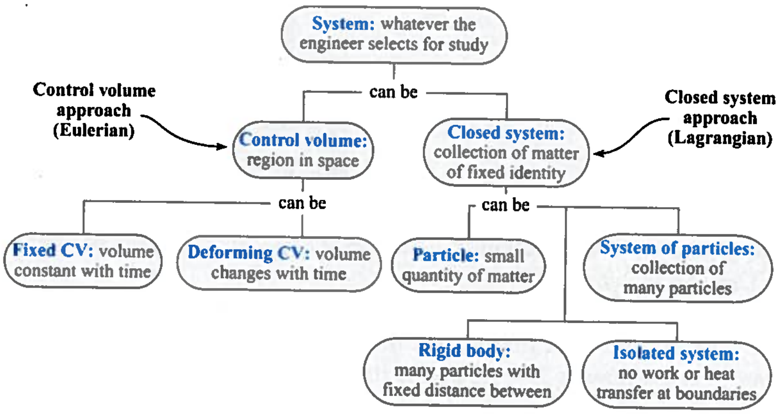


TABLE 5.4 Comparison of the Control Volume and the Closed System Approaches

Feature	Closed System Approach	Control Volume Approach
Basic idea	Analyze a body or fixed collection of matter.	Analyze a spatial region.
Lagrangian versus Eulerian	Lagrangian approach.	Eulerian approach.
Mass crossing the boundaries	Mass cannot cross the boundaries.	Mass is allowed to cross the boundaries.
Mass (quantity)	The mass of the closed system must stay constant with time; always the same number of kilograms.	The mass of the materials inside the CV can stay constant or can change with time.
Mass (identity)	Always contains the same matter.	Can contain the same matter at all times. Or the identity of the matter can vary with time.
Application	Solid mechanics, fluid mechanics, thermodynamics, and other thermal sciences.	Fluid mechanics, thermodynamics, and other thermal sciences.

Intensive and Extensive Properties

Properties, which are measurable characteristics of a system, can be classified into two catego ries. An **extensive property** is any property that depends on the amount of matter present. A **intensive property** is any property that is independent of the amount of matter present.

Examples (extensive). Mass, momentum, energy, and weight are extensive properti because each of these properties depends on the amount of matter present. **Examples (intensive** Pressure, temperature, and density are intensive properties because each of these properti are independent on the amount of matter present.

Many intensive properties are obtained by taking the ratio of two extensive properties. For example, density is the ratio of mass to volume. Similarly, specific energy e is the ratio of energy to mass.

To develop a general equation to relate intensive and extensive properties, define a generic extensive property, B. Also, define a corresponding intensive property b.

$$b = \left(\frac{B}{\text{mass}}\right)_{\text{point in space}}$$

The amount of extensive property B contained in a control volume at a given instant is

$$B_{cv} = \int_{cv} b\,dm = \int_{cv} b\rho\,d\mathcal{V} \qquad (5.12)$$

where dm and $d\mathcal{V}$ are the differential mass and differential volume, respectively, and the integral is carried out over the control volume.

Property Transport across the Control Surface

Because flow transports mass, momentum, and energy across the control surface, the next step is to describe this transport. Consider flow through a duct (Fig. 5.9) and assume that the velocity is uniformly distributed across the control surface. Then, the mass flow rate through each section is given by

$$\dot{m}_1 = \rho_1 A_1 V_1 \qquad \dot{m}_2 = \rho_2 A_2 V_2$$

The rate of outflow minus the rate of inflow is

$$(\text{outflow minus inflow}) = (\text{net mass outflow rate}) = \dot{m}_2 - \dot{m}_1 = \rho_2 A_2 V_2 - \rho_1 A_1 V_1$$

Next, we'll introduce velocity. The same control volume is shown in Fig. 5.10 with each control surface area represented by a vector, $\mathbf{A}$, oriented outward from the control volume and with magnitude equal to the cross-sectional area. The velocity is represented by a vector, $\mathbf{V}$. Taking the dot product of the velocity and area vectors at both stations gives

$$\mathbf{V}_1 \cdot \mathbf{A}_1 = -V_1 A_1 \qquad \mathbf{V}_2 \cdot \mathbf{A}_2 = V_2 A_2$$

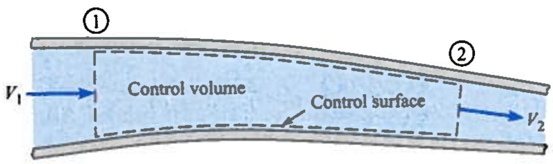


FIGURE 5.9
Flow through control volume in a duct.

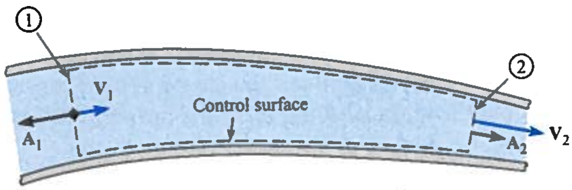


FIGURE 5.10
Control surfaces represented by area vectors and velocities by velocity vectors.

because at station 1 the velocity and area have the opposite directions while at station 2 velocity and area vectors are in the same direction. Now the net mass outflow rate ca written as

$$\begin{aligned}\text{net mass outflow rate} &= \rho_2 V_2 A_2 - \rho_1 V_1 A_1 \\ &= \rho_2 \mathbf{V}_2 \cdot \mathbf{A}_2 + \rho_1 \mathbf{V}_1 \cdot \mathbf{A}_1 \\ &= \sum_{cs} \rho \mathbf{V} \cdot \mathbf{A}\end{aligned} \quad (5$$

Equation (5.13) states that if the dot product $\rho\mathbf{V} \cdot \mathbf{A}$ is summed for all flows into and out of control volume, the result is the net mass flow rate out of the control volume, or the net m efflux (*efflux* means outflow). If the summation is positive, the net mass flow rate is out of control volume. If it is negative, the net mass flow rate is into the control volume. If the inf and outflow rates are equal, then

$$\sum_{cs} \rho \mathbf{V} \cdot \mathbf{A} = 0$$

To obtain the net rate of flow of an extensive property B across a section, write

$$\overbrace{\left(\frac{B}{\text{mass}}\right)}^{b} \overbrace{\left(\frac{\text{mass}}{\text{time}}\right)}^{\dot{m}} = \overbrace{\left(\frac{B}{\text{time}}\right)}^{\dot{B}}$$

Next, include all inlet and outlet ports:

$$\dot{B}_{\text{net}} = \sum_{cs} b \overbrace{\rho \mathbf{V} \cdot \mathbf{A}}^{\dot{m}} \quad (5.1$$

Equation (5.14) is applicable for all flows where the properties are uniformly distribut across the flow area. To account for property variation, replace the sum with an integral:

$$\dot{B}_{\text{net}} = \int_{cs} b\rho \mathbf{V} \cdot \mathbf{dA} \quad (5.1$$

Eq. (5.15) will be used in the derivation of the Reynolds transport theorem.

Reynolds Transport Theorem

The Reynolds transport theorem is an equation that relates a derivative for a *closed system* t the corresponding terms for a *control volume*. The reason for the theorem is that the conserva tion laws of science were originally formulated for closed systems. Over time, researchers fig ured out how to modify the equations so that they apply to a control volume. The result is th Reynolds transport theorem.

To derive the Reynolds transport theorem, consider a flowing fluid; see Fig. 5.11. Th darker shaded region is a *closed system*. As shown, the boundaries of the closed system chang with time so that the system always contains the same matter. Also, define a CV as identifie by the dashed line. At time t the closed system consists of the material inside the control vol ume and the material going in, so the property B of the system at this time is

$$B_{\text{closed system}}(t) = B_{\text{cv}}(t) + \Delta B_{\text{in}} \quad (5.16$$

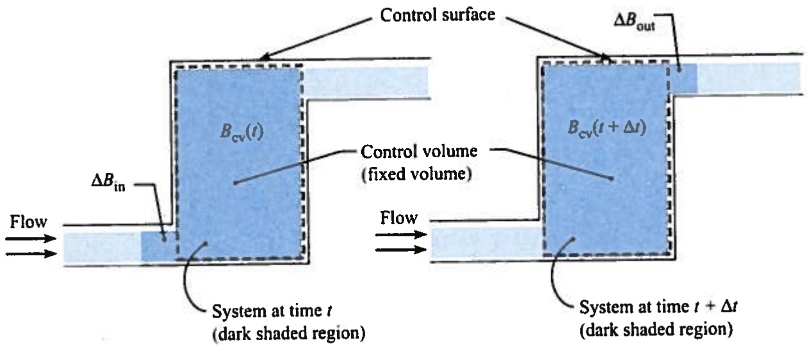


FIGURE 5.11
Progression of a closed system through a control volume.

At time $t + \Delta t$ the closed system has moved and now consists of the material in the control volume and the material passing out, so B of the system is

$$B_{\text{closed system}}(t + \Delta t) = B_{\text{cv}}(t + \Delta t) + \Delta B_{\text{out}} \tag{5.17}$$

The rate of change of the property B is

$$\frac{dB_{\text{closed system}}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{\text{closed system}}(t + \Delta t) - B_{\text{closed system}}(t)}{\Delta t} \right] \tag{5.18}$$

Substituting in Eqs. (5.16) and (5.17) results in

$$\frac{dB_{\text{closed system}}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{\text{cv}}(t + \Delta t) - B_{\text{cv}}(t) + \Delta B_{\text{out}} - \Delta B_{\text{in}}}{\Delta t} \right] \tag{5.19}$$

Rearranging terms yields

$$\frac{dB_{\text{closed system}}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{\text{cv}}(t + \Delta t) - B_{\text{cv}}(t)}{\Delta t} \right] + \lim_{\Delta t \to 0} \frac{\Delta B_{\text{out}}}{\Delta t} - \lim_{\Delta t \to 0} \frac{\Delta B_{\text{in}}}{\Delta t} \tag{5.20}$$

The first term on the right side of Eq. (5.20) is the rate of change of the property B inside the control volume, or

$$\lim_{\Delta t \to 0} \left[\frac{B_{\text{cv}}(t + \Delta t) - B_{\text{cv}}(t)}{\Delta t} \right] = \frac{dB_{\text{cv}}}{dt} \tag{5.21}$$

The remaining terms are

$$\lim_{\Delta t \to 0} \frac{\Delta B_{\text{out}}}{\Delta t} = \dot{B}_{\text{out}} \quad \text{and} \quad \lim_{\Delta t \to 0} \frac{\Delta B_{\text{in}}}{\Delta t} = \dot{B}_{\text{in}}$$

These two terms can be combined to give

$$\dot{B}_{\text{net}} = \dot{B}_{\text{out}} - \dot{B}_{\text{in}} \tag{5.22}$$

or the net efflux, or net outflow rate, of the property B through the control surface. Equation (5.20) can now be written as

$$\frac{dB_{\text{closed system}}}{dt} = \frac{d}{dt} B_{\text{cv}} + \dot{B}_{\text{net}}$$

Substituting in Eq. (5.15) for $\dot{B}_{net}$ and Eq. (5.12) for B_{cv} results in the general form of Reynolds transport theorem:

$$\underbrace{\frac{dB_{\text{closed system}}}{dt}}_{\text{Lagrangian}} = \underbrace{\frac{d}{dt}\int_{cv} b\rho d\forall + \int_{cs} b\rho \mathbf{V} \cdot \mathbf{dA}}_{\text{Eulerian}} \quad (5.$$

Eq. (5.23) may be expressed in words as

$$\left\{\begin{matrix}\text{Rate of change}\\ \text{of property } B\\ \text{in closed system}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of change}\\ \text{of property } B\\ \text{in control volume}\end{matrix}\right\} + \left\{\begin{matrix}\text{Net outflow}\\ \text{of property } B\\ \text{through control surface}\end{matrix}\right\}$$

The left side of the equation is the Lagrangian form, that is, the rate of change of propert for the closed system. The right side is the Eulerian form, that is, the change of property evaluated in the control volume and the flux measured at the control surface. This equati applies at the instant the system occupies the control volume and provides the connecti between the Lagrangian and Eulerian descriptions of fluid flow. The velocity **V** is alwa measured with respect to the control surface because it relates to the mass flux across t surface.

A simplified form of the Reynolds transport theorem can be written if the mass cros ing the control surface occurs through a number of inlet and outlet ports, and the veloci density and intensive property b are uniformly distributed (constant) across each po Then

$$\frac{dB_{\text{closed system}}}{dt} = \frac{d}{dt}\int_{cv} b\rho d\forall + \sum_{cs} \rho b \mathbf{V} \cdot \mathbf{A} \quad (5.2$$

where the summation is carried out for each port crossing the control surface.

An alternative form can be written in terms of the mass flow rates:

$$\frac{dB_{\text{closed system}}}{dt} = \int_{cv} \rho b d\forall + \sum_{cs} \dot{m}_o b_o - \sum_{cs} \dot{m}_i b_i \quad (5.2$$

where the subscripts i and o refer to the inlet and outlet ports, respectively, located on tl control surface. This form of the equation does not require that the velocity and density t uniformly distributed across each inlet and outlet port, but the property b must be.

5.3 Continuity Equation (Theory)

The continuity equation is the law of *conservation of mass* applied to a control volum Because this equation is commonly used by engineers, this section presents the relevar topics.

Derivation

The law of conservation of mass for a closed system can be written as

$$\frac{d(\text{mass of a closed system})}{dt} = \frac{dm_{\text{closed system}}}{dt} = 0 \quad (5.26$$

To transform (Eq. 5.26) into an equation for a control volume, apply the Reynolds transport theorem, Eq. (5.23). In Eq. (5.23), the extensive property is mass, $B_{cv} = m_{\text{closed system}}$. The corresponding value intensive property is mass per unit mass, or simply, unity.

$$b = \frac{m_{\text{closed system}}}{m_{\text{closed system}}} = 1$$

Substituting for B_{cv} and b in Eq. (5.23) gives

$$\frac{dm_{\text{closed system}}}{dt} = \frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot \mathbf{dA} \tag{5.27}$$

Combining Eq. (5.26) to Eq. (5.27) gives the *general form of the continuity equation.*

$$\frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot \mathbf{dA} = 0 \tag{5.28}$$

If mass crosses the boundaries at a number of inlet and exit ports, then Eq. (5.28) reduces to give the *simplified form of the continuity equation*:

$$\frac{d}{dt} m_{cv} + \sum_{cs} \dot{m}_o - \sum_{cs} \dot{m}_i = 0 \tag{5.29}$$

Physical Interpretation of the Continuity Equation

Fig. 5.12 shows the meaning of the terms in the continuity equation. The top row gives the general form (Eq. 5.28), and the second row gives the simplified form (Eq. 5.29). The arrows shows which terms have the same conceptual meaning.

The **accumulation** term describes the changes in the quantity of mass inside the control volume (CV) with respect to time. Mass inside a CV can increase with time (accumulation is positive), decrease with time (accumulation is negative) or stay the same (accumulation is zero).

The **inflow and outflow** terms describe the rates at which mass is flowing across the surfaces of the control volume. Sometimes inflow and outflow are combined to give **efflux**, which is defined as the net positive rate at which is mass is flowing out of a CV. That is, (efflux) = (outflow) − (inflow). When efflux is positive, there is a net flow of mass out of the CV, and accumulation is negative. When efflux is negative, then accumulation is positive.

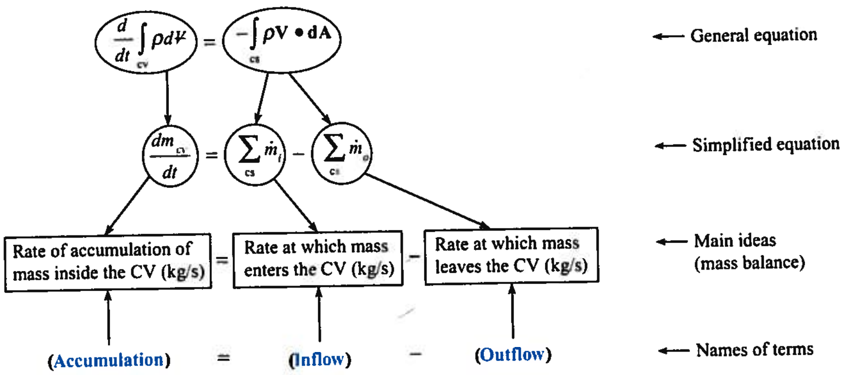


FIGURE 5.12
This figure shows the conceptual meaning of the continuity equation.

As shown in Fig. 5.12, the physics of the continuity equation can be summarized as:

$$\text{accumulation} = \text{inflow} - \text{outflow} \quad (5$$

where all terms in Eq. (5.30) are rates (see Fig. 5.12)

Eq. (5.30) is called a balance equation because the ideas relate to our everyday experie with how things balance. For example, the accumulation of cash in a bank account equal inflows (deposits) minus the outflows (withdrawals). Because the continuity equation is a ance equation, it is sometimes called the *mass balance equation.*

The continuity equation is applied at an instant in time and the units are kg/s. Someti the continuity equation is integrated with respect to time and the units are kg. To recogni problem that will involve integration, *look for a change in state during a time interval.*

5.4 Continuity Equation (Application)

This section describes how to apply the continuity equation and presents example problen

Working Equations

Three useful forms of the continuity equations are summarized in Table 5.5.

TABLE 5.5 Summary of the Continuity Equation

Description	Equations	Terms
General form: valid for any problem.	$\frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot \mathbf{dA} = 0$ (Eq. 5.28)	t = time (s) ρ = density (kg/m³) $d\forall$ = differential volume (m³) $\mathbf{V}$ = fluid velocity vector (m/s) (reference frame is the control surface)
Simplified form: useful when there are well defined inlet and exit ports.	$\frac{d}{dt}m_{cv} + \sum_{cs} \dot{m}_o - \sum_{cs} \dot{m}_i = 0$ (Eq. 5.29)	
Pipe flow form; valid for flow in a pipe. (*gases*: density can vary but the density must be uniform across sections 1 and 2). (*liquids*: the equation reduces to $A_2V_2 = A_1V_1$ for a constant density assumption).	$\rho_2 A_2 V_2 = \rho_1 A_1 V_1$ (Eq. 5.33)	$\mathbf{dA}$ = differential area vector (m²) (positive direction of **dA** is outward from CS m_{cv} = mass inside the control volume (kg) $\dot{m} = \rho A V$ = mass/time crossing CS (kg/s) A = area of flow (m²) V = mean velocity (m/s)

The process for applying the continuity equation is

Step 1. Selection. Select the continuity equation when flow rates, velocity, or ma accumulation are involved in the problem.

Step 2. Sketching. Select a CV by locating CSs that cut through where (a) you know info mation or (b) you want information. Sketch the CV and label it appropriately. Note that is common to label the inlet port as section 1 and the outlet port as section 2.

Step 3. Analysis. Write the continuity equation and perform a term-by-term analysis t simplify the starting equation to the reduced equation.

Step 4. Validation. Check units. Check the basic physics; that is, check that (inflow minu outflow) = (accumulation).

Example Problems

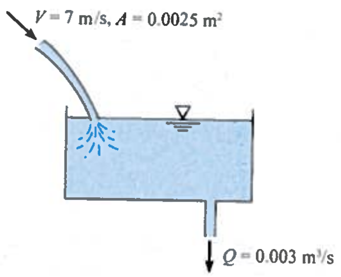
The first example problem (Example 5.4) shows how continuity is applied to a problem that involves accumulation of mass.

EXAMPLE 5.4

Applying the Continuity Equation to a Tank with an Inflow and an Outflow

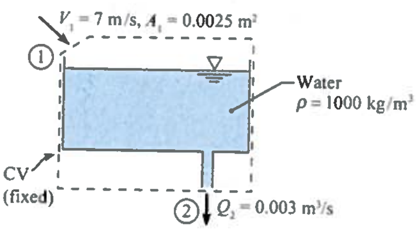
Problem Statement

A stream of water flows into an open tank. The speed of the incoming water is $V = 7$ m/s, and the section area is $A = 0.0025$ m^2. Water also flows out of the tank at rate of $Q = 0.003$ m^3/s. Water density is 1000 kg/m^3. What is the rate at which water is being stored (or removed from) the tank?



Define the Situation

Water flows into a tank at the top and out at the bottom.



State the Goal

(dm_{cv}/dt) (kg/s) ⬅ rate of accumulation of water in tank

Generate Ideas and Make a Plan

Selection. Select the simplified form of the continuity equation (Eq. 5.29).

Sketching. Modify the situation diagram to show the CV and sections 1 and 2. Notice that the CV in the upper left corner is sketched so that it is at a right angle to the inlet flow.

Analysis. Write the continuity equation (simplified form)

$$\frac{d}{dt}m_{cv} + \sum_{cs}\dot{m}_o - \sum_{cs}\dot{m}_i = 0$$

Analyze the outflow and inflow terms.

$$\sum_{cs}\dot{m}_o = \rho Q_2$$

$$\sum_{cs}\dot{m}_i = \rho A_1 V_1$$

Combine Eqs. (a), (b), and (c).

$$\frac{d}{dt}m_{cv} = \rho A_1 V_1 - \rho Q_2$$

Validate. Each term has units of kilograms per second. Eq. (d) makes physical sense: (rate of accumulation of mass) = (rate of mass flow in) − (rate of mass flow out).

Because variables on the right side of Eq. (d) are known, the problem can be solved. The plan is:

1. Calculate the flow rates on the right side of Eq. (d).
2. Apply Eq. (d) to calculate the rate of accumulation.

Take Action (Execute the Plan)

1. Mass flow rates (inlet and outlet).

$$\rho A_1 V_1 = (1000\ \text{kg/m}^3)(0.0025\ \text{m}^2)(7\ \text{m/s}) = 17.5\ \text{kg/s}$$

$$\rho Q_2 = (1000\ \text{kg/m}^3)(0.003\ \text{m}^3/\text{s}) = 3\ \text{kg/s}$$

2. Accumulation.

$$\frac{dm_{cv}}{dt} = 17.5\ \text{kg/s} - 3\ \text{kg/s}$$

$$= \boxed{14.5\ \text{kg/s}}$$

Review the Solution and the Process

1. *Discussion.* Because the accumulation is positive, the quantity of mass within the control volume is increasing with time.
2. *Discussion.* The rising level of water in the tank causes air to flow out of the CV. Because air has a density that is about 1/1000 of the density of water, this effect is negligible.

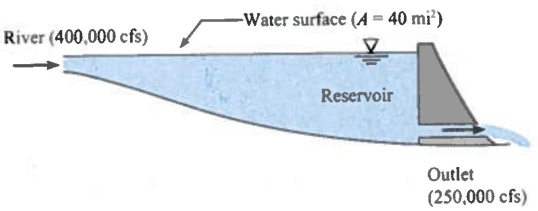
Example 5.5 shows how to solve a problem that involves accumulation by using fixed CV.

EXAMPLE 5.5

Applying the Continuity Equation to Calculate the Rate of Water Rise in a Reservoir

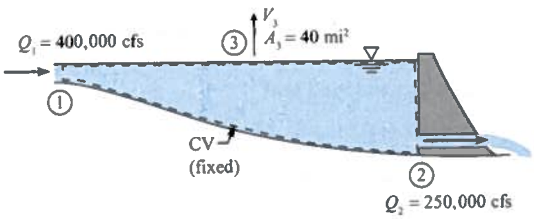
Problem Statement

A river discharges into a reservoir at a rate of 400,000 ft^3/s (cfs), and the outflow rate from the reservoir through the flow passages in a dam is 250,000 cfs. If the reservoir surface area is 40 mi^2, what is the rate of rise of water in the reservoir?



Define the Situation

A reservoir is filling with water.



State the Goal

V_3(ft/h) ⬅ Speed at which the water surface is rising

Generate Ideas and Make a Plan

Selection. Select the continuity equation because the problem involves flow rates and accumulation of mass in a reservoir.

Sketching. Select a fixed control volume and sketch this CV on the situation diagram. The control surface at section 3 is just below the water surface and is stationary. Mass passes through control surface 3 as the water level in the reservoir rises (or falls). The mass within the control volume is constant because the volume of the CV is constant.

Analysis. Write the continuity equation (simplified form):

$$\frac{d}{dt}m_{cv} + \sum_{cs}\dot{m}_o - \sum_{cs}\dot{m}_i = 0 \quad \text{(a)}$$

Next, analyze each term

- Mass in the control volume is constant. Thus,

$$dm_{cv}/dt = 0. \quad \text{(b)}$$

- There are two outflows, at sections 2 and 3. Thus,

$$\sum_{cs}\dot{m}_o = \rho Q_2 + \rho A_3 V_3 \quad \text{(c)}$$

- There is one inflow, at section 1. Thus,

$$\sum_{cs}\dot{m}_i = \rho Q_1. \quad \text{(d)}$$

Substitute Eqs. (b), (c), and (d) into Eq. (a). Then, divide each term by density

$$Q_2 + A_3 V_3 = Q_1 \quad \text{(e)}$$

Validation. Eq. (e) is dimensionally homogeneous because each term has dimensions of volume per time. Eq. (e) makes physical sense: (outflow through sections 2 and 3) equals (inflow from section 1).

Because Eq. (e) contains the problem goal and all other variables are known, the problem is cracked. The plan is to

1. Use Eq. (e) to derive an equation for V_3.
2. Solve for V_3.

Take Action (Execute the Plan)

1. Continuity Equation

$$V_3 = \frac{Q_1 - Q_2}{A_3}$$

2. Calculations

$$V_{rise} = \frac{400,000 \text{ cfs} - 250,000 \text{ cfs}}{40 \text{ mi}^2 \times (5280 \text{ ft/mi})^2}$$
$$= 1.34 \times 10^{-4} \text{ ft/s} = \boxed{0.482 \text{ ft/hr}}$$

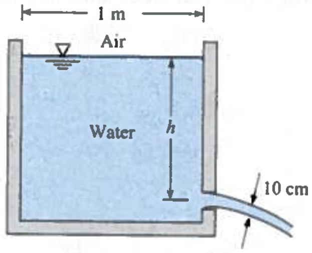
Example 5.6 shows (a) how to use a deforming CV and (b) how to integrate the continu equation.

EXAMPLE 5.6

Applying the Continuity Equation to Predict the Time for a Tank to Drain

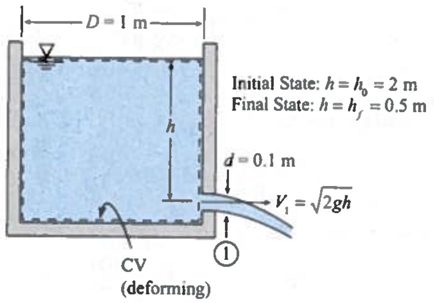
Problem Statement

A 10 cm jet of water issues from a 1-m-diameter tank. Assume the Bernoulli equation applies so the velocity in the jet is $\sqrt{2gh}$ m/s where h is the elevation of the water surface above the outlet jet. How long will it take for the water surface in the tank to drop from $h_0 = 2$ m to $h_f = 0.50$ m?



Define the Situation

Water is draining from a tank.



State the Goal

$t_f(s)$ ⬅ Time for the tank to drain from h_o to h_f

Generate Ideas and Make a Plan

Selection. Select the continuity equation by recognizing that the problem involves outflow and accumulation of mass in a tank.

Also note that the continuity equation will need to be integrated because this problem involves time and a defined initial state and final state.

Sketching. Select a deforming CV that is defined so that the top surface area is coincident with the surface level of the water. Sketch this CV in the situation diagram.

Analysis. Write the continuity equation.

$$\frac{d}{dt}m_{cv} + \sum_{cs}\dot{m}_o - \sum_{cs}\dot{m}_i = 0 \qquad \text{(a)}$$

Analyze each term in a step-by-step fashion.

- Mass in the control volume is given by*

$$m_{cv} = (\text{density})(\text{volume}) = \rho\left(\frac{\pi D^2}{4}\right)h$$

- Differentiate Eq. (b) with respect to time. Note that th only variable that changes with time is water depth h s the other variables can come out of the derivative.

$$\frac{dm_{cv}}{dt} = \frac{d}{dt}\left(\rho\left(\frac{\pi D^2}{4}\right)h\right) = \rho\left(\frac{\pi D^2}{4}\right)\frac{dh}{dt}$$

- The inflow is zero and the outflow is

$$\sum_{cs}\dot{m}_o = \rho A_1 V_1 = \rho\left(\frac{\pi d^2}{4}\right)\sqrt{2gh}$$

Substitute Eqs. (b), (c), and (d) into Eq. (a).

$$\rho\left(\frac{\pi D^2}{4}\right)\frac{dh}{dt} = -\rho\left(\frac{\pi d^2}{4}\right)\sqrt{2gh}$$

Validation. In Eq. (e), each term has units of kg/s. Also, this equation makes physical sense: (accumulation rate) = (the negative of the outflow rate).

Integration. To begin, simplify Eq. (e)

$$\left(\frac{D}{d}\right)^2\frac{dh}{dt} = -\sqrt{2gh}$$

Next, apply the method of *separation of variables.* Put the variables involving h on the left side and the other variables on the right side. Integrate using definite integrals

$$-\int_{h_0}^{h_f}\frac{dh}{\sqrt{2gh}} = \int_0^{t_f}\left(\frac{d}{D}\right)^2 dt \qquad ($$

Perform the integration to give:

$$\frac{2(\sqrt{h_o} - \sqrt{h_f})}{\sqrt{2g}} = \left(\frac{d}{D}\right)^2 t_f \qquad ($$

Because Eq. (h) contains the problem goal (t_f) and all other variables in this equation are known, the plan is to use Eq. (h to calculate (t_f).

Take Action (Execute the Plan)

$$t_f = \left(\frac{D}{d}\right)^2\left(\frac{2(\sqrt{h_o} - \sqrt{h_f})}{\sqrt{2g}}\right)$$

$$= \left(\frac{1\text{ m}}{0.1\text{ m}}\right)^2\left(\frac{2(\sqrt{(2\text{ m})} - \sqrt{(0.5\text{ m})})}{\sqrt{2(9.81\text{ m/s}^2)}}\right)$$

$$\boxed{t_f = 31.9\text{ s}}$$

*The mass in the CV also include the mass of the water below the outlet. However, when dm_{cv}/dt is evaluated, this term will go to zero.

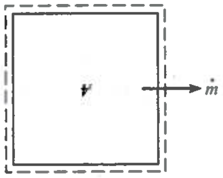
Example 5.7 shows another instance in which the continuity equation is integrated wi respect to time.

EXAMPLE 5.7

Depressurization of Gas in Tank

Problem Definition

Methane escapes through a small (10^{-7} m^2) hole in a 10 m^3 tank. The methane escapes so slowly that the temperature in the tank remains constant at 23°C. The mass flow rate of methane through the hole is given by $\dot{m} = 0.66\, pA/\sqrt{RT}$, where p is the pressure in the tank, A is the area of the hole, R is the gas constant, and T is the temperature in the tank. Calculate the time required for the absolute pressure in the tank to decrease from 500 to 400 kPa.



Define the Situation

Methane leaks through a 10^{-7} m^2 hole in 10 m^3 tank.

Assumptions.

1. Gas temperatures constant at 23°C during leakage.
2. Ideal gas law is applicable.

Properties: Table A.2, $R = 518$ J/kgK.

State the Goal

Find: Time (in seconds) for pressure to decrease from 500 kPa to 400 kPa.

Generate Ideas and Make a Plan

Select a CV that encloses whole tank.

1. Apply continuity equation, Eq. (5.29).
2. Analyze term by term.
3. Solve equation for elapsed time.
4. Calculate time.

Take Action (Execute the Plan)

1. Continuity equation

$$\frac{d}{dt}m_{cv} + \sum_{cs}\dot{m}_o - \sum_{cs}\dot{m}_i = 0$$

2. Term-by-term analysis.
   - Rate of accumulation term. The mass in the control volume is the sum of the mass of the tank shell, M_{shell}, and the mass of methane in the tank,

$$m_{cv} = m_{shell} + \rho\mathcal{V}$$

   where $\mathcal{V}$ is the internal volume of the tank, which is constant. The mass of the tank shell is constant, so

$$\frac{dm_{cv}}{dt} = \mathcal{V}\frac{dp}{dt}$$

   - There is no mass inflow:

$$\sum_{cs}\dot{m}_i = 0$$

   - Mass out flow rate is

$$\sum_{cs}\dot{m}_o = 0.66\frac{pA}{\sqrt{RT}}$$

   Substituting terms into continuity equation

$$\mathcal{V}\frac{dp}{dt} = -0.66\frac{pA}{\sqrt{RT}}$$

3. Equation for elapsed time:
   - Use ideal gas law for ρ,

$$\mathcal{V}\frac{d}{dt}\left(\frac{p}{RT}\right) = -0.66\frac{pA}{\sqrt{RT}}$$

   - Because R and T are constant,

$$\frac{dp}{dt} = -0.66\frac{pA\sqrt{RT}}{\mathcal{V}}$$

   - Next, separate variables

$$\frac{dp}{p} = -0.66\frac{A\sqrt{RT}dt}{\mathcal{V}}$$

   - Integrating equation and substituting limits for initial and final pressure

$$t = \frac{1.52\,\mathcal{V}}{A\sqrt{RT}}\ln\frac{p_0}{p_f}$$

4. Elapsed time

$$t = \frac{1.52\,(10\text{ m}^3)}{(10^{-7}\text{ m}^2)\left(518\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\times 300\text{ K}\right)^{1/2}}\ln\frac{500}{400} = \boxed{8.6\times 10^4\text{ s}}$$

Review the Solution and the Process

1. *Discussion.* The time corresponds to approximately one day.
2. *Knowledge.* Because the ideal gas law is used, the pressure and temperature have to be in absolute values.

Continuity Equation for Flow in a Conduit

A conduit is a pipe or duct or channel that is completely filled with a flowing fluid. Because flow in conduits is common, it is useful to derive an equation that applies to this case. To begin the derivation, recognize that in a conduit (see Fig. 5.13), there is no place for mass to accumulate, so Eq. (5.28) simplifies to

$$\int_{cs} \rho \mathbf{V} \cdot \mathbf{dA} = 0 \tag{5.31}$$

Mass is crossing the control surface at sections 1 and 2, so Eq. (5.31) simplifies to

$$\int_{\text{section 2}} \rho V dA - \int_{\text{section 1}} \rho V dA = 0 \tag{5.32}$$

If density is assumed to be constant across each section, Eq. (5.32) simplifies to

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{5.33}$$

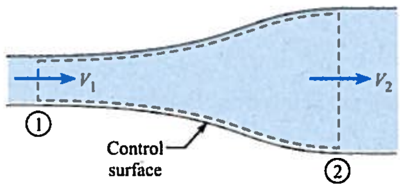


FIGURE 5.13
Flow through a conduit

Eq. (5.33), which is called the *pipe flow form* of the continuity equation, is the final result. The meaning of this equation is (rate of inflow of mass at section 1) = (rate of outflow of mass at section 2).

There are other useful ways of writing the continuity equation. For example, Eq. (5.33) can be written in several equivalent forms:

$$\rho_1 Q_1 = \rho_2 Q_2 \tag{5.34}$$

$$\dot{m}_1 = \dot{m}_2 \tag{5.35}$$

If density is assumed to be constant, then Eq. (5.34) reduces to

$$Q_2 = Q_1 \tag{5.36}$$

Eq. (5.34) is valid for both steady and unsteady incompressible flow in a pipe. If there are more than two ports and the accumulation term is zero, then Eq. (5.29) can be reduced to

$$\sum_{cs} \dot{m}_i = \sum_{cs} \dot{m}_o \tag{5.37}$$

If the flow is assumed to have constant density, Eq. (5.37) can be written in terms of discharge:

$$\sum_{cs} Q_i = \sum_{cs} Q_o \tag{5.38}$$

Summary Depending on the assumptions of the problem, there are many ways to write the continuity equation. However, one can analyze any problem using the three equations

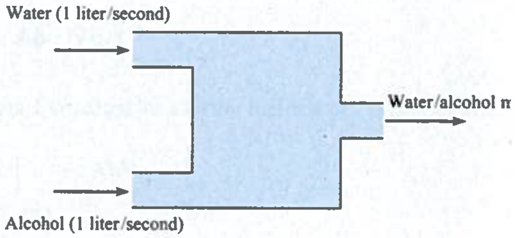
summarized in Table 5.5. Thus, we recommend starting with one of these three equati[ons]
because this is simpler than remembering many different equations.

✔CHECKPOINT PROBLEM 5.3

Water and alcohol mix in a tank. Can the continuity equation be used to show that the outlet flow rate is 2 liters per second?

a. yes

b. no



Example 5.8 shows how to apply continuity to flow in a pipe.

EXAMPLE 5.8

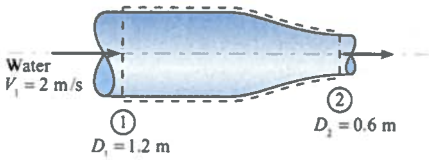
Applying the Continuity Equation to Flow in a Variable Area Pipe

Problem Statement

A 120 cm pipe is in series with a 60 cm pipe. The speed of the water in the 120 cm pipe is 2 m/s. What is the water speed in the 60 cm pipe?

Define the Situation

Water flows through a contraction in a pipe.



State the Goal

V_2(m/s) ⬅ Mean velocity at section 2

Generate Ideas and Make a Plan

Selection. Select the continuity equation because the problem variables are velocity and pipe diameter.

Sketch. Select a fixed CV. Sketch this CV on the situation diagram. Label the inlet as section 1 and outlet as section 2.

Analysis. Select the pipe flow form of continuity (i.e., Eq. 5.33) because the problem involves flow in a pipe.

$$\rho A_1 V_1 = \rho A_2 V_2 \quad \text{(a)}$$

Assume density is constant (this is standard practice for steady flow of a liquid). The continuity equation reduces to

$$A_1 V_1 = A_2 V_2 \quad \text{(b)}$$

Validate. To validate Eq (b), notice that the primary dimensions of each term are L^3/T. Also, this equation makes physical sense because it can be interpreted as (inflow) = (outflow).

Plan. Eq (b) contains the goal (V_2) and all other variables are known. Thus, the plan is to substitute numbers into this equation.

Take Action (Execute the Plan)

Continuity Equation:

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D_1}{D_2}\right)^2$$

$$V_2 = (2\ \text{m/s})\left(\frac{1.2\ \text{m}}{0.6\ \text{m}}\right)^2 = \boxed{8\ \text{m/s}}$$

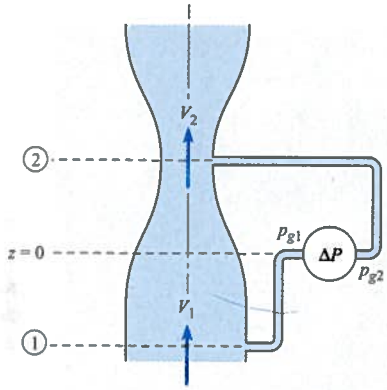
Example 5.9 shows how the continuity equation can be applied together with the Bernoulli equation

EXAMPLE 5.9

Applying the Bernoulli and Continuity Equations to Flow through a Venturi

Problem Statement

Water with a density of 1000 kg/m^3 flows through a vertical venturimeter as shown. A pressure gage is connected across two taps in the pipe (station 1) and the throat (station 2). The area ratio A_{throat}/A_{pipe} is 0.5. The velocity in the pipe is 10 m/s. Find the pressure difference recorded by the pressure gage. Assume the flow has a uniform velocity distribution and that viscous effects are not important.



Define the Situation

Water flows in venturimeter. Area ratio = 0.5. $V_1 = 10$ m/s.

Assumptions:

1. Velocity distribution is uniform.
2. Viscous effects are unimportant.

Properties: $\rho = 1000$ kg/m^3.

State the Goal

Find: Pressure difference measured by gage.

Generate Ideas and Make a Plan

1. Because viscous effects are unimportant, apply the Bernoulli equation between stations 1 and 2.
2. Combine the continuity equation (5.33) with the results of step 1.
3. Find the pressure on the gage by applying the hydrostatic equation.

Take Action (Execute the Plan)

1. The Bernoulli equation

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

Rewrite the equation in terms of piezometric pressure.

$$p_{z_1} - p_{z_2} = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$= \frac{\rho V_1^2}{2}\left(\frac{V_2^2}{V_1^2} - 1\right)$$

2. Continuity equation $V_2/V_1 = A_1/A_2$

$$p_{z_1} - p_{z_2} = \frac{\rho V_1^2}{2}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$= \frac{1000\text{ kg/m}^3}{2} \times (10\text{ m/s})^2 \times (2^2 - 1)$$

$$= 150\text{ kPa}$$

3. Apply the hydrostatic equation between the gage attachment point where the pressure is p_{g_1} and station 1 where the gage line is tapped into the pipe,

$$p_{z_1} = p_{g_1}$$

Also $p_{z_2} = p_{g_2}$ so

$$\Delta p_{gage} = p_{g_1} - p_{g_2} = p_{z_1} - p_{z_2} = \boxed{150\text{ kPa}}$$

5.5 Predicting Cavitation

Designers can encounter a phenomenon, called cavitation, in which a liquid starts to boil due to low pressure. This situation is beneficial for some applications, but it is usually a problem that should be avoided by thoughtful design. Thus, this section describes cavitation and discusses how to design systems to minimize the possibility of harmful cavitation.

Description of Cavitation

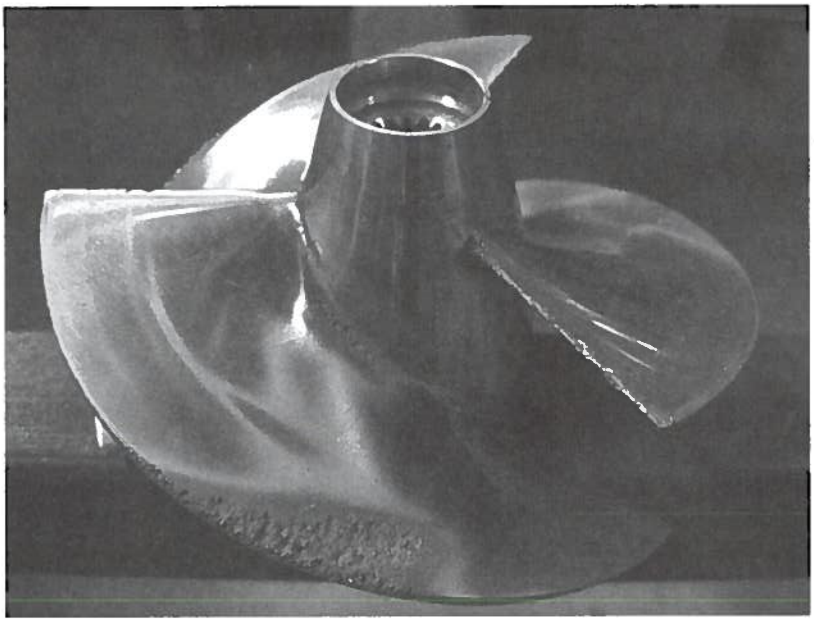
Cavitation is when fluid pressure at a given point in a system drops to the vapor pressure an boiling occurs.

EXAMPLE. Consider water flowing at 15°C in a piping system. If the pressure of the wat drops to the vapor pressure, the water will boil, and engineers will say that the system is cav tating. Because the vapor pressure of water at 15°C, which can be looked up in Appendix A. is $p_v = 1.7$ kPa abs, the condition required for cavitation is known. To avoid cavitation, th designer can configure the system so that pressures at all locations are above 1.7 kPa absolut

Cavitation can damage equipment and degrade performance. Boiling causes vapor bul bles to form, grow, and then collapse, producing shock waves, noise, and dynamic effects th lead to decreased equipment performance and, frequently, equipment failure. Cavitation dam age to a propeller (see Fig. 5.14) occurs because the spinning propeller creates low pressur near the tips of the blades where the velocity is high. Serious erosion produced by cavitation i a spillway tunnel of Hoover Dam is shown in Fig. 5.15.

FIGURE 5.14

Cavitation damage to a propeller. (Photo by Erik Axdahl)



Cavitation degrades materials because of the high pressures associated with the collapse vapor bubbles. Experimental studies reveal that very high intermittent pressure, as high 800 MPa (115,000 psi), develops in the vicinity of the bubbles when they collapse (1). There fore, if bubbles collapse close to boundaries such as pipe walls, pump impellers, valve casing and dam slipway floors, they can cause considerable damage. Usually this damage occurs the form of fatigue failure brought about by the action of millions of bubbles impacting (i effect, imploding) against the material surface over a long period of time, thus producing material pitting in the zone of cavitation.

In some applications, cavitation is beneficial. Cavitation is responsible for the effectivene of ultrasonic cleaning. Supercavitating torpedoes have been developed in which a large bubb envelops the torpedo, significantly reducing the contact area with the water and leading significantly faster speeds. Cavitation plays a medical role in shock wave lithotripsy for th destruction of kidney stones.

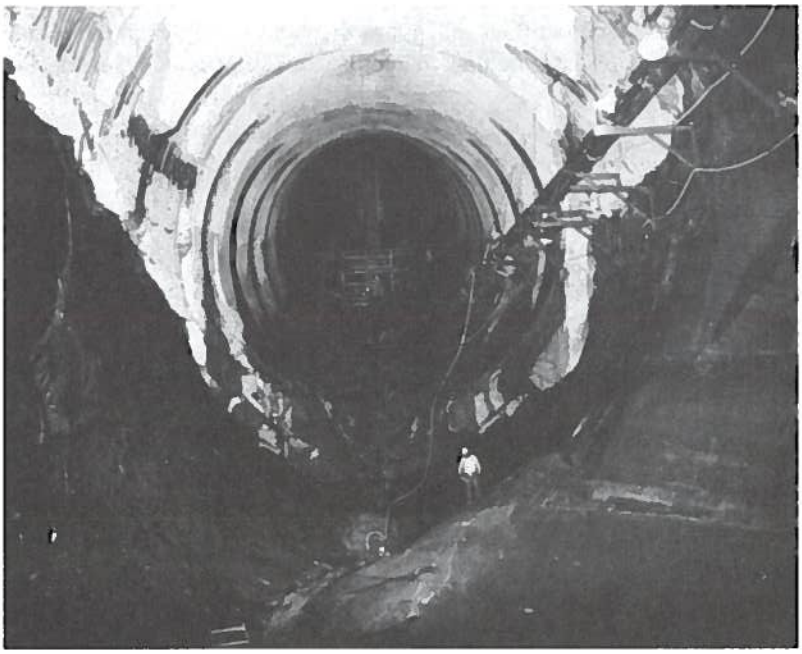


FIGURE 5.15
Cavitation damage to a hydroelectric power dam spillway tunnel. (I
Bureau of Reclamation

The world's largest and most technically advanced water tunnel for studying cavitation is located in Memphis, Tennessee—the William P. Morgan Large Cavitation Tunnel. This facility is used to test large-scale models of submarine systems and full-scale torpedoes as well as applications in the maritime shipping industry. More detailed discussions of cavitation can be found in Brennen (2) and Young (3).

Identifying Cavitation Sites

To predict cavitation, engineers looks for locations with low pressures. For example, when water flows through a pipe restriction (Fig. 5.16), the velocity increases according to the continuity equation, and in turn, the pressure decreases as dictated by the Bernoulli equation. For low flow rates, there is a relatively small drop in pressure at the restriction, so the water remains well above the vapor pressure, and boiling does not occur. However, as the flow rate increases, the pressure at the restriction becomes progressively lower until a flow rate is reached where the pressure is equal to the vapor pressure as shown in Fig. 5.16. At this point, the liquid boils to form bubbles, and cavitation ensues. The onset of cavitation can also be affected by the presence of contaminant gases, turbulence and by viscous effects.

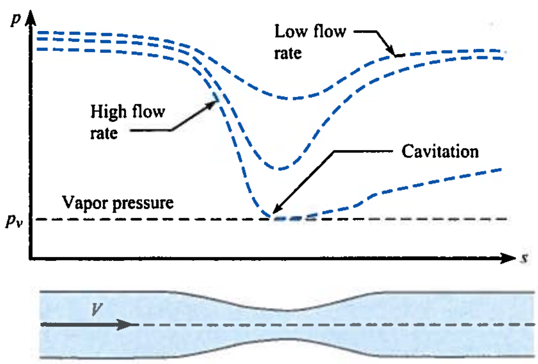


FIGURE 5.16
Flow through pipe restriction: variation of pressure for three diffe
flow rates.

The formation of vapor bubbles at the restriction in Fig. 5.16 is shown in Fig. 5.17a. T vapor bubbles form and then collapse as they move into a region of higher pressure and a swept downstream with the flow. When the flow velocity is increased further, the minimu pressure is still the local vapor pressure, but the zone of bubble formation is extended as sho in Fig. 5.17b. In this case, the entire vapor pocket may intermittently grow and collapse, pr ducing serious vibration problems.

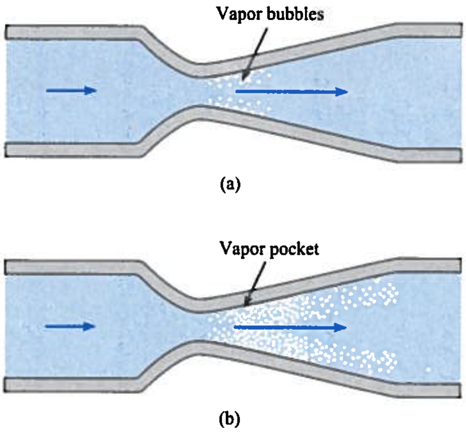


FIGURE 5.17 Formation of vapor bubbles in the process of cavitation. (a) Cavitation. (b) Cavitation—higher flow rate.

Summary Cavitation, which is caused by boiling of liquids at low pressures, is usually problema in an engineered system. Cavitation is most likely to occur at locations with low pressures such a

- High elevation points.
- Locations with high velocities. (e.g. constrictions in pipes, tips of propeller blades)
- The suction (inlet) side of pumps.

5.6 Summarizing Key Knowledge

Characterizing Flow Rate ($\dot{m}$ and Q)

- *Volume flow rate*, Q (m^3/s) is defined by

$$Q = \left(\frac{\text{volume of fluid passing through a cross sectional area}}{\text{interval of time}}\right)_{\substack{\text{instant}\\ \text{in time}}} = \lim_{\Delta t \to 0} \frac{\Delta \mathcal{V}}{\Delta t}$$

- Volume flow rate is also called *discharge*.
- Q can be calculated with four equations:

$$Q = \overline{V}A = \frac{\dot{m}}{\rho} = \int_A V\,dA = \int_A \mathbf{V} \cdot \mathbf{dA}$$

- *Mass flow rate*, $\dot{m}$ (kg/s), is defined as

$$\dot{m} = \left(\frac{\text{mass of fluid passing through a cross sectional area}}{\text{interval of time}}\right)_{\substack{\text{instant}\\ \text{in time}}} = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t}$$

- $\dot{m}$ can be calculated with four equations:

$$\dot{m} = \rho A\overline{V} = \rho Q = \int_A \rho V\,dA = \int_A \rho \mathbf{V} \cdot \mathbf{dA}$$

- *Mean-velocity*, $\overline{V}$ or V, is the value of velocity averaged over the section area at an instant in time. This concept is different than *time-averaged velocity*, which involves velocity averaged over time at a point in space.
- Typical values of mean velocity:
  - $\overline{V}/V_{max} = 0.5$ for laminar flow in a round pipe
  - $\overline{V}/V_{max} = 2/3 = 0.667$ for laminar flow in a rectangular conduit
  - $\overline{V}/V_{max} \approx 0.79$ to 0.86 for turbulent flow in a round pipe.
- Problems solvable with the flow rate equations can be organized into three categories:
  - *Algebraic Equations.* Problems in this category are solved by straightforward application of the equations (see Example 5.1).
  - *Dot Product.* When the area is not aligned with the velocity vector, then apply the dot product ($\mathbf{V} \cdot \mathbf{A}$) (see Example 5.2).
  - *Integration.* When velocity is given as a function of position, one integrates velocity over area (see Example 5.3).

The Control Volume Approach and Reynolds Transport Theorem

- A *system* is what the engineer selects to analyze. Systems can be classified into two categories: the closed system and the control volume.
  - A *closed system* is a given quantity of matter of fixed identity. Fixed identity means the closed system is always comprised of the same matter. Thus, mass cannot cross the boundary of a closed system.
  - A *control volume* (cv or CV) is a geometric region defined in space and enclosed by a *control surface* (cs or CS).
  - The Reynolds transport theorem is a mathematical tool for converting an equation written for a closed system to an equation written for a control volume.

The Continuity Equation

- The *law of conservation of mass* for a control volume is called the *continuity equation*.
- The physics of the continuity equation are

$$\begin{pmatrix}\text{rate of}\\ \text{accumulation of mass}\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{inflow of mass}\end{pmatrix} - \begin{pmatrix}\text{rate of}\\ \text{outflow of mass}\end{pmatrix}$$

- The continuity equation can be applied at an instant in time, and the units are kg/s. Also, the continuity equation can be integrated and applied over a finite time interval (e.g., 5 minutes), in which case the units are kg.
- Three useful forms of the continuity equation (see Table 5.5 on page 184) are
  - The general equation (always applies)
  - The simplified form (useful when there are well defined inlet and outlet ports)
  - The pipe flow form (applies to flow in a pipe)

Cavitation

- Cavitation occurs in a flowing liquid when the pressure drops to the local vapor pressure of the liquid.
- Vapor pressure is discussed in Chapter 2. Data for water are presented in Table A.5.

- Cavitation is usually undesirable because it can cause reduced performance Cavitation ca cause erosion or pitting of solid materials, noise, vibrations, and structural failures.
- Cavitation is most likely to occur in regions of high velocity, in inlet regions of centrifug pumps, and at locations of high elevations.
- To reduce the probability of cavitation, designers can specify that components that are susceptible to cavitation (e.g., values and centrifugal pumps) be situated at low elevations

REFERENCES

1. Knapp, R.T., J. W. Daily, and F. G. Hammitt. *Cavitation*. New York: McGraw-Hill, 1970.

2. Brennen, C. E. *Cavitation and Bubble Dynamics*. New York: Oxford University Press, 1995.

3. Young, F. R. *Cavitation*. New York: McGraw-Hill, 1989.

PROBLEMS

PLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Characterizing Flow Rates (§5.1)

5.1 Consider filling the gasoline tank of an automobile at a gas station. (a) Estimate the discharge in gpm. (b) Using the same nozzle, estimate the time to put 50 gallons in the tank. (c) Estimate the cross-sectional area of the nozzle and calculate the velocity at the nozzle exit.

5.2 The average flow rate (release) through Grand Coulee Dam is 110,000 ft^3/s. The width of the river downstream of the dam is 100 yards. Making a reasonable estimate of the river velocity, estimate the river depth.

5.3 Taking a jar of known volume, fill with water from your household tap and measure the time to fill. Calculate the discharge from the tap. Estimate the cross-sectional area of the faucet outlet, and calculate the water velocity issuing from the tap.

5.4 PLUS Another name for the volume flow rate equation could be:

a. the discharge equation

b. the mass flow rate equation

c. either a or b

5.5 A liquid flows through a pipe with a constant velocity. If a pipe twice the size is used with the same velocity, will the flow rate be (a) halved, (b) doubled, (c) quadrupled? Explain.

5.6 PLUS For flow of a gas in a pipe, which form of the continuity equation is more general?

a. $V_1A_1 = V_2A_2$

b. $\rho_1V_1A_1 = \rho_2V_2A_2$

c. both are equally applicable

5.7 PLUS The discharge of water in a 35-cm-diameter pipe is 0.06 m^3/s. What is the mean velocity?

5.8 PLUS A pipe with a 18 in. diameter carries water having a velocity of 4 ft/s. What is the discharge in cubic feet per second and in gallons per minute (1 cfs equals 449 gpm)?

5.9 A pipe with a 2 m diameter carries water having a velocity of 4 m/s. What is the discharge in cubic meters per second and in cubic feet per second?

5.10 PLUS A pipe whose diameter is 6 cm transports air with a temperature of 20°C and pressure of 180 kPa absolute at 19 m/s. Determine the mass flow rate.

5.11 PLUS Natural gas (methane) flows at 25 m/s through a pipe with a 0.84 m diameter. The temperature of the methane is 15°C, and the pressure is 160 kPa gage. Determine the mass flow rate.

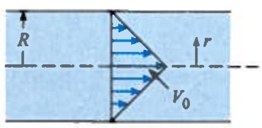
5.12 An aircraft engine test pipe is capable of providing a flow rate of 180 kg/s at altitude conditions corresponding to an absolute pressure of 50 kPa and a temperature of −18°C. The velocity of air through the duct attached to the engine is 255 m/ Calculate the diameter of the duct.

5.13 A heating and air-conditioning engineer is designing a system to move 1000 m^3 of air per hour at 100 kPa abs, and 30° The duct is rectangular with cross-sectional dimensions of 1 m by 20 cm. What will be the air velocity in the duct?

5.14 The hypothetical velocity distribution in a circular duct is

$$\frac{V}{V_0} = 1 - \frac{r}{R}$$

where r is the radial location in the duct, R is the duct radius, an V_0 is the velocity on the axis. Find the ratio of the mean velocity to the velocity on the axis.

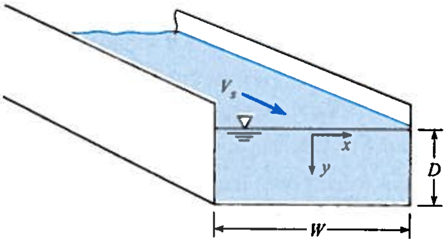


PROBLEM 5.14

5.15 Water flows in a two-dimensional channel of width W and depth D as shown in the diagram. The hypothetical velocity profile for the water is

$$V(x, y) = V_s\left(1 - \frac{4x^2}{W^2}\right)\left(1 - \frac{y^2}{D^2}\right)$$

where V_s is the velocity at the water surface midway between the channel walls. The coordinate system is as shown; x is measured from the center plane of the channel and y downward from the water surface. Find the discharge in the channel in terms of V_s, D, and W.



PROBLEM 5.15

5.16 GO Water flows in a pipe that has a 4 ft diameter and the following hypothetical velocity distribution: The velocity is maximum at the centerline and decreases linearly with r to a minimum at the pipe wall. If $V_{max} = 15$ ft/s and $V_{min} = 12$ ft/s, what is the discharge in cubic feet per second and in gallons per minute?

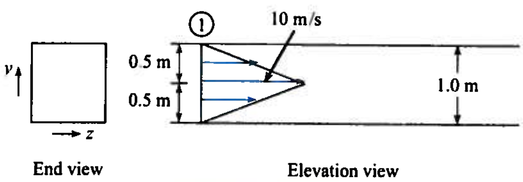
5.17 In Prob. 5.16, if $V_{max} = 8$ m/s, $V_{min} = 6$ m/s, and $D = 2$ m, what is the discharge in cubic meters per second and the mean velocity?

5.18 GO Air enters this square duct at section 1 with the velocity distribution as shown. Note that the velocity varies in the y direction only (for a given value of y, the velocity is the same for all values of z).

a. What is the volume rate of flow?

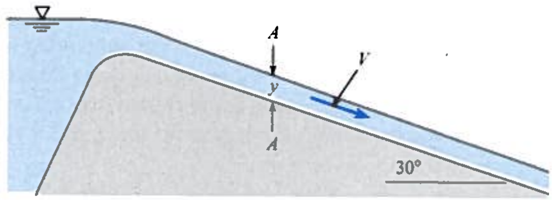
b. What is the mean velocity in the duct?

c. What is the mass rate of flow if the mass density of the air is 1.2 kg/m^3?



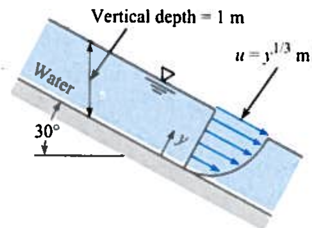
PROBLEM 5.18

5.19 PLUS The velocity at section A-A is 15 ft/s, and the vertical depth y at the same section is 4 ft. If the width of the channel is 28 ft, what is the discharge in cubic feet per second?



PROBLEM 5.19

5.20 PLUS The rectangular channel shown is 1.2 m wide. Wh the discharge in the channel?



PROBLEM 5.20

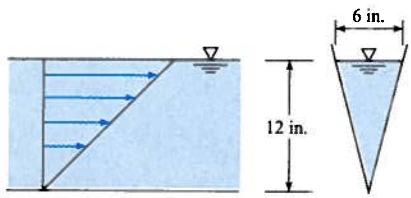
5.21 If the velocity in the channel of Prob. 5.20 is given as $u = 8[\exp(y) - 1]$ m/s and the channel width is 2 m, what is the discharge in the channel and what is the mean velocity?

5.22 PLUS Water from a pipe is diverted into a weigh tank for exactly 20 min. The increased weight in the tank is 20 kN. What is the discharge in cubic meters per second? Assume $T = 20$°C.

5.23 Water enters the lock of a ship canal through 180 ports, each port having a 2 ft by 2 ft cross section. The lock is 900 ft long and 105 ft wide. The lock is designed so that the water surface in it will rise at a maximum rate of 6 ft/min. For this condition, what will be the mean velocity in each port?

5.24 GO An empirical equation for the velocity distribution in a horizontal, rectangular, open channel is given by $u = u_{max}$ (y, where u is the velocity at a distance y feet above the floor of tl channel. If the depth d of flow is 1.2 m, $u_{max} = 3$ m/s, and $n =$ what is the discharge in cubic meters per second per meter of width of channel? What is the mean velocity?

5.25 The hypothetical water velocity in a V-shaped channel (the accompanying figure) varies linearly with depth from zer the bottom to maximum at the water surface. Determine the discharge if the maximum velocity is 6 ft/s.



PROBLEM 5.25

5.26 The velocity of flow in a circular pipe varies according to the equation $V/V_c = (1 - r^2/r_0^2)^n$, where V_c is the centerline velocity, r_0 is the pipe radius, and r is the radial distance from the centerline. The exponent n is general and is chosen to fit a given profile ($n = 1$ for laminar flow). Determine the mean velocity as a function of V_c and n.

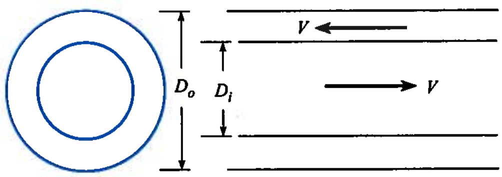
5.27 Plot the velocity distribution across the pipe, and determine the discharge of a fluid flowing through a pipe 1 m in diameter that has a velocity distribution given by $V = 12(1 - r^2/r_0^2)$ m/s. Here r_0 is the radius of the pipe, and r is the radial distance from the centerline. What is the mean velocity?

5.28 Water flows through a 4.0-in.-diameter pipeline at 75 lbm/min. Calculate the mean velocity. Assume $T = 60°$F.

5.29 WILEY PLUS Water flows through a 15 cm pipeline at 700 kg/min. Calculate the mean velocity in meters per second if $T = 20°$C.

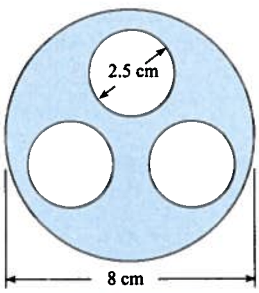
5.30 Water from a pipeline is diverted into a weigh tank for exactly 15 min. The increased weight in the tank is 4765 lbf. What is the average flow rate in gallons per minute and in cubic feet per second? Assume $T = 60°$F.

5.31 A shell and tube heat exchanger consists of a one pipe inside another pipe as shown. The liquid flows in opposite directions in each pipe. If the speed of the liquid is the same in each pipe, what is the ratio of the outer pipe diameter to the inner pipe diameter if the discharge in each pipe is the same?



PROBLEM 5.31

5.32 WILEY PLUS The cross section of a heat exchanger consists of three circular pipes inside a larger pipe. The internal diameter of the three smaller pipes is 2.5 cm, and the pipe wall thickness is 3 mm. The inside diameter of the larger pipe is 8 cm. If the velocity of the fluid in region between the smaller pipes and larger pipe is 10 m/s, what is the discharge in m^3/s?



PROBLEM 5.32

5.33 WILEY PLUS The mean velocity of water in a 6-in. pipe is 8.5 ft/s. Determine the flow in slugs per second, gallons per minute, and cubic feet per second if $T = 60°$F.

Lagrangian and Eulerian Approaches (§5.2)

5.34 Read §4.2, §5.2 and the internet to find answers to the following questions.

a. What does the Lagrangian approach mean? What are three real-world examples that illustrate the Lagrangian approach? (Use examples that are not in the text.)
b. What does the Eulerian approach mean? What are three real-world examples that illustrate the Eulerian approach? (Use examples that are not in the text.)
c. What are three important differences between the Eulerian and the Lagrangian approaches?
d. Why use an Eulerian approach? What are the benefits?
e. What is a field? How is a field related to the Eulerian approach?
f. What are the shortcomings of describing a flow field using the Lagrangian description?

5.35 What is the difference between an intensive and extensive property? Give an example of each.

5.36 WILEY PLUS State whether each of the following quantities is extensive or intensive:

a. mass
b. volume
c. density
d. energy
e. specific energy

5.37 WILEY PLUS What type of property do you get when you divide an extensive property by another extensive property—extensive or intensive? Hint: Consider density.

The Control Volume Approach (§5.2)

5.38 What is a control surface and a control volume? Can mass pass through a control surface?

5.39 WILEY PLUS In Fig 5.11 on p. 181 of §5.2,

a. the CV is passing through the system.
b. the system is passing through the CV.

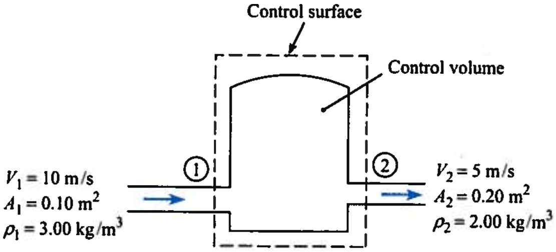
5.40 What is the purpose of the Reynolds transport theorem?

5.41 WILEY PLUS Gas flows into and out of the chamber as shown. For the conditions shown, which of the following statement(s) are true of the application of the control volume equation to the continuity principle?

a. $B_{sys} = 0$
b. $dB_{sys}/dt = 0$
c. $\sum_{cs} b\rho \mathbf{V} \cdot \mathbf{A} = 0$

d. $\frac{d}{dt}\int_{cv} \rho\, d\text{V} = 0$

e. $b = 0$



PROBLEM 5.41

5.42 WILEY PLUS The piston in the cylinder is moving up. Assume that the control volume is the volume inside the cylinder above the piston (the control volume changes in size as the piston moves). A gaseous mixture exists in the control volume. For the given conditions, indicate which of the following statements are true.

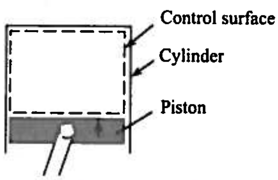
a. $\sum_{cs} \rho \mathbf{V} \cdot \mathbf{A}$ is equal to zero.

b. $\frac{d}{dt}\int_{cv} \rho\, d\text{V}$ is equal to zero.

c. The mass density of the gas in the control volume is increasing with time.

d. The temperature of the gas in the control volume is increasing with time.

e. The flow inside the control volume is unsteady.



PROBLEM 5.42

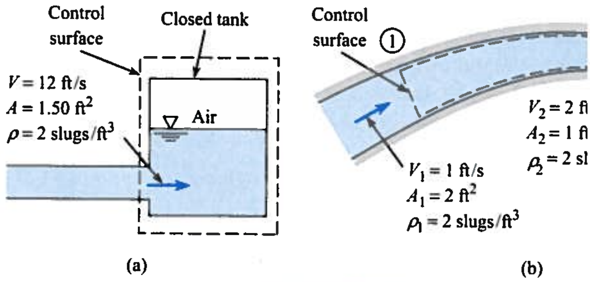
5.43 WILEY PLUS For cases *a* and *b* shown in the figure, respond to the following questions and statements concerning the application of the Reynolds transport theorem to the continuity equation.

a. What is the value of *b*?

b. Determine the value of dB_{sys}/dt.

c. Determine the value of $\sum_{cs} b\rho \mathbf{V} \cdot \mathbf{A}$.

d. Determine the value of $d/dt\int_{cv} b\rho\, d\text{V}$.



PROBLEM 5.43

Continuity Equation (Theory) (§5.3)

5.44 WILEY PLUS The law of conservation of mass for a closed syst requires that the mass of the system is

a. constant

b. zero

Applying the Continuity Equation (§5.4)

5.45 WILEY PLUS **part a only** Consider the simplified form of the continuity equation, Eq. 5.29 on p. 183 of §5.3. An engineer using this equation to find the Q_C of a creek at the confluen with a large river because she has automatic electronic measurements of the river discharge upstream, Q_{Ru}, and downstream, Q_{Rd}, of the creek confluence.

a. Which of the three terms on the left-hand side of Ec will the engineer assume is zero? Why?

b. Sketch the creek and the river and sketch the CV yc would select to solve this problem.

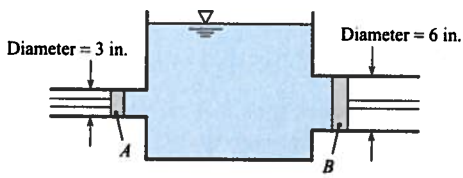
5.46 A pipe flows full with water. Is it possible for the volun flow rate into the pipe to be different than the flow rate out c pipe? Explain.

5.47 Air is pumped into one end of a tube at a certain mass rate. Is it necessary that the same mass flow rate of air comes the other end of the tube? Explain.

5.48 If an automobile tire develops a leak, how does the ma air and density change inside the tire with time? Assuming t temperature remains constant, how is the change in density related to the tire pressure?

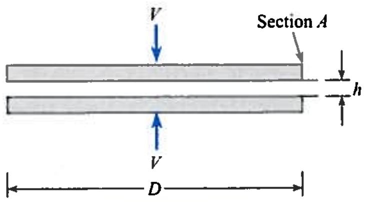
5.49 WILEY PLUS Two pipes are connected together in series. The diameter of one pipe is twice the diameter of the second pip With liquid flowing in the pipes, the velocity in the large pip is 4 m/s. What is the velocity in the smaller pipe?

5.50 Both pistons are moving to the left, but piston *A* has a speed twice as great as that of piston *B*. Then is the water lev the tank (a) rising, (b) not moving up or down, or (c) falling?



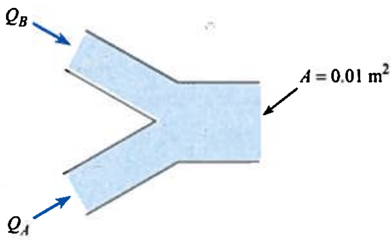
PROBLEM 5.50

5.51 Two parallel disks of diameter D are brought together, each with a normal speed of V. When their spacing is h, what is the radial component of convective acceleration at the section just inside the edge of the disk (section A) in terms of V, h, and D? Assume uniform velocity distribution across the section.



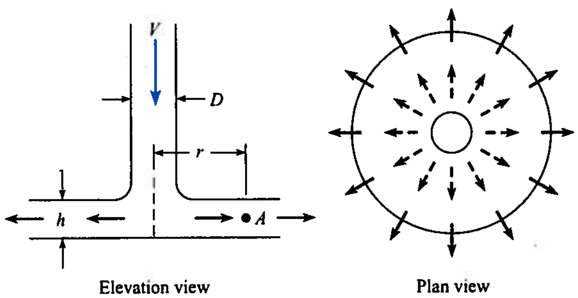
PROBLEM 5.51

5.52 WILEY PLUS Two streams discharge into a pipe as shown. The flows are incompressible. The volume flow rate of stream A into the pipe is given by $Q_A = 0.04t$ m^3/s and that of stream B by $Q_B = 0.006\,t^2$ m^3/s, where t is in seconds. The exit area of the pipe is 0.01 m^2. Find the velocity and acceleration of the flow at the exit at $t = 1$ s.



PROBLEM 5.52

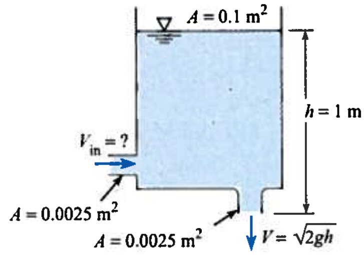
5.53 Air discharges downward in the pipe and then outward between the parallel disks. Assuming negligible density change in the air, derive a formula for the acceleration of air at point A, which is a distance r from the center of the disks. Express the acceleration in terms of the constant air discharge Q, the radial distance r, and the disk spacing h. If $D = 10$ cm, $h = 0.6$ cm, and $Q = 0.380$ m^3/s, what are the velocity in the pipe and the acceleration at point A where $r = 20$ cm?



PROBLEMS 5.53, 5.54

5.54 All the conditions of Prob. 5.53 are the same except that $h = 1$ cm and the discharge is given as $Q = Q_0(t/t_0)$, where $Q_0 = 0.1$ m^3/s and $t_0 = 1$ s. For the additional conditions, what will be the acceleration at point A when $t = 2$ s and $t = 3$ s?

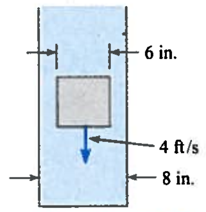
5.55 WILEY GO A tank has a hole in the bottom with a cross-sectional area of 0.0025 m^2 and an inlet line on the side with a cross-sectional area of 0.0025 m^2, as shown. The cross-sectional area of the tank is 0.1 m^2. The velocity of the liquid flowing out the bottom hole is $V = \sqrt{2gh}$, where h is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1 m and rising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet.



PROBLEM 5.55

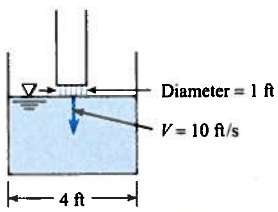
5.56 WILEY PLUS A mechanical pump is used to pressurize a bicycle tire. The inflow to the pump is 0.8 cfm. The density of the air entering the pump is 0.075 lbm/ft^3. The inflated volume of a bicycle tire is 0.035 ft^3. The density of air in the inflated tire is 0.4 lbm/ft^3. How many seconds does it take to pressurize the tire if there initially was no air in the tire?

5.57 A 6-in.-diameter cylinder falls at a rate of 4 ft/s in an 8-in.-diameter tube containing an incompressible liquid. What is the mean velocity of the liquid (with respect to the tube) in the space between the cylinder and the tube wall?



PROBLEM 5.57

5.58 WILEY PLUS This circular tank of water is being filled from a pipe as shown. The velocity of flow of water from the pipe is 10 ft/s. What will be the rate of rise of the water surface in the tank?



PROBLEM 5.58

5.59 A sphere 8 inches in diameter falls at 4 ft/s downward axially through water in a 1-ft-diameter container. Find the upward speed of the water with respect to the container wall at the midsection of the sphere.

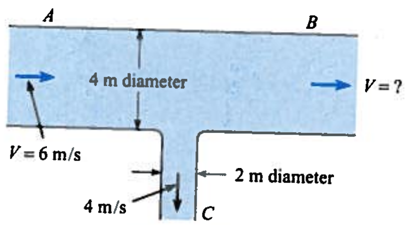
5.60 WILEY PLUS A rectangular air duct 20 cm by 60 cm carries a flow of 1.44 m^3/s. Determine the velocity in the duct. If the duct tapers to 10 cm by 40 cm, what is the velocity in the latter section? Assume constant air density.

5.61 WILEY PLUS A 30 cm pipe divides into a 20 cm branch and a 18 cm branch. If the total discharge is 0.40 m^3/s and if the same mean velocity occurs in each branch, what is the discharge in each branch?

5.62 The conditions are the same as in Prob. 5.61 except that the discharge in the 20 cm branch is twice that in the 15 cm branch. What is the mean velocity in each branch?

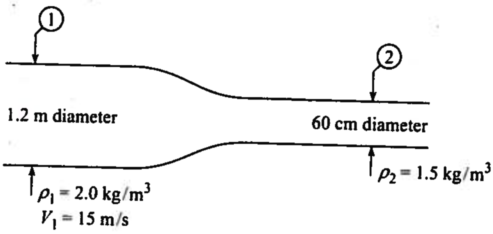
5.63 WILEY PLUS Water flows in a 10 in. pipe that is connected in series with a 6 in. pipe. If the rate of flow is 898 gpm (gallons per minute), what is the mean velocity in each pipe?

5.64 What is the velocity of the flow of water in leg B of the tee shown in the figure?



PROBLEM 5.64

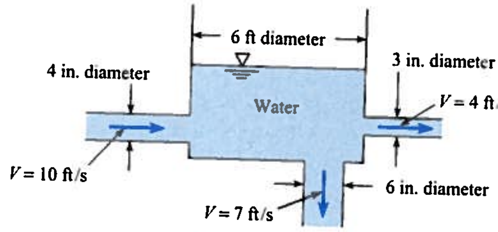
5.65 WILEY PLUS For a steady flow of gas in the conduit shown, what is the mean velocity at section 2?



PROBLEM 5.65

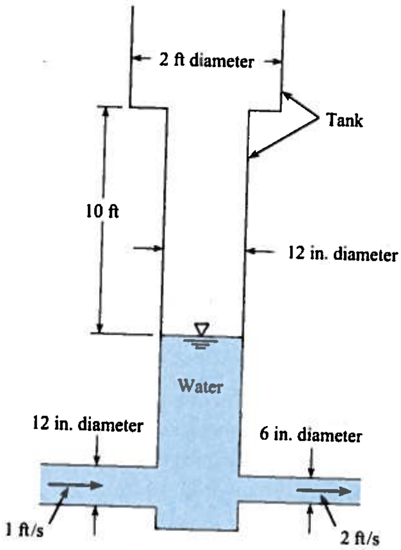
5.66 Two pipes, A and B, are connected to an open water tank. The water is entering the bottom of the tank from pipe A at 10 cfm. The water level in the tank is rising at 1.0 in./min, and the surface area of the tank is 80 ft^2. Calculate the discharge in a second pipe, pipe B, that is also connected to the bottom of the tank. Is the flow entering or leaving the tank from pipe B?

5.67 Is the tank in the figure filling or emptying? At what rate is the water level rising or falling in the tank?



PROBLEM 5.67

5.68 WILEY GO Given: Flow velocities as shown in the figure and water surface elevation (as shown) at $t = 0$ s. At the end of will the water surface in the tank be rising or falling, and at speed?



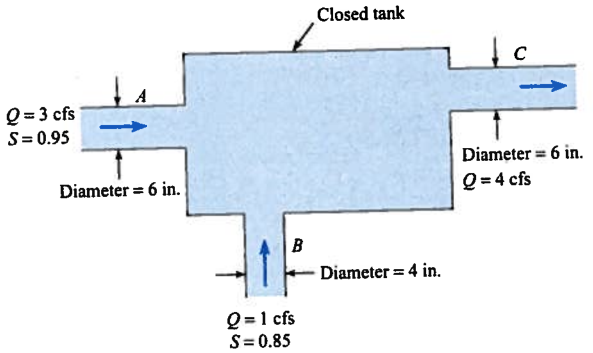
PROBLEM 5.68

5.69 WILEY GO A lake with no outlet is fed by a river with a consta flow of 1200 ft^3/s. Water evaporates from the surface at a cons rate of 13 ft^3/s per square mile surface area. The area varies wi depth h (feet) as A (square miles) = 4.5 + 5.5h. What is the equilibrium depth of the lake? Below what river discharge wil the lake dry up?

5.70 A stationary nozzle discharges water against a plate mov toward the nozzle at half the jet velocity. When the discharge from the nozzle is 5 cfs, at what rate will the plate deflect wate

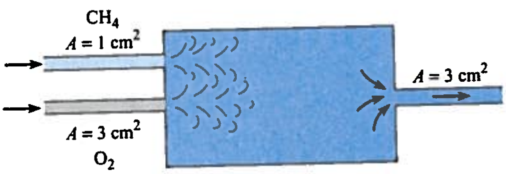
5.71 An open tank has a constant inflow of 20 ft^3/s. A 1.0-ft-diameter drain provides a variable outflow velocity V_{out} equal $\sqrt{(2gh)}$ ft/s. What is the equilibrium height h_{eq} of the liquid in the tank?

5.72 Assuming that complete mixing occurs between the two inflows before the mixture discharges from the pipe at C, find the mass rate of flow, the velocity, and the specific gravity of the mixture in the pipe at C.



PROBLEM 5.72

5.73 WILEY PLUS Oxygen and methane are mixed at 200 kPa absolute pressure and 100°C. The velocity of the gases into the mixer is 5 m/s. The density of the gas leaving the mixer is 1.9 kg/m^3. Determine the exit velocity of the gas mixture.

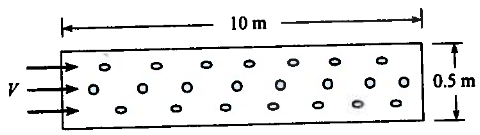


PROBLEM 5.73

5.74 WILEY PLUS A pipe with a series of holes as shown in the figure is used in many engineering systems to distribute gas into a system. The volume flow rate through each hole depends on the pressure difference across the hole and is given by

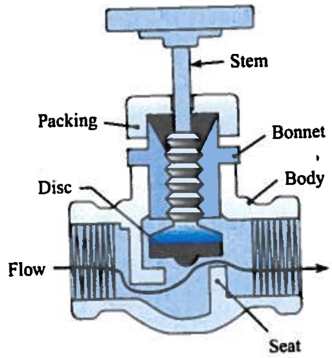
$$Q = 0.67\,A_o\left(\frac{2\Delta p}{\rho}\right)^{1/2}$$

where A_o is the area of the hole, Δp is the pressure difference across the hole, and ρ is the density of the gas in the pipe. If the pipe is sufficiently large, the pressure will be uniform along the pipe. A distribution pipe for air at 20° C is 0.5 meters in diameter and 10 m long. The gage pressure in the pipe is 100 Pa. The pressure outside the pipe is atmospheric at 1 bar. The hole diameter is 2.5 cm, and there are 50 holes per meter length of pipe. The pressure is constant in the pipe. Find the velocity of the air entering the pipe.



PROBLEM 5.74

5.75 The globe valve shown in the figure is a very common device to control flow rate. The flow comes through the pipe at the left and then passes through a minimum area formed by the disc and valve seat. As the valve is closed, the area for flow between the disc and valve is reduced. The flow area can be approximated by the annular region between the disc and the seat. The pressure drop across the valve can be estimated by application of the Bernoulli equation between the upstream pipe and the opening between the disc and valve seat. Assume there is a 10 gpm (gallons per minute) flow of water at 60°F through the valve. The inside diameter of the upstream pipe is 1 inch. The distance across the opening from the disc to the seat is 1/8th of an inch, and the diameter of the opening is 1/2 inch. What is the pressure drop across the valve in psid?

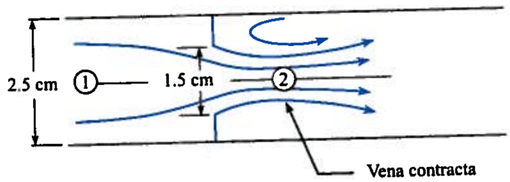


PROBLEM 5.75

5.76 In the flow through an orifice shown in the diagram the flow goes through a minimum area downstream of the orifice. This is called the "vena contracta." The ratio of the flow area at the vena contracta to the area of the orifice is 0.64.

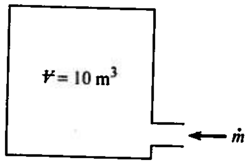
a. Derive an equation for the discharge through the orific in the form $Q = CA_o(2\Delta p/\rho)^{1/2}$, where A_o is the area of the orifice, Δp is the pressure difference between the upstream flow and the vena contracta, and ρ is the flui density. C is a dimensionless coefficient.

b. Evaluate the discharge for water at 1000 kg/m^3 and a pressure difference of 10 kPa for a 1.5 cm orifice centered in a 2.5-cm-diameter pipe.



PROBLEM 5.76

5.77 WILEY PLUS A compressor supplies gas to a 10 m^3 tank. The inl mass flow rate is given by $\dot{m} = 0.5\,\rho_0/\rho$ (kg/s), where ρ is the density in the tank and ρ_0 is the initial density. Find the time i would take to increase the density in the tank by a factor of 2 the initial density is 2 kg/m^3. Assume the density is uniform throughout the tank.

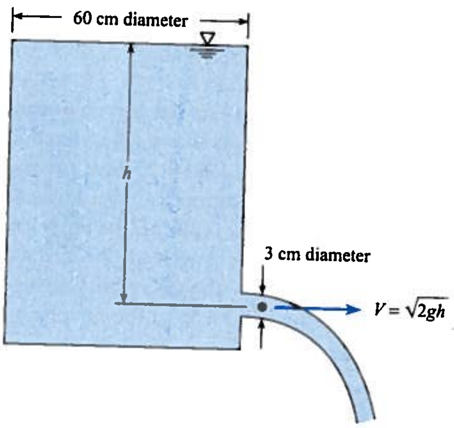


PROBLEM 5.77

5.78 A slow leak develops in a tire (assume constant volume), in which it takes 3 hr for the pressure to decrease from 30 psig to 25 psig. The air volume in the tire is 0.5 ft^3, and the temperature remains constant at 60°F. The mass flow rate of air is given by $\dot{m} = 0.68\,pA/\sqrt{RT}$. Calculate the area of the hole in the tire. Atmospheric pressure is 14 psia.

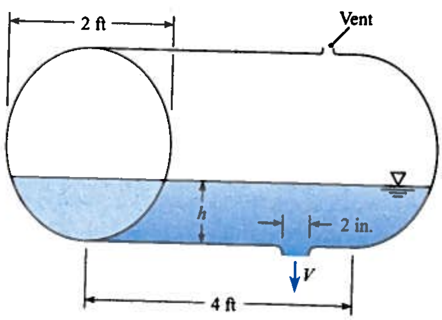
5.79 WILEY PLUS Oxygen leaks slowly through a small orifice in an oxygen bottle. The volume of the bottle is 0.1 m^3, and the diameter of the orifice is 0.12 mm. The temperature in the tank remains constant at 18°C, and the mass-flow rate is given by $\dot{m} = 0.68\,pA/\sqrt{RT}$. How long will it take the absolute pressure to decrease from 10 to 5 MPa?

5.80 How long will it take the water surface in the tank shown to drop from $h = 3$ m to $h = 50$ cm?



PROBLEM 5.80

5.81 A cylindrical drum of water, lying on its side, is being emptied through a 2 in.–diameter short pipe at the bottom of the drum. The velocity of the water out of the pipe is $V = \sqrt{2gh}$, where g is the acceleration due to gravity and h is the height of the water surface above the outlet of the tank. The tank is 4 long and 2 ft in diameter. Initially the tank is half full. Find time for the tank to empty.

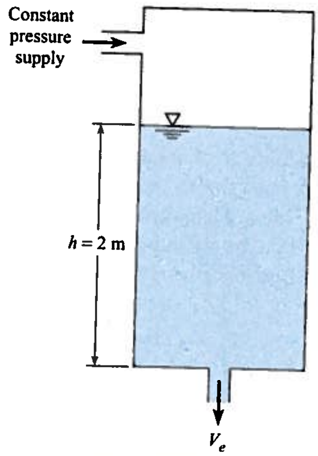


PROBLEM 5.81

5.82 WILEY GO Water is draining from a pressurized tank as sho the figure. The exit velocity is given by

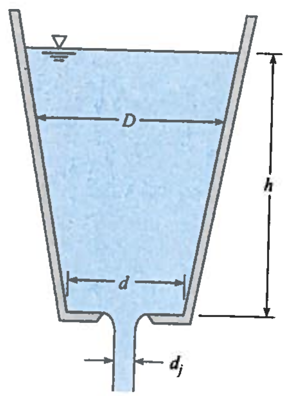
$$V_e = \sqrt{\frac{2p}{\rho} + 2gh}$$

where p is the pressure in the tank, ρ is the water density, an the elevation of the water surface above the outlet. The dept the water in the tank is 2 m. The tank has a cross-sectional a of 1 m^2, and the exit area of the pipe is 10 cm^2. The pressure the tank is maintained at 10 kPa. Find the time required to e the tank. Compare this value with the time required if the ta not pressurized.



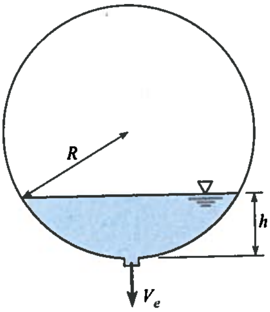
PROBLEM 5.82

5.83 For the type of tank shown, the tank diameter is given a $D = d + C_1 h$, where d is the bottom diameter and C_1 is a constant. Derive a formula for the time of fall of liquid surfac from $h = h_0$ to $h = h$ in terms of d_j, d, h_0, h, and C_1. Solve for $h_0 = 1$ m, $h = 20$ cm, $d = 20$ cm, $C_1 = 0.3$, and $d_j = 5$ cm. The velocity of water in the liquid jet exiting the tank is $V_e = \sqrt{2g}$



PROBLEM 5.83

5.84 A spherical tank with a diameter of 1 m is half filled with water. A port at the bottom of the tank is opened to drain the tank. The hole diameter is 1 cm, and the velocity of the water draining from the hole is $V_e = \sqrt{2gh}$, where h is the elevation of the water surface above the hole. Find the time required for the tank to empty.



PROBLEM 5.84

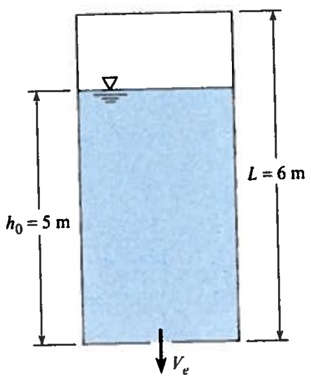
5.85 A tank containing oil is to be pressurized to decrease the draining time. The tank, shown in the figure, is 2 m in diameter and 6 m high. The oil is originally at a level of 5 m. The oil has a density of 880 kg/m³. The outlet port has a diameter of 2 cm, and the velocity at the outlet is given by

$$V_e = \sqrt{2gh + \frac{2p}{\rho}}$$

where p is the gage pressure in the tank, ρ is the density of the oil, and h is the elevation of the surface above the hole. Assume during the emptying operation that the temperature of the air in the tank is constant. The pressure will vary as

$$p = (p_0 + p_{atm})\frac{(L - h_0)}{(L - h)} - p_{atm}$$

where L is the height of the tank, p_{atm} is the atmospheric pressure, and the subscript 0 refers to the initial conditions. The initial pressure in the tank is 300 kPa gage, and the atmospheric pressure is 100 kPa.



PROBLEM 5.85

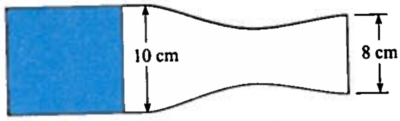
Applying the continuity equation to this problem, one finds

$$\frac{dh}{dt} = -\frac{A_e}{A_T}\sqrt{2gh + \frac{2p}{\rho}}$$

Integrate this equation to predict the depth of the oil with time for a period of one hour.

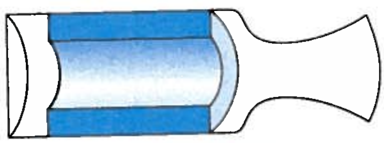
5.86 *Rocket Propulsion*. To prepare for problems 5.87, 5.88, and 5.89, use the Internet or other resources and define the following terms in the context of rocket propulsion: (a) solid fuel, (b) grain and (c) surface regression. Also explain how a solid-fuel rocket engine works.

5.87 An end-burning rocket motor has a chamber diameter of 10 cm and a nozzle exit diameter of 8 cm. The density of the solid propellant is 1800 kg/m³, and the propellant surface regresses at the rate of 1.5 cm/s. The gases crossing the nozzle exit plane have a pressure of 10 kPa abs and a temperatur of 2200°C. The gas constant of the exhaust gases is 415 J/kg K. Calculate the gas velocity at the nozzle exit plane.



PROBLEM 5.87

5.88 A cylindrical-port rocket motor has a grain design consisting of a cylindrical shape as shown. The curved internal surface and both ends burn. The solid propellant surface regresses uniformly at 1 cm/s. The propellant density is 2000 kg/n The inside diameter of the motor is 20 cm. The propellant grain is 40 cm long and has an inside diameter of 12 cm. The diamete of the nozzle exit plane is 20 cm. The gas velocity at the exit pla is 1800 m/s. Determine the gas density at the exit plane.



PROBLEM 5.88

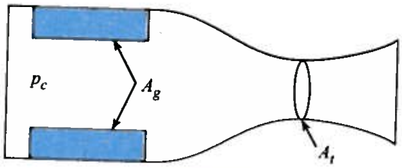
5.89 The mass flow rate through a rocket nozzle (shown) is given by

$$\dot{m} = 0.65\frac{p_c A_t}{\sqrt{RT_c}}$$

where p_c and T_c are the pressure and temperature in the rocket chamber and R is the gas constant of the gases in the chamber The propellant burning rate (surface regression rate) can be expressed as $\dot{r} = ap_c^n$, where a and n are two empirical constar Show, by application of the continuity equation, that the cham pressure can be expressed as

$$p_c = \left(\frac{a\rho_p}{0.65}\right)^{1/(1-n)}\left(\frac{A_g}{A_t}\right)^{1/(1-n)}(RT_c)^{1/[2(1-n)]}$$

where ρ_p is the propellant density and A_g is the grain surface burning area. If the operating chamber pressure of a rocket motor is 3.5 MPa and $n = 0.3$, how much will the chamber pressure increase if a crack develops in the grain, increasing the burning area by 20%?

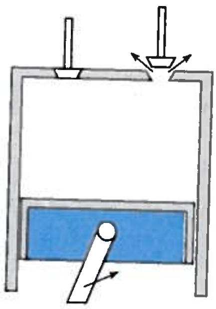


PROBLEM 5.89

5.90 The piston shown is moving up during the exhaust stroke of a four-cycle engine. Mass escapes through the exhaust port at a rate given by

$$\dot{m} = 0.65\frac{p_c A_v}{\sqrt{RT_c}}$$

where p_c and T_c are the cylinder pressure and temperature, A_v is the valve opening area, and R is the gas constant of the exhaust gases. The bore of the cylinder is 10 cm, and the piston is moving upward at 30 m/s. The distance between the piston and the head is 10 cm. The valve opening area is 1 cm^2, the chamber pressure is 300 kPa abs, the chamber temperature is 600°C, and the gas constant is 350 J/kg K. Applying the continuity equation, determine the rate at which the gas density is changing in the cylinder. Assume the density and pressure are uniform in the cylinder and the gas is ideal.



PROBLEM 5.90

5.91 WILEY PLUS Gas is flowing from Location 1 to 2 in the pipe expansion shown. The inlet density, diameter and velocity are ρ_1, D_1, and V_1 respectively. If D_2 is $2D_1$ and V_2 is half of V_1, what is the magnitude of ρ_2?

a. $\rho_2 = 4\,\rho_1$

b. $\rho_2 = 2\,\rho_1$

c. $\rho_2 = \frac{1}{2}\,\rho_1$

d. $\rho_2 = \rho_1$

5.92 WILEY PLUS Air is flowing from a ventilation duct (cross section 1) as shown, and is expanding to be released into a room at cross section 2. The area at cross section 2, A_2, is 3 times A_1. Assume that the is constant. The relation between Q_1 and Q_2 is:

a. $Q_2 = \frac{1}{3}\,Q_1$

b. $Q_2 = Q_1$

c. $Q_2 = 3\,Q_1$

d. $Q_2 = 9\,Q_1$

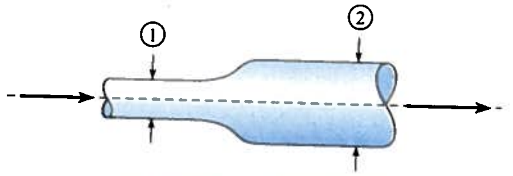
5.93 WILEY PLUS Water is flowing from Location 1 to 2 in this pi expansion. D_1 and V_1 are known at the inlet. D_2 and P_2 are known at the outlet. What equation(s) do you need to solv the inlet pressure P_1? Neglect viscous effects.

a. The continuity equation

b. The continuity equation and the flow rate equatio

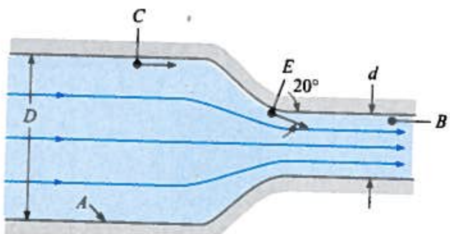
c. The continuity equation, the flow rate equation, an Bernoulli equation

d. There is insufficient information to solve the probl



PROBLEMS 5.91, 5.92, 5.93

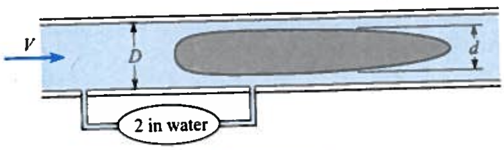
5.94 The flow pattern through the pipe contraction is as sh and the Q of water is 60 cfs. For $d = 2$ ft and $D = 6$ ft, what pressure at point B if the pressure at point C is 3200 psf?



PROBLEM 5.94

5.95 Water flows through a rigid contraction section of circul pipe in which the outlet diameter is one-half the inlet diameter. velocity of the water at the inlet varies with time as $V_{in} = (10$ m/ $[1 - \exp(-t/10)]$. How will the velocity vary with time at the ou

5.96 WILEY PLUS The annular venturimeter is useful for metering flows in pipe systems for which upstream calming distances a limited. The annular venturimeter consists of a cylindrical section mounted inside a pipe as shown. The pressure differen is measured between the upstream pipe and at the region adjacent to the cylindrical section. Air at standard conditions flows in the system. The pipe diameter is 6 in. The ratio of the cylindrical section diameter to the inside pipe diameter is 0.8. pressure difference of 2 in of water is measured. Find the volu flow rate. Assume the flow is incompressible, inviscid, and stea and that the velocity is uniformly distributed across the pipe.

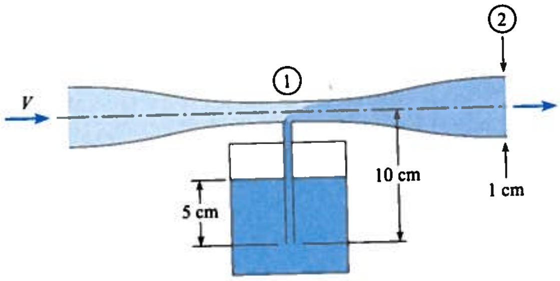


PROBLEM 5.96

5.97 Venturi-type applicators are frequently used to spray liquid fertilizers. Water flowing through the venturi creates a subatmospheric pressure at the throat, which in turn causes the liquid fertilizer to flow up the feed tube and mix with the water in the throat region. The venturi applicator shown uses water at 20°C to spray a liquid fertilizer with the same density. The venturi exhausts to the atmosphere, and the exit diameter is 1 cm. The ratio of exit area to throat area (A_2/A_1) is 2. The flow rate of water through the venturi is 8 L/m (liters/min). The bottom of the feed tube in the reservoir is 5 cm below the liquid fertilizer surface and 10 cm below the centerline of the venturi. The pressure at the liquid fertilizer surface is atmospheric. The flow rate through the feed tube between the reservoir and venturi throat is

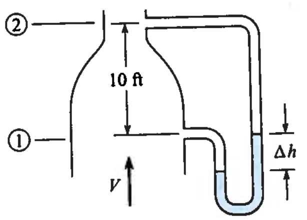
$$Q_l(\text{L/min}) = 0.5\sqrt{\Delta h}$$

where Δh is the drop in piezometric head (in meters) between the feed tube entrance and the venturi centerline. Find the flow rate of liquid fertilizer in the feed tube, Q_l. Also find the concentration of liquid fertilizer in the mixture, $[Q_l/(Q_l + Q_w)]$, at the end of the sprayer.



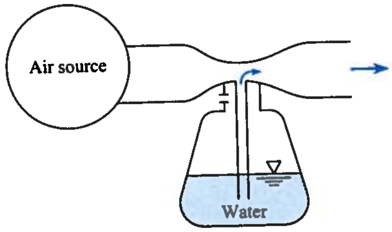
PROBLEM 5.97

5.98 WILEY PLUS Air with a density of 0.0644 lbm/ft^3 is flowing upward in the vertical duct, as shown. The velocity at the inlet (station 1) is 80 ft/s, and the area ratio between stations 1 and 2 is 0.5 ($A_2/A_1 = 0.5$). Two pressure taps, 10 ft apart, are connected to a manometer, as shown. The specific weight of the manometer liquid is 120 lbf/ft^3. Find the deflection, Δh, of the manometer.



PROBLEM 5.98

5.99 An atomizer utilizes a constriction in an air duct as shown. Design an operable atomizer making your own assumptions regarding the air source.



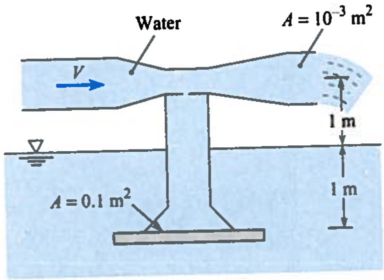
PROBLEM 5.99

5.100 WILEY PLUS A suction device is being designed based on the venturi principle to lift objects submerged in water. The operating water temperature is 15°C. The suction cup is located 1 m below the water surface, and the venturi throat is located 1 m above the water. The atmospheric pressure is 100 kPa. The ratio of the throat area to the exit area is 1/4, and the exit area is 0.001 m^2. The area of the suction cup is 0.1 m^2.

a. Find the velocity of the water at the exit for maximum lift condition.

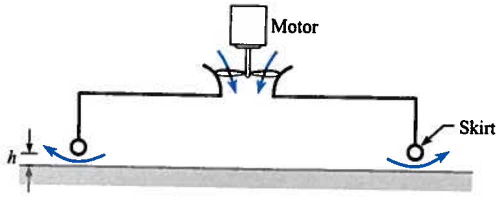
b. Find the discharge through the system for maximum lift condition.

c. Find the maximum load the suction cup can support.



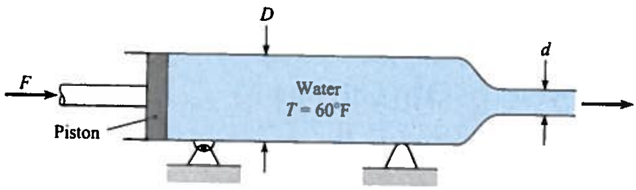
PROBLEM 5.100

5.101 WILEY PLUS A design for a hovercraft is shown in the figure. A fan brings air at 60°F into a chamber, and the air is exhausted between the skirts and the ground. The pressure inside the chamber is responsible for the lift. The hovercraft is 15 ft long and 7 ft wide. The weight of the craft including crew, fuel, and load is 2000 lbf. Assume that the pressure in the chamber is the stagnation pressure (zero velocity) and the pressure where the air exits around the skirt is atmospheric. Assume the air is incompressible, the flow is steady, and viscous effects are negligible. Find the airflow rate necessary to maintain the skirts at a height of 3 inches above the ground.



PROBLEM 5.101

5.102 Water is forced out of this cylinder by the piston. If the piston is driven at a speed of 6 ft/s, what will be the speed of efflux of the water from the nozzle if $d = 2$ in. and $D = 4$ in.? Neglecting friction and assuming irrotational flow, determine the force F that will be required to drive the piston. The exit pressure is atmospheric pressure.



PROBLEM 5.102

5.103 Air flows through a constant-area heated pipe. At the entrance, the velocity is 10 m/s, the pressure is 100 kPa absolute, and the temperature is 20°C. At the outlet, the pressure is 80 kPa absolute, and the temperature is 50°C. What is the velocity at the outlet? Can the Bernoulli equation be used to relate the pressure and velocity changes? Explain.

Predicting Cavitation (§5.5)

5.104 Sometimes driving your car on a hot day, you may encounter a problem with the fuel pump called pump cavitation. What is happening to the gasoline? How does this affect the operation of the pump?

5.105 What is cavitation? Why does the tendency for cavitation in a liquid increase with increased temperatures?

5.106 PLUS The following questions have to do with cavitation.

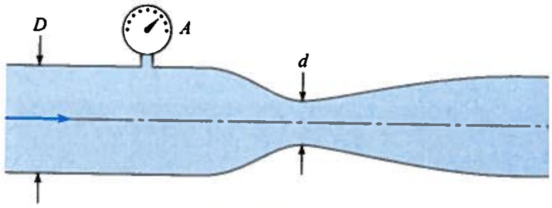
a. Is it more correct to say that cavitation has to do with (i) vacuum pressures, or (ii) vapor pressures?

b. Is cavitation more likely to occur on the low pressure (suction) side of a pump, or the high pressure (discharge) side? Why?

c. What does the word cavitation have to do with cavities, like the ones we get in our teeth? Is this aspect of cavitation the (i) cause, or the (ii) result of the phenomenon?

d. When water goes over a waterfall, and one can see lots of bubbles in the water, is that due to cavitation? Why, or why not?

5.107 GO When gage A indicates a pressure of 130 kPa gage, then cavitation just starts to occur in the venturi meter. If $D = 50$ cm and $d = 10$ cm, what is the water discharge in the system fo condition of incipient cavitation? The atmospheric pressure 100 kPa gage, and the water temperature is 10°C. Neglect gravitational effects.



PROBLEM 5.107

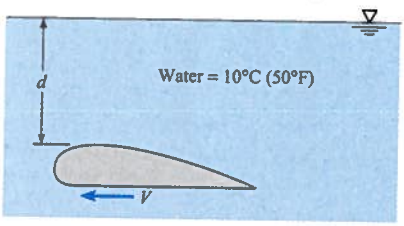
5.108 A sphere 1 ft in diameter is moving horizontally at a d of 12 ft below a water surface where the water temperature is 50°F. $V_{max} = 1.5\ V_o$, where V_o is the free stream velocity and occurs at the maximum sphere width. At what speed in still water will cavitation first occur?

5.109 GO When the hydrofoil shown was tested, the minim pressure on the surface of the foil was found to be 70 kPa abs when the foil was submerged 1.80 m and towed at a speed of 8 m/s. At the same depth, at what speed will cavitation first occur? Assume irrotational flow for both cases and $T = 10$°C.

5.110 For the hydrofoil of Prob. 5.109, at what speed will cavitation begin if the depth is increased to 3 m?

5.111 PLUS When the hydrofoil shown was tested, the minimum pressure on the surface of the foil was found to be 2.5 psi vacuum when the foil was submerged 4 ft and towe at a speed of 25 ft/s. At the same depth, at what speed will cavitation first occur? Assume irrotational flow for both case and $T = 50$°F.

5.112 For the conditions of Prob. 5.111, at what speed will cavitation begin if the depth is increased to 10 ft?



PROBLEMS 5.109, 5.110, 5.111, 5.112

5.113 A sphere is moving in water at a depth where the absol pressure is 18 psia. The maximum velocity on a sphere occurs from the forward stagnation point and is 1.5 times the free-stream velocity. The density of water is 62.4 lbm/ft³. Calculate t speed of the sphere at which cavitation will occur. $T = 50$°F.

5.114 The minimum pressure on a cylinder moving horizonta in water ($T = 10$°C) at 5 m/s at a depth of 1 m is 80 kPa absolu At what velocity will cavitation begin? Atmospheric pressure is 100 kPa absolute.