3. Velocity evaluation:

$$V = \left[2 \times 32.2\ \text{ft/s}^2 \times \frac{7}{12}\,\text{ft}\left(\frac{13.55}{0.81} - 1\right)\right]^{1/2}$$

$$= \left[2 \times 32.2 \times \frac{7}{12}(16.7 - 1)\,\text{ft}^2/\text{s}^2\right]^{1/2}$$

$$= \boxed{24.3\ \text{ft/s}}$$

Review the Solution and the Process

Discussion. The −1 in the quantity (16.7 − 1) reflects the effec of the column of kerosene in the right leg of the manometer, which tends to counterbalance the mercury in the left leg. Thus with a gas–liquid manometer, the counterbalancing effect is negligible.

EXAMPLE 4.8

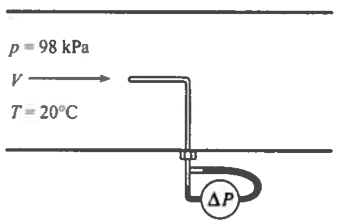
Applying a Pitot-Static Tube (pressure measured with a pressure gage)

Problem Statement

A differential pressure gage is connected across the taps of a Pitot-static tube. When this Pitot-static tube is used in a wind tunnel test, the gage indicates a Δp of 730 Pa. What is the air velocity in the tunnel? The pressure and temperature in the tunnel are 98 kPa absolute and 20°C, respectively.

Define the Situation

A differential pressure gage is attached to a Pitot-static tube for velocity measurement in a wind tunnel.



Assumptions:

- Airflow is steady.
- Pitot-tube equation applicable.

Properties: Table A.2, R_{air} = 287 J/kg K.

State the Goal

Find the air velocity (in m/s).

Generate Ideas and Make a Plan

1. Using the ideal gas law, calculate air density.
2. Using the Pitot-static tube equation, calculate the velocity.

Take Action (Execute the Plan)

1. Density calculation:

$$\rho = \frac{p}{RT} = \frac{98 \times 10^3\ \text{N/m}^2}{(287\ \text{J/kg K}) \times (20 + 273\ \text{K})} = 1.17\ \text{kg/m}^3$$

2. Pitot-static tube equation with differential pressure gage:

$$V = \sqrt{2\Delta p/\rho}$$

$$V = \sqrt{(2 \times 730\ \text{N/m}^2)/(1.17\ \text{kg/m}^3)} = \boxed{35.3\ \text{m/s}}$$

4.8 Characterizing Rotational Motion of a Flowing Fluid

In addition to velocity and acceleration, engineers also describe the rotation of a fluid. Th this topic is introduced in this section. At this point, we recommend the online vorticity fi (6) because this film shows the concepts in this section using laboratory experiments.

Concept of Rotation

Rotation of a fluid particle is defined as the average rotation of two initially mutually p pendicular faces of a fluid particle. The test is to look at the rotation of the line that bise both faces (a-a and b-b in Fig. 4.32). The angle between this line and the horizontal axis the rotation, θ.

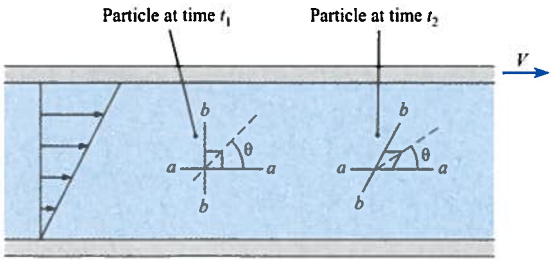


FIGURE 4.32
Rotation of a fluid pa
in flow between a mo
and stationary paralle
plate.

The general relationship between θ and the angles defining the sides is shown in Fig. 4.33, where θ_A is the angle of one side with the x-axis and the angle θ_B is the angle of the other side with the y-axis. The angle between the sides is $\beta = \frac{\pi}{2} + \theta_B - \theta_A$, so the orientation of the particle with respect to the x-axis is

$$\theta = \frac{1}{2}\beta + \theta_A = \frac{\pi}{4} + \frac{1}{2}(\theta_A + \theta_B)$$

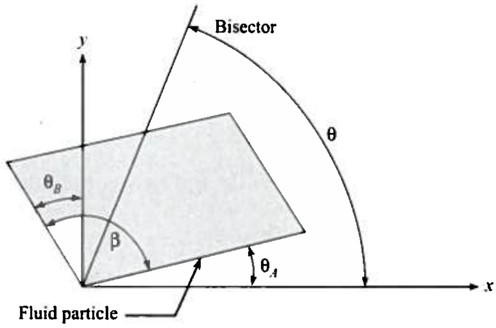


FIGURE 4.33
Orientation of rotatec
fluid particle.

The rotational rate of the particle is

$$\dot{\theta} = \frac{1}{2}(\dot{\theta}_A + \dot{\theta}_B) \qquad \textbf{(4.33)}$$

If $\dot{\theta} = 0$, the flow is **irrotational**, which means that the rotation rate (as defined by Eq. 4.33) is zero for all points in the velocity field.

Next, we derive an equation for $\dot{\theta}$ in terms of the velocity field. Consider the particle shown in Fig. 4.34. The sides of the particle are initially perpendicular with lengths Δx and Δy. Then the particle moves with time and deforms as shown with point 0 going to 0′, point 1 to 1′, and point 2 to 2′. The lengths of the sides are unchanged. After time Δt the horizontal side has rotated counterclockwise by $\Delta\theta_A$ and the vertical side clockwise (negative direction) by $-\Delta\theta_B$.

The y velocity component of point 1 is $v + (\partial v/\partial x)\Delta x$, and the x component of point 2 is $u + (\partial u/\partial y)\Delta y$. The net displacements of points 1 and 2 are*

$$\begin{aligned} \Delta y_1 &\sim \left[\left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta t - v\Delta t\right] = \frac{\partial v}{\partial x}\Delta x \Delta t \\ \Delta x_2 &\sim \left[\left(u + \frac{\partial u}{\partial y}\Delta y\right)\Delta t - u\Delta t\right] = \frac{\partial u}{\partial y}\Delta y \Delta t \end{aligned} \qquad \textbf{(4.34)}$$

*The symbol ~ means that the quantities are approximately equal but become exactly equal as the quantities approach zero.

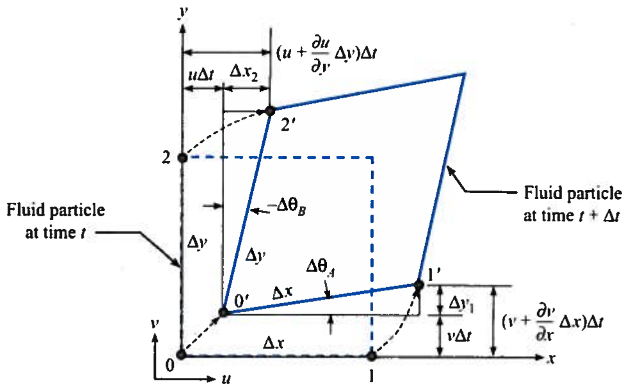


FIGURE 4.34
Translation and deformation of a fluid particle.

Referring to Fig. 4.34, the angles $\Delta\theta_A$ and $\Delta\theta_B$ are given by

$$\Delta\theta_A = \text{asin}\left(\frac{\Delta y_1}{\Delta x}\right) \sim \frac{\Delta y_1}{\Delta x} \sim \frac{\partial v}{\partial x}\Delta t$$
$$-\Delta\theta_B = \text{asin}\left(\frac{\Delta x_2}{\Delta y}\right) \sim \frac{\Delta x_2}{\Delta y} \sim \frac{\partial u}{\partial y}\Delta t \qquad \text{(4.3}$$

Dividing the angles by Δt and taking the limit as $\Delta t \rightarrow 0$,

$$\dot{\theta}_A = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta_A}{\Delta t} = \frac{\partial v}{\partial x}$$
$$\dot{\theta}_B = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta_B}{\Delta t} = -\frac{\partial u}{\partial y} \qquad \text{(4.3}$$

Substituting these results into Eq. (4.33) gives the rotational rate of the particle about the z-a
(normal to the page),

$$\dot{\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

This component of rotational velocity is defined as Ω_z, so

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(4.37}$$

Likewise, the rotation rates about the other axes are

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \text{(4.37}$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \text{(4.37}$$

The rate-of-rotation vector is

$$\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k} \qquad \text{(4.3}$$

An irrotational flow ($\Omega = 0$) requires that

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad \text{(4.39a)}$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \quad \text{(4.39b)}$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \quad \text{(4.39c)}$$

The most extensive application of these equations is in ideal flow theory. An ideal flow is the flow of an irrotational, incompressible fluid. Flow fields in which viscous effects are small can often be regarded as irrotational. In fact, if a flow of an incompressible, inviscid fluid is initially irrotational, it will remain irrotational.

Vorticity

The most common way to describe rotation is to use **vorticity**, which is a vector equal to twice the rate-of-rotation vector. The magnitude of the vorticity indicates the rotationality of a flow and is very important in flows where viscous effects dominate, such as boundary layer, separated, and wake flows. The vorticity equation is

$$\begin{aligned}\boldsymbol{\omega} &= 2\boldsymbol{\Omega}\\ &= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k} \\ &= \nabla \times \mathbf{V}\end{aligned} \quad \text{(4.40)}$$

where $\nabla \times \mathbf{V}$ is the curl of the velocity field.

An irrotational flow signifies that the vorticity vector is everywhere zero. Example 4.9 illustrates how to evaluate the rotationality of a flow field, and Example 4.10 evaluates the rotation of a fluid particle.

EXAMPLE 4.9

Evaluating Rotation

Problem Statement

The vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ represents a two-dimensional velocity field. Is the flow irrotational?

Define the Situation

Velocity field is given.

State the Goal

Determine if flow is irrotational.

Generate Ideas and Make a Plan

Because $w = 0$ and $\frac{\partial}{\partial z} = 0$, apply Eq. (4.39a) to evaluate rotationality.

Take Action (Execute the Plan)

Velocity components and derivatives

$$u = 10x \qquad \frac{\partial u}{\partial y} = 0$$

$$v = -10y \qquad \frac{\partial v}{\partial x} = 0$$

Thus, flow is irrotational.

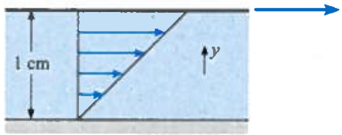
EXAMPLE 4.10

Rotation of a Fluid Particle

Problem Definition

A fluid exists between stationary and moving parallel flat plates, and the velocity is linear as shown. The distance between the plates is 1 cm, and the upper plate moves at 2 cm/s. Find the amount of rotation that the fluid particle located at 0.5 cm will undergo after it has traveled a distance of 1 cm.

Sketch:



Define the Situation

This problem involves Couette flow.

Assumptions: Planar flow ($w = 0$ and $\frac{\partial}{\partial z} = 0$).

State the Goal

Find the rotation of a fluid particle (in radians) at the midpoint after traveling 1 cm.

Generate Ideas and Make a Plan

1. Use Eq. (4.37a) to evaluate rotational rate with $v = 0$.
2. Find time for particle to travel 1 cm.
3. Calculate amount of rotation.

Take Action (Execute the Plan)

1. Velocity distribution

$$u = 0.02\ \text{m/s} \times \frac{y}{0.01\ \text{m}} = 2y\ (1/\text{s})$$

Rotational rate

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -1\ \text{rad/s}$$

2. Time to travel 1 cm:

$$u = 2\ (1/\text{s}) \times 0.005\ \text{m} = 0.01\ \text{m/s}$$

$$\Delta t = \frac{\Delta x}{u} = \frac{0.01\ \text{m}}{0.01\ \text{m/s}} = 1\ \text{s}$$

3. Amount of rotation

$$\Delta\theta = \Omega_z \times \Delta t = -1 \times 1 = -1\ \text{rad}$$

Review the Solution and the Process

Discussion. Note that the rotation is negative (in clockwise direction).

4.9 The Bernoulli Equation for Irrotational Flow

When flow is irrotational, the Bernoulli equation can be applied between any two points in tl flow. That is, the points do not need to be on the same streamline. This *irrotational form* of t Bernoulli equation is used extensively in applications such as classical hydrodynamics, t aerodynamics of lifting surfaces (wings), and atmospheric winds. Thus, this section descrit how to derive the Bernoulli equation for an irrotational flow.

To begin the derivation, apply the Euler equation, Eq. (4.15), in the n direction (normal the streamline)

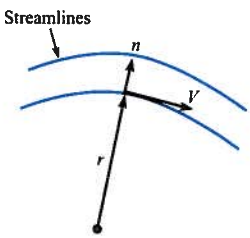
$$-\frac{d}{dn}(p + \gamma z) = \rho a_n \qquad \text{(4.4}$$

where the partial derivative of n is replaced by the ordinary derivative because the flow assumed steady (no time dependence). Two adjacent streamlines and the direction n is shov in Fig. 4.35. The local fluid speed is V, and the local radius of curvature of the streamline i: The acceleration normal to the streamline is the centripetal acceleration, so

$$a_n = -\frac{V^2}{r} \qquad \text{(4.4}$$

FIGURE 4.35

Two adjacent streamlines showing direction n between lines.



where the negative sign occurs because the direction n is outward from the center of curvature and the centripetal acceleration is toward the center of curvature. Using the irrotationality condition, the acceleration can be written as

$$a_n = -\frac{V^2}{r} = -V\left(\frac{V}{r}\right) = V\frac{dV}{dr} = \frac{d}{dr}\left(\frac{V^2}{2}\right) \tag{4.43}$$

Also the derivative with respect to r can be expressed as a derivative with respect to n by

$$\frac{d}{dr}\left(\frac{V^2}{2}\right) = \frac{d}{dn}\left(\frac{V^2}{2}\right)\frac{dn}{dr} = \frac{d}{dn}\left(\frac{V^2}{2}\right)$$

because the direction of n is the same as r so $dn/dr = 1$. Eq. (4.43) can be rewritten as

$$a_n = \frac{d}{dn}\left(\frac{V^2}{2}\right) \tag{4.44}$$

Substituting the expression for acceleration into Euler's equation, Eq. (4.41), and assuming constant density results in

$$\frac{d}{dn}\left(p + \gamma z + \rho\frac{V^2}{2}\right) = 0 \tag{4.45}$$

or

$$p + \gamma z + \rho\frac{V^2}{2} = C \tag{4.46}$$

which is the Bernoulli equation, and C is constant in the n direction (across streamlines).

Summary For an irrotational flow, the constant C in the Bernoulli equation is the same across streamlines as well as along streamlines, so it is the same everywhere in the flow field. Thus, *when applying the Bernoulli equation for irrotational flow, one can select points 1 and 2 at any locations, not just along a streamline.*

4.10 Describing the Pressure Field for Flow over a Circular Cylinder

Flow over a circular cylinder is a paradigm (i.e., model) for external flow over many objects. Thus, this flow is described in this section.

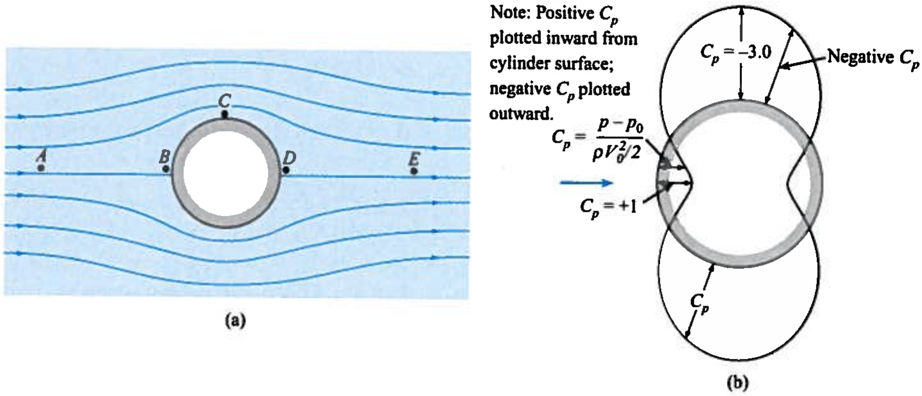
The Pressure Coefficient

To describe the pressure field, engineers often use a dimensionless group called the **pressure coefficient**:

$$C_p = \frac{p_z - p_{zo}}{\rho V_o^2/2} = \frac{h - h_o}{V_o^2/(2g)} \tag{4.47}$$

FIGURE 4.36

Irrotational flow past a cylinder. (a) Streamline pattern. (b) Pressure distribution.



Pressure Distribution for an Ideal Fluid

An ideal fluid is defined as a fluid that is nonviscous and that has constant density. If assume an irrotational flow of an ideal fluid, then calculations reveal the results shown Fig. 4.36a. Features to notice in this figure are

- The pressure distribution is symmetric on the front and back of the cylinder.
- The pressure coefficient is sometimes negative (plotted outward), which corresponds to negative gage pressure.
- The pressure coefficient is sometimes positive (plotted inward), which corresponds to positive gage pressure.
- The maximum pressure ($C_p = +1.0$) occurs on the front and back of the cylinder at the stagnation points (points B and D).
- The minimum pressure ($C_p = -3.0$) occurs at the midsection, where the velocity is highest (point C).

Next, we introduce the concepts of a favorable and adverse pressure gradient. To begin, apı Euler's equation while neglecting gravitational effects:

$$\rho a_t = -\frac{\partial p}{\partial s}$$

One notes that $a_t > 0$ if $\partial p/\partial s < 0$; that is, the fluid particle accelerates if the pressu decreases with distance along a pathline. This is a favorable pressure gradient. On the oth hand, $a_t < 0$ if $\partial p/\partial s > 0$, so the fluid particle decelerates if the pressure increases along pathline. This is an adverse pressure gradient. The definitions of pressure gradient are sur marized in the table.

Favorable pressure gradient	$\partial p/\partial s < 0$	$a_t > 0$ (acceleration)
Adverse pressure gradient	$\partial p/\partial s > 0$	$a_t < 0$ (deceleration)

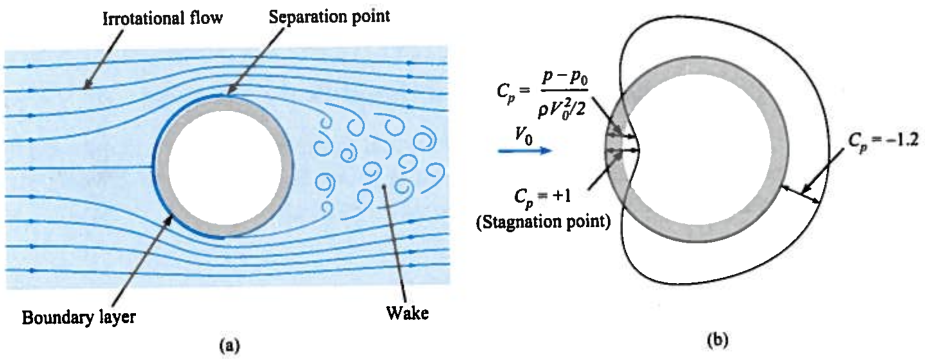
Visualize the motion of a fluid particle in Fig. 4.36a as it travels around the cylinder fro *A* to *B* to *C* to *D* and finally to *E*. Notice that it first decelerates from the free-stream velocity zero velocity at the forward stagnation point as it travels in an adverse pressure gradient. Then as it passes from B to C, it is in a favorable pressure gradient, and it accelerated to its highest speed. From C to D the pressure increases again toward the rearward stagnation point, and the particle decelerates but has enough momentum to reach D. Finally, the pressure decreases from D to E, and this favorable pressure gradient accelerates the particle back to the free-stream velocity.

Pressure Distribution for a Viscous Flow

Consider the flow of a real (viscous) fluid past a cylinder as shown in Fig. 4.37. The flow pattern upstream of the midsection is very similar to the pattern for an ideal fluid. However, in a viscous fluid the velocity at the surface is zero (no-slip condition), whereas with the flow of an inviscid fluid the surface velocity need not be zero. Because of viscous effects, a boundary layer forms next to the surface. The velocity changes from zero at the surface to the free-stream velocity across the boundary layer. Over the forward section of the cylinder, where the pressure gradient is favorable, the boundary layer is quite thin.

FIGURE 4.37

Flow of a real fluid past a circular cylinder. (a) Flow pattern. (b) Pressure distribution.



Downstream of the midsection, the pressure gradient is adverse, and the fluid particles in the boundary layer, slowed by viscous effects, can only go so far and then are forced to detour away from the surface. The particle is pushed off the wall by pressure force associated with the adverse pressure gradient. The point where the flow leaves the wall is called the separation point. A recirculatory flow called a wake develops behind the cylinder. The flow in the wake region is called separated flow. The pressure distribution on the cylinder surface in the wake region is nearly constant, as shown in Fig. 4.37b. The reduced pressure in the wake leads to increased drag.

4.11 Calculating the Pressure Field for a Rotating Flow

This section describes how to relate pressure and velocity for a *fluid in a solid body rotation.* To understand solid body rotation, consider a cylindrical container of water (Fig. 4.38a) which is stationary. Imagine that the container is placed into rotational motion about an axis (Fig. 4.38b)

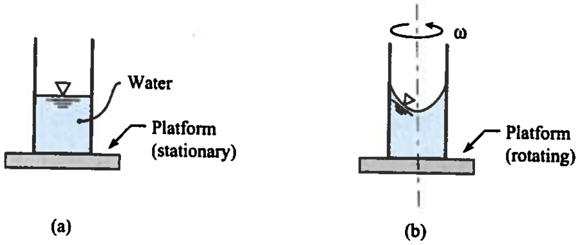


FIGURE 4.38
Sketch used to define a fluid in solid body rotation.

and allowed to reach steady state with an angular speed of ω. At steady state, the fluid partic will be at rest with respect to each other. That is, the distance between any two fluid partic will be constant. This condition also describes rotation of a rigid body; thus, this type of n tion is defined as a **fluid in a solid body rotation.**

Situations in which a fluid rotates as a solid body are found in many engineering appli tions. One common application is the centrifugal separator. The centripetal accelerations sulting from rotating a fluid separate the heavier particles from the lighter particles as t heavier particles move toward the outside and the lighter particles are displaced toward t center. A milk separator operates in this fashion, as does a cyclone separator for removi particulates from an air stream.

Derivation of an Equation for a Fluid in Solid Body Rotation

To begin, apply Euler's equation in the direction normal to the streamlines and outward fro the center of rotation. In this case the fluid particles rotate as the spokes of a wheel, so t direction ℓ in Euler's equation, Eq. (4.15), is replaced by r giving

$$-\frac{d}{dr}(p + \gamma z) = \rho a_r \quad (4.4$$

where the partial derivative has been replaced by an ordinary derivative because the flow steady and a function only of the radius r. From Eq. (4.11), the acceleration in the radial dire tion (away from the center of curvature) is

$$a_r = -\frac{V^2}{r}$$

and Euler's equation becomes

$$-\frac{d}{dr}(p + \gamma z) = -\rho\frac{V^2}{r} \quad (4.4$$

For solid body rotation about a fixed axis,

$$V = \omega r$$

Substituting this velocity distribution into Euler's equation results in

$$\frac{d}{dr}(p + \gamma z) = \rho r \omega^2 \quad (4.5$$

Integrating Eq. (4.50) with respect to r gives

$$p + \gamma z = \frac{\rho r^2 \omega^2}{2} + \text{const} \qquad (4.51)$$

or

$$\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g} = C \qquad (4.52a)$$

This equation can also be written as

$$p + \gamma z - \rho\frac{\omega^2 r^2}{2} = C \qquad (4.52b)$$

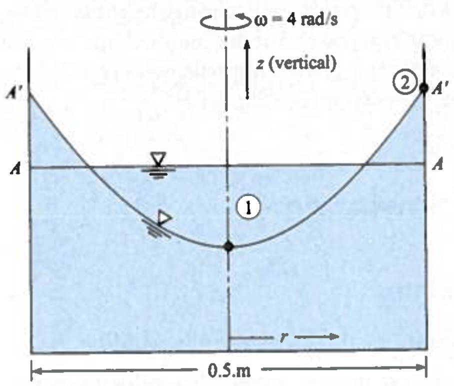
These equivalent equations describe the *pressure variation in rotating flow*. Example 4.11 shows how to apply the equation.

EXAMPLE 4.11

Calculating the Surface Profile of a Rotating Liquid

Problem Statement

A cylindrical tank of liquid shown in the figure is rotating as a solid body at a rate of 4 rad/s. The tank diameter is 0.5 m. The line AA depicts the liquid surface before rotation, and the line $A'A'$ shows the surface profile after rotation has been established. Find the elevation difference between the liquid at the center and the wall during rotation.



Define the Situation

A liquid is rotating in a cylindrical tank.

State the Goal

Calculate the elevation difference (in meters) between liquid at the center and at the wall.

Generate Ideas and Make a Plan

1. Apply Eq. (4.52a), between points 1 and 2.
2. Calculate the elevation difference.

Take Action (Execute the Plan)

1. Equation (4.52a).

$$\frac{p_1}{\gamma} + z_1 - \frac{\omega^2 r_1^2}{2g} = \frac{p_2}{\gamma} + z_2 - \frac{\omega^2 r_2^2}{2g}$$

The pressure at both points is atmospheric, so $p_1 = p_2$ and the pressure terms cancel out. At point 1, $r_1 = 0$, and at point 2, $r = r_2$. The equation reduces to

$$z_2 - \frac{\omega^2 r_2^2}{2g} = z_1$$

$$z_2 - z_1 = \frac{\omega^2 r_2^2}{2g}$$

2. Elevation difference:

$$z_2 - z_1 = \frac{(4\ \text{rad/s})^2 \times (0.25\ \text{m})^2}{2 \times 9.81\ \text{m/s}^2}$$

$$= \boxed{0.051\ \text{m or } 5.1\ \text{cm}}$$

Review the Solution and the Process

Notice that the surface profile is parabolic.

Example 4.12 illustrates the analysis of a rotating flow in a manometer.

EXAMPLE 4.12

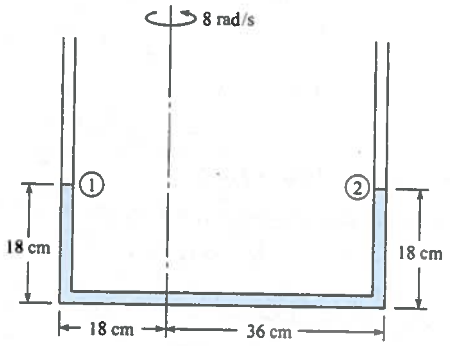
Evaluating a Rotating Manometer Tube

Problem Statement

When the U-tube is not rotated, the water stands in the tube as shown. If the tube is rotated about the eccentric axis at a rate of 8 rad/s, what are the new levels of water in the tube?

Define the Situation

A manometer tube is rotated around an eccentric axis.



Assumptions: Liquid is incompressible.

State the Goal

Find the levels of water in each leg.

Generate Ideas and Make a Plan

The total length of the liquid in the manometer must be the same before and after rotation, namely 90 cm. Assume, to start with, that liquid remains in the bottom leg. The pressure at the top of the liquid in each leg is atmospheric.

1. Apply the equation for pressure variation in rotating flows Eq. (4.52a), to evaluate difference in elevation in each leg.
2. Using constraint of total liquid length, find the level in each leg.

Take Action (Execute the Plan)

1. Application of Eq. (4.52a) between top of leg on left (1) and on right (2):

$$z_1 - \frac{r_1^2\omega^2}{2g} = z_2 - \frac{r_2^2\omega^2}{2g}$$

$$z_2 - z_1 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)$$

$$= \frac{(8\ \text{rad/s})^2}{2 \times 9.81\ \text{m/s}^2}(0.36^2\ \text{m}^2 - 0.18^2\ \text{m}^2) = 0.317\ \text{m}$$

2. The sum of the heights in each leg is 36 cm.

$$z_2 + z_1 = 0.36\ \text{m}$$

Solution for the leg heights:

$$z_2 = 0.338\ \text{m}$$
$$z_1 = 0.022\ \text{m}$$

Review the Solution and the Process

Discussion. If the result was a negative height in one leg, it would mean that one end of the liquid column would be in the horizontal leg, and the problem would have to be reworked to reflect this configuration.

4.12 Summarizing Key Knowledge

Pathline, Streamlines, and Streaklines

- To visualize flow, engineers use the streamline, streakline, and the pathline.
  - The *streamline* is a curve that is everywhere tangent to the local velocity vector.
  - The *streamline* is a mathematical entity that cannot be observed in the physical world.
  - The configuration of streamlines in a flow field is called the *flow pattern*.
  - The *pathline* is the line (straight or curved) that a particle follows.
  - A *streakline* is the line produced by a dye or other marker fluid introduced at a point.
- In *steady flow*, pathlines, streaklines, and streamlines are coincident (i.e., on top of each other) if they share a common point.
- In *unsteady flow*, pathlines, streaklines, and streamlines are not coincident.

Velocity and Velocity Field

- In a flowing fluid, *velocity* is defined as the speed and direction of travel of a fluid particle.
- A *velocity field* is a mathematical or graphical description that shows the velocity at each point (i.e., spatial location) within a flow.

Eulerian and Lagrangian Descriptions

There are two ways to describe motion (Lagrangian and Eulerian).

- In the *Lagrangian approach*, the engineer identifies a specified collection of matter and describes its motion. For example, when a engineer is describing the motion of a fluid particle this is a Lagrangian-based description.
- In the *Eulerian approach*, the engineer identifies a region in space and describes the motion of matter that is passing by in terms of what is happening at various spatial locations. For example, the velocity field is an Eulerian-based concept.
  - The Eulerian approach uses fields. A *field* is a mathematical or graphical description that shows how a variable is distributed spatially. A field can be a *scalar field* or a *vector field.*
  - The Eulerian approach uses the divergence, gradient, and curl operators.
  - The Eulerian approach uses more complicated mathematics (e.g., partial derivatives) than the Lagrangian approach.

Describing Flow

Engineers describe flowing fluids using the ideas summarized in Table 4.4.

TABLE 4.4 How Engineers Describe Flowing Fluids

Description	Key Knowledge
Engineers classify flows as *uniform* or *nonuniform*.	• Uniform and nonuniform flow describe how velocity varies spati • *Uniform flow* means that the velocity at each point on a given streamline is the same. Uniform flow requires rectilinear streamli (straight and parallel). • *Nonuniform flow* means that velocity at various points on a given streamline differs.
Engineers classify flows as *steady* or *unsteady*.	• *Steady flow* means the velocity is constant with respect to time at every point in space. • *Unsteady flow* means the velocity is changing with time at some o points in space. • Engineers often idealize unsteady flows as steady flow. Example: A draining tank of water is commonly assumed to be a steady flo
Engineers classify flows as *laminar* or *turbulent*.	• *Laminar flow* involves flow in smooth layers (laminae) with low l of mixing between layers. • *Turbulent flow* involves flow that is dominated by eddies of various Flow is chaotic, unsteady, 3D. High levels of mixing. • Occasionally, engineers describe a flow as *transitional*. This mean that the flow is changing from a laminar flow to a turbulent flow.

(Cont

TABLE 4.4 How Engineers Describe Flowing Fluids (*Continued*)

Description	Key Knowledge
Engineers classify flows as *1D*, *2D*, or *3D*.	• *One-dimensional (1-D) flow* means the velocity depends on one spatial variable. E.g., velocity depends on radius r only. • *Three-dimensional (3-D) flow* means the velocity depends on three spatial variables. E.g., velocity depends on three position coordinates $\mathbf{V} = \mathbf{V}(x, y, z)$.
Engineers classify flows as *viscous flow* or *inviscid flow*.	• In a *viscous flow*, the forces associated with viscous shear stresses are significant. Thus, viscous terms are included when solving the equations of motion. • In an *inviscid flow*, the forces associated with viscous shear stresses are insignificant. Thus, viscous terms are neglected when solving the equations of motion. The fluid behaves as if its viscosity were zero.
Engineers describe flows by describing an *inviscid flow region*, a *boundary layer*, and a *wake*.	• In the *inviscid flow region*, the streamlines are smooth and the flow can be analyzed with Euler's equation. • The *boundary layer* is a thin region of fluid next to wall. Viscous effects are significant in the boundary layer. • The *wake* is the region of separated flow behind a body.
Engineers describe flows as separated or attached.	• *Flow separation* is when fluid particles move away from the wall. • *Attached flow* is when fluid particles are moving along a wall or boundary. • The region of separated flow inside a pipe or duct is often called a *recirculation zone*.

Acceleration

- *Acceleration* is a property of a fluid particle that characterizes
  - The change in speed of the particle
  - The change in direction of travel of the particle
- *Acceleration* is defined mathematically as the derivative of the velocity vector.
- *Acceleration* of a fluid particle can be described qualitatively. Guidelines:
  - If a particle is traveling on a curved streamline, there will be a component of acceleration that is normal to the streamline and directed inwards toward the center of curvature.
  - If the particle is changing speed, there will be a component of acceleration that is tangent to the streamline.
- In an *Eulerian representation of acceleration*,
  - Terms that involve derivatives with respect to time are *local acceleration* terms.
  - All other terms are *convective acceleration* terms. Most of these terms involve derivative with respect to position.

Euler's Equation

- *Euler's equation* is *Newton's second law of motion* applied to a fluid particle when the flow i inviscid and incompressible.

- Euler's equation can be written as a *vector equation*:

$$-\nabla p_z = \rho \mathbf{a}$$

- This vector form can be also be written as a *scalar equation* in an arbitrary ℓ direction.

$$-\frac{\partial}{\partial \ell}(p + \gamma z) = -\left(\frac{\partial p_z}{\partial \ell}\right) = \rho a_\ell$$

- *Physics of Euler's equation*: The gradient of piezometric pressure is colinear with acceleration and opposite in direction. This reveals how pressure varies:
    - When streamlines are curved, pressure will increase outward from the center of curvature.
    - When a streamline is rectilinear and a particle on the streamline is changing speed, then the pressure will change in a direction tangent to the streamline. The direction of increasing pressure is opposite of the acceleration vector.
    - When streamlines are rectilinear, pressure variation normal to the streamlines is hydrostatic.

The Bernoulli Equation

- The *Bernoulli equation* is *conservation of energy* applied to a fluid particle. It is derived by integrating Euler's equation for steady, inviscid, and constant density flow.
- For the assumptions just stated, the Bernoulli equation is applied between any two points on the same streamline.
- The Bernoulli equations has two forms:
    - *Head Form*: $p/\gamma + z + V^2/(2g) =$ constant
    - *Pressure Form*: $p + \rho g z + (\rho V^2)/2 =$ constant
- There are two equivalent ways to describe the physics of the Bernoulli equation:
    - When speed increases, then piezometric pressure decreases (along a streamline).
    - The total head (velocity head plus piezometric head) is constant along a streamline. This means that energy is conserved as a fluid particle moves along a streamline.

Measuring Velocity and Pressure

- When pressure is measured at a *pressure tap* on the wall of a pipe, this provides a measurement of static pressure. This same measurement can also be used to determine pressure head or piezometric head.
- *Static pressure* is defined as the pressure in a flowing fluid. Static pressure must be measured in a way that does not change the value of the measured pressure.
- *Kinetic pressure* is $(\rho V^2)/2$.
- A *stagnation tube* provides a measurement of (static pressure) + (kinetic pressure):

$$p + (\rho V^2)/2$$

- The *Pitot-static* tube, provides a method to measure both static pressure and kinetic pressure at a point in a flowing fluid. Thus, this instrument provides a way to measure fluid velocity.

Fluid Rotation, Vorticity, and Irrotational Flow

- Rate of rotation Ω
    - Is a property of a fluid particle that describes how fast the particle is rotating.
    - Is defined by placing two perpendicular lines on a fluid particle and then averaging the rotational rate of these lines.
    - Is a vector quantity with the direction of the vector given by the right-hand rule.

- A common way to describe rotation is to use the vorticity vector ω, which is twice the rotation vector: $\omega = 2\Omega$
- In Cartesian coordinates, the vorticity is given by

$$\omega = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$$

- An irrotational flow is one in which vorticity is everywhere zero.
- When applying the Bernoulli equation for irrotational flow, one can select points 1 and 2 at any locations, not just along a streamline.

Describing the Pressure Field

- The pressure field is often described using a π-group called the pressure coefficient.
- The pressure gradient near a body is related to flow separation.
  - An adverse pressure gradient is associated with flow separation.
  - A positive pressure gradient is associated with attached flow.
- The pressure field for flow over a circular cylinder is a paradigm for understanding external flows. The pressure along the front of the cylinder is high, and the pressure in the wake is lov
- When flow is rotating as a solid body, the pressure field p can be described using

$$p + \gamma z - \rho\frac{\omega^2 r^2}{2} = C$$

where ω is the rotational speed, and r is the distance from the axis of rotation to the point the field.

Describing the Pressure Field (Summary)

Pressure variations in a flowing fluid are associated with three phenomenon:

- **Weight.** Due to the weight of a fluid, pressure increases with increasing depth (i.e., decreasing elevation). This topic is presented in Chapter 3 (Hydrostatics)
- **Acceleration.** When fluid particles are accelerating, there are usually pressure variations associated with the acceleration. In inviscid flow, the gradient of the pressure field is aligned in a direction opposite of the acceleration vector.
- **Viscous Effects.** When viscous effects are significant, there can be associated pressure changes. For example, there are pressure drops associated with flows in horizontal pipes and ducts. This topic is presented in Chapter 10 (Conduit Flow).

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5. Miller, R.W. (ed) *Flow Measurement Engineering Handbook*, New York: McGraw-Hill, 1996.

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PROBLEMS

PLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Streamlines, Streaklines, and Pathlines (§4.1)

4.1 If somehow you could attach a light to a fluid particle and take a time exposure photo, would the image you photographed be a pathline or streakline? Explain from definition of each.

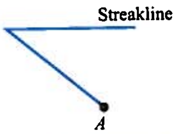
4.2 Is the pattern produced by smoke rising from a chimney on a windy day analogous to a pathline or streakline? Explain from the definition of each.

4.3 PLUS A windsock is a sock-shaped device attached to a swivel on top of a pole. Windsocks at airports are used by pilots to see instantaneous shifts in the direction of the wind. If one drew a line co-linear with a windsock's orientation at any instant, the line would be best approximate a (a) pathline, (b) streakline, or (c) streamline.

4.4 PLUS For streamlines, streaklines, and streamlines to all be co-linear, the flow must be

- **a.** dividing
- **b.** stagnant
- **c.** steady
- **d.** a tracer

4.5 At time $t = 0$, dye was injected at point A in a flow field of a liquid. When the dye had been injected for 4 s, a pathline for a particle of dye that was emitted at the 4 s instant was started. The streakline at the end of 10 s is shown below. Assume that the speed (but not the velocity) of flow is the same throughout the 10 s period. Draw the pathline of the particle that was emitted at $t = 4$ s. Make your own assumptions for any missing information.



PROBLEM 4.5

4.6 For a given hypothetical flow, the velocity from time $t = 0$ to $t = 5$ s was $u = 2$ m/s, $v = 0$. Then, from time $t = 5$ s to $t = 10$ s, the velocity was $u = +3$ m/s, $v = -4$ m/s. A dye streak was started at a point in the flow field at time $t = 0$, and the path of a particle in the fluid was also traced from that same point starting at the same time. Draw to scale the streakline, pathline of the particle, and streamlines at time $t = 10$ s.

4.7 At time $t = 0$, a dye streak was started at point A in a flow field of liquid. The speed of the flow is constant over a 10 s period, but the flow direction is not necessarily constant. At any particular instant the velocity in the entire field of flow is the same. The streakline produced by the dye is shown above. Draw (and label) a streamline for the flow field at $t = 8$ s.

Draw (and label) a pathline that one would see at $t = 10$ s for a particle of dye that was emitted from point A at $t = 2$ s.

PROBLEM 4.7

Velocity and the Velocity Field (§4.2)

4.8 PLUS A velocity field is given mathematically as $\mathbf{V} = 2\mathbf{i} +$
The velocity field is:

- **a.** 1D in x
- **b.** 1D in y
- **c.** 2D in x and y

The Eulerian and Lagrangian Approaches (§4.2)

4.9 PLUS There is a gasoline spill in a major river. The mayo large downstream city demands an estimate of how many ho it will take for the spill to get to the water supply plant intak The emergency responders measure the speed of the leading edge of the spill, effectively focusing on one particle of fluid. Meanwhile, environmental engineers at the local university employ a computer model, which simulates the velocity fiel any stage of the river, and for all locations (including steep narrow canyon sections with fast velocities, and an extremel wide reach with slow velocities). To compare these two mathematical approaches, which statement is most correct?

- **a.** The responders have an Eulerian approach, and the engineers have a Lagrangian one
- **b.** The responders have a Lagrangian approach, and th engineers have an Eulerian one.

Describing Flow (§4.3)

4.10 Identify five examples of an unsteady flow and explai features classify them as an unsteady flow.

4.11 You are pouring a heavy syrup on your pancakes. As t syrup spreads over the pancake, would the thin film of syru laminar or turbulent flow? Why?

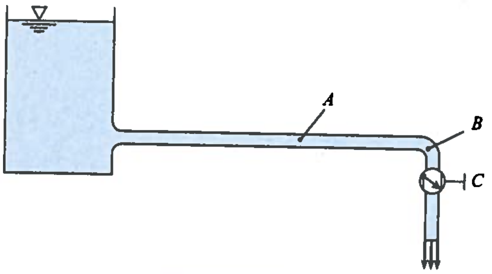
4.12 PLUS A velocity field is given by $\mathbf{V} = 10xy\mathbf{i}$. It is

- **a.** 1-D and steady
- **b.** 1-D and unsteady
- **c.** 2-D and steady
- **d.** 2-D and unsteady

4.13 Which is the most correct way to characterize turbulen

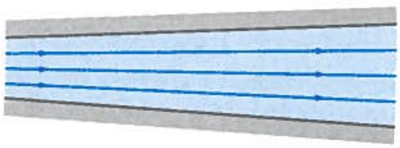
- **a.** 1D
- **b.** 2D
- **c.** 3D

4.14 In the system in the figure, the valve at C is gradually opened in such a way that a constant rate of increase in discharge is produced. How would you classify the flow at B while the valve is being opened? How would you classify the flow at A?



PROBLEM 4.14

4.15 Water flows in the passage shown. If the flow rate is decreasing with time, the flow is classified as (a) steady, (b) unsteady, (c) uniform, or (d) nonuniform.



PROBLEM 4.15

4.16 If a flow pattern has converging streamlines, how would you classify the flow?

4.17 Consider flow in a straight conduit. The conduit is circular in cross section. Part of the conduit has a constant diameter, and part has a diameter that changes with distance. Then, relative to flow in that conduit, correctly match the items in column A with those in column B.

A	B
Steady flow	$\partial V_s/\partial s = 0$
Unsteady flow	$\partial V_s/\partial s \neq 0$
Uniform flow	$\partial V_s/\partial t = 0$
Nonuniform flow	$\partial V_s/\partial t \neq 0$

4.18 Classify each of the following as a one-dimensional, two-dimensional, or three-dimensional flow.

a. Water flow over the crest of a long spillway of a dam.

b. Flow in a straight horizontal pipe.

c. Flow in a constant-diameter pipeline that follows the contour of the ground in hilly country.

d. Airflow from a slit in a plate at the end of a large rectangular duct.

e. Airflow past an automobile.

f. Airflow past a house.

g. Water flow past a pipe that is laid normal to the flow across the bottom of a wide rectangular channel.

Acceleration (§4.4)

4.19 Acceleration is the rate of change of velocity with time. the acceleration vector always aligned with the velocity vecto Explain.

4.20 For a rotating body, is the acceleration toward the cente rotation a centripetal or centrifugal acceleration? Look up wo meanings and word roots.

4.21 In a flowing fluid, acceleration means that a fluid particle is

a. changing direction

b. changing speed

c. changing both speed and direction

d. any of the above

4.22 The flow passing through a nozzle is steady. The sp of the fluid increases between the entrance and the exit of the nozzle. The acceleration halfway between the entrance and the nozzle is

a. convective

b. local

c. both

4.23 Local acceleration

a. is close to the origin

b. is quasi nonuniform

c. occurs in unsteady flow

4.24 Figure 4.36 on p. 148 in §4.10 shows the flow patter for flow past a circular cylinder. Assume that the approach velocity at A is constant (does not vary with time).

a. Is the flow past the cylinder steady or unsteady?

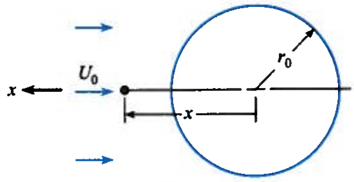
b. Is this a case of one-dimensional, two-dimensional, or three-dimensional flow?

c. Are there any regions of the flow where local acceleration is present? If so, show where they are and show vectors representing the local acceleration in the regio where it occurs.

d. Are there any regions of flow where convective acceleration is present? If so, show vectors representing the convective acceleration in the regions where it occurs.

4.25 The velocity along a pathline is given by $V\,(\text{m/s}) = s^2t^{1/2}$ where s is in meters and t is in seconds. The radius of curvature is 0.4 m. Evaluate the acceleration tangent and normal to the path at $s = 1.5$ m and $t = 0.5$ seconds.

4.26 Tests on a sphere are conducted in a wind tunnel at an air speed of U_0. The velocity of flow toward the sphere along the longitudinal axis is found to be $u = -U_0\,(1 - r_0^3/x^3)$, where r_0 is the radius of the sphere and x the distance from its center. Determine the acceleration of an air particle on the x-axis upstream of the sphere in terms of x, r_0, and U_0.



PROBLEM 4.26

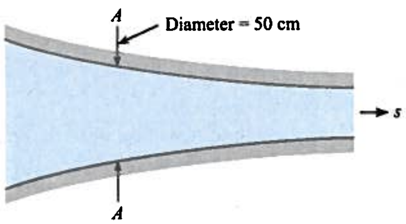
4.27 In this flow passage the velocity is varying with time. The velocity varies with time at section *A*-*A* as

$$V = 5\text{ m/s} - 2.25\frac{t}{t_0}\text{ m/s}$$

At time $t = 0.50$ s, it is known that at section *A*-*A* the velocity gradient in the *s* direction is +2 m/s per meter. Given that t_0 is 0.5 s and assuming quasi-one-dimensional flow, answer the following questions for time $t = 0.5$ s.

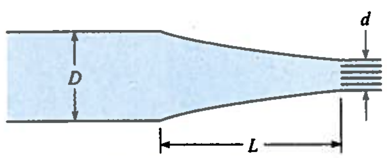
a. What is the local acceleration at *A*-*A*?

b. What is the convective acceleration at *A*-*A*?



PROBLEM 4.27

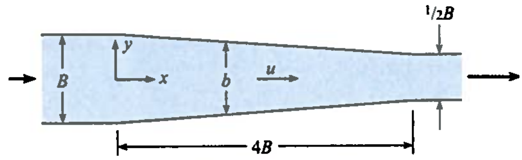
4.28 The nozzle in the figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming quasi-one-dimensional flow, what is the convective acceleration midway between the base and the tip if the velocity is 1 ft/s at the base and 4 ft/s at the tip? Nozzle length is 18 inches.



PROBLEMS 4.28, 4.29

4.29 In Prob. 4.28 the velocity varies linearly with time throughout the nozzle. The velocity at the base is $2t$ (ft/s) and at the tip is $6t$ (ft/s). What is the local acceleration midway along the nozzle when $t = 2$ s?

4.30 Liquid flows through this two-dimensional slot with a velocity of $V = 2(q_0/b)(t/t_0)$, where q_0 and t_0 are reference values. What will be the local acceleration at $x = 2B$ and $y = 0$ in terms of B, t, t_0, and q_0?



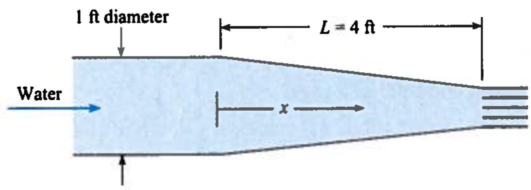
PROBLEMS 4.30, 4.31

4.31 What will be the convective acceleration for the condi of Prob. 4.30?

4.32 The velocity of water flow in the nozzle shown is given by the following expression:

$$V = 2t/(1 - 0.5x/L)^2,$$

where V = velocity in feet per second, t = time in seconds, x = distance along the nozzle, and L = length of nozzle = 4 When $x = 0.5L$ and $t = 3$ s, what is the local acceleration al the centerline? What is the convective acceleration? Assume quasi-one-dimensional flow prevails.



PROBLEM 4.32

Euler's Equation and Pressure Variation (§4.5)

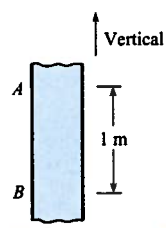
4.33 State Newton's second law of motion. What are the limitations on the use of Newton's second law? Explain.

4.34 What is the differences between a force due to weight force due to pressure? Explain.

4.35 A pipe slopes upward in the direction of liquid flow a angle of 30° with the horizontal. What is the pressure gradi the flow direction along the pipe in terms of the specific we of the liquid if the liquid is decelerating (accelerating oppo flow direction) at a rate of 0.4 *g*?

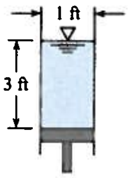
4.36 What pressure gradient is required to accelerate kerosene (S = 0.81) vertically upward in a vertical pipe at a of 0.5 *g*?

4.37 The hypothetical liquid in the tube shown in the figure ha viscosity and a specific weight of 10 kN/m³. If $p_B - p_A$ is equal kPa, one can conclude that the liquid in the tube is being accele (a) upward, (b) downward, or (c) neither: acceleration = 0.



PROBLEM 4.37

4.38 If the piston and water ($\rho = 62.4$ lbm/ft^3) are accelerated upward at a rate of $0.4g$, what will be the pressure at a depth of 2 ft in the water column?



PROBLEMS 4.38, 4.39

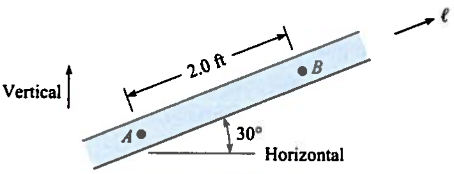
4.39 Water ($\rho = 62.4$ lbm/ft^3) stands at a depth of 10 ft in a vertical pipe that is open at the top and closed at the bottom by a piston. What upward acceleration of the piston is necessary to create a pressure of 8 psig immediately above the piston?

4.40 What pressure gradient is required to accelerate water ($\rho = 1000$ kg/m^3) in a horizontal pipe at a rate of 8 m/s^2?

4.41 Water ($\rho = 1000$ kg/m^3) is accelerated from rest in a horizontal pipe that is 80 m long and 30 cm in diameter. If the acceleration rate (toward the downstream end) is 5 m/s^2, what is the pressure at the upstream end if the pressure at the downstream end is 90 kPa gage?

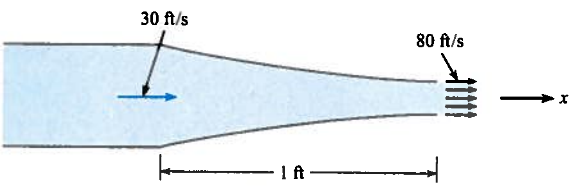
4.42 Water ($\rho = 62.4$ lbm/ft^3) stands at a depth of 10 ft in a vertical pipe that is closed at the bottom by a piston. Assuming that the vapor pressure is zero (abs), determine the maximum downward acceleration that can be given to the piston without causing the water immediately above it to vaporize.

4.43 A liquid with a specific weight of 100 lbf/ft^3 is in the conduit. This is a special kind of liquid that has zero viscosity. The pressures at points A and B are 170 psf and 100 psf, respectively. Which one (or more) of the following conclusions can one draw with certainty? (a) The velocity is in the positive ℓ direction. (b) The velocity is in the negative ℓ direction. (c) The acceleration is in the positive ℓ direction. (d) The acceleration is in the negative ℓ direction.



PROBLEM 4.43

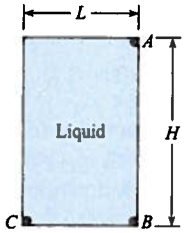
4.44 If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient, dp/dx, halfway through the nozzle? ($\rho = 62.4$ lbm/ft^3).



PROBLEM 4.44

4.45 The closed tank shown, which is full of liquid, is accelerat downward at $1.5g$ and to the right at $0.9g$. Here $L = 3$ ft, $H = 4$ and the specific gravity of the liquid is 1.2. Determine $p_C - p_A$ and $p_B - p_A$.

4.46 The closed tank shown, which is full of liquid, is accelerated downward at $\frac{2}{3}g$ and to the right at $1g$. Here $L = 2.5$ $H = 3$ m, and the liquid has a specific gravity of 1.3. Determine $p_C - p_A$ and $p_B - p_A$.



PROBLEMS 4.45, 4.46

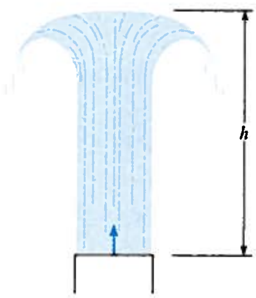
Applying the Bernoulli Equation (§4.6)

4.47 Describe in your own words how an aspirator works.

4.48 When the Bernoulli Equation applies to a venturi, such as in Fig. 4.27 on p. 134 in §4.6, which of the following are true? (Select all that apply.)

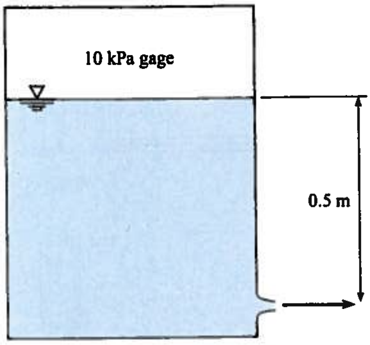
- **a.** If the velocity head and elevation head increase, then the pressure head must decrease.
- **b.** Pressure always decreases in the direction of flow alon a streamline.
- **c.** The total head of the flowing fluid is constant along a streamline.

4.49 A water jet issues vertically from a nozzle, as shown. The water velocity as it exits the nozzle is 18 m/s. Calculate how high h the jet will rise. (*Hint:* Apply the Bernoulli equation alon the centerline.)



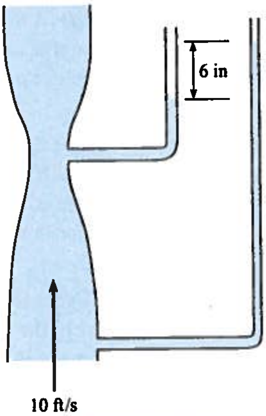
PROBLEM 4.49

4.50 A pressure of 10 kPa, gage, is applied to the surface of wate in an enclosed tank. The distance from the water surface to the outlet is 0.5 m. The temperature of the water is 20°C. Find the velocity (m/s) of water at the outlet. The speed of the water surface is much less than the water speed at the outlet.



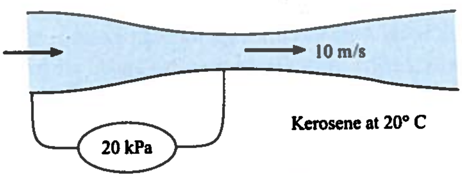
PROBLEM 4.50

4.51 Water flows through a vertical contraction (venturi) section. Piezometers are attached to the upstream pipe and minimum area section as shown. The velocity in the pipe is 10 ft/s. The difference in elevation between the two water levels in the piezometers is 6 inches. The water temperature is 68°F. What is the velocity (ft/s) at the minimum area?



PROBLEM 4.51

4.52 Kerosene at 20°C flows through a contraction section as shown. A pressure gage connected between the upstream pipe and throat section shows a pressure difference of 20 kPa. The gasoline velocity in the throat section is 8 m/s. What is the velocity (m/s) in the upstream pipe?



PROBLEM 4.52

Stagnation Tubes and Pitot-Static Tubes (§4.7)

4.53 A stagnation tube placed in a river (select all that apply)

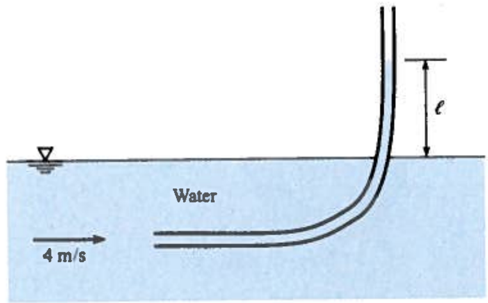
a. can be used to determine air pressure

b. can be used to determine fluid velocity

c. measures kinetic pressure

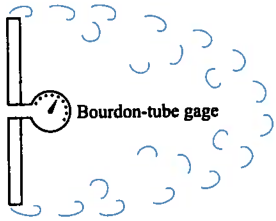
4.54 A Pitot-static tube is mounted on an airplane to measure airspeed. At an altitude of 10,000 ft, where the temper is 23°F and the pressure is 10 psia, a pressure difference corresponding to 10 in of water is measured. What is the airspe

4.55 A glass tube is inserted into a flowing stream of wat with one opening directed upstream and the other end vertical. the water velocity is 5 m/s, how high will the water rise in the vertical leg relative to the level of the water surface of the strean



PROBLEM 4.55

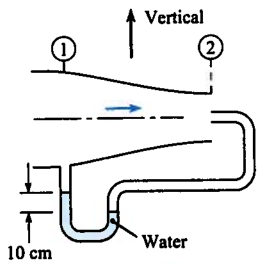
4.56 A Bourdon-tube gage is taped into the center of a disk shown. Then for a disk that is about 1 ft in diameter and for approach velocity of air (V_0) of 40 ft/s, the gage would read pressure intensity that is (a) less than $\rho V_0^2/2$, (b) equal to ρV or (c) greater than $\rho V_0^2/2$.



PROBLEM 4.56

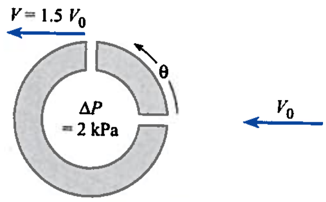
4.57 An air-water manometer is connected to a Pitot-static used to measure air velocity. If the manometer deflects 2 in. what is the velocity? Assume $T = 60$°F and $p = 15$ psia.

4.58 The flow-metering device shown consists of a stagnatic probe at station 2 and a static pressure tap at station 1. The velo station 2 is 1.5 times that at station 1. Air with a density of 1.2 k flows through the duct. A water manometer is connected bet the stagnation probe and the pressure tap, and a deflection of 10 cm is measured. What is the velocity at station 2?



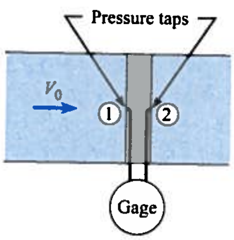
PROBLEM 4.58

4.59 The "spherical" Pitot probe shown is used to measure the flow velocity in water ($\rho = 1000 \text{ kg/m}^3$). Pressure taps are located at the forward stagnation point and at 90° from the forward stagnation point. The speed of fluid next to the surface of the sphere varies as $1.5\ V_0 \sin \theta$, where V_0 is the free-stream velocity and θ is measured from the forward stagnation point. The pressure taps are at the same level; that is, they are in the same horizontal plane. The piezometric pressure difference between the two taps is 2 kPa. What is the free-stream velocity V_0?



PROBLEM 4.59

4.60 PLUS A device used to measure the velocity of fluid in a pipe consists of a cylinder, with a diameter much smaller than the pipe diameter, mounted in the pipe with pressure taps at the forward stagnation point and at the rearward side of the cylinder. Data show that the pressure coefficient at the rearward pressure tap is −0.3. Water with a density of 1000 kg/m^3 flows in the pipe. A pressure gage connected by lines to the pressure taps shows a pressure difference of 500 Pa. What is the velocity in the pipe?

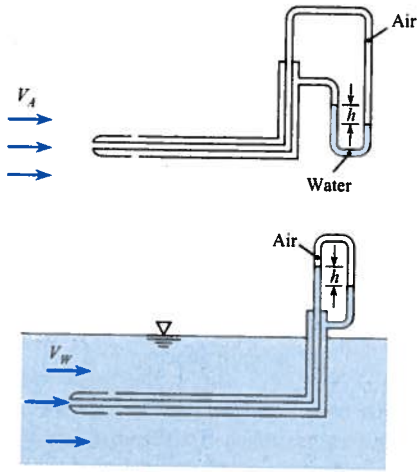


PROBLEM 4.60

4.61 Explain how you might design a spherical Pitot-static probe to provide the direction and velocity of a flowing stream. The Pitot-static probe will be mounted on a string that can be oriented in any direction.

4.62 PLUS Two Pitot-static tubes are shown. The one on the top is used to measure the velocity of air, and it is connected to an air-water manometer as shown. The one on the bottom is used to measure the velocity of water, and it too is connected to an air-water manometer as shown. If the deflection h is the same for both manometers, then one can conclude that (a) $V_A = V_w$, (b) $V_A > V_w$, or (c) $V_A < V_w$.

4.63 A Pitot-static tube is used to measure the velocity at the center of a 12 in. pipe. If kerosene at 68°F is flowing and the deflection on a mercury-kerosene manometer connected to the Pitot tube is 4 in., what is the velocity?



PROBLEM 4.62

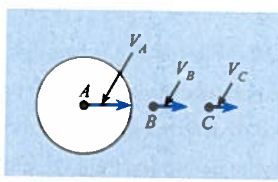
4.64 PLUS A Pitot-static tube used to measure air velocity is connected to a differential pressure gage. If the air temperature is 20°C at standard atmospheric pressure at sea level, and if the differential gage reads a pressure difference of 2 kPa, what is th air velocity?

4.65 A Pitot-static tube used to measure air velocity is connected to a differential pressure gage. If the air temperature is 60°F at standard atmospheric pressure at sea level, and if the differential gage reads a pressure difference of 15 psf, what is the air velocity?

4.66 A Pitot-static tube is used to measure the gas velocity in a duct. A pressure transducer connected to the Pitot tube registe a pressure difference of 2.0 psi. The density of the gas in the du is 0.14 lbm/ft^3. What is the gas velocity in the duct?

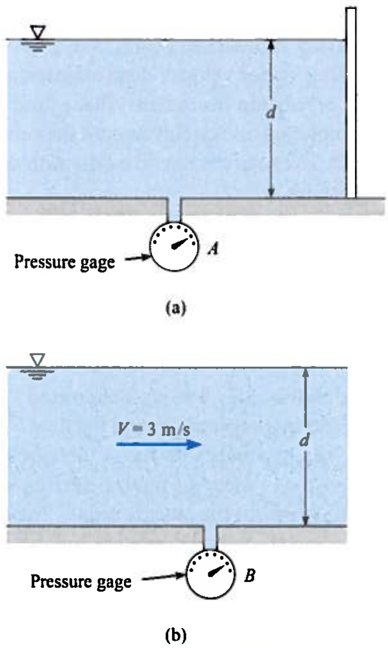
4.67 A sphere moves horizontally through still water at a spee of 11 ft/s. A short distance directly ahead of the sphere (call it point A), the velocity, with respect to the earth, induced by the sphere is 1 ft/s in the same direction as the motion of the sphe If p_0 is the pressure in the undisturbed water at the same depth as the center of the sphere, then the value of the ratio p_A/p_0 wil be (a) less than unity, (b) equal to unity, or (c) greater than unity.

4.68 PLUS Body A travels through water at a constant speed of 13 m/s as shown. Velocities at points B and C are induced by the moving body and are observed to have magnitudes of 5 m/s and 3 m/s, respectively. What is $p_B - p_C$?



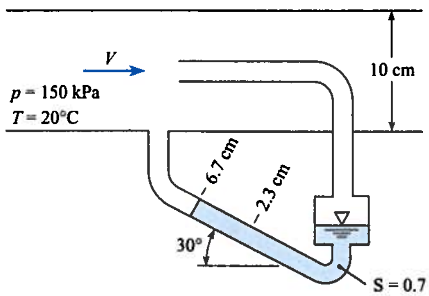
PROBLEM 4.68

4.69 Water in a flume is shown for two conditions. If the depth d is the same for each case, will gage A read greater or less than gage B? Explain.



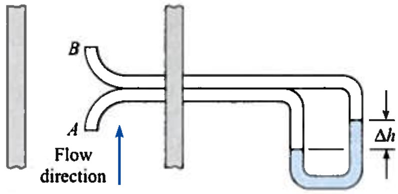
PROBLEM 4.69

4.70 WILEY GO The apparatus shown in the figure is used to measure the velocity of air at the center of a duct having a 10 cm diameter. A tube mounted at the center of the duct has a 2 mm diameter and is attached to one leg of a slant-tube manometer. A pressure tap in the wall of the duct is connected to the other end of the slant-tube manometer. The well of the slant-tube manometer is sufficiently large that the elevation of the fluid in it does not change significantly when fluid moves up the leg of the manometer. The air in the duct is at a temperature of 20°C, and the pressure is 150 kPa. The manometer liquid has a specific gravity of 0.7, and the slope of the leg is 30°. When there is no flow in the duct, the liquid surface in the manometer lies at 2.3 cm on the slanted scale. When there is flow in the duct, the liquid moves up to 6.7 cm on the slanted scale. Find the velocity of the air in the duct. Assuming a uniform velocity profile in the duct, calculate the rate of flow of the air.



PROBLEM 4.70

4.71 WILEY GO A rugged instrument used frequently for monit gas velocity in smokestacks consists of two open tubes orie to the flow direction as shown and connected to a manom The pressure coefficient is 1.0 at A and -0.3 at B. Assume water, at 20°C, is used in the manometer and that a 5 mm deflection is noted. The pressure and temperature of the st gases are 101 kPa and 250°C. The gas constant of the stack is 200 J/kg K. Determine the velocity of the stack gases.



PROBLEM 4.71

4.72 The pressure in the wake of a bluff body is approxim equal to the pressure at the point of separation. The velocit distribution for flow over a sphere is $V = 1.5\ V_0 \sin\theta$, whe is the free-stream velocity and θ is the angle measured fro forward stagnation point. The flow separates at $\theta = 120°$. I free-stream velocity is 100 m/s and the fluid is air ($\rho = 1.2$ k find the pressure coefficient in the separated region next tc sphere. Also, what is the gage pressure in this region if the free-stream pressure is atmospheric?

4.73 WILEY PLUS A Pitot-static tube is used to measure the airsp of an airplane. The Pitot tube is connected to a pressure-se device calibrated to indicate the correct airspeed when the temperature is 17°C and the pressure is 101 kPa. The airpla flies at an altitude of 3000 m, where the pressure and temp are 70 kPa and -6.3°C. The indicated airspeed is 70 m/s. V the true airspeed?

4.74 An aircraft flying at 10,000 feet uses a Pitot-static tub measure speed. The instrumentation on the aircraft provid differential pressure as well as the local static pressure and local temperature. The local static pressure is 9.8 psig, and temperature is 25°F. The differential pressure is 0.5 psid. Fi speed of the aircraft in mph.

4.75 You need to measure air flow velocity. You order a commercially available Pitot-static tube, and the accompar instructions state that the airflow velocity is given by

$$V(\text{ft/min}) = 1096.7\sqrt{\frac{h_v}{d}}$$

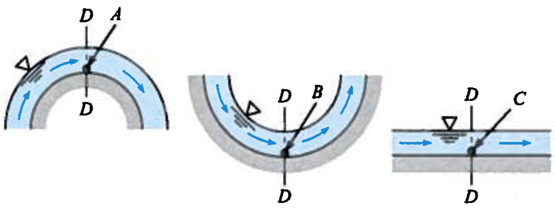
where h_v is the "velocity pressure" in inches of water and d density in pounds per cubic foot. The velocity pressure is t deflection measured on a water manometer attached to the and total pressure ports. The instructions also state the der can be calculated using

$$d\ (\text{lbm/ft}^3) = 1.325\frac{p_a}{T}$$

where P_a is the barometric pressure in inches of mercury and T is the absolute temperature in degrees Rankine. Before you use the Pitot tube you want to confirm that the equations are correct. Determine if they are correct.

4.76 Consider the flow of water over the surfaces shown. For each case the depth of water at section D-D is the same (1 ft), and the mean velocity is the same and equal to 10 ft/s. Which of the following statements are valid?

a. $p_C > p_B > p_A$
b. $p_B > p_C > p_A$
c. $p_A = p_B = p_C$
d. $p_B < p_C < p_A$
e. $p_A < p_B < p_C$



PROBLEM 4.76

Characterizing Rotational Motion of a Fluid (§4.8)

4.77 What is meant by rotation of a fluid particle? Use a sketch to explain.

4.78 Consider a spherical fluid particle in an inviscid fluid (no shear stresses). If pressure and gravitational forces are the only forces acting on the particle, can they cause the particle to rotate? Explain.

4.79 WILEY PLUS The vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ represents a two-dimensional velocity field. Is the flow irrotational?

4.80 The u and v velocity components of a flow field are given by $u = -\omega y$ and $v = \omega x$. Determine the vorticity and the rate of rotation of flow field.

4.81 The velocity components for a two-dimensional flow are

$$u = \frac{Cx}{(y^2 + x^2)} \qquad v = \frac{Cy}{(x^2 + y^2)}$$

where C is a constant. Is the flow irrotational?

4.82 WILEY PLUS A two-dimensional flow field is defined by $u = x^2 - y^2$ and $v = -2xy$. Is the flow rotational or irrotational?

4.83 Fluid flows between two parallel stationary plates. The distance between the plates is 1 cm. The velocity profile between the two plates is a parabola with a maximum velocity at the centerline of 2 cm/s. The velocity is given by

$$u = 2(1 - 4y^2)$$

where y is measured from the centerline. The cross-flow component of velocity, v, is zero. There is a reference line located 1 cm downstream. Find an expression, as a function of y, for the amount of rotation (in radian) a fluid particle will undergo when it travels a distance of 1 cm downstream.

4.84 A combination of a forced and a free vortex is represented by the velocity distribution

$$v_\theta = \frac{1}{r}[1 - \exp(-r^2)]$$

For $r \rightarrow 0$ the velocity approaches a rigid body rotation, and as r becomes large, a free-vortex velocity distribution is approached. Find the amount of rotation (in radians) that a fluid particle will experience in completing one circuit around the center as a function of r. *Hint:* The rotation rate in a flow with concentric streamlines is given by

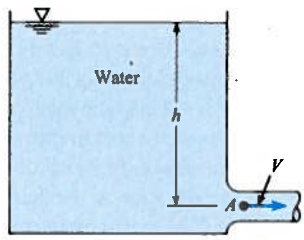
$$2\dot{\theta} = \frac{dv_\theta}{dr} + \frac{v_\theta}{r} = \frac{1}{r}\frac{d}{dr}(v_\theta r)$$

Evaluate the rotation for $r = 0.5$, 1.0, and 1.5.

The Bernoulli Equation (Irrotational Flow) (§4.9)

4.85 WILEY PLUS Liquid flows with a free surface around a bend. The liquid is inviscid and incompressible, and the flow is steady and irrotational. The velocity varies with the radius across the flow a $V = 1/r$ m/s, where r is in meters. Find the difference in depth o the liquid from the inside to the outside radius. The inside radiu of the bend is 1 m and the outside radius is 3 m.

4.86 The velocity in the outlet pipe from this reservoir is 30 ft/s and $h = 18$ ft. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A?

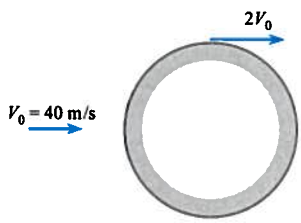


PROBLEMS 4.86, 4.87

EXAM 2

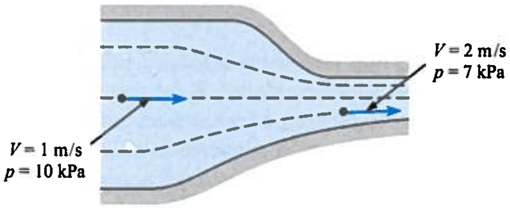
4.87 WILEY PLUS The velocity in the outlet pipe from this reservoir is 8 m/s and $h = 19$ m. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A?

4.88 The maximum velocity of the flow past a circular cylinder, as shown, is twice the approach velocity. What is Δp between the point of highest pressure and the point of lowest pressure in a 40 m/s wind? Assume irrotational flow and standard atmospheric conditions.



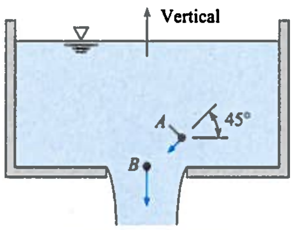
PROBLEM 4.88

4.89 The velocity and pressure are given at two points in the flow field. Assume that the two points lie in a horizontal plane and that the fluid density is uniform in the flow field and is equal to 1000 kg/m^3. Assume steady flow. Then, given these data, determine which of the following statements is true. (a) The flow in the contraction is nonuniform and irrotational. (b) The flow in the contraction is uniform and irrotational. (c) The flow in the contraction is nonuniform and rotational. (d) The flow in the contraction is uniform and rotational.



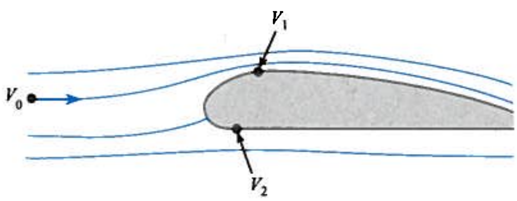
PROBLEM 4.89

4.90 Water (ρ = 62.4 lbm/ft^3) flows from the large orifice at the bottom of the tank as shown. Assume that the flow is irrotational. Point B is at zero elevation, and point A is at 1 ft elevation. If V_A = 4 ft/s at an angle of 45° with the horizontal and if V_B = 12 ft/s vertically downward, what is the value of $p_A - p_B$?



PROBLEM 4.90

4.91 Ideal flow theory will yield a flow pattern past an airfoil similar to that shown. If the approach air velocity V_0 is 80 m/s, what is the pressure difference between the bottom and the top of this airfoil at points where the velocities are V_1 = 85 m/s and V_2 = 75 m/s? Assume ρ_{air} is uniform at 1.2 kg/m^3.



PROBLEM 4.91

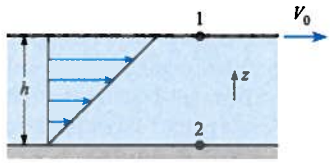
4.92 Consider the flow of water between two parallel plates in which one plate is fixed as shown. The distance between the plates is h, and the speed of the moving plate is V. A person wishes to calculate the pressure difference between the plates and applies the Bernoulli equation between points 1 and 2,

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

and concludes that

$$p_1 - p_2 = \gamma(z_2 - z_1) + \rho\frac{V_2^2}{2}$$
$$= \gamma h + \rho\frac{V^2}{2}$$

Is this correct? Provide the reason for your answer.



PROBLEM 4.92

4.93 Euler's equations for a planar (two-dimensional) flow in xy-plane are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial h}{\partial x} \qquad x = \text{direction}$$
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial h}{\partial y} \qquad y = \text{direction}$$

a. The slope of a streamline is given by

$$\frac{dy}{dx} = \frac{v}{u}$$

Using this relation in Euler's equation, show that

$$d\left(\frac{u^2 + v^2}{2g} + h\right) = 0$$

or

$$d\left(\frac{V^2}{2g} + h\right) = 0$$

which means that $V^2/2g + h$ is constant along a streaml

b. For an irrotational flow,

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

Substituting this equation into Euler's equation, show t

$$\frac{\partial}{\partial x}\left(\frac{V^2}{2g} + h\right) = 0$$
$$\frac{\partial}{\partial y}\left(\frac{V^2}{2g} + h\right) = 0$$

which means that $V^2/2g + h$ is constant in all directic

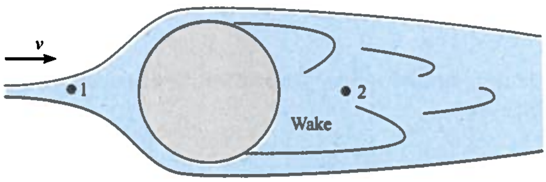
Pressure Field for a Circular Cylinder (§4.10)

4.94 WILEY PLUS A fluid is flowing around a cylinder as shown in Fig 4.37 on p. 149 in §4.10. A favorable pressure gradient can be found

- **a.** upstream of the stagnation point
- **b.** at the stagnation point
- **c.** between the stagnation point and separation point

4.95 The velocity distribution over the surface of a sphere upstream of the separation point is $u_\theta = 1.5\,U \sin\theta$, where U is the free stream velocity and θ is the angle measured from the forward stagnation point. A pressure of -2.5 in H_2O gage is measured at the point of separation on a sphere in a 100 ft/s airflow with a density of 0.07 lbm/ft^3. The pressure far upstream of the sphere in atmospheric. Estimate the location of the stagnation point (θ). Separation occurs on the windward side of the sphere.

4.96 Knowing the speed at point 1 of a fluid upstream of a sphere and the average speed at point 2 in the wake of in the sphere, can one use the Bernoulli equation to find the pressure difference between the two points? Provide the rationale for your decision.

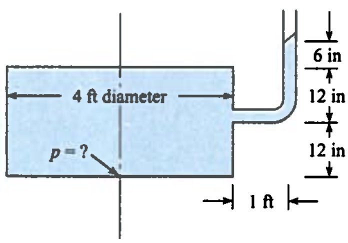


PROBLEM 4.96

Pressure Field for a Rotating Flow (§4.11)

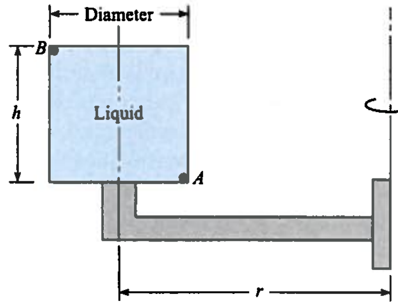
4.97 Take a spoon and rapidly stir a cup of liquid. Report on the contour of the surface. Provide an explanation for the observed shape.

4.98 This closed tank, which is 4 ft in diameter, is filled with water ($\rho = 62.4$ lbm/ft^3) and is spun around its vertical centroidal axis at a rate of 10 rad/s. An open piezometer is connected to the tank as shown so that it is also rotating with the tank. For these conditions, what is the pressure at the center of the bottom of the tank?



PROBLEM 4.98

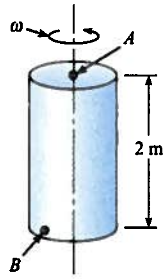
4.99 A tank of liquid (S = 0.80) that is 1 ft in diameter and 1.0 ft high (h = 1.0 ft) is rigidly fixed (as shown) to a rotating arm having a 2 ft radius. The arm rotates such that the speed at point A is 20 ft/s. If the pressure at A is 25 psf, what is the pressure at B?



PROBLEM 4.99

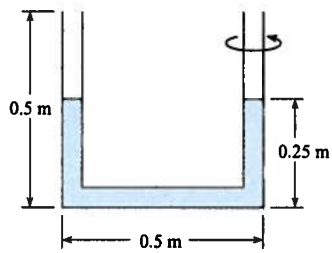
4.100 WILEY PLUS Separators are used to separate liquids of different densities, such as cream from skim milk, by rotating the mixtur at high speeds. In a cream separator the skim milk goes to the outside while the cream migrates toward the middle. A factor of merit for the centrifuge is the centrifugal acceleration force (RCF), which is the radial acceleration divided by the acceleratic due to gravity. A cream separator can operate at 9000 rpm (rev/min If the bowl of the separator is 20 cm in diameter, what is the centripetal acceleration if the liquid rotates as a solid body and what is the RCF?

4.101 A closed tank of liquid (S = 1.2) is rotated about a vertical axis (see the figure), and at the same time the entire tank is accelerated upward at 4 m/s^2. If the rate of rotation is 10 rad/s, wh is the difference in pressure between points A and B ($p_B - p_A$)? Point B is at the bottom of the tank at a radius of 0.5 m from the axis of rotation, and point A is at the top on the axis of rotation.



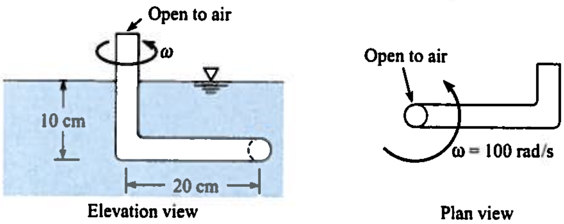
PROBLEM 4.101

4.102 WILEY GO A U-tube is rotated about one leg, as shown. Before being rotated the liquid in the tube fills 0.25 m of each leg. The length of the base of the U-tube is 0.5 m, and each leg is 0.5 m long. What would be the maximum rotation rate (in rad/s to ensure that no liquid is expelled from the outer leg?



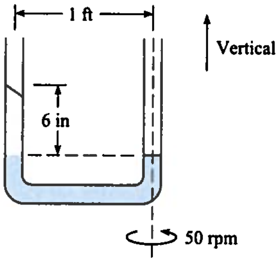
PROBLEM 4.102

4.103 An arm with a stagnation tube on the end is rotated at 100 rad/s in a horizontal plane 10 cm below a liquid surface as shown. The arm is 20 cm long, and the tube at the center of rotation extends above the liquid surface. The liquid in the tube is the same as that in the tank and has a specific weight of 10,000 N/m³. Find the location of the liquid surface in the central tube.



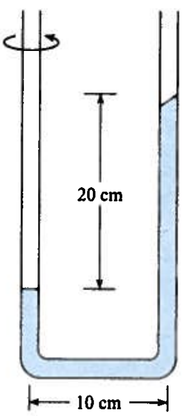
PROBLEM 4.103

4.104 A U-tube is rotated at 50 rev/min about one leg. The fluid at the bottom of the U-tube has a specific gravity of 3.0. The distance between the two legs of the U-tube is 1 ft. A 6 in. height of another fluid is in the outer leg of the U-tube. Both legs are open to the atmosphere. Calculate the specific gravity of the other fluid.



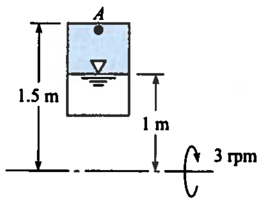
PROBLEM 4.104

4.105 WILEY PLUS A manometer is rotated around one leg, as shown. The difference in elevation between the liquid surfaces in the legs is 20 cm. The radius of the rotating arm is 10 cm. The liquid in the manometer is oil with a specific gravity of 0.8. Find the number of *g*'s of acceleration in the leg with greatest amount of oil.



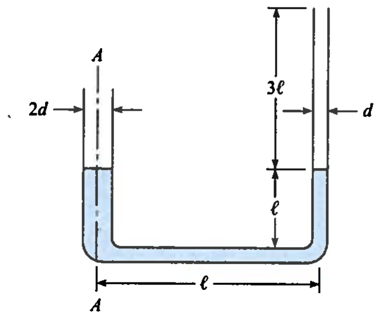
PROBLEM 4.105

4.106 A fuel tank for a rocket in space under a zero-*g* environment is rotated to keep the fuel in one end of the tan
The system is rotated at 3 rev/min. The end of the tank (poin
is 1.5 m from the axis of rotation, and the fuel level is 1 m fro
the rotation axis. The pressure in the nonliquid end of the ta
0.1 kPa, and the density of the fuel is 800 kg/m³. What is the pressure at the exit (point *A*)?



PROBLEM 4.106

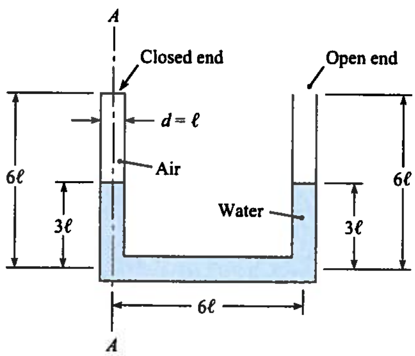
4.107 Water stands in these tubes as shown when no rotatio
occurs. Derive a formula for the angular speed at which wate
will just begin to spill out of the small tube when the entire system is rotated about axis *A*-*A*.



PROBLEM 4.107

4.108 WILEY PLUS Water (ρ = 1000 kg/m³) fills a slender tube 1 cm i
diameter, 40 cm long, and closed at one end. When the tube is rotated in the horizontal plane about its open end at a constant speed of 50 rad/s, what force is exerted on the closed end?

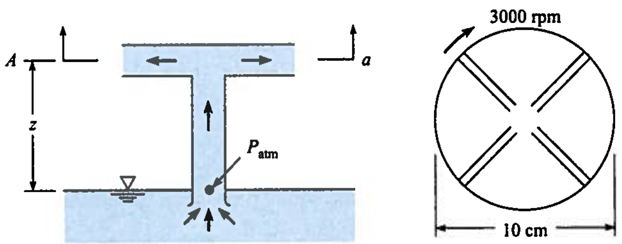
4.109 Water (ρ = 1000 kg/m³) stands in the closed-end U-tu
as shown when there is no rotation. If ℓ = 2 cm and if the ent
system is rotated about axis *A*-*A*, at what angular speed will



PROBLEM 4.109

water just begin to spill out of the open tube? Assume that the temperature for the system is the same before and after rotation and that the pressure in the closed end is initially atmospheric.

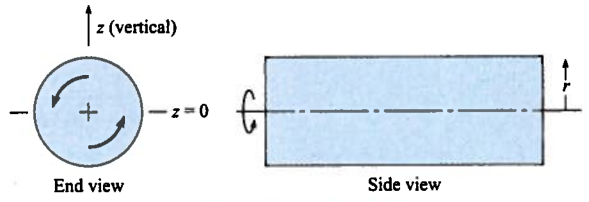
4.110 WILEY PLUS A simple centrifugal pump consists of a 10 cm disk with radial ports as shown. Water is pumped from a reservoir through a central tube on the axis. The wheel spins at 3000 rev/min, and the liquid discharges to atmospheric pressure. To establish the maximum height for operation of the pump, assume that the flow rate is zero and the pressure at the pump intake is atmospheric pressure. Calculate the maximum operational height z for the pump.



PROBLEM 4.110

Exam 2

4.111 A closed cylindrical tank of water ($\rho = 1000$ kg/m^3) is rotated about its horizontal axis as shown. The water inside the tank rotates with the tank ($V = r\omega$). Derive an equation for dp/dz along a vertical-radial line through the center of rotation. What is dp/dz along this line for $z = -1$ m, $z = 0$, and $z = +1$ m when $\omega = 5$ rad/s? Here $z = 0$ at the axis.



PROBLEMS 4.111, 4.112

4.112 The tank shown is 4 ft in diameter and 12 ft long and is closed and filled with water ($\rho = 62.4$ lbm/ft^3). It is rotated about its horizontal-centroidal axis, and the water in the tank rotates with the tank ($V = r\omega$). The maximum velocity is 25 ft/s. What is the maximum difference in pressure in the tank? Where is the point of minimum pressure?