## Assumptions:

- 1. Deceleration is constant.
- 2. Gasoline is incompressible.

Properties:  $\gamma = 42 \text{ lbf/ft}^3 (6.60 \text{ kN/m}^3)$ 

#### State the Goal

#### Find:

- 1. Pressure (psfg and kPa, gage) at top front of tank.
- 2. Maximum pressure (psfg and kPa, gage) in tank.

## Make a Plan

- Apply Euler's equation, Eq. (4.15), along top of tank. Elevation, z, is constant.
- 2. Evaluate pressure at top front.
- Maximum pressure will be at front bottom. Apply Euler's equation from top to bottom at front of tank.
- 4. Using result from step 2, evaluate pressure at front bottom.

# Take Action (Execute the Plan)

1. Euler's equation along the top of the tank

$$\frac{dp}{d\ell} = -\rho a_{\ell}$$

Integration from back (1) to front (2)

$$p_2 - p_1 = -\rho a_\ell \Delta \ell = -\frac{\gamma}{g} a_\ell \Delta \ell$$

2. Evaluation of  $p_2$  with  $p_1 = 0$ 

$$p_2 = -\left(\frac{42 \text{ lbf/ft}^3}{32.2 \text{ ft/s}^2}\right) \times (-10 \text{ ft/s}^2) \times 20 \text{ ft}$$
  
= 261 psfg

In SI units

$$p_2 = -\left(\frac{6.60 \text{ kN/m}^3}{9.81 \text{ m/s}^2}\right) \times (-3.05 \text{ m/s}^2) \times 6.1 \text{ m}$$
$$= 12.5 \text{ (kPa gage)}$$

3. Euler's equation in vertical direction

$$\frac{\mathrm{d}}{\mathrm{d}z}(p+\gamma z)=-\rho a_z$$

**4.** For vertical direction,  $a_z = 0$ . Integration from top of tank (2) to bottom (3):

$$p_2 + \gamma z_2 = p_3 + \gamma z_3$$
  
 $p_3 = p_2 + \gamma (z_2 - z_3)$   
 $p_3 = 261 \, \text{lbf/ft}^2 + 42 \, \text{lbf/ft}^3 \times 6 \, \text{ft} = 513 \, \text{psfg}$ 

n SI units

$$p_3 = 12.5 \text{ kN/m}^2 + 6.6 \text{ kN/m}^3 \times 1.83 \text{ m}$$
  
 $p_3 = 24.6 \text{ kPa gage}$ 

# 4.6 Applying the Bernoulli Equation along a Streamline

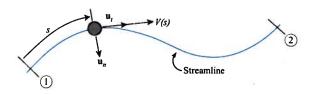
Because the Bernoulli equation is used frequently in fluid mechanics, this section introducthis topic.

# **Derivation of the Bernoulli Equation**

Select a particle on a streamline (Fig. 4.26). The position coordinate s gives the particle's potion. The unit vector  $\mathbf{u}_t$  is tangent to the streamline, and the unit vector  $\mathbf{u}_n$  is normal to t streamline. Assume steady flow so the velocity of the particle depends on position only. That V = V(s).

#### **FIGURE 4.26**

Sketch used for the derivation of the Bernoulli equation.



Assume that viscous forces on the particle can be neglected. Then, apply Euler's equation (Eq. 4.15) to the particle in the  $u_t$  direction.

$$-\frac{\partial}{\partial s}(p+\gamma z)=\rho a_t \tag{4.17}$$

Acceleration is given by Eq. (4.11). Because the flow is steady,  $\partial V/\partial t = 0$ , and Eq. (4.11) gives

$$a_t = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = V \frac{\partial V}{\partial s}$$
 (4.18)

Because p, z, and V in Eqs. (4.17) and (4.18) depend only on position s, the partial derivatives become ordinary derivatives (i.e., functions only of a single variable). Thus, write the these derivatives as ordinary derivatives and combine Eqs. (4.17) and (4.18) to give

$$-\frac{d}{ds}(p+\gamma z) = \rho V \frac{dV}{ds} = \rho \frac{d}{ds} \left(\frac{V^2}{2}\right)$$
 (4.19)

Move all the terms to one side:

$$\frac{d}{ds}\left(p + \gamma z + \rho \frac{V^2}{2}\right) = 0 \tag{4.20}$$

When the derivative of an expression is zero, the expression is equal to a constant. Thus, rewrite Eq. (4.20) as:

$$p + \gamma z + \rho \frac{V^2}{2} = C \tag{4.21a}$$

where C is a constant. Eq. (4.21a) is the pressure form of the Bernoulli equation. This is called the pressure form because all terms have units of pressure. Dividing Eq. (4.21a) by the specific weight yields the head form of the Bernoulli equation, which is given as Eq. (4.21b). In the head form, all terms have units of length.

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C \tag{4.21b}$$

# Physical Interpretation #1 (Energy Is Conserved)

One way to interpret the Bernoulli equation leads to the idea that when the Bernoulli equation applies, the total head of the flowing fluid is a constant along a streamline. To develop this interpretation, recall that piezometric head, introduced in Chapter 3, is defined as

piezometric head = 
$$h \equiv \frac{p}{\gamma} + z$$
 (4.22)

Introduce Eq. (4.22) into Eq. (4.21b)

$$h + \frac{V^2}{2g} = \text{Constant} \tag{4.23}$$

Now, velocity head is defined by

velocity head 
$$\equiv \frac{V^2}{2\sigma}$$
 (4.24)

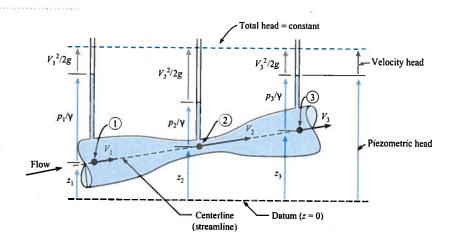
Combine Eqs. (4.22) to (4.24) to give

$$\begin{pmatrix}
\text{Piezometric} \\
\text{head}
\end{pmatrix} + \begin{pmatrix}
\text{Velocity} \\
\text{head}
\end{pmatrix} = \begin{pmatrix}
\text{Constant along} \\
\text{streamline}
\end{pmatrix}$$
(4.2)

Eq. (4.25) is shown visually in Fig. 4.27. Notice that piezometric head (blue lines) and th velocity head (gray lines) are changing, but the sum of the piezometric head plus velocity hea is everywhere constant. Thus, the total head is constant for all points along a streamline whe the Bernoulli equation applies.

#### **FIGURE 4.27**

Water flowing through a Venturi nozzle. The piezometers shows the piezometric head at locations 1, 2, and 3.



The previous discussion introduced head. Head is a concept that is used to characteriz the balance of work and energy in a flowing fluid. As shown in Fig. 4.27, head can be visualized as the height of a column of liquid. Each type of head describes a work or energy term. Velocit head characterizes the kinetic energy in a flowing fluid, elevation head characterizes the grav itational potential energy of a fluid, and pressure head is related to work done by the pressurforce. As shown in Fig. 4.27, the total head is constant. This means that when the Bernoull equation applies, the fluid is not losing energy as it flows. The reason is that viscous effects are the cause of energy loses, and viscous effects are negligible when the Bernoulli equation applies.

# Physical Interpretation #2 (Velocity and Pressure Vary Inversely)

A second way to interpret the Bernoulli equation leads to the idea that when velocity increases then pressure will decrease. To develop this interpretation, recall that piezometric pressure introduced in Chapter 3, is defined as

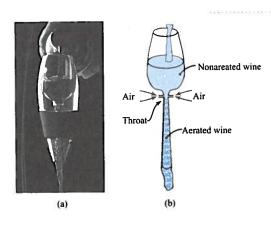
piezometric pressure = 
$$p_z \equiv p + \gamma z$$
 (4.26)

Introduce Eq. (4.26) into Eq. (4.21a)

$$p_z + \frac{\rho V^2}{2} = \text{Constant} \tag{4.27}$$

For Eq. (4.27) to be true, piezometric pressure and velocity must vary inversely so that the sum of  $p_z$  and  $(V^2/2g)$  is a constant. Thus, the pressure form of the Bernoulli equation shows that piezometric pressure varies inversely with velocity. In regions of high velocity, piezometric pressure will be low; in regions of low velocity, piezometric pressure will be high.

**EXAMPLE.** Fig. 4.28 shows a Vinturi<sup>™</sup> red wine aerator, which is a product that is used to add air to wine. When wine flows through the Vinturi<sup>™</sup>, the shape of the device causes an increase in the wine's velocity and a corresponding decrease in its pressure. At the throat, the pressure is below atmospheric pressure so air flows inward through two inlet ports and mixes with the wine to create aerated wine, which tastes better to most people.



#### **FIGURE 4.28**

(a) The Vinturi™ wine aerator, and (b) a sketcillustrating the operating principle. (Photo courte of Vinturi Inc.)

# **Working Equations and Process**

Table 4.3 summarizes the Bernoulli equation.

TABLE 4.3 Summary of the Bernoulli Equation

Description	Equation		Terms
Bernoulli equation (head form) Recommend form to use for liquids	$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) $ Eq	q. (4.21b)	$p = { m static pressure (Pa)}$ (use gage pressure or abs pressure) (ave vacuum pressure; will be wrong) $\gamma = { m specific weight (N/m^3)}$ $V = { m speed (m/s)}$ $g = { m gravitational constant} = 9.81 { m m/s}^3$ $z = { m elevation or elevation head (m)}$ $\frac{p}{\gamma} = { m pressure head (m)}$ $\frac{V^2}{2g} = { m velocity head (m)}$ $\frac{p}{\gamma} + z = { m piezometric head (m)}$ $p + \gamma z = { m piezometric pressure (Pa)}$ $\frac{\rho V^2}{2} = { m kinetic pressure (Pa)}$
Bernoulli equation (pressure form) Recommend form to use for gases	$\left(p_1 + \frac{\rho V_1^2}{2} + \rho g z_1\right) = \left(p_2 + \frac{\rho V_2^2}{2} + \rho g z_2\right) \qquad E$	.q. (4.21a)	

The process for applying the Bernoulli equation is

- Step 1. Selection. Select the head form or the pressure form. Check that the assumption are satisfied.
- Step 2. Sketching. Select a streamline. Then, select points 1 and 2 where you know inf mation or where you want to find information. Annotate your documentation to show streamline and points.
- Step 3. General Equation. Write the general form of the Bernoulli equation. Perform term-by-term analysis to simplify the general equation to a reduced equation that appl to the problem at hand.
- Step 4. Validation. Check the reduced equation to ensure that it makes physical sense.

Example 4.4 shows how to apply the Bernoulli equation to a draining tank of water.

# **EXAMPLE 4.4**

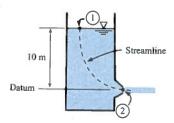
Applying the Bernoulli Equation to Water Draining out a Tank

# **Problem Statement**

Water in an open tank drains through a port at the bottom of the tank. The elevation of the water in the tank is 10 m above the drain. Find the velocity of the liquid in the drain port.

#### Define the Situation

Water flows out of a tank.



#### Assumptions:

- · Steady flow.
- · Viscous effects are negligible.

# State the Goal

 $V_2$  (m/s)  $\leftarrow$  Velocity at the exit port.

## Generate Ideas and Make a Plan

Selection. Select the head form of the Bernoulli equation because the fluid is a liquid. Document assumptions (see above).

Sketching. Select point 1 where information is known and point 2 where information is desired. On the situation diagram (see above), sketch the streamline, label points 1 and 2, and label the datum.

# General Equation.

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right)$$
 (a)

### Term-by-term analysis.

- $p_1 = p_2 = 0 \text{ kPa gage}$
- Let  $V_1 = 0$  because  $V_1 \ll V_2$
- Let  $z_1 = 10 \text{ m}$  and  $z_2 = 0 \text{ m}$

Reduce Eq. (a) so it applies to the problem at hand.

$$(0 + 0 + 10 \text{ m}) = \left(0 + \frac{V_2^2}{2g} + 0\right)$$
 (b)

Simplify Eq. (b):

$$V_2 = \sqrt{2g(10 \text{ m})}$$
 (c)

Because Eq. (c) has only one unknown, the plan is to use this equation to solve for  $V_2$ .

## Take Action (Execute the Plan)

$$V_2 = \sqrt{2g(10 \text{ m})}$$

$$V_2 = \sqrt{2(9.81 \text{ m/s}^2)(10 \text{ m})}$$

$$V_2 = 14 \text{ m/s}$$

# Review the Solution and the Process

- 1. Knowledge. Notice that the same answer would be calculated for an object dropped from the same elevation as the water in the tank. This is because both problems involve equating gravitational potential energy at 1 with kinetic energy at 2.
- 2. Validate. The assumption of the small velocity at the liquid surface is generally valid. It can be shown (Chapter 5) that

$$\frac{V_1}{V_2} = \frac{D_2^2}{D_2^2}$$

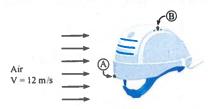
For example, a diameter ratio of 10 to 1  $(D_2/D_1 = 0.1)$ results in the velocity ratio of 100 to 1 ( $V_1/V_2 = 1/100$ ). When the Bernoulli equation is applied to a gas, it is common to neglect the elevation terms because these terms are negligibly small as compared to the pressure and velocity terms. An example of applying the Bernoulli equation to a flow of air is presented in Example 4.5.

# **EXAMPLE 4.5**

Applying the Bernoulli Equation to Air Flowing around a Bicycle Helmet

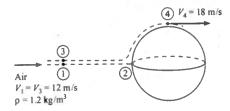
#### **Problem Statement**

The problem is to estimate the pressure at locations A and B so these values can be used to estimate the ventilation in a bicycle helmet that is being designed. Assume an air density of  $\rho=1.2$  kg/m<sup>3</sup> and an air speed of 12 m/s relative to the helmet. Point A is a stagnation point, and the velocity of air at point B is 18 m/s.



#### Define the Situation

Idealize flow around a bike helmet as flow around the upper half of a sphere. Assume steady flow. Assume that point B is outside the boundary layer. Relabel the points as shown in the situation diagram because this makes application of the Bernoulli equation easier.



#### State the Goal

 $p_2(Pa \text{ gage}) \blacktriangleleft Pressure at the forward stagnation point.$  $<math>p_4(Pa \text{ gage}) \blacktriangleleft Pressure at the shoulder.}$ 

## Generate Ideas and Make a Plan

Selection. Select the pressure form of the Bernoulli equation because the flow is air. Then write the Bernoulli equation along the stagnation streamline (i.e., from point 1 to point 2).

$$\left(p_1 + \frac{\rho V_1^2}{2} + \rho g z_1\right) = \left(p_2 + \frac{\rho V_2^2}{2} + \rho g z_2\right)$$
 (a)

### Conduct a term-by-term analysis.

- p<sub>1</sub> = 0 kPa gage because the external flow is at atmospheri pressure.
- $V_1 = 12 \text{ m/s}$
- let z<sub>1</sub> = z<sub>2</sub> = 0 because elevation terms are negligibly smal for a gas flow such as a flow of air
- let  $V_2 = 0$  because this is a stagnation point.

Now, simplify Eq. (a).

$$0+\frac{\rho V_1^2}{2}+0=p_2+0+0$$

Eq. (b) has only a single unknown  $(p_2)$ .

Next, apply the Bernoulli equation to the streamline that connects points 3 and 4.

$$\left(p_3 + \frac{\rho V_3^2}{2} + \rho g z_3\right) = \left(p_4 + \frac{\rho V_4^2}{2} + \rho g z_4\right)$$

Do a term-by-term analysis to give:

$$\left(0 + \frac{\rho V_3^2}{2} + 0\right) = \left(p_4 + \frac{\rho V_4^2}{2} + 0\right)$$

Eq. (d) has only one unknown  $(p_4)$ . The plan is

- 1. Calculate  $(p_2)$  using Eq. (b).
- 2. Calculate (p<sub>4</sub>) using Eq. (d).

# Take Action (Execute the Plan)

1. Bernoulli equation (point 1 to point 2)

$$p_2 = \frac{\rho V_1^2}{2} = \frac{(1.2 \text{ kg/m}^3)(12 \text{ m/s})^2}{2}$$
$$p_2 = 86.4 \text{ Pa gage}$$

2. Bernoulli equation (point 3 to point 4)

$$p_2 = \frac{\rho(V_3^2 - V_4^2)}{2} = \frac{(1.2 \text{ kg/m}^3)(12^2 - 18^2)(\text{m/s})^2}{2}$$

$$p_2 = -108 \text{ Pa gage}$$

#### Review the Solution and the Process

- Discussion. Notice that where the velocity is high (i.e., point 4), the pressure is low (negative gage pressure).
- Knowledge. Remember to specify pressure units in gage pressure or absolute pressure.
- 3. *Knowledge*. Theory shows that the velocity at the should of a sphere is 3/2 the velocity in the free stream.

Example 4.6 involves a venturi. A venturi (also called a venturi nozzle) is a constric section as shown in this example. As fluid flows through a venturi, the pressure is reduced the narrow area, called the throat. This drop in pressure is called the venturi effect.

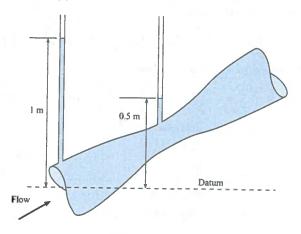
The venturi can be used to entrain liquid drops into a flow of gas as in a carburetor.' venturi can also be used to measure the flow rate. The venturi is commonly analyzed with Bernoulli equation.

# **EXAMPLE 4.6**

Applying the Bernoulli Equation to Flow through a Venturi Nozzle

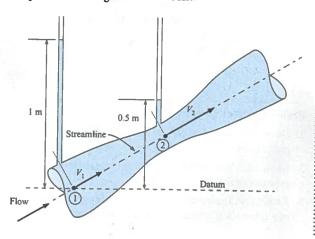
# **Problem Statement**

Piezometric tubes are tapped into a venturi section as shown in the figure. The liquid is incompressible. The upstream piezometric head is 1 m, and the piezometric head at the throat is 0.5 m. The velocity in the throat section is twice as large as in the approach section. Find the velocity in the throat section.



#### Define the Situation

A liquid flows through a venturi nozzle.



#### State the Goal

 $V_2(m/s) \leftarrow Velocity at point 2.$ 

# Generate Ideas and Make a Plan

Select the Bernoulli equation because the problem involves flow through a nozzle. Select the head form because a liquid is involved. Select a streamline and points 1 and 2. Sketch these choices on the situation diagram.

Write the general form of the Bernoulli equation.

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$
 (a)

Introduce piezometric head because this is what the piezometer measures:

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g}$$

$$(1.0 \text{ m}) + \frac{V_1^2}{2g} = (0.5 \text{ m}) + \frac{V_2^2}{2g}$$

Let  $V_1 = 0.5 V_2$ 

$$(1.0 \text{ m}) + \frac{(0.5 V_2)^2}{2g} = (0.5 \text{ m}) + \frac{V_2^2}{2g}$$
 (b)

**Plan.** Use Eq. (b) to solve for  $V_2$ .

## Take Action (Execute the Plan)

Bernoulli equation (i.e., Eq. b):

$$(0.5 \text{ m}) = \frac{0.75 V_2^2}{2g}$$

Thus,

$$V_2 = \sqrt{\frac{2g(0.5 \text{ m})}{0.75}}$$

$$V_2 = \sqrt{\frac{2(9.81 \text{ m/s}^2)(0.5 \text{ m})}{0.75}}$$

$$V_2 = \boxed{3.62 \text{ m/s}}$$

#### Review the Solution and the Process

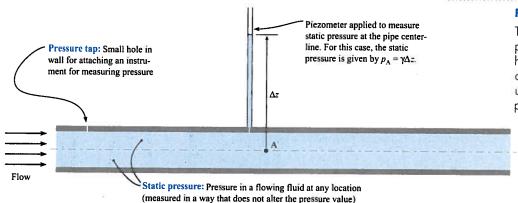
- 1. Knowledge. Notice how a piezometer is used to measure piezometric head in the nozzle.
- Knowledge. A piezometer could not be used to measure the piezometric head if the pressure anywhere in the line were subatmospheric. In this case, pressure gages or manometers could be used.

# 4.7 Measuring Velocity and Pressure

The piezometer, stagnation tube, and Pitot-static tube have long been used to measure pressure and velocity. Indeed, many concepts in measurement are based on these instruments. Thus, this section describes these instruments.

# **Static Pressure**

Static pressure is the pressure in a flowing fluid. A common way to measure static pressure is to drill a small hole in the wall of a pipe and then connect a piezometer or pressure gage to this port (see Fig. 4.29). This port is called a **pressure tap**. The reason that a pressure tap is useful is that it provides a way to measure static pressure that does not disturb the flow.

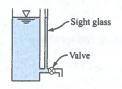


#### **FIGURE 4.29**

This figure defines a pressure port and sho how a piezometer is connected to a wall c used to measure static pressure.

# **✓ CHECKPOINT PROBLEM 4.3**

Restaurants often use large coffee dispensers (see sketch). The sight glass shows the level of coffee. If the valve is opened, what happens to the level of coffee that is visible in the sight glass? Will the level go up, go down, or stay the same? Why?



# Stagnation Tube

A stagnation tube (also known as a total head tube) is an open-ended tube directed upstream in a flow (see Fig. 4.30). A stagnation tube measures the sum of static pressure and kinetic pressure. Kinetic pressure is defined at an arbitrary point A as:

$$\begin{pmatrix} \text{kinetic pressure} \\ \text{at point A} \end{pmatrix} = \frac{\rho V_A^2}{2}$$

Next, we will derive an equation for velocity in an open channel flow. For the stagnation tube in Fig. 4.30, select points 0 and 1 on the streamline, and let  $z_0 = z_1$ . The Bernoulli equation reduces to

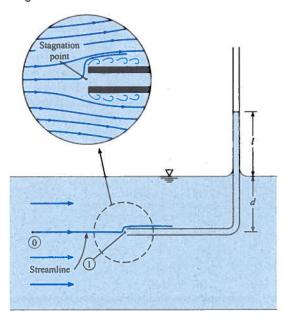
$$p_1 + \frac{\rho V_1^2}{2} = p_0 + \frac{\rho V_0^2}{2} \tag{4.28}$$

The velocity at point 1 is zero (stagnation point). Hence, Eq. (4.28) simplifies to

$$V_0^2 = \frac{2}{\rho}(p_1 - p_0) \tag{4.29}$$

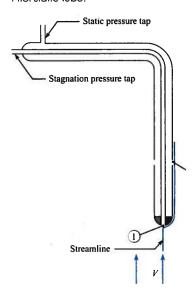
**FIGURE 4.30** 

Stagnation tube.



#### FIGURE 4.31

Pitot-static tube.



Next, apply the hydrostatic equation:  $p_0 = \gamma d$  and  $p_1 = \gamma (l + d)$ . Therefore, Eq. (4.29) can written as

$$V_0^2 = \frac{2}{\rho} (\gamma(l+d) - \gamma d)$$

which reduces to

$$V_0 = \sqrt{2gl} \tag{4.3}$$

# Pitot-Static Tube

The Pitot-static tube, named after the eighteenth-century French hydraulic engineer who vented it, is based on the same principle as the stagnation tube, but it is much more versal than the stagnation tube. The Pitot-static tube, shown in Fig. 4.31, has a pressure tap at 1 upstream end of the tube for sensing the kinetic pressure. There are also ports located seve tube diameters downstream of the front end of the tube for sensing the static pressure in 1 fluid where the velocity is essentially the same as the approach velocity. When the Bernot equation, Eq. (4.21a), is applied between points 1 and 2 along the streamline shown in Fig. 4. the result is

$$p_1 + \gamma z_1 + \frac{\rho V_1^2}{2} = p_2 + \gamma z_2 + \frac{\rho V_2^2}{2}$$

But  $V_1 = 0$ , so solving that equation for  $V_2$  gives an equation for velocity.

$$V_2 = \left[\frac{2}{9}(p_{z,1} - p_{z,2})\right]^{1/2} \tag{4.3}$$

Here  $V_2 = V$ , where V is the velocity of the stream and  $p_{z,1}$  and  $p_{z,2}$  are the piezometric pressures at points 1 and 2, respectively.

By connecting a pressure gage or manometer between the pressure taps shown in Fig. 4.31 that lead to points 1 and 2, one can easily measure the flow velocity with the Pitot-static tube. A major advantage of the Pitot-static tube is that it can be used to measure velocity in a pressurized pipe; a stagnation tube is not convenient to use in such a situation.

If a differential pressure gage is connected across the taps, the gage measures the difference in piezometric pressure directly. Therefore Eq. (4.31) simplifies to

$$V = \sqrt{2\Delta p/\rho} \tag{4.32}$$

where  $\Delta p$  is the pressure difference measured by the gage.

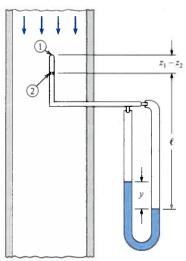
More information on Pitot-static tubes and flow measurement is available in the Flow Measurement Engineering Handbook (5). Example 4.7 illustrates the application of the Pitot-static tube with a manometer. Then, Example 4.8 illustrates application with a pressure gage.

# **EXAMPLE 4.7**

Applying a Pitot-Static Tube (pressure measured with a manometer).

#### **Problem Statement**

A mercury manometer is connected to the Pitot-static tube in a pipe transporting kerosene as shown. If the deflection on the manometer is 7 in., what is the kerosene velocity in the pipe? Assume that the specific gravity of the kerosene is 0.81.



#### Define the Situation

A Pitot-static tube is mounted in a pipe and connected to a manometer.

Assumptions: Pitot-static tube equation is applicable.

**Properties:**  $S_{\text{kero}} = 0.81$ , from Table A.4,  $S_{\text{Hg}} = 13.55$ .

#### State the Goal

Find: Flow velocity (m/s).

# Generate Ideas and Make a Plan

- Find difference in piezometric pressure using the manometer equation.
- 2. Substitute in Pitot-static tube equation.
- 3. Evaluate velocity.

#### Take Action (Execute the Plan)

Manometer equation between points 1 and 2 on Pitot-statube:

$$p_1 + (z_1 - z_2)\gamma_{\rm kero} + \ell\gamma_{\rm kero} - y\gamma_{\rm Hg} - (\ell - y)\gamma_{\rm kero} = p$$
or

$$p_1 + \gamma_{\text{kero}} z_1 - (p_2 + \gamma_{\text{kero}} z_2) = y(\gamma_{\text{Hg}} - \gamma_{\text{kero}})$$
$$p_{z,1} - p_{z,2} = y(\gamma_{\text{Hg}} - \gamma_{\text{kero}})$$

2. Substitution into the Pitot-static tube equation:

$$V = \left[\frac{2}{\rho_{\text{kero}}} y (\gamma_{\text{Hg}} - \gamma_{\text{kero}})\right]^{1/2}$$
$$= \left[2gy \left(\frac{\gamma_{\text{Hg}}}{\gamma_{\text{kero}}} - 1\right)\right]^{1/2}$$

3. Velocity evaluation:

$$V = \left[ 2 \times 32.2 \text{ ft/s}^2 \times \frac{7}{12} \text{ ft} \left( \frac{13.55}{0.81} - 1 \right) \right]^{1/2}$$
$$= \left[ 2 \times 32.2 \times \frac{7}{12} (16.7 - 1) \text{ ft}^2/\text{s}^2 \right]^{1/2}$$
$$= 24.3 \text{ ft/s}$$

#### Review the Solution and the Process

Discussion. The -1 in the quantity (16.7 -1) reflects the effect of the column of kerosene in the right leg of the manometer, which tends to counterbalance the mercury in the left leg. Thus with a gas-liquid manometer, the counterbalancing effect is negligible.

# **EXAMPLE 4.8**

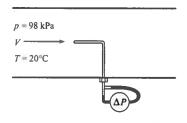
Applying a Pitot-Static Tube (pressure measured with a pressure gage)

#### Problem Statement

A differential pressure gage is connected across the taps of a Pitot-static tube. When this Pitot-static tube is used in a wind tunnel test, the gage indicates a  $\Delta p$  of 730 Pa. What is the air velocity in the tunnel? The pressure and temperature in the tunnel are 98 kPa absolute and 20°C, respectively.

#### Define the Situation

A differential pressure gage is attached to a Pitot-static tube for velocity measurement in a wind tunnel.



#### Assumptions:

- · Airflow is steady.
- Pitot-tube equation applicable.

Properties: Table A.2,  $R_{air} = 287 \text{ J/kg K}$ .

#### State the Goal

Find the air velocity (in m/s).

#### Generate Ideas and Make a Plan

- 1. Using the ideal gas law, calculate air density.
- 2. Using the Pitot-static tube equation, calculate the velocity.

#### Take Action (Execute the Plan)

1. Density calculation:

$$\rho = \frac{p}{RT} = \frac{98 \times 10^3 \text{ N/m}^2}{(287 \text{ J/kg K}) \times (20 + 273 \text{ K})} = 1.17 \text{ kg/m}^3$$

2. Pitot-static tube equation with differential pressure gage:

$$V = \sqrt{2\Delta p/\rho}$$

$$V = \sqrt{(2 \times 730 \text{ N/m}^2)/(1.17 \text{ kg/m}^3)} = 35.3 \text{ m/s}$$

# 4.8 Characterizing Rotational Motion of a Flowing Fluid

In addition to velocity and acceleration, engineers also describe the rotation of a fluid. The this topic is introduced in this section. At this point, we recommend the online vorticity fit (6) because this film shows the concepts in this section using laboratory experiments.

# **Concept of Rotation**

**Rotation** of a fluid particle is defined as the average rotation of two initially mutually pendicular faces of a fluid particle. The test is to look at the rotation of the line that bise both faces (a-a and b-b in Fig. 4.32). The angle between this line and the horizontal axis the rotation,  $\theta$ .