9 PREDICTING SHEAR FORCE

FIGURE 9.1

When engineers design sailboats for racing, they consider the drag force on the hull. This drag force is caused by the pressure and shear stress distributions. This chapter is concerned with the shear stress and the shear force. (Foucras G./StockImage/Getty Images.)

Chapter Road Map

This chapter describes how to predict shear stress and shear force on a flat surface. The emphasis is c the theory because this theory provides the foundati for more advanced study in fluid mechanics.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Describe Couette flow. Show how to derive and apply th working equations. (§9.1)
- Describe Hele-Shaw flow. Show how to derive and apply the working equations. (§9.1)
- Sketch the development of the boundary layer on a flat plate. Label and explain the main features. (§9.2)
- Define the local shear stress coefficient, c_f, and the average shear stress coefficient, C_f. (§9.3)
- Define or calculate Re_x and Re_L. (§9.3)
- For the laminar boundary, calculate the boundary layer thickness, the shear stress, and the shear force using suitable correlations. (§9.3)
- Describe the transition Reynolds number. (§9.4)
- Describe or apply the power law equation for the turbulen boundary layer. (§9.5)
- Sketch a turbulent boundary layer. Label and describe the three zones of flow. (§9.5)
- For the turbulent boundary layer, calculate the boundary layer thickness, the shear stress, and the shear force using suitable correlations. (§9.5)

9.1 Uniform Laminar Flow

In this section, Newton's second law of motion is used to derive a differential equation that governs a 1-D, steady, viscous flow. Then, the equation is solved for two classic problems: Couette flow (see Section 2.6) and Hele-Shaw flow (fully developed laminar flow between two parallel plates). The rationale for this section is to introduce fundamentals that are useful for analyzing viscous flows.

The equation derived in this section is a special case of the Navier-Stokes equation. The Navier-Stokes equation is probably the single most important equation in fluid mechanics.

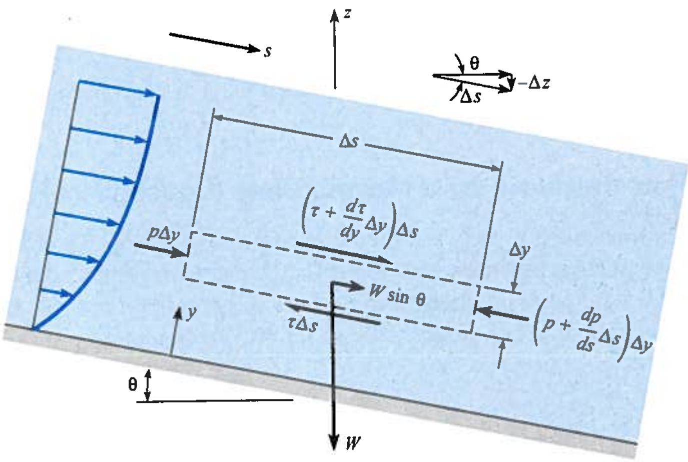
The Equation of Motion for Steady and Uniform Laminar Flow

Consider a CV (Fig. 9.2), which is aligned with the flow direction s. The streamlines are inclined at an angle θ with respect to the horizontal plane. The control volume has dimensions $\Delta s \times \Delta y \times$ unity; that is, the control volume has a unit length into the page. By application of the momentum equation, the sum of the forces acting in the s-direction is equal to the net outflow of momentum from the control volume. The flow is uniform, so the outflow of momentum is equal to the inflow and the momentum equation reduces to

$$\sum F_s = 0 \tag{9.1}$$

FIGURE 9.2

Control volume for analysis of uniform flow with parallel streamlines.



There are three forces acting on the matter in the control volume: the forces due to pressure, shear stress, and gravity. The net pressure force is

$$p\Delta y - \left(p + \frac{dp}{ds}\Delta s\right)\Delta y = -\frac{dp}{ds}\Delta s \Delta y$$

The net force due to shear stress is

$$\left(\tau + \frac{d\tau}{dy}\Delta y\right)\Delta s - \tau\Delta s = \frac{d\tau}{dy}\Delta y \Delta s$$

The component of gravitational force is $\rho g \Delta s \Delta y \sin\theta$. However, $\sin\theta$ can be related to the r at which the elevation, z, decreases with increasing s and is given by $-dz/ds$. Thus the gravi tional force becomes

$$\rho g \Delta s \Delta y \sin\theta = -\gamma \Delta y \Delta s \frac{dz}{ds}$$

Summing the forces and dividing by volume ($\Delta s \Delta y$) results in

$$\frac{d\tau}{dy} = \frac{d}{ds}(p + \gamma z) \tag{9.2}$$

where it is noted that the gradient of the shear stress is equal to the gradient in piezometric pre sure in the flow direction. The shear stress is equal to $\mu\, du/dy$, so the basic equation becomes

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d}{ds}(p + \gamma z) \tag{9.3}$$

where μ is constant. Eq. (9.3) is the Navier-Stokes equation applied to a uniform and stea flow. The general form of this equation is introduced in Chapter 16. This equation is no applied to the two flow configurations.

✔CHECKPOINT PROBLEM 9.1

The sketch identifies terms that appear in the Navier-Stokes equation.

$$\boxed{\mu \frac{d^2u}{dy^2}} = \boxed{\frac{d(p + \gamma z)}{ds}}$$

Term A   Term B

a. What are the secondary dimensions of each term? Primary dimensions?

b. What does Term A mean?

c. What does Term B mean?

d. What does this equation mean holistically? That is, what is the physical interpretation?

Flow Produced by a Moving Plate (Couette Flow)

Consider the flow between the two plates shown in Fig. 9.3. The lower plate is fixed, and th upper plate is moving with a speed U. The plates are separated by a distance L. In this probler there is no pressure gradient in the flow direction ($dp/ds = 0$), and the streamlines are in th horizontal direction ($dz/ds = 0$), so Eq. (9.3) reduces to

$$\frac{d^2u}{dy^2} = 0$$

The two boundary conditions are

$$u = 0 \quad \text{at} \quad y = 0$$
$$u = U \quad \text{at} \quad y = L$$

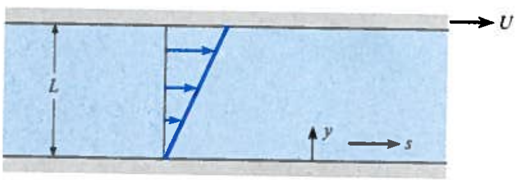


FIGURE 9.3

Flow generated by a moving plate (Couette flow).

Integrating this equation twice gives

$$u = C_1 y + C_2$$

Applying the boundary conditions results in

$$u = \frac{y}{L}U \tag{9.4}$$

which shows that the velocity profile is linear between the two plates. The shear stress is constant and equal to

$$\tau = \mu\frac{du}{dy} = \mu\frac{U}{L} \tag{9.5}$$

This flow is known as a **Couette flow** after a French scientist, M. Couette, who did pioneering work on the flow between parallel plates and rotating cylinders. It has application in the design of lubrication systems.

Example 9.1 illustrates the application of Couette flow in calculating shear stress.

EXAMPLE 9.1

Shear Stress in Couette Flow

Problem Statement

SAE 30 lubricating oil at $T = 38°C$ flows between two parallel plates, one fixed and the other moving at 1.0 m/s. Plates are spaced 0.3 mm apart. What is the shear stress on the plates?

Define the Situation

SAE 30 lubricating oil is flowing between parallel plates

Properties: From Table A.4, $\mu = 1.0 \times 10^{-1}\ \text{N}\cdot\text{s/m}^2$

State the Goal

Find: Shear stress (in N/m^2) on top plate.

Generate Ideas and Make a Plan

Calculate shear stress using Eq. (9.5).

Take Action (Execute the Plan)

$$\tau = \mu\frac{du}{dy} = \mu\frac{U}{L}$$

$$= (1.0 \times 10^{-1}\ \text{N}\cdot\text{s/m}^2)(1.0\ \text{m/s})/(3 \times 10^{-4}\ \text{m})$$

$$\tau = \boxed{333\ \text{N/m}^2}$$

Review the Solution and the Process

Knowledge. Because the velocity gradient is constant, the she stress is constant throughout the flow. Thus, the magnitude c the shear stress is the same for the bottom plate as the top pl:

Flow Between Stationary Parallel Plates (Hele-Shaw Flow)

Consider the two parallel plates separated by a distance B in Fig. 9.4. In this case, the flow velocity is zero at the surface of both plates, so the boundary conditions for Eq. (9.3) are

$$u = 0 \quad \text{at} \quad y = 0$$
$$u = 0 \quad \text{at} \quad y = B$$

Because the flow is uniform (i.e., there is no change in velocity in the streamwise direction), u is a function of y only. Therefore, d^2u/dy^2 in Eq. (9.3), as well as the gradient in piezometric

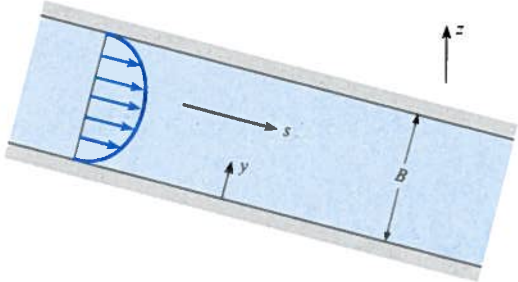


FIGURE 9.4
Uniform flow between two stationary plates (Hele-Shaw flow).

pressure, must also be equal to a constant in the streamwise direction. Integrating Eq. (9.3 twice gives

$$u = \frac{y^2}{2\mu}\frac{d}{ds}(p + \gamma z) + C_1 y + C_2$$

To satisfy the boundary condition at $y = 0$, set $C_2 = 0$. Applying the boundary condition a $y = B$ requires that C_1 be

$$C_1 = -\frac{B}{2\mu}\frac{d}{ds}(p + \gamma z)$$

so the final equation for the velocity is

$$u = -\frac{1}{2\mu}\frac{d}{ds}(p + \gamma z)(By - y^2) = -\frac{1}{2\mu}(By - y^2)\frac{d(p + \gamma z)}{ds} \qquad (9.6$$

which is a parabolic profile with the maximum velocity occurring on the centerline between the plates, as shown in Fig. 9.3. The maximum velocity is

$$u_{\max} = -\left(\frac{B^2}{8\mu}\right)\frac{d}{ds}(p + \gamma z) \qquad (9.7a$$

or in terms of piezometric head

$$u_{\max} = -\left(\frac{B^2\gamma}{8\mu}\right)\frac{dh}{ds} \qquad (9.7b)$$

The fluid always flows in the direction of decreasing piezometric pressure or piezometric head, so dh/ds is negative, giving a positive value for $u_{\max}$.

The discharge per unit width, q, is obtained by integrating the velocity over the distance between the plates:

$$q = \int_0^B u\,dy = -\left(\frac{B^3}{12\mu}\right)\frac{d}{ds}(p + \gamma z) = -\left(\frac{B^3\gamma}{12\mu}\right)\frac{dh}{ds} \qquad (9.8)$$

The average velocity is

$$V = \frac{q}{B} = -\left(\frac{B^2}{12\mu}\right)\frac{d}{ds}(p + \gamma z) = \frac{2}{3}u_{\max} \qquad (9.9)$$

Note that flow is the result of a change of the piezometric head, not just a change of p or z alone. Experiments reveal that if the Reynolds number (VB/ν) is less than 1000, the flow is laminar. For a Reynolds number greater than 1000, the flow may be turbulent, and the equations developed in this section are invalid.

The flow between parallel plates is often called **Hele-Shaw flow**. It has application in flow visualization studies and in microchannel flows.

A significant difference between Couette flow and Hele-Shaw flow is that the motion of a plate is responsible for Couette flow, whereas a gradient in piezometric pressure provides the force to move a Hele-Shaw flow.

Example 9.2 illustrates how to calculate the pressure gradient required for flow between two parallel plates.

EXAMPLE 9.2

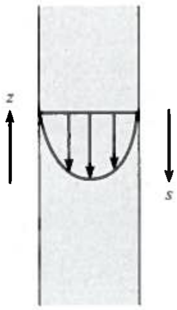
Pressure Gradient for Flow Between Parallel Plates

Problem Statement

Oil having a specific gravity of 0.8 and a viscosity of 2×10^{-2} N · s/m² flows downward between two vertical smooth plates spaced 10 mm apart. If the discharge per meter of width is 0.01 m²/s, what is the pressure gradient dp/ds for this flow?

Define the Situation

Oil flows downward between two vertical smooth plates spaced 10 mm apart. The discharge per meter of width is 0.01 (m²/s).



State the Goal

Find: Pressure gradient dp/ds (in Pa/m) for this flow.

Properties: $S = 0.8$, $\mu = 2 \times 10^{-2}$ N · s/m².

Generate Ideas and Make a Plan

1. Check to see if the flow is laminar using $VB/\nu < 1000$. If it is laminar, continue.
2. Calculate piezometric head gradient using Eq. (9.8).
3. Subtract elevation gradient to obtain the pressure gradient.

Take Action (Execute the Plan)

1. Check for laminar flow

$$\mathrm{Re} = \frac{VB}{\nu} = \frac{VB\rho}{\mu} = \frac{q\rho}{\mu}$$

$$= \frac{(0.01\ \mathrm{m^2/s}) \times 800\ \mathrm{kg/m^3}}{0.02\ \mathrm{N \cdot s/m^2}} = 400$$

$VB/\nu < 1000$. Flow is laminar, equations apply.

2. Kinematic viscosity:

$$\nu = \mu/\rho = \frac{2 \times 10^{-2}\ \mathrm{N \cdot s/m^2}}{0.8 \times 1000\ \mathrm{kg/m^3}} = 2.5 \times 10^{-5}\ \mathrm{m^2/s}$$

Piezometric head gradient is

$$\frac{dh}{ds} = -\frac{12\mu}{B^3\gamma}q = -\frac{12\nu}{B^3 g}q$$

$$\frac{dh}{ds} = -\frac{12 \times 2.5 \times 10^{-5}\ \mathrm{m^2/s}}{(0.01\ \mathrm{m})^3 \times 9.81\ \mathrm{m^4/s^2}} \times 0.01\ \mathrm{m^2/s} = -0.306$$

3. Plates are oriented vertically, s is positive downward, so $dz/ds = -1$. Thus

$$\frac{dh}{ds} = \frac{d}{ds}\left(\frac{p}{\gamma}\right) + \frac{dz}{ds}$$

$$\frac{d}{ds}\left(\frac{p}{\gamma}\right) = \frac{dh}{ds} - \frac{dz}{ds} = -0.306 + 1 = 0.694$$

or

$$\frac{dp}{ds} = (0.8 \times 9810\ \mathrm{N/m^3}) \times 0.694 = \boxed{5450\ \mathrm{N/m^2}\ \text{per met}}$$

Review the Solution and the Process

Note that the pressure increases in the downward direction, which means that the pressure, in part, supports the weight the fluid.

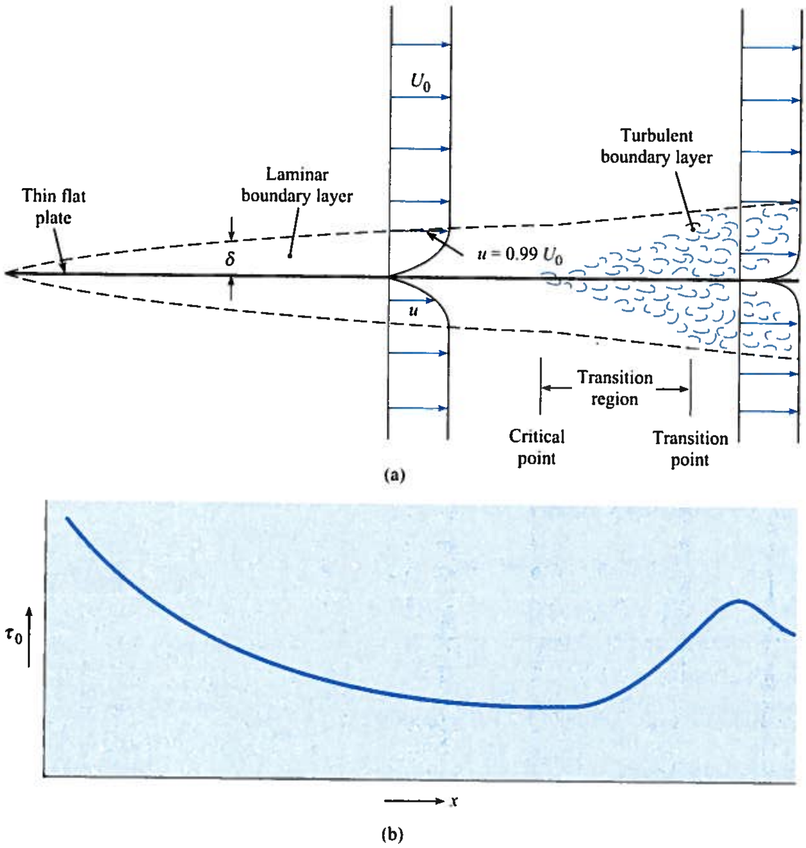
9.2 Qualitative Description of the Boundary Layer

The purpose of this section is to provide a qualitative description of the **boundary layer**, whi is the region adjacent to a surface over which the velocity changes from the free-stream val (with respect to the object) to zero at the surface. This region, which is generally very thi occurs because of the viscosity of the fluid. The velocity gradient at the surface is responsit for the viscous shear stress and shear force.

The boundary-layer development for flow past a thin plate oriented parallel to the flc direction shown in Fig. 9.5a. The thickness of the boundary layer, δ, is defined as the distan from the surface where the velocity is 99% of the free-stream velocity. The actual thickness a boundary layer may be 2% to 3% of the plate length, so the boundary-layer thickness shov in Fig. 9.5a is exaggerated at least by a factor of five to show details of the flow field. Fluid pass over the top and underneath the plate, so two boundary layers are depicted (one above and o below the plate). For convenience, the surface is assumed to be stationary, and the free-strea fluid is moving at a velocity U_0.

FIGURE 9.5

Development of boundary layer and shear stress along a thin, flat plate. (a) Flow pattern above and below the plate. (b) Shear-stress distribution on either side of plate.



The development and growth of the boundary layer occurs because of the "no-slip" condi tion at the surface; that is, the fluid velocity at the surface must be zero. As the fluid particle next to the plate pass close to the leading edge of the plate, a retarding force (from the shea stress) begins to act on the particles to slow them down. As these particles progress farther downstream, they continue to be subjected to shear stress from the plate, so they continue to decelerate. In addition, these particles (because of their lower velocity) retard other particles adjacent to them but farther out from the plate. Thus the boundary layer becomes thicker, or "grows," in the downstream direction. The broken line in Fig. 9.5a identifies the outer limit of the boundary layer. As the boundary layer becomes thicker, the velocity gradient at the wall becomes smaller and the local shear stress is reduced.

The initial section of the boundary layer is the laminar boundary layer. In this region the flow is smooth and steady. Thickening of the laminar boundary layer continues smoothly in the downstream direction until a point is reached where the boundary layer becomes unstable. Beyond this point, the critical point, small disturbances in the flow will grow and spread, leading to turbulence. The boundary becomes fully turbulent at the transition point. The region between the critical point and the transition point is called the transition region.

In most problems of practical interest, the extent of the laminar boundary layer is small and contributes little to the total drag force on a body. Still it is important for flow of very viscous liquids and for flow problems with small length scales.

The turbulent boundary layer is characterized by intense cross-stream mixing as turbulent eddies transport high-velocity fluid from the boundary layer edge to the region close to the wall. This cross-stream mixing gives rise to a high effective viscosity, which can be three orders of magnitude higher than the actual viscosity of the fluid itself. The effective viscosity, due to turbulent mixing is not a property of the fluid but rather a property of the flow, namely, the mixing process. Because of this intense mixing, the velocity profile is much "fuller" than the laminar-flow velocity profile as shown in Fig. 9.5a. This situation leads to an increased velocity gradient at the surface and a larger shear stress.

The shear-stress distribution along the plate is shown in Fig. 9.4b. It is easy to visualize that the shear stress must be relatively large near the leading edge of the plate where the velocity gradient is steep, and that it becomes progressively smaller as the boundary layer thickens in the downstream direction. At the point where the boundary layer becomes turbulent, the shear stress at the boundary increases because the velocity profile changes producing a steeper gradient at the surface.

These qualitative aspects of the boundary layer serve as a foundation for the quantitative relations presented in the next section.

9.3 Laminar Boundary Layer

This section summarizes the equations for the velocity profile and shear stress in a laminar boundary layer and describes how to calculate shear stress and shear forces on a surface. This information can be used to estimate drag forces on surfaces in low Reynolds-number flows.

Boundary-Layer Equations

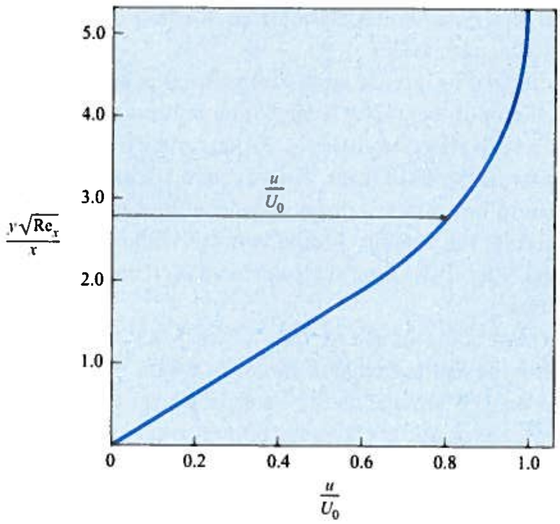
In 1904 Prandtl (1) first stated the essence of the boundary-layer hypothesis, which is that viscous effects are concentrated in a thin layer of fluid (the boundary layer) next to solid boundaries. Along with his discussion of the qualitative aspects of the boundary layer, he also simplified the general equations of motion of a fluid (Navier-Stokes equations) for application to the boundary layer.

In 1908, Blasius, one of Prandtl's students, obtained a solution for the flow in a laminar boundary layer (2) on a flat plate with a constant free-stream velocity. One of Blasius's key assumptions was that the shape of the nondimensional velocity distribution did not vary fro section to section along the plate. That is, he assumed that a plot of the relative velocity, u/U versus the relative distance from the boundary, y/δ, would be the same at each section. W this assumption and with Prandtl's equations of motion for boundary layers, Blasius obtain a numerical solution for the relative velocity distribution, shown in Fig. 9.6.* In this plot, x the distance from the leading edge of the plate, and Re_x is the Reynolds number based on t free-stream velocity and the length along the plate ($\mathrm{Re}_x = U_0 x/\nu$). In Fig. 9.6 the outer limit the boundary layer ($u/U_0 = 0.99$) occurs at approximately $y\mathrm{Re}_x^{1/2}/x = 5$. Because $y = \delta$ at t point, the following relationship is derived for the *boundary-layer thickness* in laminar flow a flat plate:

$$\frac{\delta}{x}\mathrm{Re}_x^{1/2} = 5 \quad \text{or} \quad \delta = \frac{5x}{\mathrm{Re}_x^{1/2}} \qquad (9.1$$

FIGURE 9.6

Velocity distribution in laminar boundary layer. [After Blasius (2).]



The Blasius solution also showed that

$$\left.\frac{d(u/U_0)}{d[(y/x)\mathrm{Re}_x^{1/2}]}\right|_{y=0} = 0.332$$

which can be used to find the shear stress at the surface. The velocity gradient at the bounda becomes

$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\frac{U_0}{x}\mathrm{Re}_x^{1/2}$$

$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\frac{U_0^{3/2}}{x^{1/2}\nu^{1/2}} \qquad (9.1$$

*Experimental evidence indicates that the Blasius solution is valid except very near the leading edge of the plate. the vicinity of the leading edge, an error results because of certain simplifying assumptions. However, the discrepan is not significant for most engineering problems.

Equation (9.11) shows that the velocity gradient (and shear stress) decreases with increasing distance x along the plate.

Shear Stress

The shear stress at the boundary is obtained from

$$\tau_0 = \mu \frac{du}{dy}\bigg|_{y=0} = 0.332\mu \frac{U_0}{x} \mathrm{Re}_x^{1/2} \tag{9.12}$$

Equation (9.12) is used to obtain the local shear stress at any given section (any given value of x) for the laminar boundary layer.

Example 9.3 illustrates the application of the laminar boundary layer equations for calculating boundary layer thickness and shear stress.

Shear Force

Consider one side of a flat plate with width B and length L. Because the shear stress at the boundary, τ_0, varies along the plate, it is necessary to integrate this stress over the entire surface to obtain the total shear force, F_s.

$$F_s = \int_0^L \tau_0 B\, dx \tag{9.13}$$

EXAMPLE 9.3

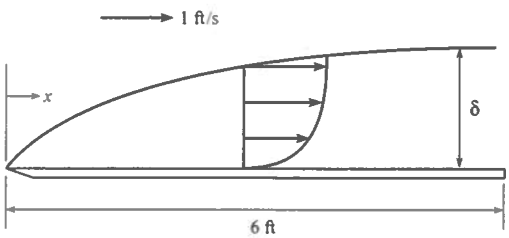
Laminar Boundary-Layer Thickness and Shear Stress

Problem Statement

Crude oil at 70°F ($\nu = 10^{-4}$ ft²/s, S = 0.86) with a free-stream velocity of 1 ft/s flows past a thin, flat plate that is 4 ft wide and 6 ft long in a direction parallel to the flow. The flow is laminar. Determine and plot the boundary-layer thickness and the shear stress distribution along the plate.

Define the Situation

Crude oil flows past a thin, flat plate. Free-stream velocity is 1 ft/s.



Oil. $\nu = 10^{-4}$ ft²/s, S = 0.86

Assumptions:

1. Plate is smooth, flat with sharp leading edge.
2. Boundary layer is laminar.

State the Goal

Surface shear stress, τ_0, as function of x.
Boundary-layer thickness, δ, as function of x.

Generate Ideas and Make a Plan

1. Calculate boundary-layer thickness with Eq. (9.10).
2. Calculate shear-stress distribution with Eq. (9.12).
3. Summarize results using a table and a plot.

Take Action (Execute the Plan)

1. Reynolds-number variation with distance

$$\mathrm{Re}_x = \frac{U_0 x}{\nu} = \frac{1 \times x}{10^{-4}} = 10^4 x$$

Boundary-layer thickness

$$\delta = \frac{5x}{\mathrm{Re}_x^{1/2}} = \frac{5x}{10^2 x^{1/2}} = 5 \times 10^{-2} x^{1/2}\ \text{ft}$$

2. Shear-stress distribution

$$\tau_0 = 0.332\mu \frac{U_0}{x} \mathrm{Re}_x^{1/2}$$

$$\mu = \rho\nu = 1.94\ \text{slugs/ft}^3 \times 0.86 \times 10^{-4}\ \text{ft}^2/\text{s}$$
$$= 1.67 \times 10^{-4}\ \text{lbf-s/ft}^2$$

$$\tau_0 = 0.332(1.67 \times 10^{-4})\frac{1}{x}(10^2 x^{1/2}) = \frac{5.54 \times 10^{-3}}{x^{1/2}}\ \text{psf}$$

3. Summary (make a plot and build a table).

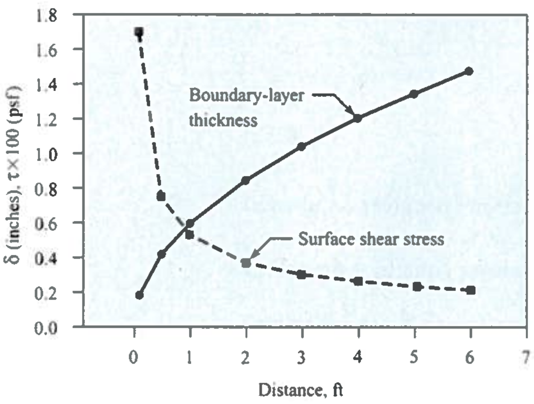


TABLE 9.1 Results: δ and τ_0 for Different Values of x

	$x = 0.1$ ft	$x = 1.0$ ft	$x = 2$ ft	$x = 4$ ft	$x = 6$ ft
$x^{1/2}$	0.316	1.00	1.414	2.00	2.45
τ_0, psf	0.018	0.0055	0.0037	0.0028	0.0023
δ, ft	0.016	0.050	0.071	0.10	0.122
δ, in	0.190	0.600	0.848	1.200	1.470

Review the Solution and the Process

1. Notice that the boundary-layer thickness increases with distance. At the end of the plate $\delta/x = 0.02$, or the boundary-layer thickness is 2% of the distance from leading edge.
2. Notice also that shear stress decreases with distance from leading edge of the plate.

Substituting in Eq. (9.12) for τ_0 and integrating gives

$$F_s = \int_0^L 0.332B\mu \frac{U_0 U_0^{1/2} x^{1/2}}{x\nu^{1/2}}\, dx$$
$$= 0.664 B\mu U_0 \frac{U_0^{1/2} L^{1/2}}{\nu^{1/2}} \quad \text{(9.1}$$
$$= 0.664 B\mu U_0 \mathrm{Re}_L^{1/2}$$

where Re_L is the Reynolds number based on the approach velocity and the length of the plat

Shear-Stress Coefficients

It is convenient to express the shear stress at the boundary, τ_0, and the total shearing force F_s i terms of π-groups involving the kinetic pressure of the free stream, $\rho U_0^2/2$. The **local shea stress coefficient**, c_f, is defined as

$$c_f = \frac{\tau_0}{\rho U_0^2/2} \quad \text{(9.1}$$

Substituting Eq. (9.12) into Eq. (9.15) gives c_f as a function of Reynolds number based on th distance from the leading edge.

$$c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}} \quad \text{where} \quad Re_x = \frac{Ux}{\nu} \quad \text{(9.1}$$

The total shearing force, as given by Eq. (9.13), can also be expressed as a π-group

$$C_f = \frac{F_s}{(\rho U_0^2/2)A} \quad \text{(9.1}$$

where A is the plate area. This π-group is called the average shear-stress coefficient. Substituting Eq. (9.14) into Eq. (9.17) gives C_f:

$$C_f = \frac{1.33}{\text{Re}_L^{1/2}} \quad \text{where} \quad Re_L = \frac{UL}{\nu} \tag{9.18}$$

Example 9.4 shows how to calculate the total shear force for a laminar boundary layer on a flat plate.

EXAMPLE 9.4

Resistance Calculation for Laminar Boundary Layer on a Flat Plate

Problem Statement

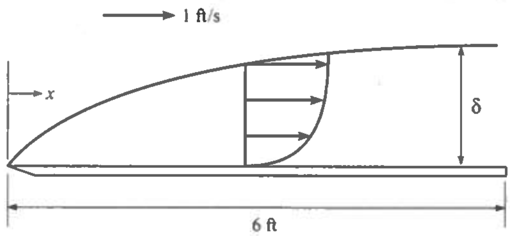
Crude oil at 70°F ($\nu = 10^{-4}$ ft²/s, S = 0.86.) with a free-stream velocity of 1 ft/s flows past a thin, flat plate that is 4 ft wide and 6 ft long in a direction parallel to the flow. The flow is laminar. Determine the resistance on one side of the plate.

Define the Situation

Crude oil flows past a thin, flat plate. Free-stream velocity is 1 ft/s.

Properties: For oil, $\nu = 10^{-4}$ ft²/s, S = 0.86.

Assumptions: Flow is laminar.



State the Goal

Find: Shear force (in lbf) on one side of plate.

Generate Ideas and Make a Plan

1. Calculate the Reynolds number based on plate length.
2. Evaluate C_f using Eq. (9.18).
3. Calculate the shear force using Eq. (9.17).

Take Action (Execute the Plan)

1. Reynolds number.

$$\text{Re}_L = \frac{U_0 L}{\nu} = \frac{1\text{ ft/s} \times 6\text{ ft}}{10^{-4}\text{ ft}^2\text{/s}} = 6 \times 10^4$$

2. Value for C_f:

$$C_f = \frac{1.33}{\text{Re}_L^{1/2}} = \frac{1.33}{(6 \times 10^4)^{1/2}} = 0.0054$$

3. Shear force.

$$F_s = \frac{C_f B L \rho U_0^2}{2}$$
$$= 0.0054 \times 4\text{ ft} \times 6\text{ ft} \times 0.86 \times 1.94\text{ slugs/ft}^3 \times \frac{1^2\,(\text{ft/s})^2}{2} = \boxed{0.108\text{ lbf}}$$

9.4 Boundary Layer Transition

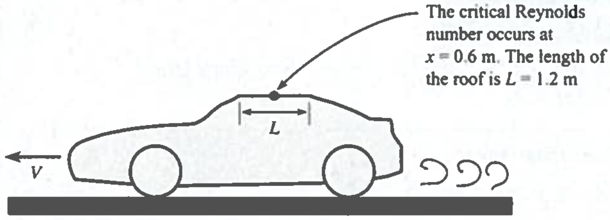
Transition is the zone where the laminar boundary layer changes into a turbulent boundary layer as shown in Fig. 9.5a. As the laminar boundary layer continues to grow, the viscous stresses are less capable of damping disturbances in the flow. A point is then reached where disturbances occurring in the flow are amplified, leading to turbulence. The critical point occurs at a Reynolds number of about 10^5 ($\text{Re}_{cr} \cong 10^5$) based on the distance from the leading edge. Vortices created near the wall grow and mutually interact, ultimately leading to a fully turbulent boundary layer at the transition point, which nominally occurs at a Reynolds number of 3×10^6 ($\text{Re}_{tr} \cong 3 \times 10^6$). For purposes of simplicity in this text, it will be assumed that the boundary layer changes from laminar to turbulent flow at a Reynolds number 500,000. The details of the transition region can be found in White (3).

Transition to a turbulent boundary layer can be influenced by several other flow conditions, such as free-stream turbulence, pressure gradient, wall roughness, wall heating, and wall cooling. With appropriate roughness elements at the leading edge, the boundary layer c become turbulent at the very beginning of the plate. In this case it is said that the bound layer is "**tripped**" at the leading edge.

✔CHECKPOINT PROBLEM 9.2

Suppose the roof of an automobile is idealized as a flat plate. Given the data in the figure, what is th speed V of the car in mph? Assume $T = 20°C$ and $p = 1$ atm.

a. 12.6
b. 14.1
c. 16.9
d. 28.1
e. 34.7



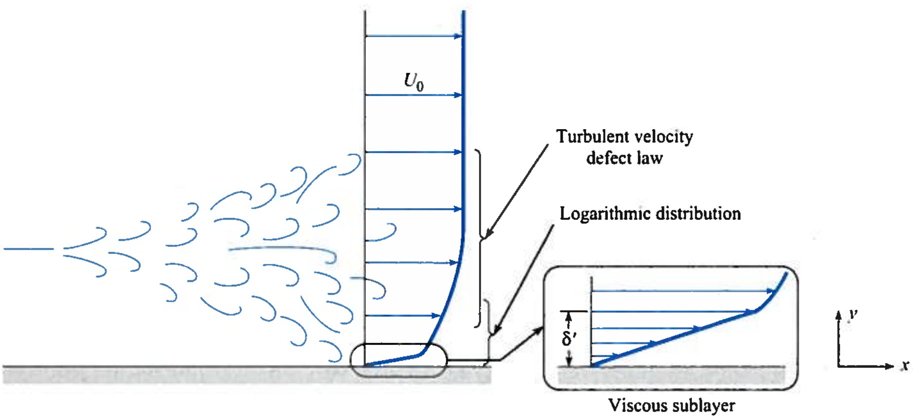
9.5 Turbulent Boundary Layer

Understanding the mechanics of the turbulent boundary layer is important because in t majority of practical problems, the turbulent boundary layer is primarily responsible for she force. In this section the velocity distribution in the turbulent boundary layer on a flat pl oriented parallel to the flow is presented. The correlations for boundary-layer thickness a shear stress are also included.

Velocity Distribution

The velocity distribution in the turbulent boundary layer is more complicated than the lan nar boundary layer. The turbulent boundary has three zones of flow that require differe equations for the velocity distribution in each zone, as opposed to the single relationship the laminar boundary layer. Figure 9.7 shows a portion of a turbulent boundary layer which the three different zones of flow are identified. The zone adjacent to the wall is the v cous sublayer; the zone immediately above the viscous sublayer is the logarithmic region; a

FIGURE 9.7
Sketch of zones in turbulent boundary layer.



finally, beyond that region is the velocity defect region. Each of these velocity zones will be discussed separately.

Viscous Sublayer The zone immediately adjacent to the wall is a layer of fluid that is essentially laminar because the presence of the wall dampens the cross-stream mixing and turbulent fluctuations. This very thin layer is called the **viscous sublayer**. This thin layer behaves as a Couette flow introduced in Section 9.1. In the viscous sublayer, τ is virtually constant and equal to the shear stress at the wall, τ_0. Thus $du/dy = \tau_0/\mu$, which on integration yields

$$u = \frac{\tau_0 y}{\mu} \tag{9.19}$$

Dividing the numerator and denominator by ρ gives

$$u = \frac{\tau_0/\rho}{\mu/\rho} y$$

$$\frac{u}{\sqrt{\tau_0/\rho}} = \frac{\sqrt{\tau_0/\rho}}{\nu} y \tag{9.20}$$

The combination of variables $\sqrt{\tau_0/\rho}$ has the dimensions of velocity and recurs again and again in derivations involving boundary-layer theory. It has been given the special name **shear velocity**. The shear velocity (which is also sometimes called **friction velocity**) is symbolized as u_*. Thus, by definition,

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \tag{9.21}$$

Now, substituting u_* for $\sqrt{\tau_0/\rho}$ in Eq. (9.20), yields the nondimensional velocity distribution in the viscous sublayer:

$$\frac{u}{u_*} = \frac{y}{\nu/u_*} \tag{9.22}$$

Experimental results show that the limit of viscous sublayer occurs when yu_*/ν is approximately 5. Consequently, the thickness of the viscous sublayer, identified by δ', is given as

$$\delta' = \frac{5\nu}{u_*} \tag{9.23}$$

The thickness of the viscous sublayer is very small (typically less than one-tenth the thickness of a dime). The thickness of the viscous sublayer increases as the wall shear stress decreases in the downstream direction.

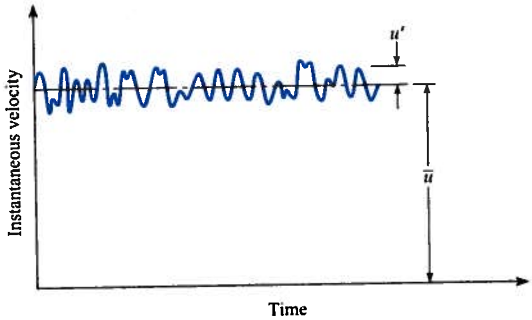
The Logarithmic Velocity Distribution The flow zone outside the viscous sublayer is turbulent; therefore, a completely different type of flow is involved. The mixing action of turbulence causes small fluid masses to be swept back and forth in a direction transverse to the mean flow direction. A small mass of fluid swept from a low-velocity zone next to the viscous sublayer into a higher-velocity zone farther out in the stream has a retarding effect on the higher-velocity stream. Similarly, a small mass of fluid that originates farther out in the boundary layer in a high-velocity flow zone and is swept into a region of low velocity has the effect of accelerating the lower-velocity fluid. Although the process just described is primarily a momentum exchange phenomenon, it has the same effect as applying a shear stress to the fluid; thus in turbulent flow these "stresses" are termed **apparent shear stresses**, or **Reynolds stresses** after the British scientist–engineer who first did extensive research in turbulent flow in the late 1800s.

The mixing action of turbulence causes the velocities at a given point in a flow to fluctuat with time. If one places a velocity-sensing device, such as a hot-wire anemometer, in a turbu lent flow, one can measure a fluctuating velocity, as illustrated in Fig. 9.8. It is convenient t think of the velocity as composed of two parts: a mean value, $\bar{u}$, plus a fluctuating part, u'. Th fluctuating part of the velocity is responsible for the mixing action and the momentun exchange, which manifests itself as an apparent shear stress as noted previously. In fact, th apparent shear stress is related to the fluctuating part of the velocity by

$$\tau_{\text{app}} = -\rho\overline{u'v'} \quad \text{(9.24}$$

FIGURE 9.8

Velocity fluctuations in turbulent flow.



where u' and v' refer to the x and y components of the velocity fluctuations, respectivel and the bar over these terms denotes the product of $u'v'$ averaged over a peric of time.* The expression for apparent shear stress is not very useful in this form, Prandtl developed a theory to relate the apparent shear stress to the temporal mean veloci distribution.

The theory developed by Prandtl is analogous to the idea of molecular transport creati shear stress presented in Chapter 2. In the turbulent boundary layer, the principal flow is para lel to the boundary. However, because of turbulent eddies, there are fluctuating componer transverse to the principal flow direction. These fluctuating velocity components are asso ated with small masses of fluid, as shown in Fig. 9.8, that move across the boundary layer. the mass moves from the lower-velocity region to the higher-velocity region, it tends to reta its original velocity. The difference in velocity between the surrounding fluid and the trar ported mass is identified as the fluctuating velocity component u', For the mass shown Fig. 9.8, u' would be negative and approximated by†

$$u' \approx \ell\frac{du}{dy}$$

where du/dy is the mean velocity gradient and ℓ is the distance the small fluid mass travels the transverse direction. Prandtl identified this distance as the "mixing length." Prandtl sumed that the magnitude of the transverse fluctuating velocity component is proportional the magnitude of the fluctuating component in the principal flow direction: $|v'| \cong |u$ which seems to be a reasonable assumption because both components arise from the same

*Equation (9.24) can be derived by considering the momentum exchange that results when the transverse compor of turbulent flow passes through an area parallel to the x-z plane. Or, by including the fluctuating velocity compon in the Navier-Stokes equations, one can obtain the apparent shear stress terms, one of which is Eq. (9.24). Detail these derivations appear in Chapter 18 of Schlichting (4).

†For convenience, the bar used to denote time-averaged velocity is deleted.

of eddies. Also, it should be noted that a positive v' will be associated with a negative u', so the product $\overline{u'v'}$ will be negative. Thus the apparent shear stress can be expressed as

$$\tau_{\text{app}} = -\rho\overline{u'v'} = \rho\ell^2\left(\frac{du}{dy}\right)^2 \tag{9.25}$$

A more general form of Eq. (9.25) is

$$\tau_{\text{app}} = \rho\ell^2\left|\frac{du}{dy}\right|\frac{du}{dy}$$

which ensures that the sign for the apparent shear stress is correct.

The theory leading to Eq. (9.25) is called Prandtl's mixing-length theory and is used extensively in analyses involving turbulent flow.* Prandtl also made the important and clever assumption that the mixing length is proportional to the distance from the wall ($\ell = \kappa y$) for the region close to the wall. If one considers the velocity distribution in a boundary layer where du/dy is positive, as is shown in Fig. 9.9, and substitutes κy for ℓ, then Eq. (9.25) reduces to

$$\tau_{\text{app}} = \rho\kappa^2 y^2\left(\frac{du}{dy}\right)^2$$

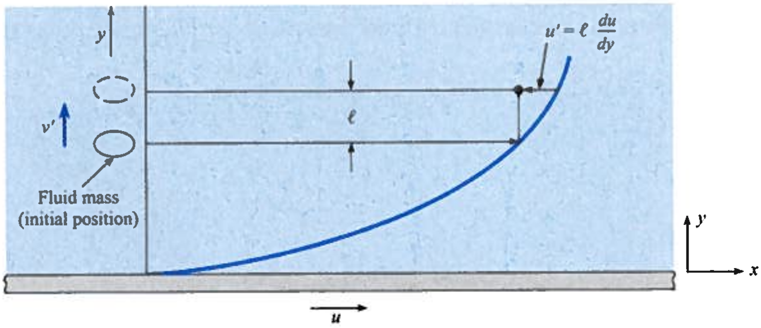


FIGURE 9.9
Concept of mixing leng

For the zone of flow near the boundary, it is assumed that the shear stress is uniform and approximately equal to the shear stress at the wall. Thus the foregoing equation becomes

$$\tau_0 = \rho\kappa^2 y^2\left(\frac{du}{dy}\right)^2 \tag{9.26}$$

Taking the square root of each side of Eq. (9.26) and rearranging yields

$$du = \frac{\sqrt{\tau_0/\rho}}{\kappa}\frac{dy}{y}$$

Integrating the above equation and substituting u_* for $\sqrt{\tau_0/\rho}$ gives

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln y + C \tag{9.27}$$

*Prandtl published an account of his mixing-length concept in 1925. G. I. Taylor (5) published a similar concept in 1915, but the idea has been traditionally attributed to Prandtl.

Experiments on smooth boundaries indicate that the constant of integration C can be given terms of u_*, ν, and a pure number as

$$C = 5.56 - \frac{1}{\kappa}\ln\frac{\nu}{u_*}$$

When this expression for C is substituted into Eq. (9.27), the result is

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\frac{yu_*}{\nu} + 5.56$$ (9.2

In Eq. (9.28), κ has sometimes been called the universal turbulence constant, or Karman's consta Experiments show that this constant is approximately 0.41 (3) for the turbulent zone next to t viscous sublayer. Introducing this value for κ into Eq. (9.28) gives the *logarithmic velocity distributi*

$$\frac{u}{u_*} = 2.44\ln\frac{yu_*}{\nu} + 5.56$$ (9.2

Obviously the region where this model is valid is limited because the mixing length cann continuously increase to the boundary layer edge. This distribution is valid for values of yu_* ranging from approximately 30 to 500.

The region between the viscous sublayer and the logarithmic velocity distribution is t buffer zone. There is no equation for the velocity distribution in this zone, although vario empirical expressions have been developed (6). However, it is common practice to extrapola the velocity profile for the viscous sublayer to larger values of yu_*/ν and the logarithmic veloci profile to smaller values of yu_*/ν until the velocity profiles intersect as shown in Fig. 9.10.

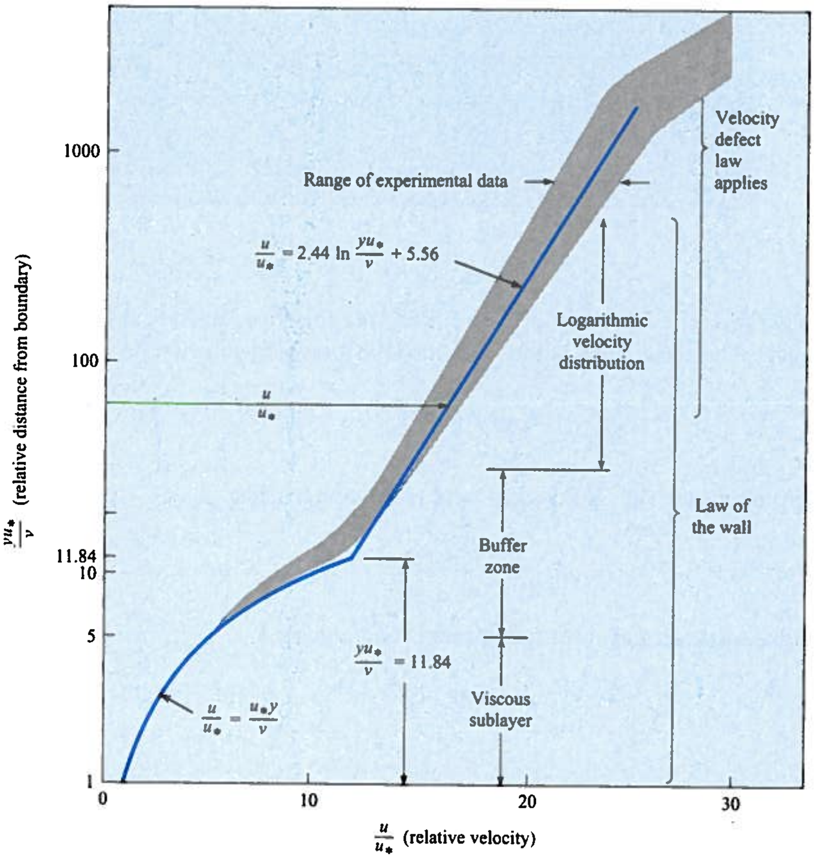


FIGURE 9.10
Velocity distribution in a turbulent boundary layer.

The intersection occurs at $yu_*/\nu = 11.84$ and is regarded as the demarcation between the viscous sublayer and the logarithmic profile. The "nominal" thickness of the viscous sublayer is

$$\delta'_N = 11.84\frac{\nu}{u_*} \quad (9.30)$$

The combination of the viscous and logarithmic velocity profile for the range of yu_*/ν from 0 to approximately 500 is called the **law of the wall.**

Making a semilogarithmic plot of the velocity distribution in a turbulent boundary layer, as shown in Fig. 9.10, makes it straightforward to identify the velocity distribution in the viscous sublayer and in the region where the logarithmic equation applies. However, the logarithmic nature of this plot accentuates the nondimensional distance yu_*/ν near the wall. A better perspective of the relative extent of the regions is obtained by plotting the graph on a linear scale, as shown in Fig. 9.11. From this plot one notes that the laminar sublayer and buffer zone are a very small part of the thickness of the turbulent boundary layer.

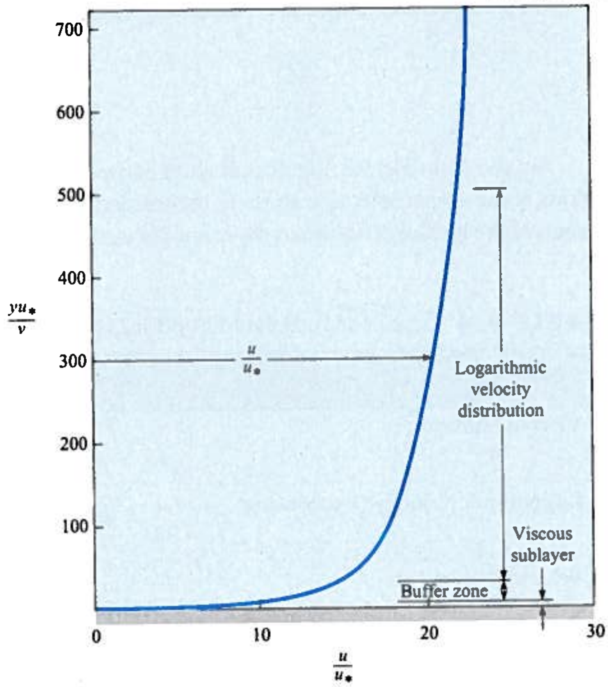


FIGURE 9.11
Velocity distribution in a turbulent boundary layer, linear scales.

Velocity Defect Region For $y/\delta > 0.15$ and $yu_*/\nu > 500$ the velocity profile corresponding to the law of the wall becomes increasingly inadequate to match experimental data, so a third zone, called the velocity defect region, is identified. The velocity in this region is represented by the **velocity defect law**, which for a flat plate with zero pressure gradient is simply expressed as

$$\frac{U_0 - u}{u_*} = f\left(\frac{y}{\delta}\right) \quad (9.31)$$

and the correlation with experimental data is plotted in Fig. 9.12. At the edge of the boundary layer $y = \delta$ and $(U_0 - u)/u_* = 0$, so $u = U_0$, or the free-stream velocity. This law applies to rough as well as smooth surfaces. However, the functional relationship has to be modified for flows with free-stream pressure gradients.

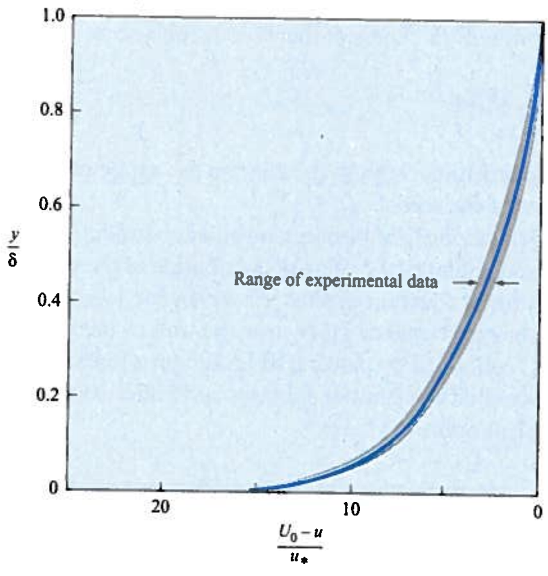


FIGURE 9.12 Velocity defect law for boundary layers on flat plate (zero pressure gradient). [After Rouse (6).]

As shown in Fig. 9.9, the demarcation between the law of the wall and the velocity defect r gions is somewhat arbitrary, so there is considerable overlap between the two regions. The thr zones of the turbulent boundary layer and their range of applicability are summarized in Table 9

TABLE 9.2 Zones for Turbulent Boundary Layer on Flat Plate

Zone	Velocity Distribution	Range
Viscous Sublayer	$\frac{u}{u_*} = \frac{yu_*}{\nu}$	$0 < \frac{yu_*}{\nu} < 11.84$
Logarithmic Velocity Distribution	$\frac{u}{u_*} = 2.44\ln\frac{yu_*}{\nu} + 5.56$	$11.84 \leq \frac{yu_*}{\nu} < 500$
Velocity Defect Law	$\frac{U_0 - u}{u_*} = f\left(\frac{y}{\delta}\right)$	$500 \leq \frac{yu_*}{\nu}, \frac{y}{\delta} > 0.15$

Power-Law Formula for Velocity Distribution Analyses have shown that for a wide range c Reynolds numbers ($10^5 < \text{Re} < 10^7$), the velocity profile in the turbulent boundary layer on flat plate is approximated reasonably by the *power-law* equation

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7} \quad \text{(9.3}$$

Comparisons with experimental results show that this formula conforms to those result very closely over about 90% of the boundary layer ($0.1 < y/\delta < 1$). Obviously it is not valid a the surface because $(du)/(dy)|_{y=0} \to \infty$, which implies infinite surface shear stress. For th inner 10% of the boundary layer, one must resort to equations for the law of the wall (se Fig. 9.10) to obtain a more precise prediction of velocity. Because Eq. (9.32) is valid ove the major portion of the boundary layer, it is used to advantage in deriving the overall thicknes of the boundary layer as well as other relations for the turbulent boundary layer.

Example 9.5 illustrates the application of various equations to calculate the velocity in the turbulent boundary layer.

EXAMPLE 9.5

Turbulent Boundary-Layer Properties

Problem Statement

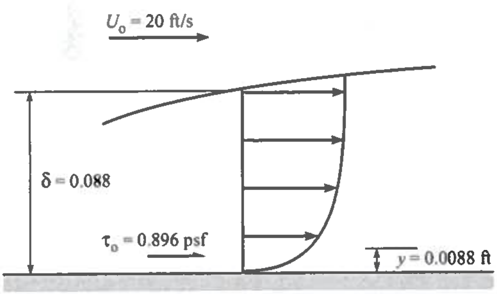
Water (60°F) flows with a velocity of 20 ft/s past a flat plate. The plate is oriented parallel to the flow. At a particular section downstream of the leading edge of the plate, the boundary layer is turbulent, the shear stress on the plate is 0.896 lbf/ft², and the boundary-layer thickness is 0.0880 ft. Find the velocity of the water at a distance of 0.0088 ft from the plate as determined by

a. The logarithmic velocity distribution
b. The velocity defect law
c. The power-law formula

Also, what is the nominal thickness of the viscous sublayer?

Define the Situation

Water flows past a flat plate oriented parallel to the flow. At a point downstream of the leading edge of the plate, shear stress on the plate is 0.896 lbf/ft², and boundary layer thickness is 0.0880 ft.



Properties:

From Table A.5, $\rho = 1.94$ slugs/ft³, $\nu = 1.22 \times 10^{-5}$ ft²/s.

State the Goal

1. V(ft/s) ⬅ Velocity at $y = 0.0088$ ft using:
   a. Logarithmic velocity distribution
   b. Velocity defect law
   c. Power-law formula
2. Calculate the nominal thickness of the viscous sublayer

Generate Ideas and Make a Plan

1. Calculate shear velocity, u_*, from Eq. (9.21).
2. Calculate u using Eq. (9.29) for logarithmic profile.
3. Calculate y/δ and find $(U_0 - u)/u_*$ from Fig. 9.12.
4. Calculate u from $(U_0 - u)/u_*$ for velocity defect law.
5. Calculate u from Eq. (9.32) for power law.
6. Calculate δ'_N from Eq. (9.30).

Take Action (Execute the Plan)

1. Shear velocity

$$u_* = (\tau_0/\rho)^{1/2}$$
$$= [(0.896 \text{ lbf/ft}^2)/(1.94 \text{ slugs/ft}^3)]^{1/2} = 0.680 \text{ ft/s}$$

2. Logarithmic velocity distribution

$$yu_*/\nu = (0.0088 \text{ ft})(0.680 \text{ ft/s})/(1.22 \times 10^{-5} \text{ ft}^2/\text{s}) = 490$$
$$u/u_* = 2.44 \ln(yu_*/\nu) + 5.56$$
$$= 2.44 \times \ln(490) + 5.56 = 20.7$$
$$u = 20.7 \times 0.680 \text{ ft/s} = \boxed{14.1 \text{ ft/s}}$$

3. Nondimensional distance

$$y/\delta = 0.0088 \text{ ft}/0.088 \text{ ft} = 0.10$$

From Fig. 9.12

$$\frac{U_0 - u}{u_*} = 8.2$$

4. Velocity from defect law

$$u = U_0 - 8.2u_*$$
$$= 20 \text{ ft/s} - (8.2)(0.68) \text{ ft/s}$$
$$= \boxed{14.4 \text{ ft/s}}$$

5. Power-law formula

$$u/U_0 = (y/\delta)^{1/7}$$
$$u = (U_0)(0.10)^{1/7}$$
$$= (20 \text{ ft/s})(0.7197)$$
$$= \boxed{14.4 \text{ ft/s}}$$

6. Nominal sublayer thickness

$$\delta'_N = 11.84\nu/u_* = (11.84)(1.22 \times 10^{-5} \text{ ft}^2/\text{s})/(0.68 \text{ ft/s})$$
$$= 2.12 \times 10^{-4} \text{ ft} = \boxed{2.54 \times 10^{-3} \text{ in}}$$

Review the Solution and the Process

Notice that the velocity obtained using logarithmic distribution and defect law are nearly the same, which indicates that the point is in the overlap region.

Boundary-Layer Thickness and Shear-Stress Correlations

Unlike the laminar boundary layer, there is no analytically derived equation for the thickn of the turbulent boundary layer. There is a way to obtain an equation by using momentu principles and empirical data for the local shear stress and by assuming the 1/7 power veloc profile (3). The result is

$$\delta = \frac{0.16x}{\mathrm{Re}_x^{1/7}} \quad \text{(9.:}$$

where x is the distance from the leading edge of the plate and Re_x is $U_0 x/\nu$.

Many empirical expressions have been proposed for the local shear-stress distribution i the turbulent boundary layer on a flat plate. One of the simplest correlations is

$$c_f = \frac{\tau_0}{\rho U_0^2/2} = \frac{0.027}{\mathrm{Re}_x^{1/7}} \quad \text{(9.34}$$

and the corresponding average shear-stress coefficient is

$$C_f = \frac{0.032}{\mathrm{Re}_L^{1/7}} \quad \text{(9.34}$$

where Re_L is the Reynolds number of the plate based on the length of the plate in the strear wise direction.

Even though the variation of c_f with Reynolds number given by Eq. (9.34a) provides reasonably good fit with experimental data for Reynolds numbers less than 10^7, it tends underpredict the skin friction at higher Reynolds numbers. Several correlations have be proposed in the literature; see the review by Schlichting (4). A correlation proposed by White (that fits the data for turbulent Reynolds numbers up to 10^{10} is

$$c_f = \frac{0.455}{\ln^2(0.06\mathrm{Re}_x)} \quad \text{(9.3}$$

The corresponding average shear-stress coefficient is

$$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} \quad \text{(9.3}$$

These are the correlations for shear-stress coefficients recommended here.

The boundary layer on a flat plate is composed of both a laminar and turbulent par The purpose here is to develop a correlation valid for the combined boundary layer. A noted in Section 9.3, the boundary layer on a flat plate consists first of a laminar boundar layer that grows in thickness, develops instability, and becomes turbulent. A turbulen boundary layer develops over the remainder of the plate. As discussed earlier in Section 9. the transition from a laminar to turbulent boundary layer is not immediate but take place over a transition length. However for the purposes of analysis here it is assumed th transition occurs at a point corresponding to a transition Reynolds number, Re_{tr}, of abou 500,000.

The idea here is to take the turbulent shear force for length L, $F_{s,\,\mathrm{turb}}(L)$, assuming th boundary layer is turbulent from the leading edge, subtract the portion up to the transitio point, $F_{s,\,\mathrm{turb}}(L_{tr})$ and replace it with the laminar shear force up to the transition poin $F_{s,\,\mathrm{lam}}(L_{tr})$. Thus the composite shear force on the plate is

$$F_s = F_{s,\,\mathrm{turb}}(L) - F_{s,\,\mathrm{turb}}(L_{tr}) + F_{s,\,\mathrm{lam}}(L_{tr})$$

Substituting in Eq. (9.18) for laminar flow and Eq. (9.36) for turbulent flow over a plate of width B gives

$$F_s = \left(\frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)}BL - \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_{tr})}BL_{tr} + \frac{1.33}{\mathrm{Re}_{tr}^{1/2}}BL_{tr}\right)\rho\frac{U_0^2}{2} \tag{9.37}$$

where Re_{tr} is the Reynolds number at the transition, Re_L is the Reynolds number at the end of the plate, and L_{tr} is the distance from the leading edge of the plate to the transition zone.

Expressing the resistance force in terms of the average shear-stress coefficient, $C_f = F_s/(BL\rho U_0^2/2)$, gives

$$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} + \frac{L_{tr}}{L}\left(\frac{1.33}{\mathrm{Re}_{tr}^{1/2}} - \frac{0.523}{\ln^2(0.06\mathrm{Re}_{tr})}\right)$$

Here $L_{tr}/L = \mathrm{Re}_{tr}/\mathrm{Re}_L$. Therefore,

$$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} + \frac{\mathrm{Re}_{tr}}{\mathrm{Re}_L}\left(\frac{1.33}{\mathrm{Re}_{tr}^{1/2}} - \frac{0.523}{\ln^2(0.06\mathrm{Re}_{tr})}\right)$$

Finally, for $\mathrm{Re}_{tr} = 500{,}000$, the equation for average shear-stress coefficient becomes

$$C_f = \frac{0.523}{\ln^2(0.06\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \tag{9.38}$$

The variation of C_f with Reynolds number is shown by the solid line in Fig. 9.13. This curve corresponds to a boundary layer that begins as a laminar boundary layer and then changes to a turbulent boundary layer after the transition Reynolds number. This is the normal condition for a flat-plate boundary layer. Table 9.3 summarizes the equations for boundary-layer thickness, and for local shear-stress and average shear-stress coefficients for the boundary layer on a flat plate.

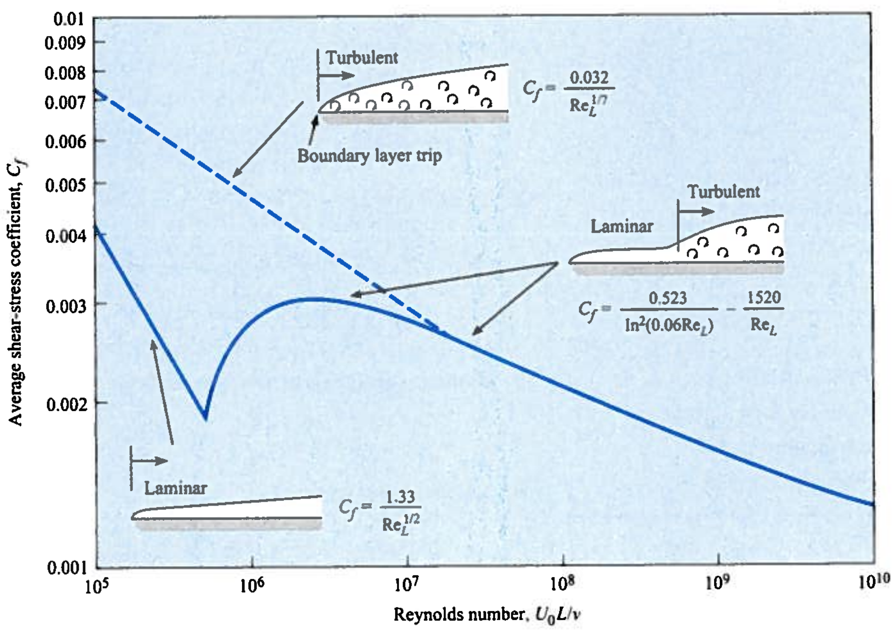


FIGURE 9.13
Average shear-stress coefficients.

Example 9.6 shows the calculation of shear force due to a boundary layer on a flat plate.

TABLE 9.3 Summary of Equations for Boundary Layer on a Flat Plate

	Laminar Flow $Re_x, Re_L < 5 \times 10^5$	Turbulent Flow $Re_x, Re_L \geq 5 \times 10^5$
Boundary-Layer Thickness, δ	$\delta = \dfrac{5x}{Re_x^{1/2}}$	$\delta = \dfrac{0.16x}{Re_x^{1/7}}$
Local Shear-Stress Coefficient, c_f	$c_f = \dfrac{0.664}{Re_x^{1/2}}$	$c_f = \dfrac{0.455}{\ln^2(0.06Re_x)}$
Average Shear-Stress Coefficient, C_f (mixed boundary layer)	$C_f = \dfrac{1.33}{Re_L^{1/2}}$	$C_f = \dfrac{0.523}{\ln^2(0.06Re_L)} - \dfrac{1520}{Re_L}$
Average Shear-Stress Coefficient, C_f (tripped boundary layer)	$C_f = \dfrac{0.032}{Re_L^{1/7}}$	

EXAMPLE 9.6

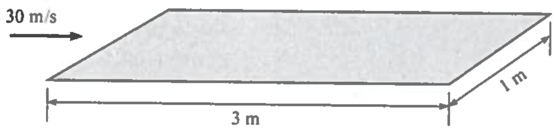
Calculating Shear Force on a Flat Plate

Problem Statement

Assume that air 20°C and normal atmospheric pressure flows over a smooth, flat plate with a velocity of 30 m/s. The initial boundary layer is laminar and then becomes turbulent at a transitional Reynolds number of 5×10^5. The plate is 3 m long and 1 m wide. What will be the average resistance coefficient C_f for the plate? Also, what is the total shearing resistance of one side of the plate, and what will be the resistance due to the turbulent part and the laminar part of the boundary layer?

Define the Situation

Air flows past a flat plate



Assumptions: The leading edge of the plate is sharp, and the boundary is not tripped on the leading edge.

Properties: From Table A.3,

$$\rho = 1.2\ \text{kg/m}^3,\ \nu = 1.51 \times 10^{-5}\ \text{m}^2/\text{s}.$$

State the Goal

1. Average shear-stress coefficient, C_f, for the plate
2. Total shear force (in newtons) on one side of plate
3. Shear force (in newtons) due to laminar part
4. Shear force (in newtons) due to turbulent part

Generate Ideas and Make a Plan

1. Calculate the Reynolds number based on plate length, Re_L.
2. Calculate C_f using Eq. (9.38).
3. Calculate the shear force on one side of plate using $F_s = (1/2)\rho U_0^2 C_f BL$.
4. Using value for transition Reynolds number, find transition point.
5. Use Eq. (9.18) to find average shear-stress coefficient for laminar portion.
6. Calculate shear force for laminar portion.
7. Subtract laminar portion from total shear force.

Take Action (Execute the Plan)

1. Reynolds number based on plate length

$$Re_L = \frac{30\ \text{m/s} \times 3\ \text{m}}{1.51 \times 10^{-5}\ \text{m}^2/\text{s}} = 5.96 \times 10^6$$

2. Average shear-stress coefficient

$$C_f = \frac{0.523}{\ln^2(0.06Re_L)} - \frac{1520}{Re_L} = \boxed{0.00294}$$

3. Total shear force

$$F_s = C_f BL\rho(U_0^2/2)$$

$$= 0.00294 \times 1\,\text{m} \times 3\,\text{m} \times 1.2\,\text{kg/m}^3 \times \frac{(30\,\text{m/s})^2}{2} = \boxed{4.76\ \text{N}}$$

4. Transition point

$$\frac{Ux_{tr}}{\nu} = 500{,}000$$

$$x_{tr} = \frac{500{,}000 \times 1.51 \times 10^{-5}}{30} = 0.252\ \text{m}$$

5. Laminar average shear-stress coefficient

$$C_f = \frac{1.33}{Re_{tr}^{1/2}} = 0.00188$$

6. Laminar shear force

$$F_{s,\,lam} = 0.00188 \times 1\ \text{m} \times 0.252\ \text{m} \times 1.2\ \text{kg/m}^3 \times \frac{(30\ \text{m/s})^2}{2}$$

$$= \boxed{0.256\ \text{N}}$$

7. Turbulent shear force

$$F_{s,\,turb} = 4.76\ \text{N} - 0.26\ \text{N} = \boxed{4.50\ \text{N}}$$

If the boundary layer is "tripped" by some roughness or leading-edge disturbance (such as a wire across the leading edge), the boundary layer is turbulent from the leading edge. This is shown by the dashed line in Fig. 9.13. For this condition the boundary layer thickness, local shear-stress coefficient, and average shear-stress coefficient are fit reasonably well by Eqs. (9.33), (9.34a), and (9.34b).

$$\delta = \frac{0.16x}{\mathrm{Re}_x^{1/7}} \qquad c_f = \frac{0.027}{\mathrm{Re}_x^{1/7}} \qquad C_f = \frac{0.032}{\mathrm{Re}_L^{1/7}} \tag{9.39}$$

which are valid up to a Reynolds number of 10^7. For Reynolds numbers beyond 10^7, the average shear-stress coefficient given by Eq. (9.36) can be used. It is of interest to note that marine engineers incorporate tripping mechanisms for the boundary layer on ship models to produce a boundary layer that can be predicted more precisely than a combination of laminar and turbulent boundary layers.

Example 9.7 illustrates calculating shear force with a tripped boundary layer.

EXAMPLE 9.7

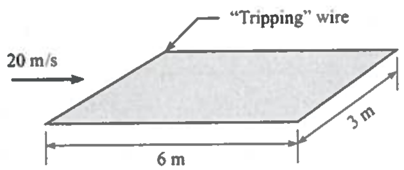
Shear Force with a Tripped Boundary Layer

Problem Statement

Air at 20°C flows past a smooth, thin plate with a free-stream velocity of 20 m/s. Plate is 3 m wide and 6 m long in the direction of flow, and boundary layer is tripped at the leading edge.

Define the Situation

Air flows past a smooth, thin plate. Boundary layer is tripped at leading edge.



Properties: From Table A.3,

$$\rho = 1.2\ \mathrm{kg/m^3}, \quad \mu = 1.81 \times 10^{-5}\ \mathrm{N \cdot s/m^2}.$$

State the Goal

Find: Total shear force (in newtons) on both sides of plate.

Generate Ideas and Make a Plan

1. Calculate the Reynolds number based on plate length.
2. Find average shear-stress coefficient from Eq. (9.39).
3. Calculate shear force for both sides of plate.

Take Action (Execute the Plan)

1. Reynolds number

$$\mathrm{Re}_L = \frac{\rho U L}{\mu} = \frac{1.2 \times 20 \times 6}{1.81 \times 10^{-5}} = 7.96 \times 10^6$$

Reynolds number is less than 10^7.

2. Average shear-stress coefficient

$$C_f = \frac{0.032}{\mathrm{Re}_L^{1/7}} = \frac{0.032}{(7.96 \times 10^6)^{1/7}} = 0.0033$$

3. Shear force

$$F_s = 2 \times C_f A \frac{\rho U_0^2}{2}$$
$$= 0.0033 \times 3\ \mathrm{m} \times 6\ \mathrm{m} \times 1.2\ \mathrm{kg/m^3} \times (20\ \mathrm{m/s})^2$$
$$= \boxed{28.5\ \mathrm{N}}$$

Even though the equations in this chapter have been developed for flat plates, they are useful for engineering estimates for some surfaces that are not truly flat plates. For example, the skin friction drag of the submerged part of the hull of a ship can be estimated with Eq. (9.38).

9.6 Pressure Gradient Effects on Boundary Layers

In the preceding sections the features of a boundary layer on a flat plate where the external pressure gradient is zero have been presented. The boundary layer begins as laminar, goes through transition, and becomes turbulent with a "fuller" velocity profile and an increase in local shear stress. The purpose of this section is to present some features of the boundary la over a curved surface where the external pressure gradient is not zero.

The flow over an airfoil section is shown in Fig. 9.14. The variation in static pressure w distance, s, along the surface is also shown on the figure. The point corresponding to $s =$ (the forward stagnation point where the pressure is equal to the stagnation pressure. The pr sure then decreases toward a minimum value at the midsection. This minimum pressı corresponds to the location of maximum speed as predicted by the Bernoulli equation. T pressure then rises again as the flow decelerates toward the trailing edge. When the pressı decreases with increasing distance ($dp/ds < 0$), the pressure gradient is referred to as a fav able pressure gradient as introduced in Chapter 4. This means that the direction of the foı due to the pressure gradient is in the flow direction. In other words, the effect of the pressı gradient is to accelerate the flow. This is the condition between the forward stagnation po and the point of minimum pressure. A rise in pressure with distance ($dp/ds > 0$) is called adverse pressure gradient and occurs between the point of minimum pressure and the traili edge. The pressure force due to the adverse pressure gradient acts in the direction opposite the flow direction and tends to decelerate the flow.

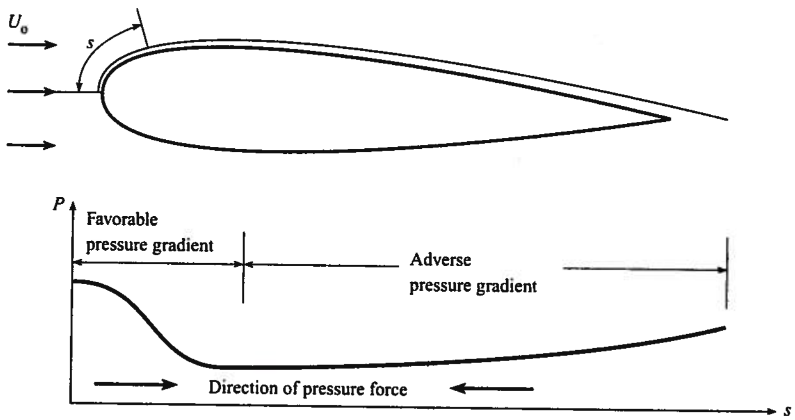


FIGURE 9.14 Surface pressure distribution on airfoil section.

The external pressure gradient effects the properties of the boundary layer. Compared ı a flat plate, the laminar boundary layer in a favorable pressure gradient grows more slowly an is more stable. This means that the boundary-layer thickness is less and the local shear stress increased. Also the transition region is moved downstream, so the boundary layer become turbulent somewhat later. Of course, free-stream turbulence and surface roughness will sti promote the early transition to a fully turbulent boundary layer.

The effect of external pressure gradient on the boundary layer is most pronounced for th adverse pressure gradient. The development of the velocity profiles for the laminar and turbu lent boundary layers in an adverse pressure gradient are shown in Fig. 9.15. The retarding forc associated with the adverse pressure gradient decelerates the flow, especially near the surfac where the velocities are the lowest. Ultimately there is a reversal of flow at the wall, which give rise to a recirculatory pattern and the formation of an eddy. This phenomenon is called **boundary layer separation**. The point of separation is defined where the velocity gradient $\partial u/\partial$ becomes zero as indicated on the figure. The separation point for the turbulent boundary laye occurs farther downstream because the velocity profile is much fuller (higher velocities persis closer to the wall) than the laminar profile, and it takes longer for the adverse pressure gradien to decelerate the flow. Thus the turbulent boundary layer is less affected by the adverse pressur gradient.

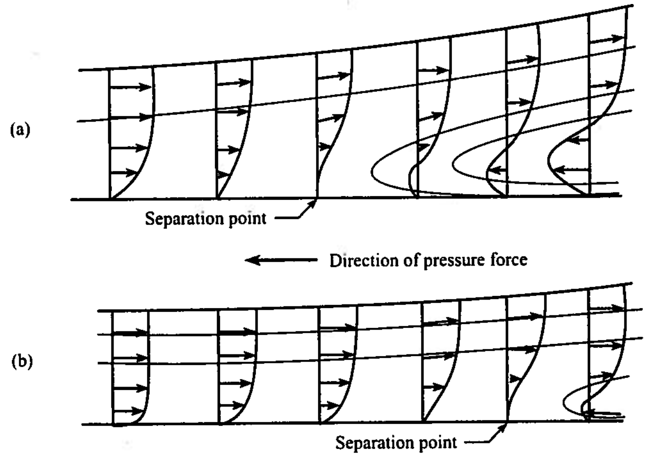


FIGURE 9.15
Velocity distribution anc streamlines for bounda layer separation.
(a) Laminar boundary l
(b) Turbulent boundary layer.

Even though shear stresses on a body in a flow may not contribute significantly to the total drag force, the effect of boundary-layer separation can be very important. When boundary-layer separation takes place on airfoils at a high angle of attack, "stall" occurs, which means the airfoil loses its capability to provide lift. A photograph illustrating boundary-layer separation on an airfoil section is shown in Fig. 4.26. Boundary-layer separation on a cylinder was discussed and illustrated in Section 4.8. Understanding and controlling boundary-layer separation is important in the design of fluid dynamic shapes for maximum performance.

9.7 Summarizing Key Knowledge

Uniform Laminar Flow

- The variation in velocity for a planar, viscous, steady flow with parallel streamlines is governed by the equation

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{d}{ds}(p + \gamma z)$$

where the distance y is normal to the streamlines and the distance s is along the streamlines.

- In this chapter, this equation is used to analyze two flow configurations:
  - Couette flow (flow generated by a moving plate)
  - Hele-Shaw flow (flow between stationary parallel plates).

Boundary Layer

- The boundary layer is the region where the viscous stresses are responsible for the velocity change between the wall and the free stream.
- The boundary-layer thickness is the distance from the wall to the location where the velocity is 99% of the free-stream velocity.
- The laminar boundary layer is characterized by smooth (nonturbulent) flow where the momentum transfer between fluid layers occurs because of viscosity.
- As the boundary layer thickness grows, the laminar boundary layer becomes unstable, and a turbulent boundary layer ensues.

- The transition point for a boundary layer on a flat plate occurs at a nominal Reynolds number of 5×10^5 based on the free stream velocity and the distance from the leading edg
- The turbulent boundary layer is characterized by an unsteady flow where the momentur exchange between fluid layers occurs because of the mixing of fluid elements normal to the direction of fluid motion. This effect, known as the Reynolds stress, significantly enhances the momentum exchange and leads to a much higher "effective" shear stress.

Predicting Shear Stress and Shear Force

- The local shear-stress coefficient is defined as

$$c_f = \frac{\tau_0}{\frac{1}{2}\rho U_0^2}$$

where τ_0 is the wall shear stress and U_0 is the free-stream velocity.
- The value for the local shear-stress coefficient on a flat plate depends on the Reynolds number based on the distance from the leading edge.
- The average shear-stress coefficient is

$$C_f = \frac{F_s}{\frac{1}{2}\rho U_0^2 A}$$

where F_s is the force due to shear-stress, or shear force, on a surface with area A.
- The value for the average shear-stress coefficient for a flat plate depends on the nature of the boundary layer as related to the Reynolds number based on the length of the plate in the flow direction.
- The laminar boundary layer near the leading edge and the subsequent turbulent boundar layer contribute to the average shear stress on a flat plate.
- Through leading-edge roughness or other flow disturbance, the boundary layer can be "tripped" at the plate's leading edge, effecting a turbulent boundary layer over the entire plate

Effects of Pressure Gradient

- The boundary layer for flow over a curved body is subjected to an external pressure gradient.
- A favorable pressure gradient produces a force in the flow direction and tends to keep the boundary layer stable.
- An adverse pressure gradient decelerates the flow and can lead to boundary layer separation.

REFERENCES

1. Prandtl, L. "Über Flussigkeitsbewegung bei sehr kleiner Reibung." *Verhandlungen des III. Internationalen Mathematiker-Kongresses.* Leipzig, 1905.

2. Blasius, H. "Grenzschichten in Flüssigkeiten mit kleiner Reibung." *Z. Mat. Physik.* (1908), English translation in NACA TM 1256.

3. White, F. M. *Viscous Fluid Flow.* New York: McGraw-Hill, 1991.

4. Schlichting, H. *Boundary Layer Theory.* New York: McGraw-Hil 1979.

5. Taylor, G. I. "The Transport of Vorticity and Heat through Fluids in Turbulent Motion." *Phil. Trans A*, vol. 215 (1915), p. 1.

6. Rouse, H. and Appel, D. W. *Advanced Mechanics of Fluids.* New York: John Wiley, 1959.

PROBLEMS

PLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* instructor's discretion.

Uniform Laminar Flow (§9.1)

9.1 PLUS In which case is the flow caused by a pressure gradient?

a. Couette flow

b. Hele-Shaw flow

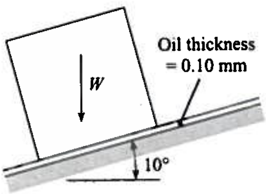
9.2 The velocity distribution in a Couette flow is linear if the viscosity is constant. If the moving plate is heated and the viscosity of the liquid is decreased near the hot plate, how will the velocity distribution change? Give a qualitative description and the rationale for your argument.

9.3 Consider the flow of various fluids between two parallel plates.

a. Assume the fluid is a liquid, its viscosity is constant along the flow direction, and the pressure gradient is linear with distance. How would the pressure gradient differ if the viscosity of the fluid decreased (due to temperature rise) along the flow direction. The density is unchanged. Give a qualitative description of pressure distribution and provide rationale for your answer.

b. Assume the fluid is a gas flowing between two parallel plates. If there were an increase in temperature due to heat transfer along the flow direction, the gas density would decrease. Assume the viscosity is unaffected. How will the velocity and pressure distribution change from the case with constant density? Sketch the pressure distribution and give the rationale for your result.

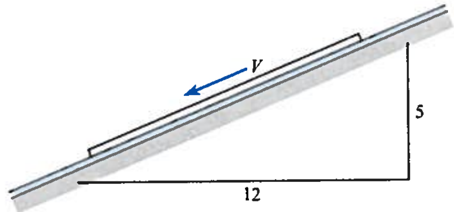
9.4 PLUS The cube shown weighing 110 N and measuring 39 cm on a side is allowed to slide down an inclined surface on which there is a film of oil having a viscosity of 10^{-2} N · s/m^2. What is the velocity of the block if the oil has a thickness of 0.11 mm?



PROBLEM 9.4

9.5 PLUS A board 3 ft by 3 ft that weighs 40 lbf slides down an inclined ramp with a velocity of 0.5 fps. The board is separated from the ramp by a layer of oil 0.02 in. thick. Neglecting the edge effects of the board, calculate the approximate dynamic viscosity μ of the oil.

9.6 A board 1 m by 1 m that weighs 30 N slides down an inclined ramp with a velocity of 17 cm/s. The board is separated from the ramp by a layer of oil 0.8 mm thick. Neglecting the edge effects of the board, calculate the approximate dynamic viscosity μ of the oil.



PROBLEMS 9.5, 9.6

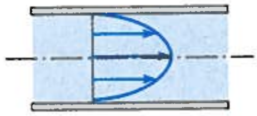
9.7 PLUS Uniform, steady flow is occurring between horizo parallel plates as shown.

a. The flow is Hele-Shaw; therefore, what is causing the to move?

b. Where is the maximum velocity located?

c. Where is the maximum shear stress located?

d. Where is the minimum shear stress located?



PROBLEM 9.7

9.8 Uniform, steady flow is occurring between horizontal parallel plates as shown.

a. In a few words, tell what other condition must be pr to cause the odd velocity distribution.

b. Where is the minimum shear stress located?

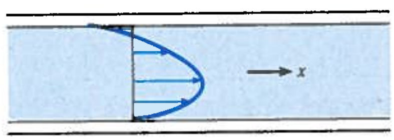
9.9 PLUS Under certain conditions (pressure decreasing in *x*-direction, the upper plate fixed, and the lower plate movin to the right in the positive *x*-direction), the laminar velocity distribution will be as shown. For such conditions, indicate whether each of the following statements is true or false.

a. The shear stress midway between the plates is zero.

b. The minimum shear stress in the liquid occurs next the moving plate.

c. The shear stress is greatest where the velocity is the greatest.

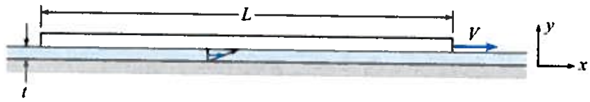
d. The minimum shear stress occurs where the velocit the greatest.



PROBLEMS 9.8, 9.9

9.10 A flat plate is pulled to the right at a speed of 30 cm/s. Oil with a viscosity of 4 $N \cdot s/m^2$ fills the space between the plate and the solid boundary. The plate is 1 m long ($L = 1$ m) by 30 cm wide, and the spacing between the plate and boundary is 2.0 mm.

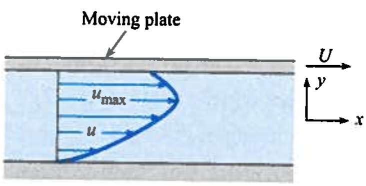
a. Express the velocity mathematically in terms of the coordinate system shown.
b. By mathematical means, determine whether this flow is rotational or irrotational.
c. Determine whether continuity is satisfied, using the differential form of the continuity equation.
d. Calculate the force required to produce this plate motion.



PROBLEM 9.10

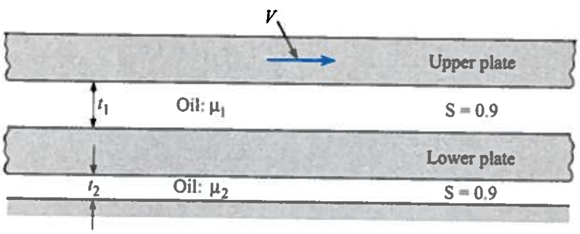
9.11 The velocity distribution that is shown represents laminar flow. Indicate which of the following statements are true.

a. The velocity gradient at the boundary is infinitely large.
b. The maximum shear stress in the liquid occurs midway between the walls.
c. The maximum shear stress in the liquid occurs next to the boundary.
d. The flow is irrotational.
e. The flow is rotational.



PROBLEM 9.11

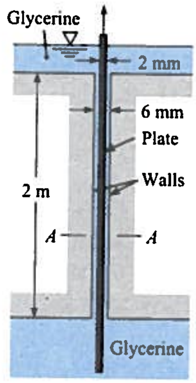
9.12 The upper plate shown is moving to the right with a velocity V, and the lower plate is free to move laterally under the action of the viscous forces applied to it. For steady-state conditions, derive an equation for the velocity of the lower plate. Assume that the area of oil contact is the same for the upper plate, each side of the lower plate, and the fixed boundary.



PROBLEM 9.12

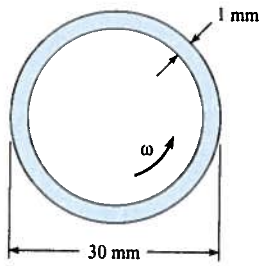
9.13 A circular horizontal disk with a 27 cm diameter has a clearance of 3.0 mm from a horizontal plate. What torqu is required to rotate the disk about its center at an angular speed of 31 rad/s when the clearance space contains oil ($\mu = 8$ $N \cdot s/m^2$)?

9.14 A plate 2 mm thick and 1 m wide (normal to the page) is pulled between the walls shown in the figure at a spee of 0.40 m/s. Note that the space that is not occupied by the plat is filled with glycerine at a temperature of 20°C. Also, the plate is positioned midway between the walls. Sketch the velocity distribution of the glycerine at section A-A. Neglecting the weight of the plate, estimate the force required to pull the plate at the speed given.



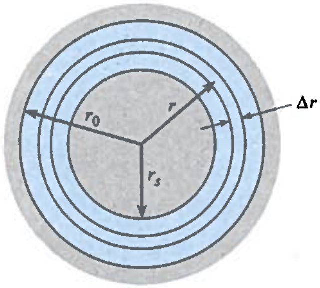
PROBLEM 9.14

9.15 A bearing uses SAE 30 oil with a viscosity of 0.1 $N \cdot s/m^2$. The bearing is 30 mm in diameter, and the gap between the shaft and the casing is 1 mm. The bearing has a length of 1 cm. The shaft turns at $\omega = 200$ rad/s. Assuming that the flow between the shaft and the casing is a Couette flow, find the torque required to turn the bearing.



PROBLEM 9.15

9.16 An important application of viscous flow is found in lubrication theory. Consider a shaft that turns inside a stationary cylinder, with a lubricating fluid in the annular region. By considering a system consisting of an annulus of fluid of radius r and width Δr, and realizing that under steady-state operation the net torque on this ring is zero, show that $d(r^2\tau)/dr = 0$, where τ is the viscous shear stress. For a flow that has a tangential component of velocity only, the shear stress is related to the velocity by $\tau = \mu r d(V/r)/dr$. Show that the torque per unit length acting on the inner cylinder is given by $T = 4\pi\mu\omega r_s^2/(1 - r_s^2/r_0^2)$, where ω is the angular velocity of the shaft.



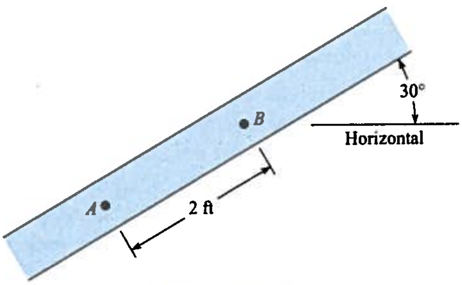
PROBLEM 9.16

9.17 Using the equation developed in Prob. 9.16, find the power necessary to rotate a 2 cm shaft at 60 rad/s if the inside diameter of the casing is 2.2 cm, the bearing is 3 cm long, and SAE 30 oil at 38°C is the lubricating fluid.

9.18 The analysis developed in Prob. 9.16 applies to a device used to measure the viscosity of a fluid. By applying a known torque to the inner cylinder and measuring the angular velocity achieved, one can calculate the viscosity of the fluid. Assume you have a 4 cm inner cylinder and a 4.5 cm outer cylinder. The cylinders are 10 cm long. When a force of 0.6 N is applied to the tangent of the inner cylinder, it rotates at 20 rpm. Calculate the viscosity of the fluid.

9.19 WILEY PLUS Two horizontal parallel plates are spaced 0.015 ft apart. The pressure decreases at a rate of 25 psf/ft in the horizontal x-direction in the fluid between the plates. What is the maximum fluid velocity in the x direction? The fluid has a dynamic viscosity of 10^{-3} lbf-s/ft^2 and a specific gravity of 0.80.

9.20 A viscous fluid fills the space between these two plates, and the pressures at A and B are 150 psf and 100 psf, respectively. The fluid is not accelerating. If the specific weight of the fluid is 100 lbf/ft^3, then one must conclude that (a) flow is downward, (b) flow is upward, or (c) there is no flow.



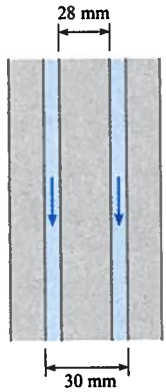
PROBLEM 9.20

9.21 Glycerine at 20°C flows downward between two vertical parallel plates separated by a distance of 0.4 cm. The ends are open, so there is no pressure gradient. Calculate the discharge per unit width, q, in m^2/s.

9.22 WILEY PLUS Two vertical parallel plates are spaced 0.01 ft apa[r] If the pressure decreases at a rate of 60 psf/ft in the vertica[l] z-direction in the fluid between the plates, what is the maxin[num] fluid velocity in the z-direction? The fluid has a viscosity of 10^{-3} lbf-s/ft^2 and a specific gravity of 0.80.

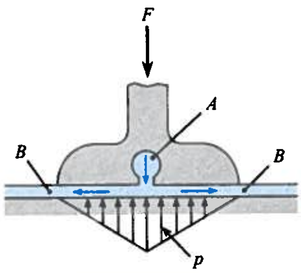
9.23 WILEY GO Two parallel plates are spaced 0.09 in. apart, and mo[tor] oil (SAE 30) with a temperature of 100°F flows at a rate of 0.00 per foot of width between the plates. What is the pressure grad[ient] in the direction of flow if the plates are inclined at 60° with the horizontal and if the flow is downward between the plates?

9.24 WILEY GO Glycerin at 20°C flows downward in the annular r[egion] between two cylinders. The internal diameter of the outer cylinder is 3 cm, and the external diameter of the inner cylin[der] 2.8 cm. The pressure is constant along the flow direction. The [flow] is laminar. Calculate the discharge. (*Hint:* The flow between t[he] two cylinders can be treated as the flow between two flat plat[es.])



PROBLEM 9.24

9.25 WILEY PLUS One type of bearing that can be used to support large structures is shown in the accompanying figure. Here [f] under pressure is forced from the bearing midpoint (slot A) to the exterior zone B. Thus a pressure distribution occurs a[s] shown. For this bearing, which is 43 cm wide, what discharg[e] of oil from slot A per meter of length of bearing is required? Assume a 190 kN load per meter of bearing length with a clearance space t between the floor and the bearing surface [of] 1.5 mm. Assume an oil viscosity of 0.20 N · s/m^2. How muc[h] oil per hour would have to be pumped per meter of bearing length for the given conditions?



PROBLEM 9.25

9.26 Often in liquid lubrication applications there is a heat generated that is transferred across the lubricating layer. Consider a Couette flow with one wall at a higher temperature than the other. The temperature gradient across the flow affects the fluid viscosity according to the relationship.

$$\mu = \mu_0 \exp\left(-0.1\frac{y}{L}\right)$$

where μ_0 is the viscosity at $y = 0$ and L is the distance between the walls. Incorporate this expression into the Couette flow equation, integrate and express the shear stress in the form

$$\tau = C\frac{U\mu_0}{L}$$

where C is a constant and U is the velocity of the moving wall. Analyze your answer. Should the shear stress be greater or less than that with uniform viscosity?

9.27 Gases form good insulating layers. Consider an application in which there is a Couette flow with the moving plate at a higher temperature than the fixed plate. The viscosity varies between the plates as

$$\mu = \mu_0\left(1 + 0.1\frac{y}{L}\right)^{1/2}$$

where μ_0 is the viscosity at $y = 0$ and L is the distance between the plates. Incorporate this expression into the Couette flow equation, integrate and express the shear stress in the form

$$\tau = C\frac{U\mu_0}{L}$$

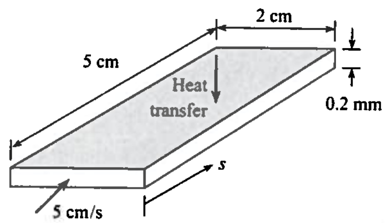
where C is a constant and U is the velocity of the moving plate. Analyze your answer. Should the shear stress be greater or less than that with uniform viscosity?

9.28 An engineer is designing a very thin, horizontal channel for cooling electronic circuitry. The channel is 2 cm wide and 5 cm long. The distance between the plates is 0.2 mm. The average velocity is 5 cm/s. The fluid used has a viscosity of 1.2 cp and a density of 800 kg/m^3. Assuming no change in viscosity or density, find the pressure drop in the channel and the power required to move the flow through the channel.

9.29 Consider the channel designed for electronic cooling in Prob. 9.28. Because of the heating, the viscosity will change through the channel. Assume the viscosity varies as

$$\mu = \mu_0 \exp\left(-0.1\frac{s}{L}\right)$$

where μ_0 is the viscosity at $s = 0$ and L is the length of the channel. Find the percentage change of the pressure drop due to viscosity variation.



PROBLEMS 9.28, 9.29

Describing the Boundary Layer (§9.2)

9.30 a. Explain in your own words what is meant by "boundary layer." b. Define "boundary layer thickness."

9.31 Which of the following are features of a laminar boundary layer? (Select all that are correct.)

a. Flow is smooth.

b. The boundary layer thickness increases in the downstream direction.

c. A decreasing boundary layer thickness correlates with decreased shear stress.

d. An increasing boundary layer thickness correlates with decreased shear stress.

Laminar Boundary Layer (§9.3)

9.32 Assume the wall adjacent to a liquid laminar boundary is heated and the viscosity of the fluid is lower near the wall and increases the free-stream value at the edge of the boundary layer. How would this variation in viscosity affect the boundary-layer thickness and local shear stress? Give the rationale for your answer.

9.33 A thin plate 6 ft long and 3 ft wide is submerged and held stationary in a stream of water ($T = 60°F$) that has a velocity of 5 ft/s. What is the thickness of the boundary layer on the plate for $Re_x = 500{,}000$ (assume the boundary layer is still laminar), and at what distance downstream of the leading edge does this Reynolds number occur? What is the shear stress on the plate at this point?

9.34 What is the ratio of the boundary-layer thickness on a smooth, flat plate to the distance from the leading edge just before transition to turbulent flow?

9.35 A model airplane has a wing span of 3.1 ft and a chord (leading edge–trailing edge distance) of 5 in. The model flies in air at 60°F and atmospheric pressure. The wing can be regarded as a flat plate so far as drag is concerned. (a) At what speed will a turbulent boundary layer start to develop on the wing? (b) What will be the total drag force on the wing just before turbulence appears?

9.36 Oil ($\mu = 10^{-2}$ N · s/m^2; $\rho = 900$ kg/m^3) flows past a plate in a tangential direction so that a boundary layer develops. If the velocity of approach is 4 m/s, then at a section 30 cm downstream of the leading edge the ratio of τ_δ (shear stress at the edge of the boundary layer) to τ_0 (shear stress at the plate surface) is approximately (a) 0, (b) 0.24, (c) 2.4, or (d) 24.

9.37 A liquid ($\rho = 1000$ kg/m^3; $\mu = 2 \times 10^{-2}$ N · s/m^2; $\nu = 2 \times 10^{-5}$ m^2/s) flows tangentially past a flat plate. If the approach velocity is 2 m/s, what is the liquid velocity 1 m downstream from the leading edge of the plate, at 0.8 mm away from the plate?

9.38 The plate of Prob. 9.37 has a total length of 3 m (parallel to the flow direction), and it is 1 m wide. What is the skin friction drag (shear force) on one side of the plate?

9.39 Oil ($\nu = 10^{-4}$ m^2/s) flows tangentially past a thin plate. If the free-stream velocity is 5 m/s, what is the velocity 1 m downstream from the leading edge and 3 mm away from the plate?

9.40 Oil ($\nu = 10^{-4}$ m^2/s; S = 0.9) flows past a plate in a tangential direction so that a boundary layer develops. If the velocity of approach is 0.85 m/s, what is the oil velocity 1.6 m downstream from the leading edge, 10 cm away from the plate?

9.41 A thin plate 0.7 m long and 1.5 m wide is submerged and held stationary in a stream of water ($T = 10$°C) that has a velocity of 1.5 m/s. What is the thickness of the boundary layer on the plate for $\text{Re}_x = 500{,}000$ (assume the boundary layer is still laminar), and at what distance downstream of the leading edge does this Reynolds number occur? What is the shear stress on the plate on this point?

9.42 A flat plate 1.5 m long and 1.0 m wide is towed in water at 20°C in the direction of its length at a speed of 15 cm/s. Determine the resistance of the plate and the boundary layer thickness at its aft end.

9.43 Transition from a laminar to a turbulent boundary layer occurs between the Reynolds numbers of $\text{Re}_x = 10^5$ and $\text{Re}_x = 3 \times 10^6$. The thickness of the turbulent boundary layer based on the distance from the leading edge is $\delta = 0.16x/(\text{Re}_x)^{1/7}$. Find the ratio of the thickness of the laminar boundary layer at the beginning of transition to the thickness of the turbulent boundary layer at the end of transition.

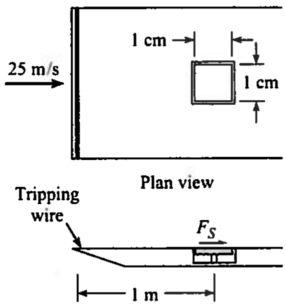
Turbulent Boundary Layer (§9.5)

9.44 Classify each of the following features into one of two categories: laminar boundary layer (L), or turbulent boundary layer (T).

- **a.** Flow is smooth
- **b.** Three differently shaped velocity distributions in 3 zones
- **c.** Velocity profile that follows a power law
- **d.** Velocity profile that is a function of $\sqrt{\text{Re}}$
- **e.** Logarithmic velocity distribution
- **f.** Thickness is inversely related to the 7th root of Re
- **g.** Thickness is inversely related to $\sqrt{\text{Re}}$
- **h.** Velocity defect region
- **i.** Mixing action causes locally unsteady velocities
- **j.** Shear stress is a function of a natural log
- **k.** Shear stress is a function of $\sqrt{\text{Re}}$

9.45 Assume that a turbulent gas boundary layer was adjac a cool wall and the viscosity in the wall region was reduced. may this affect the features of the boundary layer? Give som rationale for your answers.

9.46 An element for sensing local shear stress is positioned in a flat plate 1 meter from the leading edge. Th element simply consists of a small plate, 1 cm × 1 cm, mou flush with the wall, and the shear force is measured on the The fluid flowing by the plate is air with a free-stream veloc $V = 30$ m/s, a density of 1.2 kg/m^3, and a kinematic viscosi 1.5×10^{-5} m^2/s. The boundary layer is tripped at the leadi edge. What is the magnitude of the force due to shear stres acting on the element?



PROBLEM 9.46

9.47 For the conditions of Prob. 9.46, what is the shearing resistance on one side of the plate for the part of the plate th has a Reynolds number, Re_x, less than 500,000? What is the of the laminar shearing force to the total shearing force on plate?

9.48 An airplane wing of 2 m chord length (leading e trailing edge distance) and 11 m span flies at 200 km/hr in 30°C. Assume that the resistance of the wing surfaces is like of a flat plate.

- **a.** What is the friction drag on the wing?
- **b.** What power is required to overcome this?
- **c.** How much of the chord is laminar?
- **d.** What will be the change in drag if a turbulent bou layer is tripped at the leading edge?

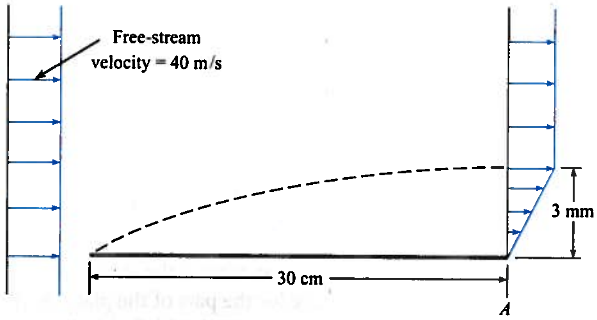
9.49 A turbulent boundary layer exists in the flow of at 20°C over a flat plate. The local shear stress measured at t surface of the plate is 0.2 N/m^2. What is the velocity at a poi 0.52 cm from the plate surface?

9.50 A liquid flows tangentially past a flat plate. The fluid properties are $\mu = 10^{-5}$ N · s/m^2 and $\rho = 1.5$ kg/m^3. Find t skin friction drag on the plate per unit width if the plate is long and the approach velocity is 16 m/s. Also, what is the velocity gradient at a point that is 1 m downstream of the leading edge and just next to the plate ($y = 0$)?

9.51 For the hypothetical boundary layer on the flat plate shown, what is the shear-stress on the plate at the downstre end (point A)? Here $\rho = 1.2$ kg/m^3 and $\mu = 1.8 \times 10^{-5}$ N ·

9.52 Assume that the velocity profile in a boundary layer is replaced by a step profile, as shown in the figure, where the velocity is zero adjacent to the surface and equal to the free-stream velocity (U) at a distance greater than δ_* from the surface. Assume also that the density is uniform and equal to the free-stream density (ρ_∞). The distance δ_* (displacement thickness) is so chosen that the mass flux corresponding to the step profile is equal to the mass flux through the actual boundary layer. Derive an integral expression for the displacement thickness as a function of u, U, y, and δ.

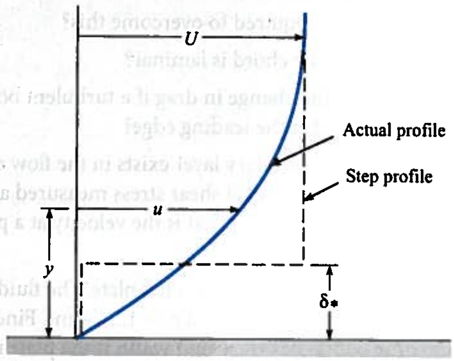
9.53 Because of the reduction of velocity associated with the boundary layer, the streamlines outside the boundary layer are shifted away from the boundary. This amount of displacement of the streamlines is defined as the displacement thickness δ_*. Using the expression developed in Prob. 9.52, evaluate the displacement thickness of the boundary layer at the downstream edge of the plate (point A) in Prob. 9.51.



PROBLEMS 9.51, 9.53

9.54 Use the expression developed in Prob. 9.52 to find the ratio of the displacement thickness to the boundary layer thickness for the turbulent boundary layer profile given by

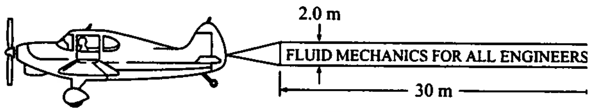
$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$$



PROBLEM 9.52

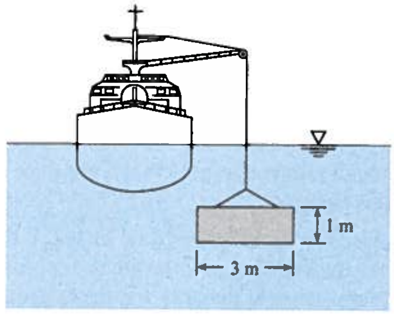
9.55 What is the ratio of the skin friction drag of a plate 30 m long and 5 m wide to that of a plate 10 m long and 5 m wide if both plates are towed lengthwise through water ($T = 20°C$) at 10 m/s?

9.56 Estimate the power required to pull the sign shown if it is towed at 41 m/s and if it is assumed that the sign has the same resistance characteristics as a flat plate. Assume standard atmospheric pressure and a temperature of 10°C.



PROBLEM 9.56

9.57 A thin plastic panel (3 mm thick) is lowered from a ship to a construction site on the ocean floor. The plastic panel weighs 300 N in air and is lowered at a rate of 3 m/s. Assuming that the panel remains vertically oriented, calculate the tension i the cable.



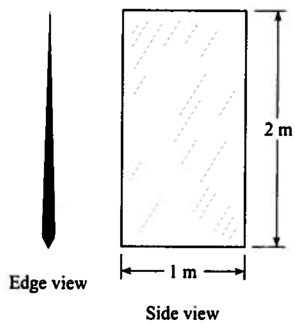
PROBLEM 9.57

9.58 The plate shown in the figure is weighted at the bottom so it will fall stably and steadily in a liquid. The weight of the plate in air is 23.5 N, and the plate has a volume of 0.002 m³. Estimate its falling speed in freshwater at 20°C. The boundary layer is normal; that is, it is not tripped at the leading edge.

In this problem, the final falling speed (terminal velocity) occurs when the weight is equal to the sum of the skin friction and buoyancy.

$$W = B + F_s = \gamma V + \frac{1}{2} C_f \rho U_0^2 S$$

Hints: Find the final falling speed. This problem requires an itera tive solution.

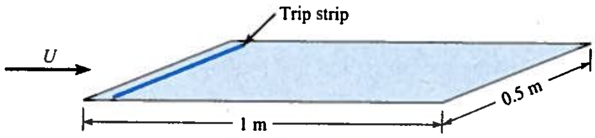


PROBLEM 9.58

9.59 A turbulent boundary layer develops from the leading edge of a flat plate with water at 20°C flowing tangentially past the plate with a free-stream velocity of 7.7 m/s. Determine the thickness of the viscous sublayer, δ', at a distance 7.8 m downstream from the leading edge.

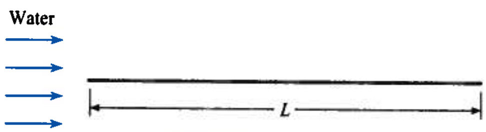
9.60 A model airplane descends in a vertical dive through air at standard conditions (1 atmosphere and 20°C). The majority of the drag is due to skin friction on the wing (like that on a flat plate). The wing has a span of 1 m (tip to tip) and a chord length (leading edge to trailing edge distance) of 10 cm. The leading edge is rough, so the turbulent boundary layer is "tripped." The model weighs 3 N. Determine the speed (in meters per second) at which the model will fall.

9.61 A flat plate is oriented parallel to a 24 m/s airflow at 20°C and atmospheric pressure. The plate is $L = 3$ m in the flow direction and 0.5 m wide. On one side of the plate, the boundary layer is tripped at the leading edge, and on the other side there is no tripping device. Find the total drag force on the plate.



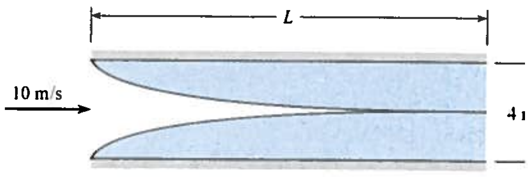
PROBLEM 9.61

9.62 An engineer is designing a horizontal, rectangular conduit that will be part of a system that allows fish to bypass a dam. Inside the conduit, a flow of water at 40°F will be divided into two streams by a flat, rectangular metal plate. Calculate the viscous drag force on this plate, assuming boundary-layer flow with free-stream velocity of 10 ft/s and plate dimensions of $L = 6$ ft and $W = 4.0$ ft.



PROBLEM 9.62

9.63 A model is being developed for the entrance region between two flat plates. As shown in the figure, it is assumed that the region is approximated by a turbulent boundary layer originating at the leading edge. The system is designed such that the plates end where the boundary layers merge. The spacing between the plates is 4 mm, and the entrance velocity is 10 m/s. The fluid is water at 20°C. Roughness at the leading edge trips the boundary layers. Find the length L where the boundary layers merge, and find the force per unit depth (into the paper) due to shear stress on both plates.

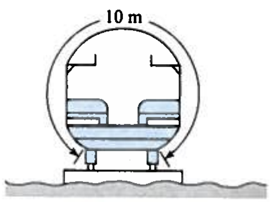


PROBLEM 9.63

9.64 An outboard racing boat "planes" at 70 mph over wat
60°F. The part of the hull in contact with the water has an a
width of 3 ft and a length of 8 ft. Estimate the power requir
overcome its shear force.

9.65 A motor boat pulls a long, smooth, water-soaked log (
in diameter and 50 m long) at a speed of 1.7 m/s. Assuming
submergence, estimate the force required to overcome the s
force of the log. Assume a water temperature of 10°C and th
boundary layer is tripped at the front of the log.

9.66 High-speed passenger trains are streamlined to r
shear force. The cross section of a passenger car of one such
is shown. For a train 81 m long, (a) estimate the shear force
speed of 81.1 km/hr and (b) for one of 204 km/hr. What po
is required for just the shear force at these speeds? These
power calculations will be answers (c) and (d) respective
Assume $T = 10$°C and that the boundary layer is tripped at
front of the train.



PROBLEM 9.66

9.67 Consider the boundary layer next to the smooth hull
ship. The ship is cruising at a speed of 45 ft/s in 60°F freshw
Assuming that the boundary layer on the ship hull develops
same as on a flat plate, determine

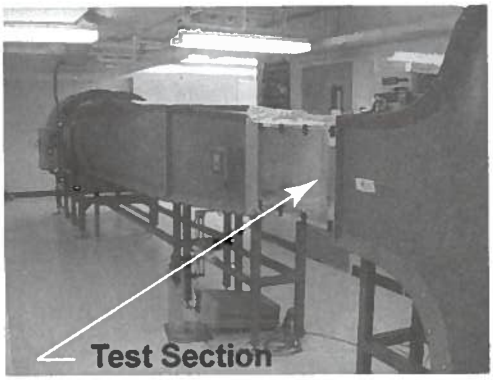
a. The thickness of the boundary layer at a distance $x = 100$ ft downstream from the bow.

b. The velocity of the water at a point in the boundary layer at $x = 100$ ft and $y/\delta = 0.50$.

c. The shear stress, τ_0, adjacent to the hull at $x = 100$

9.68 A wind tunnel operates by drawing air through a
contraction, passing this air through a test section, and then
exhausting the air using a large axial fan. Experimental data
recorded in the test section, which is typically a rectangular
section of duct that is made of clear plastic (usually acrylic).
the test section, the velocity should have a very uniform
distribution; thus, it is important that the boundary layer be
very thin at the end of the test section. For the pictured wind
tunnel, the test section is square with a dimension of $W = 457$

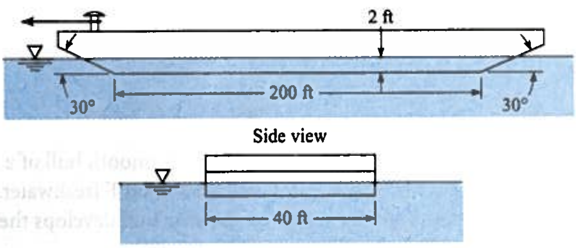
on each side and a length of $L = 914$ mm. Find the ratio of maximum boundary-layer thickness to test section width [$\delta(x = L)/W$] for two cases: minimum operating velocity (1 m/s) and maximum operating velocity (70 m/s). Assume air properties at 1 atm and 20°C.



PROBLEM 9.68 (Photo by Donald Elger)

9.69 A ship 600 ft long steams at a rate of 25 ft/s through still freshwater ($T = 50$°F). If the submerged area of the ship is 50,000 ft^2, what is the skin friction drag of this ship?

9.70 A river barge has the dimensions shown. It draws 2 ft of water when empty. Estimate the skin friction drag of the barge when it is being towed at a speed of 10 ft/s through still freshwater at 60°F.



PROBLEM 9.70

9.71 A supertanker has length, breadth, and draught (fully loaded) dimensions of 325 m, 48 m, and 19 m, respectively. In open seas the tanker normally operates at a speed of 18 kt (1 kt = 0.515 m/s). For these conditions, and assuming that flat-plate boundary-layer conditions are approximated, estimate the skin friction drag of such a ship steaming in 10°C water. What power is required to overcome the skin friction drag? What is the boundary-layer thickness at 300 m from the bow?

9.72 A model test is needed to predict the wave drag on a ship. The ship is 500 ft long and operates at 30 ft/s in seawater at 10°C. The wetted area of the prototype is 25,000 ft^2. The model/prototype scale ratio is 1/100. Modeling is done in freshwater at 60°F to match the Froude number. The viscous drag can be calculated by assuming a flat plate with the wetted area of the model and a length corresponding to the length of the model. A total drag of 0.1 lbf is measured in model tests. Calculate the wave drag on the actual ship.

9.73 A ship is designed so that it is 250 m long, its beam measures 30 m, and its draft is 12 m. The surface area of the ship below the water line is 8800 m^2. A 1/40 scale model of the ship is tested and is found to have a total drag of 26.0 N when towed at speed of 1.45 m/s. Using the methods outlined in Section 8.9, answer the following questions, assuming that model tests are made in freshwater (20°C) and that prototype conditions are seawater (10°C).

a. To what speed in the prototype does the 1.45 m/s correspond?
b. What are the model skin friction drag and wave drag?
c. What would the ship drag be in saltwater correspondin to the model test conditions in freshwater?

9.74 A hydroplane 3 m long skims across a very calm lake ($T = 20$°C) at a speed of 15 m/s. For this condition, what will be the minimum shear stress along the smooth bottom?

9.75 Estimate the power required to overcome the shear force c a water skier if he or she is towed at 30 mph and each ski is 4 ft by 6 in. Assume the water temperature is 60°F.

9.76 If the wetted area of an 80 m ship is 1500 m^2, approximate how great is the surface drag when the ship is traveling at a spee of 15 m/s. What is the thickness of the boundary layer at the stern? Assume seawater at $T = 10$°C.