Finally, it can be seen in Fig. 15.8 that a point will be reached where the specific energy is minimum and only a single depth occurs. At this point, the flow is termed critical. Thus one definition of **critical flow** is the flow that occurs when the specific energy is minimum for a given discharge. The flow for which the depth is less than critical (velocity is greater than critical) is termed **supercritical flow**, and the flow for which the depth is greater than critical (velocity is less than critical) is termed **subcritical flow**. Therefore, subcritical flow occurs upstream and supercritical flow occurs downstream of the sluice gate in Fig. 15.9. Subcritical flows corresponds to a Froude number less than one ($\mathrm{Fr} < 1$), and supercritical flow corresponds to ($\mathrm{Fr} > 1$). Some engineers refer to subcritical and supercritical flow as **tranquil** and **rapid** flow, respectively. Other aspects of critical flow are shown in the next section.

Characteristics of Critical Flow

Critical flow occurs when the specific energy is minimum for a given discharge. The depth for this condition may be determined by solving for dE/dy from $E = y + Q^2/2gA^2$ and setting dE/dy equal to zero:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} \tag{15.21}$$

However, $dA = T\,dy$, where T is the width of the channel at the water surface, as shown in Fig. 15.10. Then Eq. (15.21), with $dE/dy = 0$, will reduce to

$$\frac{Q^2 T_c}{gA_c^3} = 1 \tag{15.22}$$

or

$$\frac{A_c}{T_c} = \frac{Q^2}{gA_c^2} \tag{15.23}$$

If the **hydraulic depth**, D, is defined as

$$D = \frac{A}{T} \tag{15.24}$$

then Eq. (15.23) will yield a critical hydraulic depth D_c, given by

$$D_c = \frac{Q^2}{gA_c^2} = \frac{V^2}{g} \tag{15.25}$$

Dividing Eq. (15.25) by D_c and taking the square root yields

$$1 = \frac{V}{\sqrt{gD_c}} \tag{15.26}$$

Note: $V/\sqrt{gD_c}$ is the Froude number. Therefore, it has been shown that the Froude number is equal to unity when critical flow prevails.

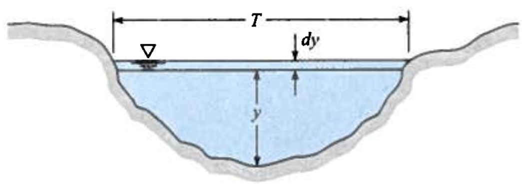


FIGURE 15.10
Open-channel relatio

If a channel is of rectangular cross section, then A/T is the actual depth, and $Q^2/A^2 = q^2/y$ so the condition for **critical depth** (Eq. 15.23) for a rectangular channel becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \quad (15.27)$$

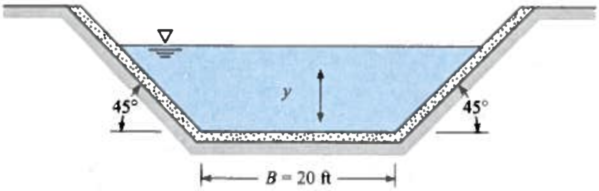
where q is the discharge per unit width of channel.

EXAMPLE 15.9

Calculating Critical Depth in a Channel

Problem Statement

Determine the critical depth in this trapezoidal channel for a discharge of 500 cfs. The width of the channel bottom is $B = 20$ ft, and the sides slope upward at an angle of 45°.



Define the Situation

Water flows in a trapezoidal channel with known geometry.

State the Goal

Calculate the critical depth.

Generate Ideas and Make a Plan

1. For critical flow, Eq. (15.22) must apply.
2. Relate this channel geometry to width T and area A in Eq. (15.22).
3. By iteration (choose y and compute A^3/T), find y that will yield A^3/T equal to 7764 ft². This y will be critical depth y_c.

Take Action (Execute the Plan)

1. Apply Eq. (15.22) or Eq. (15.23).

$$\frac{Q^2 T_c}{g A_c^3} = 1 \text{ or } \frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

2. For $Q = 500$ cfs,

$$\frac{A_c^3}{T_c} = \frac{500^2}{32.2} = 7764 \text{ ft}^2$$

For this channel, $A = y(B + y)$ and $T = B + 2y$.

3. Iterate to find y_c.

$$y_c = \boxed{2.57 \text{ ft}}$$

Critical flow may also be examined in terms of how the discharge in a channel varies with depth for a given specific energy. For example, consider flow in a rectangular channel where

$$E = y + \frac{Q^2}{2gA^2}$$

or

$$E = y + \frac{Q^2}{2gy^2B^2}$$

If one considers a unit width of the channel and lets $q = Q/B$, then the foregoing equation becomes

$$E = y + \frac{q^2}{2gy^2}$$

If one determines how q varies with y for a constant value of specific energy, one sees that critical flow occurs when the discharge is maximum (see Fig. 15.11).

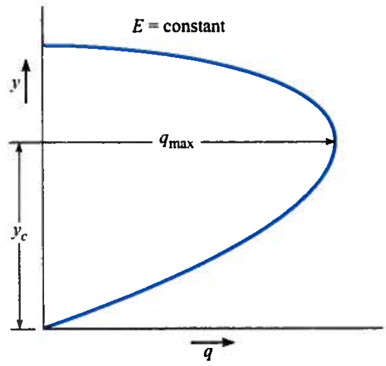


FIGURE 15.11
Variation of q and y w
constant specific ener

Originally, the term **critical flow** probably related to the unstable character of the flow for this condition. Referring to Fig. 15.8, one can see that only a slight change in specific energy will cause the depth to increase or decrease a significant amount; this is a very unstable condition. In fact, observations of critical flow in open channels show that the water surface consists of a series of standing waves. Because of the unstable nature of the depth in critical flow, designing canals so that normal depth is either well above or well below critical depth is usually best. The flow in canals and rivers is usually subcritical; however, the flow in steep chutes or over spillways is supercritical.

In this section, various characteristics of critical flow have been explored. The main ones can be summarized as follows:

1. Critical flow occurs when specific energy is minimum for a given discharge (Fig. 15.8).
2. Critical flow occurs when the discharge is maximum for a given specific energy.
3. Critical flow occurs when

$$\frac{A^3}{T} = \frac{Q^2}{g}$$

4. Critical flow occurs when Fr = 1. Subcritical flow occurs when Fr < 1. Supercritical flow occurs when Fr > 1.
5. For rectangular channels, critical depth is given as $y_c = (q^2/g)^{1/3}$.

Common Occurrence of Critical Flow

Critical flow occurs when a liquid passes over a broad-crested weir (Fig. 15.12). The principle of the broad-crested weir is illustrated by first considering a closed sluice gate that prevents water from being discharged from the reservoir, as shown in Fig. 15.12a. If the gate is opened a small amount (gate position a'-a'), the flow upstream of the gate will be subcritical, and the flow downstream will be supercritical (as in the condition shown in Fig. 15.9). As the gate is opened further, a point is finally reached where the depths immediately upstream and downstream of the gate are the same. This is the critical condition. At this gate opening and beyond, the gate has no influence on the flow; this is the condition shown in Fig. 15.12b, the broad-crested weir. If the depth of flow over the weir is measured, the rate of flow can easily be computed from Eq. (15.27):

$$q = \sqrt{gy_c^3}$$

or

$$Q = L\sqrt{gy_c^3} \tag{15.28}$$

where L is the length of the weir crest normal to the flow direction.

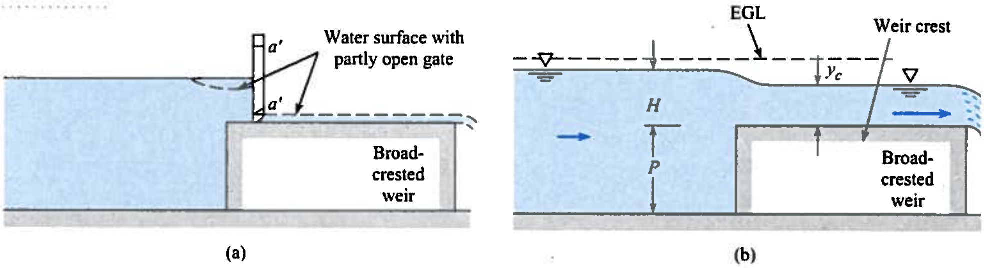


FIGURE 15.12
Flow over a broad-crested weir.
(a) Depth of flow controlled by sluice gate.
(b) Depth of flow is controlled by weir, and is y_c.

Because $y_c/2 = (V_c^2/2g)$, from Eq. (15.25), it can be shown that $y_c = (2/3E)$, where E is th total head above the crest $(H + V_{\text{approach}}^2/2g)$; hence Eq. (15.28) can be rewritten as

$$Q = L\sqrt{g}\left(\frac{2}{3}\right)^{3/2} E^{3/2}$$

or

$$Q = 0.385L\sqrt{2g}E_c^{3/2} \quad (15.29$$

For high weirs, the upstream velocity of approach is almost zero. Hence Eq. (15.29) can b expressed as

$$Q_{\text{theor}} = 0.385L\sqrt{2g}H^{3/2} \quad (15.30$$

If the height P of the broad-crested weir is relatively small, then the velocity of approach may b significant, and the discharge produced will be greater than that given by Eq. (15.30). Also, heac loss will have some effect. To account for these effects, a discharge coefficient C is defined as

$$C = Q/Q_{\text{theor}} \quad (15.31$$

Then

$$Q = 0.385CL\sqrt{2g}H^{3/2} \quad (15.32$$

where Q is the actual discharge over the weir. An analysis of experimental data by Raju (15 shows that C varies with $H/(H + P)$ as shown in Fig. 15.13. The curve in Fig. 15.13 is for a wei with a vertical upstream face and a sharp corner at the intersection of the upstream face anc the weir crest. If the upstream face is sloping at a 45° angle, the discharge coefficient should b increased 10% over that given in Fig. 15.13. Rounding of the upstream corner will also produce a coefficient of discharge as much as 3% greater.

Equation (15.32) reveals a definite relationship for Q as a function of the head, H This type of discharge-measuring device is in the broad class of discharge meters callec

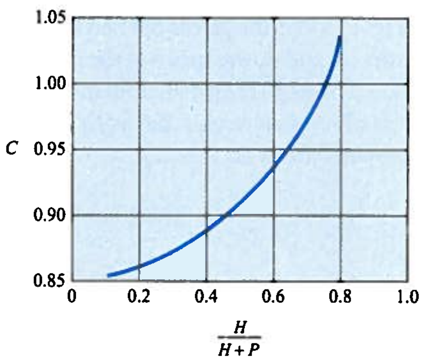


FIGURE 15.13
Discharge coefficient for a broad-crested weir for $0.1 < H/L < 0.8$.

critical-flow flumes. Another very common critical-flow flume is the **venturi flume**, which was developed and calibrated by Parshall (8). Figure 15.14 shows the essential features of the venturi flume. The discharge equation for the venturi flume is in the same form as Eq. (15.32), the only difference being that the experimentally determined coefficient C will have a different value from the C for the broad-crested weir. For more details on the venturi flume, you may refer to Roberson et al. (9), Parshall (8), and Chow (5). The venturi flume is especially useful for discharge measurement in irrigation systems because little head loss is required for its use, and sediment is easily flushed through if the water happens to be silty.

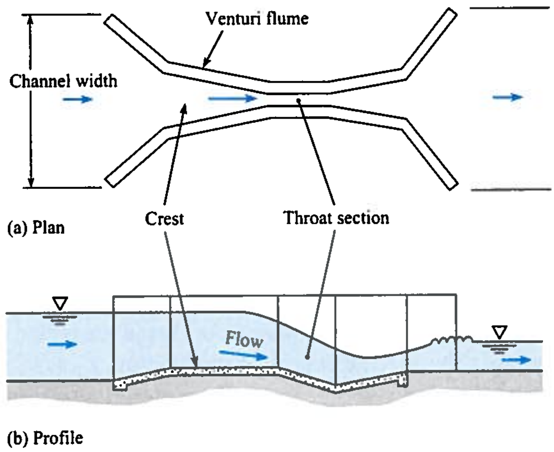


FIGURE 15.14
Flow through a venturi flume.

The depth also passes through a critical stage in channel flow where the slope changes from a mild one to a steep one. A **mild slope** is defined as a slope for which the normal depth y_n is greater than y_c. Likewise, a **steep slope** is one for which $y_n < y_c$. This condition is shown in Fig. 15.15. Note that y_c is the same for both slopes in the figure because y_c is a function of the discharge only. However, normal depth (uniform-flow depth) for the mild upstream channel is greater than critical, whereas the normal depth for the steep downstream channel is less than critical; hence it is obvious that the depth must pass through a critical stage. Experiments show that critical depth occurs a very short distance upstream of the intersection of the two channels.

Another place where critical depth occurs is upstream of a free overfall at the end of a channel with a mild slope Fig. 15.16. Critical depth will occur at a distance of $3y_c$ to $4y_c$ upstream of

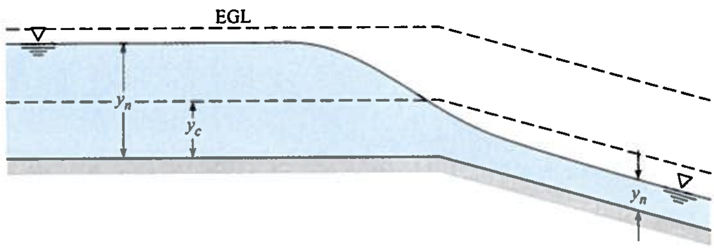


FIGURE 15.15
Critical depth at a bre in grade.

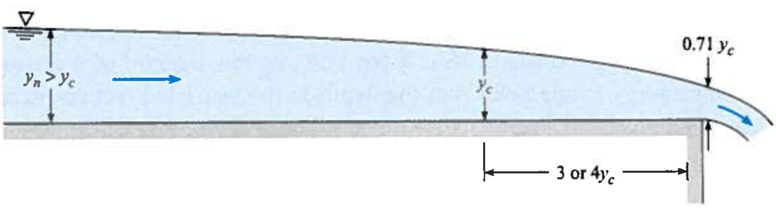


FIGURE 15.16
Critical depth at a free overfall.

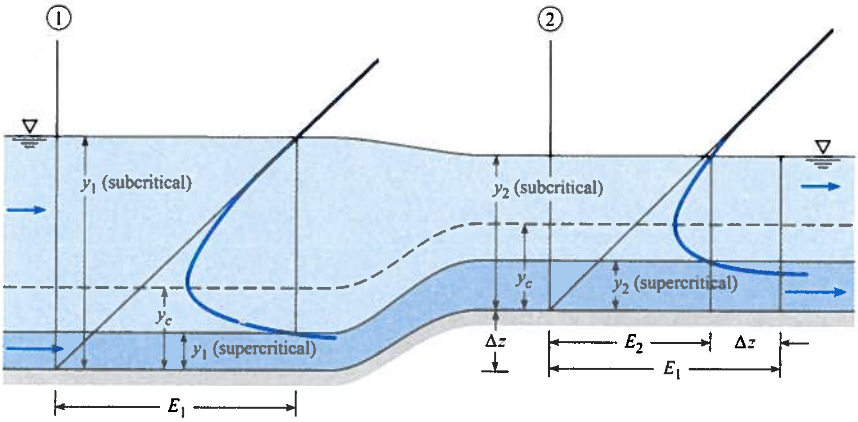
the brink. Such occurrences of critical depth (at a break in grade or at a brink) are useful i computing surface profiles because they provide a point for starting surface-profile calculations.*

Channel Transitions

Whenever a channel's cross-sectional configuration (shape or dimension) changes along it length, the change is termed a **transition**. Concepts previously presented are used to show hov the flow depth changes when the floor of a rectangular channel is increased in elevation o when the width of the channel is decreased. In these developments negligible energy losses ar assumed. First, the case where the floor of the channel is raised (an upstep) is considered. Late in this section, configurations of transitions used for subcritical flow from a rectangular to trapezoidal channel are presented.

Consider the rectangular channel shown in Fig. 15.17, where the floor rises an amount Δz To help in evaluating depth changes, one can use a diagram of specific energy versus deptl which is similar to Fig. 15.8. This diagram is applied both at the section upstream of the transitio and at the section just downstream of the transition. Because the discharge, Q, is the same at botl sections, the same diagram is valid at both sections. As noted in Fig. 15.17, the depth of flow a section 1 can be either large (subcritical) or small (supercritical) if the specific energy E_1 is greate than that required for critical flow. It can also be seen in Fig. 15.17 that when the upstream flov is subcritical, a decrease in depth occurs in the region of the elevated channel bottom. This occur because the specific energy at this section, E_2, is less than that at section 1 by the amount Δz Therefore, the specific-energy diagram indicates that y_2 will be less than y_1. In a similar manne it can be seen that when the upstream flow is supercritical, the depth as well as the actual water surface elevation increases from section 1 to section 2. A further note should be made about th effect on flow depth of a change in bottom-surface elevation. If the channel bottom at section 2 i at an elevation greater than that just sufficient to establish critical flow at section 2, then there i not enough head at section 1 to cause flow to occur over the rise under steady-flow conditions Instead, the water level upstream will rise until it is just sufficient to reestablish steady flow.

FIGURE 15.17
Change in depth with change in bottom elevation of a rectangular channel.



When the channel bottom is kept at the same elevation but the channel is decreased ir width, then the discharge per unit of width between sections 1 and 2 increases, but the specifi energy E remains constant. Thus when utilizing the diagram of q versus depth for the giver specific energy E, one notes that the depth in the restricted section increases if the upstream flow is supercritical and decreases if it is subcritical (see Fig. 15.18).

*The procedure for making these computations starts on p. 588 (water-surface profiles).

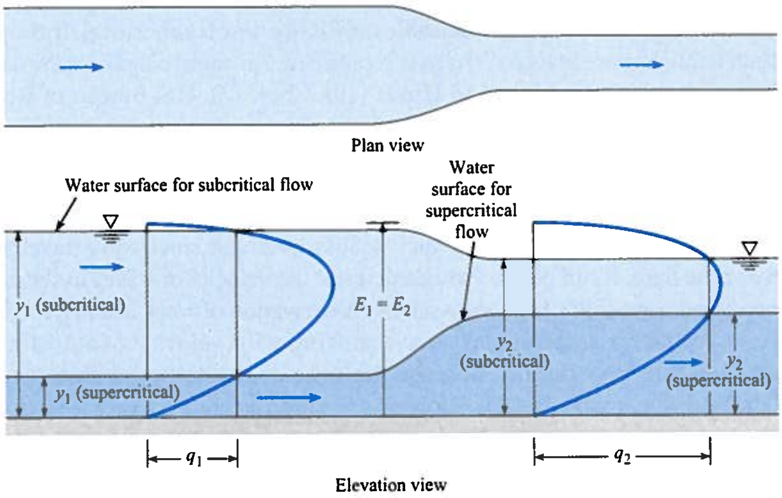


FIGURE 15.18
Change in depth with change in channel widt

The foregoing paragraphs describe gross effects for the simplest transitions. In practice, it is more common to find transitions between a channel of one shape (rectangular cross section, for example) and a channel having a different cross section (trapezoidal, for example). A very simple transition between two such channels consists of two straight vertical walls joining the two channels, as shown by half section in Fig. 15.19.

This type of transition can work, but it will produce excessive head loss because of the abrupt change in cross section and the ensuing separation that will occur. To reduce the head losses, a more gradual type of transition is used. Figure 15.20 is a half section of a transition similar to that of Fig. 15.19, but with the angle θ much greater than 90°. This is called a **wedge transition**.

The **warped-wall** transition shown in Fig. 15.21 will yield even smoother flow than either of the other two, and it will thus have less head loss. In the practical design and analysis of transitions, engineers usually use the complete energy equation, including the kinetic energy factors α_1 and α_2 as well as a head loss term h_L, to define velocity and water-surface

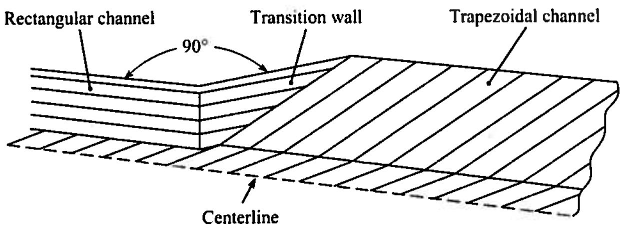


FIGURE 15.19
Simplest type of transiti between a rectangular channel and a trapezo channel.

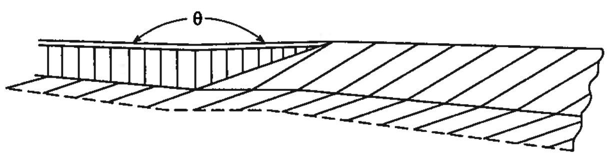


FIGURE 15.20
Half section of a wedg transition.

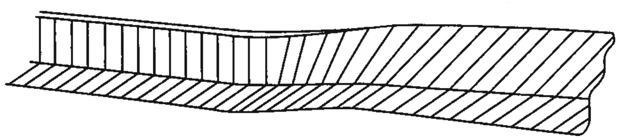


FIGURE 15.21
Half section of a warp wall transition.

elevation through the transition. Analyses of transitions utilizing the one-dimensional for of the energy equation are applicable only if the flow is subcritical. If the flow is supercritic then a much more involved analysis is required. For more details on the design and analysis transitions, you are referred to Hinds (10), Chow (5), U.S. Bureau of Reclamation (11), ar Rouse (12).

Wave Celerity

Wave celerity is the velocity at which an infinitesimally small wave travels relative to the velo ity of the fluid. It can be used to characterize the velocity of waves in the ocean or propagatic of a flood wave following a dam failure. A derivation of wave celerity, c, follows.

Consider a small solitary wave moving with velocity c in a calm body of liquid small depth (Fig. 15.22a). Because the velocity in the liquid changes with time, this is condition of unsteady flow. However, if one referred all velocities to a reference fram moving with the wave, the shape of the wave would be fixed, and the flow would be stead Then the flow is amenable to analysis with the Bernoulli equation. The steady-flow cond tion is shown in Fig. 15.22b. When the Bernoulli equation is written between a point o the surface of the undisturbed fluid and a point at the wave crest, the following equatio results:

$$\frac{c^2}{2g} + y = \frac{V^2}{2g} + y + \Delta y \qquad (15.3$$

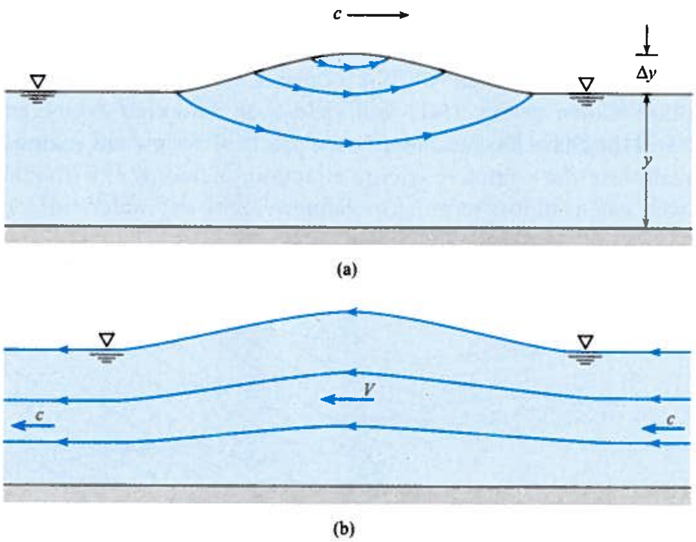


FIGURE 15.22
Solitary wave (exaggerated vertical scale).
(a) Unsteady flow.
(b) Steady flow.

In Eq. (15.33), V is the velocity of the liquid in the section where the crest of the wave located. From the continuity equation, $cy = V(y + \Delta y)$. Hence

$$V = \frac{cy}{y + \Delta y}$$

and

$$V^2 = \frac{c^2 y^2}{(y + \Delta y)^2} \qquad (15.34$$

When Eq. (15.34) is substituted into Eq. (15.33), it yields

$$\frac{c^2}{2g} + y = \frac{c^2 y^2}{2g[y^2 + 2y\Delta y + (\Delta y)^2]} + y + \Delta y \qquad (15.35)$$

Solving Eq. (15.35) for c after discarding terms with $(\Delta y)^2$, assuming an infinitesimally small wave, yields the **wave celerity equation**

$$c = \sqrt{gy} \qquad (15.36)$$

It has thus been shown that the speed of a small solitary wave is equal to the square root of the product of the depth and g.

15.6 Hydraulic Jump

Occurrence of the Hydraulic Jump

An interesting and important case of rapidly varied flow is the hydraulic jump. A **hydraulic jump** occurs when the flow is supercritical in an upstream section of a channel and is then forced to become subcritical in a downstream section (the change in depth can be forced by a sill in the downstream part of the channel or just by the prevailing depth in the stream further downstream), resulting in an abrupt increase in depth, and considerable energy loss. Hydraulic jumps (Fig. 15.23) are often considered in the design of open channels and spillways of dams. If a channel is designed to carry water at supercritical velocities, the designer must be certain that the flow will not become subcritical prematurely. If it did, overtopping of the channel walls would undoubtedly occur, with consequent failure of the structure. Because the energy loss in the hydraulic jump is initially not known, the energy equation is not a suitable tool for analysis of the velocity-depth relationships. Because there is a significant difference in hydrostatic head on both sides of the equation causing opposing pressure forces, the momentum equation can be applied to the problem, as developed in the following sections.

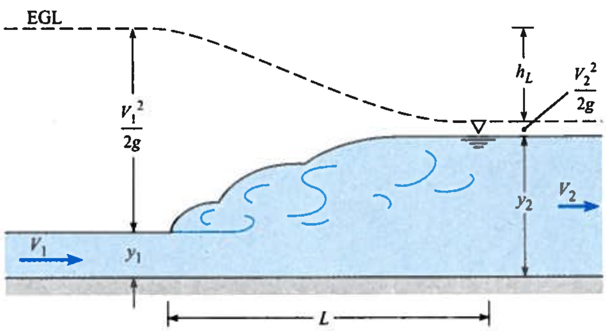
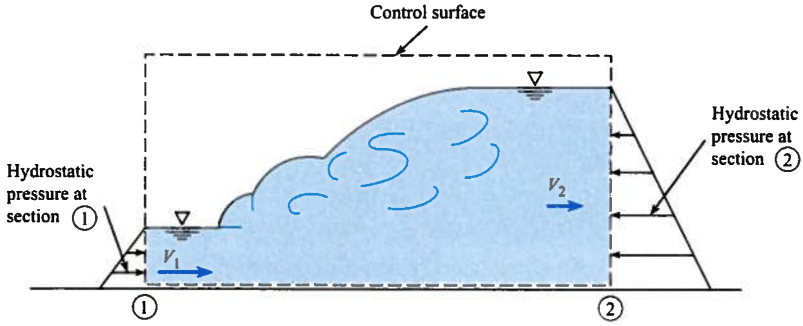


FIGURE 15.23 Definition sketch for the hydraulic jump.

Derivation of Depth Relationships in Hydraulic Jumps

Consider flow as shown in Fig. 15.23. Here it is assumed that uniform flow occurs both upstream and downstream of the jump and that the resistance of the channel bottom over the relatively short distance L is negligible. The derivation is for a horizontal channel, but experiments show that the results of the derivation will apply to all channels of moderate slope

FIGURE 15.24
Control-volume analysis for the hydraulic jump.



($S_0 < 0.02$). The derivation is started by applying the momentum equation in the x-directic to the control volume shown in Fig. 15.24:

$$\sum F_x = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

The forces are the hydrostatic forces on each end of the system; thus the following is obtaine

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho V_2 A_2 V_2 - \rho V_1 A_1 V_1$$

or

$$\bar{p}_1 A_1 + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2 \quad \text{(15.3}$$

In Eq. (15.37), $\bar{p}_1$ and $\bar{p}_2$ are the pressures at the centroids of the respective areas A_1 and A_2.

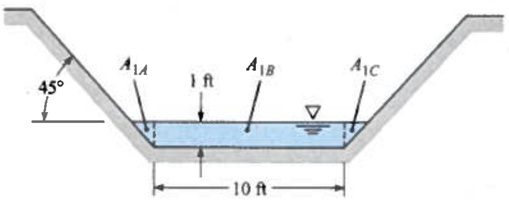
A representative problem (e.g., Example 15.10) is to determine the downstream depth given the discharge and upstream depth. The left-hand side of Eq. (15.37) would be know because V, A, and p are all functions of y and Q, and the right-hand side is a function of therefore, y_2 can be determined.

EXAMPLE 15.10

Calculating Downstream Depth for a Hydraulic Jump

Problem Statement

Water flows in a trapezoidal channel at a rate of 300 cfs. The channel has a bottom width of 10 ft and side slopes of 1 vertical to 1 horizontal. If a hydraulic jump is forced to occur where the upstream depth is 1.0 ft, what will be the downstream depth and velocity? What are the values of Fr_1 and Fr_2?



Define the Situation

A hydraulic jump is forced in a trapezoidal channel.

Properties: Water (50°F), Table A.5:

$\gamma = 62.4$ lbf/ft^3, and $\rho = 1.94$ slugs/ft^3.

State the Goal

1. Downstream depth and velocity
2. Values of Fr_1 and Fr_2

Generate Ideas and Make a Plan

1. Find cross section, velocity, and hydraulic depth in the upstream section.
2. Find pressure in the upstream section to use for left-hand side of Eq. (15.37).
3. Use channel geometry information to solve for y_2 in right-hand side of Eq. (15.37).
4. Use Eq. (15.2) to solve for the Froude number at both sections.

Take Action (Execute the Plan)

1. By inspection, for the upstream section, the cross-sectional flow area is 11 ft^2.

   Therefore, the mean velocity is $V_1 = Q/A_1 = 27.3$ ft/s.

   The hydraulic depth is $D_1 = A_1/T_1 = 11\text{ ft}^2/12\text{ ft} = 0.9167$ ft.

2. The location of the centroid ($\bar{y}$) of the area A_1 can be obtained by taking moments of the subareas about the water surface (see example sketch).

$A_1\bar{y}_1 = A_{1A} \times 0.333 \text{ ft} + A_{1B} \times 0.500 \text{ ft} + A_{1C} \times 0.333 \text{ ft}$

$(11 \text{ ft}^2)\bar{y}_1 = (0.333 \text{ ft})(0.500 \text{ ft}^2 \times 2) + (0.50 \text{ ft})(10.00 \text{ ft}^2)$

$\bar{y} = 0.485 \text{ ft}$

Pressure $p_1 = 62.4 \text{ lbf/ft}^3 \times 0.485 \text{ ft} = 30.26 \text{ lbf/ft}^2$. Therefore,

$$30.26 \times 11 + 1.94 \times 300 \times 27.3 = \bar{p}_2A_2 + \rho QV_2$$

3. Using right-hand side of Eq. (15.37), solve for y_2.

$$\bar{p}_2A_2 + \rho QV_2 = 16{,}221 \text{ lbf}$$

$$\gamma\bar{y}_2A_2 + \frac{\rho Q^2}{A_2} = 16{,}221$$

$$\bar{y}_2 = \frac{\sum A_i y_i}{A_2} = \frac{By_2^2/2 + y_2^3/3}{A_2}$$

Using $B = 10$ ft, $Q = 300 \text{ ft}^3\text{/s}$, and material properties assumed earlier,

$$y_2 = \boxed{5.75 \text{ ft}}$$

4. Froude numbers at both sections are

$$\text{Fr}_1 = \frac{V_1}{\sqrt{gD_1}} = \frac{27.3 \text{ ft/s}}{\sqrt{32.2 \text{ ft/s}^2 \times 0.9167 \text{ ft}}} = \boxed{5.02}$$

$$V_2 = \frac{Q}{A_2} = \frac{300}{57.5 + 5.75^2} = 3.31 \text{ ft/s}$$

$$D_2 = \frac{A_2}{T_2} = \frac{90.56}{21.5} = 4.21 \text{ ft}$$

$$\text{Fr}_2 = \frac{V}{\sqrt{gD}} = \frac{3.31}{\sqrt{32.2 \times 4.21}} = \boxed{0.284}$$

Hydraulic Jump in Rectangular Channels

If one writes Eq. (15.37) for a unit width of a rectangular channel where $\bar{p}_1 = \gamma y_1/2$, $\bar{p}_2 = \gamma y_2/2$, $Q = q$, $A_1 = y_1$, and $A_2 = y_2$, this will yield

$$\gamma\frac{y_1^2}{2} + \rho qV_1 = \gamma\frac{y_2^2}{2} + \rho qV_2 \qquad \textbf{(15.38a)}$$

but $q = Vy$, so Eq. (15.38a) can be rewritten as

$$\frac{\gamma}{2}(y_1^2 - y_2^2) = \frac{\gamma}{g}(V_2^2y_2 - V_1^2y_1) \qquad \textbf{(15.38b)}$$

The preceding equation can be further manipulated to yield

$$\frac{2V_1^2}{gy_1} = \left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} \qquad \textbf{(15.39)}$$

The term on the left-hand side of Eq. (15.39) will be recognized as twice Fr_1^2. Hence Eq. (15.39) is written as

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\text{Fr}_1^2 = 0 \qquad \textbf{(15.40)}$$

By use of the quadratic formula, it is easy to solve for y_2/y_1 in terms of the upstream Froude number. This yields an equation for **depth ratio** across a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{1 + 8\text{Fr}_1^2} - 1) \qquad \textbf{(15.41)}$$

or

$$y_2 = \frac{y_1}{2}(\sqrt{1 + 8\text{Fr}_1^2} - 1) \qquad \textbf{(15.42)}$$

The other solution of Eq. (15.40) gives a negative downstream depth, which is not physically possible. Hence the downstream depth is expressed in terms of the upstream depth and the upstream Froude number. In Eqs. (15.41) and (15.42), the depths y_1 and y_2 are said to be **conjugate** or **sequent** (both terms are in common use) to each other, in contrast to the alternate depths obtained from the energy equation. Numerous experiments show that the relation represented by Eqs. (15.41) and (15.42) is valid over a wide range of Froude numbers.

Although no theory has been developed to predict the length of a hydraulic jump, experiments [see Chow (5)] show that the relative length of the jump, L/y_2, is approximately 6 for Fr_1 ranging from 4 to 18.

Head Loss in a Hydraulic Jump

In addition to determining the geometric characteristics of the hydraulic jump, it is often desirable to determine the head loss produced by it. This is obtained by comparing the specific energy before the jump to that after the jump, the head loss being the difference between the two specific energies. It can be shown that the head loss for a jump in a rectangular channel is

$$h_L = \frac{(y_2 - y_1)^3}{4y_1 y_2} \tag{15.43}$$

For more information on the hydraulic jump, see Chow (5). The following example shows that Eq. (15.43) yields a magnitude that equals the difference between the specific energies at the two ends of the hydraulic jump.

EXAMPLE 15.11

Calculating Head Loss in a Hydraulic Jump

Problem Statement

Water flows in a rectangular channel at a depth of 30 cm with a velocity of 16 m/s, as shown in the sketch that follows. If a downstream sill (not shown) forces a hydraulic jump, what will be the depth and velocity downstream of the jump? What head loss is produced by the jump?

Define the Situation

A hydraulic jump is occurring in a rectangular channel.

State the Goal

- Calculate downstream depth and velocity.
- Calculate head loss produced by the jump.

Generate Ideas and Make a Plan

1. To calculate h_L using Eq. (15.43), one calculates y_2 from the depth ratio equation (Eq. 15.42). This requires Fr_1.
2. Check validity of head loss by comparing to $E_1 - E_2$.

Take Action (Execute the Plan)

1. Calculate Fr_1, y_2, V_2, and h_L from Eqs. (Eq. 15.42) and (15.43).

$$Fr_1 = \frac{V}{\sqrt{gy_1}} = \frac{16}{\sqrt{9.81\,(0.30)}} = 9.33$$

$$y_2 = \frac{0.30}{2}\left[\sqrt{1 + 8(9.33)^2} - 1\right] = \boxed{3.81 \text{ m}}$$

$$V_2 = \frac{q}{y_2} = \frac{(16 \text{ m/s})(0.30 \text{ m})}{3.81 \text{ m}} = \boxed{1.26 \text{ m/s}}$$

$$h_L = \frac{(3.81 - 0.30)^3}{4(0.30)(3.81)} = \boxed{9.46 \text{ m}}$$

2. Compare the head loss to $E_1 - E_2$.

$$h_L = \left(0.30 + \frac{16^2}{2 \times 9.81}\right) - \left(3.81 + \frac{1.26^2}{2 \times 9.81}\right) = 9.46 \text{ m}$$

The value is the same, so $\boxed{\text{validity of } h_L \text{ equation is verified.}}$

Use of Hydraulic Jump on Downstream End of Dam Spillway

Previously it was shown that the transition from supercritical to subcritical flow produces a hydraulic jump, and that the relative height of the jump (y_2/y_1) is a function of Fr_1. Because flow over the spillway of a dam invariably results in supercritical flow at the lower end of the spillway, and because flow in the channel downstream of a spillway is usually subcritical, it is obvious that a hydraulic jump must form near the base of the spillway (see Fig. 15.25). The downstream portion of the spillway, called the spillway **apron**, must be designed so that the hydraulic jump always forms on the concrete structure itself. If the hydraulic jump were allowed to form beyond the concrete structure, as in Fig. 15.26, severe erosion of the foundation material as a result of the high-velocity supercritical flow could undermine the dam and cause its complete failure. One way to solve this problem might be to incorporate a long, sloping apron into the design of the spillway, as shown in Fig. 15.27. A design like this would work

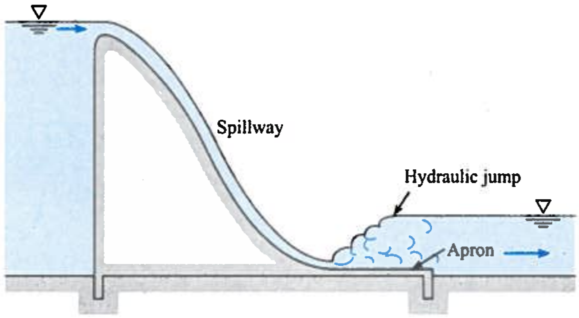


FIGURE 15.25
Spillway of dam and hydraulic jump.

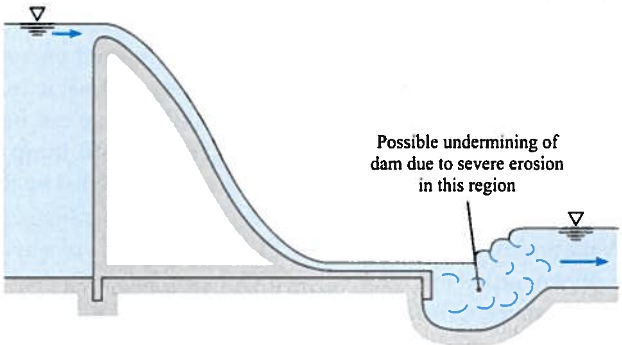


FIGURE 15.26
Hydraulic jump occurri
downstream of spillwa
apron.

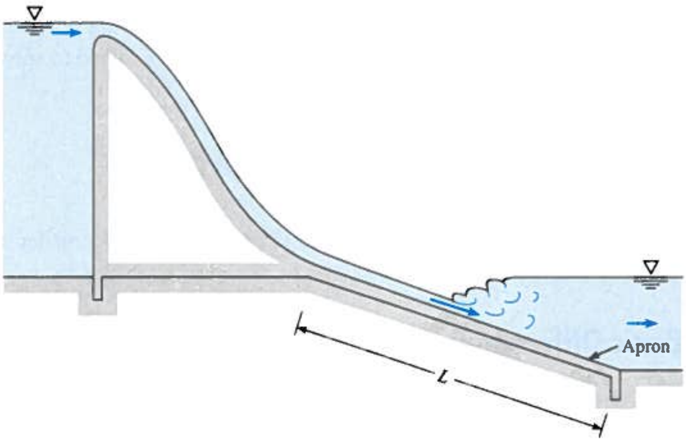


FIGURE 15.27
Long sloping apron.

very satisfactorily from the hydraulics point of view. For all combinations of Fr_1 and water surface elevation in the downstream channel, the jump would always form on the sloping apron. However, its main drawback is cost of construction. Construction costs will be reduced as the length, L, of the stilling basin is reduced. Much research has been devoted to the design of stilling basins that will operate properly for all upstream and downstream conditions and yet be relatively short to reduce construction cost. Research by the U.S. Bureau of Reclamation (13) has resulted in sets of standard designs that can be used. These designs include sills, baffle piers, and chute blocks, as shown in Fig. 15.28.

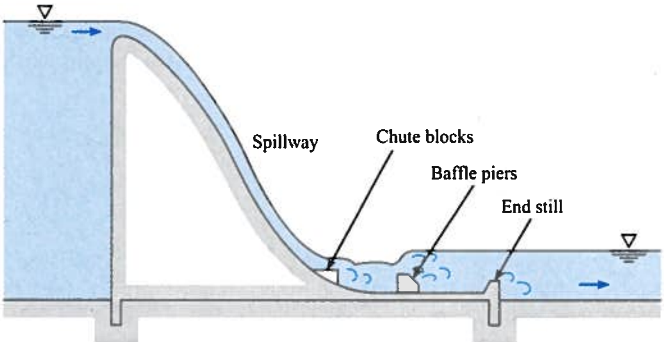


FIGURE 15.28
Spillway with stilling basin Type III as recommended by the USBR (13).

Naturally Occurring Hydraulic Jumps

Hydraulic jumps can occur naturally in creeks and rivers, providing spectacular standing waves, called rollers. Kayakers and white-water rafters must exercise considerable skill when navigating hydraulic jumps because the significant energy loss that occurs over a short distance can be dangerous, potentially engulfing the boat in turbulence. A special case of hydraulic jump, referred to as a submerged hydraulic jump, can be deadly to white-water enthusiasts because it is not easy to see. A **submerged hydraulic jump** occurs when the downstream depth predicted by conservation of momentum is exceeded by the tailwater elevation, and the jump cannot move upstream in response to this disequilibrium because of a buried obstacle [see Valle and Pasternak (14)]. Thus, the visual markers of a hydraulic jump, particularly the rolling waves depicted in Figs. 15.23 and 15.24, are hidden.

A **surge**, or **tidal bore**, is a moving hydraulic jump that may occur for a high tide entering a bay or river mouth. Tides are generally low enough that the waves they produce are smooth and nondestructive. However, in some parts of the world the tides are so high that their entry into shallow bays or mouths of rivers causes a surge to form. Surges may be very hazardous to small boats. The same analytical methods used for the jump can be used to solve for the speed of the surge.

15.7 Gradually Varied Flow

For gradually varied flow, channel resistance is a significant factor in the flow process. Therefore, the energy equation is invoked by comparing S_0 and S_f.

Basic Differential Equation for Gradually Varied Flow

There are a number of cases of open-channel flow in which the change in water-surface profile is so gradual that it is possible to integrate the relevant differential equation from one section