15 FLOW IN OPEN CHANNELS

FIGURE 15.1

Aerial view of the California Aqueduct at the southwest end of the Tehachapi Mountains. (Macduff Everton/The Image Bank/Getty Images).

Chapter Road Map

The flow of water in open channels can be observe in aqueducts, rivers, flumes, irrigation ditches, and other contexts. Although these contexts are quite different, a small set of concepts and a few equatic generalize to most applications of open channel flc These ideas are introduced in this chapter.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define an open channel. Define uniform flow and nonuniform flow. (§15.1)
- Define the Froude number, the hydraulic radius, and the Reynolds number. List the criteria for laminar and turbulen flow. (§15.1)
- For steady flow, explain the physics of the energy equation and also explain the corresponding HGL and EGL. (§15.2)
- For uniform flow, calculate flow rate with the (a) Darcy-Weisbach approach, (b) Chezy equation, and (c) Manning equation. (§15.3)
- Define and explain the best hydraulic section. (§15.3)
- Describe and compare rapidly varied flow and graduall varied flow. (§15.4)
- Describe critical depth, specific energy, supercritical flow and subcritical flow. (§15.5)
- Describe a hydraulic jump. Perform calculations. (§15.6)
- Describe the factors used to classify surface profiles that occur in gradually varied flow. (§15.7)

An **open channel** is one in which a liquid flows with a free surface. A **free surface** means that the liquid surface is exposed to the atmosphere. Examples of open channels are natural creeks and rivers, artificial channels such as irrigation ditches and canals, and pipelines or sewers flowing less than full. In most cases, water or wastewater is the flowing liquid.

15.1 Description of Open-Channel Flow

Flow in an open channel is described as uniform or nonuniform, as distinguished in Fig. 15.2. As defined in Chapter 4, **uniform flow** means that the velocity is constant along a streamline, which in open-channel flow means that depth and cross section are constant along the length of a channel. The depth for uniform-flow conditions is called **normal depth** and is designated by y_n. For **nonuniform flow**, the velocity changes from section to section along the channel, thus one observes changes in depth. The velocity change may be due to a change in channel configuration, such as a bend, change in cross-sectional shape, or change in channel slope. For example, Fig. 15.2 shows steady flow over a spillway of constant width, where the water must flow progressively faster as it goes over the brink of the spillway (from A to B), caused by the suddenly steeper slope. The faster velocity requires a smaller depth, in accordance with conservation of mass (continuity). From reach B to C, the flow is uniform because the velocity, and thus depth, are constant. After reach C the abrupt flattening of channel slope requires the velocity to suddenly, and turbulently, slow down. Thus there is a deeper depth downstream of C than in reach B to C.

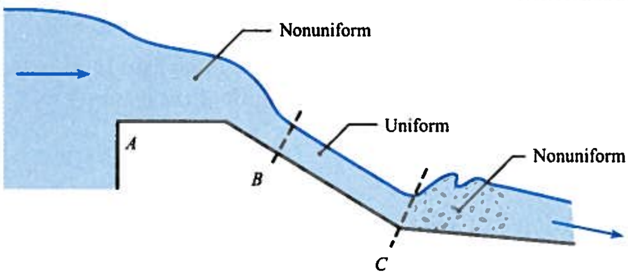


FIGURE 15.2
Distinguishing uniform and nonuniform flow. T example shows steady flow over a spillway, su as the emergency over channel of a dam.

The most complicated open-channel flow is unsteady nonuniform flow. An example of this is a breaking wave on a sloping beach. Theory and analysis of unsteady nonuniform flow are reserved for more advanced courses.

Dimensional Analysis in Open-Channel Flow

Open-channel flow results from gravity moving water from higher to lower elevations and is impeded by friction forces caused by the roughness of the channel. Thus the functional equation $Q = f(\mu, \rho, V, L)$ and dimensional analysis leads to two important π-groups: the Froude number and the Reynolds number. The Froude number squared is the ratio of kinetic force to gravity force:

$$\mathrm{Fr}^2 = \frac{\text{kinetic force}}{\text{gravity force}} = \frac{\rho L^2 V^2}{\gamma L^3} = \frac{V^2}{L\gamma/\rho} \tag{15.1}$$

$$\mathrm{Fr} = \frac{V}{\sqrt{gL}} \tag{15.2}$$

The Froude number is importan if the gravitational force influences the direction of flow, such as in flow over a spillway, or he formation of surface waves. However, it is unimportant when gravity causes only a hydro atic pressure distribution, such as in a closed conduit.

The use of Reynolds number for determining whether the flow in open channels will laminar or turbulent depends on the **hydraulic radius**, given by

$$R_h = \frac{A}{P}$$ (15

where A is the cross-sectional area of flow and P is the wetted perimeter. The characteris length R_h is analogous to diameter D in pipe flow. Recall that for pipe flow (Chapter 10), if t Reynolds number ($VD\rho/\mu = VD/\nu$) is less than 2000, the flow will be laminar, and if it greater than about 3000, one can expect the flow to be turbulent. The Reynolds number cri rion for open-channel flow would be 2000 if one replaced D in the Reynolds number by 4, where R_h is the hydraulic radius. For this definition of Reynolds number, laminar flow wou occur in open channels if $V(4R_h)/\nu < 2000$.

However, the standard convention in open-channel flow analysis is to define the Reynol number as

$$\text{Re} = \frac{VR_h}{\nu}$$ (15.

Therefore, in open channels, if the Reynolds number is less than 500, the flow is laminar, a if Re is greater than about 750, one can expect to have turbulent flow. A brief analysis of tl turbulent criterion (see Example 15.1) will show that water flow in channels will usually turbulent unless the velocity and/or the depth is very small.

It should be noted that for rectangular channels (see Fig. 15.3), the hydraulic radius is

$$R_h = \frac{A}{P} = \frac{By}{B + 2y}$$ (15,

For a wide and shallow channel, $B \gg y$ and Eq. (15.5) reduces to $R_h \approx y$ which means that t hydraulic radius approaches the depth of the channel.

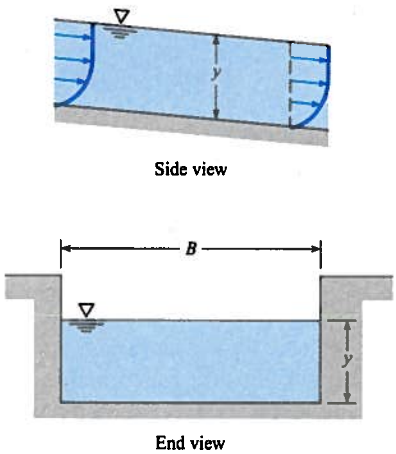
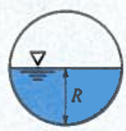


FIGURE 15.3
Open-channel relations.

✔CHECKPOINT PROBLEM 15.1

What is the hydraulic radius for this channel?



a. $\pi R/(4 + 2\pi)$
b. $\pi R/(2 + \pi)$
c. $R/4$
d. $R/2$
e. R

Most open-channel flow problems involve turbulent flow. If one calculates the conditions needed to maintain laminar flow, as in Example 15.1, one sees that laminar flow is uncommon.

EXAMPLE 15.1

Calculating Reynolds Number and Classifying Flow for a Rectangular Open Channel

Problem Statement

Water (60 °F) flows in a 10-ft-wide rectangular channel at a depth of 6 ft. What is the Reynolds number if the mean velocity is 0.1 ft/s? With this velocity, at what maximum depth can one be assured of having laminar flow?

Define the Situation

Water flows in a rectangular channel.

$$B = 10\text{ ft}, y = 6\text{ ft}, V = 0.1\text{ ft/s}.$$

Properties:

Water (60 °F, 1 atm, Table A.5): $\nu = 1.22 \times 10^{-5}\text{ ft}^2/\text{s}$.

State the Goal

1. Re ⬅ Reynolds number
2. y_m(ft) ⬅ Maximum depth for laminar flow

Generate Ideas and Make a Plan

To find Re, apply Eq. (15.4). To find y_m, apply the criteria that laminar flow occurs for Re < 500. The plan is:

1. Calculate hydraulic radius using Eq. (15.5).
2. Calculate Reynolds number using Eq. (15.4).
3. Let Re = 500, solve for R_h, and then solve for y_m.

Take Action (Execute the Plan)

1. Hydraulic radius

$$R_h = \frac{By}{B + 2y} = \frac{(10\text{ ft})(6\text{ ft})}{(10\text{ ft}) + 2(6\text{ ft})} = 2.727\text{ ft}$$

2. Reynolds number

$$\text{Re} = \frac{VR_h}{\nu} = \frac{(0.1\text{ ft/s})(2.727\text{ ft})}{(1.22 \times 10^{-5}\text{ ft}^2/\text{s})} = \boxed{22400}$$

3. Laminar Flow Criteria (Re < 500).

$$\text{Re} = VR_h/\nu = (0.10\text{ ft/s})R_h/(1.22 \times 10^{-5}\text{ ft}^2/\text{s}) = 500$$

$$R_h = (500)(1.22 \times 10^{-5}\text{ ft}^2/\text{s})/(0.10\text{ ft/s}) = 0.061\text{ ft}$$

For a rectangular channel,

$$R_h = (By)/(B + 2y)$$

$$(By)/(B + 2y) = (10y)/(10 + 2y) = 0.061\text{ ft}$$

$$y_m = \boxed{0.062\text{ ft}}$$

Review the Solution and the Process

1. *Knowledge.* Velocity or depth must be very small to yield laminar flow of water in an open channel.
2. *Knowledge.* Depth and hydraulic radius are virtually the same when depth is very small relative to width.

15.2 Energy Equation for Steady Open-Channel Flow

To derive the energy equation for flow in an open channel, begin with Eq. (7.29) and let the pump head and turbine head equal zero: $h_p = h_t = 0$. Equation (7.29) becomes

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L \qquad (15.6)$$

Use Fig. 15.4 to show that

$$\frac{p_1}{\gamma} + z_1 = y_1 + S_0\Delta x \qquad \text{and} \qquad \frac{p_2}{\gamma} + z_2 = y_2$$

where S_0 is the slope of the channel bottom, and y is the depth of flow. Assume the flow in the channel is turbulent, so $\alpha_1 = \alpha_2 \approx 1.0$. Equation (15.6) becomes

$$y_1 + \frac{V_1^2}{2g} + S_0\Delta x = y_2 + \frac{V_2^2}{2g} + h_L \qquad (15.7)$$

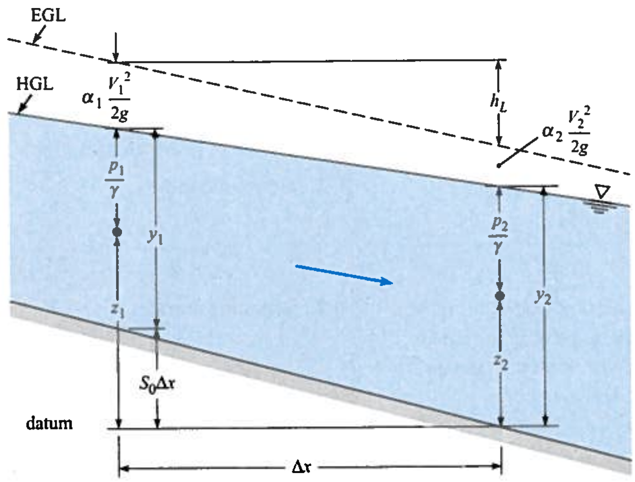


FIGURE 15.4 Definition sketch for flow in open channels.

In addition to the foregoing assumptions, Eq. (15.7) also requires that the channel have uniform cross section, and the flow be steady.

15.3 Steady Uniform Flow

Uniform flow requires that velocity be constant in the flow direction, so the shape of the chan nel and the depth of fluid is the same from section to section. Consideration of the foregoir slope equations shows that for uniform flow, the slope of the HGL will be the same as the chan nel slope because the velocity and depth are the same in both sections. The HGL, and thus tl slope of the water surface, is controlled by head loss. If one restates the Darcy-Weisbach equ tion introduced in Chapter 10 with D replaced by $4R_h$, the head loss is

$$h_f = \frac{fL}{4R_h}\frac{V^2}{2g} \quad \text{or} \quad \frac{h_f}{L} = \frac{f}{4R_h}\frac{V^2}{2g} \qquad (15.$$

From Fig. 15.4, S_0 = [slope of EGL], which is a function of the head loss, so $S_0 = (h_f/L)$, yiel ing the following equation for velocity:

$$V = \sqrt{\frac{8g}{f}R_h S_0} \qquad (15.$$

To solve Eq. (15.9) for velocity, the friction factor f can be found from the Moody diagra (Fig. 10.14) and can then be used to solve iteratively for the velocity for a given uniform-flc condition. This is demonstrated in Example 15.2.

EXAMPLE 15.2

Applying the Darcy-Weisbach Equation to Find the Flow Rate in a Rectangular Open Channel

Problem Statement

Estimate the discharge of water that a concrete channel 10 ft wide can carry if the depth of flow is 6 ft and the slope of the channel is 0.0016.

Define the Situation

- Water flows in a rectangular channel.
- $B = 10$ ft, $y = 6$ ft, $S_0 = 0.0016$.

Assumptions. Uniform flow

Properties.

- Water (60 °F, 1 atm, Table A.5): $\nu = 1.22 \times 10^{-5}$ ft²/s
- Concrete (Table 10.4): $k_s \approx 0.005$ ft

State the Goal

$Q(\text{ft}^3/\text{s})$ ⬅ Discharge in the channel

Generate Ideas and Make a Plan

Because the goal is Q, apply the flow rate equation

$$Q = VA \qquad \text{(a)}$$

To find V in Eq. (a), apply Eq. (15.9):

$$V = \sqrt{\frac{8g}{f} R_h S_0} \qquad \text{(b)}$$

To find R_h in Eq. (b), apply Eq. (15.5).

$$R_h = \frac{By}{B + 2y} = \frac{(10 \text{ ft})(6 \text{ ft})}{(10 \text{ ft}) + 2(6 \text{ ft})} = 2.727 \text{ ft} \qquad \text{(c)}$$

To find f in Eq. (b), use an iterative approach with the Moody diagram. This is a Case 2 problem from Chapter 10. The plan is:

1. Calculate relative roughness. Then, guess a value of f.
2. Calculate V using Eq. (b).
3. Calculate Reynolds number, then look up f on the Moody diagram and compare to the guess in step 1. If needed, go back to step 2.
4. Calculate Q using Eq. (a).

Take Action (Execute the Plan)

1. Calculate relative roughness.

$$\frac{k_s}{4R_h} = \frac{0.005 \text{ ft}}{4(60 \text{ ft}^2/22 \text{ ft})} = \frac{0.005 \text{ ft}}{4(2.73 \text{ ft})} = 0.00046$$

Use value of $k_s/4R_h = 0.00046$ as a guide to estimate $f = 0.016$.

2. Calculate V based on guess of f.

$$V = \sqrt{\frac{8(32.2 \text{ ft/s}^2)(2.73 \text{ ft})(0.0016)}{0.016}}$$
$$= \sqrt{70.6 \text{ ft}^2/\text{s}^2} = 8.39 \text{ ft/s}$$

3. Calculate a new value of f based on V from step 2.

$$\text{Re} = V\frac{4R_h}{\nu} = \frac{8.39 \text{ ft/s}(10.9 \text{ ft})}{1.2(10^{-5} \text{ ft}^2/\text{s})} = 7.62 \times 10^6$$

Using this new value of Re and $k_s/4R_h = 0.00046$, read f as 0.016. This value of f is the same as the previous estimate. Thus, we conclude that

$$V = 8.39 \text{ ft/s}$$

4. Flow rate equation

$$Q = VA = 8.39 \text{ ft/s}(60 \text{ ft}^2) = \boxed{503 \text{ cfs}}$$

Review the Solution and the Process

1. *Notice.* The approach to solving this problem parallels the approach presented in Chapter 10 for solving problems th involve flow in conduits.
2. *Knowledge.* Hydraulic diameter is four times the hydraulic radius. This is why the relative roughness formula in step is $k_s/(4R_h)$.

Rock-Bedded Channels

For rock-bedded channels such as those in some natural streams or unlined canals, the larger rocks produce most of the resistance to flow, and essentially none of this resistance is due to viscous effects. Thus, the friction factor is independent of the Reynolds number. This is analogous to the fully rough region of the Moody diagram for pipe flow. For a rock-bedded channel, Limerinos (1) has shown that the resistance coefficient f can be given in terms of the size of rock in the stream bed as

$$f = \frac{1}{\left[1.2 + 2.03 \log\left(\frac{R_h}{d_{84}}\right)\right]^2} \qquad \textbf{(15.10)}$$

where d_{84} is a measure of the rock size.*

*Most river-worn rocks are somewhat elliptical in shape. Limerinos (1) showed that the intermediate dimension d_{84} correlates best with f. The d_{84} refers to the size of rock (intermediate dimension) for which 84% of the rocks in the random sample are smaller than the d_{84} size. Details for choosing the sample are given by Wolman (3).

EXAMPLE 15.3

Resistance Coefficient for Boulders

Problem Statement

Determine the value of the resistance coefficient, f, for a natural rock-bedded channel that is 100 ft wide and has an average depth of 4.3 ft. The d_{84} size of boulders in the stream bed is 0.72 ft.

Define the Situation

A natural channel is lined with boulders.

State the Goal

Find the friction factor, f.

Generate Ideas and Make a Plan

1. Since the channel is wide, approximate R_h as the depth of the channel.
2. Use Eq. (15.10) to find f on the basis of the d_{84} boulder size.

Take Action (Execute the Plan)

1. R_h is 4.3 ft.
2. Evaluate f.

$$f = \frac{1}{\left[1.2 + 2.03\log\left(\frac{4.3}{0.72}\right)\right]^2} = \boxed{0.130}$$

The Chezy Equation

Leaders in open-channel research have recommended the use of the methods already pr sented (involving the Reynolds number and relative roughness k_s) for channel design (: However, many engineers continue to use two traditional methods, the Chezy equation ar the Manning equation.

As noted earlier, the depth in uniform flow, called normal depth, y_n, is constant. Cons quently, h_f/L is the slope S_0 of the channel, and Eq. (15.8) can be written as

$$R_hS_0 = \frac{f}{8g}V^2$$

or

$$V = C\sqrt{R_hS_0} \quad \text{(15.1}$$

where

$$C = \sqrt{8g/f} \quad \text{(15.1}$$

Because $Q = VA$, the discharge in a channel is given by

$$Q = CA\sqrt{R_hS_0} \quad \text{(15.1}$$

This equation is known as the Chezy equation after a French engineer of that name. For pra tical application, the coefficient C must be determined. One way to determine C is by knowir an acceptable value of the friction factor f and using Eq. (15.12).

The Manning Equation

The second, and more common, way to determine C in the SI system of units is given as:

$$C = \frac{R_h^{1/6}}{n} \quad \text{(15.1}$$

where n is a resistance coefficient called Manning's n, which has different values for differe types of boundary roughness. When this expression for C is inserted into Eq. (15.13), the resu is a common form of the discharge equation for uniform flow in open channels for SI units, referred to as the Manning equation:

$$Q = \frac{1.0}{n} A R_h^{2/3} S_0^{1/2} \tag{15.15}$$

Table 15.1 gives values of n for various types of boundary surfaces. The major limitation of this approach is that the viscous or relative-roughness effects are not present in the design formula. Hence, application outside the range of normal-sized channels carrying water is not recommended.

Manning Equation—Traditional System of Units

The form of the Manning equation depends on the system of units because Manning's equation is not dimensionally homogeneous. In Eq. (15.15), notice that the primary dimensions on the left side of the equation are L^3/T and the primary dimensions on the right side are $L^{8/3}$.

To convert the Manning equation from SI to traditional units, one must apply a factor equal to 1.49 if the same value of n is used in the two systems. Thus in the traditional system the discharge equation using Manning's n is

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2} \tag{15.16}$$

TABLE 15.1 Typical Values of Roughness Coefficient, Manning's n

Lined Canals	n
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, troweled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Corrugated metal	0.024
Unlined Canals	
Earth, straight and uniform	0.023
Earth, winding and weedy banks	0.035
Cut in rock, straight and uniform	0.030
Cut in rock, jagged and irregular	0.045
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight, with some grass	0.026
Earth, winding, no vegetation	0.030
Earth, winding, weedy banks	0.050
Earth, very weedy and overgrown	0.080

In Example 15.4, a value for Manning's n is calculated from known information about channel and compared to tabulated values for n in Table 15.1.

EXAMPLE 15.4

Apply the Chezy Equation to find Manning's Value of n for Flow in a Channel

Problem Statement

If a channel with boulders has a slope of 0.0030, is 100 ft wide, has an average depth of 4.3 ft, and is known to have a friction factor of 0.130, what is the discharge in the channel, and what is the numerical value of Manning's n for this channel?

Define the Situation

Water flows in an channel with boulders

$$S_0 = 0.003, B = 100 \text{ ft}, y = 4.3 \text{ ft}, f = 0.13$$

Assumptions. $R_h \approx y = 4.3$ ft (because the channel is wide).

State the Goal

1. Q(cfs) ⬅ Discharge in the channel
2. n ⬅ Manning's n

Generate Ideas and Make a Plan

To find Q, apply the flow rate equation

$$Q = VA \quad \text{(a)}$$

To find V in Eq. (a), apply Eq. (15.9):

$$V = \sqrt{\frac{8g}{f} R_h S_0} \quad \text{(b)}$$

To find n, apply Eq. (15.16):

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2} \quad \text{(c)}$$

Because Eqs. (a) to (c) form a set of three equations with three unknowns, they can be solved. The plan is:

1. Calculate V using Eq. (b).
2. Calculate Q using Eq. (a).
3. Calculate n using Eq. (c).

Take Action (Execute the Plan)

1. Velocity

$$V = \left[\sqrt{\frac{(8)(32.2 \text{ ft/s}^2)}{0.130}}\right]\left[\sqrt{(4.3 \text{ ft})(0.0030)}\right] = 5.06 \text{ ft/s}$$

2. Flow Rate Equation

$$Q = VA = (5.06 \text{ ft/s})(100 \times 4.3 \text{ ft}^2) = \boxed{2180 \text{ cfs}}$$

3. Manning's n (traditional units).

$$n = \frac{1.49}{Q} A R_h^{2/3} S_0^{1/2}$$

$$n = \left(\frac{1.49}{2176 \text{ ft}^3/\text{s}}\right)(100 \times 4.3 \text{ ft}^2)(4.3 \text{ ft})^{2/3}(0.003)^{1/2}$$

$$n = \boxed{0.0426}$$

Review the Solution and the Process

1. *Validation.* This calculated value of n is within the range of typical values in Table 15.1 under the category of "Unlined Canals, Cut in rock."
2. *Notice.* For uniform flow, f in the Darcy-Weisbach equation can be related to Manning's n (as shown by this example).

In Example 15.5 the Chezy equation for traditional units is used to compute discharge.

EXAMPLE 15.5

Discharge Using Chezy Equation

Problem Statement

Using the Chezy equation with Manning's n, compute the discharge in a concrete channel 10 ft wide if the depth of flow is 6 ft and the slope of the channel is 0.0016.

Define the Situation

Water flows in a concrete channel. Width = 10 ft. Depth = 6 ft. Slope = 0.0016.

Properties: n = 0.015 for concrete channel (Table 15.1).

State the Goal

Find the discharge, Q.

Generate Ideas and Make a Plan

Use the Chezy equation for traditional units, Eq. (15.16).

Take Action (Execute the Plan)

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2}$$

$$R_h = \frac{60}{22} = 2.73 \text{ ft} \quad \text{and} \quad R_h^{2/3} = 1.95$$

$$S_0^{1/2} = 0.04 \quad \text{and} \quad A = 60 \text{ ft}^2$$

$$Q = \frac{1.49}{0.015}(60)(1.96)(0.04) = \boxed{467 \text{ cfs}}$$

The two results (Examples 15.5 and 15.5) are within expected engineering accuracy for this type of problem. For a more complete discussion of the historical development of Manning's equation and the choice of n values for use in design or analysis, refer to Yen (4) and Chow (5).

Best Hydraulic Section for Uniform Flow

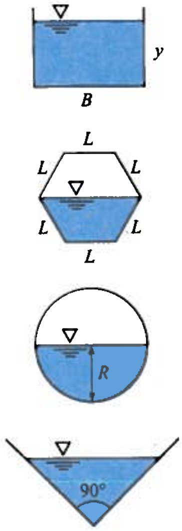
The **best hydraulic section** is the channel geometry that gives the maximum discharge for a given cross sectional area. Maximum discharge occurs when a geometry has the minimum wetted perimeter. Therefore, it yields the least viscous energy loss for a given area. Consider the quantity $AR_h^{2/3}$ in Manning's equation given in Eqs. (15.15 and 15.16), which is referred to as the section factor. Because $R_h = A/P$, the section factor relating to uniform flow is given by $A(A/P)^{2/3}$. Thus, for a channel of given resistance and slope, the discharge will increase with increasing cross-sectional area but decrease with increasing wetted perimeter P. For a given area, A, and a given shape of channel—for example, rectangular cross section—there will be a certain ratio of depth to width (y/B) for which the section factor will be maximum. This ratio is the best hydraulic section.

Example 15.6 shows that the best hydraulic section for a rectangular channel occurs when $y = \frac{1}{2}B$. It can be shown that the best hydraulic section for a trapezoidal channel is half a hexagon as shown; for the circular section, it is the half circle with depth equal to radius; and for the triangular section, it is a triangle with a vertex of 90° (Fig. 15.5). Of all the various shapes, the half circle has the best hydraulic section because it has the smallest perimeter for a given area.

The best hydraulic section can be relevant to the cost of the channel. For example, if a trapezoidal channel were to be excavated and if the water surface were to be at adjacent ground level, the minimum amount of excavation (and excavation cost) would result if the channel of best hydraulic section were used.

FIGURE 15.5

Best hydraulic sections for different geometries



✔CHECKPOINT PROBLEM 15.2

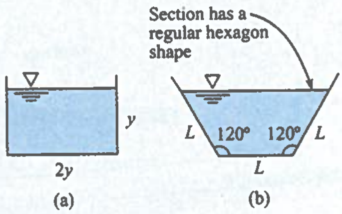
Consider uniform flow of water in two channels. Both have the same slope, the same wall roughness, and the same section area.

Which statement is true?

a. $Q_A = Q_B$

b. $Q_A < Q_B$

c. $Q_A > Q_B$



EXAMPLE 15.6

Finding the Best Hydraulic Section for a Rectangular Channel

Problem Statement

Determine the best hydraulic section for a rectangular channel with depth y and width B.

Define the Situation

Water flows in a rectangular channel. Depth $= y$. Width $= B$.

State the Goal

Find the best hydraulic section (relate B and y).

Generate Ideas and Make a Plan

1. Set $A = By$ and $P = B + 2y$ so that both are a function of y.
2. Let A be constant, and minimize P.
   - Differentiate P with respect to y and set the derivative equal to zero.
   - Express the result of minimizing P as a relation between y and B.

Take Action (Execute the Plan)

1. Relate A and P in terms of y.

$$P = \frac{A}{y} + 2y$$

2a. Minimize P.

$$\frac{dP}{dy} = \frac{-A}{y^2} + 2 = 0$$

$$\frac{A}{y^2} = 2$$

2b. Express result in terms of y and B.

$$A = By, \text{ so}$$

$$\frac{By}{y^2} = 2 \quad \text{or} \quad \boxed{y = \frac{1}{2}B}$$

Review the Solution and the Process

Knowledge. The best hydraulic section for a rectangular channel occurs when the depth is one-half the width of the channel, see Fig. 15.5.

Uniform Flow in Culverts and Sewers

Sewers are conduits that carry sewage (liquid domestic, commercial, or industrial waste) fro households, businesses, and factories to sewage disposal sites. These conduits are often circul in cross section, but elliptical and rectangular conduits are also used. The volume rate of sev age varies throughout the day and season, but of course sewers are designed to carry the ma imum design discharge flowing full or nearly full. At discharges less than the maximum, tl sewers will operate as open channels.

Sewage usually consists of about 99% water and 1% solid waste. Because most sewage is s dilute, it is assumed that it has the same physical properties as water for purposes of discharg computations. However, if the velocity in the sewer is too small, the solid particles may sett out and cause blockage of the flow. Therefore, sewers are usually designed to have a minimu velocity of about 2 ft/s (0.60 m/s) at times when the sewer is flowing full. This condition is m by choosing a slope on the sewer line to achieve the desired velocity.

A culvert is a conduit placed under a fill such as a highway embankment. It is used t convey stream-flow from the uphill side of the fill to the downhill side. Figure 15.6 shows th

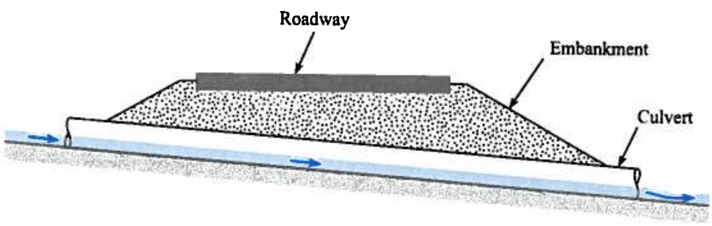


FIGURE 15.6
Culvert under a highway embankment.

essential features of a culvert. A culvert should be able to convey runoff from a **design storm** without overtopping the fill and without erosion of the fill at either the upstream or downstream end of the culvert. The design storm, for example, might be the maximum storm that could be expected to occur once in 50 years at the particular site.

The flow in a culvert is a function of many variables, including cross-sectional shape (circular or rectangular), slope, length, roughness, entrance design, and exit design. Flow in a culvert may occur as an open channel throughout its length, it may occur as a completely full pipe, or it may occur as a combination of both. The complete design and analysis of culverts are beyond the scope of this text; therefore, only simple examples are included here (Examples 15.7 and 15.8). For more extensive treatment of culverts, please refer to Chow (5), Henderson (6), and American Concrete Pipe Assoc. (7).

EXAMPLE 15.7

Sizing a Round Concrete Sewer Line

Problem Statement

A sewer line is to be constructed of concrete pipe to be laid on a slope of 0.006. If $n = 0.013$ and if the design discharge is 110 cfs, what size pipe (commercially available) should be selected for a full-flow condition? What will be the mean velocity in the sewer pipe for these conditions? (It should be noted that concrete pipe is readily available in commercial sizes of 8-in., 10-in., and 12-in. diameter and then in 3-in. increments up to 36-in. diameter. From 36-in. diameter up to 144 in. the sizes are available in 6-in. increments.)

Define the Situation

Sewer line, $S_0 = 0.006$, Q (design) = 110 cfs.

Assumptions: Can only use a standard pipe size.

State the Goal

Find: Pipe diameter large enough to carry design discharge and that allows $V \geq 2$ ft/s at full-flow condition.

Generate Ideas and Make a Plan

1. Use Chezy equation for traditional units, Eq. (15.16).
2. Solve for $AR^{2/3}$.
3. For pipe flowing full, relate A and P to diameter through R_h.
4. Solve for diameter, and use the next commercial size larger.
5. Check that velocity for full flow is greater than 2 ft/s.

Take Action (Execute the Plan)

1. Chezy equation for traditional units is

$$Q = \frac{1.49}{n} AR^{2/3} S_0^{1/2}$$

$$Q = 110 \text{ ft}^3/\text{s}$$

$$n = 0.013$$

$$S_0 = 0.006 \text{ (assume atmospheric pressure in the pipe)}$$

2. Solve for $AR^{2/3}$. Note that units of $AR^{2/3}$ are ft$^{8/3}$ because A is in ft^2 and R_h is in ft$^{2/3}$.

$$AR^{2/3} = \frac{(110 \text{ ft}^3/\text{s})(0.013)}{(1.49)(0.006)^{1/2}} = 12.39 \text{ ft}^{8/3}$$

3. Relate A and P to diameter by relating to R_h.

$$R_h = \frac{A}{P} \quad \text{and} \quad R_h^{2/3} = \left(\frac{A}{P}\right)^{2/3}$$

$$AR_h^{2/3} = \frac{A^{5/3}}{P^{2/3}} = 12.39 \text{ ft}^{8/3}$$

For a pipe flowing full, $A = \pi D^2/4$ and $P = \pi D$, or

$$\frac{(\pi D^2/4)^{5/3}}{(\pi D)^{2/3}} = 12.39 \text{ ft}^{8/3}$$

4. Solving for diameter yields $D = 3.98$ ft $= 47.8$ in. Use the next commercial size larger, which is $\boxed{D = 48 \text{ in.}}$

$$A = \frac{\pi D^2}{4} = 50.3 \text{ ft}^2 \text{ (for pipe flowing full)}$$

5. Verify that velocity of full flow is greater than 2 ft/s.

$$V = \frac{Q}{A} = \frac{(110 \text{ ft}^3/\text{s})}{(50.3 \text{ ft}^2)} = \boxed{2.19 \text{ ft/s}}$$

Example 15.8 demonstrates the calculation of necessary slope given all sources of head loss and a required discharge.

EXAMPLE 15.8

Culvert Design

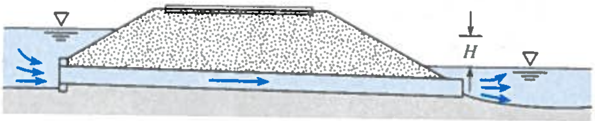
Problem Statement

A 54-in.-diameter culvert laid under a highway embankment has a length of 200 ft and a slope of 0.01. This was designed to pass a 50-year flood flow of 225 cfs under full-flow conditions (see figure). For these conditions, what head H is required? When the discharge is only 50 cfs, what will be the uniform flow depth in the culvert? Assume $n = 0.012$.

Define the Situation

Situation: Culvert has been designed to carry 225 cfs with given dimensions.

Assumptions: Uniform flow, so that pipe head loss h_f can be related to S_0.



State the Goal

Find:

1. The height H required between the two free surfaces when flowing full.
2. The uniform flow depth in the culvert when $Q = 50$ cfs.

Generate Ideas and Make a Plan

1. Use energy equation between the two end sections, accounting for head loss.
2. Document all sources of head loss.
3. Find pipe head loss h_f using Eq. (15.17) and the fact that

$$S_0 = \frac{h_f}{L}.$$

4. Use continuity equation to find V, the uniform flow velocity, needed to calculate head loss.
5. Solve for H.
6. Solve for depth of flow, for $Q = 50$ cfs, using Eq. (15.16) and pipe geometry relations for pipe flowing partly full.

Take Action (Execute the Plan)

1. Energy equation

$$\frac{p_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{g} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

Let points 1 and 2 be at the upstream and downstream water surfaces, respectively.

Thus, $(p_1 = p_2 = 0 \text{ gage} \quad \text{and} \quad V_1 = V_2 = 0)$

Also, $(z_1 - z_2 = H)$

Therefore, $(H = \sum h_L)$

2. Head losses occur at culvert entrance and exit, as well as over the length of pipe.

$$H = \text{pipe head loss} + \text{entrance head loss} + \text{exit head loss}$$

$$H = \frac{V^2}{2g}(K_e + K_E) + \text{pipe head loss}$$

$$K_e = 0.50 \text{ (from Table 10.5)}$$

$$K_E = 1.00 \text{ (from Table 10.5)}$$

3. Pipe head loss is

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2}$$

$$Q = 225 \text{ ft}^3/\text{s}$$

$$A = \frac{\pi D^2}{4} = 15.90 \text{ ft}^2$$

$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = 1.125 \text{ ft}$$

$$R_h^{2/3} = (1.125 \text{ ft})^{2/3} = 1.0817 \text{ ft}^{2/3}$$

$$S_0 = \frac{h_f}{L}$$

$$225 = \frac{1.49}{0.012}(15.90 \text{ ft}^2)(1.0817 \text{ ft}^{2/3})\left(\frac{h_f}{200}\right)^{1/2}$$

$$h_f = 2.22 \text{ ft}$$

4. Continuity equation yields

$$V = \frac{Q}{A} = \frac{225 \text{ ft}^3/\text{s}}{15.90 \text{ ft}^2} = 14.15 \text{ ft/s}$$

5. Solve for H.

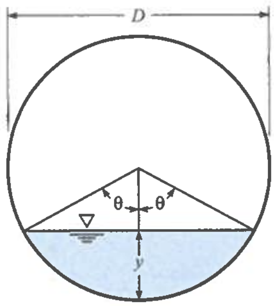
$$H = \frac{14.15^2}{64.4}(0.50 + 1.0) + 2.22$$

$$H = 4.66 \text{ ft} + 2.22 \text{ ft} = \boxed{6.88 \text{ ft}}$$

6. Depth of flow for $Q = 50$ cfs is

$$50 = \frac{1.49}{0.012} A R_h^{2/3} (0.01)^{1/2}$$

Values of A and R_h will depend on geometry of partly full pipe, as shown:



Area A if angle θ is given in degrees

$$A = \left[\left(\frac{\pi D^2}{4}\right)\left(\frac{2\theta}{360^\circ}\right)\right] - \left(\frac{D}{2}\right)^2(\sin\theta\cos\theta)$$

Wetted perimeter will be $P = \pi D(\pi/180^\circ)$, so

$$R_h = \frac{A}{P} = \left(\frac{D}{4}\right)\left[1 - \left(\frac{\sin\theta\cos\theta}{(\pi\theta/180^\circ)}\right)\right]$$

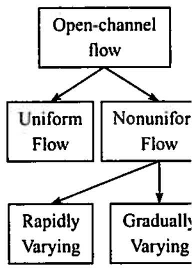
Substituting these relations for A and R_h into the dischar equation and solving for θ yields θ = 70°. Therefore, y is

$$y = \frac{D}{2} - \frac{D}{2}\cos\theta = \left(\frac{54\text{ in}}{2}\right)(1 - 0.342) = \boxed{17.8\text{ in}}$$

15.4 Steady Nonuniform Flow

As stated in the beginning of this chapter, and shown in Fig. 15.2, all open-channel flows are classified as either uniform or nonuniform. Recall that uniform flow has constant velocity along a streamline and thus has constant depth for a constant cross section. In steady nonuniform flow, the depth and velocity change over distance (but not with time). For all such cases, the energy equation as generally introduced in Section 15.2 is invoked to compare two cross sections. However, for analysis of nonuniform flow, it is useful to distinguish whether the depth and velocity change occurs over a short distance, referred to as **rapidly varied flow**, or over a long reach of the channel, referred to as **gradually varied flow** (Fig. 15.7). The head loss term is different for these two cases. For rapidly varied flow, one can neglect the resistance of the channel walls and bottom because it occurs over a short distance. For gradually varied flow, because of the long distances involved, the surface resistance is a significant variable in the energy balance.

FIGURE 15.7
Classifying nonuniforn flow.



15.5 Rapidly Varied Flow

Rapidly varied flow is analyzed with the energy equation presented previously for open-channel flow, Eq. (15.7), with the additional assumptions that the channel bottom is horizontal ($S_0 = 0$) and the head loss is zero ($h_L = 0$). Therefore, Eq. (15.7) becomes

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \tag{15.17}$$

Specific Energy

The sum of the depth of flow and the velocity head is defined as the **specific energy**:

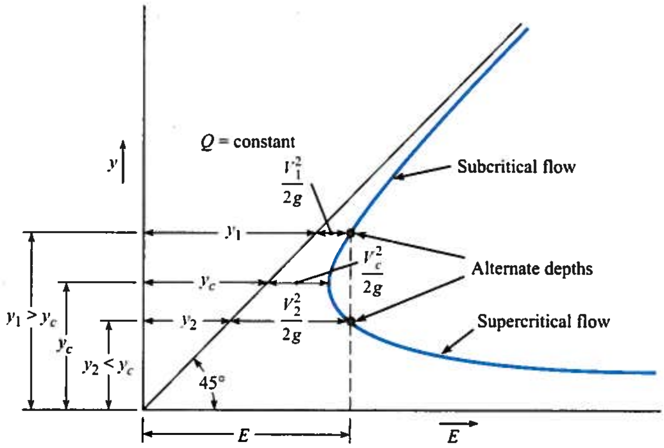
$$E = y + \frac{V^2}{2g} \tag{15.18}$$

Note that specific energy has dimensions [L]; that is, it is an energy head. Equation (15.17) states that the specific energy at section 1 is equal to the specific energy at section 2, or $E_1 = E_2$. The continuity equation between sections 1 and 2 is

$$A_1V_1 = A_2V_2 = Q \tag{15.19}$$

FIGURE 15.8

Relation between depth and specific energy.



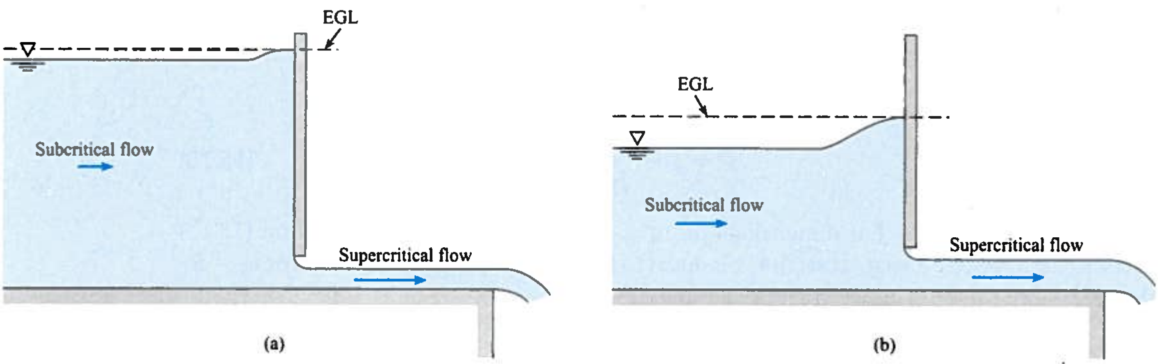
Therefore, Eq. (15.17) can be expressed as

$$y_1 + \frac{Q^2}{2gA_1^2} = y_2 + \frac{Q^2}{2gA_2^2} \qquad (15.20)$$

Because A_1 and A_2 are functions of the depths y_1 and y_2, respectively, the magnitude of the specific energy at section 1 or 2 is solely a function of the depth at each section. If, for a given channel and given discharge, one plots depth versus specific energy, a relationship such as that shown in Fig. 15.8 is obtained. By studying Fig. 15.8 for a given value of specific energy, one can see that the depth may be either large or small. This means that for the small depth, the bulk of the energy of flow is in the form of kinetic energy—that is, $Q^2/(2gA^2) \gg y$—whereas for a larger depth, most of the energy is in the form of potential energy. Flow under a sluice gate (Fig. 15.9) is an example of flow in which two depths occur for a given value of specific energy. The large depth and low kinetic energy occur upstream of the gate; the low depth and large kinetic energy occur downstream. The depths as used here are called alternate depths. That is, for a given value of E, the large depth is alternate to the low depth, or vice versa. Returning to the flow under the sluice gate, one finds that if the same rate of flow is maintained, but the gate is set with a larger opening, as in Fig. 15.9b, the upstream depth will drop, and the downstream depth will rise. This results in different alternate depths and a smaller value of specific energy than before. This is consistent with the diagram in Fig. 15.8.

FIGURE 15.9

Flow under a sluice gate. (a) Smaller gate opening. (b) Larger gate opening.



Finally, it can be seen in Fig. 15.8 that a point will be reached where the specific energy is minimum and only a single depth occurs. At this point, the flow is termed critical. Thus one definition of **critical flow** is the flow that occurs when the specific energy is minimum for a given discharge. The flow for which the depth is less than critical (velocity is greater than critical) is termed **supercritical flow**, and the flow for which the depth is greater than critical (velocity is less than critical) is termed **subcritical flow**. Therefore, subcritical flow occurs upstream and supercritical flow occurs downstream of the sluice gate in Fig. 15.9. Subcritical flows corresponds to a Froude number less than one ($\mathrm{Fr} < 1$), and supercritical flow corresponds to ($\mathrm{Fr} > 1$). Some engineers refer to subcritical and supercritical flow as **tranquil** and **rapid** flow, respectively. Other aspects of critical flow are shown in the next section.

Characteristics of Critical Flow

Critical flow occurs when the specific energy is minimum for a given discharge. The depth for this condition may be determined by solving for dE/dy from $E = y + Q^2/2gA^2$ and setting dE/dy equal to zero:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} \tag{15.21}$$

However, $dA = T\,dy$, where T is the width of the channel at the water surface, as shown in Fig. 15.10. Then Eq. (15.21), with $dE/dy = 0$, will reduce to

$$\frac{Q^2 T_c}{gA_c^3} = 1 \tag{15.22}$$

or

$$\frac{A_c}{T_c} = \frac{Q^2}{gA_c^2} \tag{15.23}$$

If the **hydraulic depth**, D, is defined as

$$D = \frac{A}{T} \tag{15.24}$$

then Eq. (15.23) will yield a critical hydraulic depth D_c, given by

$$D_c = \frac{Q^2}{gA_c^2} = \frac{V^2}{g} \tag{15.25}$$

Dividing Eq. (15.25) by D_c and taking the square root yields

$$1 = \frac{V}{\sqrt{gD_c}} \tag{15.26}$$

Note: $V/\sqrt{gD_c}$ is the Froude number. Therefore, it has been shown that the Froude number is equal to unity when critical flow prevails.

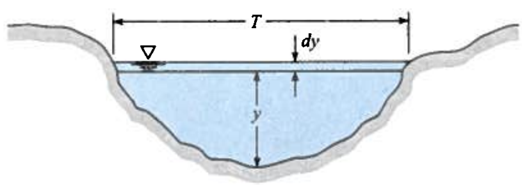


FIGURE 15.10
Open-channel relatio

If a channel is of rectangular cross section, then A/T is the actual depth, and $Q^2/A^2 = q^2/y$ so the condition for **critical depth** (Eq. 15.23) for a rectangular channel becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad (15.27$$

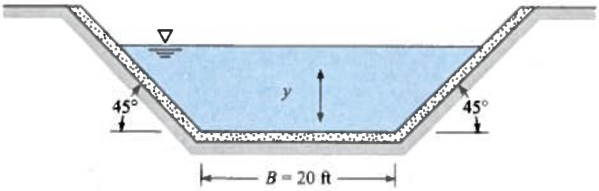
where q is the discharge per unit width of channel.

EXAMPLE 15.9

Calculating Critical Depth in a Channel

Problem Statement

Determine the critical depth in this trapezoidal channel for a discharge of 500 cfs. The width of the channel bottom is $B = 20$ ft, and the sides slope upward at an angle of 45°.



Define the Situation

Water flows in a trapezoidal channel with known geometry.

State the Goal

Calculate the critical depth.

Generate Ideas and Make a Plan

1. For critical flow, Eq. (15.22) must apply.
2. Relate this channel geometry to width T and area A in Eq. (15.22).
3. By iteration (choose y and compute A^3/T), find y that will yield A^3/T equal to 7764 ft². This y will be critical depth y_c.

Take Action (Execute the Plan)

1. Apply Eq. (15.22) or Eq. (15.23).

$$\frac{Q^2 T_c}{g A_c^3} = 1 \text{ or } \frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

2. For $Q = 500$ cfs,

$$\frac{A_c^3}{T_c} = \frac{500^2}{32.2} = 7764 \text{ ft}^2$$

For this channel, $A = y(B + y)$ and $T = B + 2y$.

3. Iterate to find y_c.

$$y_c = \boxed{2.57 \text{ ft}}$$

Critical flow may also be examined in terms of how the discharge in a channel varies with depth for a given specific energy. For example, consider flow in a rectangular channel where

$$E = y + \frac{Q^2}{2gA^2}$$

or

$$E = y + \frac{Q^2}{2gy^2B^2}$$

If one considers a unit width of the channel and lets $q = Q/B$, then the foregoing equation becomes

$$E = y + \frac{q^2}{2gy^2}$$

If one determines how q varies with y for a constant value of specific energy, one sees that critical flow occurs when the discharge is maximum (see Fig. 15.11).