TURBOMACHINERY 14

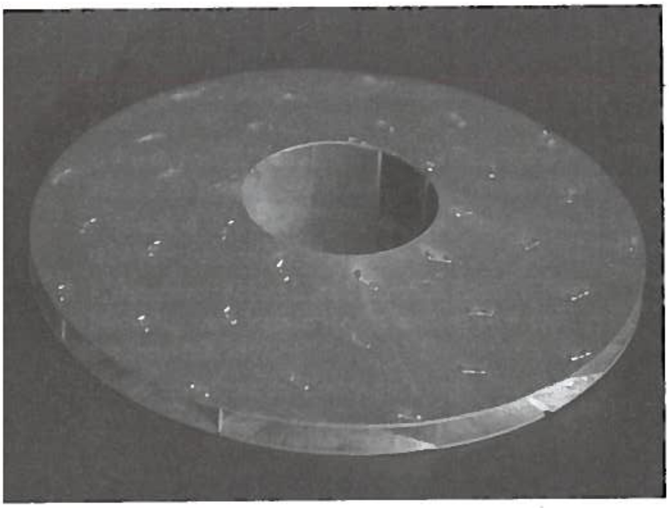


FIGURE 14.1

This figure shows the impeller from the blower inside a vacuum cleaner. This impeller rotates inside a housing. This rotational motion creates a suction pressure that draws air into the center hole. The air is flung outward by the spinning blades of the impeller.

This impeller was "liberated" from the vacuum cleaner by Jason Stirpe, while he was an engineering student. Jason used this impeller with a DC motor and a homemade housing to fabricate a blower for a design that he was creating. Being resourceful is at the heart of technology innovation. (Photo by Donald Elger.)

Chapter Road Map

Machines to move fluids or to extract power from moving fluids have been designed since the begin of recorded history. Fluid machines are everywher They are the essential components of the automob we drive, the supply systems for the water we drink, power generation plants for the electricity we use, c the air-conditioning and heating systems that provid the comfort we enjoy. Thus, this chapter introduces concepts underlying various types of machines.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Describe the factors that influence the thrust and efficier of a propeller. (§14.1)
- Calculate the thrust and efficiency of a propeller. (§14.
- Describe axial flow and radial flow pumps. (§14.2, 1
- Define the head coefficient and the discharge coefficie (§14.2)
- Sketch a pump performance curve and describe the relevant π-groups that appear. (§14.2, 14.3)
- Explain how specific speed is used to select an approp type of pump for an application. (§14.4)
- Describe an impulse turbine and a reaction turbine. (§
- Describe the maximum power that can be produced by wind turbine. (§14.8)

Fluid machines are separated into two broad categories: positive-displacement machines and turbomachines. **Positive-displacement machines** operate by forcing fluid into or out of a chamber. Examples include the bicycle tire pump, the gear pump, the peristaltic pump, and the human heart. **Turbomachines** involve the flow of fluid through rotating blades or rotors that remove or add energy to the fluid. Examples include propellers, fans, water pumps, windmills, and compressors.

Axial-flow turbomachines operate with the flow entering and leaving the machine in t direction that is parallel to the axis of rotation of blades. A radial-flow machine can have t flow either entering or leaving the machine in the radial direction that is normal to the axis rotation.

Table 14.1 provides a classification for turbomachinery. Power-absorbing machin require power to increase head (or pressure). A power-producing machine provides sh power at the expense of head (or pressure) loss. Pumps are associated with liquids, where fans (blowers) and compressors are associated with gases. Both gases and liquids produce pow through turbines. Oftentimes the gas turbine refers to an engine that has both a compressor a turbine and produces power.

TABLE 14.1 Categories of Turbomachinery

	Power Absorbing	Power Producing
Axial machines	Axial pumps Axial fans Propellers Axial compressors	Axial turbine (Kaplan) Wind turbine Gas turbine
Radial machines	Centrifugal pump Centrifugal fan Centrifugal compressor	Impulse turbine (Pelton wheel) Reaction turbine (Francis turbine)

14.1 Propellers

A propeller is a fan that converts rotational motion into thrust. The design of a propeller based on the fundamental principles of airfoil theory. For example, consider a section of th propeller in Fig. 14.2, and notice the analogy between the lifting vane and the propeller. Th propeller is rotating at an angular speed ω, and the speed of advance of the airplane an propeller is V_0. Focusing on an elemental section of the propeller, Fig. 14.2c, it is noted th the given section has a velocity with components V_0 and V_t. Here V_t is tangential velocit $V_t = r\omega$, resulting from the rotation of the propeller. Reversing and adding the velocity vecto V_0 and V_t yield the velocity of the air relative to the particular propeller section (Fig. 14.2d).

The angle θ is given by

$$\theta = \arctan\left(\frac{V_0}{r\omega}\right) \qquad (14.1$$

For a given forward speed and rotational rate, this angle is a minimum at the propeller ti $(r = r_0)$ and increases toward the hub as the radius decreases. The angle β is known as th **pitch angle**. The local angle of attack of the elemental section is

$$\alpha = \beta - \theta \qquad (14.2$$

The propeller can be analyzed as a series of elemental sections (of width dr) producing lift an drag, which provide the propeller thrust and create resistive torque. This torque multiplied b the rotational speed is the power input to the propeller.

The propeller is designed to produce thrust, and because the greatest contribution t thrust comes from the lift force F_L, the goal is to maximize lift and minimize drag, F_D. Fo

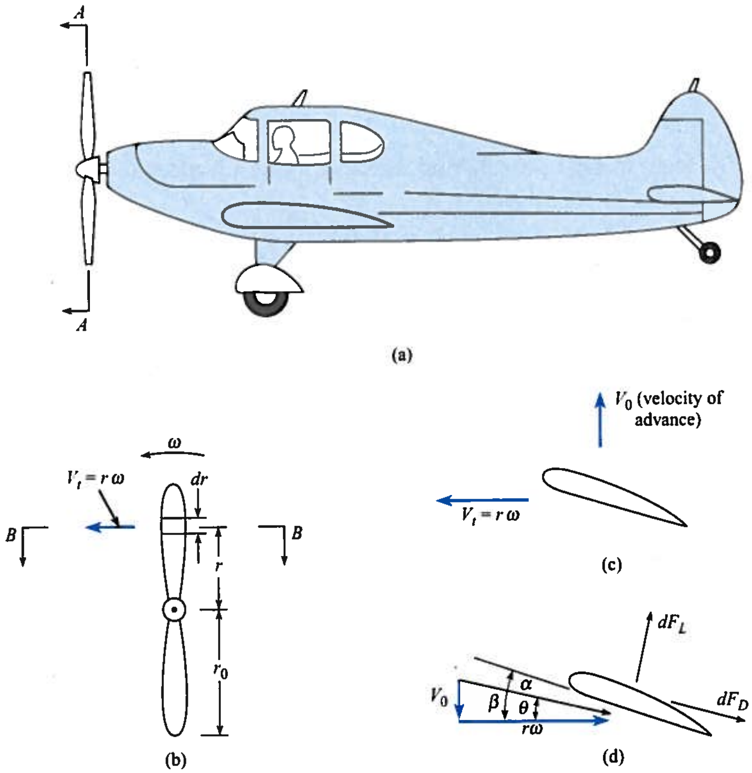


FIGURE 14.2

Propeller motion.
(a) Airplane motion.
(b) View *A-A*.
(c) View *B-B*.
(d) Velocity relative to blade element.

a given shape of propeller section, the optimum angle of attack can be determined from data such as are given in Fig. 11.24. Because the angle θ increases with decreasing radius, the local pitch angle has to change to achieve the optimum angle of attack. This is done by twisting the blade.

A dimensional analysis can be performed to determine the π-groups that characterize the performance of a propeller. For a given propeller shape and pitch distribution, the thrust of a propeller T, will depend on the propeller diameter D, the rotational speed n, the forward speed V_0, the fluid density ρ, and the fluid viscosity μ.

$$T = f(D, \omega, V_0, \rho, \mu) \tag{14.3}$$

Performing a dimensional analysis results in

$$\frac{T}{\rho n^2 D^4} = f\left(\frac{V_0}{nD}, \frac{\rho D^2 n}{\mu}\right) \tag{14.4}$$

It is conventional practice to express the rotational rate, n, as revolutions per second (rps). The π-group on the left is called the **thrust coefficient**,

$$C_T = \frac{T}{\rho n^2 D^4} \tag{14.5}$$

The first π-group on the right is the **advance ratio**. The second group is a Reynolds number based on the tip speed and diameter of the propeller. For most applications, the Reynolds number is high, and experience shows that the thrust coefficient is unaffected by the Reyno number, so

$$C_T = f\left(\frac{V_0}{nD}\right) \quad \text{(14}$$

The angle θ at the propeller tip is related to the advance ratio by

$$\theta = \arctan\left(\frac{V_0}{\omega r_0}\right) = \arctan\left(\frac{1}{\pi}\frac{V_0}{nD}\right) \quad \text{(14}$$

As the advance ratio increases and θ increases, the local angle of attack at the blade eleme decreases, the lift increases, and the thrust coefficient goes down. This trend is illustrated Fig. 14.3, which shows the dimensionless performance curves for a typical propeller. Ul mately, an advance ratio is reached where the thrust coefficient goes to zero.

Performing a dimensional analysis for the power, P, shows

$$\frac{P}{\rho n^3 D^5} = f\left(\frac{V_0}{nD}, \frac{\rho D^2 n}{\mu}\right) \quad \text{(14.}$$

The π-group on the left is the **power coefficient**,

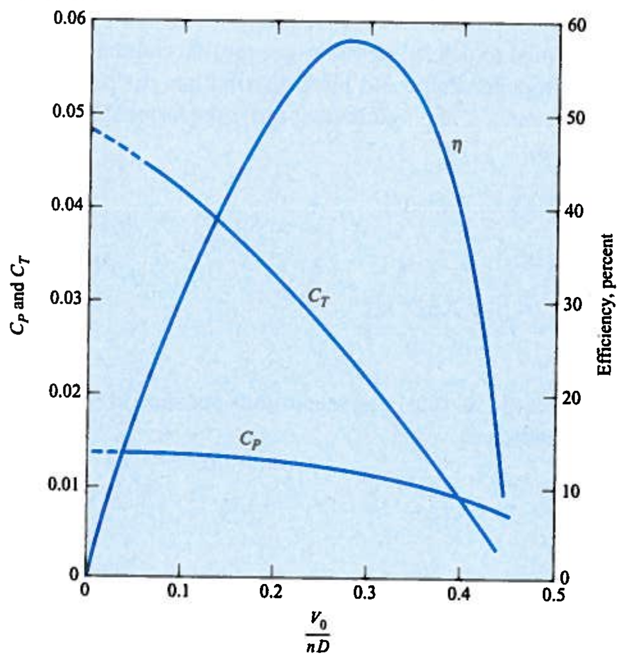
$$C_P = \frac{P}{\rho n^3 D^5} \quad \text{(14.}$$

As with the thrust coefficient, the power coefficient is not significantly influenced by the Reynol number at high Reynolds numbers, so C_P reduces to a function of the advance ratio only

$$C_P = f\left(\frac{V_0}{nD}\right) \quad \text{(14.1}$$

FIGURE 14.3

Dimensionless performance curves for a typical propeller; D = 2.90 m, n = 1400 rpm. [After Weick (1).]



The functional relationship between C_P and V_0/nD for an actual propeller is also shown in Fig. 14.3. Even though the thrust coefficient approaches zero for a given advance ratio, the power coefficient shows little decrease because it still takes power to overcome the torque on the propeller blade.

The curves for C_T and C_P are evaluated from performance characteristics of a given propeller operating at different values of V_0 as shown in Fig. 14.4. Although the data for the curves are obtained for a given propeller, the values for C_T and C_P, as a function of advance ratio, can be applied to geometrically similar propellers of different sizes and angular speeds.* Example 14.1 illustrates such an application.

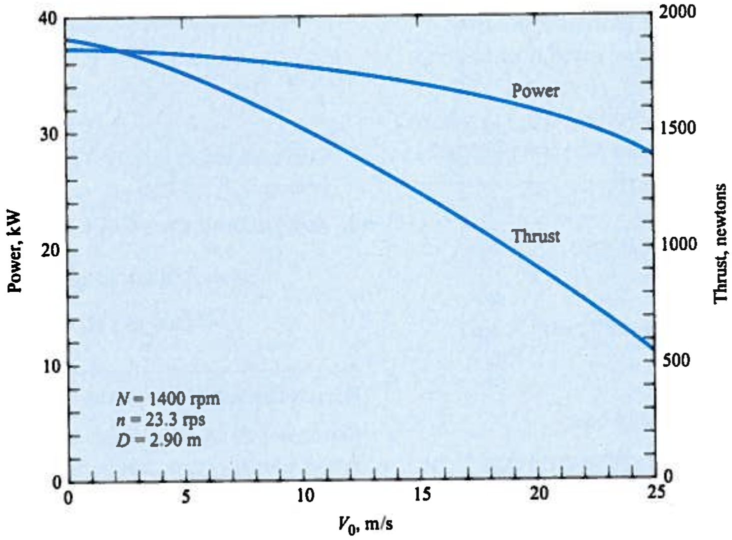


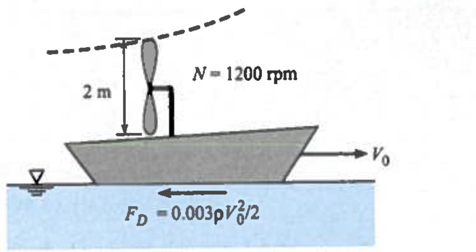
FIGURE 14.4
Power and thrust of a propeller 2.90 m in diameter at a rotational speed of 1400 rpm. [After Weick (2).]

EXAMPLE 14.1

Propeller Application

Problem Statement

A propeller having the characteristics shown in Fig. 14.3 is to be used to drive a swamp boat. If the propeller is to have a diameter of 2 m and a rotational speed of $N = 1200$ rpm, what should be the thrust starting from rest? If the boat resistance (air and water) is given by the empirical equation $F_D = 0.003\rho V_0^2/2$, where V_0 is the boat speed in meters per second, F_D is the drag, and ρ is the mass density of the water, what will be the maximum speed of the boat and what power will be required to drive the propeller? Assume $\rho_{air} = 1.20$ kg/m^3 and $\rho_{water} = 1000$ kg/m^3.



Define the Situation

A propeller is being used to drive a swamp boat.

Properties: $\rho = 1.2$ kg/m^3, $\rho_w = 1000$ kg/m^3.

*The speed of sound was not included in the dimensional analysis. However, the propeller performance is reduced because the Mach number based on the propeller tip speed leads to shock waves and other compressible-flow effects.

State the Goals

- Calculate thrust (in N) starting from rest.
- Find maximum speed (in m/s) of swamp boat.
- Calculate power required (in kW) to operate propeller.

Generate Ideas and Make a Plan

1. From Fig. 14.3, find thrust coefficient for zero advance ratio.
2. Calculate thrust using Eq. (14.5).
3. To calculate maximum speed, plot propeller thrust versus boat speed and on same graph plot resistance of swamp boat versus boat speed. The maximum speed is where the curves intersect.
4. The maximum power will be when the boat speed is zero, so use Eq. (14.9) with C_P for zero advance ratio from Fig. 14.3.

Take Action (Execute the Plan)

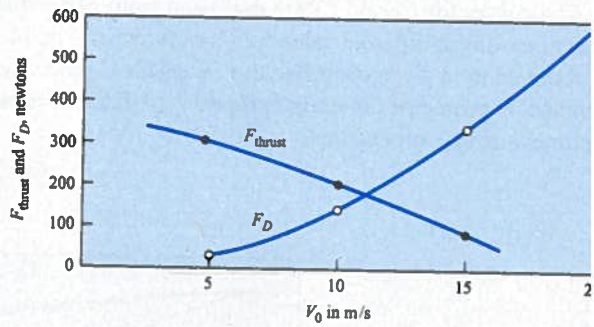
1. From Fig. 14.3, $C_T = 0.048$ for $V_0/nD = 0$.
2. Thrust

$$F_T = C_T \rho_a D^4 n^2 = 0.048(1.20\ \text{kg/m}^3)(2\ \text{m})^4(20\ \text{rps})^2$$
$$= \boxed{369\ \text{N}}$$

3. Table of thrust versus speed of swamp boat

V_0	V_0/nD	C_T	$F_T = C_T\rho_a D^4 n^2$	$F_D = 0.003\rho_w V_0^2/2$
5 m/s	0.125	0.040	307 N	37.5 N
10 m/s	0.250	0.027	207 N	150 N
15 m/s	0.375	0.012	90 N	337 N

Graph of propeller thrust and swamp boat drag versus speed



Curves intersect at $V_0 = 11$ m/s. Hence maximum speed of swamp boat is 11 m/s.

4. At $V_0/nD = 0$, $C_P = 0.014$.

$$P = 0.014(1.20\ \text{kg/m}^3)(2\ \text{m})^5(20\ \text{rps})^3$$
$$= 4300\ \text{m}\cdot\text{N/s} = \boxed{4.30\ \text{kW}}$$

Review the Solution and the Process

Discussion. In an actual application. The starting rotational rate of propeller need not be 1200 rpm but can be a lower value. After the boat is gaining speed the rotational rate can be increased to achieve maximum speed.

The efficiency of a propeller is defined as the ratio of the power output—that is, thru times velocity of advance—to the power input. Hence the efficiency η is given as

$$\eta = \frac{F_T V_0}{P} = \frac{C_T \rho D^4 n^2 V_0}{C_p \rho D^5 n^3}$$

or

$$\eta = \frac{C_T}{C_P}\left(\frac{V_0}{nD}\right) \quad \text{(14.1}$$

The variation of efficiency with advance ratio for a typical propeller is also shown in Fig. 14. The efficiency can be calculated directly from C_T and C_P performance curves. Note at lo advance ratios, the efficiency increases with advance ratio and then reaches a maximum value before the decreasing thrust coefficient causes the efficiency to drop toward zero. The maximum efficiency represents the best operating point for fuel efficiency.

Many propeller systems are designed to have variable pitch; that is, pitch angles can be changed during propeller operation. Different efficiency curves corresponding to varying pitch angles are shown in Fig. 14.5. The envelope for the maximum efficiency is also shown in the figure. During operation of the aircraft, the pitch angle can be controlled to achieve maximum efficiency corresponding to the propeller rpm and forward speed.

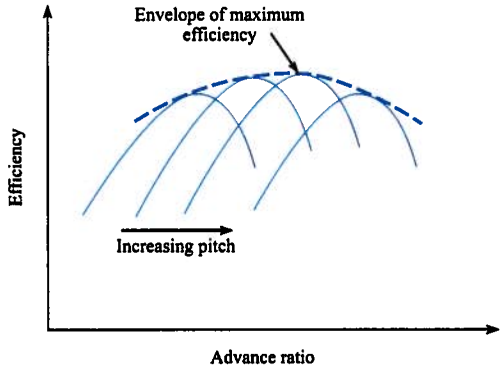


FIGURE 14.5
Efficiency curves for variable-pitch propeller.

The best source for propeller performance information is from propeller manufacturers. There are many speciality manufacturers for everything from marine to aircraft applications.

14.2 Axial-Flow Pumps

The axial flow pump acts much like a propeller enclosed in a housing as shown in Fig. 14.6. The rotating element, the impeller, causes a pressure change between the upstream and downstream sections of the pump. In practical applications, axial-flow machines are best suited to deliver relatively low heads and high flow rates. Hence pumps used for dewatering lowlands, such as those behind dikes, are almost always of the axial-flow type. Water turbines in low-head dams (less than 30 m) where the flow rate and power production are large are also generally of the axial type.

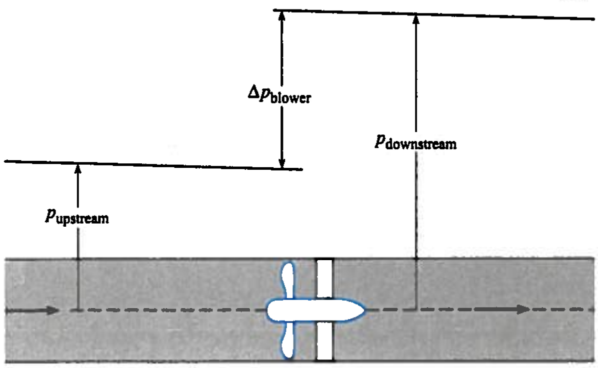


FIGURE 14.6
Axial-flow blower in a duct.

Head and Discharge Coefficients for Pumps

The thrust coefficient is defined as $F_T/\rho D^4 n^2$ for use with propellers, and if the same variat are applied to flow in an axial pump, the thrust can be expressed as $F_T = \Delta p A = \gamma \Delta H A$ or

$$C_T = \frac{\gamma \Delta H A}{\rho D^4 n^2} = \frac{\pi}{4}\frac{\gamma \Delta H D^2}{\rho D^4 n^2} = \frac{\pi}{4}\frac{g \Delta H}{D^2 n^2} \quad (14.$$

A new parameter, called the **head coefficient** C_H, is defined using the variables of Eq. (14.12)

$$C_H = \frac{4}{\pi} C_T = \frac{\Delta H}{D^2 n^2 / g} \quad (14.$$

which is a π-group that relates head delivered to fan diameter and rotational speed.

The independent π-group relating to propeller operation is V_0/nD; however, multiply the numerator and denominator by the diameter squared gives $V_0 D^2/nD^3$, and $V_0 D^2$ is prop tional to the discharge, Q. Thus the π-group for pump similarity studies is Q/nD^3 and is ide tified as the **discharge coefficient** C_Q. The power coefficient used for pumps is the same as t power coefficient used for propellers. Summarizing, the π-groups used in the similarity ana ses of pumps are

$$C_H = \frac{\Delta H}{D^2 n^2 / g} \quad (14.$$

$$C_P = \frac{P}{\rho D^5 n^3} \quad (14.1$$

$$C_Q = \frac{Q}{n D^3} \quad (14.1$$

where C_H and C_P are functions of C_Q for a given type of pump.

Figure 14.7 is a set of curves of C_H and C_P versus C_Q for a typical axial-flow pump. Al plotted on this graph is the efficiency of the pump as a function of C_Q. The dimensional curv

FIGURE 14.7
Dimensionless performance curves for a typical axial-flow pump. [After Stepanoff (3).]

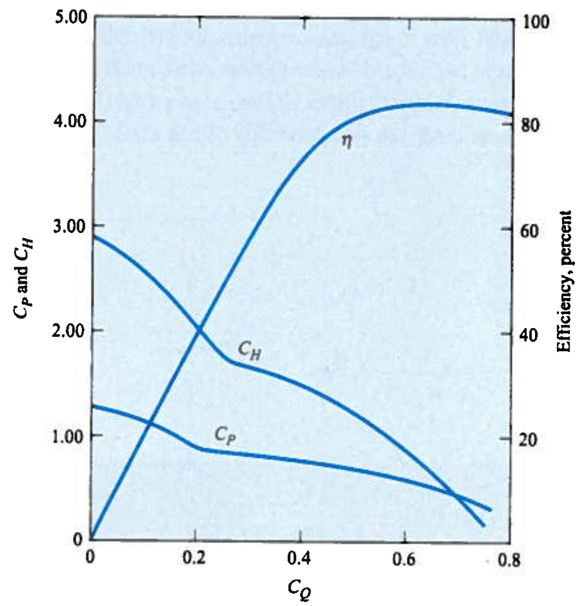
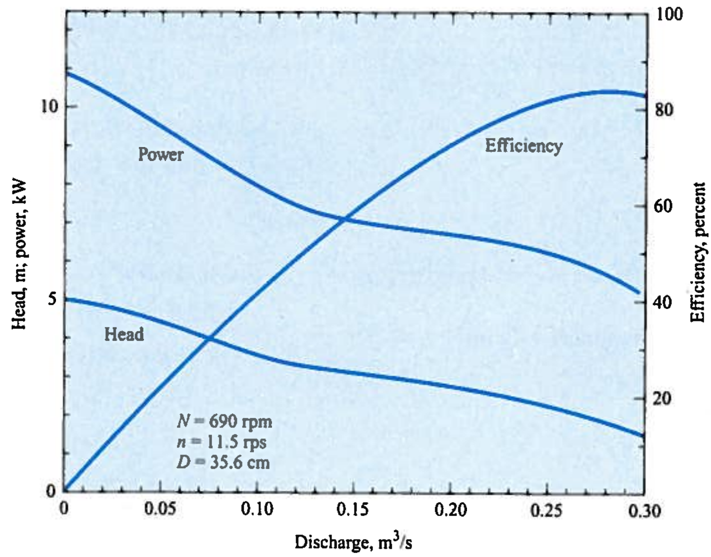


FIGURE 14.8

Performance curves for a typical axial-flow pump. [After Stepanoff (3).]



(head and power versus Q for a constant speed of rotation) from which Fig. 14.7 was developed are shown in Fig. 14.8. Because curves like those shown in Fig. 14.7 or Fig. 14.8 characterize pump performance, they are often called characteristic curves or performance curves. These curves are obtained by experiment.

There can be a problem with overload when operating axial-flow pumps. As seen in Fig. 14.7, when the pump flow is throttled below maximum-efficiency conditions, the required power increases with decreasing flow, thus leading to the possibility of overloading at low-flow conditions. For very large installations, special operating procedures are followed to avoid such overloading. For instance, the valve in the bypass from the pump discharge back to the pump inlet can be adjusted to maintain a constant flow through the pump. However, for small-scale applications, it is often desirable to have complete flexibility in flow control without the complexity of special operating procedures.

Performance curves are used to predict prototype operation from model tests or the effect of changing the speed of the pump. Example 14.2 shows how to use pump curves to calculate discharge and power.

EXAMPLE 14.2

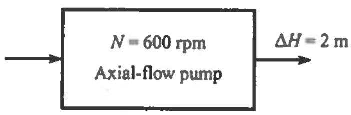
Discharge and Power for Axial-Flow Pump

Define the Situation

For the pump represented by Figs. 14.7 and 14.8, what discharge of water in cubic meters per second will occur when the pump is operating against a 2-m head and at a speed of 600 rpm? What power in kilowatts is required for these conditions?

Define the Situation

This problem involves an axial flow pump with water.



Properties: Assume $\rho = 1000$ kg/m^3.

State the Goal

- Calculate discharge (in m^3/s).
- Calculate power (in kW).

Generate Ideas and Make a Plan

1. Calculate C_H.
2. From Fig. 14.7 find C_Q and C_P.
3. Use C_Q to calculate discharge.
4. Use C_P to calculate power.

Take Action (Execute the Plan)

1. Rotational rate is (600 rev/min)/(60 s/min) = 10 rps. $D = 35.6$ cm.

$$C_H = \frac{2\text{ m}}{(0.356\text{ m})^2(10^2\text{ s}^{-2})/(9.81\text{ m/s}^2)} = 1.55$$

2. From Fig. 14.7, $C_Q = 0.40$ and $C_P = 0.72$.
3. Discharge is

$$Q = C_Q n D^3$$
$$Q = 0.40(10\text{ s}^{-1})(0.356\text{ m})^3 = \boxed{0.180\text{ m}^3/\text{s}}$$

4. Power is

$$\begin{aligned} P &= 0.72\rho D^5 n^3 \\ &= 0.72(10^3\text{ kg/m}^3)(0.356\text{ m})^5(10\text{ s}^{-1})^3 \\ &= 4.12\text{ km}\cdot\text{N/s} = 4.12\text{ kJ/s} = \boxed{4.12\text{ kW}} \end{aligned}$$

Example 14.3 illustrates how to calculate head and power for an axial-flow pump.

EXAMPLE 14.3

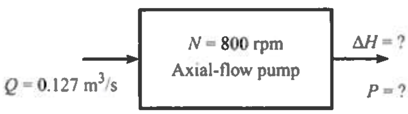
Head and Power for Axial-Flow Pump

Problem Statement

If a 30-cm axial-flow pump having the characteristics shown in Fig. 14.7 is operated at a speed of 800 rpm, what head ΔH will be developed when the water-pumping rate is 0.127 m^3/s? What power is required for this operation?

Define the Situation

This problem involves a 30-cm axial flow pump with water.



Properties. Water, $\rho = 10^3$ kg/m^3.

State the Goals

- Calculate H = head (in meters) developed.
- Calculate power (in kW) required.

Generate Ideas and Make a Plan

1. Calculate the discharge coefficient, C_Q.
2. From Fig. 14.7, read C_H, and C_P.
3. Use Eq. (14.14) to calculate head produced.
4. Use Eq. (14.15) to calculate power required.

Take Action (Execute the Plan)

1. Discharge coefficient is

$$Q = 0.127\text{ m}^3/\text{s}$$
$$n = \frac{800}{60} = 13.3\text{ rps}$$
$$D = 30\text{ cm}$$
$$C_Q = \frac{0.127\text{ m}^3/\text{s}}{(13.3\text{ s}^{-1})(0.30\text{ m})^3} = 0.354$$

2. From Fig. 14.7, $C_H = 1.70$ and $C_p = 0.80$.
3. Head produced is

$$\Delta H = \frac{C_H D^2 n^2}{g} = \frac{1.70(0.30\text{ m})^2(13.3\text{ s}^{-1})^2}{(9.81\text{ m/s}^2)} = \boxed{2.76\text{ m}}$$

4. Power required is

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= 0.80(10^3\text{ kg/m}^3)(0.30\text{ m})^5(13.3\text{ s}^{-1})^3 = \boxed{4.57\text{ kW}} \end{aligned}$$

Fan Laws

The **fan laws** are used extensively by designers and practitioners involved with axial fans and blowers. The fan laws are equations that provide the discharge, pressure rise, and power requirements for a fan that operates at different speeds. The laws are based on the discharge, head, and power coefficients being the same at any other state as at the reference state, o; namely, $C_Q = C_{Qo}$, $= C_{Ho}$, and $C_P = C_{Po}$. Because the size and design of fan is unchanged, the discharge at speed n is

$$Q = Q_o \frac{n}{n_o} \tag{14.17a}$$

and the pressure rise is

$$\Delta p = \Delta p_o \left(\frac{n}{n_o}\right)^2 \tag{14.17b}$$

and finally the power required is

$$P = P_o \left(\frac{n}{n_o}\right)^3 \tag{14.17c}$$

These fan laws cannot be applied between fans of different size and design. Of course, the fan laws do not provide exact values because of design considerations and manufacturing tolerances, but they are very useful in estimating fan performance.

14.3 Radial-Flow Machines

Radial-flow machines are characterized by the radial flow of the fluid through the machine. Radial-flow pumps and fans are better suited for larger heads at lower flow rates than axial machines.

Centrifugal Pumps

A sketch of the **centrifugal pump** is shown in Fig. 14.9. Fluid from the inlet pipe enters the pump through the eye of the impeller and then travels outward between the vanes of the impeller to its edge, where the fluid enters the casing of the pump and is then conducted to the discharge pipe. The principle of the radial-flow pump is different from that of the axial-flow

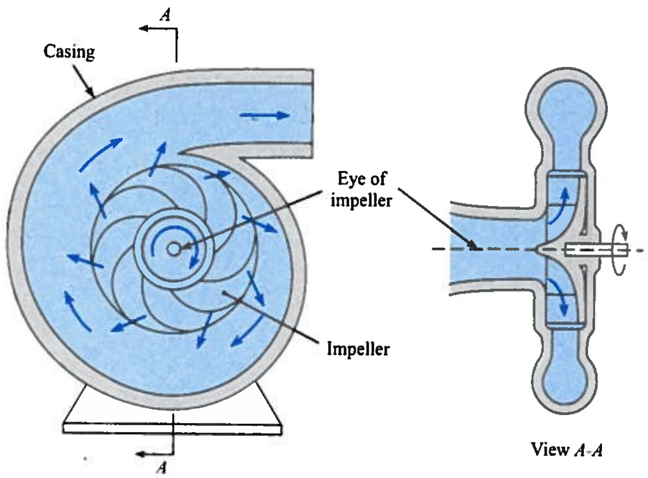


FIGURE 14.9
Centrifugal pump.

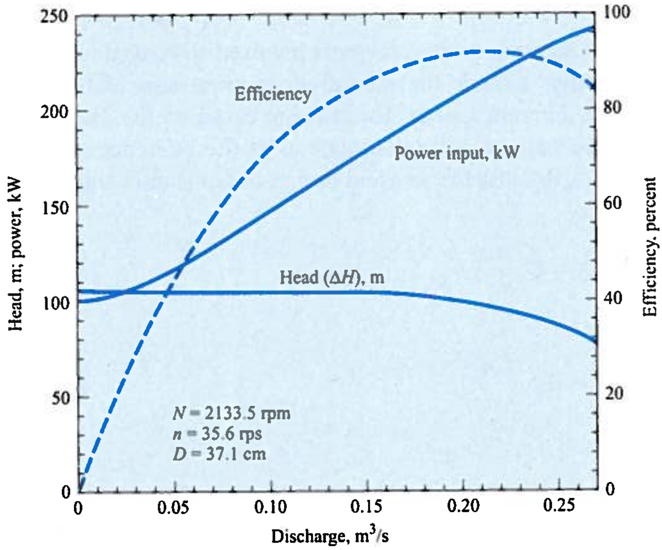


FIGURE 14.10
Performance curves for a typical centrifugal pump; D = 37.1 cm. [After Daugherty and Franzini (4). Used with the permission of the McGraw-Hill Companies.]

pump in that the change in pressure results in large part from rotary action (pressure increas outward like that in the rotating tank in Section 4.4 produced by the rotating impeller Additional pressure increase is produced in the radial-flow pump when the high velocity of th flow leaving the impeller is reduced in the expanding section of the casing.

Although the basic designs are different for radial- and axial-flow pumps, it can be show that the same similarity parameters (C_Q, C_P, and C_H) apply for both types. Thus the metho that have already been discussed for relating size, speed, and discharge in axial-flow machin also apply to radial-flow machines.

The major practical difference between axial- and radial-flow pumps so far as the user concerned is the difference in the performance characteristics of the two designs. The dime sional performance curves for a typical radial-flow pump operating at a constant speed of rot tion are shown in Fig. 14.10. The corresponding dimensionless performance curves for th same pump are shown in Fig. 14.11. Note that the power required at shutoff flow is less th

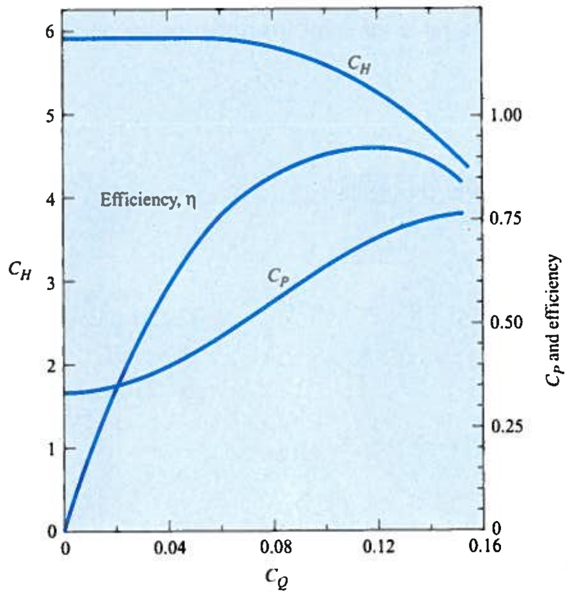


FIGURE 14.11
Dimensionless performance curves for a typical centrifugal pump, from data given in Fig. 14.9. [After Daugherty and Franzini (4).]

that required for flow at maximum efficiency. Normally, the motor used to drive the pump is chosen for conditions corresponding to maximum pump efficiency. Hence the flow can be throttled between the limits of shutoff condition and normal operating conditions with no chance of overloading the pump motor. In this latter case, a radial-flow pump offers a distinct advantage over axial-flow pumps.

Radial-flow pumps are manufactured in sizes from 1 hp or less and heads of 50 or 60 ft to thousands of horsepower and heads of several hundred feet. Figure 14.12 shows a cutaway view of a single-suction, single-stage, horizontal-shaft radial pump. Fluid enters in the direction of the rotating shaft and is accelerated outward by the rotating impeller. There are many other configurations designed for specific applications.

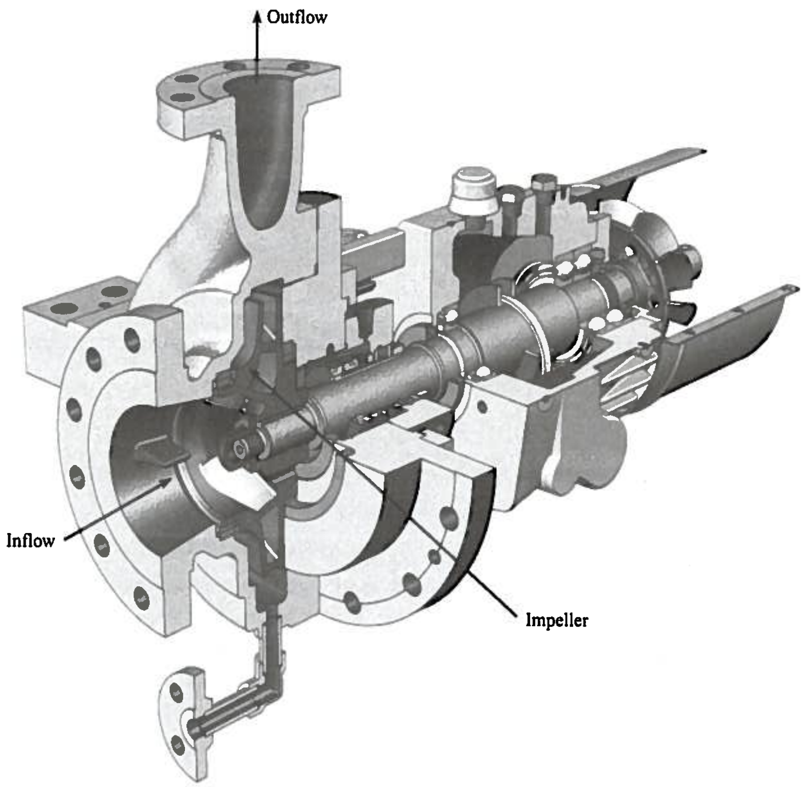


FIGURE 14.12
Cutaway view of a single-suction, single-st horizontal-shaft radial pump. Pump inlet, outl and impeller shown or photograph. (Copyrigl Sulzer Pumps)

Example 14.4 shows how to find the speed and discharge for a centrifugal pump needed to provide a given head.

EXAMPLE 14.4

Speed and Discharge of Centrifugal Pump

Problem Statement

A pump that has the characteristics given in Fig. 14.10 when operated at 2133.5 rpm is to be used to pump water at maximum efficiency under a head of 76 m. At what speed should the pump be operated, and what will the discharge be for these conditions?

Define the Situation

A centrifugal pump operated at 2133.5 rpm pumps water to head of 76 m at maximum efficiency.

Assumptions: Assume pump is the same size as that corresponding to Fig. 14.10 and water properties are the same.

State the Goal

1. Find the operational speed of pump (rpm).
2. Calculate discharge (m^3/s).

Generate Ideas and Make a Plan

The C_H, C_P, C_Q, and η are the same for any pump with the same characteristics operating at maximum efficiency. Thus

$$(C_H)_N = (C_H)_{2133.5\ \text{rpm}}$$

where N represents the unknown speed. Also $(C_Q)_N = (C_Q)_{2133.5\ \text{rpm}}$.

1. Calculate speed using same head coefficient.
2. Calculate discharge using same discharge coefficient.

Take Action (Execute the Plan)

1. Speed calculation: From Fig. 14.10, at maximum efficiency $\Delta H = 90$ m.

$$\left(\frac{g\Delta H}{n^2D^2}\right)_N = \left(\frac{g\Delta H}{n^2D^2}\right)_{2133.5}$$

$$\frac{76\text{ m}}{N^2} = \frac{90\text{ m}}{2133.5^2\text{ rpm}^2}$$

$$N = 2133.5 \times \left(\frac{76}{90}\right)^{1/2} = \boxed{1960\text{ rpm}}$$

2. Discharge calculation: From Fig. 14.10, at maximum efficiency $Q = 0.255\ \text{m}^3/\text{s}$.

$$\left(\frac{Q}{nD^3}\right)_N = \left(\frac{Q}{nD^3}\right)_{2133.5}$$

$$\frac{Q_{1960}}{Q_{2133.5}} = \frac{1960}{2133.5} = 0.919$$

$$Q_{1960} = \boxed{0.234\ \text{m}^3/\text{s}}$$

Example 14.5 shows how to scale up data for a specific centrifugal pump to predic performance.

EXAMPLE 14.5

Head, Discharge, and Power of a Centrifugal Pump

Problem Statement

The pump having the characteristics shown in Figs. 14.10 and 14.11 is a model of a pump that was actually used in one of the pumping plants of the Colorado River Aqueduct [see Daugherty and Franzini (4)]. For a prototype that is 5.33 times larger than the model and operates at a speed of 400 rpm, what head, discharge, and power are to be expected at maximum efficiency?

Define the Situation

A prototype pump is 5.33 times larger than the corresponding model. The prototype operates at 400 rpm.

Assumptions: Pumping water with $\rho = 10^3\ \text{kg/m}^3$.

State the Goal

Find (at maximum efficiency)

1. Head (in meters)
2. Discharge (in m^3/s)
3. Power (in kW)

Generate Ideas and Make a Plan

1. Find C_Q, C_H, and C_P at maximum efficiency from Fig. 14.11.
2. Evaluate speed in rps and calculate new diameter.
3. Use Eqs. (14.14) through (14.16) to calculate head, discharge, and power.

Take Action (Execute the Plan)

1. From Fig. 14.11 at maximum efficiency, $C_Q = 0.12$, $C_H = 5.2$ and $C_P = 0.69$.
2. Speed in rps: $n = (400/60)$ rps $= 6.67$ rps
$D = 0.371 \times 5.33 = 1.98$ m.
3. Pump performance
   - Head

$$\Delta H = \frac{C_H D^2 n^2}{g} = \frac{5.2(1.98\text{ m})^2(6.67\text{ s}^{-1})^2}{(9.81\text{ m/s}^2)} = \boxed{92.4\text{ m}}$$

   - Discharge

$$Q = C_Q n D^3 = 0.12(6.67\text{ s}^{-1})(1.98\text{ m})^3 = \boxed{6.21\ \text{m}^3/\text{s}}$$

   - Power

$$P = C_P \rho D^5 n^3 = 0.69((10^3\text{ kg})/\text{m}^3)(1.98\text{ m})^5(6.67\text{ s}^{-1})^3 = \boxed{6230\text{ kW}}$$

14.4 Specific Speed

From the discussion in the preceding sections it was pointed out that axial-flow pumps are best suited for high discharge and low head, whereas radial machines perform better for low discharge and high head. A tool for selecting the best pump is the value of a π-group called the specific speed, n_s. The **specific speed** is obtained by combining both C_H and C_Q in such a manner that the diameter D is eliminated:

$$n_s = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{(Q/nD^3)^{1/2}}{[\Delta H/(D^2n^2/g)]^{3/4}} = \frac{nQ^{1/2}}{g^{3/4}\Delta H^{3/4}}$$

Thus specific speed relates different types of pumps without reference to their sizes.

As shown in Fig. 14.13, when efficiencies of different types of pumps are plotted against n_s, it is seen that certain types of pumps have higher efficiencies for certain ranges of n_s. For low specific speeds, the radial-flow pump is more efficient, whereas high specific speeds favor axial-flow machines. In the range between the completely axial-flow machine and the completely radial-flow machine, there is a gradual change in impeller shape to accommodate the particular flow conditions with maximum efficiency. The boundaries between axial, mixed, and radial machines are somewhat vague, but the value of the specific speed provides some guidance on which machine would be most suitable. The final choice would depend on which pumps were commercially available as well as their purchase price, operating cost, and reliability.

It should be noted that the specific speed traditionally used for pumps in the United States is defined as $N_s = NQ^{1/2}/\Delta H^{3/4}$. Here the speed N is in revolutions per minute, Q is in gallons

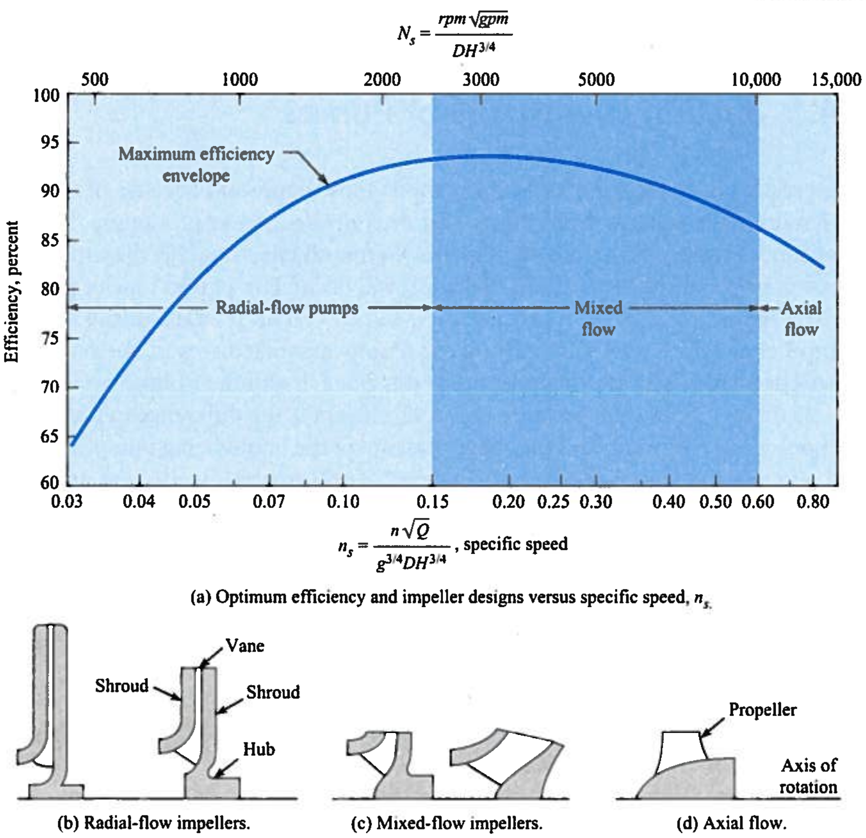


FIGURE 14.13
Optimum efficiency and impeller design versus specific speed.

per minute, and ΔH is in feet. This form is not dimensionless. Therefore its values are mu larger than those found for n_s (the conversion factor is 17,200). Most texts and referenc published before the introduction of the SI system of units use this traditional definition f specific speed.

Example 14.6 illustrates the use of specific speed to select a pump type.

EXAMPLE 14.6

Using Specific Speed to Select a Pump

Problem Statement

What type of pump should be used to pump water at the rate of 10 cfs and under a head of 600 ft? Assume $N = 1100$ rpm.

Define the Situation

A pump will be pumping water at 10 cfs for a head of 600 ft.

State the Goal

Find the best type of pump for this application.

Generate Ideas and Make a Plan

1. Calculate specific speed.
2. Use Fig. 14.13 to select pump type.

Take Action (Execute the Plan)

1. Rotational rate in rps

$$n = \frac{1100}{60} = 18.33 \text{ rps}$$

Specific speed

$$n_s = \frac{n\sqrt{Q}}{(g\Delta H)^{3/4}}$$

$$= \frac{18.33 \text{ rps} \times (10 \text{ cfs})^{1/2}}{(32.2 \text{ ft/s}^2 \times 600 \text{ ft})^{3/4}} = 0.035$$

2. From Fig. 14.13, a radial-flow pump is the best choice.

14.5 Suction Limitations of Pumps

The pressure at the suction side of a pump is most important because of the possibility th cavitation may occur. As water flows past the impeller blades of a pump, local high-velocit flow zones produce low relative pressures (Bernoulli effect), and if these pressures reach th vapor pressure of the liquid, then cavitation will occur. For a given type of pump operating a given speed and a given discharge, there will be certain pressure at the suction side of th pump below which cavitation will occur. Pump manufacturers in their testing procedur always determine this limiting pressure and include it with their pump performance curves.

More specifically, the pressure that is significant is the difference in pressure between th suction side of the pump and the vapor pressure of the liquid being pumped. Actually, in prac tice, engineers express this difference in terms of pressure head, called the **net positive suctio head**, which is abbreviated NPSH. To calculate NPSH for a pump that is delivering a give discharge, one first applies the energy equation from the reservoir from which water is bein pumped to the section of the intake pipe at the suction side of the pump. Then one subtract the vapor pressure head of the water to determine NPSH.

In Fig. 14.14, points 1 and 2 are the points between which the energy equation would b written to evaluate NPSH.

A more general parameter for indicating susceptibility to cavitation is specific speed. Howeve instead of using head produced (ΔH), one uses NPSH for the variable to the 3/4 power. This is

$$n_{ss} = \frac{nQ^{1/2}}{g^{3/4}(\text{NPSH})^{3/4}}$$

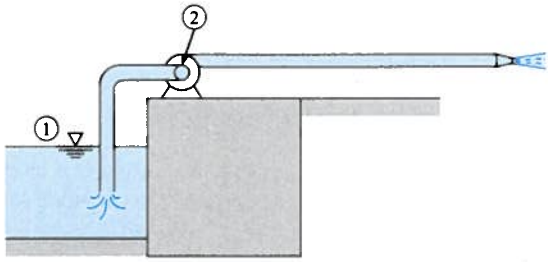


FIGURE 14.14
Locations used to evalu
NPSH for a pump.

Here n_{ss} is called the suction specific speed. The more traditional suction specific speed used in the United States is $N_{ss} = NQ^{1/2}/(\text{NPSH})^{3/4}$, where N is in rpm, Q is in gallons per minute (gpm), and NPSH is in feet. Analyses of data from pump tests show that the value of the suction specific speed is a good indicator of whether cavitation may be expected. For example, the Hydraulic Institute (5) indicates that the critical value of N_{ss} is 8500. The reader is directed to manufacturer's data or the Hydraulic Institute for more details about critical NPSH or N_{ss}.

An analysis to find NPSH for a pump system is illustrated in Example 14.7.

EXAMPLE 14.7

Calculating Net Positive Suction Head

Problem Statement

In Fig. 14.14 the pump delivers 2 cfs flow of 80°F water, and the intake pipe diameter is 8 in. The pump intake is located 6 ft above the water surface level in the reservoir. The pump operates at 1750 rpm. What are the net positive suction head and the traditional suction specific speed for these conditions?

Define the Situation

A pump delivers 2 cfs flow of 80°F water.

Assumptions:

1. Entrance loss coefficient = 0.10.
2. Bend loss coefficient = 0.20.

Properties: Table A.5, (Water at 80°F) $\gamma = 62.2$ lbf/ft³, and $p_{vap} = 0.506$ psi.

State the Goal

- Calculate the positive suction head (NPSH).
- Calculate the traditional suction specific speed (N_{ss}).

Generate Ideas and Make a Plan

The net positive suction head is the difference between pressure at pump inlet and the vapor pressure.

1. Determine the atmospheric pressure in head of water for reservoir surface.
2. Determine velocity in 8-in. pipe.
3. Apply the energy equation between the reservoir and pur entrance.
4. Calculate NPSH.
5. Calculate N_{ss} with $N_{ss} = (NQ^{1/2})/(\text{NPSH})^{3/4}$.

Take Action (Execute the Plan)

1. Pressure head at reservoir

$$\frac{p_1}{\gamma} = \frac{14.7 \text{ lbf/in}^2 \times 144 \text{ (in}^2/\text{ft}^2)}{62.2 \text{ lbf/ft}^3} = 34 \text{ ft}$$

2. Velocity in pipe

$$V_2 = \frac{Q}{A} = \frac{2 \text{ cfs}}{\pi \times ((4 \text{ in})/12)^2} = 5.73 \text{ ft/s}$$

3. Energy equation between points 1 and 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

- Input values

$$V_1 = 0, \quad z_1 = 0, \quad z_2 = 6$$

- Head loss

$$\sum h_L = (0.1 + 0.2)\frac{V_2^2}{2g}$$

- Head at pump entrance

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} - z_2 - \frac{V_2^2}{2g}(1 + 0.3)$$

$$= 34 - 6 - 1.3 \times \frac{5.73^2}{2 \times 32.2} = 27.3 \text{ ft}$$

4. Vapor pressure in feet of head

$$0.506 \times 144/62.2 = 1.17 \text{ ft.}$$

Net positive suction head

$$\text{NPSH} = 27.3 - 1.17 = 26.1 \text{ ft}$$

5. Traditional suction specific speed

$$Q = 2 \text{ cfs} = 898 \text{ gpm}$$

$$N_{ss} = (1750)(898)^{1/2}/(26.1)^{3/4} = \boxed{4540}$$

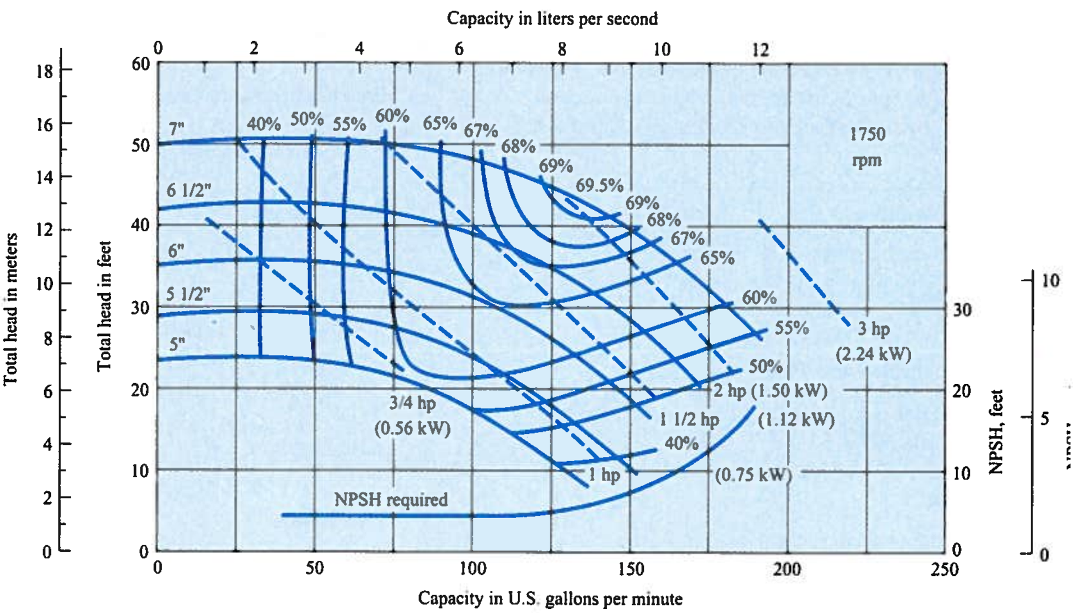
Review the Solution and the Process

1. *Discussion.* For a typical single-stage centrifugal pump with an intake diameter of 8 in. and pumping 2 cfs, the critical NPSH is normally about 10 ft; therefore, the pump of this example is operating well within the safe range with respect to cavitation susceptibility.
2. *Discussion.* This value of N_{ss} is much below the critical limit of 8500; therefore, it is in a safe operating range so far as cavitation is concerned.

A typical pump performance curve for a centrifugal pump that would be supplied by pump manufacturer is shown in Fig. 14.15. The solid lines labeled from 5 in. to 7 in. represen different impeller sizes that can be accommodated by the pump housing. These curves give th head delivered as a function of discharge. The dashed lines represent the power required by th pump for a given head and discharge. Lines of constant efficiency are also shown. Obviousl when selecting an impeller, one would like to have the operating point as close as possible t the point of maximum efficiency. The NPSH value gives the minimum head (absolute head) a the pump intake for which the pump will operate without cavitation.

FIGURE 14.15

Centrifugal pump performance curve. [After McQuiston and Parker (6). Used with permission of John Wiley and Sons.]



14.6 Viscous Effects

In the foregoing sections, similarity parameters were developed to predict prototype result from model tests, neglecting viscous effects. The latter assumption is not necessarily valic especially if the model is quite small. To minimize the viscous effects in modeling pumps, th

Hydraulic Institute standards (5) recommend that the size of the model be such that the model impeller is not less than 30 cm in diameter. These same standards state that "the model should have complete geometric similarity with the prototype, not only in the pump proper, but also in the intake and discharge conduits."

Even with complete geometric similarity, one can expect the model to be less efficient than the prototype. An empirical formula proposed by Moody (7) is used for estimating prototype efficiencies of radial- and mixed-flow pumps and turbines from model efficiencies. That formula is

$$\frac{1-e_1}{1-e}=\left(\frac{D}{D_1}\right)^{1/5} \tag{14.18}$$

Here e_1 is the efficiency of the model and e is the efficiency of the prototype.

Example 14.8 shows how to estimate the efficiency due to viscous effects.

EXAMPLE 14.8

Calculating Viscous Effects on Pump Efficiency

Problem Statement

A model having an impeller diameter of 45 cm is tested and found to have an efficiency of 85%. If a geometrically similar prototype has an impeller diameter of 1.80 m, estimate its efficiency when it is operating under conditions that are dynamically similar to those in the model test ($C_{Q,\text{model}} = C_{Q,\text{prototype}}$).

Define the Situation

A pump with a 45-cm diameter impeller has 85% efficiency.

Assumptions: The efficiency differences are due to viscous effects.

State the Goal

Find the efficiency of a pump with a 1.6-m impeller.

Generate Ideas and Make a Plan

Use Eq. (14.18) to determine viscous effects.

Take Action (Execute the Plan)

Efficiency

$$e = 1-\frac{1-e_1}{(D/D_1)^{1/5}} = 1-\frac{0.15}{1.32} = 1-0.11 = 0.89$$

or

$$\boxed{e = 89\%}$$

14.7 Centrifugal Compressors

Centrifugal compressors are similar in design to centrifugal pumps. Because the density of the air or gases used is much less than the density of a liquid, the compressor must rotate at much higher speeds than the pump to effect a sizable pressure increase. If the compression process were isentropic and the gases ideal, the power necessary to compress the gas from p_1 to p_2 would be

$$P_{\text{theo}} = \frac{k}{k-1}Q_1 p_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right] \tag{14.19}$$

where Q_1 is the volume flow rate into the compressor and k is the ratio of specific heats. The power calculated using Eq. (14.19) is referred to as the **theoretical adiabatic power**. The efficiency of a compressor with no water cooling is defined as the ratio of the theoretical adiabatic power to the actual power required at the shaft. Ordinarily the efficiency improves with higher inlet-volume flow rates, increasing from a typical value of 0.60 at 0.6 m^3/s to 0.74 at 40 m^3/s. Higher efficiencies are obtainable with more expensive design refinements.

Example 14.9 shows how to calculate shaft power required to operate a compressor.

EXAMPLE 14.9

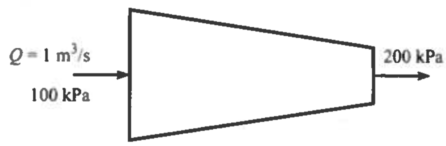
Calculating Shaft Power for a Centrifugal Compressor

Problem Statement

Determine the shaft power required to operate a compressor that compresses air at the rate of 1 m^3/s from 100 kPa to 200 kPa. The efficiency of the compressor is 65%.

Define the Situation

The inlet flow rate to a compressor is 1.0 m^3/s. The pressure change is from 100 kPa to 200 kPa.



From Table A.2, $k = 1.4$.

State the Goal

P_{shaft}(kW) ⬅ Required shaft power (in kW).

Generate Ideas and Make a Plan

1. Use Eq. (14.19) to calculate theoretical power.
2. Divide theoretical power by efficiency to find shaft (required) power.

Take Action (Execute the Plan)

1. Theoretical power

$$P_{theo} = \frac{k}{k-1} Q_1 p_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]$$

$$= (3.5)(1\ m^3/s)(10^5\ N/m^2)[(2)^{0.286} - 1]$$

$$= 0.767 \times 10^5\ N \cdot m/s = 76.7\ kW$$

2. Shaft power

$$P_{shaft} = \frac{76.7}{0.65} kW = \boxed{118\ kW}$$

Cooling is necessary for high-pressure compressors because of the high gas temperature resulting from the compression process. Cooling can be achieved through the use of wate jackets or intercoolers that cool the gases between stages. The efficiency of water-cooled com pressors is based on the power required to compress ideal gases isothermally, or

$$P_{theo} = p_1 Q_1 \ln \frac{p_2}{p_1} \tag{14.2C}$$

which is usually called the **theoretical isothermal power**. The efficiencies of water-coole compressors are generally lower than those of noncooled compressors. If a compressor i cooled by water jackets, its efficiency characteristically ranges between 55% and 60%. The us of intercoolers results in efficiencies from 60% to 65%.

Application to Fluid Systems

The selection of a pump, fan, or compressor for a specific application depends on the desire flow rate. This process requires the acquisition or generation of a system curve for the flov system of interest and a performance curve for the fluid machine. The intersection of these tw curves provides the operating point as discussed in Chapter 10.

For example, consider using the centrifugal pump with the characteristics shown in Fig. 14.1 to pump water at 60°F from a wall into a tank as shown in Fig. 14.16. A pumping capacity of a least 80 gpm is required. Two hundred feet of standard schedule-40 2-inch galvanized iro pipe are to be used. There is a check valve in the system as well as an open gate valve. There i

FIGURE 14.16
System for pumping water from a well into a tank.

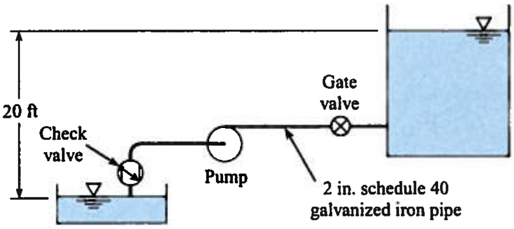
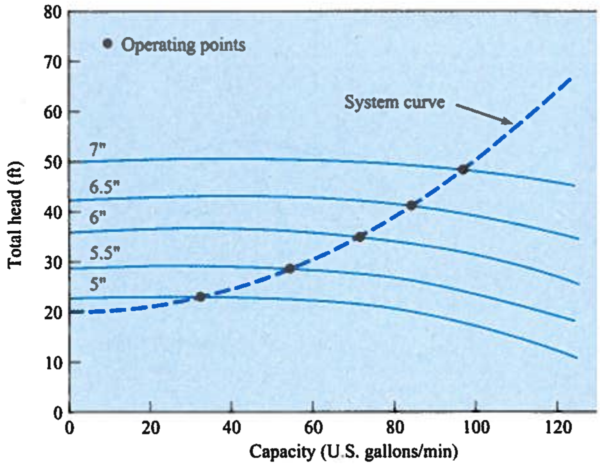


FIGURE 14.17
System and pump performance curves for pumping applicat



a 20-ft elevation between the well and the top of the fluid in the tank. Applying the energy equation, the head required by the pump is

$$h_p = \Delta z + \frac{V^2}{2g}\left(\frac{fL}{D} + \sum K_L\right)$$

where K_L represents the head loss coefficients for the entrance, check valve, gate valve, and sudden-expansion loss entering the tank. Using representative values for the loss coefficients and evaluating the friction factor from the Moody diagram in Chapter 10 leads to

$$h_p = 20 + 0.00305\, Q^2$$

where Q is the flow rate in gpm. This is the system curve.

The result of plotting the system curve on the pump performance curves is shown in Fig. 14.17. The locations where the lines cross are the operating points. One notes that a discharge of just over 80 gpm is achieved with the 6.5 in. impeller. Also, referring back to Fig. 14.15, the efficiency at this point is about 62%. To ensure that the design requirements are satisfied, the engineer may select the larger impeller, which has an operating point of 95 gpm. If the pump is to be used in continuous operation and the efficiency is important to operating costs, the engineer may choose to consider another pump that would have a higher efficiency at the operation point. An engineer experienced in the design of pump systems would be very familiar with the trade-offs for economy and performance and could make a design decision relatively quickly.

In some systems it may be advantageous to use two pumps in series or in parallel. If two pumps are used in series, the performance curve is the sum of the pump heads of the two machines at the same flow rate, as shown in Fig. 14.18a. This configuration would be desirable for a flow system with a steep system curve, as shown in the figure. If two pumps are connected in parallel, the performance curve is obtained by adding the flow rates of the two pumps at the same pump heads, as shown in Fig. 14.18b. This configuration would be advisable for flow systems with shallow system curves, as shown in the figure. The concepts presented here for pumps also apply to fans and compressors.

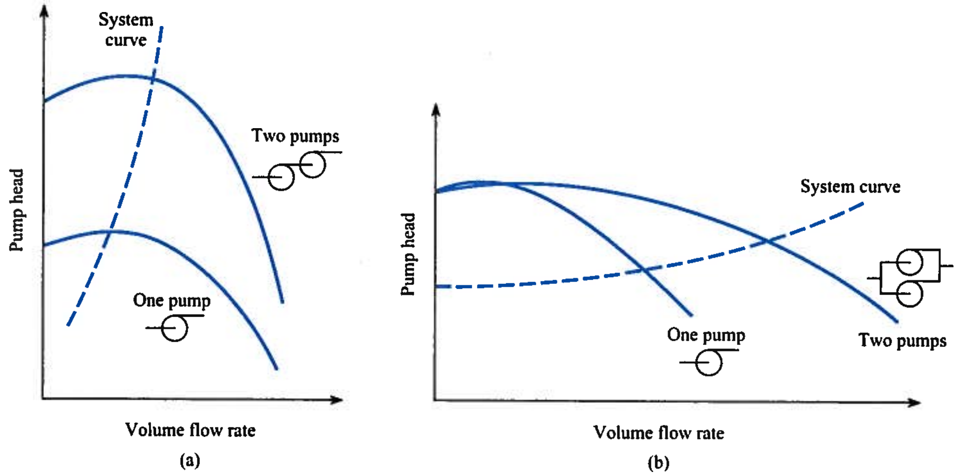


FIGURE 14.18
Pump performance curves for pumps connected in series (a) and in parallel (b).

14.8 Turbines

A **turbine** is defined as a machine that extracts energy from a moving fluid. Much of the basic theory and most similarity parameters used for pumps also apply to turbines. However, there are some differences in physical features and terminology. The details of the flow through the impellers of radial-flow machines have not yet been considered. These topics will now be addressed.

The two main categories of hydraulic machines are the **impulse** and **reaction** turbines. In a reaction turbine, the water flow is used to rotate a turbine wheel or runner through the action of vanes or blades attached to the wheel. When the blades are oriented like a propeller, the flow is axial and the machine is called a **Kaplan turbine**. When the vanes are oriented like an impeller in a centrifugal pump, the flow is radial, and the machine is called a **Francis turbine**. In an impulse turbine, the water accelerates through a nozzle and impinges on vanes attached to the rim of the wheel. This machine is called a **Pelton wheel**.

Impulse Turbine

In the impulse turbine a jet of fluid issuing from a nozzle impinges on vanes of the turbine wheel, or **runner**, thus producing power as the runner rotates (see Fig. 14.19). Figure 14.20 shows a runner for the Henry Borden hydroelectric plant in Brazil. The primary feature of the impulse turbine with respect to fluid mechanics is the power production as the jet is deflected by the moving vanes. When the momentum equation is applied to this deflected jet, it can be shown [see Daugherty and Franzini (4)] for idealized conditions that the maximum power will be developed when the vane speed is one-half of the initial jet speed. Under such conditions the exiting jet speed will be zero—all the kinetic energy of the jet will have been expended in driving the vane. Thus if one applies the energy equation, between the incoming jet and the exiting fluid (assuming negligible head loss and negligible kinetic energy at exit), it is found that the head given up to the turbine is $h_t = (V_j^2/2g)$, and the power thus developed is

$$P = Q\gamma h_t \tag{14.21}$$

FIGURE 14.19
Impulse turbine.

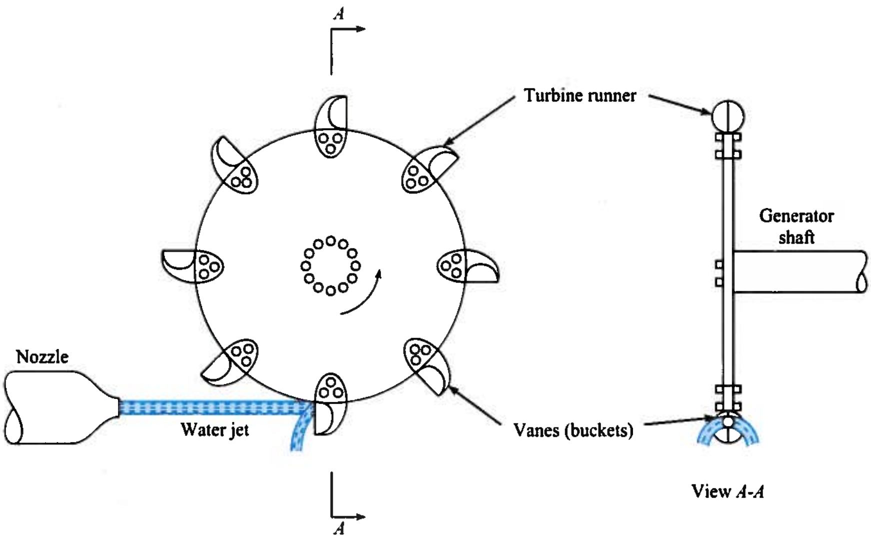


FIGURE 14.20
Grinding of a bucket on a Pelton wheel turbine located at Voith Hydro's Sao Paulo Plant. (By courtesy of Voith Hydro)

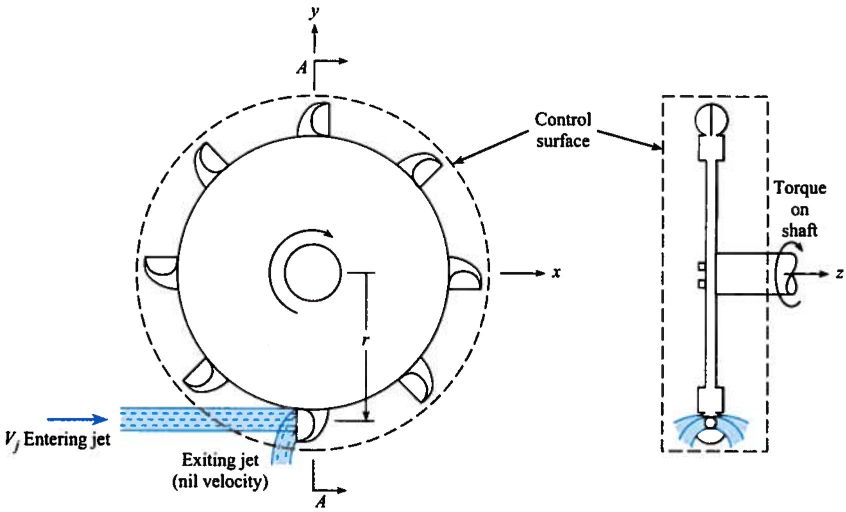


FIGURE 14.21
Control-volume approach for the impulse turbine using the angular-momentum principle.

where Q is the discharge of the incoming jet, γ is the specific weight of jet fluid, and $h_t = V_j^2/2g$ or the velocity head of the jet. Thus Eq. (14.21) reduces to

$$P = \rho Q \frac{V_j^2}{2} \tag{14.22}$$

To obtain the torque on the turbine shaft, the angular-momentum equation is applied to a control volume, as shown in Fig. 14.21. For steady flow

$$\sum M = \sum_{cs} \mathbf{r}_o \times (\dot{m}_o \mathbf{v}_o) - \sum_{cs} \mathbf{r}_i \times (\dot{m}_i \mathbf{v}_i)$$

Generally it is assumed that the exiting fluid has negligible angular momentum. The moment acting on the system is the torque T acting on the shaft. Thus the angular-momentum equation reduces to

$$T = -\dot{m} r V_j \tag{14.23}$$

The mass flow rate across the control surface is ρQ, so the torque is

$$T = -\rho Q V_j r$$

The minus sign indicates that the torque applied to the system (to keep it rotating at constant angular velocity) is in the clockwise direction. However, the torque applied by the system to the shaft is in the counterclockwise direction, which is the direction of wheel rotation, so

$$T = \rho Q V_j r \tag{14.24}$$

The power developed by the turbine is $T\omega$, or

$$P = \rho Q V_j r \omega \tag{14.25}$$

Furthermore, if the velocity of the turbine vanes is $(1/2)V_j$ for maximum power, as noted earlier, then $P = \rho Q V_j^2/2$, which is the same as Eq. (14.22).

The calculation of torque for an impulse turbine is illustrated in Example 14.10.

EXAMPLE 14.10

Analyzing an Impulse Turbine

Problem Statement

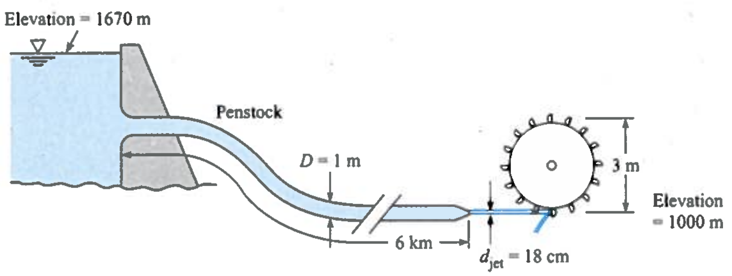
What power in kilowatts can be developed by the impulse turbine shown if the turbine efficiency is 85%? Assume that the resistance coefficient f of the penstock is 0.015 and the head loss in the nozzle itself is negligible. What will be the angular speed of the wheel, assuming ideal conditions ($V_j = 2V_{\text{bucket}}$), and what torque will be exerted on the turbine shaft?

Define the Situation

This problem involves an impulse turbine with an efficiency of 85%.

Assumptions:

1. There is no entrance loss.
2. Head loss in nozzle is negligible.
3. Water density is 1000 kg/m³.



State the Goal

Find:

- Power (kW) developed by turbine
- Angular speed (rpm) of wheel for maximum efficiency
- Torque (kN · m) on turbine shaft

Generate Ideas and Make a Plan

1. Apply energy equation, to find nozzle velocity.
2. Use Eq. (14.22) for power.
3. For maximum efficiency, $\omega r = (V_j/2)$.
4. Calculate torque from $P = T\omega$.

Take Action (Execute the Plan)

1. Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_j}{\gamma} + \frac{V_j^2}{2g} + z_j + h_L$$

- Values in energy equation

$$p_1 = 0, z_1 = 1670 \text{ m}, V_1 = 0, p_j = 0, z_j = 1000 \text{ m}$$

- Penstock-supply pipe velocity ratio

$$V_{\text{penstock}} = \frac{V_j A_j}{A_{\text{penstock}}} = V_j \left(\frac{0.18 \text{ m}}{1 \text{ m}} \right)^2 = 0.0324\, V_j$$

- Head loss

$$h_L = f \frac{L}{D} \frac{1}{2g} V_{\text{penstock}}^2$$

$$= \frac{0.015 \times 6000}{1} (0.0324)^2 \frac{V_j^2}{2g} = 0.094 \frac{V_j^2}{2g}$$

- Jet velocity

$$z_1 - z_2 = 1.094 \frac{V_j^2}{2g}$$

$$V_j = \left(\frac{2 \times 9.81 \text{ m/s}^2 \times 670 \text{ m}}{1.094} \right)^{1/2} = 109.6 \text{ m/s}$$

2. Gross power

$$P = Q\gamma \frac{V_j^2}{2g} = \frac{\gamma A_j V_j^3}{2g}$$

$$= \frac{9810(\pi/4)(0.18)^2(109.6)^3}{2 \times 9.81} = 16{,}750 \text{ kW}$$

Power delivered

$$P = 16{,}750 \times \text{efficiency} = \boxed{14{,}240 \text{ kW}}$$

3. Angular speed of wheel

$$V_{\text{bucket}} = \frac{1}{2}(109.6 \text{ m/s}) = 54.8 \text{ m/s}$$

$$r\omega = 54.8 \text{ m/s}$$

$$\omega = \frac{54.8 \text{ m/s}}{1.5 \text{ m}} = 36.5 \text{ rad/s}$$

Wheel speed

$$N = (36.5 \text{ rad/s}) \frac{1 \text{ rev}}{2\pi \text{ rad}} (60 \text{ s/min}) = \boxed{349 \text{ rpm}}$$

4. Torque

$$T = \frac{\text{power}}{\omega} = \frac{14{,}240 \text{ kW}}{36.5 \text{ rad/s}} = \boxed{390 \text{ kN} \cdot \text{m}}$$

Reaction Turbine

In contrast to the impulse turbine, where a jet under atmospheric pressure impinges on on one or two vanes at a time, flow in a reaction turbine is under pressure and reacts on all van of the impeller turbine simultaneously. Also, this flow completely fills the chamber in whic the impeller is located (see Fig. 14.22). There is a drop in pressure from the outer radius of tl impeller, r_1, to the inner radius, r_2. This is another point of difference with the impulse turbin in which the pressure is the same for the entering and exiting flows. The original form of tl reaction turbine, first extensively tested by J. B. Francis, had a completely radial-flow impell (Fig. 14.23). That is, the flow passing through the impeller had velocity components only in plane normal to the axis of the runner. However, more recent impeller designs, such as tl mixed-flow and axial-flow types, are still called reaction turbines.

Torque and Power Relations for the Reaction Turbine

As for the impulse turbine, the angular-momentum equation is used to develop formulas for tl torque and power for the reaction turbine. The segment of turbine runner shown in Fig. 14.2

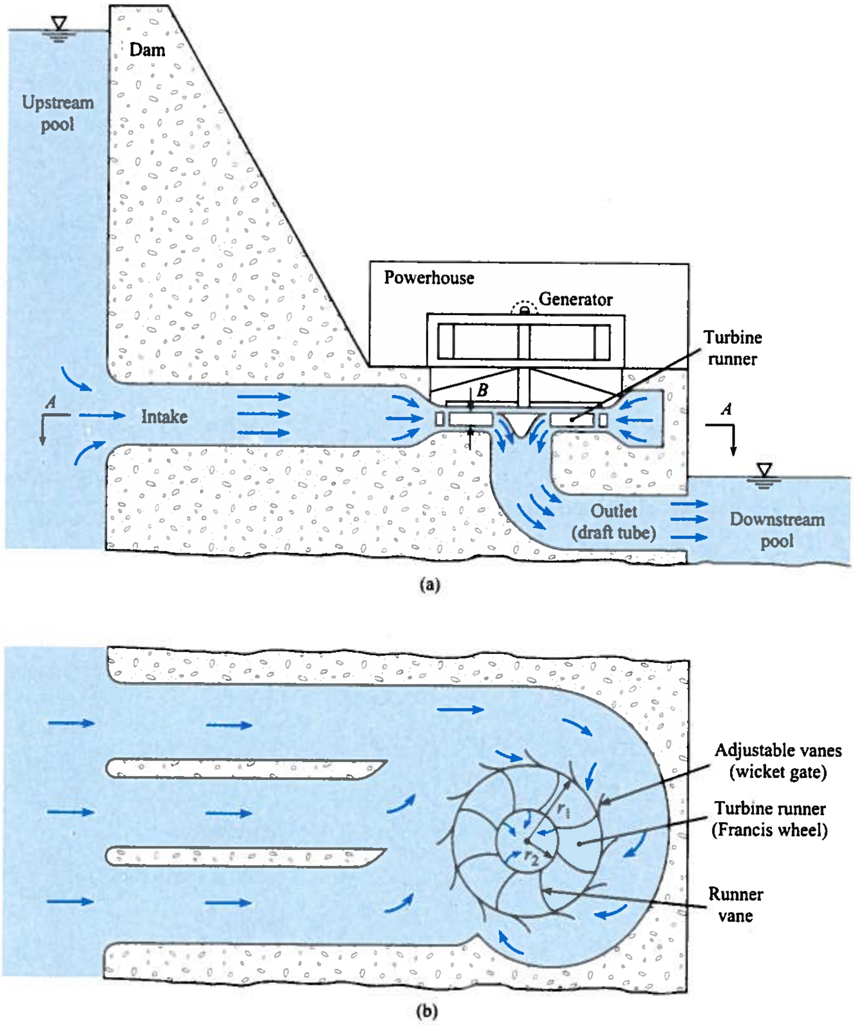


FIGURE 14.22
Schematic view of a reaction-turbine installation. (a) Elevation view. (b) Plan view, section *A-A*.

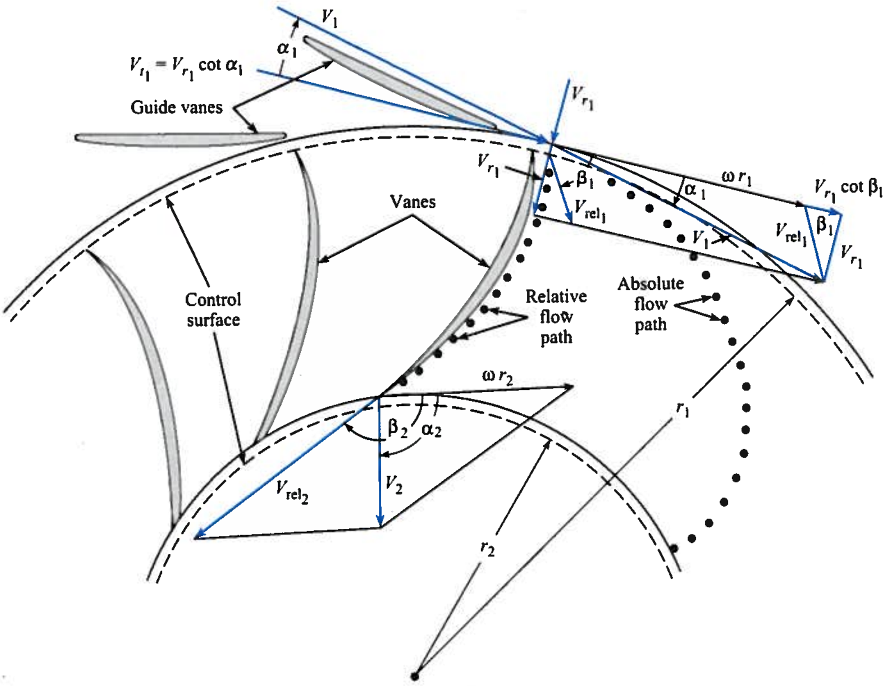


FIGURE 14.23
Velocity diagrams for the impeller for a Franc turbine.

depicts the flow conditions that occur for the entire runner. The guide vanes outside the runner itself cause the fluid to have a tangential component of velocity around the entire circumference of the runner. Thus the fluid has an initial amount of angular momentum with respect to the turbine axis when it approaches the turbine runner. As the fluid passes through the passages of the runner, the runner vanes effect a change in the magnitude and direction of its velocity. Thus the angular momentum of the fluid is changed, which produces a torque on the runner. This torque drives the runner, which, in turn, generates power.

To quantify the preceding, let V_1 and α_1 represent the incoming velocity and the angle of the velocity vector with respect to a tangent to the runner, respectively. Similar terms at the inner-runner radius are V_2 and α_2. Applying the angular-momentum equation for steady flow, Eq. (6.27), to the control volume shown in Fig. 14.23 yields

$$
\begin{aligned}
T &= \dot{m}(-r_2 V_2 \cos \alpha_2) - \dot{m}(-r_1 V_1 \cos \alpha_1) \\
&= \dot{m}(r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2)
\end{aligned}
\qquad \textbf{(14.26)}
$$

The power from this turbine will be $T\omega$, or

$$
P = \rho Q \omega (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2) \qquad \textbf{(14.27)}
$$

Equation (14.27) shows that the power production is a function of the directions of the flow velocities entering and leaving the impeller—that is, α_1 and α_2.

It is interesting to note that even though the pressure varies within the flow in a reaction turbine, it does not enter into the expressions derived using the angular-momentum equation. The reason it does not appear is that the chosen outer and inner control surfaces are concentric with the axis about which the moments and angular momentum are evaluated. The pressure forces acting on these surfaces all pass through the given axis; therefore they do not produce moments about the given axis.

Vane Angles

It should be apparent that the head loss in a turbine will be less if the flow enters the runr with a direction tangent to the runner vanes than if the flow approaches the vane with angle of attack. In the latter case, separation will occur with consequent head loss. Th vanes of an impeller designed for a given speed and discharge and with fixed guide var will have a particular optimum blade angle β_1. However, if the discharge is changed from t condition of the original design, the guide vanes and impeller vane angles will not "matc the new flow condition. Most turbines for hydroelectric installations are made with movat guide vanes on the inlet side to effect a better match at all flows. Thus α_1 is increased or d creased automatically through governor action to accommodate fluctuating power deman on the turbine.

To relate the incoming-flow angle α_1 and the vane angle β_1, first assume that the flc entering the impeller is tangent to the blades at the periphery of the impeller. Likewise, tl flow leaving the stationary guide vane is assumed to be tangent to the guide vane. To develop tl desired equations, consider both the radial and the tangential components of velocity at the out periphery of the wheel ($r = r_1$). It is easy to compute the radial velocity, given Q and tl geometry of the wheel, by the continuity equation:

$$V_{r_1} = \frac{Q}{2\pi r_1 B} \quad (14.2$$

where B is the height of the turbine blades. The tangential (tangent to the outer surface of tl runner) velocity of the incoming flow is

$$V_{t_1} = V_{r_1} \cot \alpha_1 \quad (14.2$$

However, this tangential velocity is equal to the tangential component of the relative velocit in the runner, $V_{r_1} \cot \beta_1$, plus the velocity of the runner itself, ωr_1. Thus the tangential velocit when viewed with respect to the runner motion, is

$$V_{t_1} = r_1\omega + V_{r_1} \cot \beta_1 \quad (14.3($$

Now, eliminating V_{t_1} between Eqs. (14.29) and (14.30) results in

$$V_{r_1} \cot \alpha_1 = r_1\omega + V_{r_1} \cot \beta_1 \quad (14.31$$

Equation (14.31) can be rearranged to yield

$$\alpha_1 = \text{arccot}\left(\frac{r_1\omega}{V_{r_1}} + \cot \beta_1\right) \quad (14.32$$

Example 14.11 illustrates how to calculate the inlet blade angle to avoid separation.

EXAMPLE 14.11

Analyzing a Francis Turbine

Problem Statement

A Francis turbine is to be operated at a speed of 600 rpm and with a discharge of 4.0 m^3/s. If $r_1 = 0.60$ m, $\beta_1 = 110°$, and the blade height B is 10 cm, what should be the guide vane angle α_1 for a nonseparating flow condition at the runner entrance?

Define the Situation

A Francis turbine is operating with an angular speed of 600 rpm and a discharge of 4.0 m^3/s.

State the Goal

Find the inlet guide vane angle, α_1.

Generate Ideas and Make a Plan

Use Eq. (14.32) for inlet guide angle.

Take Action (Execute the Plan)

Radial velocity at inlet

$$\alpha_1 = \text{arccot}\left(\frac{r_1\omega}{V_{r_1}} + \cot\beta_1\right)$$

$$r_1\omega = 0.6 \times 600 \text{ rpm} \times 2\pi \text{ rad/rev} \times 1/60 \text{ min/s} = 37.7 \text{ m/s}$$

Inlet guide vane angle

$$V_{r_1} = \frac{Q}{2\pi r_1 B} = \frac{4.00 \text{ m}^3/\text{s}}{2\pi \times 0.6 \text{ m} \times 0.10 \text{ m}} = 10.61 \text{ m/s}$$

$$\cot\beta_1 = \cot(110^\circ) = -0.364$$

$$\alpha_1 = \text{arccot}\left(\frac{37.7}{10.61} - 0.364\right) = \boxed{17.4^\circ}$$

Specific Speed for Turbines

Because of the attention focused on the production of power by turbines, the specific speed for turbines is defined in terms of power:

$$n_s = \frac{nP^{1/2}}{g^{3/4}\gamma^{1/2}h_t^{5/4}}$$

It should also be noted that large water turbines are innately more efficient than pumps. The reason for this is that as the fluid leaves the impeller of a pump, it decelerates appreciably over a relatively short distance. Also, because guide vanes are generally not used in the flow passages with pumps, large local velocity gradients develop, which in turn cause intense mixing and turbulence, thereby producing large head losses. In most turbine installations, the flow that exits the turbine runner is gradually reduced in velocity through a gradually expanding draft tube, thus producing a much smoother flow situation and less head loss than for the pump. For additional details of hydropower turbines, see Daugherty and Franzini (4).

Gas Turbines

The conventional gas turbine consists of a compressor that pressurizes the air entering the turbine and delivers it to a combustion chamber. The high-temperature, high-pressure gases resulting from combustion in the combustion chamber expand through a turbine, which both drives the compressor and delivers power. The theoretical efficiency (power delivered/rate of energy input) of a gas turbine depends on the pressure ratio between the combustion chamber and the intake; the higher the pressure ratio, the higher the efficiency. The reader is directed to Cohen et al. (8) for more detail.

Wind Turbines

Wind energy is discussed frequently as an alternative energy source. The application of wind turbines* as potential sources for power becomes more attractive as utility power rates increase and the concern over greenhouse gases grows. In many European countries, especially northern Europe, the wind turbine is playing an ever-increasing role in power generation.

In essence, the wind turbine is just a reverse application of the process of introducing energy into an airstream to derive a propulsive force. The wind turbine extracts energy from

*The phrase "wind turbine" is used to convey the idea of conversion of wind to electrical energy. A windmill converts wind energy to mechanical energy.

the wind to produce power. There is one significant difference, however. The theoretical upp limit of efficiency of a propeller supplying energy to an airstream is 100%; that is, it is the retically possible, neglecting viscous and other effects, to convert all the energy supplied tc propeller into energy of the airstream. This is not the case for a wind turbine.

A sketch of a horizontal-axis wind turbine is shown in Fig. 14.24. The wind blows alo the axis of the turbine. The area of the circle traced out by the rotating blades is the **captu area**. The power associated with the wind passing through the capture area is

$$P = \rho Q \frac{V^2}{2} = \rho A \frac{V^3}{2} \quad \text{(14.3}$$

where ρ is the air density and V is the wind speed. In an analysis attributed to Glauert/Betz (9), tl theoretical maximum power attainable from a wind turbine is 16/27 or 59.3% of this power or

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho V^3 A\right) \quad \text{(14.3}$$

Other factors, such as swirl of the airstream and viscous effects, further reduce the pow achievable from a wind turbine.

FIGURE 14.24

Horizontal-axis wind turbine showing capture area.

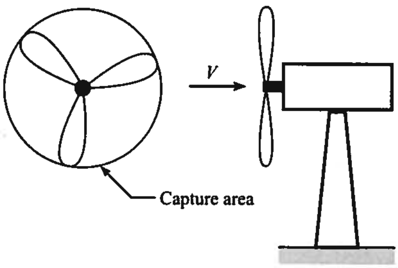
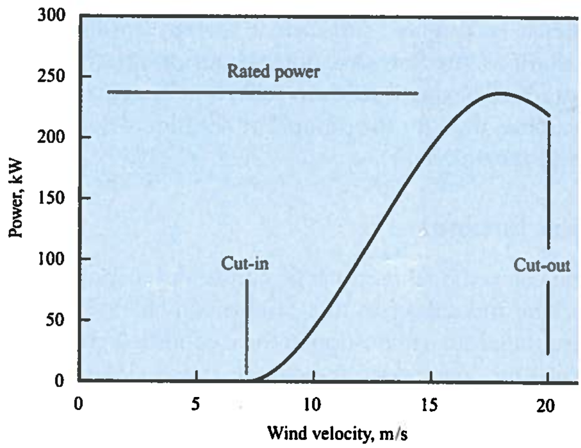


FIGURE 14.25

Typical wind turbine power curve.



The power output of any wind turbine is related to the wind speed through the winc turbine power curve. A typical curve is shown in Fig. 14.25. This curve can usually be obtaine from the manufacturer. The wind turbine is inoperative below the cut-in speed. After cut-i the power increases with wind speed reaching a maximum value, which is the rated pow output for the turbine. Engineering design and safety constraints impose an upper limit on th rotational velocity and establish the cutout speed. A braking system is used to prevent opera tion of the wind turbine beyond this velocity.

The conventional horizontal-axis wind turbine has been the focus of most research an design. Considerable effort has also been devoted to assessment of the Savonius rotor and th Darrieus turbine, both of which are vertical-axis turbines, as shown in Fig. 14.26. The Savoni rotor consists of two curved blades forming an S-shaped passage for the air flow. The Darrieu turbine consists of two or three airfoils attached to a vertical shaft; the unit resembles an eg beater. The advantage of vertical-axis turbines is that their operation is independent of win

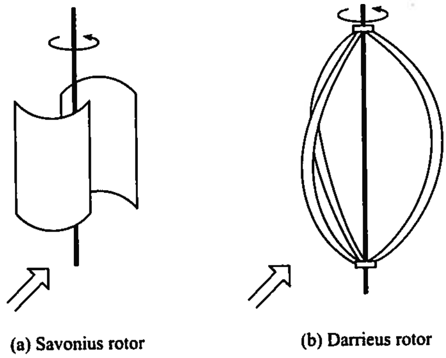


FIGURE 14.26 Wind turbine configurations. (a) Savonius turbine. (b) Darrieus turbine.

direction. The Darrieus wind turbine is considered superior in performance but has a disadvantage in that it is not self-starting. Frequently, a Savonius rotor is mounted on the axis of a Darrieus turbine to provide the starting torque.

For more information on wind turbines and wind turbine systems, refer to *Wind Energy Explained* (10).

EXAMPLE 14.12

Calculating the Capture Area of a Wind Turbine

Problem Statement

Calculate the minimum capture area necessary for a windmill that has to operate five 100-watt bulbs if the wind velocity is 20 km/h and the air density is 1.2 kg/m^3.

Define the Situation

A wind turbine needs to produce produces of 500 watts.

State the Goal

Find the minimum capture area of the windmill.

Generate Ideas and Make a Plan

Use equation for maximum power of windmill.

Take Action (Execute the Plan)

Capture area for maximum power

$$A = P_{max}\frac{54}{16}\frac{1}{\rho V^3}$$

Wind velocity in m/s

$$20\text{ km/h} = \frac{20 \times 1000}{3600} = 5.56\text{ m/s}$$

Minimum capture area

$$A = 500\text{ W} \times \frac{54}{16} \times \frac{1}{1.2\text{ kg/m}^3 \times (5.56\text{ m/s})^3}$$

$$= \boxed{8.18\text{ m}^2}$$

Review the Solution and the Process

Discussion. This area corresponds to a windmill diameter o 3.23 m, or about 10.6 ft.

14.9 Summarizing Key Knowledge

The Propeller

- The thrust of a propeller is calculated using

$$F_T = C_T \rho n^2 D^4$$

where ρ is the fluid density, n is the rotational rate of the propeller, and D is the propeller diameter. The thrust coefficient C_T is a function of the advance ratio V_0/nD.

- The efficiency of a propeller is the ratio of the power delivered by the propeller to the power provided to the propeller.

$$\eta = \frac{F_T V_0}{P}$$

Pumps

- Pumps can be *axial flow* or *radial flow*
  - An axial-flow pump consists of an impeller, much like a propeller, mounted in a housing.
  - In a radial-flow pump, fluid enters near the eye of the impeller, passes through the vanes, and exits at the edge of the vanes.
- The head provided by a pump is quantified by the *head coefficient*, C_H, defined as

$$C_H = \frac{g\Delta H}{n^2 D^2}$$

  where ΔH is the head across the pump.
- The head coefficient is a function of the *discharge coefficient*, which is

$$C_Q = \frac{Q}{nD^3}$$

  where Q is the discharge.
- Pump performance curves show head delivered, power required, and efficiency as a function of discharge.
- The specific speed of a pump can be used to select an appropriate type of pump for a given application.
  - Axial-flow pumps are best suited for high-discharge, low-head applications.
  - Radial-flow pumps are best suited for low-discharge, high-head applications.

Water Turbines

- Turbines convert the energy associated with a moving fluid to shaft work.
- Turbines are classified into two categories.
  - The *impulse turbine* consists of a liquid jet impinging on vanes of a turbine wheel or runner.
  - A *reaction turbine* consists of a series of rotating vanes immersed in a flowing fluid. The pressure on the vanes provides the torque for the power.

Wind Turbines

- Wind turbines are classified based on the axis of the rotor
  - The rotor of a turbine can revolve around a *horizontal axis*. Most commercial wind turbines use this design.
  - The rotor of a turbine can revolve around a *vertical axis*. Two types of turbine in this category are the Darrieus turbine and the Savonius turbine.

- The maximum power derivable from a wind turbine is

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho V_0^3 A\right)$$

where A is the capture area of the wind turbine (projected area from direction of wind) and V_0 is the wind speed.

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PROBLEMS

WileyPLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLU* instructor's discretion.

Propellers (§14.1)

14.1 Explain why the thrust of a fixed-pitch propeller decreases with increasing forward speed.

14.2 What limits the rotational speed of a propeller?

14.3 WileyPLUS What thrust is obtained from a propeller 3 m in diameter that has the characteristics given in Fig. 14.3 on p. 520 of §14.1 when the propeller is operated at an angular speed of 1100 rpm and an advance velocity of zero? Assume $\rho = 1.05$ kg/m^3.

14.4 WileyPLUS What thrust is obtained from a propeller 3 m in diameter that has the characteristics given in Fig. 14.3 on p. 520 of §14.1 when the propeller is operated at an angular speed of 1400 rpm and an advance velocity of 80 km/h? What power is required to operate the propeller under these conditions? Assume $\rho = 1.05$ kg/m^3.

14.5 A propeller 8 ft in diameter has the characteristics shown in Fig. 14.3 on p. 520 of §14.1. What thrust is produced by the propeller when it is operating at an angular speed of 1200 rpm and a forward speed of 30 mph? What power input is required under these operating conditions? If the forward speed is reduced to zero, what is the thrust? Assume $\rho = 0.0024$ slugs/ft^3.

14.6 WileyPLUS A propeller 8 ft in diameter, like the one for which characteristics are given in Fig. 14.3, on p. 520 of §14.1, is to be used on a swamp boat and is to operate at maximum efficiency when cruising. If the cruising speed is to be 30 mph, what should the angular speed of the propeller be? Assume $\rho = 0.0024$ slugs/ft^3.

14.7 For the propeller and conditions described in Prob. 1 determine the thrust and the power input.

14.8 GO A propeller is being selected for an airplane tha cruise at 2000 m altitude, where the pressure is 60 kPa abs and the temperature is 10°C. The mass of the airplane is 1200 kg, and the planform area of the wing is 10 m^2. The lift-to-drag ratio is 30:1. The lift coefficient is 0.4. The engi speed at cruise conditions is 3000 rpm. The propeller is to operate at maximum efficiency, which corresponds to a th coefficient of 0.025. Calculate the diameter of the propelle the speed of the aircraft.

14.9 WileyPLUS If the tip speed of a propeller is to be kept below where c is the speed of sound, what is the maximum allowa angular speed of propellers having diameters of 2 m (6.56 f 3 m (9.84 ft), and 4 m (13.12 ft)? Take the speed of sound a 335 m/s (1099 ft/s).

14.10 A propeller 2 m in diameter, like the one for which characteristics are given in Fig. 14.3, on p. 520 of §14.1, is to used on a swamp boat and is to operate at maximum efficie when cruising. If the cruising speed is to be 40 km/h, what should the angular speed of the propeller be?

14.11 For the propeller and conditions described in Prob. 14.10, determine the thrust and the power input. Assume $\rho = 1.2$ kg/m^3.

14.12 WILEY PLUS A propeller 2 m in diameter and like the one for which characteristics are given in Fig. 14.3 on p. 520 of §14.1 is used on a swamp boat. If the angular speed is 1000 rpm and if the boat and passengers have a combined mass of 300 kg, estimate the initial acceleration of the boat when starting from rest. Assume $\rho = 1.1\ \text{kg/m}^3$.

Axial Flow Pumps and Fans (§14.2)

14.13 Answer the following questions about axial-flow pumps.

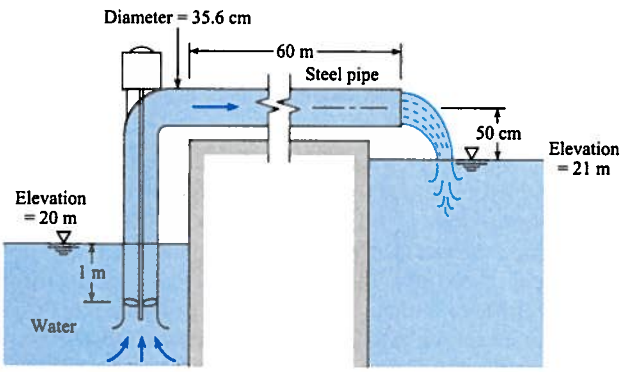
a. Axial-flow pumps are best suited for what conditions of head produced and discharge?

b. For an axial-flow pump, how does the head produced by the pump and the power required to operate a pump vary with flow rate through the pump?

14.14 WILEY PLUS If a pump having the characteristics shown in Fig. 14.7 on p. 524 of §14.2 has a diameter of 40 cm and is operated at a speed of 1000 rpm, what will be the discharge when the head is 3 m?

14.15 The pump used in the system shown has the characteristics given in Fig. 14.8 on p. 525 of §14.2. What discharge will occur under the conditions shown, and what power is required?

14.16 If the conditions are the same as in Prob. 14.15 except that the speed is increased to 900 rpm, what discharge will occur, and what power is required for the operation?



PROBLEMS 14.15, 14.16

14.17 For a pump having the characteristics given in Fig. 14.7 or 14.8 of §14.2, what water discharge and head will be produced at maximum efficiency if the pump diameter is 20 in. and the angular speed is 1100 rpm? What power is required under these conditions?

14.18 WILEY GO A pump has the characteristics given by Fig. 14.7 on p. 524 of §14.2. What discharge and head will be produced at maximum efficiency if the pump size is 50 cm and the angular speed is 45 rps? What power is required when pumping water at 10°C under these conditions?

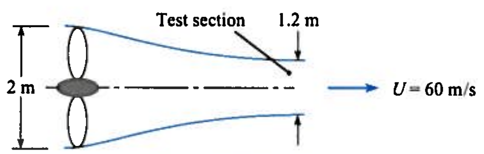
14.19 For a pump having the characteristics of Fig. 14.7 on p. 524 of §14.2, plot the head-discharge curve if the pump is 14 in. in diameter and is operated at a speed of 1000 rpm.

14.20 For a pump having the characteristics of Fig. 14.7 on p. 52 of §14.2, plot the head-discharge curve if the pump diameter is 60 cm and the speed is 690 rpm.

14.21 An axial-flow blower is used for a wind tunnel that has a test section measuring 60 cm by 60 cm and is capable of airspeeds up to 40 m/s. If the blower is to operate at maximum efficiency at the highest speed and if the rotational speed of the blower is 2000 rpm at this condition, what are th diameter of the blower and the power required? Assume that the blower has the characteristics shown in Fig. 14.7 on p. 524 of §14.2. Assume $\rho = 1.2\ \text{kg/m}^3$.

14.22 WILEY PLUS An axial-flow blower is used to air-condition an office building that has a volume of $10^5\ \text{m}^3$. It is decided that the air at 60°F in the building must be completely changed every 15 min. Assume that the blower operates at 600 rpm at maximum efficiency and has the characteristics shown in Fig. 14.' on p. 524 of §14.2. Calculate the diameter and power requirements for two blowers operating in parallel.

14.23 An axial fan 2 m in diameter is used in a wind tunnel as shown (test section 1.2 m in diameter; test section velocity of 60 m/s). The rotational speed of the fan is 1800 rpm. Assume th density of the air is constant at $1.2\ \text{kg/m}^3$. There are negligible losses in the tunnel. The performance curve of the fan is identic: to that shown in Fig. 14.7 on p. 524 of §14.2. Calculate the powe: needed to operate the fan.



PROBLEM 14.23

Radial Flow Pumps (§14.3)

14.24 The radial flow pump is best suited for what conditions o head produced and discharge?

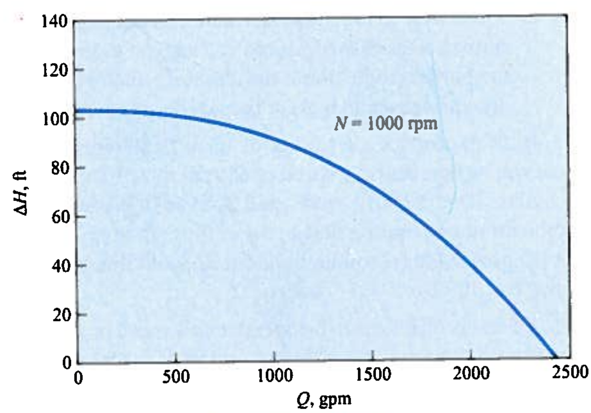
14.25 A pump is used to pump water out of a reservoir. What limits the depth for which the pump can draw water?

14.26 If a pump having the characteristics given in Fig. 14.10 o p. 528 of §14.3 is doubled in size but halved in speed, what will be the head and discharge at maximum efficiency?

14.27 A pump having the characteristics given in Fig. 14.10 on p. 528 of §14.3 pumps water at 20°C from a reservoir at an elevation of 366 m to a reservoir at an elevation of 450 m through a 36-cm steel pipe. If the pipe is 610 m long, what will b the discharge through the pipe?

14.28 WILEY GO If a pump having the characteristics given in Fig. 14.1 or 14.11 (both in §14.3) is operated at a speed of 1600 rpm, wha will be the discharge when the head is 135 ft?

14.29 If a pump having the performance curve shown is operated at a speed of 1600 rpm, what will be the maximum possible head developed?



PROBLEM 14.29

14.30 If a pump having the characteristics given in Fig. 14.10 on p. 528 of §14.3 is operated at a speed of 30 rps, what will be the shutoff head?

14.31 If a pump having the characteristics given in Fig. 14.11 on p. 528 of §14.3 is 40 cm in diameter and is operated at a speed of 25 rps, what will be the discharge when the head is 50 m?

14.32 WILEY PLUS A centrifugal pump 20 cm in diameter is used to pump kerosene at a speed of 5000 rpm. Assume that the pump has the characteristics shown in Fig. 14.11 on p. 528 of §14.3. Calculate the flow rate, the pressure rise across the pump, and the power required if the pump operates at maximum efficiency.

Pump Selection (§14.4)

14.33 Answer the following questions regarding pump sizing and selection.

a. What is the difference between a system curve and a pump curve? Explain.

b. The operating point for a pump system is established by what condition?

14.34 The value of the specific speed suggests the type of pump to be used for a given application. A high specific speed suggests the use of what kind of pump?

14.35 WILEY PLUS The pump curve for a given pump is represented by

$$h_{p,\text{pump}} = 20\left[1 - \left(\frac{Q}{100}\right)^2\right]$$

where $h_{p,\text{pump}}$ is the head provided by the pump in feet and Q is the discharge in gpm. The system curve for a pumping application is

$$h_{p,\text{sys}} = 5 + 0.002Q^2$$

where $h_{p,\text{sys}}$ is the head in feet required to operate the system and Q is the discharge in gpm. Find the operating point (Q) for (a) one pump, (b) two identical pumps connected in series, and (c) two identical pumps connected in parallel.

14.36 What is the suction specific speed for the pump that is operating under the conditions given in Prob. 14.15? Is this a safe operation with respect to susceptibility to cavitation?

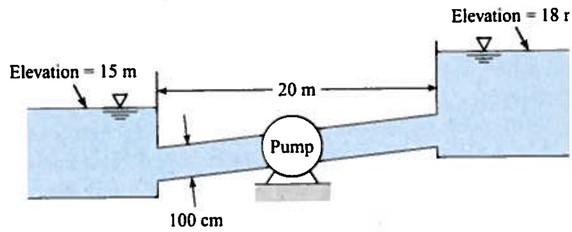
14.37 What type of pump should be used to pump water at a rate of 10 cfs and under a head of 30 ft? Assume N = 1500 rp

14.38 For the most efficient operation, what type of pump should be used to pump water at a rate of 0.10 m^3/s and unde head of 30 m? Assume n = 25 rps.

14.39 What type of pump should be used to pump water at a of 0.40 m^3/s and under a head of 70 m? Assume N = 1100 rp

14.40 An axial-flow pump is to be used to lift water against a head (friction and static) of 15 ft. If the discharge is to be 4000 gpm, what maximum speed in revolutions per minute is allowed if the suction head is 5 ft?

14.41 A pump is needed to pump water at a rate of 0.2 m^3/s from the lower to the upper reservoir shown in the figure. W type of pump would be best for this operation if the impeller speed is to be 600 rpm? Assume f = 0.02 and K_e = 0.5.



PROBLEM 14.41

14.42 Plot the five performance curves from Fig. 14.15 on p of §14.5 for the different impeller diameters in terms of the and discharge coefficients. Use impeller diameter for D.

Compressors (§14.7)

14.43 WILEY PLUS The pressure rise associated with gases in a compressor causes the gas temperature to increase as well. T ratio of final temperature to initial temperature is less than t ratio of final pressure to initial pressure. Will the final densit (a) less or (b) greater than the initial density?

14.44 WILEY PLUS Methane flowing at the rate of 1 kg/s is to be compressed by a noncooled centrifugal compressor from 100 kPa to 165 kPa. The temperature of the methane enterir the compressor is 27°C. The efficiency of the compressor is Calculate the shaft power necessary to run the compressor.

14.45 A 36 kW (shaft output) motor is available to run a noncooled compressor for carbon dioxide. The pressure is t be increased from 100 kPa to 150 kPa. If the compressor is efficient, calculate the volume flow rate into the compressor.

14.46 WILEY PLUS A water-cooled centrifugal compressor is used compress air from 100 kPa to 600 kPa at the rate of 2 kg/s. The temperature of the inlet air is 15°C. The efficiency of th compressor is 50%. Calculate the necessary shaft power.

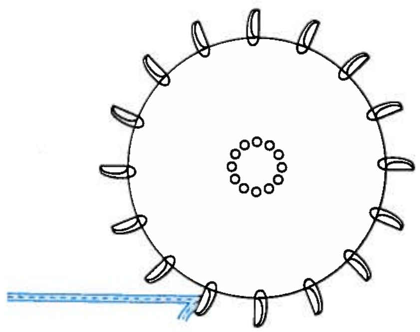
Impulse Turbines (§14.8)

14.47 An impulse turbine will produce no power if the vel of the jet striking the bucket is the same as the bucket veloc Explain.

14.48 GO A penstock 1 m in diameter and 10 km long carries water at 10°C from a reservoir to an impulse turbine. If the turbine is 85% efficient, what power can be produced by the system if the upstream reservoir elevation is 650 m above the turbine jet and the jet diameter is 16.0 cm? Assume that $f = 0.016$ and neglect head losses in the nozzle. What should the diameter of the turbine wheel be if it is to have an angular speed of 360 rpm? Assume ideal conditions for the bucket design $[V_{bucket} = (1/2)V_j]$.

14.49 Consider an idealized bucket on an impulse turbine that turns the water through 180°. Prove that the bucket speed should be one-half the incoming jet speed for a maximum power production. (*Hint:* Set up the momentum equation to solve for the force on the bucket in terms of V_j and V_{bucket}; then the power will be given by this force times V_{bucket}. You can use your mathematical talent to complete the problem.)

14.50 Consider a single jet of water striking the buckets of the impulse wheel as shown. Assume ideal conditions for power generation $[V_{bucket} = (1/2)V_j$ and the jet is turned through 180° of arc]. With the foregoing conditions, solve for the jet force on the bucket and then solve for the power developed. Note that this power is not the same as that given by Eq. (14.24)! Study the figure to resolve the discrepancy.



PROBLEM 14.50

Reaction Turbines (§14.8)

14.51 Answer the following questions about reaction turbines.

a. How does a reaction turbine differ from a centrifugal pump?

b. What is meant by the "runner" in a reaction turbine?

14.52 For a given Francis turbine, $\beta_1 = 60°$, $\beta_2 = 90°$, $r_1 = 5$ m, $r_2 = 3$ m, and $B = 1$ m. The discharge is 126 m³/s, and the rotational speed is 60 rpm. Assume $T = 10°$C.

a. What should α_1 be for a nonseparating flow condition at the entrance to the runner?

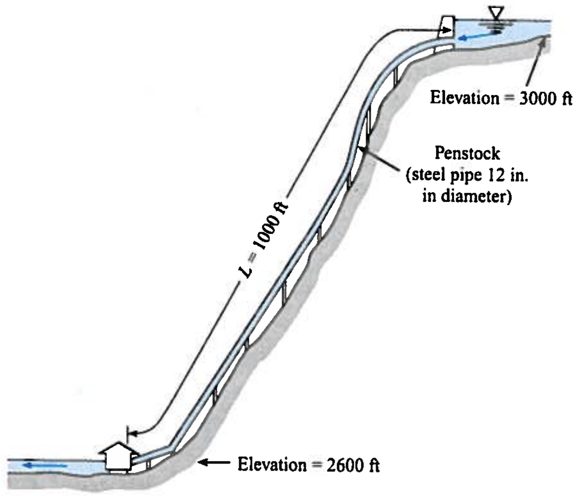
b. What is the maximum attainable power with the conditions noted?

c. If you were to redesign the turbine blades of the runner, what changes would you suggest to increase the power production if the discharge and overall dimensions are to be kept the same?

14.53 PLUS To produce a discharge of 3.3 m³/s, a Francis turbine will be operated at a speed of 60 rpm, $r_1 = 1.5$ m, $r_2 = 1.20$ m, $B = 33$ cm, $\beta_1 = 85°$, and $\beta_2 = 165°$. What should (a) α_1 be for nonseparating flow to occur through the runner? What (b) power and (c) torque should result with this operatio Assume $T = 10°$C.

14.54 A Francis turbine is to be operated at a speed of 120 rpn and with a discharge of 200 m³/s. If $r_1 = 3$ m, $B = 0.90$ m, and $\beta_1 = 45°$, what should α_1 be for nonseparating flow at the runner inlet?

14.55 Shown is a preliminary layout for a proposed small hydroelectric project. The initial design calls for a discharge of 8 cfs through the penstock and turbine. Assume 80% turbine efficiency. For this setup, what power output could be expected from the power plant? Draw the HGL and EGL for the system.



PROBLEM 14.55

Wind Turbines (§14.8)

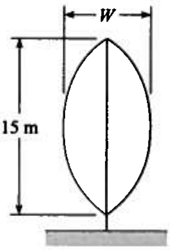
14.56 What determines the minimum and maximum wind speeds at which a wind turbine can operate?

14.57 Using the Internet and other resources, identify at least four types of wind turbines. For each type, describe its distinguishing characteristics and its relative advantages and disadvantages.

14.58 PLUS Calculate the minimum capture area necessary for a wind turbine that will be required to power the 2 kW demands of an energy-efficient home. Assume a wind velocity of 10 mph and an air density of 1.2 kg/m³.

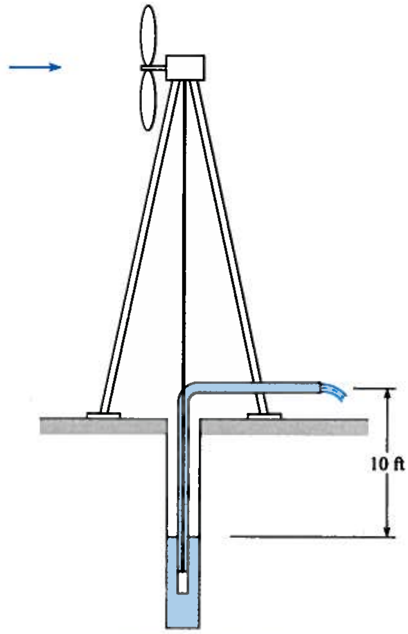
14.59 PLUS Calculate the maximum power derivable from a conventional horizontal-axis wind turbine with a propeller 2.3 m in diameter in a 47 km/h wind with density 1.2 kg/m³.

14.60 A wind farm consists of 20 Darrieus turbines, each 15 m high. The total output from the turbines is to be 2 MW in a wind of 20 m/s and an air density of 1.2 kg/m³. The Darrieus turbine shown has the shape of an arc of a circle. Find the minimum width, *W*, of the turbine needed to provide this power output.



PROBLEM 14.60

14.61 A windmill is connected directly to a mechanical pump that is to pump water from a well 10 ft deep as shown. The windmill is a conventional horizontal-axis type with a fan diameter of 10 ft. The efficiency of the mechanical pump is 80%. The density of the air is 0.07 lbm/ft³. Assume the windmill delivers the maximum power available. There is 20 ft of 2-inch galvanized pipe in the system. What would the discharge of the pump be (in gallons per minute) for a 30 mph wind? (1 cfm = 7.48 gpm)



PROBLEM 14.61