3. Because $Re_D < 2000$, the flow is laminar.
4. Head loss (laminar flow).

$$h_f = \frac{32\mu LV}{\gamma D^2} = \frac{32\rho\nu LV}{\rho g D^2} = \frac{32\nu LV}{gD^2}$$

$$= \frac{32(6 \times 10^{-4}\ \text{m}^2/\text{s})(100\ \text{m})(1.13\ \text{m/s})}{(9.81\ \text{m/s}^2)(0.15\ \text{m})^2}$$

$$= \boxed{9.83\ \text{m}}$$

Review the Solution and the Process

Knowledge. An alternative way to calculate head loss for lamina flow is to use the Darcy-Weisbach equation (10.12) as follows:

$$f = \frac{64}{Re_D} = \frac{64}{283} = 0.226$$

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.226\left(\frac{100\ \text{m}}{0.15\ \text{m}}\right)\left(\frac{(1.13\ \text{m/s})}{2 \times 9.81\ \text{m/s}^2}\right)^2$$

$$= 9.83\ \text{m}$$

10.6 Turbulent Flow and the Moody Diagram

This section describes the characteristics of turbulent flow, presents equations for calculating the friction factor f, and presents a famous graph called the Moody diagram. This information is important because most flows in conduits are turbulent.

Qualitative Description of Turbulent Flow

Turbulent flow is a flow regime in which the movement of fluid particles is chaotic, eddying, and unsteady, with significant movement of particles in directions transverse to the flow direction. Because of the chaotic motion of fluid particles, turbulent flow produces high levels of mixing and has a velocity profile that is more uniform or flatter than the corresponding laminar velocity profile. According to Eq. (10.2), turbulent flow occurs when $Re \geq 3000$.

Engineers and scientists model turbulent flow by using an empirical approach. This is because the complex nature of turbulent flow has prevented researchers from establishing a mathematical solution of general utility. Still, the empirical information has been used successfully and extensively in system design. Over the years, researchers have proposed many equations for shear stress and head loss in turbulent pipe flow. The empirical equations that have proven to be the most reliable and accurate for engineering use are presented in the next section.

Equations for the Velocity Distribution

The time-average velocity distribution is often described using an equation called the power-law formula.

$$\frac{u(r)}{u_{max}} = \left(\frac{r_0 - r}{r_0}\right)^m \quad (10.35)$$

where u_{max} is velocity in the center of the pipe, r_0 is the pipe radius, and m is an empirically determined variable that depends on Re as shown in Table 10.2. Notice in Table 10.2 that the velocity in the center of the pipe is typically about 20% higher than the mean velocity V. Although Eq. (10.35) provides an accurate representation of the velocity profile, it does not predict an accurate value of wall shear stress.

An alternative approach to Eq. (10.35) is to use the turbulent boundary-layer equations presented in Chapter 9. The most significant of these equations, called the logarithmic velocity distribution, is given by Eq. (9.29) and repeated here:

$$\frac{u(r)}{u_*} = 2.44 \ln \frac{u_*(r_0 - r)}{\nu} + 5.56 \quad (10.36)$$

where u_*, the shear velocity, is given by $u_* = \sqrt{\tau_0/\rho}$.

TABLE 10.2 Exponents for Power-Law Equation and Ratio of Mean to Maximum Velocity

Re	4×10^3	2.3×10^4	1.1×10^5	1.1×10^6	3.2×10^6
m	$\frac{1}{6.0}$	$\frac{1}{6.6}$	$\frac{1}{7.0}$	$\frac{1}{8.8}$	$\frac{1}{10.0}$
u_{max}/V	1.26	1.24	1.22	1.18	1.16

Source: Schlichting (2).

Equations for the Friction Factor, *f*

To derive an equation for *f in turbulent flow*, substitute the log law in Eq. (10.36) into the de nition of mean velocity given by Eq. (5.10):

$$V = \frac{Q}{A} = \left(\frac{1}{\pi r_0^2}\right)\int_0^{r_0} u(r)2\pi r dr = \left(\frac{1}{\pi r_0^2}\right)\int_0^{r_0} u_*\left[2.44 \ln\frac{u_*(r_0 - r)}{\nu} + 5.56\right]2\pi r dr$$

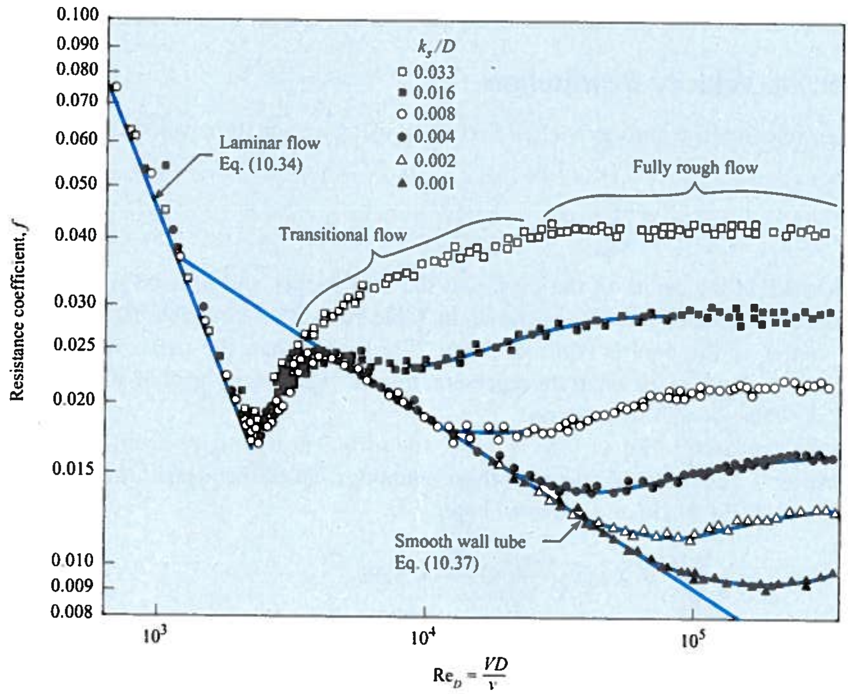
After integration, algebra, and tweaking the constants to better fit experimental data, the result

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(\mathrm{Re}\sqrt{f}) - 0.8 \qquad \textbf{(10.3}$$

Equation (10.37), first derived by Prandtl in 1935, gives the friction factor for turbulent flow tubes that have smooth walls. The details of the derivation of Eq. (10.37) are presented l White (21). To determine the influence of roughness on the walls, Nikuradse (4), one Prandtl's graduate students, glued uniform-sized grains of sand to the inner walls of a tube ar then measured pressure drops and flow rates.

Nikuradse's data, Fig. 10.13, shows the friction factor *f* plotted as function of Reynol number for various sizes of sand grains. To characterize the size of sand grains, Nikuradse use

FIGURE 10.13 Resistance coefficient *f* versus Reynolds number for sand-roughened pipe. [After Nikuradse (4)].



a variable called the **sand roughness height** with the symbol k_s. The π-group, k_s/D, is given the name **relative roughness**.

In laminar flow, the data in Fig. 10.13 show that wall roughness does not influence f. In particular, notice how the data corresponding to various values of k_s/D collapse into a single blue line that is labeled "laminar flow."

In turbulent flow, the data in Fig. 10.13 show that wall roughness has a major impact on f. When $k_s/D = 0.033$, then values of f are about 0.04. As the relative roughness drops to 0.002, values of f decrease by a factor of about 3. Eventually wall roughness does not matter, and the value of f can be predicted by assuming that the tube has a smooth wall. This latter case corresponds to the blue curve in Fig. 10.13 that is labeled "smooth wall tube." The effects of roughness are summarized by White (5) and presented in Table 10.3. These regions are also labeled in Fig. 10.13.

TABLE 10.3 Effects of Wall Roughness

Type of Flow	Parameter Ranges		Influence of Parameters on f
Laminar Flow	$\mathrm{Re}_D < 2000$	NA	f depends on Reynolds number f is independent of wall roughness (k_s/D)
Turbulent Flow, Smooth Tube	$\mathrm{Re}_D > 3000$	$\left(\frac{k_s}{D}\right)\mathrm{Re}_D < 10$	f depends on Reynolds number f is independent of wall roughness (k_s/D)
Transitional Turbulent Flow	$\mathrm{Re}_D > 3000$	$10 < \left(\frac{k_s}{D}\right)\mathrm{Re}_D < 1000$	f depends on Reynolds number f depends on wall roughness (k_s/D)
Fully Rough Turbulent Flow	$\mathrm{Re}_D > 3000$	$\left(\frac{k_s}{D}\right)\mathrm{Re}_D > 1000$	f is independent of Reynolds number f depends on wall roughness (k_s/D)

Moody Diagram

Colebrook (6) advanced Nikuradse's work by acquiring data for commercial pipes and then developing an empirical equation, called the Colebrook-White formula, for the friction factor. Moody (3) used the Colebrook-White formula to generate a design chart similar to that shown in Fig. 10.14. This chart is now known as the **Moody diagram** for commercial pipes.

In the Moody diagram, Fig. 10.14, the variable k_s denotes the **equivalent sand roughness**. That is, a pipe that has the same resistance characteristics at high Re values as a sand-roughened pipe is said to have a roughness equivalent to that of the sand-roughened pipe. Table 10.4 gives the equivalent sand roughness for various kinds of pipes. This table can be used to calculate the relative roughness for a given pipe diameter, which, in turn, is used in Fig. 10.14, to find the friction factor.

In the Moody diagram, Fig. 10.14, the abscissa is the Reynolds number Re, and the ordinate is the resistance coefficient f. Each blue curve is for a constant relative roughness k_s/D, and the values of k_s/D are given on the right at the end of each curve. To find f, given Re and k_s/D, one goes to the right to find the correct relative roughness curve. Then one looks at the bottom of the chart to find the given value of Re and, with this value of Re, moves vertically upward until the given k_s/D curve is reached. Finally, from this point one moves horizontally to the left scale to read the value of f. If the curve for the given value of k_s/D is not plotted in Fig. 10.14, then one simply finds the proper position on the graph by interpolation between the k_s/D curves that bracket the given k_s/D.

To provide a more convenient solution to some types of problems, the top of the Moody diagram presents a scale based on the parameter $\mathrm{Re} f^{1/2}$. This parameter is useful when h_f and k_s/D are known but the velocity V is not. Using the Darcy-Weisbach equation given in Eq. (10.12) and the definition of Reynolds number, one can show that

$$\mathrm{Re} f^{1/2} = \frac{D^{3/2}}{\nu}(2gh_f/L)^{1/2} \qquad (10.38)$$

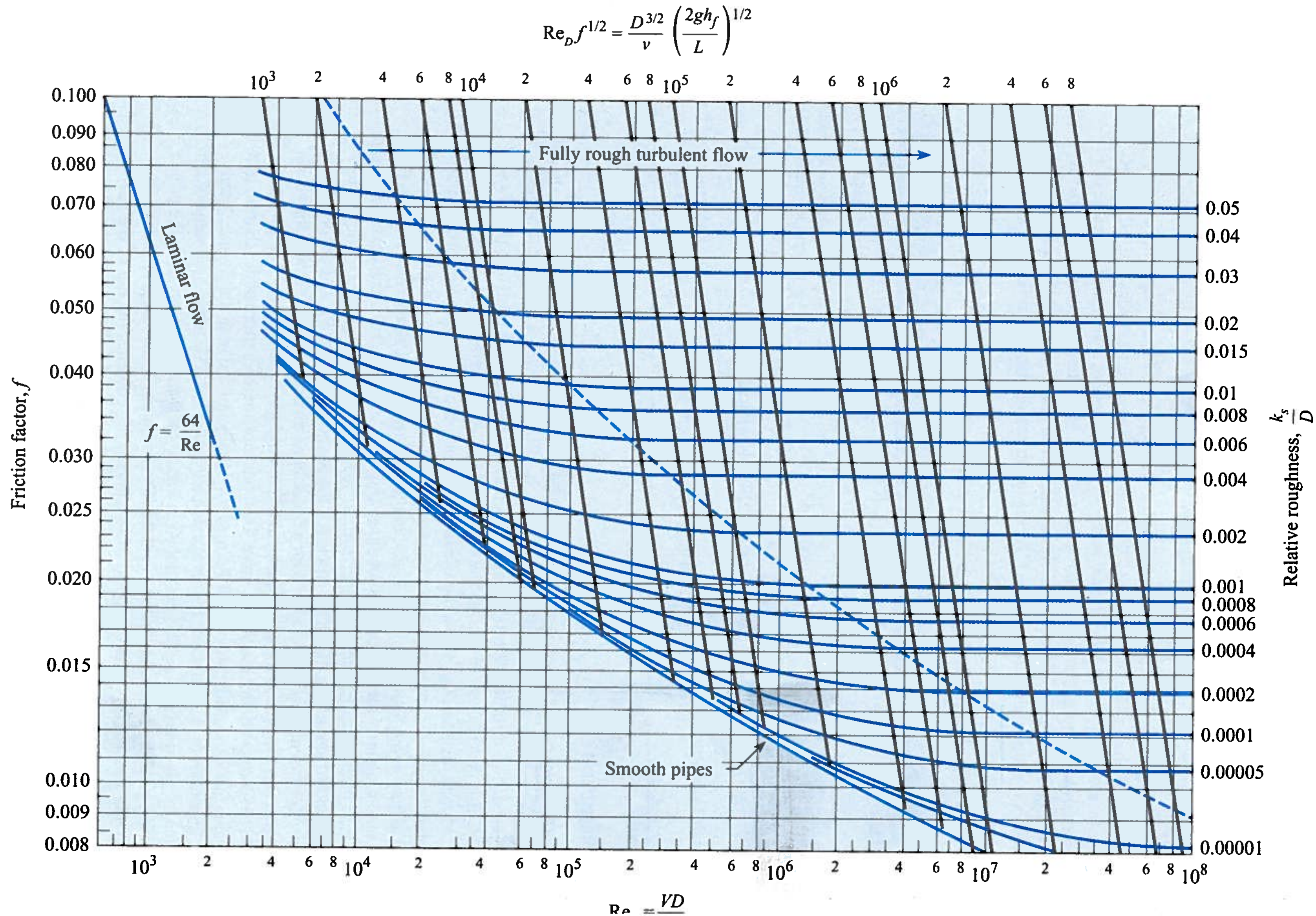


FIGURE 10.14

Friction Factor f versus Reynolds number. Reprinted with minor variations. [After Moody (3). Reprinted with permission from the ASME.]

TABLE 10.4 Equivalent Sand-Grain Roughness, (k_s), for Various Pipe Materials

Boundary Material	k_s, Millimeters	k_s, Inches
Glass, plastic	Smooth	Smooth
Copper or brass tubing	0.0015	6×10^{-5}
Wrought iron, steel	0.046	0.002
Asphalted cast iron	0.12	0.005
Galvanized iron	0.15	0.006
Cast iron	0.26	0.010
Concrete	0.3 to 3.0	0.012–0.12
Riveted steel	0.9–9	0.035–0.35
Rubber pipe (straight)	0.025	0.001

In the Moody diagram, Fig. 10.14, curves of constant $\mathrm{Re} f^{1/2}$ are plotted using heavy black lines that slant from the left to right. For example, when $\mathrm{Re} f^{1/2} = 10^5$ and $k_s/D = 0.004$, then $f = 0.029$. When using computers to carry out pipe-flow calculations, it is much more convenient to have an equation for the friction factor as a function of Reynolds number and relative roughness. By using the Colebrook-White formula, Swamee and Jain (7) developed an explicit equation for friction factor, namely

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}_D^{0.9}}\right)\right]^2} \tag{10.39}$$

It is reported that this equation predicts friction factors that differ by less than 3% from those on the Moody diagram for $4 \times 10^3 < \mathrm{Re}_D < 10^8$ and $10^{-5} < k_s/D < 2 \times 10^{-2}$.

✔CHECKPOINT PROBLEM 10.2

Water (15°C) flows in a 100 m length of cast iron pipe. The pipe inside diameter is 0.15 m, and the mean velocity is 0.6 m/s.

a. What is the value of Reynolds number?

b. What is the value of k_s/D?

c. What is the value of f from the Moody diagram?

d. What is the value of f from the Swamee-Jain correlation?

e. What is the value of head loss?

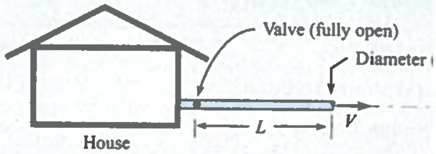
10.7 Strategy for Solving Problems

Analyzing flow in conduits can be challenging because the equations often cannot be solved with algebra. Thus, this section presents a strategy.

Conduit problems are solved with the energy equation together with equations for head-loss. Thus, the next checkpoint problem allows you to test your understanding of the energy equation.

✔CHECKPOINT PROBLEM 10.3

The sketch shows an idealization of a garden hose of diameter D and length L connected to a pipe bib at a residence. Assume that the supply pressure p_s upstream of the valve is constant. Assume that the faucet valve has no head loss because it is fully open. Thus, the only head loss is in the garden hose.



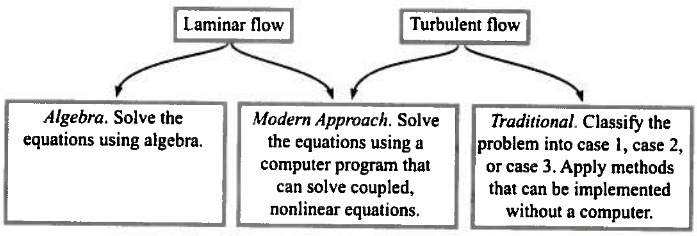
a. Derive an equation for the mean velocity V of the water in terms of the friction factor and oth
relevant variables.

b. How much will V change if L is doubled? Assume f remains constant.

Fig. 10.15 provides a strategy for problem solving. When flow is laminar, solutions are straig forward because head loss is linear with velocity V and the equations are simple enough solve with algebra. When flow is turbulent, head loss is nonlinear with V and the equations too complex to solve with algebra. Thus for turbulent flow, engineers use computer solutic or the traditional approach.

FIGURE 10.15

A strategy for solving conduit flow problems.



To solve a turbulent flow problem using the *traditional approach*, one classifies the pro lems into three cases:

Case 1 is when the goal is to find the *head loss*, given the pipe length, pipe diameter, and flc rate. This problem is straightforward because it can be solved using algebra; see Example 10

Case 2 is when the goal is to find the *flow rate*, given the head loss (or pressure drop), tl pipe length, and the pipe diameter. This problem usually requires an iterative approac See Examples 10.4 and 10.5.

Case 3 is when the goal is to find the *pipe diameter*, given the flow rate, length of pipe, and he loss (or pressure drop). This problem usually requires an iterative approach; see Example 10.

There are several approaches that sometimes eliminate the need for an iterative approac For case 2, an iterative approach can sometimes be avoided by using an explicit equatic developed by Swamee and Jain (7):

$$Q = -2.22\, D^{5/2}\sqrt{gh_f/L}\,\log\left(\frac{k_s}{3.7\,D} + \frac{1.78\,\nu}{D^{3/2}\sqrt{gh_f/L}}\right) \qquad \text{(10.4}$$

Using Eq. (10.40) is equivalent to using the top of the Moody diagram, which presents a sca for Re $f^{1/2}$. For case 3, one can sometimes use an explicit equation developed by Swamee an Jain (7) and modified by Streeter and Wylie (8):

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \qquad \text{(10.4}$$

Example 10.3 shows an example of a case 1 problem.

EXAMPLE 10.3

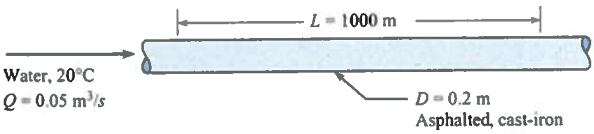
Head Loss in a Pipe (Case 1)

Problem Statement

Water ($T = 20°C$) flows at a rate of 0.05 m^3/s in a 20 cm asphalted cast-iron pipe. What is the head loss per kilometer of pipe?

Define the Situation

Water is flowing in a pipe.



Assumptions: Fully developed flow

Properties: Water (20°C), Table A.5: $\nu = 1 \times 10^{-6}\ m^2/s$

State the Goal

Calculate the head loss (in meters) for $L = 1000$ m.

Generate Ideas and Make a Plan

Because this is a case 1 problem (head loss is the goal), the solution is straightforward.

1. Calculate the mean velocity using the flow rate equation.
2. Calculate the Reynolds number using Eq. (10.1).
3. Calculate the relative roughness and then look up f on the Moody diagram.
4. Find head loss by applying the Darcy-Weisbach equation (10.1

Take Action (Execute the Plan)

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.05\ m^3/s}{(\pi/4)(0.02\ m)^2} = 1.59\ m/s$$

2. Reynolds number

$$Re_D = \frac{VD}{\nu} = \frac{(1.59\ m/s)(0.20\ m)}{10^{-6}\ m^2/s} = 3.18 \times 10^5$$

3. Resistance coefficient
   - Equivalent sand roughness (Table 10.4):

$$k_s = 0.12\ mm$$

   - Relative roughness:

$$k_s/D = (0.00012\ m)/(0.2\ m) = 0.0006$$

   - Look up f on the Moody diagram for $Re = 3.18 \times 10^5$ and $k_s/D = 0.0006$:

$$f = 0.019$$

4. Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.019\left(\frac{1000\ m}{0.20\ m}\right)\left(\frac{1.59^2\ m^2/s^2}{2(9.81\ m/s^2)}\right)$$
$$= \boxed{12.2\ m}$$

Example 10.4 shows an example of a case 2 problem. Notice that the solution involves application of the scale on the top of the Moody diagram, thereby avoiding an iterative solution.

EXAMPLE 10.4

Flow Rate in a Pipe (Case 2)

Problem Statement

The head loss per kilometer of 20 cm asphalted cast-iron pipe is 12.2 m. What is the flow rate of water through the pipe?

Define the Situation

This is the same situation as Example 10.3 except that the head loss is now specified and the discharge is unknown.

State the Goal

Calculate the discharge (m^3/s) in the pipe.

Generate Ideas and Make a Plan

This is a case 2 problem because flow rate is the goal. However, a direct (i.e., noniterative) solution is possible because head loss is specified. The strategy will be to use the horizontal scale on the top of the Moody diagram.

1. Calculate the parameter on the top of the Moody diagram.
2. Using the Moody diagram, find the friction factor f.
3. Calculate mean velocity using the Darcy-Weisbach equation (10.12).
4. Find discharge using the flow rate equation.

Take Action (Execute the Plan)

1. Compute the parameter $D^{3/2}\sqrt{2gh_f/L}/\nu$.

$$D^{3/2}\frac{\sqrt{2gh_f/L}}{\nu} = (0.20\ m)^{3/2} \times \frac{[2(9.81\ m/s^2)(12.2\ m/1000\ m)]^{1/2}}{1.0 \times 10^{-6}\ m^2/s}$$
$$= 4.38 \times 10^4$$

2. Determine resistance coefficient.
   - Relative roughness: $k_s/D = (0.00012\ m)/(0.2\ m) = 0.0006$
   - Look up f on the Moody diagram for $D^{3/2}\sqrt{2gh_f/L}/\nu = 4.4 \times 10^4$ and $k_s/D = 0.0006$:

$$f = 0.019$$

3. Find V using the Darcy-Weisbach equation.

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$12.2 \text{ m} = 0.019\left(\frac{1000 \text{ m}}{0.20 \text{ m}}\right)\left(\frac{V^2}{2(9.81 \text{ m/s}^2)}\right)$$

$$V = 1.59 \text{ m/s}$$

4. Use flow rate equation to find discharge.

$$Q = VA = (1.59 \text{ m/s})(\pi/4)(0.2 \text{ m})^2 = \boxed{0.05 \text{ m}^3/\text{s}}$$

Review the Solution and the Process

Validation. The calculated flow rate matches the value from Example 10.3. This is expected because the data are the same.

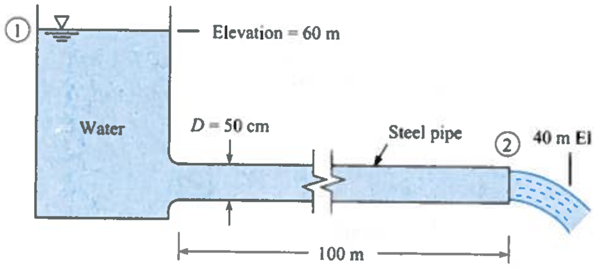
When case 2 problems require iteration, several methods can be used to find a solution. O of the easiest ways is a method called "successive substitution," which is illustrated Example 10.5.

EXAMPLE 10.5

Flow Rate in a Pipe (Case 2)

Problem Statement

Water ($T = 20°\text{C}$) flows from a tank through a 50 cm diameter steel pipe. Determine the discharge of water.



Define the Situation

Water is draining from a tank through a steel pipe.

Assumptions:

- Flow is fully developed.
- Include only the head loss in the pipe.

Properties:

- Water (20°C), Table A.5: $\nu = 1 \times 10^{-6}$ m²/s.
- Steel pipe, Table 10.4, equivalent sand roughness: $k_s = 0.046$ mm. Relative roughness (k_s/D) is 9.2×10^{-5}.

State the Goal

Find: Discharge (m³/s) for the system.

Generate Ideas and Make a Plan

This is a case 2 problem because flow rate is the goal. An iterative solution is used because V is unknown, so there is no direct way to use the Moody diagram.

1. Apply the energy equation from section 1 to section 2.
2. First trial. Guess a value of f and then solve for V.
3. Second trial. Using V from the first trial, calculate a new value of f.
4. Convergence. If the value of f is constant within a few percent between trials, then stop. Otherwise, continue with more iterations.
5. Calculate flow rate using the flow rate equation.

Take Action (Execute the Plan)

1. Energy equation (reservoir surface to outlet)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 60 = 0 + \frac{V_2^2}{2g} + 40 + f\frac{L}{D}\frac{V_2^2}{2g}$$

or

$$V = \left(\frac{2g \times 20}{1 + 200f}\right)^{1/2} \quad \text{(a)}$$

2. First trial (iteration 1)
   - Guess a value of $f = 0.020$.
   - Use Eq. (a) to calculate $V = 8.86$ m/s.
   - Use $V = 8.86$ m/s to calculate $\text{Re} = 4.43 \times 10^6$.
   - Use $\text{Re} = 4.43 \times 10^6$ and $k_s/D = 9.2 \times 10^{-5}$ on the Moody diagram to find that $f = 0.012$.
   - Use Eq. (a) with $f = 0.012$ to calculate $V = 10.7$ m/s.
3. Second trial (iteration 2)
   - Use $V = 10.7$ m/s to calculate $\text{Re}_D = 5.35 \times 10^6$.
   - Use $\text{Re}_D = 5.35 \times 10^6$ and $k_s/D = 9.2 \times 10^{-5}$ on the Moody diagram to find that $f = 0.012$.
4. Convergence. The value of $f = 0.012$ is unchanged between the first and second trials. Therefore, there is no need for more iterations.
5. Flow rate

$$Q = VA = (10.7 \text{ m/s}) \times (\pi/4) \times (0.50)^2 \text{ m}^2 = 2.10 \text{ m}^3/\text{s}$$

In a case 3 problem, derive an equation for diameter D and then use the method of successive substitution to find a solution. Iterative approaches, as illustrated in Example 10.6, can employ a spreadsheet program to perform the calculations.

EXAMPLE 10.6

Finding Pipe Diameter (Case 3)

Problem Statement

What size of asphalted cast-iron pipe is required to carry water (60°F) at a discharge of 3 cfs and with a head loss of 4 ft per 1000 ft of pipe?

Define the Situation

Water is flowing in a asphalted cast-iron pipe. $Q = 3\ \text{ft}^3/\text{s}$.

Assumptions: Fully developed flow

Properties:

- Water (60°F), Table A.5: $\nu = 1.22 \times 10^{-5}\ \text{ft}^2/\text{s}$
- Asphalted cast-iron pipe, Table 10.4, equivalent sand roughness: $k_s = 0.005$ in.

State the Goal

Calculate the pipe diameter (in ft) so that head loss is 4 ft per 1000 ft of pipe length.

Generate Ideas and Make a Plan

Because this is a case 3 problem (pipe diameter is the goal), use an iterative approach.

1. Derive an equation for pipe diameter by using the Darcy-Weisbach equation.
2. For iteration 1, guess f, solve for pipe diameter, and then recalculate f.
3. To complete the problem, build a table in a spreadsheet program.

Take Action (Execute the Solution)

1. Develop an equation to use for iteration.

- Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = f\left(\frac{L}{D}\right)\left(\frac{Q^2/A^2}{2g}\right) = \frac{fLQ^2}{2g(\pi/4)^2 D^5}$$

- Solve for pipe diameter

$$D^5 = \frac{fLQ^2}{0.785^2(2gh_f)} \qquad ($$

2. Iteration 1

- Guess $f = 0.015$.
- Solve for diameter using Eq. (a):

$$D^5 = \frac{0.015(1000\ \text{ft})(3\ \text{ft}^3/\text{s})^2}{0.785^2(64.4\ \text{ft/s}^2)(4\ \text{ft})} = 0.852\ \text{ft}^5$$

$$D = 0.968\ \text{ft}$$

- Find parameters needed for calculating f:

$$V = \frac{Q}{A} = \frac{3\ \text{ft}^3/\text{s}}{(\pi/4)(0.968^2\ \text{ft}^2)} = 4.08\ \text{ft/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(4.08\ \text{ft/s})(0.968\ \text{ft})}{1.22 \times 10^{-5}\ \text{ft}^2/\text{s}} = 3.26 \times 10^5$$

$$k_s/D = 0.005/(0.97 \times 12) = 0.00043$$

- Calculate f using Eq. (10.39): $f = 0.0178$.

3. In the table below, the first row contains the values from iteration 1. The value of $f = 0.0178$ from iteration 1 is use for the initial value for iteration 2. Notice how the solutio has converged by iteration 2.

Iteration #	Initial f	D	V	Re	k_s/D	Nev
		(ft)	(ft/s)			
1	0.0150	0.968	4.08	3.26E+05	4.3E−04	0.01
2	0.0178	1.002	3.81	3.15E+05	4.2E−04	0.01
3	0.0178	1.001	3.81	3.15E+05	4.2E−04	0.01
4	0.0178	1.001	3.81	3.15E+05	4.2E−04	0.01

Specify a pipe with a 12-inch inside diameter.

10.8 Combined Head Loss

Previous sections have described how to calculate head loss in pipes. This section completes the story by describing how to calculate head loss in components. This knowledge is essential for modeling and design of systems.

The Minor Loss Coefficient, *K*

When fluid flows through a component such as a partially open value or a bend in a pipe, v
cous effects cause the flowing fluid to lose mechanical energy. For example, Fig. 10.16 sho
flow through a "generic component." At section 2, the head of the flow will be less than at s
tion 1. To characterize component head loss, engineers use a π-group called the **minor l
coefficient** K

$$K \equiv \frac{(\Delta h)}{(V^2/2g)} = \frac{(\Delta p_z)}{(\rho V^2/2)} \quad \text{(10.4}$$

where Δh is the drop in piezometric head that is caused by a component, Δp_z is the drop
pizeometric pressure, and V is mean velocity. As shown in Eq. (10.42), the minor loss coe
cient has two useful interpretations:

$$K = \frac{\text{drop in piezometric head across component}}{\text{velocity head}} = \frac{\text{pressure drop due to component}}{\text{kinetic pressure}}$$

Thus, the head loss across a single component or transition is $h_L = K(V^2/(2g))$, where K is t
minor loss coefficient for that component or transition.

Most values of K are found by experiment. For example, consider the setup show
in Fig. 10.17. To find K, flow rate is measured and mean velocity is calculated usi
$V = (Q/A)$. Pressure and elevation measurements are used to calculate the change in piez
metric head.

$$\Delta h = h_2 - h_1 = \left(\frac{p_2}{\gamma} + z_2\right) - \left(\frac{p_1}{\gamma} + z_1\right) \quad \text{(10.4}$$

Then, values of V and Δh are used in Eq. (10.42) to calculate K. The next section presen
typical data for K.

FIGURE 10.16
Flow through a generic component.

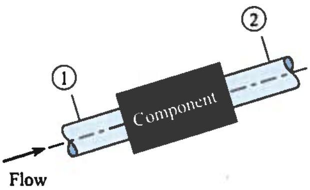
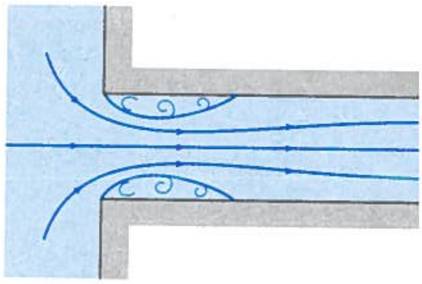


FIGURE 10.17
Flow at a sharp-edged inlet.

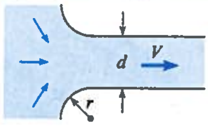
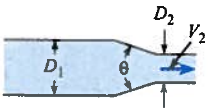
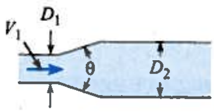
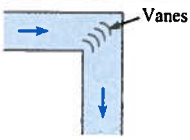
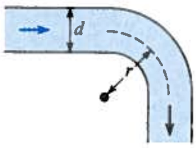


Data for the Minor Loss Coefficient This section presents K data and relates thes
data to flow separation and wall shear stress. This information is useful for syster
modeling.

Pipe inlet. Near the entrance to a pipe when the entrance is rounded, flow is devel
oping as shown in Fig. 10.3 and the wall shear stress is higher than that found in full
developed flow. Alternatively, if the pipe inlet is abrupt, or sharp-edged, as in Fig. 10.1
separation occurs just downstream of the entrance. Hence the streamlines converge an
then diverge with consequent turbulence and relatively high head loss. The loss coefficien

for the abrupt inlet is $K_e = 0.5$. This value is found in Table 10.5 using the row labeled "Pipe entrance" and the criteria of $r/d = 0.0$. Other values of head loss are summarized in Table 10.5.

TABLE 10.5 Loss Coefficients for Various Transitions and Fittings

Description	Sketch	Additional Data		K	Source
Pipe entrance		r/d		K_e	(10)†
$h_L = K_e V^2/2g$		0.0		0.50	
		0.1		0.12	
		>0.2		0.03	
Contraction			K_C $\theta = 60°$	K_C $\theta = 180°$	(10)
$h_L = K_C V_2^2/2g$		D_2/D_1			
		0.00	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
		0.90	0.06	0.10	
Expansion			K_E $\theta = 20°$	K_E $\theta = 180°$	(9)
$h_L = K_E V_1^2/2g$		D_1/D_2			
		0.00		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
		0.80	0.10	0.15	
90° miter bend		Without vanes	$K_b = 1.1$		(15)
90° smooth bend		With vanes	$K_b = 0.2$		(15)
		r/d			(16) and (
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings	Globe valve—wide open			$K_v = 10.0$	(15)
	Angle valve—wide open			$K_v = 5.0$	
	Gate valve—wide open			$K_v = 0.2$	
	Gate valve—half open			$K_v = 5.6$	
	Return bend			$K_b = 2.2$	
	Tee				
	Straight-through flow			$K_t = 0.4$	
	Side-outlet flow			$K_t = 1.8$	
	90° elbow			$K_b = 0.9$	
	45° elbow			$K_b = 0.4$	

†Reprinted by permission of the American Society of Heating, Refrigerating and Air Conditioning Engineers, Atlanta, Georgia, from the 1981 *ASHRAE Handbook—Fundamen*

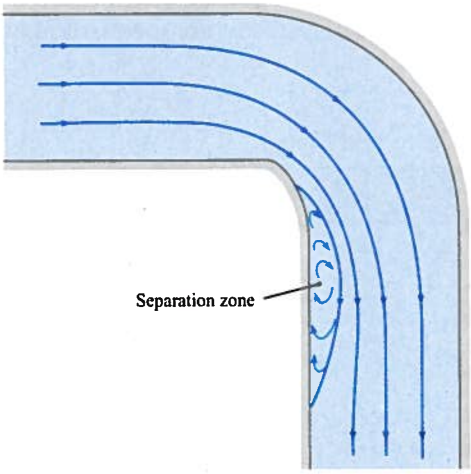


FIGURE 10.18
Flow pattern in an elbow.

Flow in an elbow. In an elbow (90° smooth bend), considerable head loss is produced b secondary flows and by separation that occurs near the inside of the bend and downstream (the midsection as shown in Fig. 10.18.

The loss coefficient for an elbow at high Reynolds numbers depends primarily on th shape of the elbow. For a very short-radius elbow, the loss coefficient is quite high. For large radius elbows, the coefficient decreases until a minimum value is found at an r/d value of about (see Table 10.5). However, for still larger values of r/d, an increase in loss coefficient occu because the elbow itself is significantly longer.

Other components. The loss coefficients for a number of other fittings and flow trans tions are given in Table 10.5. This table is representative of engineering practice. For more e: tensive tables, see references (10–15).

In Table 10.5, the K was found by experiment, so one must be careful to ensure th Reynolds number values in the application correspond to the Reynolds number values use to acquire the data.

Combined Head Loss Equation

The total head loss is given by Eq. (10.4), which is repeated here:

$$\{\text{Total head loss}\} = \{\text{Pipe head loss}\} + \{\text{Component head loss}\} \quad (10.4$$

To develop an equation for the combined head loss, substitute Eqs. (10.12) and (10.42) Eq. (10.44):

$$h_L = \sum_{\text{pipes}} f \frac{L}{D} \frac{V^2}{2g} + \sum_{\text{components}} K \frac{V^2}{2g} \quad (10.4$$

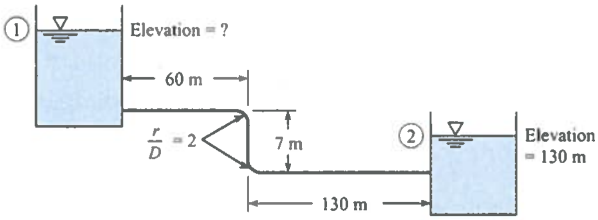
Equation (10.45) is called the *combined head loss equation*. To apply this equation, follow tl same approaches that were used for solving pipe problems. That is, classify the flow as case 2, or 3 and apply the usual equations: the energy, Darcy-Weisbach, and flow rate equatio Example 10.7 illustrates this approach for a case 1 problem.

EXAMPLE 10.7

Pipe System with Combined Head Loss

Problem Statement

If oil ($\nu = 4 \times 10^{-5}$ m²/s; $S = 0.9$) flows from the upper to the lower reservoir at a rate of 0.028 m³/s in the 15 cm smooth pipe, what is the elevation of the oil surface in the upper reservoir?



Define the Situation

Oil is flowing from a upper reservoir to a lower reservoir.

Properties:

- Oil: $\nu = 4 \times 10^{-5}$ m²/s, $S = 0.9$
- Minor head loss coefficients, Table 10.5: entrance $= K_e = 0.5$; bend $= K_b = 0.19$; outlet $= K_E = 1.0$

State the Goal

Calculate the elevation (in meters) of the free surface of the upper reservoir.

Generate Ideas and Make a Plan

This is a case 1 problem because flow rate and pipe dimensions are known. Thus, the solution is straightforward.

1. Apply the energy equation from 1 to 2.
2. Apply the combined head loss equation (10.45).
3. Develop an equation for z_1 by combining results from steps 1 and 2.
4. Calculate the resistance coefficient f.
5. Solve for z_1 using the equation from step 3.

Take Action (Execute the Plan)

1. Energy equation and term-by-term analysis

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + z_1 + 0 = 0 + 0 + z_2 + 0 + h_L$$

$$z_1 = z_2 + h_L$$

Interpretation: Change in elevation head is balanced by the total head loss.

2. Combined head loss equation

$$h_L = \sum_{\text{pipes}} f \frac{L}{D} \frac{V^2}{2g} + \sum_{\text{components}} K \frac{V^2}{2g}$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} + \left(2K_b \frac{V^2}{2g} + K_e \frac{V^2}{2g} + K_E \frac{V^2}{2g}\right)$$

$$= \frac{V^2}{2g}\left(f \frac{L}{D} + 2K_b + K_e + K_E\right)$$

3. Combine eqs. (1) and (2).

$$z_1 = z_2 + \frac{V^2}{2g}\left(f \frac{L}{D} + 2K_b + K_e + K_E\right)$$

4. Resistance coefficient

- Flow rate equation

$$V = \frac{Q}{A} = \frac{(0.028 \text{ m}^3/\text{s})}{(\pi/4)(0.15 \text{ m})^2} = 1.58 \text{ m/s}$$

- Reynolds number

$$\text{Re}_D = \frac{VD}{\nu} = \frac{1.58 \text{ m/s}(0.15 \text{ m})}{4 \times 10^{-5} \text{ m}^2/\text{s}} = 5.93 \times 10^3$$

Thus, flow is turbulent.

- Swamee-Jain equation (10.39)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{5930^{0.9}}\right)\right]^2} = 0.0$$

5. Calculate z_1 using the equation from step (3):

$$z_1 = (130 \text{ m}) + \frac{(1.58 \text{ m/s})^2}{2(9.81)\text{m/s}^2}$$

$$\left(0.036 \frac{(197 \text{ m})}{(0.15 \text{ m})} + 2(0.19) + 0.5 + 1.0\right)$$

$$\boxed{z_1 = 136 \text{ m}}$$

Review the Solution and the Process

1. *Discussion.* Notice the difference is the magnitude of the pipe head loss versus the magnitude of the component head loss:

$$\text{Pipe head loss} \sim \Sigma f \frac{L}{D} = 0.036 \frac{(197 \text{ m})}{(0.15 \text{ m})} = 47.2$$

$$\text{Component head loss} \sim \Sigma K = 2(0.19) + 0.5 + 1.0 = 1.8$$

Thus pipe losses $\gg$ component losses for this problem.

2. *Skill.* When pipe head loss is dominant, make simple estimates of K because these estimates will not impact the prediction very much.

10.9 Nonround Conduits

Previous sections have considered round pipes. This section extends this information l describing how to account for conduits that are square, triangular, or any other nonrour shape. This information is important for applications such as sizing ventilation ducts in buil ings and for modeling flow in open channels.

When a conduit is noncircular, then engineers modify the Darcy-Weisbach equatio Eq. (10.12), to use hydraulic diameter D_h in place of diameter.

$$h_L = f\frac{L}{D_h}\frac{V^2}{2g} \quad \text{(10.4}$$

Equation (10.46) is derived using the same approach as Eq. (10.12), and the **hydraulic diamet** that emerges from this derivation is

$$D_h \equiv \frac{4 \times \text{cross-section area}}{\text{wetted perimeter}} \quad \text{(10.4}$$

where the "wetted perimeter" is that portion of the perimeter that is physically touching tl fluid. The wetted perimeter of a rectangular duct of dimension $L \times w$ is $2L + 2w$. Thus, tl hydraulic diameter of this duct is:

$$D_h \equiv \frac{4 \times Lw}{2L + 2w} = \frac{2Lw}{L + w}$$

Using Eq. (10.47), the hydraulic diameter of a round pipe is the pipe's diameter D. Whe Eq. (10.46) is used to calculate head loss, the resistance coefficient f is found using a Reynolc number based on hydraulic diameter. Use of hydraulic diameter is an approximation. Accorc ing to White (20), this approximation introduces an uncertainty of 40% for laminar flow an 15% for turbulent flow.

$$f = \left(\frac{64}{\mathrm{Re}_{D_h}}\right) \pm 40\% \text{ (laminar flow)}$$

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D_h} + \frac{5.74}{\mathrm{Re}_{D_h}^{0.9}}\right)\right]^2} \pm 15\% \text{ (turbulent flow)} \quad \text{(10.4}$$

In addition to hydraulic diameter, engineers also use **hydraulic radius**, which is defined as

$$R_h \equiv \frac{\text{section area}}{\text{wetted perimeter}} = \frac{D_h}{4} \quad \text{(10.4}$$

Notice that the ratio of R_h to D_h is 1/4 instead of 1/2. Although this ratio is not logical, it is th convention used in the literature and is useful to remember. Chapter 15, which focuses o open-channel flow, will present examples of hydraulic radius.

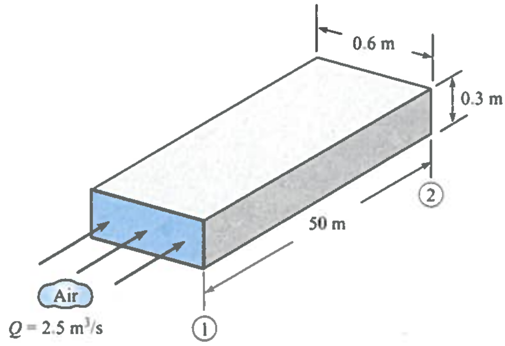
Summary. To model flow in a nonround conduit, the approaches of the previous section are followed with the only difference being the use of hydraulic diameter in place of diamete This is illustrated by Example 10.8.

EXAMPLE 10.8

Pressure Drop in an HVAC Duct

Problem Statement

Air ($T = 20°C$ and $p = 101$ kPa absolute) flows at a rate of 2.5 m^3/s in a horizontal, commercial steel, HVAC duct. (Note that HVAC is an acronym for heating, ventilating, and air conditioning.) What is the pressure drop in inches of water per 50 m of duct?



Define the Situation

Air is flowing through a duct.

Assumptions:

- Fully developed flow, meaning that $V_1 = V_2$. Thus, the velocity head terms in the energy equation cancel out.
- No sources of component head loss.

Properties:

- Air (20°C, 1 atm, Table A.2:) $\rho = 1.2$ kg/m^3, $\nu = 15.1 \times 10^{-6}$ m^2/s
- Steel pipe, Table 10.4: $k_s = 0.046$ mm

State the Goal

Find: Pressure drop (inch H_2O) in a length of 50 m.

Generate Ideas and Make a Plan

This is a case 1 problem because flow rate and duct dimensions are known. Thus, the solution is straightforward.

1. Derive an equation for pressure drop by using the energy equation.
2. Calculate parameters needed to find head loss.
3. Calculate head loss by using the Darcy-Weisbach equatioı (10.12).
4. Calculate pressure drop Δp by combining steps 1, 2, and 3.

Take Action (Execute the Plan)

1. Energy equation (after term-by-term analysis)

$$p_1 - p_2 = \rho g h_L$$

2. Intermediate calculations
   - Flow rate equation

$$V = \frac{Q}{A} = \frac{2.5\ \text{m}^3/\text{s}}{(0.3\ \text{m})(0.6\ \text{m})} = 13.9\ \text{m/s}$$

   - Hydraulic diameter

$$D_h = \frac{4 \times \text{section area}}{\text{wetted perimeter}} = \frac{4(0.3\ \text{m})(0.6\ \text{m})}{(2 \times 0.3\ \text{m}) + (2 \times 0.6\ \text{m})} = 0.4$$

   - Reynolds number

$$\text{Re} = \frac{VD_h}{\nu} = \frac{(13.9\ \text{m/s})(0.4\ \text{m})}{(15.1 \times 10^{-6}\ \text{m}^2/\text{s})} = 368{,}000$$

   Thus, flow is turbulent.

   - Relative roughness

$$k_s/D_h = (0.000046\ \text{m})/(0.4\ \text{m}) = 0.000115$$

   - Resistance coefficient (Moody diagram): $f = 0.015$

3. Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D_h}\right)\left(\frac{V^2}{2g}\right) = 0.015\left(\frac{50\ \text{m}}{0.4\ \text{m}}\right)\left\{\frac{(13.9\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}\right\}$$

$$= 18.6\ \text{m}$$

4. Pressure drop (from step 1)

$$p_1 - p_2 = \rho g h_L = (1.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(18.6\ \text{m}) = 220\ \text{F}$$

$$\boxed{p_1 - p_2 = 0.883\ \text{inch H}_2\text{O}}$$

10.10 Pumps and Systems of Pipes

This section explains how to model flow in a network of pipes and how to incorporate performance data for a centrifugal pump. These topics are important because pumps and pipe networks are common.