FLOW IN CONDUITS 10

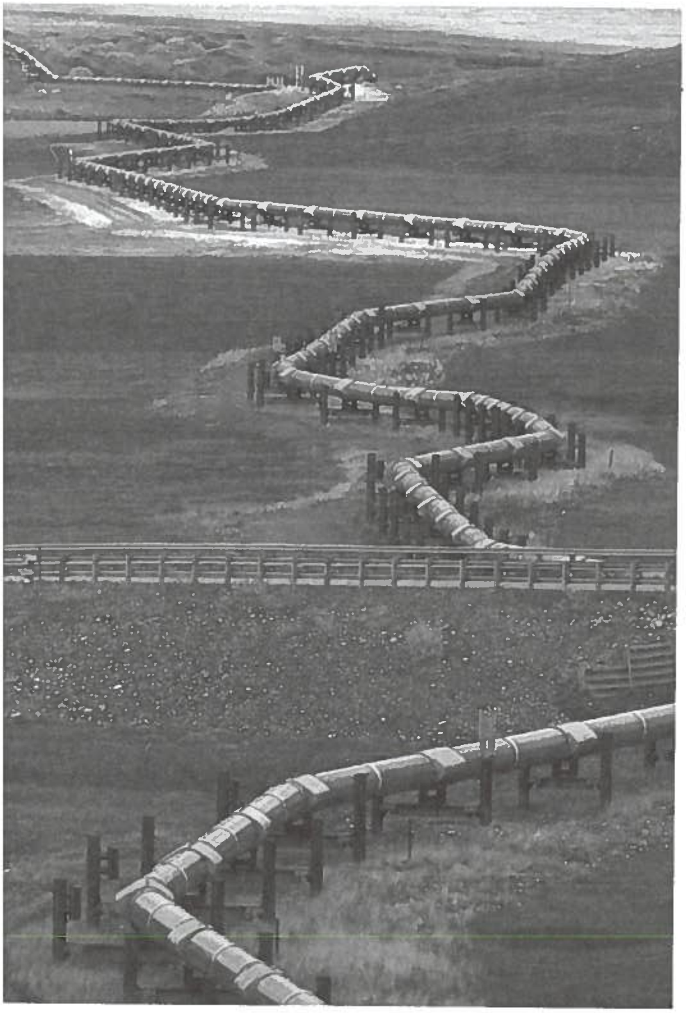


FIGURE 10.1

The Alaskan pipeline, a significant accomplishment of the engineering profession, transports oil 1286 km across the state of Alaska. The pipe diameter is 1.2 m, and 44 pumps are used to drive the flow. This chapter presents information for designing systems involving pipes, pumps, and turbines. (© Eastcott/Momatiuk/The Image Works.)

Chapter Road Map

This chapter explains how to analyze flow in con duits. The primary tool, the energy equation, was presented in Chapter 7. This chapter expands on knowledge by describing how to calculate head l In addition, this chapter explains how to design p into systems and how to analyze a network of pip

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define a conduit. Classify a flow as laminar or turbule Define or calculate the Reynolds number. (§10.1)
- Describe developing flow and fully developed flow. C a flow into these categories. (§10.1)
- Specify a pipe size using the NPS standard. (§10.2)
- Describe total head loss, pipe head loss, and compor head loss. (§10.3)
- Define the friction factor f. List the steps to derive the Darcy-Weisbach equation. (§10.3)
- Describe the physics of the Darcy-Weisbach equation the meaning of the variables that appear in the equati Apply this equation. (§10.3)
- Calculate h_f or f for laminar flow. (§10.5)
- Describe the main features of the Moody diagram. Calculate f for turbulent flow using the Moody diagram the Swamee-Jain correlation. (§10.6)
- Solve turbulent flow problems when the equations cann be solved by algebra alone. (§10.7)
- Define the minor loss coefficient. Describe and apply the combined head loss equation. (§10.8)
- Define hydraulic diameter and hydraulic radius and so relevant problems. (§10.9)
- Solve problems that involve pumps and pipe networks. (§1

A **conduit** is any pipe, tube, or duct that is completely filled with a flowing fluid. Example include a pipeline transporting liquefied natural gas, a microchannel transporting hydrogen i a fuel cell, and a duct transporting air for heating of a building. A pipe that is partially fille with a flowing fluid, for example a drainage pipe, is classified as an open-channel flow and i analyzed using ideas from Chapter 15.

10.1 Classifying Flow

This section describes how to classify flow in a conduit by considering (a) whether the flow i laminar or turbulent, and (b) whether the flow is developing or fully developed. Classifyin flow is essential for selecting the proper equation for calculating head loss.

Laminar Flow and Turbulent Flow

Flow in a conduit is classified as being either laminar or turbulent, depending on the magni tude of the Reynolds number. The original research involved visualizing flow in a glass tube a shown in Fig. 10.2a. Reynolds (1) in the 1880s injected dye into the center of the tube an observed the following:

- When the velocity was low, the streak of dye flowed down the tube with little expansion, a shown in Fig. 10.2b. However, if the water in the tank was disturbed, the streak would shif about in the tube.
- If velocity was increased, at some point in the tube, the dye would all at once mix with the water as shown in Fig. 10.2c.
- When the dye exhibited rapid mixing (Fig. 10.2c), illumination with an electric spark revealed eddies in the mixed fluid as shown in Fig. 10.2d.

The flow regimes shown in Fig. 10.2 are laminar flow (Fig. 10.2b) and turbulent flow (Fig 10.2c and 10.2d). Reynolds showed that the onset of turbulence was related to a π-group th is now called the Reynolds number ($\text{Re} = \rho VD/\mu$) in honor of Reynolds' pioneering work.

The Reynolds number is often written as Re_D, where the subscript "D" denotes that diam eter is used in the formula. This subscript is called a *length scale*. Indicating the length scale fo Reynolds number is good practice because muliple values are used. For example, Chapter introduced Re_x and Re_L.

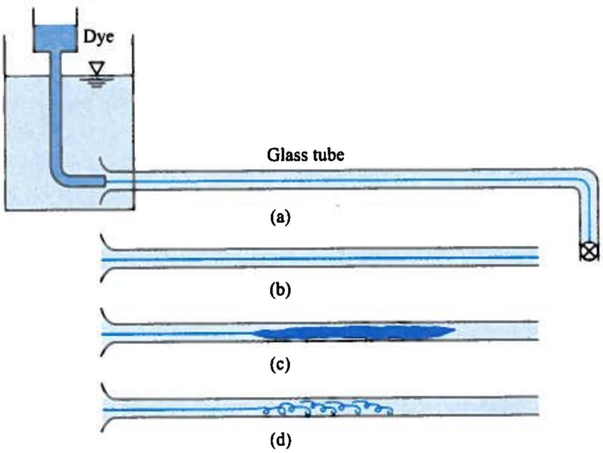


FIGURE 10.2
Reynolds' experiment.
(a) Apparatus.
(b) Laminar flow of dye in tube.
(c) Turbulent flow of dye in tube.
(d) Eddies in turbulent flow.

Reynolds number can be calculated with four different equations. These equations are equivalent because one can start with one formula and derive the others. The formulas are

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{4Q}{\pi D \nu} = \frac{4\dot{m}}{\pi D \mu} \tag{10.1}$$

Reynolds discovered that if the fluid in the upstream reservoir was not completely still or if the pipe had some vibrations, then the change from laminar to turbulent flow occurred at $\mathrm{Re}_D \sim 2100$. However, if conditions were ideal, it was possible to reach a much higher Reynolds number before the flow became turbulent. Reynolds also found that, when going from high velocity to low velocity, the change back to laminar flow occurred at $\mathrm{Re}_D \sim 2000$. Based on Reynolds' experiments, engineers use guidelines to establish whether or not flow in a conduit will be laminar or turbulent. The guidelines used in this text are as follows:

$$\begin{aligned} &\mathrm{Re}_D \leq 2000 && \text{laminar flow} \\ 2000 \leq\ &\mathrm{Re}_D \leq 3000 && \text{unpredictable} \\ &\mathrm{Re}_D \geq 3000 && \text{turbulent flow} \end{aligned} \tag{10.2}$$

In Eq. (10.2), the middle range ($2000 \leq \mathrm{Re}_D \leq 3000$) corresponds to a type of flow that is unpredictable because it can change back and forth between laminar and turbulent states. Recognize that precise values of Reynolds number versus flow regime do not exist. Thus, the guidelines given in Eq. (10.2) are approximate, and other references may give different values. For example, some references use $\mathrm{Re}_D = 2300$ as the criteria for turbulence.

Developing Flow and Fully Developed Flow

Flow in a conduit is classified as either developing flow or fully developed flow. For example, consider laminar fluid entering a pipe from a reservoir as shown in Fig. 10.3. As the fluid moves down the pipe, the velocity profile changes in the streamwise direction as viscous effects cause the plug-type profile to gradually change into a parabolic profile. This region of changing velocity profile is called **developing flow**. After the parabolic distribution is achieved, the flow profile remains unchanged in the streamwise direction, and flow is called **fully developed flow**.

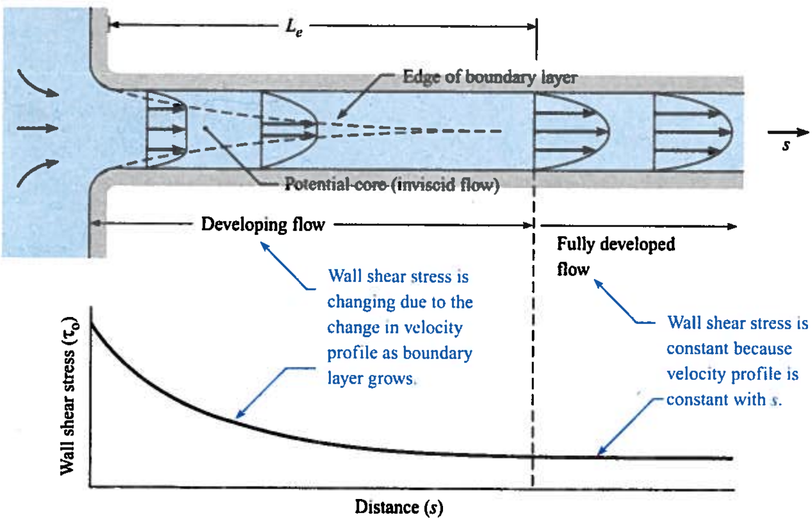


FIGURE 10.3
In developing flow, the wall shear stress is changing. In fully developed flow, the w shear stress is constant

The distance required for flow to develop is called the **entry or entrance length** (L_e). I the entry length, the wall shear stress is decreasing in the streamwise (i.e. s) direction. Fc laminar flow, the wall shear-stress distribution is shown in Fig. 10.3. Near the pipe entranc the radial velocity gradient (change in velocity with distance from the wall) is high, so th shear stress is large. As the velocity profile progresses to a parabolic shape, the velocity gra dient and the wall shear stress decrease until a constant value is achieved. The entry length defined as the distance at which the shear stress reaches 2% of the fully developed valu Correlations for entry length are

$$\frac{L_e}{D} = 0.05\,\mathrm{Re}_D \qquad \text{(laminar flow: } \mathrm{Re}_D \leq 2000) \qquad (10.3\text{a}$$

$$\frac{L_e}{D} = 50 \qquad \text{(turbulent flow: } \mathrm{Re}_D \geq 3000) \qquad (10.3\text{b}$$

Eq. (10.3) is valid for flow entering a circular pipe from a reservoir under quiescent cond tions. Other upstream components such as valves, elbows, and pumps produce complex flo fields that require different lengths to achieve fully developing flow.

In summary, flow in a conduit is classified into four categories: laminar developing, lam nar fully developed, turbulent developing, or turbulent fully developed. The key to classifica tion is to calculate the Reynolds number as shown by Example 10.1.

EXAMPLE 10.1

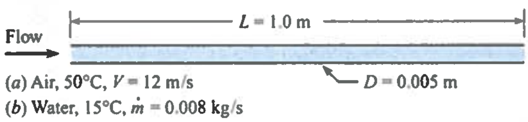
Classifying Flow in Conduits

Problem Statement

Consider fluid flowing in a round tube of length 1 m and diameter 5 mm. Classify the flow as laminar or turbulent and calculate the entrance length for (a) air (50°C) with a speed of 12 m/s and (b) water (15°C) with a mass flow rate of $\dot{m}$ = 8 g/s.

Define the Situation

Fluid is flowing in a round tube (two cases given).



Properties:

1. Air (50°C), Table A.3, $\nu = 1.79 \times 10^{-5}\ \mathrm{m^2/s}$
2. Water (15°C), Table A.5, $\mu = 1.14 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$

Assumptions:

1. The pipe is connected to a reservoir.
2. The entrance is smooth and tapered.

State the Goal

- Determine whether each flow is laminar or turbulent.
- Calculate the entrance length (in meters) for each case.

Generate Ideas and Make a Plan

- Calculate the Reynolds number using Eq. (10.1).
- Establish whether the flow is laminar or turbulent using Eq. (10.2).
- Calculate the entrance length using Eq. (10.3).

Take Action (Execute the Plan)

a. Air

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(12\ \mathrm{m/s})(0.005\ \mathrm{m})}{1.79 \times 10^{-5}\ \mathrm{m^2/s}} = 3350$$

Because $\mathrm{Re}_D > 3000$, the $\boxed{\text{flow is turbulent.}}$

$$L_e = 50D = 50(0.005\,\mathrm{m}) = \boxed{0.25\ \mathrm{m}}$$

b. Water

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.008\ \mathrm{kg/s})}{\pi(0.005\ \mathrm{m})(1.14 \times 10^{-3}\ \mathrm{N \cdot s/m^2})} = 1787$$

Because $\mathrm{Re}_D < 2000$, the $\boxed{\text{flow is laminar.}}$

$$L_e = 0.05\mathrm{Re}_D D = 0.05(1787)(0.005\ \mathrm{m}) = \boxed{0.447\ \mathrm{m}}$$

10.2 Specifying Pipe Sizes

This section describes how to specify pipes using the Nominal Pipe Size (NPS) standard. This information is useful for specifying a size of pipe that is available commercially.

Standard Sizes for Pipes (NPS)

One of the most common standards for pipe sizes is called the Nominal Pipe Size (NPS) system. The terms used in the NPS system are introduced in Fig. 10.4. The ID (pronounced "eye dee") indicates the inner pipe diameter, and the OD ("oh dee") indicates the outer pipe diameter. As shown in Table 10.1, an NPS pipe is specified using two values: a nominal pipe size (NPS) and a schedule. The nominal pipe size determines the outside diameter or OD. For example, pipes with a nominal size of 2 inches have an OD of 2.375 inches. Once the nominal size reaches 14 inches, the nominal size and the OD are equal. That is, a pipe with a nominal size of 24 inches will have an OD of 24 in.

Pipe schedule is related to the thickness of the wall. The original meaning of schedule was the ability of a pipe to withstand pressure, thus pipe schedule correlates with wall thickness. Each nominal pipe size has many possible schedules that range from schedule 5 to schedule 160. The data in Table 10.1 show representative ODs and schedules; more pipe sizes are specified in engineering handbooks and on the Internet.

FIGURE 10.4
Section view of a pip

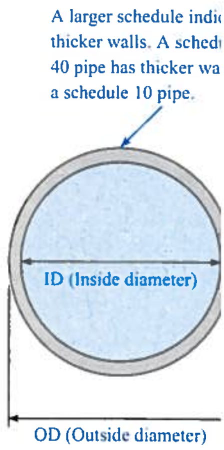


TABLE 10.1 Nominal Pipe Sizes

NPS (in)	OD (in)	Schedule	Wall Thickness (in)	ID (in)
1/2	0.840	40	0.109	0.622
		80	0.147	0.546
1	1.315	40	0.133	1.049
		80	0.179	0.957
2	2.375	40	0.154	2.067
		80	0.218	1.939
4	4.500	40	0.237	4.026
		80	0.337	3.826
8	8.625	40	0.322	7.981
		80	0.500	7.625
14	14.000	10	0.250	13.500
		40	0.437	13.126
		80	0.750	12.500
		120	1.093	11.814
24	24.000	10	0.250	23.500
		40	0.687	22.626
		80	1.218	21.564
		120	1.812	20.376

10.3 Pipe Head Loss

This section presents the Darcy-Weisbach equation, which is used for calculating head loss in a straight run of pipe. This equation is one of the most useful equations in fluid mechanics.

Combined (Total) Head Loss

Pipe head loss is one type of head loss; the other type is called component head loss. All hea loss is classified using these two categories:

$$\text{(Total head loss)} = \text{(Pipe head loss)} + \text{(Component head loss)} \quad (10.$$

Component head loss is associated with flow through devices such as valves, bends, and tee **Pipe head loss** is associated with fully developed flow in conduits, and it is caused by she stresses that act on the flowing fluid. Note that pipe head loss is sometimes called major hea loss, and component head loss is sometimes called minor head loss. Pipe head loss is predicte with the Darcy-Weisbach equation.

Derivation of the Darcy-Weisbach Equation

To derive the Darcy-Weisbach equation, start with the situation shown in Fig. 10.5. Assun fully developed and steady flow in a round tube of constant diameter D. Situate a cylindric control volume of diameter D and length ΔL inside the pipe. Define a coordinate system wi an axial coordinate in the streamwise direction (s direction) and a radial coordinate in tl r direction.

Apply the momentum equation to the control volume shown in Fig. 10.5.

$$\sum \mathbf{F} = \frac{d}{dt}\int_{cv} \mathrm{v}\rho d\forall + \int_{cs} \mathrm{v}\rho \mathbf{V} \cdot d\mathbf{A} \quad (10.$$

(Net forces) = (Momentum accumulation rate) + (Net efflux of momentum)

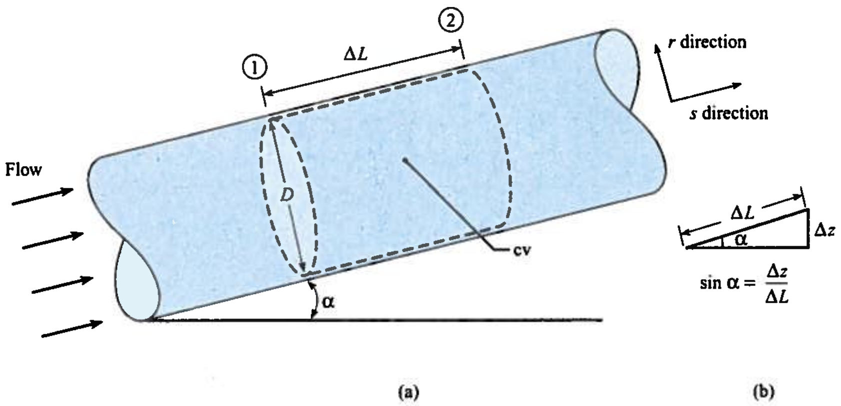


FIGURE 10.5
Initial situation for the derivation of the Darcy-Weisbach equation.

Select the streamwise direction and analyze each of the three terms in Eq. (10.5). The n efflux of momentum is zero because the velocity distribution at section 2 is identical to tl velocity distribution at section 1. The momentum accumulation term is also zero because tl flow is steady. Thus, Eq. (10.5) simplifies to $\Sigma F = 0$. Forces are shown in Fig. 10.6. Summing forces in the streamwise direction gives

$$F_{\text{pressure}} + F_{\text{shear}} + F_{\text{weight}} = 0$$

$$(p_1 - p_2)\left(\frac{\pi D^2}{4}\right) - \tau_0(\pi D \Delta L) - \gamma\left[\left(\frac{\pi D^2}{4}\right)\Delta L\right]\sin\alpha = 0 \quad (10.$$

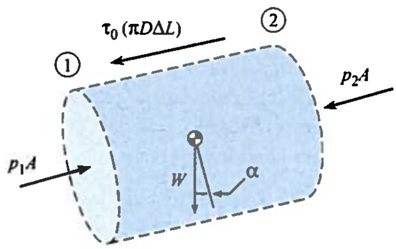


FIGURE 10.6
Force diagram.

Figure 10.5b shows that $\sin\alpha = (\Delta z/\Delta L)$. Equation (10.6) becomes

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \frac{4\Delta L \tau_0}{D} \tag{10.7}$$

Next, apply the energy equation to the control volume shown in Fig. 10.5. Recognize that $h_p = h_t = 0$, $V_1 = V_2$, and $\alpha_1 = \alpha_2$. Thus, the energy equation reduces to

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$
$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma h_L \tag{10.8}$$

Combine Eqs. (10.7) and (10.8) and replace ΔL by L. Also, introduce a new symbol h_f to represent head loss in pipe.

$$h_f = \begin{pmatrix}\text{head loss}\\ \text{in a pipe}\end{pmatrix} = \frac{4L\tau_0}{D\gamma} \tag{10.9}$$

Rearrange the right side of Eq. (10.9).

$$h_f = \left(\frac{L}{D}\right)\left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left\{\frac{\rho V^2/2}{\gamma}\right\} = \left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left(\frac{L}{D}\right)\left\{\frac{V^2}{2g}\right\} \tag{10.10}$$

Define a new π-group called the **friction factor** f that gives the ratio of wall shear stress (τ_0) to kinetic pressure ($\rho V^2/2$):

$$f \equiv \frac{(4 \cdot \tau_0)}{(\rho V^2/2)} \approx \frac{\text{shear stress acting at the wall}}{\text{kinetic pressure}} \tag{10.11}$$

In the technical literature, the friction factor is identified by several different labels that are synonymous: friction factor, Darcy friction factor, Darcy-Weisbach friction factor, and the resistance coefficient. There is also another coefficient called the Fanning friction factor, often used by chemical engineers, which is related to the Darcy-Weisbach friction factor by a factor of 4.

$$f_{\text{Darcy}} = 4 f_{\text{Fanning}}$$

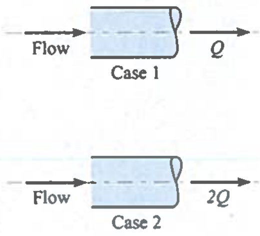
This text uses only the Darcy-Weisbach friction factor. Combining Eqs. (10.10) and (10.11) gives the Darcy-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{10.12}$$

To use the Darcy-Weisbach equation, the flow should be fully developed and steady. The Darcy-Weisbach equation is used for either laminar flow or turbulent flow and for either round pipes or nonround conduits such as a rectangular duct.

✔CHECKPOINT PROBLEM 10.1

The figure shows flow through two pipes. Case 1 has half the flow of Case 2. Both cases involve the same length of pipe, the same friction factor, and the same diameter. What is the ratio of head loss for Case 1 to head loss for Case 2?



a. 1:4

b. 1:2

c. head loss is the same

d. 2:1

e. 4:1

The Darcy-Weisbach equation shows that head loss depends on the friction factor, th pipe length-to-diameter ratio, and the mean velocity squared. The key to using the Darcy Weisbach equation is calculating a value of the friction factor f. This topic is addressed in th next sections of this text.

10.4 Stress Distributions in Pipe Flow

This section derives equations for the stress distributions on a plane that is oriented normal t stream lines. These equations, which apply to both laminar and turbulent flow, provide insigh about the nature of the flow. Also, these equations are used for subsequent derivations.

In pipe flow the pressure acting on a plane that is normal to the direction of flow is hydro static. This means that the pressure distribution varies linearly as shown in Fig. 10.7. The reaso that the pressure distribution is hydrostatic can be explained with Euler's equation (see p. 130).

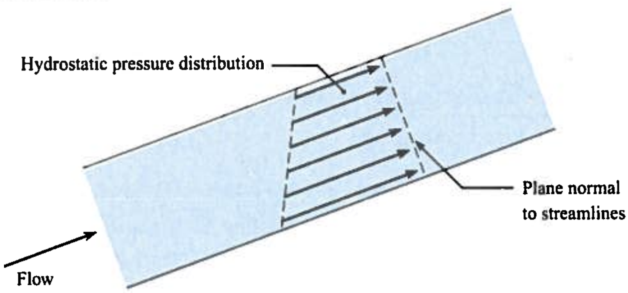


FIGURE 10.7
For fully developed flow in a pipe, the pressure distribution on an area normal to streamlines is hydrostatic.

To derive an equation for the shear-stress variation, consider flow of a Newtonian fluid i a round tube that is inclined at an angle α with respect to the horizontal as shown in Fig. 10. Assume that the flow is fully developed, steady, and laminar. Define a cylindrical control vo ume of length ΔL and radius r.

Apply the momentum equation in the s direction. The net momentum efflux is ze because the flow is fully developed; that is, the velocity distribution at the inlet is the same a the velocity distribution at the exit. The momentum accumulation is also zero because the flo is steady. The momentum equation simplifies to force equilibrium.

$$\sum F_s = F_{\text{pressure}} + F_{\text{weight}} + F_{\text{shear}} = 0 \qquad \textbf{(10.1}$$

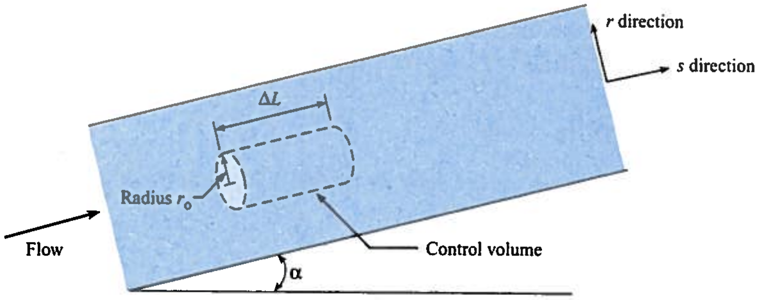


FIGURE 10.8
Sketch for derivation o
equation for shear stre

Analyze each term in Eq. (10.13) using the force diagram shown in Fig. 10.9:

$$pA - \left(p + \frac{dp}{ds}\Delta L\right)A - W\sin\alpha - \tau(2\pi r)\Delta L = 0 \qquad \text{(10.14)}$$

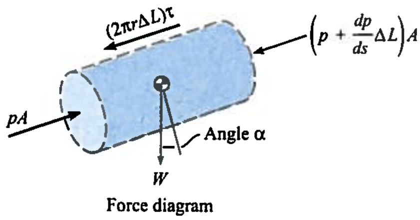


FIGURE 10.9
Force diagram corresponding to the control volume defined in Fig. 10.8.

Let $W = \gamma A\Delta L$, and let $\sin\alpha = \Delta z/\Delta L$ as shown in Fig. 10.5b. Next, divide Eq. (10.14) by $A\Delta L$:

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right] \qquad \text{(10.15)}$$

Equation (10.15) shows that the shear-stress distribution varies linearly with r as shown in Fig. 10.10. Notice that the shear stress is zero at the centerline, it reaches a maximum value of τ_0 at the wall, and the variation is linear in between. This linear shear stress variation applies to both laminar and turbulent flow.

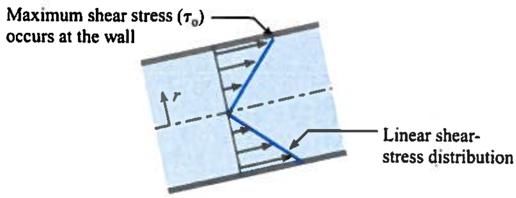


FIGURE 10.10
In fully developed flow (laminar or turbulent), th
shear-stress distribution
an area that is normal t
streamlines is linear.

10.5 Laminar Flow in a Round Tube

This section describes laminar flow and derives relevant equations. Laminar flow is important for flow in small conduits called microchannels, for lubrication flow, and for analyzing other flows in which viscous forces are dominant. Also, knowledge of laminar flow provides a foundation for the study of advanced topics.

Laminar flow is a flow regime in which fluid motion is smooth, the flow occurs in layers (laminae), and the mixing between layers occurs by molecular diffusion, a process that is much

slower than turbulent mixing. According to Eq. (10.2), laminar flow occurs when $\mathrm{Re}_D \leq 200$
Laminar flow in a round tube is called **Poiseuille flow** or **Hagen-Poiseuille flow** in honor
researchers who studied low-speed flows in the 1840s.

Velocity Profile

To derive an equation for the velocity profile in laminar flow, begin by relating stress to rate-o
strain using the viscosity equation:

$$\tau = \mu \frac{dV}{dy}$$

where y is the distance from the pipe wall. Change variables by letting $y = r_0 - r$, where r_0
pipe radius and r is the radial coordinate. Next, use the chain rule of calculus:

$$\tau = \mu\left(\frac{dV}{dy}\right) = \mu\left(\frac{dV}{dr}\right)\left(\frac{dr}{dy}\right) = -\left(\mu\frac{dV}{dr}\right) \qquad \text{(10.1}$$

Substitute Eq. (10.16) into Eq. (10.15).

$$-\left(\frac{2\mu}{r}\right)\left(\frac{dV}{dr}\right) = \frac{d}{ds}(p + \gamma z) \qquad \text{(10.1}$$

In Eq. (10.17), the left side of the equation is a function of radius r, and the right side is
function of axial location s. This can be true if and only if each side of Eq. (10.17) is equal tc
constant. Thus,

$$\text{constant} = \frac{d}{ds}(p + \gamma z) = \left(\frac{\Delta(p + \gamma z)}{\Delta L}\right) = \left(\frac{\gamma \Delta h}{\Delta L}\right) \qquad \text{(10.1}$$

where Δh is the change in piezometric head over a length ΔL of conduit. Combine Eqs. (10.1
and (10.18):

$$\frac{dV}{dr} = -\left(\frac{r}{2\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) \qquad \text{(10.1}$$

Integrate Eq. (10.19):

$$V = -\left(\frac{r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) + C \qquad \text{(10.2}$$

To evaluate the constant of integration C in Eq. (10.20), apply the no-slip condition, whi
states that the velocity of the fluid at the wall is zero. Thus,

$$V(r = r_0) = 0$$

$$0 = -\frac{r_0^2}{4\mu}\left(\frac{\gamma \Delta h}{\Delta L}\right) + C$$

Solve for C and substitute the result into Eq. (10.20):

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) \qquad \text{(10.2}$$

The maximum velocity occurs at $r = r_0$:

$$V_{\max} = -\left(\frac{r_0^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) \qquad \text{(10.2}$$

Combine Eqs. (10.21) and (10.22):

$$V(r) = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) = V_{max}\left(1 - \left(\frac{r}{r_0}\right)^2\right) \quad (10.23)$$

Equation (10.23) shows that velocity varies as radius squared ($V \sim r^2$), meaning that the velocity distribution in laminar flow is parabolic as plotted in Fig. 10.11.

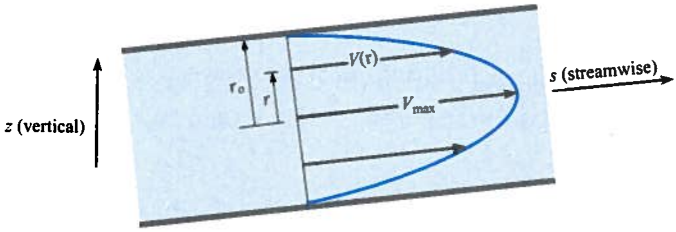


FIGURE 10.11
The velocity profile in Poiseuille flow is paral

Discharge and Mean Velocity V

To derive an equation for discharge Q, introduce the velocity profile from Eq. (10.23) into the flow rate equation.

$$Q = \int V\,dA$$
$$= -\int_0^{r_0} \frac{(r_0^2 - r^2)}{4\mu}\left(\frac{\gamma \Delta h}{\Delta L}\right)(2\pi r\,dr) \quad (10.24)$$

Integrate Eq. (10.24):

$$Q = -\left(\frac{\pi}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)\frac{(r^2 - r_0^2)^2}{2}\Bigg|_0^{r_0} = -\left(\frac{\pi r_0^4}{8\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) \quad (10.25)$$

To derive an equation for mean velocity, apply $Q = \overline{V}A$ and use Eq. (10.25).

$$\overline{V} = -\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) \quad (10.26)$$

Comparing Eqs. (10.26) and (10.22) reveals that $\overline{V} = V_{max}/2$. Next, substitute $D/2$ for r_0 in Eq. (10.26). The final result is an equation for mean velocity in a round tube.

$$\overline{V} = -\left(\frac{D^2}{32\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) = \frac{V_{max}}{2} \quad (10.27)$$

Head Loss and Friction Factor f

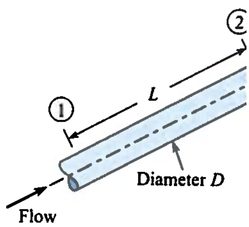
To derive an equation for head loss in a round tube, assume fully developed flow in the pipe shown in Fig. 10.12. Apply the energy equation from section 1 to 2 and simplify to give

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + h_L \quad (10.28)$$

Let $h_L = h_f$ and then Eq. (10.28) becomes

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + h_f \quad (10.29)$$

FIGURE 10.12
Flow in a pipe.



Expand Eq. (10.27).

$$\overline{V} = -\left(\frac{\gamma D^2}{32\mu}\right)\left(\frac{\Delta h}{\Delta L}\right) = -\left(\frac{\gamma D^2}{32\mu}\right)\frac{\left(\frac{p_2}{\gamma} + z_2\right) - \left(\frac{p_1}{\gamma} + z_1\right)}{\Delta L} \quad \text{(10.3(}$$

Reorganize Eq. (10.30) and replace ΔL with L.

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + \frac{32\mu \overline{V} L}{\gamma D^2} \quad \text{(10.3'}$$

Comparing Eqs. (10.29) and (10.31) gives an equation for head loss in a pipe.

$$h_f = \frac{32\mu L \overline{V}}{\gamma D^2} \quad \text{(10.3:}$$

Key assumptions on Eq. (10.32) are (a) laminar flow, (b) fully developed flow, (c) steady flo and (d) Newtonian fluid.

Equation (10.32) shows that head loss in laminar flow varies linearly with velocity. Als head loss is influenced by viscosity, pipe length, specific weight, and pipe diameter squared.

To derive an equation for the friction factor f, combine Eq. (10.32) with the Darcy-Weisbac equation (10.12).

$$h_f = \frac{32\,\mu L V}{\gamma D^2} = f\frac{L}{D}\frac{V^2}{2g} \quad \text{(10.3}$$

$$\text{or } f = \left(\frac{32\mu L V}{\gamma D^2}\right)\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) = \frac{64\mu}{\rho D V} = \frac{64}{\text{Re}_D} \quad \text{(10.3}$$

Equation (10.34) shows that the friction factor for laminar flow depends only on Reynol number. Example 10.2 illustrates how to calculate head loss.

EXAMPLE 10.2

Head Loss for Laminar Flow

Problem Statement

Oil ($S = 0.85$) with a kinematic viscosity of 6×10^{-4} m²/s flows in a 15 cm diameter pipe at a rate of 0.020 m³/s. What is the head loss for a 100 m length of pipe?

Define the Situation

- Oil is flowing in a pipe at a flow rate of $Q = 0.02$ m³/s.
- Pipe diameter is $D = 0.15$ m.

Assumptions: Fully developed, steady flow

Properties: Oil: $S = 0.85$, $\nu = 6 \times 10^{-4}$ m²/s

State the Goal

Calculate head loss (in meters) for a pipe length of 100 m.

Generate Ideas and Make a Plan

1. Calculate the mean velocity using the flow rate equation.
2. Calculate the Reynolds number using Eq. (10.1).
3. Check whether the flow is laminar or turbulent using Eq. (10.2).
4. Calculate head loss using Eq. (10.32).

Take Action (Execute the Plan)

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.020\ \text{m}^3/\text{s}}{(\pi D^2)/4} = \frac{0.020\ \text{m}^3/\text{s}}{\pi((0.15\ \text{m})^2/4)} = 1.13\ \text{m/s}$$

2. Reynolds number

$$\text{Re}_D = \frac{VD}{\nu} = \frac{(1.13\ \text{m/s})(0.15\ \text{m})}{6 \times 10^{-4}\ \text{m}^2/\text{s}} = 283$$

3. Because $\mathrm{Re}_D < 2000$, the flow is laminar.
4. Head loss (laminar flow).

$$h_f = \frac{32\mu LV}{\gamma D^2} = \frac{32\rho\nu LV}{\rho g D^2} = \frac{32\nu LV}{gD^2}$$
$$= \frac{32(6 \times 10^{-4}\ \mathrm{m^2/s})(100\ \mathrm{m})(1.13\ \mathrm{m/s})}{(9.81\ \mathrm{m/s^2})(0.15\ \mathrm{m})^2}$$
$$= \boxed{9.83\ \mathrm{m}}$$

Review the Solution and the Process

Knowledge. An alternative way to calculate head loss for lamina flow is to use the Darcy-Weisbach equation (10.12) as follows:

$$f = \frac{64}{\mathrm{Re}_D} = \frac{64}{283} = 0.226$$
$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.226\left(\frac{100\ \mathrm{m}}{0.15\ \mathrm{m}}\right)\left(\frac{(1.13\ \mathrm{m/s})^2}{2 \times 9.81\ \mathrm{m/s^2}}\right)$$
$$= 9.83\ \mathrm{m}$$

10.6 Turbulent Flow and the Moody Diagram

This section describes the characteristics of turbulent flow, presents equations for calculating the friction factor f, and presents a famous graph called the Moody diagram. This information is important because most flows in conduits are turbulent.

Qualitative Description of Turbulent Flow

Turbulent flow is a flow regime in which the movement of fluid particles is chaotic, eddying, and unsteady, with significant movement of particles in directions transverse to the flow direction. Because of the chaotic motion of fluid particles, turbulent flow produces high levels of mixing and has a velocity profile that is more uniform or flatter than the corresponding laminar velocity profile. According to Eq. (10.2), turbulent flow occurs when $\mathrm{Re} \geq 3000$.

Engineers and scientists model turbulent flow by using an empirical approach. This is because the complex nature of turbulent flow has prevented researchers from establishing a mathematical solution of general utility. Still, the empirical information has been used successfully and extensively in system design. Over the years, researchers have proposed many equations for shear stress and head loss in turbulent pipe flow. The empirical equations that have proven to be the most reliable and accurate for engineering use are presented in the next section.

Equations for the Velocity Distribution

The time-average velocity distribution is often described using an equation called the power-law formula.

$$\frac{u(r)}{u_{\max}} = \left(\frac{r_0 - r}{r_0}\right)^m \qquad \textbf{(10.35)}$$

where $u_{\max}$ is velocity in the center of the pipe, r_0 is the pipe radius, and m is an empirically determined variable that depends on Re as shown in Table 10.2. Notice in Table 10.2 that the velocity in the center of the pipe is typically about 20% higher than the mean velocity V. Although Eq. (10.35) provides an accurate representation of the velocity profile, it does not predict an accurate value of wall shear stress.

An alternative approach to Eq. (10.35) is to use the turbulent boundary-layer equations presented in Chapter 9. The most significant of these equations, called the logarithmic velocity distribution, is given by Eq. (9.29) and repeated here:

$$\frac{u(r)}{u_*} = 2.44 \ln \frac{u_*(r_0 - r)}{\nu} + 5.56 \qquad \textbf{(10.36)}$$

where u_*, the shear velocity, is given by $u_* = \sqrt{\tau_0/\rho}$.