8 DIMENSIONAL ANALYSIS AND SIMILITUDE

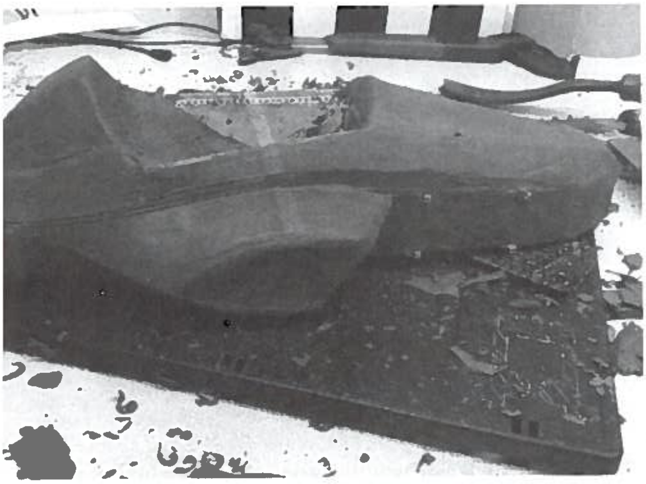


FIGURE 8.1
The photo shows a model of a formula racing car that was built out of clay for testing in a small wind tunnel. The purpose of the testing was to assess the drag characteristics. The work was done by Josh Hartung, while he was an undergraduate engineering student. (Photo courtesy of Josh Hartung.)

Chapter Road Map

Because of the complexity of flows, designs are ofte based on experimental results, which are commonly done using scale models. The theoretical basis of experimental testing is called dimensional analysis, the topic of this chapter. This topic is also used to simplify analysis and to present results.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Explain why dimensional analysis is needed. (§8.1)
- Explain or apply the Buckingham Π theorem. (§8.2)
- Find π-groups using the step-by-step method. (§8.3)
- Find π-groups using the exponent method. (§8.3)
- Define and describe common π-groups. (§8.4).
- Define a model and a prototype. (§8.5)
- Explain what similitude means, including geometric similitude and dynamic similitude. Describe the criteria for acheiving similitude. (§8.5)

8.1 Need for Dimensional Analysis

Fluid mechanics is more heavily involved with experimental testing than other discipli because the analytical tools currently available to solve the momentum and energy equatic are not capable of providing accurate results. This is particularly evident in turbulent, separat flows. The solutions obtained by utilizing techniques from computational fluid dynamics w the largest computers available yield only fair approximations for turbulent flow problem hence the need for experimental evaluation and verification.

For analyzing model studies and for correlating the results of experimental research, it is essential that researchers employ dimensionless groups. To appreciate the advantages of using dimensionless groups, consider the flow of water through the unusual orifice illustrated in Fig. 8.2. Actually, this is much like a nozzle used for flow metering except that the flow is in the opposite direction. An orifice operating in this flow condition will have a much different performance than one operating in the normal mode. However, it is not unlikely that a firm or city water department might have such a situation where the flow may occur the "right way" most of the time and the "wrong way" part of the time—hence the need for such knowledge.

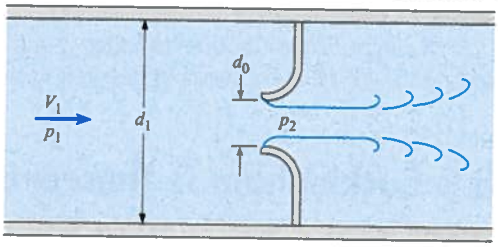


FIGURE 8.2 Flow through inverted flow nozzle.

Because of size and expense it is not always feasible to carry out tests on a full-scale prototype. Thus engineers will test a subscale model and measure the pressure drop across the model. The test procedure may involve testing several orifices, each with a different throat diameter d_0. For purposes of discussion, assume that three nozzles are to be tested. The Bernoulli equation, introduced in Chapter 4, suggests that the pressure drop will depend on flow velocity and fluid density. It may also depend on the fluid viscosity.

The test program may be carried out with a range of velocities and possibly with fluids of different density (and viscosity). The pressure drop, $p_1 - p_2$, is a function of the velocity V_1, density ρ, and diameter d_0. By carrying out numerous measurements at different values of V_1 and ρ for the three different nozzles, the data could be plotted as shown in Fig. 8.3a for tests using water. In addition, further tests could be planned with different fluids at considerably more expense.

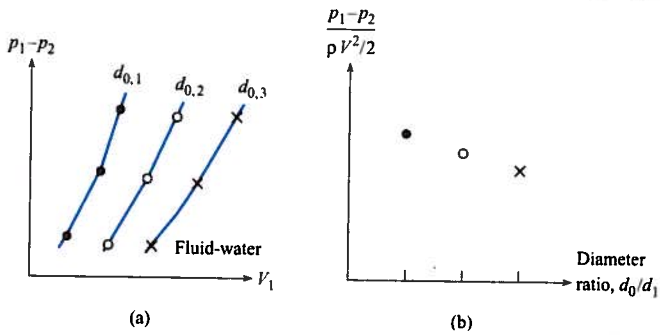


FIGURE 8.3 Relations for pressure, velocity, and diameter. (a) Using dimensional variables. (b) Using dimensionless groups.

The material introduced in this chapter leads to a much better approach. Through dimensional analysis it can be shown that the pressure drop can be expressed as

$$\frac{p_1 - p_2}{(\rho V^2)/2} = f\left(\frac{d_0}{d_1}, \frac{\rho V_1 d_0}{\mu}\right) \tag{8.1}$$

which means that dimensionless group for pressure, $(p_1 - p_2)/(\rho V^2/2)$, is a function of the dimensionless throat/pipe diameter ratio d_0/d_1 and the dimensionless group, $(\rho V_1 d_0)/\mu$, which will be identified later as the Reynolds number. The purpose of the experimental program is to establish the functional relationship. As will be shown later, if the Reynolds number is suffi ciently large, the results are independent of Reynolds number. Then

$$\frac{p_1 - p_2}{(\rho V^2)/2} = f\left(\frac{d_0}{d_1}\right) \quad (8.2$$

Thus for any specific orifice design (same d_0/d_1) the pressure drop, $p_1 - p_2$, divided by $\rho V_1^2/$ for the model is same for the prototype. Therefore the data collected from the model tests ca be applied directly to the prototype. Only one test is needed for each orifice design. Conse quently only three tests are needed, as shown in Fig. 8.2b. The fewer tests result in considerabl savings in effort and expense.

The identification of dimensionless groups that provide correspondence between mode and prototype data is carried out through **dimensional analysis.**

8.2 Buckingham Π Theorem

In 1915 Buckingham (1) showed that the number of independent dimensionless groups c variables (dimensionless parameters) needed to correlate the variables in a given process i equal to $n - m$, where n is the number of variables involved and m is the number of basi dimensions included in the variables.

Buckingham referred to the dimensionless groups as Π, which is the reason the theorem is called the Π theorem. Henceforth dimensionless groups will be referred to as **π-groups.** the equation describing a physical system has n dimensional variables and is expressed as

$$y_1 = f(y_2, y_3, \dots y_n)$$

then it can be rearranged and expressed in terms of $(n - m)$ π-groups as

$$\pi_1 = \varphi(\pi_2, \pi_3, \dots \pi_{n-m})$$

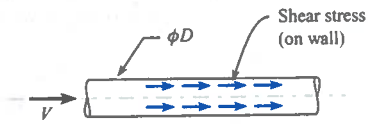
Thus if the drag force F of a fluid flowing past a sphere is known to be a function of the velocity mass density ρ, viscosity μ, and diameter D, then five variables (F, V, ρ, μ, and D) and thre basic dimensions (L, M, and T) are involved.* By the Buckingham Π theorem there will b $5 - 3 = 2$ π-groups that can be used to correlate experimental results in the form

$$\pi_1 = \varphi(\pi_2)$$

✔ CHECKPOINT PROBLEM 8.1

When a fluid flows in a round pipe, the shear stress on the walls of the pipe depends on the viscosity and density of the fluid, the mean velocity, the pipe diameter, and on the roughness of the pipe wall The wall roughness is characterized by a variable that has units of meters that is called the sand roughness height. How many π-groups are needed to correlate experimental data?

a. 1
b. 2
c. 3
d. 4
e. 5



*Note that only three basic dimensions will be considered here. Temperature will not be included.

8.3 Dimensional Analysis

Dimensional analysis is the process for applying π-groups to analysis, experiment design, and the presentation of results. This section presents two methods for finding π-groups:

The Step-by-Step Method

Several methods may be used to carry out the process of finding the π-groups, but the step-by-step approach, very clearly presented by Ipsen (2), is one of the easiest and reveals much about the process. The process for the step-by-step method follows in Table 8.1.

The final result can be expressed as a functional relationship of the form

$$\pi_1 = f(\pi_2, \pi_2, \dots \pi_n) \tag{8.3}$$

The selection of the dependent and independent π-groups depends on the application. Also the selection of variables used to eliminate dimensions is arbitrary.

TABLE 8.1 The Step-by-Step Approach

Step	Action Taken during This Step
1	Identify the significant dimensional variables and write out the primary dimensions of each.
2	Apply the Buckingham Π theorem to find the number of π-groups.*
3	Set up table with the number of rows equal to the number of dimensional variables and the number of columns equal to the number of basic dimensions plus one ($m + 1$).
4	List all the dimensional variables in the first column with primary dimensions.
5	Select a dimension to be eliminated, choose a variable with that dimension in the first column, and combine with remaining variables to eliminate the dimension. List combined variables in the second column with remaining primary dimensions.
6	Select another dimension to be eliminated, choose from variables in the second column that have that dimension, and combine with the remaining variables. List the new combinations with remaining primary dimensions in the third column
7	Repeat Step 6 until all dimensions are eliminated. The remaining dimensionless groups are the π-groups. List the π-groups in the last column

*Note that, in rare instances, the number of π-groups may be one more than predicted by the Buckingham Π theorem. This anomaly can occur because it is possible that two-dimensional categories can be eliminated when dividing (or multiplying) by a given variable. See Ipsen (2) for an example of this.

Example 8.1 shows how to use the step-by-step method to find the π-groups for a body falling in a vacuum.

EXAMPLE 8.1

Finding π-Group for a Body Falling in a Vacuum

Problem Statement

There are three significant dimensional variables for a body falling in a vacuum (no viscous effects): the velocity V; the acceleration due to gravity, g; and the distance through which the body falls, h. Find the π-groups using the step-by-step method.

Define the Situation

A body is falling in a vacuum, $V = f(g, h)$.

State the Goal

Find the π-groups.

Generate Ideas and Make a Plan

Apply the step-by-step method in Table 8.1.

Take Action (Execute the Plan)

1. Significant variables and dimensions

$$[V] = L/T$$
$$[g] = L/T^2$$
$$[h] = L$$

There are only two dimensions, L and T.

2. From the Buckingham Π theorem, there is only one (three variables–two dimensions) π-group.
3. Set up table with three rows (number of variables) and three (dimensions + 1) columns.
4. List variables and primary dimensions in first column.

Variable	[]	Variable	[]	Variable	[]
V	$\frac{L}{T}$	$\frac{V}{h}$	$\frac{1}{T}$	$\frac{V}{\sqrt{gh}}$	0
g	$\frac{L}{T^2}$	$\frac{g}{h}$	$\frac{1}{T^2}$		
h	L				

5. Select h to eliminate L. Divide g by h, enter in second column with dimension $1/T^2$. Divide V by h, enter in second column with dimension $1/T$.
6. Select g/h to eliminate T. Divide V/h by $\sqrt{g/h}$ and enter in third column.

As expected, there is only one π-group,

$$\pi = \frac{V}{\sqrt{gh}}$$

The final functional form of equation of the equation is

$$\frac{V}{\sqrt{gh}} = C$$

Review the Solution and the Process

1. *Knowledge*. From physics, one can show that $C = \sqrt{2}$.
2. *Knowledge*. The proper relationship between V, h, and g was found with dimensionless analysis. If the value of C was not known, it could be determined from experiment.

Example 8.2 illustrates the application of the step-by-step method for finding π-groups for a problem with five variables and three primary dimensions.

EXAMPLE 8.2

Finding π-Groups for Drag on a Sphere Using Step-by-Step Method

Problem Statement

The drag F_D of a sphere in a fluid flowing past the sphere is a function of the viscosity μ, the mass density ρ, the velocity of flow V, and the diameter of the sphere D. Use the step-by-step method to find the π-groups.

Define the Situation

The functional relationship is $F_D = f(V, \rho, \mu, D)$.

State the Goal

Find the π-groups using the step-by-step method.

Generate Ideas and Make a Plan

Apply the step-by-step procedure from Table 8.1.

Take Action (Execute the Plan)

1. Dimensions of significant variables

$$F = \frac{ML}{T^2}, V = \frac{L}{T}, \rho = \frac{M}{L^3}, \mu = \frac{M}{LT}, D = L$$

2. Number of π-groups, $5 - 3 = 2$.

3. Set up table with five rows and four columns.
4. Write variables and dimensions in first column.

Variable	[]	Variable	[]	Variable	[]	Variable	[]
F_D	$\frac{ML}{T^2}$	$\frac{F_D}{D}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 D^2}$	0
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VD}$	0
D	L						

5. Eliminate L using D and write new variable combinations with corresponding dimensions in the second column.
6. Eliminate M using ρD^3 and write new variable combinations with dimensions in the third column.
7. Eliminate T using V/D and write new combinations in the fourth column.

The final two π-groups are

$$\pi_1 = \frac{F_D}{\rho V^2 D^2} \quad \text{and} \quad \pi_2 = \frac{\mu}{\rho VD}$$

The functional equation can be written as

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho VD}\right)$$

The form of the π-groups obtained will depend on the variables selected to eliminate dimensions. For example, if in Example 8.2, $\mu/\rho D^2$ had been used to eliminate the time dimension, the two π-groups would have been

$$\pi_1 = \frac{\rho F_D}{\mu^2} \quad \text{and} \quad \pi_2 = \frac{\mu}{\rho VD}$$

The result is still valid but may not be convenient to use. The form of any π-group can be altered by multiplying or dividing by another π-group. Multiplying the π_1 by the square of π_2 yields the original π_1 in Example 8.2.

$$\frac{\rho F_D}{\mu^2} \times \left(\frac{\mu}{\rho VD}\right)^2 = \frac{F_D}{\rho V^2 D^2}$$

By so doing the two π-groups would be the same as in Example 8.2.

The Exponent Method

An alternative method for finding the π-groups is the exponent method. This method involves solving a set of algebraic equations to satisfy dimensional homogeneity. The process for the exponent method is listed in Table 8.2.

TABLE 8.2 The Exponent Method

Step	Action Taken During This Step
1	Identify the significant dimensional variables, y_i, and write out the primary dimensions of each, $[y_i]$.
2	Apply the Buckingham Π theorem to find the number of π-groups.
3	Write out the product of the primary dimensions in the form $$[y_1] = [y_2]^a \times [y_3]^b \times \cdots \times [y_n]^k$$ where n is the number of dimensional variables and a, b, etc. are exponents.
4	Find the algebraic equations for the exponents that satisfy dimensional homogeneity (same power for dimensions on each side of equation).
5	Solve the equations for the exponents.
6	Express the dimensional equation in the form $y_1 = y_2^a y_3^b \dots y_n^k$ and identify the π-groups.

Example 8.3 illustrates how to apply the exponent method to find the π-groups of tl same problem addressed in Example 8.2.

EXAMPLE 8.3

Finding π-Groups for Drag on a Sphere Using Exponent Method

Problem Statement

The drag of a sphere, F_D, in a flowing fluid is a function of the velocity V, the fluid density ρ, the fluid viscosity μ, and the sphere diameter D. Find the π-groups using the exponent method.

Define the Situation

The functional equation is $F_D = f(V, \rho, \mu, D)$.

State the Goal

Find the π-groups using the exponent method.

Generate Ideas and Make a Plan

Apply the process for the exponent method from Table 8.2.

Take Action (Execute the Plan)

1. Dimensions of significant variables are

$$[F] = \frac{ML}{T^2}, [V] = \frac{L}{T}, [\rho] = \frac{M}{L^3}, [\mu] = \frac{M}{LT}, [D] = L$$

2. Number of π-groups is $5 - 3 = 2$.
3. Form product with dimensions.

$$\frac{ML}{T^2} = \left[\frac{L}{T}\right]^a \times \left[\frac{M}{L^3}\right]^b \times \left[\frac{M}{LT}\right]^c \times [L]^d$$
$$= \frac{L^{a-3b-c+d}M^{b+c}}{T^{a+c}}$$

4. Dimensional homogeneity. Equate powers of dimensions on each side.

$$L:\ a - 3b - c + d = 1$$
$$M:\ b + c = 1$$
$$T:\ a + c = 2$$

5. Solve for exponents a, b, and c in terms of d.

$$\begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1-d \\ 1 \\ 2 \end{pmatrix}$$

The value of the determinant is -1 so a unique solution is achievable. Solution is $a = d, b = d - 1, c = 2 - d$

6. Write dimensional equation with exponents.

$$F = V^d \rho^{d-1} \mu^{2-d} D^d$$
$$F = \frac{\mu^2}{\rho}\left(\frac{\rho VD}{\mu}\right)^d$$
$$\frac{F\rho}{\mu^2} = \left(\frac{\rho VD}{\mu}\right)^d$$

There are two π-groups:

$$\pi_1 = \frac{F\rho}{\mu^2} \quad \text{and} \quad \pi_2 = \frac{\rho VD}{\mu}$$

By dividing π_1 by the square of π_2, the π_1 group can be written as $F_D/(\rho V^2 D^2)$, so the functional form of the equation can be written as

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho VD}{\mu}\right)$$

Review the Solution and the Process

Discussion. The functional relationship between the two π-groups can be obtained from experiments.

Selection of Significant Variables

All the foregoing procedures deal with straightforward situations. However, some problems c occur. To apply dimensional analysis one must first decide which variables are significa If the problem is not sufficiently well understood to make a good choice of the significa variables, dimensional analysis seldom provides clarification.

A serious shortcoming might be the omission of a significant variable. If this is done, o of the significant π-groups will likewise be missing. In this regard, it is often best to identi a list of variables that one regards as significant to a problem and to determine if only o dimensional category (such as M or L or T) occurs. When this happens, it is likely that the is an error in choice of significant variables because it is not possible to combine two variables to eliminate the lone dimension. Either the variable with the lone dimension should not have been included in the first place (it is not significant), or another variable should have been included.

How does one know if a variable is significant for a given problem? Probably the truest answer is by experience. After working in the field of fluid mechanics for several years, one develops a feel for the significance of variables to certain kinds of applications. However, even the inexperienced engineer will appreciate the fact that free-surface effects have no significance in closed-conduit flow; consequently, surface tension, σ, would not be included as a variable. In closed-conduit flow, if the velocity is less than approximately one-third the speed of sound, compressibility effects are usually negligible. Such guidelines, which have been observed by previous experimenters, help the novice engineer develop confidence in her or his application of dimensional analysis and similitude.

8.4 Common π-Groups

The most common π-groups can be found by applying dimensional analysis to the variables that might be significant in a general flow situation, The purpose of this section is to develop these common π-groups and discuss their significance.

Variables that have significance in a general flow field are the velocity V, the density ρ, the viscosity μ, and the acceleration due to gravity g. In addition, if fluid compressibility were likely, then the bulk modulus of elasticity, E_v, should be included. If there is a liquid-gas interface, the surface tension effects may also be significant. Finally the flow field will be affected by a general length, L, such as the width of a building or the diameter of a pipe. These variables will be regarded as the independent variables. The primary dimensions of the significant independent variables are

$$[V] = L/T \qquad [\rho] = M/L^3 \qquad [\mu] = M/LT$$
$$[g] = L/T^2 \qquad [E_v] = M/LT^2 \qquad [\sigma] = M/T^2 \qquad [L] = L$$

There are several other independent variables that could be identified for thermal effects, such as temperature, specific heat, and thermal conductivity. Inclusion of these variables is beyond the scope of this text.

Products that result from a flowing fluid are pressure distributions (p), shear stress distributions (τ), and forces on surfaces and objects (F) in the flow field. These will be identified as the dependent variables. The primary dimensions of the dependent variables are

$$[p] = M/LT^2 \qquad [\tau] = [\Delta p] = M/LT^2 \qquad [F] = (ML)/T^2$$

There are other dependent variables not included here, but they will be encountered and introduced for specific applications.

Altogether there are 10 significant variables, which, by application of the Buckingham Π theorem, means there are seven π-groups. Utilizing either the step-by-step method or the exponent method yields

$$\frac{p}{\rho V^2} \qquad \frac{\tau}{\rho V^2} \qquad \frac{F}{\rho V^2 L^2}$$
$$\frac{\rho V L}{\mu} \qquad \frac{V}{\sqrt{E_v/\rho}} \qquad \frac{\rho L V^2}{\sigma} \qquad \frac{V^2}{gL}$$

The first three groups, the dependent π-groups, are identified by specific names. For these groups it is common practice to use the kinetic pressure, $\rho V^2/2$, instead of ρV^2. In most applications one is concerned with a pressure difference, so the pressure π-group expressed as

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho V^2}$$

where C_p is called the pressure coefficient and p_0 is a reference pressure. The pressure coeffi cient was introduced earlier in Chapter 4 and Section 8.1. The π-group associated with she stress is called the shear-stress coefficient and defined as

$$c_f = \frac{\tau}{\frac{1}{2}\rho V^2}$$

where the subscript f denotes "friction." The π-group associated with force is referred to, her as a force coefficient and defined as

$$C_F = \frac{F}{\frac{1}{2}\rho V^2 L^2}$$

This coefficient will be used extensively in Chapter 11 for lift and drag forces on airfoils ar hydrofoils.

The independent π-groups are named after earlier contributors to fluid mechanics. Tl π-group $VL\rho/\mu$ is called the Reynolds number, after Osborne Reynolds, and designated by R The group $V/(\sqrt{E_v/\rho})$ is rewritten as (V/c) because $\sqrt{E_v/\rho}$ is the speed of sound, c. Th π-group is called the Mach number and designated by M. The π-group $\rho LV^2/\sigma$ is called t Weber number and designated by We. The remaining π-group is usually expressed as $V/\sqrt{}$ and identified as the Froude (rhymes with "food") number* and written as Fr.

The general functional form for all the π-groups is

$$C_p, c_f, C_F = f(\text{Re}, \text{M}, \text{We}, \text{Fr}) \qquad \text{(8.}$$

which means that either of the three dependent π-groups are functions of the four independe π-groups; that is, the pressure coefficient, the shear-stress coefficient, or the force coefficient a functions of the Reynolds number, Mach number, Weber number, and Froude number.

The π-groups, their symbols, and their names are summarized in Table 8.3. Each indepe dent π-group has an important physical interpretation as indicated by the ratio column. T Reynolds number can be viewed as the ratio of kinetic to viscous forces. The kinetic forces a the forces associated with fluid motion. The Bernoulli equation indicates that the pressure d ference required to bring a moving fluid to rest is the kinetic pressure, $\rho V^2/2$, so the kine forces,† F_k, should be proportional to

$$F_k \propto \rho V^2 L^2$$

The shear force due to viscous effects, F_v, is proportional to the shear stress and area

$$F_v \propto \tau A \propto \tau L^2$$

and the shear stress is proportional to

$$\tau \propto \mu \frac{dV}{dy} \propto \frac{\mu V}{L}$$

*Sometimes the Froude number is written as $V/\sqrt{(\Delta\gamma g L)/\gamma}$ and called the densimetric Froude number. It has ap cation in studying the motion of fluids in which there is density stratification, such as between saltwater and freshw in an estuary or heated-water effluents associated with thermal power plants.

†Traditionally the kinetic force has been identified as the "inertial" force.

TABLE 8.3 Common Π-Groups

π-Group	Symbol	Name	Ratio
$\frac{p - p_0}{(\rho V^2)/2}$	C_p	Pressure coefficient	$\frac{\text{Pressure differenece}}{\text{Kinetic pressure}}$
$\frac{\tau}{(\rho V^2)/2}$	c_f	Shear-stress coefficient	$\frac{\text{Shear stress}}{\text{Kinetic pressure}}$
$\frac{F}{(\rho V^2 L^2)/2}$	C_F	Force coefficient	$\frac{\text{Force}}{\text{Kinetic force}}$
$\frac{\rho L V}{\mu}$	Re	Reynolds number	$\frac{\text{Kinetic force}}{\text{Viscous force}}$
$\frac{V}{c}$	M	Mach number	$\sqrt{\frac{\text{Kinetic force}}{\text{Compressive force}}}$
$\frac{\rho L V^2}{\sigma}$	We	Weber number	$\frac{\text{Kinetic force}}{\text{Surface-tension force}}$
$\frac{V}{\sqrt{gL}}$	Fr	Froude number	$\sqrt{\frac{\text{Kinetic force}}{\text{Gravitational force}}}$

so $F_v \propto \mu VL$. Taking the ratio of the kinetic to the viscous forces

$$\frac{F_k}{F_v} \propto \frac{\rho VL}{\mu} = \text{Re}$$

yields the Reynolds number. The magnitude of the Reynolds number provides important information about the flow. A low Reynolds number implies viscous effects are important; a high Reynolds number implies kinetic forces predominate. The Reynolds number is one of the most widely used π-groups in fluid mechanics. It is also often written using kinematic viscosity, $\text{Re} = \rho VL/\mu = VL/\nu$.

The ratios of the other independent π-groups have similar significance. The Mach number is an indicator of how important compressibility effects are in a fluid flow. If the Mach number is small, then the kinetic force associated with the fluid motion does not cause a significant density change, and the flow can be treated as incompressible (constant density). On the other hand, if the Mach number is large, there are often appreciable density changes that must be considered in model studies.

The Weber number is an important parameter in liquid atomization. The surface tension of the liquid at the surface of a droplet is responsible for maintaining the droplet's shape. If a droplet is subjected to an air jet and there is a relative velocity between the droplet and the gas, kinetic forces due to this relative velocity cause the droplet to deform. If the Weber number is too large, the kinetic force overcomes the surface-tension force to the point that the droplet shatters into even smaller droplets. Thus a Weber-number criterion can be useful in predicting the droplet size to be expected in liquid atomization. The size of the droplets resulting from liquid atomization is a very significant parameter in gas-turbine and rocket combustion.

The Froude number is unimportant when gravity causes only a hydrostatic pressure distribution, such as in a closed conduit. However, if the gravitational force influences the pattern of flow, such as in flow over a spillway or in the formation of waves created by a ship as it cruises over the sea, the Froude number is a most significant parameter.

8.5 Similitude

Scope of Similitude

Similitude is the theory and art of predicting prototype performance from model observa tions. Whenever it is necessary to perform tests on a model to obtain information that cann be obtained by analytical means alone, the rules of similitude must be applied. The theory o similitude involves the application of π-groups, such as the Reynolds number or the Frouc number, to predict prototype performance from model tests. The art of similitude enters th problem when the engineer must make decisions about model design, model constructio performance of tests, or analysis of results that are not included in the basic theory.

Present engineering practice makes use of model tests more frequently than most peop realize. For example, whenever a new airplane is being designed, tests are made not only on th general scale model of the prototype airplane but also on various components of the plan Numerous tests are made on individual wing sections as well as on the engine pods and ta sections.

Models of automobiles and high-speed trains are also tested in wind tunnels to predict th drag and flow patterns for the prototype. Information derived from these model studies ofte indicates potential problems that can be corrected before the prototype is built, thereby savi considerable time and expense in development of the prototype.

In civil engineering, model tests are always used to predict flow conditions for the spi ways of large dams. In addition, river models assist the engineer in the design of flood-contr structures as well as in the analysis of sediment movement in the river. Marine engineers ma extensive tests on model ship hulls to predict the drag of the ships. Much of this type of testi is done at the David Taylor Model Basin, Naval Surface Warfare Center, Carderock Divisio near Washington, D.C. (see Fig. 8.4). Tests are also regularly performed on models of tall buildin

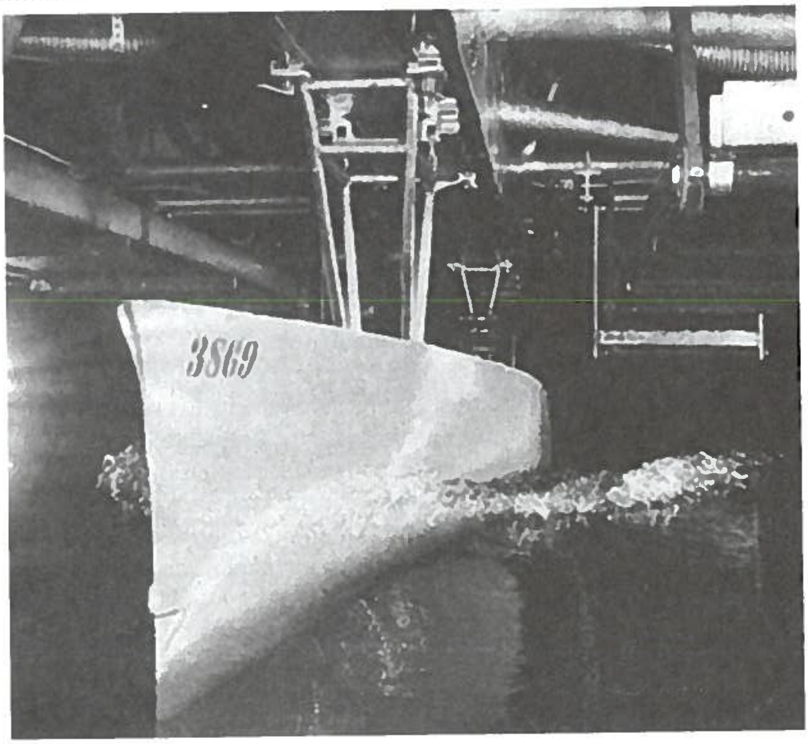


FIGURE 8.4
Ship-model test at the David Taylor Model Basin, Naval Surface Warfare Center, Carderock Division. (Naval Surface Warfare Center Carderock Division)

to help predict the wind loads on the buildings, the stability characteristics of the buildings, and the airflow patterns in their vicinity. The latter information is used by the architects to design walkways and passageways that are safer and more comfortable for pedestrians to use.

Geometric Similitude

Geometric similitude means that the model is an exact geometric replica of the prototype.* Consequently, if a 1:10 scale model is specified, all linear dimensions of the model must be 1/10 of those of the prototype. In Fig. 8.5 if the model and prototype are geometrically similar, the following equalities hold:

$$\frac{\ell_m}{\ell_p} = \frac{w_m}{w_p} = \frac{c_m}{c_p} = L_r \tag{8.5}$$

Here ℓ, w, and c are specific linear dimensions associated with the model and prototype, and L_r is the scale ratio between model and prototype. It follows that the ratio of corresponding areas between model and prototype will be the square of the length ratio: $A_r = L_r^2$. The ratio of corresponding volumes will be given by $\forall_m/\forall_p = L_r^3$.

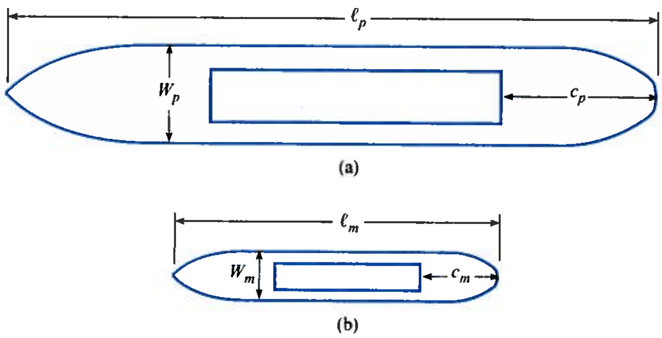


FIGURE 8.5
(a) Prototype. (b) Mod

Dynamic Similitude

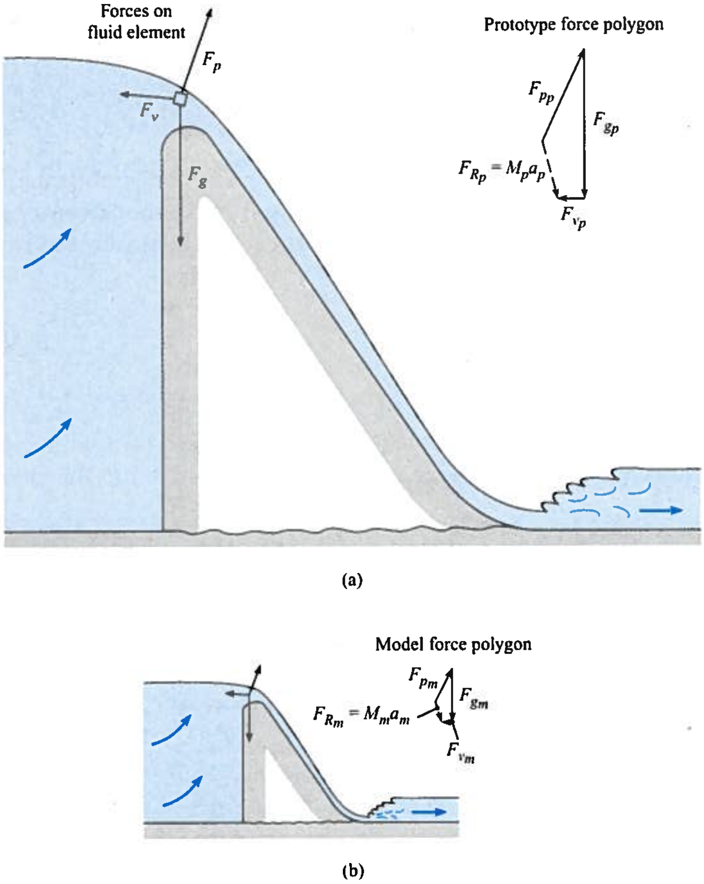
Dynamic similitude means that the forces that act on corresponding masses in the model and prototype are in the same ratio (F_m/F_p = constant) throughout the entire flow field. For example, the ratio of the kinetic to viscous forces must be the same for the model and the prototype. Because the forces acting on the fluid elements control the motion of those elements, it follows that dynamic similarity will yield similarity of flow patterns. Consequently, the flow patterns for the model and the prototype will be the same if geometric similitude is satisfied and if the relative forces acting on the fluid are the same in the model as in the prototype. This latter condition requires that the appropriate π-groups introduced in Section 8.4 be the same for the model and prototype because these π-groups are indicators of relative forces within the fluid.

A more physical interpretation of the force ratios can be illustrated by considering the flow over the spillway shown in Fig. 8.6a. Here corresponding masses of fluid in the model and prototype are acted on by corresponding forces. These forces are the force of gravity F_g, the pressure force F_p, and the viscous resistance force F_v. These forces add vectorially as shown in Fig. 8.6 to yield a resultant force F_R, which will in turn produce an acceleration of the volume of fluid in accordance with Newton's second law of motion. Hence, because the force polygons in the

*For most model studies this is a basic requirement. However, for certain types of problems, such as river models, distortion of the vertical scale is often necessary to obtain meaningful results.

FIGURE 8.6

Model-prototype relations: prototype view (a) and model view (b).



prototype and model are similar, the magnitudes of the forces in the prototype and model w be in the same ratio as the magnitude of the vectors representing mass times acceleration:

$$\frac{m_m a_m}{m_p a_p} = \frac{F_{gm}}{F_{gp}}$$

or

$$\frac{\rho_m L_m^3 (V_m / t_m)}{\rho_p L_p^3 (V_p / t_p)} = \frac{\gamma_m L_m^3}{\gamma_p L_p^3}$$

which reduces to

$$\frac{V_m}{g_m t_m} = \frac{V_p}{g_p t_p}$$

But

$$\frac{t_m}{t_p} = \frac{L_m / V_m}{L_p / V_p}$$

so

$$\frac{V_m^2}{g_m L_m} = \frac{V_p^2}{g_p L_p} \quad (8$$

Taking the square root of each side of Eq. (8.6) gives

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}} \qquad \text{or} \qquad \mathrm{Fr}_m = \mathrm{Fr}_p \tag{8.7}$$

Thus the Froude number for the model must be equal to the Froude number for the prototype to have the same ratio of forces on the model and the prototype.

Equating the ratio of the forces producing acceleration to the ratio of viscous forces,

$$\frac{m_m a_m}{m_p a_p} = \frac{F_{vm}}{F_{vp}} \tag{8.8}$$

where $F_v \propto \mu VL$ leads to

$$\mathrm{Re}_m = \mathrm{Re}_p$$

The same analysis can be carried out for the Mach number and the Weber number. To summarize, if the independent π-groups for the model and prototype are equal, then the condition for dynamic similitude is satisfied.

Referring back to Eq. (8.4) for the general functional relationship,

$$C_p, c_f, C_F = f(\mathrm{Re}, \mathrm{M}, \mathrm{We}, \mathrm{Fr})$$

if the independent π-groups are the same for the model and the prototype, then dependent π-groups must also be equal so

$$C_{p,m} = C_{p,p} \qquad c_{f,m} = c_{f,p} \qquad C_{F,m} = C_{F,p} \tag{8.9}$$

To have complete similitude between the model and the prototype, it is necessary to have both geometric and dynamic similitude.

In many situations it may not be possible nor necessary to have all the independent π-groups the same for the model and the prototype to carry out useful model studies. For the flow of a liquid in a horizontal pipe, for example, in which the fluid completely fills the pipe (no free surface), there would be no surface tension effects, so the Weber number would be inappropriate. Compressibility effects would not be important, so the Mach number would not be needed. In addition, gravity would not be responsible for the flow, so the Froude number would not have to be considered. The only significant π-group would be the Reynolds number; thus dynamic similitude would be achieved by matching the Reynolds number between the model and the prototype.

On the other hand if a model test were to be done for the flow over a spillway, the Froude number would be a significant π-group because gravity is responsible for the motion of the fluid. Also, the action of viscous stresses due to the spillway surface could possibly affect the flow pattern, so the Reynolds number may be a significant π-group. In this situation, dynamic similitude may require that both the Froude number and the Reynolds number be the same for the model and prototype.

The choice of significant π-groups for dynamic similitude and their actual use in predicting prototype performance are considered in the next two sections.

8.6 Model Studies for Flows without Free-Surface Effects

Free-surface effects are absent in the flow of liquids or gases in closed conduits, including control devices such as valves, or in the flow about bodies (e.g., aircraft) that travel through air or are deeply submerged in a liquid such as water (submarines). Free-surface effects are also absent where a structure such as a building is stationary and wind flows past it. In all these cases, given relatively low Mach numbers, the Reynolds-number criterion is the most si nificant for dynamic similarity. That is, the Reynolds number for the model must equal t Reynolds number for the prototype.

Example 8.4 illustrates the application of Reynolds-number similitude for the flow ov a blimp.

EXAMPLE 8.4

Reynolds-Number Similitude

Problem Statement

The drag characteristics of a blimp 5 m in diameter and 60 m long are to be studied in a wind tunnel. If the speed of the blimp through still air is 10 m/s, and if a 1/10 scale model is to be tested, what airspeed in the wind tunnel is needed for dynamically similar conditions? Assume the same air pressure and temperature for both model and prototype.

Define the Situation

A 1/10 scale model blimp is being testing in a wind tunnel. Prototype speed is 10 m/s.

Assumptions: Same air pressure and temperature for model and prototype, therefore $\nu_m = \nu_p$.

State the Goal

Find the air speed (m/s) in the wind tunnel for dynamic similitude.

Generate Ideas and Make a Plan

The only π-group that is appropriate is the Reynolds number (there are no compressibility effects, free-surface effects, or gravitation effects). Thus equating the model and prototype Reynolds number satisfies dynamic similitude.

1. Equate the Reynolds number of the model and the prototype.
2. Calculate model speed.

Take Action (Execute the Plan)

1. Reynolds-number similitude

$$\text{Re}_m = \text{Re}_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

2. Model velocity

$$V_m = V_p \frac{L_p}{L_m} \frac{\nu_m}{\nu_p} = 10 \text{ m/s} \times 10 \times 1 = \boxed{100 \text{ m/s}}$$

Example 8.4 shows that the airspeed in the wind tunnel must be 100 m/s for true Reynol number similitude. This speed is quite large, and in fact Mach-number effects may start become important at such a speed. However, it will be shown in Section 8.8 that it is not alwa necessary to operate models at true Reynolds-number similitude to obtain useful results.

If the engineer feels that it is essential to maintain Reynolds-number similitude, then o a few alternatives are available. One way to produce high Reynolds numbers at nomi airspeeds is to increase the density of the air. A NASA wind tunnel at the Ames Research Cen at Moffett Field in California is one such facility. It has a 12-ft-diameter test section, it can pressurized up to 90 psia (620 kPa), it can be operated to yield a Reynolds number per foot to 1.2×10^7, and the maximum Mach number at which a model can be tested in this wi tunnel is 0.6. The airflow in this wind tunnel is produced by a single-stage, 20-blade axial-fl fan, which is powered by a 15,000-horsepower, variable-speed, synchronous electric motor (Several problems are peculiar to a pressurized tunnel. First, a shell (essentially a pressuri bottle) must surround the entire tunnel and its components, adding to the cost of the tun Second, it takes a long time to pressurize the tunnel in preparation for operation, increasi the time from the start to the finish of runs. In this regard it should be noted that the origi pressurized wind tunnel at the Ames Research Center was built in 1946; however, because extensive use, the tunnel's pressure shell began to deteriorate, so a new facility (the one pre ously described) was built and put in operation in 1995. Improvements over the old facil include a better data collection system, very low turbulence, and capability of depressurizi only the test section instead of the entire 620,000 ft^3 wind tunnel circuit when installing a removing models. The original pressurized wind tunnel was used to test most models of U.S. commercial aircraft over the past half-century, including the Boeing 737, 757, and 767; Lockheed L-1011; and McDonnell Douglas DC-9 and DC-10.

The Boeing 777 was tested in the low-speed, pressurized 5 m-by-5 m tunnel in Farnborough, England. This tunnel, operated by the Defence Evaluation and Research Agency (DERA) of Great Britain, can operate at three atmospheres with Mach numbers up to 0.2. Approximately 15,000 hours of total testing time was required for the Boeing 777 (4).

Another method of obtaining high Reynolds numbers is to build a tunnel in which the test medium (gas) is at a very low temperature, thus producing a relatively high-density–low-viscosity fluid. NASA has built such a tunnel and operates it at the Langley Research Center. This tunnel, called the National Transonic Facility, can be pressurized up to 9 atmospheres. The test medium is nitrogen, which is cooled by injecting liquid nitrogen into the system. In this wind tunnel it is possible to reach Reynolds numbers of 10^8 based on a model size of 0.25 m (5). Because of its sophisticated design, its initial cost was approximately $100,000,000 (6), and its operating expenses are high.

Another modern approach in wind-tunnel technology is the development of magnetic or electrostatic suspension of models. The use of the magnetic suspension with model airplanes has been studied (6), and the electrostatic suspension for the study of single-particle aerodynamics has been reported (7).

The use of wind tunnels for aircraft design has grown significantly as the size and sophistication of aircraft have increased. For example, in the 1930s the DC-3 and B-17 each had about 100 hours of wind-tunnel tests at a rate of $100 per hour of run time. By contrast the F-15 fighter required about 20,000 hours of tests at a cost of $20,000 per hour (6). The latter test time is even more staggering when one realizes that a much greater volume of data per hour at higher accuracy is obtained from the modern wind tunnels because of the high-speed data acquisition made possible by computers.

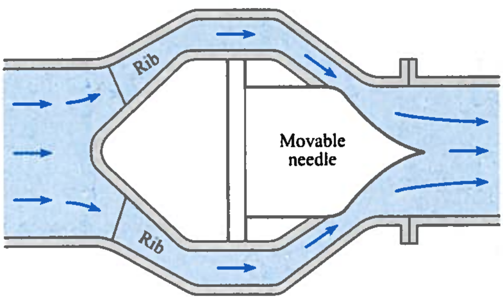
Example 8.5 illustrates the use of Reynolds-number similitude to design a test for a valve.

EXAMPLE 8.5

Reynolds-Number Similitude of a Valve

Problem Statement

The valve shown is the type used in the control of water in large conduits. Model tests are to be done, using water as the fluid, to determine how the valve will operate under wide-open conditions. The prototype size is 6 ft in diameter at the inlet. What flow rate is required for the model if the prototype flow is 700 cfs? Assume that the temperature for model and prototype is 60°F and that the model inlet diameter is 1 ft.



Define the Situations

A 1/6 scale model of a valve will be tested in a water tunnel. Prototype flow rate is 700 cfs.

Assumptions:

1. No compressibility, free surface or gravitational effects.
2. Temperature of water in model and prototype is the sam Therefore kinematic viscosity for model and prototype a equal.

State the Goal

Find the flow rate through the model in cfs.

Generate Ideas and Make a Plan

Dynamic similitude is obtained by equating the model and prototype Reynolds number. The model/prototype area rati is the square of the scale ratio.

1. Equate Reynolds number of model and prototype.
2. Calculate the velocity ratio.
3. Calculate the discharge ratio using model/prototype area ra

Take Action (Execute the Plan)

1. Reynolds-number similitude

$$\text{Re}_m = \text{Re}_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

2. Velocity ratio

$$\frac{V_m}{V_p} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}$$

Since $\nu_p = \nu_m$,

$$\frac{V_m}{V_p} = \frac{L_p}{L_m}$$

3. Discharge

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \frac{L_p}{L_m}\left(\frac{L_m}{L_p}\right)^2 = \frac{L_m}{L_p}$$

$$Q_m = 700 \text{ cfs} \times \frac{1}{6} = \boxed{117 \text{ cfs}}$$

Review the Solution and the Process

Discussion. This discharge is very large and serves to emphasize that very few model studies are made that completely satisfy the Reynolds-number criterion. This subject will be discussed further in the next sections.

8.7 Model-Prototype Performance

Geometric (scale model) and dynamic (same π-groups) similitude mean that the depende π-groups are the same for both the model and the prototype. For this reason, measuremen made with the model can be applied directly to the prototype. Such correspondence is illu trated in this section.

Example 8.6 shows how the pressure difference measured in a model test can be used find the pressure difference between the corresponding two points on the prototype.

EXAMPLE 8.6

Application of Pressure Coefficient

Problem Statement

A 1/10 scale model of a blimp is tested in a wind tunnel under dynamically similar conditions. The speed of the blimp through still air is 10 m/s. A 17.8 kPa pressure difference is measured between two points on the model. What will be the pressure difference between the two corresponding points on the prototype? The temperature and pressure in the wind tunnel is the same as the prototype.

Define the Situation

A 1/10 scale of a blimp is tested in a wind tunnel under dynamically similar conditions. A pressure difference of 17.8 kPa is measured on the model.

Properties: Pressure and temperature are the same for wind tunnel test and prototype, so $\nu_m = \nu_p$.

State the Goal

Find the corresponding pressure difference (Pa) on prototype.

Generate Ideas and Make a Plan

Eq. (8.4) reduces to

$$C_p = f(\text{Re})$$

1. Equate the Reynolds numbers to find the velocity ratio.
2. Equate the coefficient of pressure to find the pressure difference.

Take Action (Execute the Plan)

1. Reynolds-number similitude

$$\text{Re}_m = \text{Re}_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\frac{V_p}{V_m} = \frac{L_m}{L_p} = \frac{1}{10}$$

2. Pressure coefficient correspondence

$$\frac{\Delta p_m}{\frac{1}{2}\rho_m V_m^2} = \frac{\Delta p_p}{\frac{1}{2}\rho_p V_p^2}$$

$$\frac{\Delta p_p}{\Delta p_m} = \left(\frac{V_p}{V_m}\right)^2 = \left(\frac{L_m}{L_p}\right)^2 = \frac{1}{100}$$

Pressure difference on prototype

$$\Delta p_p = \frac{\Delta p_m}{100} = \frac{17.8 \text{ kPa}}{100} = \boxed{178 \text{ Pa}}$$

Example 8.7 illustrates calculating the fluid dynamic force on a prototype blimp from wind tunnel data using similitude.

EXAMPLE 8.7

Drag Force from Wind Tunnel Testing

Problem Statement

A 1/10 scale of a blimp is tested in a wind tunnel under dynamically similar conditions. If the drag force on the model blimp is measured to be 1530 N, what corresponding force could be expected on the prototype? The air pressure and temperature are the same for both model and prototype.

Define the Situation

A 1/10 scale model of blimp is tested in a wind tunnel, and a drag force of 1530 N is measured.

Properties: Pressure and temperature are the same, $\nu_m = \nu_p$.

State the Goal

Find the drag force (in newtons) on the prototype.

Generate Ideas and Make a Plan

Reynolds number is the only significant π-group, so Eq. (8.4) reduces to $C_F = f(\text{Re})$.

1. Find velocity ratio by equating Reynolds numbers.
2. Find the force by equating the force coefficients.

Take Action (Execute the Plan)

1. Reynolds-number similitude

$$\text{Re}_m = \text{Re}_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\frac{V_p}{V_m} = \frac{V_m}{L_p} = \frac{1}{10}$$

2. Force coefficient correspondence

$$\frac{F_p}{\frac{1}{2}\rho_p V_p^2 L_p^2} = \frac{F_m}{\frac{1}{2}\rho_m V_m^2 L_m^2}$$

$$\frac{F_p}{F_m} = \frac{V_p^2}{V_m^2}\frac{L_p^2}{L_m^2} = \frac{L_m^2}{L_p^2}\frac{L_p^2}{L_m^2} = 1$$

Therefore

$$F_p = 1530 \text{ N}$$

Review the Solution and the Process

Discussion. The result that the model force is the same as the prototype force is interesting. When Reynolds-number similitude is used, and the fluid properties are the same, th forces on the model will always be the same as the forces o the prototype.

8.8 Approximate Similitude at High Reynolds Numbers

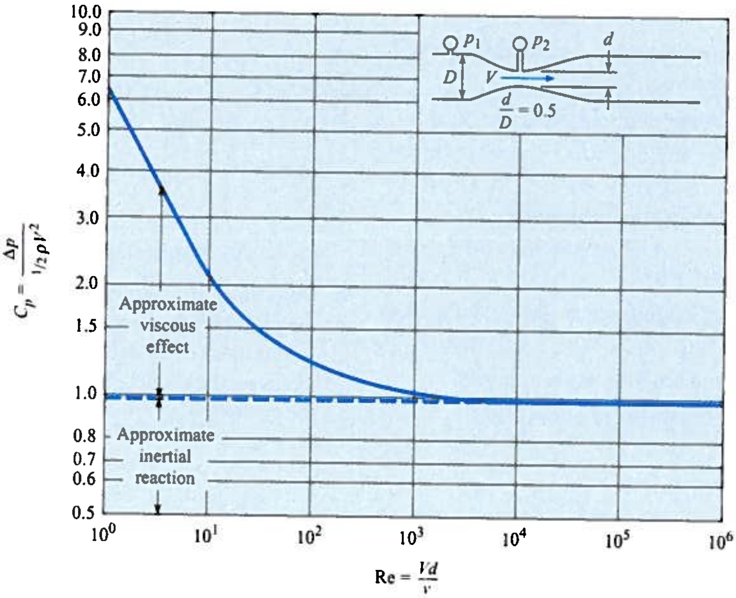
The primary justification for model tests is that it is more economical to get answers needed for engineering design by such tests than by any other means. However, as revealed by Examples 8.3, 8.4, and 8.6, Reynolds-number similitude requires expensive model tests (high-pressure facilities, large test sections, or using different fluids). This section shows that approxim: similitude is achievable even though high Reynolds numbers cannot be reached in model tes

Consider the size and power required for wind-tunnel tests of the blimp in Example 8 The wind tunnel would probably require a section at least 2 m by 2 m to accommodate t model blimp. With a 100 m/s airspeed in the tunnel, the power required for producing co tinuously a stream of air of this size and velocity is in the order of 4 MW. Such a test is n prohibitive, but it is very expensive. It is also conceivable that the 100 m/s airspeed wou introduce Mach-number effects not encountered with the prototype, thus generating conce over the validity of the model data. Furthermore, a force of 1530 N is generally larger than th usually associated with model tests. Therefore, especially in the study of problems involvi non-free-surface flows, it is desirable to perform model tests in such a way that large magr tudes of forces or pressures are not encountered.

For many cases, it is possible to obtain all the needed information from abbreviated tes Often the Reynolds-number effect (relative viscous effect) either becomes insignificant at hi Reynolds numbers or becomes independent of the Reynolds number. The point where testi can be stopped often can be detected by inspection of a graph of the pressure coefficient C_p vers the Reynolds number Re. Such a graph for a venturi meter in a pipe is shown in Fig. 8.7. In th meter, Δp is the pressure difference between the points shown, and V is the velocity in tl restricted section of the venturi meter. Here it is seen that viscous forces affect the value of (below a Reynolds number of approximately 50,000. However, for higher Reynolds numbers, (is virtually constant. Physically this means that at low Reynolds numbers (relatively high visco forces), a significant part of the change in pressure comes from viscous resistance, and the remai der comes from the acceleration (change in kinetic energy) of the fluid as it passes through th venturi meter. However, with high Reynolds numbers (resulting from either small viscosity or large product of V, D, and ρ), the viscous resistance is negligible compared with the force require to accelerate the fluid. Because the ratio of Δp to the kinetic pressure does not change (constant C for high Reynolds numbers, there is no need to carry out tests at higher Reynolds numbers. Th is true in general, so long as the flow pattern does not change with the Reynolds number.

FIGURE 8.7

C_p for a venturi meter as a function of the Reynolds numbers.



In a practical sense, whoever is in charge of the model test will try to predict from previou works approximately what maximum Reynolds number will be needed to reach the point of insig- nificant Reynolds-number effect and then will design the model accordingly. After a series of test has been made on the model, C_p versus Re will be plotted to see whether the range of constant C_p has indeed been reached. If so, then no more data are needed to predict the prototype performance. However, if C_p has not reached a constant value, the test program has to be expanded or results extrapolated. Thus the results of some model tests can be used to predict prototype performance, even though the Reynolds numbers are not the same for the model and the prototype. This is especially valid for angular-shaped bodies, such as model buildings, tested in wind tunnels.

In addition, the results of model testing can be combined with analytic results. Computational fluid dynamics (CFD) may predict the change in performance with Reynolds number but may not be reliable to predict the performance level. In this case, the model testing would be used to establish the level and of performance, and the trends predicted by CFD would be used to extrapolate the results to other conditions.

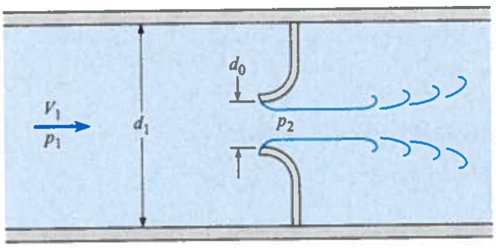
Example 8.8 is an illustration on the approximate similitude at high Reynolds number for flow through a constriction.

EXAMPLE 8.8

Measuring Head Loss in a Nozzle in Reverse Flow

Problem Statement

Tests are to be performed to determine the head loss in a nozzle under a reverse-flow situation. The prototype operates with water at 50°F and with a nominal reverse-flow velocity of 5 ft/s. The diameter of the prototype is 3 ft. The tests are done in a 1/12 scale model facility with water at 60°F. A head loss (pressure drop) of 1 psid is measured with a velocity of 20 ft/s. What will be the head loss in the actual nozzle?



Define the Situation

A 1/12 scale model tests for head loss in a reverse-flow nozzle. A pressure difference of 1 psid is measured with model at 20 ft/s.

Properties: Table F.5.: Water at 50°F, $\rho = 1.94$ slugs/ft^3, $\nu = 1.41 \times 10^{-5}$ ft^2/s; water at 60°F, $\rho = 1.94$ slugs/ft^3, $\nu = 1.22 \times 10^{-5}$ ft^2/s

State the Goal

Find the pressure drop (psid) for the prototype nozzle.

Generate Ideas and Make a Plan

The only significant π-group is the Reynolds number, so Eq. (8.4) reduces to $C_p = f(\text{Re})$. Dynamic similitude achieved if $\text{Re}_m = \text{Re}_p$, then $C_{p,m} = C_{p,p}$. From Fig. 8.7, if Re_m, $\text{Re}_p > 10^3$, then $C_{p,m} = C_{p,p}$.

1. Calculate Reynolds number for model and prototype.
2. Check if both exceed 10^3. If not, model tests need to be reevaluated.
3. Calculate pressure coefficient.
4. Evaluate pressure drop in prototype.

Take Action (Execute the Plan)

1. Reynolds numbers

$$\text{Re}_m = \frac{VD}{\nu} = \frac{20\text{ ft/s} \times (3/12\text{ ft})}{1.22 \times 10^{-5}\text{ ft}^2/\text{s}} = 4.10 \times 10^5$$

$$\text{Re}_p = \frac{5\text{ ft/s} \times 3\text{ ft}}{1.41 \times 10^{-5}\text{ ft}^2/\text{s}} = 1.06 \times 10^6$$

2. Both Reynolds numbers exceed 10^3. Therefore $C_{p,m} = C_{p,p}$. The test is valid.

3. Pressure coefficient from model tests

$$C_{p,m} = \frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{1\text{ lbf/in}^2 \times 144\text{ in}^2/\text{ft}^2}{\frac{1}{2} \times 1.94\text{ slug/ft}^3 \times (20\text{ ft/s})^2} = 0.371$$

4. Pressure drop in prototype

$$\Delta p_p = 0.371 \times \frac{1}{2}\rho V^2 = 0.371 \times 0.5 \times 1.94\text{ slug/ft}^3 \times (5\text{ ft/s})^2$$

$$= 9.0\text{ lbf/ft}^2 = \boxed{0.0625\text{ psid}}$$

Review the Solution and the Process

1. *Knowledge.* Because the Reynolds numbers are so much greater than 10^3, the equation for pressure drop is valid over a wide range of velocities.
2. *Discussion.* This example justifies the independence of Reynolds number referred to in Section 8.1.

In some situations viscous and compressibility effects may both be important, but it is possible to have dynamic similitude with both π-groups. Which π-group is chosen for sin tude depends a great deal on what information the engineer is seeking. If the engineer is in ested in the viscous motion of fluid near a wall in shock-free supersonic flow, then the Reyn number should be selected as the significant π-group. However, if the shock wave pattern c a body is of interest, then the Mach number should be selected for similitude. A useful rul thumb is that compressibility effects are unimportant for $M < 0.3$.

Example 8.9 shows the difficulty in having Reynolds-number similitude and avoid Mach-number effects in wind tunnel tests of an automobile.

EXAMPLE 8.9

Model Tests for Drag Force on an Automobile

Problem Statement

A 1/10 scale of an automobile is tested in a wind tunnel with air at atmospheric pressure and 20°C. The automobile is 4 m long and travels at a velocity of 100 km/hr in air at the same conditions. What should the wind-tunnel speed be such that the measured drag can be related to the drag of the prototype? Experience shows that the dependent π-groups are independent of Reynolds numbers for values exceeding 10^5. The speed of sound is 1235 km/hr.

Define the Situation

A 1/10 scale model of a 4 m–long automobile moving at 100 km/hr is tested in wind tunnel.

Properties: Air (20°C), Table A.3, $\rho = 1.2\ \text{kg/m}^3$, $\nu = 1.51 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$

State the Goal

Find the wind tunnel speed to achieve similitude.

Generate Ideas and Make a Plan

Mach number of the prototype is about 0.08 (100/1235), so Mach-number effects are unimportant. Dynamic similitude is achieved with Reynolds numbers, $\text{Re}_m = \text{Re}_p$. With dynamic similitude, $C_{F,m} = C_{F,p}$, and model measurements can be applied to prototype.

1. Determine the model speed for dynamic similitude.
2. Evaluate the model speed. If it is not feasible, continue to next step.
3. Calculate the prototype Reynolds number. If $\text{Re}_p > 10^5$, then $\text{Re}_m \geq 10^5$, for $C_{F,m} = C_{F,p}$.
4. Find the speed for which $\text{Re}_m \geq 10^5$.

Take Action (Execute the Plan)

1. Velocity from Reynolds-number similitude

$$\left(\frac{VL}{\nu}\right)_m = \left(\frac{VL}{\nu}\right)_p$$

$$\frac{V_m}{V_p} = \frac{L_p}{L_m} = 10$$

$$V_m = 10 \times 100\ \text{km/hr} = 1000\ \text{km/hr}$$

2. With this velocity, $M = 1000/1235 = 0.81$. This is too high for model tests because it would introduce unwanted compressibility effects.
3. Reynolds number of prototype

$$\text{Re}_p = \frac{VL\rho}{\mu} = \frac{100\ \text{km/hr} \times 0.278\ (\text{m/s})(\text{km/hr}) \times 4\ \text{m}}{1.51 \times 10^{-5}\ \text{m}^2/\text{s}}$$

$$= 7.4 \times 10^6$$

Therefore $C_{F,m} = C_{F,p}$, if $\text{Re}_m \geq 10^5$.

4. Wind tunnel speed

$$V_m \geq \text{Re}_m \frac{\nu_m}{L_m} = 10^5 \times \frac{1.51 \times 10^{-5}\ \text{m}^2/\text{s}}{0.4\ \text{m}}$$

$$\geq \boxed{3.8\ \text{m/s}}$$

Review the Solution and the Process

Discussion. The wind-tunnel speed must exceed 3.8 m/s. From a practical point of view, the speed will be chosen to provide sufficiently large forces for reliable and accurate measurements.

8.9 Free-Surface Model Studies

Spillway Models

The flow over a spillway is a classic case of a free-surface flow. The major influence, besides th spillway geometry itself, on the flow of water over a spillway is the action of gravity. Hence th Froude-number similarity criterion is used for such model studies. It can be appreciated fo large spillways with depths of water on the order of 3 m or 4 m and velocities on the order of 10 m/s or more, that the Reynolds number is very large. At high values of the Reynolds number, the relative viscous forces are often independent of the Reynolds number, as noted in the foregoing section (Sec. 8.8). However, if the reduced-scale model is made too small, the viscous forces as well as the surface-tension forces would have a larger relative effect on the flow in the model than in the prototype. Therefore, in practice, spillway models are made large enough so that the viscous effects have about the same relative effect in the model as in the prototype (i.e., the viscous effects are nearly independent of the Reynolds number). Then the Froude number is the significant π-group. Most model spillways are made at least 1 m high, and for precise studies, such as calibration of individual spillway bays, it is not uncommon to design and construct model spillway sections that are 2 m or 3 m high. Figures 8.8 and 8.9 show a comprehensive model and spillway model for Hell's Canyon Dam in Idaho.

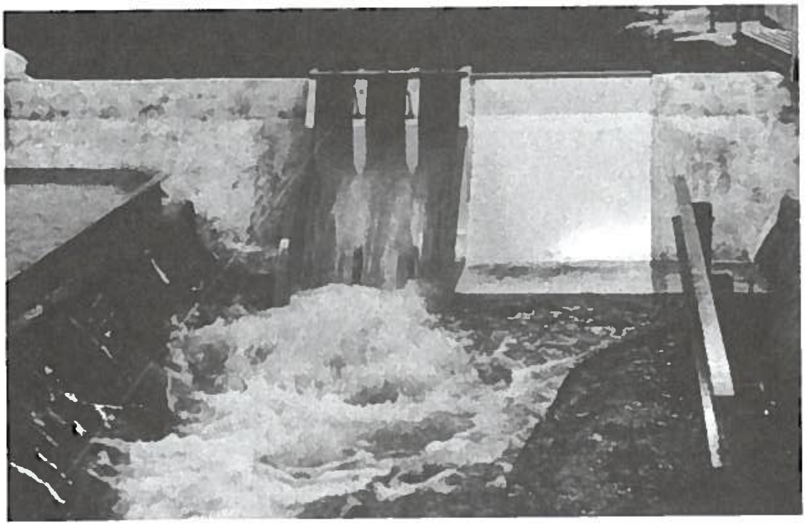


FIGURE 8.8

Comprehensive model Hell's Canyon Dam. Te were made at the Alb Hydraulic Laboratory, Washington State University. (Photo cour of Albrook Hydraulic Laboratory, Washingt State University)

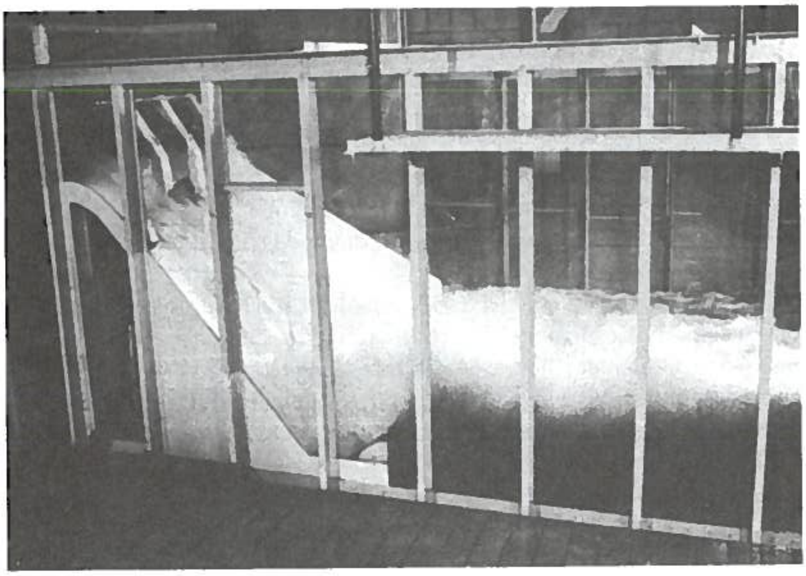


FIGURE 8.9

Spillway model for He Canyon Dam. Tests w made at the Albrook Hydraulic Laboratory, Washington State University. (Photo cour of Albrook Hydraulic Laboratory, Washingt State University)

Example 8.10 is an application of Froude-number similitude in modeling discharge o' a spillway.

EXAMPLE 8.10

Modeling Flood Discharge Over a Spillway

Problem Statement

A 1/49 scale model of a proposed dam is used to predict prototype flow conditions. If the design flood discharge over the spillway is 15,000 m^3/s, what water flow rate should be established in the model to simulate this flow? If a velocity of 1.2 m/s is measured at a point in the model, what is the velocity at a corresponding point in the prototype?

Define the Situation

A 1/49 scale model of spillway will be tested.
Prototype discharge is 15,000 m^3/s.

State the Goal

Find:

1. Flow rate over model.
2. Velocity on prototype at point where velocity is 1.2 m/s on model.

Generate Ideas and Make a Plan

Gravity is responsible for the flow, so the significant π-group is the Froude number. For dynamic similitude, $\text{Fr}_m = \text{Fr}_p$.

1. Calculate velocity ratio from Froude-number similitude.
2. Calculate discharge ratio using scale ratio and calculate model discharge.
3. Use velocity ratio from step 1 to find velocity at point on prototype.

Take Action (Execute the Plan)

1. Froude-number similitude

$$\text{Fr}_m = \text{Fr}_p$$

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

The acceleration due to gravity is the same, so

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

2. Discharge ratio

$$\frac{Q_m}{Q_p} = \frac{A_m}{A_p}\frac{V_m}{V_p} = \frac{L_m^2}{L_p^2}\sqrt{\frac{L_m}{L_p}} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

Discharge for model

$$Q_m = Q_p\left(\frac{1}{49}\right)^{5/2} = 15{,}000\frac{\text{m}^3}{\text{s}} \times \frac{1}{16{,}800} = \boxed{0.89\ \text{m}^3/\text{s}}$$

3. Velocity on prototype

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$$

$$V_p = \sqrt{49} \times 1.2\ \text{m/s} = \boxed{8.4\ \text{m/s}}$$

Ship Model Tests

The largest facility for ship testing in the United States is the David Taylor Model Basin, Nava Surface Warfare Center, Carderock Division, near Washington, D.C. Two of the core facilitie are the towing basins and the rotating arm facility. In the rotating arm facility, models are sus pended from the end of a rotating arm in a larger circular basin. Forces and moments ca be measured on ship models up to 9 m in length at steady-state speeds as high as 15.4 m/ (30 knots). In the high-speed towing basin, models 1.2 m to 6.1 m can be towed at speeds u to 16.5 m/s (32 knots).

The aim of the ship model testing is to determine the resistance that the propulsion systen of the ship must overcome. This resistance is the sum of the wave resistance and the surfac resistance of the hull. The wave resistance is a free-surface, or Froude-number, phenomenon and the hull resistance is a viscous, or Reynolds-number, phenomenon. Because both wav and viscous effects contribute significantly to the overall resistance, it would appear that bot the Froude and Reynolds criteria should be used. However, it is impossible to satisfy both if th model liquid is water (the only practical test liquid), because the Reynolds-number similitud dictates a higher velocity for the model than for the prototype [equal to $V_p(L_p/L_m)$], wherea the Froude-number similitude dictates a lower velocity for the model [equal to $V_p(\sqrt{L_m}/\sqrt{L_p})$]. To circumvent such a dilemma, the procedure is to model for the phenomenon that is the most difficult to predict analytically and to account for the other resistance by analytical means. Because the wave resistance is the most difficult problem, the model is operated according to the Froude-number similitude, and the hull resistance is accounted for analytically.

To illustrate how the test results and the analytical solutions for surface resistance are merged to yield design data, the following necessary sequential steps are indicated.

1. Make model tests according to Froude-number similitude, and the total model resistance is measured. This total model resistance will be equal to the wave resistance plus the surface resistance of the hull of the model.
2. Estimate the surface resistance of the model by analytical calculations.
3. Subtract the surface resistance calculated in step 2 from the total model resistance of step 1 to yield the wave resistance of the model.
4. Using the Froude-number similitude, scale the wave resistance of the model up to yield the wave resistance of the prototype.
5. Estimate the surface resistance of the hull of the prototype by analytical means.
6. The sum of the wave resistance of the prototype from step 4 and the surface resistance of the prototype from step 5 yields the total prototype resistance, or drag.

8.10 Summarizing Key Knowledge

Rationale and Description of Dimensional Analysis

- Dimensional analysis involves combining dimensional variables to form dimensionless groups. These groups, called π-groups, can be regarded as the scaling parameters for fluid flow. Dimensional analysis is applied to analysis, experiment design, and to the presentation of results.
- The *Buckingham* Π *theorem* states that the number of independent π-groups is $n - m$, where n is the number of dimensional variables and m is the number of basic dimensions included in the variables.

Rationale and Description of Dimensional Analysis

- The π-groups can be found by either the *step-by-step method* or the *exponent method.*
  - In the *step-by-step method* each dimension is removed by successively using a dimensional variable until the π-groups are obtained.
  - In the *exponent method*, each variable is raised to a power, they are multiplied together, and three simultaneous algebraic equations formulated for dimensional homogeneity, are solved to yield the π-groups.

Common π-groups

- Four common *independent* π*-groups* are

$$\text{Reynolds number Re} = \frac{\rho V L}{\mu} \qquad \text{Mach number} \quad \text{M} = \frac{V}{c}$$

$$\text{Weber number} \quad \text{We} = \frac{\rho V^2 L}{\sigma} \qquad \text{Froude number Fr} = \frac{V}{\sqrt{gL}}$$

- Three common *dependent π-groups* are

$$\text{Pressure coefficient, } C_p = \frac{\Delta p}{(\rho V^2)/2}$$

$$\text{Shear stress coefficient, } c_f = \frac{\tau}{(\rho V^2)/2}$$

$$\text{Force coefficient, } C_F = \frac{F}{(\rho V^2 L^2)/2}$$

- The general functional form of the common π-groups is

$$C_F, c_f, C_p = f(\text{Re, M, We, Fr})$$

Dimensional Analysis in Experimental Testing

- Experimental testing is often performed with a small-scale replica (*model*) of the full-scal structure (*prototype*).
- *Similitude* is the art and theory of predicting prototype performance from model observations. To achieve exact similitude:
  - The model must be a scale model of the prototype (*geometric similitude*).
  - Values of the π-groups must be the same for the model and the prototype (*dynamic similitude*).
- In practice, it is not always possible to have complete dynamic similitude, so *only the most important π-groups are matched.*

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PROBLEMS

WILEY PLUS Problem available in *WileyPLUS* at instructor's discretion.

WILEY GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Dimensional Analysis (§8.2)

8.1 Find the primary dimensions of density ρ, viscosity μ, and pressure *p*.

8.2 According to the Buckingham Π theorem, if there are six dimensional variables and three primary dimensions, how many dimensionless variables will there be?

8.3 Explain what is meant by dimensional homogeneity.

8.4 WILEY PLUS Determine which of the following equations are dimensionally homogeneous:

a. $Q = \frac{2}{3} CL\sqrt{2g}H^{3/2}$

where Q is discharge, C is a pure number, L is length, g is acceleration due to gravity, and H is head.

b. $V = \frac{1.49}{n} R^{2/3} S^{1/2}$

where V is velocity, n is length to the one-sixth power, R is length, and S is slope.

c. $h_f = f \frac{L}{D} \frac{V^2}{2g}$

where h_f is head loss, f is a dimensionless resistance coefficient, L is length, D is diameter, V is velocity, and g is acceleration due to gravity.

d. $D = \frac{0.074}{\text{Re}^{0.2}} \frac{Bx\rho V^2}{2}$

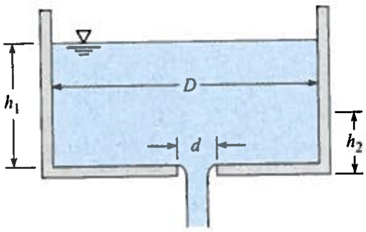
where D is drag force, Re is Vx/ν, B is width, x is length, ρ is mass density, ν is the kinematic viscosity, and V is velocity.

8.5 Determine the dimensions of the following variables and combinations of variables in terms of primary dimensions.

a. T (torque)
b. $\rho V^2/2$, where V is velocity and ρ is mass density
c. $\sqrt{\tau/\rho}$, where τ is shear stress
d. Q/ND^3, where Q is discharge, D is diameter, and N is angular speed of a pump

8.6 It takes a certain length of time for the liquid level in a tank of diameter D to drop from position h_1 to position h_2 as the tank is being drained through an orifice of diameter d at the bottom. Determine the π-groups that apply to this problem. Assume that the liquid is nonviscous. Express your answer in the functional form.

$$\frac{\Delta h}{d} = f(\pi_1, \pi_2, \pi_3)$$



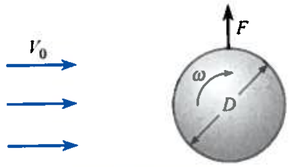
PROBLEM 8.6

8.7 The maximum rise of a liquid in a small capillary tube is a function of the diameter of the tube, the surface tension, and the specific weight of the liquid. What are the significant π-groups for the problem?

8.8 For very low velocities it is known that the drag force F_D of a small sphere is a function solely of the velocity V of flow past the sphere, the diameter d of the sphere, and the viscosity μ of the fluid. Determine the π-groups involving these variables.

8.9 Observations show that the side thrust F_s for a rough spinning ball in a fluid is a function of the ball diameter D, the free-stream velocity V_0, the density ρ, the viscosity μ, the roughness height k_s, and the angular velocity of spin ω. Determine the dimensionless parameter(s) that would be usec to correlate the experimental results of a study involving the variables noted above. Express your answer in the functional form

$$\frac{F}{\rho V_0^2 D^2} = f(\pi_1, \pi_2, \pi_3)$$



PROBLEM 8.9

8.10 Consider steady viscous flow through a small horizonta tube. For this type of flow, the pressure gradient along the tube, $\Delta p/\Delta \ell$ should be a function of the viscosity μ, the mean veloci V, and the diameter D. By dimensional analysis, derive a functional relationship relating these variables.

8.11 A flow-metering device, called a vortex meter, consists c square element mounted inside a pipe. Vortices are generated the element, which gives rise to an oscillatory pressure measu on the leeward side of the element. The fluctuation frequency related to the flow velocity. The discharge in the pipe is a function of the frequency of the oscillating pressure ω, the pi diameter D, the size of the element l, the density ρ, and the viscosity μ. Thus

$$Q = f(\omega, D, l, \rho, \mu)$$

Find the π-groups in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

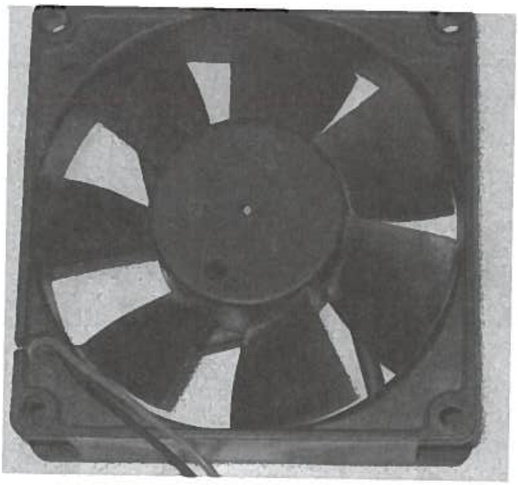
8.12 It is known that the pressure developed by a centrifuga pump, Δp, is a function of the diameter D of the impeller, the speed of rotation n, the discharge Q, and the fluid density ρ. dimensional analysis, determine the π-groups relating these variables.

8.13 The force on a satellite in the earth's upper atmosphere depends on the mean path of the molecules λ (a length), the density ρ, the diameter of the body D, and the molecular spe $F = f(\lambda, \rho, D, c)$. Find the nondimensional form of this equa

8.14 A general study is to be made of the height of rise of li in a capillary tube as a function of time after the start of a te Other significant variables include surface tension, mass der specific weight, viscosity, and diameter of the tube. Determi the dimensionless parameters that apply to the problem. Ex your answer in the functional form

$$\frac{h}{d} = f(\pi_1, \pi_2, \pi_3)$$

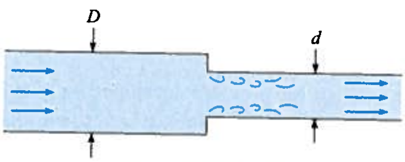
8.15 An engineer is using an experiment to characterize the power P consumed by a fan (see photo) to be used in an electronics cooling application. Power depends on four variables: $P = f(\rho, D, Q, n)$, where ρ is the density of air, D is the diameter of the fan impeller, Q is the flow rate produced by the fan, and n is the rotation rate of the fan. Find the relevant π-groups and suggest a way to plot the data.



PROBLEM 8.15 (Photo by Donald Elger)

8.16 By dimensional analysis, determine the π-groups for the change in pressure that occurs when water or oil flows through a horizontal pipe with an abrupt contraction as shown. Express your answer in the functional form

$$\frac{\Delta p d^4}{\rho Q^2} = f(\pi_1, \pi_2)$$



PROBLEM 8.16

8.17 A solid particle falls through a viscous fluid. The falling velocity V, is believed to be a function of the fluid density ρ_f, the particle density ρ_p, the fluid viscosity μ, the particle diameter D, and the acceleration due to gravity g:

$$V = f(\rho_f, \rho_p, \mu, D, g)$$

By dimensional analysis, develop the π-groups for this problem. Express your answer in the form

$$\frac{V}{\sqrt{gD}} = f(\pi_1, \pi_2)$$

8.18 An experimental test program is being set up to calibrate a new flow meter. The flow meter is to measure the mass flow rate of liquid flowing through a pipe. It is assumed that the mass fl rate is a function of the following variables:

$$\dot{m} = f(\Delta p, D, \mu, \rho)$$

where Δp is the pressure difference across the meter, D is the pipe diameter, μ is the liquid viscosity, and ρ is the liquid dens Using dimensional analysis, find the π-groups. Express your answer in the form

$$\frac{\dot{m}}{\sqrt{\rho \Delta p D^4}} = f(\pi)$$

8.19 A torpedo-like device is being designed to travel just bel the water surface. Which dimensionless numbers in Section 8. would be significant in this problem? Give a rationale for your answer.

8.20 Experiments are to be done on the drag forces on an oscillating fin in a water tunnel. It is assumed that the drag force F_D, is a function of the liquid density ρ, the fluid velocity V, the surface area of the fin S, and the frequency of oscillation ω:

$$F_D = f(\rho, V, S, \omega)$$

By dimensional analysis, find the dimensionless parameters for this problem. Express your answer in the form

$$\frac{F_D}{\rho V^2 S} = f(\pi)$$

8.21 Flow situations in biofluid mechanics involve the flow through tubes that change in size with time (such as blood vessels) or are supplied by an oscillatory source. The volume flo rate Q in the tube will be a function of the frequency ω, the tub diameter D, the fluid density ρ, viscosity μ, and the pressure gradient $(\Delta p)/(\Delta l)$. Find the π-groups for this situation in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

8.22 The rise velocity V_b of a bubble with diameter D in a liquic of density ρ_l and viscosity μ depends on the acceleration due to gravity, g, and the density difference between the bubble and the fluid, $\rho_l - \rho_b$. Find the π-groups in the form

$$\frac{V_b}{\sqrt{gD}} = f(\pi_1, \pi_2)$$

8.23 The discharge of a centrifugal pump is a function of the rotational speed of the pump N, the diameter of the impeller D, the head across the pump h_p, the viscosity of the fluid μ, the density of the fluid ρ, and the acceleration due to gravity g. The functional relationship is

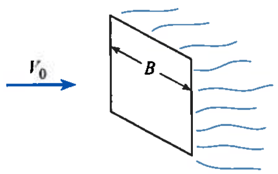
$$Q = f(N, D, h_p, \mu, \rho, g)$$

By dimensional analysis, find the dimensionless parameters. Express your answer in the form

$$\frac{Q}{ND^3} = f(\pi_1, \pi_2, \pi_3)$$

8.24 Drag tests show that the drag of a square plate placed normal to the free-stream velocity is a function of the velocity V, the density ρ, the plate dimensions B, the viscosity μ, the free-stream turbulence root mean square velocity u_{rms}, and the turbulence length scale L_x. Here u_{rms} and L_x are in ft/s and ft, respectively. By dimensional analysis, develop the π-groups that could be used to correlate the experimental results. Express your answer in the functional form

$$\frac{F_D}{\rho V^2 B^2} = f(\pi_1, \pi_2, \pi_3)$$



PROBLEM 8.24

8.25 Using Wikipedia, read about the Womersley number (α) and answer the following questions.

a. Is α dimensionless? How do you know? Show that all the terms in fact cancel out.

b. Like other independent π-groups, α is the ratio of two forces. Of what two forces is it the ratio?

c. What does the velocity profile in a blood vessel look like for $\alpha \leq 1$? For $\alpha \geq 10$?

d. What is the aorta, and where in the human body is it located? What is a typical value for α in the aorta? What might you conclude about the velocity profile there?

8.26 The Womersley number (α) is a π-group given by the ratio of [pulsatile transient force]/[viscous force]. Biomedical engineers have applied this to characterize flow in blood vessels. The Womersley number is given by:

$$\alpha = r\sqrt{\frac{\omega\rho}{\mu}}$$

where r = blood vessel radius, and ω = frequency, typically the heart rate. Just as does Re, α has different practical implications in critical ranges. In the range a $\alpha \leq 1$, a parabolic (laminar) velocity distribution has time to develop in a tube during each heartbeat cycle. When $\alpha \geq 10$, the velocity profile is relatively flat (called plug flow) in the blood vessel. For a human research subject, assume the heart rate is 70 beats/s, the radius of the aorta is 17 mm, the density of blood is 1060 kg/m^3, and the radius of a capillary is 7 μm. The viscosity of blood is normally 3×10^{-3} Pa · s.

a. Find α for the aorta of this subject.

b. Find α for the capillary of this subject.

c. Does either the aorta or the capillary have an α that would predict plug flow? Does either have an α indicating a parabolic velocity distribution?

Common π-Groups (§8.4)

8.27 For each item below, which π-group (Re, We, M, or Fr) would best match the given description?

a. (Kinetic force)/(Surface-tension force)

b. (Kinetic force)/(Viscous force)

c. (Kinetic force)/(Gravitational force)

d. (Kinetic force)/(Compressive force)

e. Used for modeling water flowing over a spillway or a dam

f. Used for designing laser jet printers

g. Used for analyzing the drag on a car in a wind tunne

h. Used to analyze the flight of supersonic jets

Similitude (§8.5)

8.28 What is meant by geometric similitude?

8.29 Many automobile companies advertise products with l drag for improved performance. Gather all the information can find on wind-tunnel testing of automobiles, and summa your findings in a concise, informative manner on two page or less.

8.30 One of the shortcomings of mounting a model of an automobile in a wind tunnel and measuring drag is that the effect of the road is not included. Give some thought as to yo impressions of what the effect of the road may be on autom drag and your reasoning. Also list some variables that may influence the effect of the ground on automobile drag.

8.31 One of the largest wind tunnels in the United States is NASA facility in Moffat Field, California. Look up informati on this facility (size, test section velocity, etc.) and summari your findings in a concise, informative manner.

8.32 The hydrodynamic drag on a sailboat is very importar the performance of the craft, especially in competitive races as the America's Cup. Investigate on the Internet or other so the extent and types of simulations that have been carried o high-performance sailboats.

8.33 The drag on a submarine moving below the free surface is to be determined by a test on a 1/18 scale model in a water tunnel. The velocity of the prototype in seawater (ρ = 1015 kg/m^3, $\nu = 1.4 \times 10^{-6}$ m^2/s) is 3 m/s. The test is in pure water at 20°C. Determine the speed of the water in t water tunnel for dynamic similitude and the ratio of the dra force on the model to the drag force on the prototype.

8.34 Water with a kinematic viscosity of 10^{-6} m^2/s flc through a 4 cm pipe. What would the velocity of water have to be for the water flow to be dynamically similar to oil ($\nu = 10^{-5}$ m^2/s) flowing through the same pipe at a velocity of 0.5 m/s?

8.35 Oil with a kinematic viscosity of 4×10^{-6} m^2/s through a smooth pipe 12 cm in diameter at 2.3 m/s. What velocity should water have at 20°C in a smooth pipe 5 cm ir diameter to be dynamically similar?

8.36 A large venturi meter is calibrated by means of a 1/10 scale model using the prototype liquid. What is the discharge ratio Q_m/Q_p for dynamic similarity? If a pressure difference of 400 kPa is measured across ports in the model for a given discharge, what pressure difference will occur between similar ports in the prototype for dynamically similar conditions?

8.37 A 1/5 scale model of an experimental deep sea submersible that will operate at great depths is to be tested to determine its drag characteristic by towing it behind a submarine. For true similitude, what should be the towing speed relative to the speed of the prototype?

8.38 A spherical balloon that is to be used in air at 60°F and atmospheric presssure is tested by towing a 1/12 scale model in a lake. The model is 1.4 ft in diameter, and a drag of 37 lbf is measured when the model is being towed in deep water at 5 ft/s. What drag (in pounds force and newtons) can be expected for the prototype in air under dynamically similar conditions? Assume that the water temperature is 60°F.

8.39 An engineer needs a value of lift force for an airplane that has a coefficient of lift (C_L) of 0.4. The π-group is defined as

$$C_L = 2\frac{F_L}{\rho V^2 S}$$

where F_L is the lift force, ρ is the density of ambient air, V is the speed of the air relative to the airplane, and S is the area of the wings from a top view. Estimate the lift force in newtons for a speed of 80 m/s, an air density of 1.1 kg/m^3, and a wing area (planform area) of 15 m^2.

PROBLEM 8.39 (© Daniel Karlsson/Stocktrek Images, Inc.)

8.40 An airplane travels in air (p = 100 kPa, T = 10°C) at 150 m/s. If a 1/8 scale model of the plane is tested in a wind tunnel at 25°C, what must the density of the air in the tunnel be so that both the Reynolds-number and the Mach-number criteria are satisfied? The speed of sound varies with the square root of the absolute temperature. (*Note:* The dynamic viscosity is independent of pressure.)

8.41 The Airbus A380-300 has a wing span of 79.8 m. The cruise altitude is 10,000 m in a standard atmosphere. Assume you are designing a wind tunnel to operate with air at 20°C. The span of the scale model A380 in the wind tunnel is 1 m. Assume Mach-number correspondence between model and prototype Both the speed of sound and the dynamic viscosity vary linearly with the square root of the absolute temperature. Wha would the pressure of the air in the wind tunnel have to be to have Reynolds-number similitude? Use the properties for a standard atmosphere in Chapter 3 to find properties at 10,000 m altitude.

8.42 The Boeing 787-3 Dreamliner has a wing span of 52 m. It flies at a cruise Mach number of 0.85, which corresponds t a velocity of 945 km/hr at an altitude of 10,000 m. You are going to estimate the drag on the prototype by measuring the drag on 1 m wing span scale model in a wind tunnel with air where the speed of sound is 340 m/s and the density is 0.98 kg/m^3. What i the ratio of the force on the prototype to the force on the model Only Mach-number similitude is considered. Use the properties of the standard atmosphere in Chapter 3 to evaluate the density of air for the prototype.

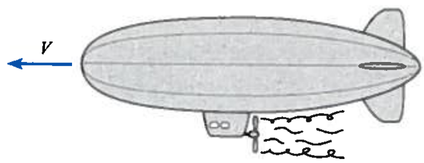
8.43 Flow in a given pipe is to be tested with air and then with water. Assume that the velocities (V_A and V_W) are such that the flow with air is dynamically similar to the flow with water. Then for this condition, the magnitude of the ratio of the velocities, V_A/V_W, will be (a) less than unity, (b) equal to unity, or (c) greater than unity.

8.44 A smooth pipe designed to carry crude oil (D = 47 in. ρ = 1.75 slugs/ft^3, and μ = 4 × 10^{-4} lbf-s/ft^2) is to be modeled with a smooth pipe 4 in. in diameter carrying water (T = 60°F). If the mean velocity in the prototype is 2 ft/s, what should be the mean velocity of water in the model to ensure dynamically similar conditions?

8.45 A student is competing in a contest to design a radio-controlled blimp. The drag force acting on the blimp depends on the Reynolds number, Re = $(\rho VD)/\mu$, where V is the speed of the blimp, D is the maximum diameter, ρ is the density of air, and μ is the viscosity of air. This blimp has a coefficient of drag (C_D) of 0.3. This π-group is defined as

$$C_D = 2\frac{F_D}{\rho V^2 A_p}$$

where F_D is the drag force ρ is the density of ambient air, V is the speed of the blimp, and $A_p = \pi D^2/4$ is the maximum section are of the blimp from a front view. Calculate the Reynolds number, the drag force in newtons, and the power in watts required to move the blimp through the air. Blimp speed is 800 mm/s, and the maximum diameter is 475 mm. Assume that ambient air is at 20°C.



PROBLEM 8.45

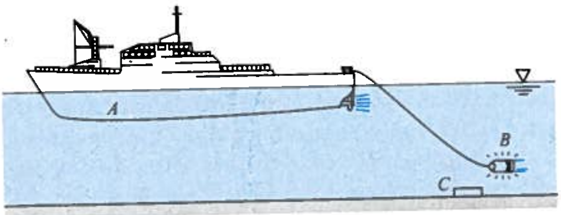
8.46 Colonization of the moon will require an improved understanding of fluid flow under reduced gravitational forces. The gravitational force on the moon is 1/5 that on the surface of the earth. An engineer is designing a model experiment for flow in a conduit on the moon. The important scaling parameters are the Froude number and the Reynolds number. The model will be full scale. The kinematic viscosity of the fluid to be used on the moon is 0.5×10^{-5} m²/s. What should be the kinematic viscosity of the fluid to be used for the model on earth?

8.47 A drying tower at an industrial site is 10 m in diameter. The air inside the tower has a kinematic viscosity of 4×10^{-5} m²/s and enters at 12 m/s. A 1/15 scale model of this tower is fabricated to operate with water that has a kinematic viscosity of 10^{-6} m²/s. What should the entry velocity of the water be to achieve Reynolds-number scaling?

8.48 A flow meter to be used in a 40 cm pipeline carrying oil ($\nu = 10^{-5}$ m²/s, $\rho = 860$ kg/m³) is to be calibrated by means of a model (1/9 scale) carrying water ($T = 20$°C and standard atmospheric pressure). If the model is operated with a velocity of 1.6 m/s, find the velocity for the prototype based on Reynolds-number scaling. For the given conditions, if the pressure difference in the model was measured as 3.0 kPa, what pressure difference would you expect for the discharge meter in the oil pipeline?

8.49 Water at 10°C flowing through a rough pipe 10 cm in diameter is to be simulated by air (20°C) flowing through the same pipe. If the velocity of the water is 1.5 m/s, what will the air velocity have to be to achieve dynamic similarity? Assume the absolute air pressure in the pipe to be 150 kPa. If the pressure difference between two sections of the pipe during air flow was measured as 780 Pa, what pressure difference occurs between these two sections when water is flowing under dynamically similar conditions?

8.50 The "noisemaker" B is towed behind the minesweeper A to set off enemy acoustic mines such as that shown at C. The drag force of the "noisemaker" is to be studied in a water tunnel at a 1/5 scale (the model is 1/5 the size of the full scale). If the full-scale towing speed is 5 m/s, what should be the water velocity in the water tunnel for the two tests to be exactly similar? What will be the prototype drag force if the model drag force is found to be 2400 N? Assume that seawater at the same temperature is used in both the full-scale and the model tests.



PROBLEM 8.50

8.51 An experiment is being designed to measure aerodynamic forces on a building. The model is a 1/500 scale replica of the prototype. The wind velocity on the prototype is 47 ft/s, and the density is 0.0024 slugs/ft³. The maximum veloci in the wind tunnel is 300 ft/s. The viscosity of the air flowing fo the model and the prototype is the same. Find the density neec in the wind tunnel for dynamic similarity. A force of 50 lbf is measured on the model. What will the force be on the prototyp

8.52 A 60 cm valve is designed for control of flow in a petrole pipeline. A 1/3 scale model of the full-size valve is to be tested with water in the laboratory. If the prototype flow rate is to be 0.5 m³/s, what flow rate should be established in the laborator test for dynamic similitude to be established? Also, if the press coefficient C_p in the model is found to be 1.07, what will be th corresponding C_p in the full-scale valve? The relevant fluid proper for the petroleum are $S = 0.82$ and $\mu = 3 \times 10^{-3}$ N · s/m². The viscosity of water is 10^{-3} N · s/m².

8.53 The moment acting on a submarine rudder is studied by a 1/40 scale model. If the test is made in a water tunnel and if the moment measured on the model is 2 N · m when the freshwater speed in the tunnel is 6.6 m/s, what are t corresponding moment and speed for the prototype? Assume prototype operates in sea water. Assume $T = 10$°C for both th freshwater and the seawater.

8.54 A model hydrofoil is tested in a water tunnel. For given angle of attack, the lift of the hydrofoil is measured to be 25 when the water velocity is 15 m/s in the tunnel. If the prototy hydrofoil is to be twice the size of the model, what lift force would be expected for the prototype for dynamically similar conditions? Assume a water temperature of 20°C for both model and prototype.

8.55 A 1/10 scale model of an automobile is tested in a pressurized wind tunnel. The test is to simulate the automobil traveling at 100 km/h in air at atmospheric pressure and 25°C The wind tunnel operates with air at 25°C. At what pressure i test section must the tunnel operate to have the same Mach a Reynolds numbers? The speed of sound in air at 25°C is 345 r

8.56 If the tunnel in Prob. 8.55 were to operate at atmosphe pressure and 25°C, what speed would be needed to achieve t same Reynolds number for the prototype? At this speed, wou you conclude that Mach-number effects were important?

8.57 Experimental studies have shown that the condit for breakup of a droplet in a gas stream is

$$\mathrm{We}/\mathrm{Re}^{1/2} = 0.5$$

where Re is the Reynolds number and We is the Weber num based on the droplet diameter. What diameter water droplet would break up in a 12 m/s airstream at 20°C and standard atmospheric pressure? The surface tension of water is 0.041

8.58 Water is sprayed from a nozzle at 30 m/s into air at atmospheric pressure and 20°C. Estimate the size of the dro produced if the Weber number for breakup is 6.0 based on t droplet diameter.

8.59 Determine the relationship between the kinematic viscosity ratio ν_m/ν_p and the scale ratio if both the Reynolds-number and the Froude-number criteria are to be satisfied in a given model test.

8.60 WILEY GO A hydraulic model, 1/20 scale, is built to simulate the flow conditions of a spillway of a dam. For a particular run, the waves downstream were observed to be 8 cm high. How high would be similar waves on the full-scale dam operating under the same conditions? If the wave period in the model is 2 s, what would the wave period in the prototype be?

8.61 The scale ratio between a model dam and its prototype is 1/25. In the model test, the velocity of flow near the crest of the spillway was measured to be 2.5 m/s. What is the corresponding prototype velocity? If the model discharge is 0.10 m^3/s, what is the prototype discharge?

8.62 WILEY PLUS A seaplane model is built at a 1/6 scale. To simulate takeoff conditions at 117 km/h, what should be the corresponding model speed to achieve Froude-number scaling?

8.63 If the scale ratio between a model spillway and its prototype is 1/36, what velocity and discharge ratio will prevail between model and prototype? If the prototype discharge is 3000 m^3/s, what is the model discharge?

8.64 The depth and velocity at a point in a river are measured to be 20 ft and 15 ft/s, respectively. If a 1/64 scale model of this river is constructed and the model is operated under dynamically similar conditions to simulate the free-surface conditions, then what velocity and depth can be expected in the model at the corresponding point?

8.65 WILEY PLUS A 1/40 scale model of a spillway is tested in a laboratory. If the model velocity and discharge are 3.2 ft/s and 3.53 cfs, respectively, what are the corresponding values for the prototype?

8.66 Flow around a bridge pier is studied using a model at 1/12 scale. When the velocity in the model is 0.9 m/s, the standing wave at the pier nose is observed to be 2.5 cm in height. What are the corresponding values of velocity and wave height in the prototype?

8.67 A 1/25 scale model of a spillway is tested. The discharge in the model is 0.1 m^3/s. To what prototype discharge does this correspond? If it takes 1 min for a particle to float from one point to another in the model, how long would it take a similar particle to traverse the corresponding path in the prototype?

8.68 WILEY PLUS A tidal estuary is to be modeled at 1/600 scale. In the actual estuary, the maximum water velocity is expected to be 3.6 m/s, and the tidal period is approximately 12.5 h. What corresponding velocity and period would be observed in the model?

8.69 WILEY PLUS The maximum wave force on a 1/36 model seawall was found to be 80 N. For a corresponding wave in the full-scale wall, what full-scale force would you expect? Assume freshwater is used in the model study. Assume $T = 10°C$ for both model prototype water.

8.70 WILEY PLUS A model of a spillway is to be built at 1/80 scale. If prototype has a discharge of 800 m^3/s, what must be the water discharge in the model to ensure dynamic similarity? The tota force on part of the model is found to be 51 N. To what protot force does this correspond?

8.71 WILEY PLUS A newly designed dam is to be modeled in the laboratory. The prime objective of the general model study is to determine the adequacy of the spillway design and to observe the water velocities, elevations, and pressures at critical points of the structure. The reach of the river to be modeled is 1200 m long, the width of the dam (also the maximum width of the reservoir upstream) is to be 300 m, and the maximum flood discharge to be modeled is 5000 m^3. The maximum laboratory discharge is limited to 0.90 m^3/s, and the floor space available for the model construction is 50 m long and 20 m wide. Determine the largest feasible scale ratio (model/prototype) for such a study.

8.72 A ship model 4 ft long is tested in a towing tank at a speec that will produce waves that are dynamically similar to those observed around the prototype. The test speed is 5 ft/s. What should the prototype speed be, given that the prototype length i 100 ft? Assume both the model and the prototype are to operate in freshwater.

8.73 WILEY PLUS The wave resistance of a model of a ship at 1/25 scale 2 lbf at a model speed of 5 ft/s. What are the corresponding velocity and wave resistance of the prototype?

8.74 A 1/20 scale model building that is rectangular in plan vie and is three times as high as it is wide is tested in a wind tunnel. If the drag of the model in the wind tunnel is measured to be 200 N for a wind speed of 20 m/s, then the prototype building ir a 40 m/s wind (same temperature) should have a drag of about (a) 40 kN, (b) 80 kN, (c) 230 kN, or (d) 320 kN.

8.75 WILEY PLUS A model of a high-rise office building at 1/550 scale is tested in a wind tunnel to estimate the pressures and forces on the full-scale structure. The wind-tunnel air speed is 20 m/s at 20°C and atmospheric pressure, and the full-scale structure is expected to withstand winds of 200 km/h (10°C). If the extreme values of the pressure coefficient are found to be 1.0, −2.7, and −0.8 on the windward wall, side wall, and leeward wall of the model, respectively, what corresponding pressures could be expected to act on the prototype? If the lateral wind force (wind force on building normal to wind direction) was measured as 20 N in the model, what lateral force might be expected in the prototype in the 200 km/h wind?

8.76 Experiments were carried out in a water tunnel and a wind tunnel to measure the drag force on an object. The water tunnel was operated with freshwater at 20°C, and the wind tunnel was operated at 20°C and atmospheric pressure. Three models were used with dimensions of 5 cm, 8 cm, and 15 cm. The drag force on each model was measured at different velocities. The following data were obtained.

Data for the water tunnel

Model Size, cm	Velocity, m/s	Force, N
5	1.0	0.064
5	4.0	0.69
5	8.0	2.20
8	1.0	0.135
8	4.0	1.52
8	8.0	4.52

Data for the wind tunnel

Model Size, cm	Velocity, m/s	Force, N
8	10	0.025
8	40	0.21
8	80	0.64
15	10	0.06
15	40	0.59
15	80	1.82

The drag force is a function of the density, viscosity, velocity, and model size,

$$F_D = f(\rho, \mu, V, D)$$

Using dimensional analysis, express this equation using π-groups and then write a computer program or use a spreadsheet to reduce the data. Plot the data using the dimensionless parameters.

8.77 Experiments are performed to measure the pressure drop pipe with water at 20°C and crude oil at the same temperature. are gathered with pipes of two diameters, 5 cm and 10 cm. The following data were obtained for pressure drop per unit length.

For water

Pipe Diameter, cm	Velocity, m/s	Pressure Drop, N/m^3
5	1	210
5	2	730
5	5	3750
10	1	86
10	2	320
10	5	1650

For crude oil

Pipe Diameter, cm	Velocity, m/s	Pressure Drop, N/m^3
5	1	310
5	2	1040
5	5	5300
10	1	130
10	2	450
10	5	2210

The pressure drop per unit length is assumed to be a functio the pipe diameter, liquid density and viscosity, and the veloc

$$\frac{\Delta p}{L} = f(\rho, \mu, V, D)$$

Perform a dimensional analysis to obtain the π-groups and write a computer program or use a spreadsheet to reduce th data. Plot the results using the dimensionless parameters.