7 THE ENERGY EQUATION

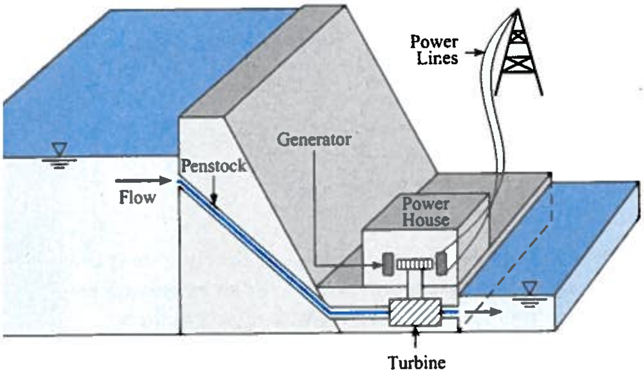


FIGURE 7.1

The energy equation can be applied to hydroelectric power generation. In addition, the energy equation can be applied to thousands of other applications. It is one of the most useful equations in fluid mechanics.

Chapter Road Map

This chapter describes how conservation of energy can be applied to a flowing fluid. The resulting equation is called the *energy equation*.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Explain the meaning of energy, work, and power. (§7.1)
- Classify energy into categories. (§7.1)
- Define a pump and a turbine. (§7.1)
- Explain conservation of energy for a closed system and a CV. (§7.2)
- List the steps to derive the energy equation. (§7.3)
- Explain flow work and shaft work. (§7.3)
- Define head loss and the kinetic energy correction factor. (§7.3)
- Describe the physics of the energy equation and the meaning of the variables that appear in the equation. Describe the process for applying the energy equation. Apply the energy equation. (§7.3)
- Apply the power equation. (§7.4)
- Define mechanical efficiency and apply this concept. (§7.5)
- Contrast the energy equation and the Bernoulli equation. (§7.6)
- Calculate head loss for a sudden expansion. (§7.7)
- Explain the conceptual foundations of the energy grade line and hydraulic grade line. Sketch these lines. (§7.8)

7.1 Energy Concepts

The energy equation is built on foundational concepts that are introduced in this section.

Energy

Energy is the property of a system that characterizes the amount of work that this system can do on its environment. In simple terms, if matter (i.e., the system) can be used to lift a weight, then that matter has energy.

Examples

- Water behind a dam has energy because the water can be directed through a pipe (i.e., a penstock), then used to rotate a wheel (i.e., a water turbine) that lifts a weight. Of course this work can also rotate the shaft of an electrical generator, which is used to produce electrical power.
- Wind has energy because the wind can pass across a set of blades (e.g., a windmill), rotate the blades, and lift a weight that is attached to a rotating shaft. This shaft can also do work to rotate the shaft of an electrical generator.
- Gasoline has energy because it can be placed into a cylinder (e.g., a gas engine), burned and expanded to move a piston in a cylinder. This moving cylinder can then be connected to a mechanism that is used to lift a weight.

The SI unit of energy, the *joule*, is the energy associated with a force of one newton acting through a distance of one meter. For example, if a person with a weight of 700 newtons travels up a 10-meter flight of stairs, their gravitational potential energy has changed by $\Delta PE = (700\text{ N})(10\text{ m}) = 700\text{ N}\cdot\text{m} = 700\text{ J}$. In traditional units, the unit of energy, the *foot-pound-force* (lbf) is defined as energy associated with a force of 1.0 lbf moving through a distance of 1.0 foot.

Another way to define a unit of energy is describe the heating of water. A *small calorie* (cal) is the amount of energy required to increase the temperature of 1.0 gram of water by 1°C. The unit conversion between small calories and joules is 1.0 cal = 4.187 J. The *large calorie* (Cal), is the amount of energy to raise 1.0 kg of water by 1°C. Thus, 1.0 Cal = 4187 J. The large calorie is used in the United States to characterize the energy in food. Thus, a food item with 100 calories has an energy content of 0.4187 MJ. Energy in the traditional system is often measured using the British thermal unit (Btu). One Btu is the amount of energy required to raise the temperature of 1.0 lbm of water by 1.0°F.

Energy can be classified into categories.

- **Mechanical Energy.** This is the energy associated with motion (i.e., *kinetic energy*) plus the energy associated with position in a field. Regarding position in a field, this refers to position in a gravitational field (i.e., gravitational potential energy) and to deflection of an elastic object such as a spring (i.e., spring potential energy).
- **Thermal Energy.** This is energy associated with temperature changes and phase changes. For example, select a system comprised of 1 kg of ice (about 1 liter). The energy to melt the ice is 334 kJ. The energy to raise the temperature of the liquid water from 0°C to 100°C is 419 kJ.
- **Chemical Energy.** This is the energy associated with chemical bonds between elements. For example, when methane (CH_4) is burned, there is a chemical reaction that involves the breaking of the bonds in the methane and formation of new bonds to produce CO_2 and water. This chemical reaction releases heat, which is another way of saying the chemical energy is converted to thermal energy during combustion.

- **Electrical Energy.** This is the energy associated with electrical change. For example, a charged capacitor contains the amount of electrical energy $\Delta E = 1/2\, CV^2$ where C is capacitance and V is voltage.
- **Nuclear Energy.** This is energy associated with the binding of the particles in the nucleus of an atom. For example, when the uranium atom divides into two other atoms during fission, energy is released.

Work

Work is done on a system when a force acts on the system over a distance. In the absence o completing effects, the effect of work is to increase the energy of the system.

EXAMPLE. For the spray bottle in Fig. 7.2a, work is done when a finger acts through distance as the trigger is displaced. Work is also done by the piston as it exerts a force on th liquid as the piston is displaced. The magnitude of work done W can be evaluated using.

$$W = \int_{s_1}^{s_2} \mathbf{F} \cdot \mathbf{ds} \quad (7.$$

where **s** is position, and **F** is force. The effect of this work is to increase the energy of th system in several ways: water is lifted through a elevation, thereby increasing its potenti energy, and water is sprayed out the nozzle, thereby increasing its kinetic energy.

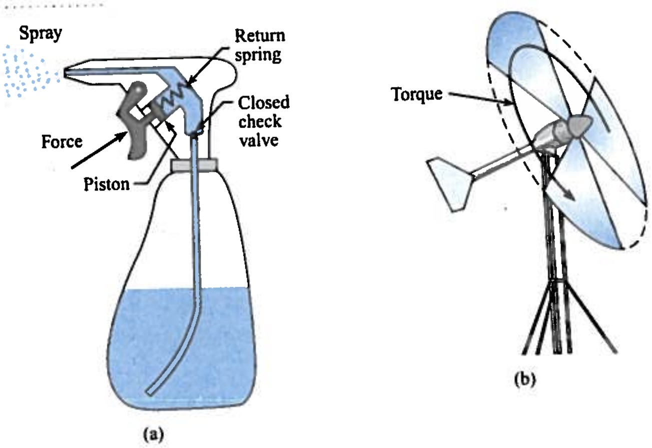


FIGURE 7.2
(a) In a spray bottle, a piston pump does work on the fluid, thereby increasing the energy of the liquid. (b) For a wind turbine, the air does work on the blade, thereby allowing the wind turbine to be used to produce electrical power.

EXAMPLE. For the wind turbine in Fig 7.2b, work is done by air that exerts a force on blades and causes the blades to rotate through a distance.

Work has the same units as energy: joules or newton-meters in SI and ft-lbf in traditio units.

Power

Power, which expresses a rate of work or energy, is defined by

$$P \equiv \frac{\text{quantity of work (or energy)}}{\text{interval of time}} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \dot{W} \quad ($$

Equation (7.2) is defined at an instant in time because power can vary with time. To calculate power, engineers use several different equations. For rectilinear motion, such as a car or bicycle, the amount of work is the product of force and displacement $\Delta W = F\Delta x$: Then, power can be found using

$$P = \lim_{\Delta t \to 0} \frac{F\Delta x}{\Delta t} = FV \tag{7.3a}$$

where V is the velocity of the moving body.

When a shaft is rotating (Fig. 7.2b), the amount of work is given by the product of torque and angular displacement $\Delta W = T\Delta\theta$. In this case, the power equation is

$$P = \lim_{\Delta t \to 0} \frac{T\Delta\theta}{\Delta t} = T\omega \tag{7.3b}$$

where ω is the angular speed. The SI units of angular speed are rad/s.

Because power has units of energy per time, the SI unit is a joule/second, which is called a watt. Common units for power are the watt (W), horsepower (hp), and the ft-lbf/s. Some typical values of power:

- A incandescent lightbulb can use 60 to 100 J/s of energy.
- A well-conditioned athlete can sustain a power output of about 300 J/s for an hour. This is about four-tenths of a horsepower. The horsepower is the approximate power that a draft horse can supply.
- A typical midsize car (2011 Toyota Camry) has a rated power of 126 kW (169 hp).
- A large hydroelectric facility (i.e., Bonneville Dam on the Columbia River 40 miles east of Portland, Oregon) has a rated power of 1080 MW.

Pumps and Turbines

A **turbine** is a machine that is used to extract energy from a flowing fluid.* Examples of turbines include the horizontal-axis wind turbine shown in Fig. 7.2b, the gas turbine, the Kaplan turbine, the Francis turbine, and the Pelton wheel.

A **pump** is a machine that is used to provide energy to a flowing fluid. Examples of pumps include the piston pump shown in Fig. 7.2a, the centrifugal pump, the diaphragm pump, and the gear pump.

7.2 Conservation of Energy

When James Prescott Joule died, his obituary in *The Electrical Engineer* (1) stated that

> *Very few indeed who read this announcement will realize how great of a man has passed away; and yet it must be admitted by those most competent to judge that his name must be classed among the greatest original workers in science.*

Joule was a brewer who did science as a hobby, yet he formulated one of the most important scientific laws ever developed. However, Joule's theory of conservation of energy was so

*The engine on a jet, which is called a gas turbine, is a notable exception. The jet engine adds energy to a flowing fluid, thereby increasing the momentum of a fluid jet and producing thrust.

controversial that he could not get a scientific journal to publish it. So his theory first ap peared in a local Manchester newspaper (2). What a fine example of persistence! Nowaday Joule's ideas about work and energy are foundational to engineering. This section introduce Joule's theory.

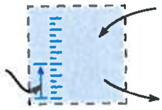
Joule's Theory of Energy Conservation

Joule recognized that the energy of a *closed system* can be changed in only two ways.

- **Work.** The energy of the system can be changed by work interactions at the boundary.
- **Heat Transfer.** The energy of the system can change by heat transfer across the boundary. Heat transfer can be defined as the transfer of thermal energy from hot to cold by mechanisms of conduction, convection, and radiation.

Joule's idea of energy conservation is illustrated in Fig. 7.3. The system is represented b the blue box. The scale on the left side of the figure represents the quantity of energy in th system. The arrows on the right side illustrate that energy can increase or decrease via work c heat transfer interactions. Note that energy is a property of a system, whereas work and he transfer are interactions that occur on system boundaries.

FIGURE 7.3
The law of conservation of energy for a closed system.



The work and energy balance proposed by Joule is captured with an equation called the fir law of thermodynamics:

$$\Delta E = Q - W$$

$$\begin{Bmatrix}\text{increase in}\\ \text{energy stored}\\ \text{in the system}\end{Bmatrix} = \begin{Bmatrix}\text{amount of energy}\\ \text{that entered system}\\ \text{by heat transfer}\end{Bmatrix} - \begin{Bmatrix}\text{amount of energy}\\ \text{that left system}\\ \text{due to work}\end{Bmatrix} \quad (7.$$

Terms in Eq. (7.4) have units of joules, and the equation is applied during a time interval wh the system undergoes a process to move from state 1 to state 2. To modify Eq. (7.4) so that applies at an instant in time, take the derivative to give

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \quad (7.$$

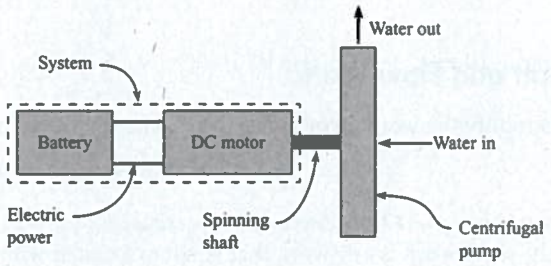
Eq. (7.5) applies at an instant in time and has units of joules per second or watts. The wo and time terms have sign conventions:

- W and $\dot{W}$ are positive if work is done by the system on the surroundings.
- W and $\dot{W}$ are negative if work is done by the surroundings on the system.
- Q and $\dot{Q}$ are positive if heat (i.e., thermal energy) is transferred into the system.
- Q and $\dot{Q}$ are negative if heat (i.e., thermal energy) is transferred out of the system.

✔CHECKPOINT PROBLEM 7.1

A battery is used to power a DC motor, which is then used to drive a centrifugal pump. For the indicated system, which statements are true? Circle all that apply. Assume steady state operation.

a. $\dot{W} > 0$
b. $\dot{W} < 0$
c. $\dot{Q} > 0$
d. $\dot{Q} < 0$
e. $\frac{dE}{dt} > 0$
f. $\frac{dE}{dt} < 0$



Control Volume (Open System)

Eq. (7.5) applies to a *closed system*. To extend it to a CV, apply the Reynolds transport theorem Eq. (5.23). Let the extensive property be energy ($B_{sys} = E$), and let $b = e$ to obtain

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv} e\rho d\forall + \int_{cs} e\rho \mathbf{V} \cdot d\mathbf{A} \tag{7.6}$$

where e is energy per mass in the fluid. Eq. (7.6) is the general form of conservation of energy for a control volume. However, most problems in fluid mechanics can be solved with a simpler form of this equation. This simpler equation will be derived in the next section.

7.3 The Energy Equation

This section shows how to simplify Eq. (7.6) to a form that is convenient for problems that occur in fluid mechanics.

Select Eq. (7.6). Then, let $e = e_k + e_p + u$ where e_k is the kinetic energy per unit mass, e_p is the gravitational potential energy per unit mass, and u is the internal energy* per unit mass.

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv} (e_k + e_p + u)\rho d\forall + \int_{cs} (e_k + e_p + u)\rho \mathbf{V} \cdot d\mathbf{A} \tag{7.7}$$

Next, let†

$$e_k = \frac{\text{kinetic energy of a fluid particle}}{\text{mass of this fluid particle}} = \frac{mV^2/2}{m} = \frac{V^2}{2} \tag{7.8}$$

Similarly, let

$$e_p = \frac{\text{gravitational potential energy of a fluid particle}}{\text{mass of this fluid particle}} = \frac{mgz}{m} = gz \tag{7.9}$$

*By definition, internal energy contains all forms of energy that are not kinetic energy or gravitational potential energy.
†It is assumed that the control surface is not accelerating, so V, which is referenced to the control surface, is also referenced to an inertial reference frame.

where z is the elevation measured relative to a datum. When Eqs. (7.8) and (7.9) are substitut into Eq. (7.7), the result is

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv}\left(\frac{V^2}{2} + gz + u\right)\rho\, d\forall + \int_{cs}\left(\frac{V^2}{2} + gz + u\right)\rho \mathbf{V} \cdot d\mathbf{A} \qquad (7.$$

Shaft and Flow Work

To simplify the work term in Eq. (7.10), classify work into two categories:

(work) = (flow work) + (shaft work)

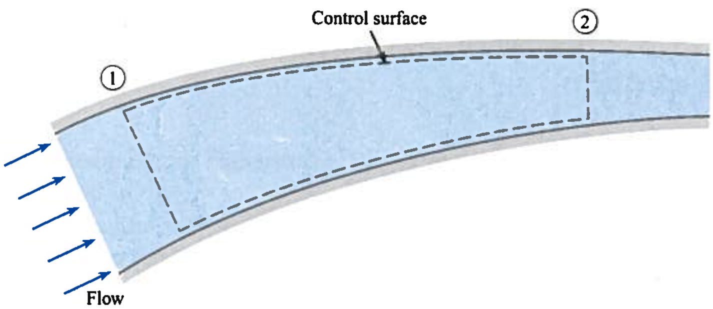
When the work is associated with a pressure force, then the work is called **flow work**. Altern tively, **shaft work** is any work that is not associated with a pressure force. Shaft work is usua done through a shaft (from which the term originates) and is commonly associated with pump or turbine. According to the sign convention for work, pump work is negative. Similar turbine work is positive. Thus,

$$\dot{W}_{\text{shaft}} = \dot{W}_{\text{turbines}} - \dot{W}_{\text{pumps}} = \dot{W}_t - \dot{W}_p \qquad (7.1$$

To derive an equation for flow work, use the idea that work equals force times distan Begin the derivation by defining a control volume situated inside a converging pipe (Fig. 7. At section 2, the fluid that is inside the control volume will push on the fluid that is outside t control volume. The magnitude of the pushing force is p_2A_2. During a time interval Δt, t displacement of the fluid at section 2 is $\Delta x_2 = V_2\Delta t$. Thus, the amount of work is

$$\Delta W_2 = (F_2)(\Delta x_2) = (p_2A_2)(V_2\Delta t) \qquad (7.1$$

FIGURE 7.4
Sketch for deriving flow work.



Convert the amount of work given by Eq. (7.12) into a rate of work:

$$\dot{W}_2 = \lim_{\Delta t \to 0} \frac{\Delta W_2}{\Delta t} = p_2A_2V_2 = \left(\frac{p_2}{\rho}\right)(\rho A_2 V_2) = \dot{m}\left(\frac{p_2}{\rho}\right) \qquad (7.1$$

This work is positive because the fluid inside the control volume is doing work on the enviro ment. In a similar manner, the flow work at section 1 is negative and is given by

$$\dot{W}_1 = -\dot{m}\left(\frac{p_1}{\rho}\right)$$

The net flow work for the situation pictured in Fig. 7.4 is

$$\dot{W}_{\text{flow}} = \dot{W}_2 + \dot{W}_1 = \dot{m}\left(\frac{p_2}{\rho}\right) - \dot{m}\left(\frac{p_1}{\rho}\right) \qquad (7.1$$

Equation (7.14) can be generalized to a situation involving multiple streams of fluid passing across a control surface:

$$\dot{W}_{\text{flow}} = \sum_{\text{outlets}} \dot{m}_{\text{out}}\left(\frac{p_{\text{out}}}{\rho}\right) - \sum_{\text{inlets}} \dot{m}_{\text{in}}\left(\frac{p_{\text{in}}}{\rho}\right) \tag{7.15}$$

To develop a general equation for flow work, use integrals to account for velocity and pressure variations on the control surface. Also, use the dot product to account for flow direction. The general equation for flow work is

$$\dot{W}_{\text{flow}} = \int_{cs} \left(\frac{p}{\rho}\right) \rho \mathbf{V} \cdot d\mathbf{A} \tag{7.16}$$

In summary, the work term is the sum of flow work [Eq. (7.16)] and shaft work [Eq. (7.11)]:

$$\dot{W} = \dot{W}_{\text{flow}} + \dot{W}_{\text{shaft}} = \left(\int_{cs} \left(\frac{p}{\rho}\right) \rho \mathbf{V} \cdot d\mathbf{A}\right) + \dot{W}_{\text{shaft}} \tag{7.17}$$

Introduce the work term from Eq. (7.17) into Eq. (7.10) and let $\dot{W}_{\text{shaft}} = \dot{W}_s$

$$\begin{aligned} \dot{Q} - \dot{W}_s - \int_{cs} \frac{p}{\rho} \rho \mathbf{V} \cdot d\mathbf{A} \\ = \frac{d}{dt}\int_{cv} \left(\frac{V^2}{2} + gz + u\right) \rho \, d\mathcal{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u\right) \rho \mathbf{V} \cdot d\mathbf{A} \end{aligned} \tag{7.18}$$

In Eq. (7.18), combine the last term on the left side with the last term on the right side:

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{cv} \left(\frac{V^2}{2} + gz + u\right) \rho \, d\mathcal{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u + \frac{p}{\rho}\right) \rho \mathbf{V} \cdot d\mathbf{A} \tag{7.19}$$

Replace $p/\rho + u$ by the specific enthalpy, h. The integral form of the energy principle is

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{cv} \left(\frac{V^2}{2} + gz + u\right) \rho \, d\mathcal{V} + \int_{cs} \left(\frac{V^2}{2} + gz + h\right) \rho \mathbf{V} \cdot d\mathbf{A} \tag{7.20}$$

Kinetic Energy Correction Factor

The next simplification is to extract the velocity terms out of the integrals on the right side of Eq. (7.20). This is done by introducing the kinetic energy correction factor.

Figure 7.5 shows fluid that is pumped through a pipe. At sections 1 and 2, kinetic energy is transported across the control surface by the flowing fluid. To derive an equation for this kinetic energy, start with the mass flow rate equation.

$$\dot{m} = \rho A \bar{V} = \int_A \rho V dA$$

This integral can be conceptualized as adding up the mass of each fluid particle that is crossing the section area and then dividing by the time interval associated with this crossing. To convert this integral to kinetic energy (KE), multiply the mass of each fluid particle by $(V^2/2)$.

$$\left\{\begin{matrix}\text{Rate of KE} \\ \text{transported} \\ \text{across a section}\end{matrix}\right\} = \int_A \rho V\left(\frac{V^2}{2}\right) dA = \int_A \frac{\rho V^3 dA}{2}$$

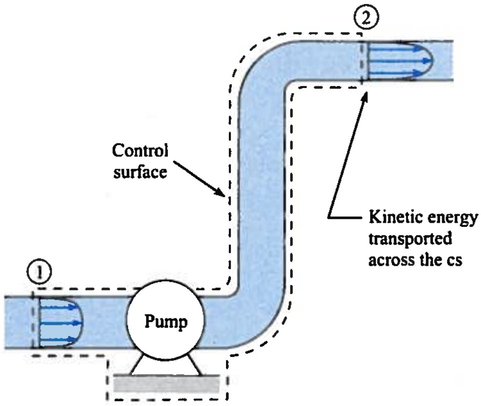


FIGURE 7.5
Flow carries kinetic energy into and out of a control volume.

The **kinetic energy correction factor** is defined as

$$\alpha = \frac{\text{actual KE/time that crosses a section}}{\text{KE/time by assuming a uniform velocity distribution}} = \frac{\displaystyle\int_A \frac{\rho V^3 dA}{2}}{\displaystyle\frac{\bar{V}^3}{2}\int_A \rho\, dA}$$

For a constant density fluid, this equation simplifies to

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \qquad \text{(7.2}$$

For theoretical development, α is found by integrating the velocity profile using Eq. (7.21 This approach, illustrated in Example 7.1, is a lot of work. Thus in application, engineers com monly estimate a value of α. Some *guidelines* are listed here.

- For fully developed laminar flow in a pipe, the velocity distribution is parabolic. Use $\alpha = 2.0$ because this is the correct value as shown by Example 7.1.
- For fully developed turbulent flow in a pipe, $\alpha \approx 1.05$ because the velocity profile is pluglike. Use $\alpha = 1.0$ for this case.
- For flow at the exit of a nozzle or converging section, use $\alpha = 1.0$ because converging flov leads to a uniform velocity profile. This is why wind tunnels use converging sections.
- For a uniform flow such as air flow in a wind tunnel or air flow incident on a wind turbine, use $\alpha = 1.0$.

EXAMPLE 7.1

Calculating the Kinetic Energy Correction Factor for Laminar Flow

Problem Statement

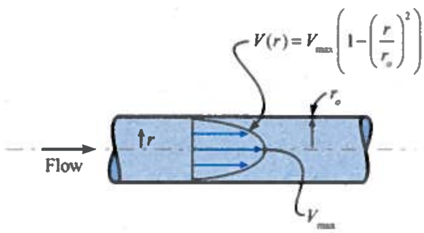
The velocity distribution for laminar flow in a pipe is given by the equation

$$V(r) = V_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

where V_{max} is the velocity in the center of the pipe, r_0 is the radius of the pipe, and r is the radial distance from the center. Find the kinetic-energy correction factor α.

Define the Situation

There is laminar flow in a round pipe.



State the Goal

α ⬅ Find the kinetic-energy correction factor (no units)

Generate Ideas and Make a Plan

Because the goal is α, apply the definition given by Eq. (7.21).

$$\alpha = \frac{1}{A}\int_A \left(\frac{V(r)}{\overline{V}}\right)^3 dA \quad \text{(a)}$$

Eq. (a) has one known (A) and two unknowns (dA, $\overline{V}$). To find dA use Fig. 5.3 (see page 175).

$$dA = 2\pi r dr \quad \text{(b)}$$

To find $\overline{V}$, apply the *flow rate equation*,

$$\overline{V} = \frac{1}{A}\int_A V(r)dA = \frac{1}{\pi r_0^2}\int_{r=0}^{r=r_0} V(r)2\pi r dr \quad \text{(c)}$$

Now the problem is cracked. There are three equations and three unknowns. The plan is:

1. Find the mean velocity $\overline{V}$ using Eq. (c)
2. Plug $\overline{V}$ into Eq. (a) and integrate

Take Action (Execute the Plan)

1. Flow Rate Equation (find mean velocity)

$$\overline{V} = \frac{1}{\pi r_0^2}\left[\int_0^{r_0} V_{max}\left(1 - \frac{r^2}{r_0^2}\right)2\pi r\, dr\right]$$

$$= \frac{2V_{max}}{r_0^2}\left[\int_0^{r_0}\left(1 - \frac{r^2}{r_0^2}\right) r\, dr\right] = \frac{2V_{max}}{r_0^2}\left[\int_0^{r_0}\left(r - \frac{r^3}{r_0^2}\right) dr\right]$$

$$= \frac{2V_{max}}{r_0^2}\left[\left(\frac{r^2}{2} - \frac{r^4}{4r_0^2}\right)\Big|_0^{r_0}\right] = \frac{2V_{max}}{r_0^2}\left[\frac{r_0^2}{2} - \frac{r_0^2}{4}\right] = V_{max}/2$$

2. Definition of α

$$\alpha = \frac{1}{A}\left[\int_A\left(\frac{V(r)}{\overline{V}}\right)^3 dA\right] = \frac{1}{\pi r_0^2 \overline{V}^3}\left[\int_0^{r_0} V(r)^3 2\pi r\, dr\right]$$

$$= \frac{1}{\pi r_0^2 (V_{max}/2)^3}\left[\int_0^{r_0}\left[V_{max}\left(1 - \frac{r^2}{r_0^2}\right)\right]^3 2\pi r\, dr\right]$$

$$= \frac{16}{r_0^2}\left[\int_0^{r_0}\left(1 - \frac{r^2}{r_0^2}\right)^3 r dr\right]$$

To evaluate the integral, make a change of variable by lettir $u = (1 - r^2/r_0^2)$. The integral becomes

$$\alpha = \left(\frac{16}{r_0^2}\right)\left(-\frac{r_0^2}{2}\right)\left(\int_1^0 u^3\, du\right) = 8\left(\int_0^1 u^3\, du\right)$$

$$= 8\left(\frac{u^4}{4}\Big|_0^1\right) = 8\left(\frac{1}{4}\right)$$

$$\boxed{\alpha = 2}$$

Review the Solution and the Process

1. *Knowledge.* Laminar fully developed flow in a round pip called Poiseuille flow. Useful facts:
   - The velocity profile is parabolic.
   - The mean velocity is one-half of the maximum (centerline) velocity: $\overline{V} = V_{max}/2$.
   - The kinetic energy correction factor is $\alpha = 2$.
2. *Knowledge.* In practice, engineers commonly estimate c The purpose of this example is to illustrate how to calculate α.

Last Steps of the Derivation

Now that the KE correction factor is available, the derivation of the energy equation may be completed. Begin by applying Eq. (7.20) to the control volume shown in Fig. 7.5. Assume steady flow and that velocity is normal to the control surfaces. Then, Eq. (7.20) simplifies to:

$$\dot{Q} - \dot{W}_s + \int_{A_1}\left(\frac{p_1}{\rho} + gz_1 + u_1\right)\rho V_1 dA_1 + \int_{A_1}\frac{\rho V_1^3}{2}dA_1 = \int_{A_2}\left(\frac{p_2}{\rho} + gz_2 + u_2\right)\rho V_2 dA_2 + \int_{A_2}\frac{\rho V_2^3}{2}dA_2 \quad \text{(7.22)}$$

Assume that piezometric head $p/\gamma + z$ is constant across sections 1 and 2.* If temperature is also assumed constant across each section, then $p/\rho + gz + u$ can be taken outside the integral to yield

$$\dot{Q} - \dot{W}_s + \left(\frac{p_1}{\rho} + gz_1 + u_1\right)\int_{A_1}\rho V_1 dA_1 + \int_{A_1}\rho\frac{V_1^3}{2}dA_1 = \left(\frac{p_2}{\rho} + gz_2 + u_2\right)\int_{A_2}\rho V_2 dA_2 + \int_{A_2}\rho\frac{V_2^3}{2}dA_2 \quad \text{(7.23)}$$

*Euler's equation can be used to show that pressure variation normal to rectilinear streamlines is hydrostatic.

Next, factor out $\int \rho V\,dA = \rho \bar{V} A = \dot{m}$ from each term in Eq. (7.23). Because $\dot{m}$ does not ap pear as a factor of $\int(\rho V^3/2)dA$, express $\int(\rho V^3/2)dA$ as $\alpha(\rho \bar{V}^3/2)A$, where α is the kinet energy correction factor:

$$\dot{Q} - \dot{W}_s + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1\frac{\bar{V}_1^2}{2}\right)\dot{m} = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2\frac{\bar{V}_2^2}{2}\right)\dot{m} \qquad (7.2$$

Divide through by $\dot{m}$:

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1\frac{\bar{V}_1^2}{2} = \frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2\frac{\bar{V}_2^2}{2} \qquad (7.2$$

Introduce Eq. (7.11) into Eq. (7.25):

$$\frac{\dot{W}_p}{\dot{m}g} + \frac{p_1}{\gamma} + z_1 + \alpha_1\frac{\bar{V}_1^2}{2g} = \frac{\dot{W}_t}{\dot{m}g} + \frac{p_2}{\gamma} + z_2 + \alpha_2\frac{\bar{V}_2^2}{2g} + \frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g} \qquad (7.2$$

Introduce **pump head** and **turbine head**:

$$\textit{Pump head} = h_p = \frac{\dot{W}_p}{\dot{m}g} = \frac{\text{work/time done by pump on flow}}{\text{weight/time of flowing fluid}}$$

$$\textit{Turbine head} = h_t = \frac{\dot{W}_t}{\dot{m}g} = \frac{\text{work/time done by flow on turbine}}{\text{weight/time of flowing fluid}} \qquad (7.2$$

Equation (7.26) becomes:

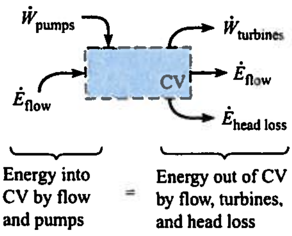
$$\frac{p_1}{\gamma} = \alpha_1\frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2 + h_t + \left[\frac{1}{g}(u_2 - u_1) - \frac{\dot{Q}}{\dot{m}g}\right] \qquad (7.2$$

Equation (7.28) is separated into terms that represent mechanical energy (nonbrackete terms) and terms that represent thermal energy (the bracketed term). This bracketed term always positive because of the second law of thermodynamics. This term is called head loss an is represented by h_L. **Head loss** is the conversion of useful mechanical energy to waste therm energy through viscous action. Head loss is analogous to thermal energy (heat) that is pr duced by Coulomb friction. When the bracketed term is replaced by head loss h_L, Eq. (7.2 becomes the *energy equation.*

$$\left(\frac{p_1}{\gamma} + \alpha_1\frac{\bar{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2\right) + h_t + h_L \qquad (7.2$$

FIGURE 7.6

The energy balance for a CV when the energy equation is applied.



Physical Interpretation of the Energy Equation

The energy equation describes an energy balance for a control volume (Fig. 7.6). The inflov of energy are balanced with the outflows of energy.* Regarding inflows, energy can be tran ported across the control surface by the flowing fluid or a pump can do work on the fluid ar thereby add energy to the fluid. Regarding outflows, energy within the flow can be used to c work on a turbine, energy can be transported across the control surface by the flowing fluid, mechanical energy can be converted to waste thermal heat via head loss.

*The term "$\dot{E}_{\text{flow}}$" includes a work term, namely flow work. Remember that energy is a property of a system, where work and heat transfer are interactions that occur on system boundaries. Here, we are using the term "energy balanc to describe (energy terms) + (work terms) + (heat transfer terms).

The energy balance can also be expressed using head:

$$\left(\frac{p_1}{\gamma} + \alpha_1\frac{\overline{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2\frac{\overline{V}_2^2}{2g} + z_2\right) + h_t + h_L$$

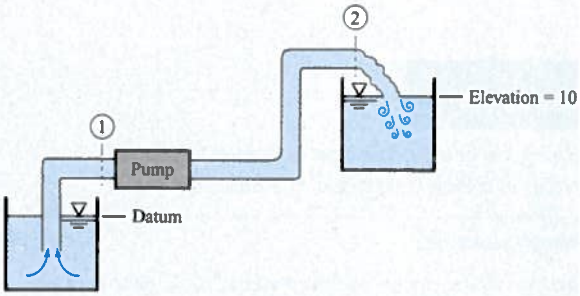
$$\begin{pmatrix}\text{pressure head}\\ \text{velocity head}\\ \text{elevation head}\end{pmatrix}_1 + \begin{pmatrix}\text{pump}\\ \text{head}\end{pmatrix} = \begin{pmatrix}\text{pressure head}\\ \text{velocity head}\\ \text{elevation head}\end{pmatrix}_2 + \begin{pmatrix}\text{turbine}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{head}\\ \text{loss}\end{pmatrix}$$

Head can be thought of as the *ratio of energy to weight for a fluid particle.* Or, head can describe the *energy per time that is passing across a section* because head and power are related by $P = \dot{m}gh$.

✔ CHECKPOINT PROBLEM 7.2

As shown, a pump moves water from a lower reservoir to a higher reservoir. The pipe has a constant diameter. Which statements are true? (circle all that apply)

a. Pressure head at 1 is zero.
b. Pressure head at 2 is zero.
c. Velocity head at 1 > velocity head at 2.
d. Velocity head at 2 < velocity head at 1.
e. Pump head is negative.
f. Pump head is positive.
g. Head loss is positive.
h. Head loss is negative.



Working Equations

Table 7.1 summarizes the energy equation, its variables, and the main assumptions.

TABLE 7.1 Summary of the Energy Equation

Description	Equation	Terms
The energy equation has only one form. Major assumptions • Steady state; no energy accumulation in CV • CV has one inlet and one outlet • Constant density flow • All thermal energy terms (except for head loss) can be neglected. • Streamlines are straight and parallel at each section • Temperature is constant across each section.	$\left(\frac{p_1}{\gamma} + \alpha_1\frac{\overline{V}_1^2}{2g} + z_1\right) + h_p =$ $\left(\frac{p_2}{\gamma} + \alpha_2\frac{\overline{V}_2^2}{2g} + z_2\right) + h_t + h_L$ Eq. (7.29)	$\left(\frac{p}{\gamma} + \alpha\frac{\overline{V}^2}{2g} + z\right) = \begin{pmatrix}\text{energy/weight transported}\\ \text{into or out of cv}\\ \text{by fluid flow}\end{pmatrix}$ p/γ = pressure head at cs (m) $\alpha\frac{\overline{V}^2}{2g}$ = velocity head at cs (m) (α = kinetic energy (KE) correction factor at cs) ($\alpha \approx 1.0$ for turbulent flow) ($\alpha \approx 1.0$ for nozzles) ($\alpha \approx 2.0$ for full-developed laminar flow in round pipe z = elevation head at cs (m) h_p = head added by a pump (m) h_t = head removed by a turbine (m) h_L = head loss (m) (to predict head loss, apply Eq. (10.45))

The process for applying the energy equation is

Step 1. Selection. Select the continuity equation when the problem involves pumps, tu bine, or head loss. Check to ensure that the assumptions used to derive the energy equa tion are satisfied. The assumptions are steady flow, one inlet port and one outlet poı constant density, and negligible thermal energy terms (except for head loss).

Step 2. CV Selection. Select and label section 1 (inlet port) and section 2 (outlet port). Loca sections 1 and 2 where (a) you know information or (b) where you want information. E convention, engineers usually do not sketch a CV when applying the energy equation.

Step 3. Analysis. Write the general form of the energy equation. Conduct a term-by-terı analysis. Simplify the general equation to the reduced equation.

Step 4. Validation. Check units. Check the physics: (head in via fluid flow and pump) (head out via fluid flow, turbine, and head loss)

EXAMPLE 7.2

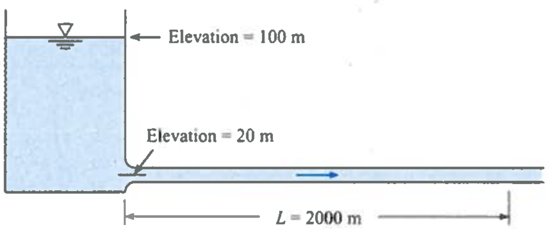
Applying the Energy Equation to Predict the Speed of Water in a Pipe Connected to a Reservoir

Problem Statement

A horizontal pipe carries cooling water at 10°C for a thermal power plant. The head loss in the pipe is

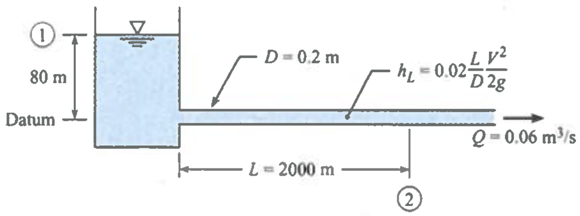
$$h_L = \frac{0.02(L/D)V^2}{2g}$$

where L is the length of the pipe from the reservoir to the point in question, V is the mean velocity in the pipe, and D is the diameter of the pipe. If the pipe diameter is 20 cm and the rate of flow is 0.06 m³/s, what is the pressure in the pipe at $L = 2000$ m. Assume $\alpha_2 = 1$.



Define the Situation

Water flows in a system.



Assumptions:
- $\alpha_2 = 1.0$
- Steady Flow

Water (10°C, 1 atm., Table A.5): $\gamma = 9810$ N/m³

State the Goal

p_2(kPa) ⬅ Pressure at section 2

Generate Ideas and Make a Plan

Select the energy equation because (a) the situation involves water flowing through a pipe, and (b) the energy equation contains the goal (p_2). Locate section 1 at the surface and section 2 at the location where we want to know pressure. The plan is to:

1. Write the general form of the energy equation (7.29)
2. Analyze each term in the energy equation.
3. Solve for p_2.

Take Action (Execute the Plan)

1. Energy equation (general form)

$$\frac{p_1}{\gamma} + \alpha_1\frac{\overline{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{\overline{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term-by-term analysis
   - $p_1 = 0$ because the pressure at top of a reservoir is $p_{atm} = 0$ gage.
   - $V_1 \approx 0$ because the level of the reservoir is constant or changing very slowly.
   - $z_1 = 100$ m; $z_2 = 20$ m.
   - $h_p = h_t = 0$ because there are no pumps or turbines in the system.
   - Find V_2 using the flow rate equation (5.3).

$$V_2 = \frac{Q}{A} = \frac{0.06 \text{ m}^3/\text{s}}{(\pi/4)(0.2 \text{ m})^2} = 1.910 \text{ m/s}$$

- Head loss is

$$h_L = \frac{0.02(L/D)V^2}{2g} = \frac{0.02(2000\text{ m}/0.2\text{ m})(1.910\text{ m/s})^2}{2(9.81\text{ m/s}^2)}$$

$$= 37.2\text{ m}$$

3. Combine steps 1 and 2.

$$(z_1 - z_2) = \frac{p_2}{\gamma} + \alpha_2\frac{\overline{V}_2^2}{2g} + h_L$$

$$80\text{ m} = \frac{p_2}{\gamma} + 1.0\frac{(1.910\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 37.2\text{ m}$$

$$80\text{ m} = \frac{p_2}{\gamma} + (0.186\text{ m}) + (37.2\text{ m})$$

$$p_2 = \gamma(42.6\text{ m}) = (9810\text{ N/m}^3)(42.6\text{ m}) = \boxed{418\text{ kPa}}$$

Review the Solution and the Process

1. *Skill.* Notice that section 1 was set at the free surface because properties are known there. Section 2 was set where we want to find information.
2. *Knowledge.* Regarding selection of an equation, we could have chosen the Bernoulli equation. However, it would have been a lousy choice because the Bernoulli equation assumes inviscid flow.
   - *Key Idea:* Select the Bernoulli equation if viscous effect can be neglected; select the energy equation if viscous effects are significant.
   - *Rule of Thumb:* When fluid is flowing through a pipe tl is more than about five diameters long, i.e., $(L/D > 5)$, viscous effects are significant.

7.4 The Power Equation

Depending on context, engineers use various equations for calculating power. This section shows how to calculate power associated with pumps and turbines. An equation for pump power follows from the definition of pump head given in Eq. (7.27):

$$\dot{W}_p = \gamma Q h_p = \dot{m} g h_p \tag{7.30a}$$

Similarly, the power delivered from a flow to a turbine is

$$\dot{W}_t = \gamma Q h_t = \dot{m} g h_t \tag{7.30b}$$

Equations (7.30a) and (7.30b) can be generalized to give an equation for calculating power associated with a pump or turbine.

$$P = \dot{m} g h = \gamma Q h \tag{7.31}$$

Equations for calculating power are summarized in Table 7.2.

You can try out the equations in Table 7.2 in the next checkpoint problem.

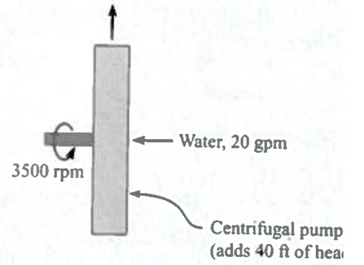
TABLE 7.2 Summary of the Power Equation

Description	Equation	Terms
Rectilinear motion of an object such as an airplane, a submarine, or a car	$P = FV$ (7.3a)	P = power (W) F = force doing work (N) V = speed of object (m/s)
Rotational motion such as a shaft driving a pump or an output shaft from a turbine	$P = T\omega$ (7.3b)	T = torque (N · m) ω = angular speed (rad/s)
Power supplied from a pump to a flowing fluid Power supplied from a flowing fluid to a turbine	$P = \dot{m}gh = \gamma Q h$ (7.31)	$\dot{m}$ = mass flow rate through machine (g = gravitational constant = 9.81 (m/s² h = head of pump or head of turbine (r γ = specific weight (N/m³) Q = volume flow rate (m³/s)

✔ CHECKPOINT PROBLEM 7.3

The shaft on a centrifugal pump is spinning at a rate of 3500 rpm. The pump discharge is 20 gpm, and the pump head is 40 ft. What is the torque in the shaft of the pump in units of ft-lbf? [Assume that the mechanical efficiency of the pump is 100%].

a. 0.0032
b. 0.30
c. 9.80
d. 27.2



EXAMPLE 7.3

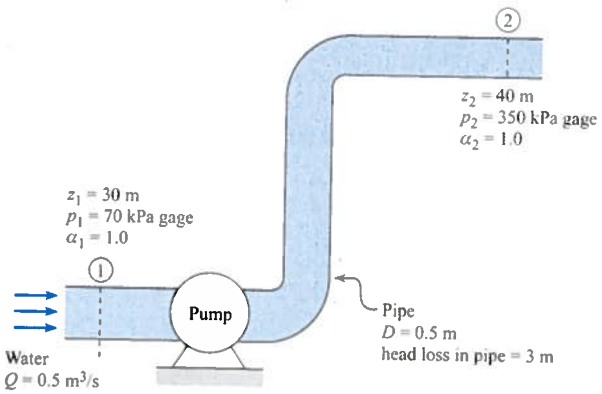
Applying the Energy Equation to Calculate the Power Required by a Pump

Problem Statement

A pipe 50 cm in diameter carries water (10°C) at a rate of 0.5 m^3/s. A pump in the pipe is used to move the water from an elevation of 30 m to 40 m. The pressure at section 1 is 70 kPa gage, and the pressure at section 2 is 350 kPa gage. What power in kilowatts and in horsepower must be supplied to the flow by the pump? Assume $h_L = 3$ m of water and $\alpha_1 = \alpha_2 = 1$.

Define the Situation

Water is being pumped through a system.



Water (10°C, 1 atm., Table A.5): $\gamma = 9810\ N/m^3$

State the Goal

P(W and hp) ⇐ Power the pump is supplying to the water in units of watts and horsepower.

Generate Ideas and Make a Plan

Because this problem involves water being pumped through a system, it is an energy equation problem. However, the goal is to find power, so the power equation will also be needed. The steps are

1. Write the energy equation between section 1 and section 2
2. Analyze each term in the energy equation.
3. Calculate the head of the pump h_p.
4. Find the power by applying the power equation (7.30a).

Take Action (Execute the Plan)

1. Energy equation (general form)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\overline{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\overline{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term-by-term analysis
   - Velocity head cancels because $V_1 = V_2$.
   - $h_t = 0$ because there are no turbines in the system.
   - All other head terms are given.
   - Inserting terms into the general equation gives

$$\frac{p_1}{\gamma} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + h_L$$

3. Pump head (from step 2)

$$h_p = \left(\frac{p_2 - p_1}{\gamma}\right) + (z_2 - z_1) + h_L$$

$$= \left(\frac{(350{,}000 - 70{,}000)\ N/m^2}{9810\ N/m^3}\right) + (10\ m) + (3\ m)$$

$$= (28.5\ m) + (10\ m) + (3\ m) = 41.5\ m$$

Physics: The head provided by the pump (41.5 m) is balanced by the increase in pressure head (28.5 m) plus the increase in elevation head (10 m) plus the head loss (3 m).

4. Power equation

$$P = \gamma Q h_p$$

$$= (9810\ N/m^3)(0.5\ m^3/s)(41.5\ m)$$

$$= \boxed{204\ kW} = (204\ kW)\left(\frac{1.0\ hp}{0.746\ kW}\right) = \boxed{273\ hp}$$

Review the Solution and the Process

Discussion. The calculated power represents the (work/time) being done by the pump impeller on the water. The electrical power supplied to the pump would need to be larger than this because of energy losses in the electrical motor and because the pump itself is not 100% efficient. Both of these factors c be accounted for using pump efficiency (η_{pump}) and motor efficiency (η_{motor}), respectively.

7.5 Mechanical Efficiency

Fig. 7.7 shows an electric motor connected to a centrifugal pump. Motors, pump, turbines, and similar device have energy losses. In pump and turbines, energy losses are due to factors such as mechanical friction, viscous dissipation, and leakage. Energy losses are accounted for by using efficiency.

Mechanical efficiency is defined as the ratio of power output to power input:

$$\eta \equiv \frac{\text{power output from a machine or system}}{\text{power input to a machine or system}} = \frac{P_{output}}{P_{input}} \quad (7.32)$$

The symbol for mechanical efficiency is the Greek letter η, which is pronounced as "eta." In addition to mechanical efficiency, engineers also use *thermal efficiency*, which is defined using thermal energy input into a system. In this text, only mechanical efficiency is used, and we sometimes use the label "efficiency" instead of "mechanical efficiency."

FIGURE 7.7
CAD drawing of a centrifugal pump and electric motor. (Image courtesy of Ted Kyte; www.ted-kyte.com.)

EXAMPLE. Suppose an electric motor like the one shown in Fig. 7.7 is drawing 1000 W of electrical power from a wall circuit. As shown in Fig. 7.8, the motor provides 750 J/s of power to its output shaft. This power drives the pump, and the pump supplies 450 J/s to the fluid.

In this example, the efficiency of the electric motor is

$$\eta_{motor} = (750\ \text{J/s})/(1000\ \text{J/s}) = 0.75 = 75\%$$

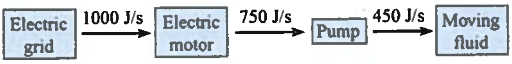


FIGURE 7.8
The energy flow through a pump that is powered by an electric motor.

Similarly, the efficiency of the pump is

$$\eta_{\text{pump}} = (450\ \text{J/s})/(750\ \text{J/s}) = 0.60 = 60\%$$

and the combined efficiency is

$$\eta_{\text{combined}} = (450\ \text{J/s})/(1000\ \text{J/s}) = 0.45 = 45\%$$

EXAMPLE. Suppose that wind incident on a wind turbine contains 1000 J/s of energy shown in Fig. 7.9. Because a wind turbine cannot extract all the energy and because of loss the work that the wind turbine does on its output shaft is 360 J/s. This power drives an elect generator, and the generator produces 324 J/s of electrical power, which is supplied to t power grid. Calculate the system efficiency and the efficiency of the components.

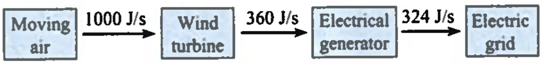


FIGURE 7.9
The energy flow associated with generating electrical power from a wind turbine.

The efficiency of the wind turbine is

$$\eta_{\text{wind turbine}} = (360\ \text{J/s})/(1000\ \text{J/s}) = 0.36 = 36\%$$

The efficiency of the electric generator is

$$\eta_{\text{electric generator}} = (324\ \text{J/s})/(360\ \text{J/s}) = 0.90 = 90\%$$

The combined efficiency is

$$\eta_{\text{combined}} = (324\ \text{J/s})/(1000\ \text{J/s}) = 0.324 = 32.4\%$$

We can generalize the results of the last two examples to summarize the efficiency equ tions (Table 7.3). Example 7.4 shows how efficiency enters into a calculation of power.

TABLE 7.3 Summary of the Efficiency Equation

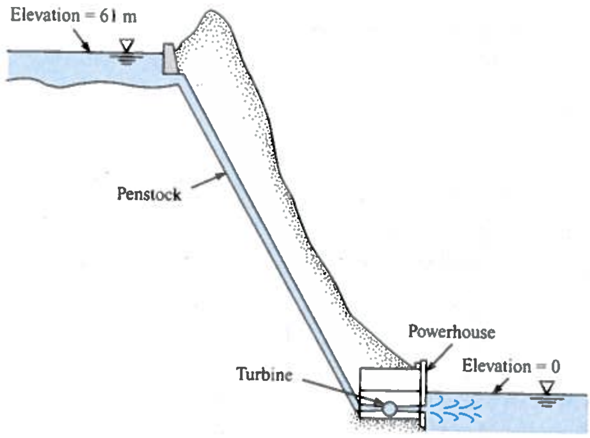
Description	Equation	Terms
Pump	$P_{\text{pump}} = \eta_{\text{pump}} P_{\text{shaft}}$ (7.33a)	P_{pump} = power that the pump supplies to the fluid (W) $[P_{\text{pump}} = \dot{m}gh_p = \gamma Q h_p]$ η_{pump} = efficiency of pump () P_{shaft} = power that is supplied to the pump shaft (V
Turbine	$P_{\text{shaft}} = \eta_{\text{turbine}} P_{\text{turbine}}$ (7.33b)	P_{turbine} = power that the fluid supplies to a turbine (W) $[P_{\text{turbine}} = \dot{m}gh_t = \gamma Q h_t]$ η_{turbine} = efficiency of turbine () P_{shaft} = power that is supplied by the turbine shaft (

EXAMPLE 7.4

Applying the Energy Equation to Predict the Power Produced by a Turbine

Problem Statement

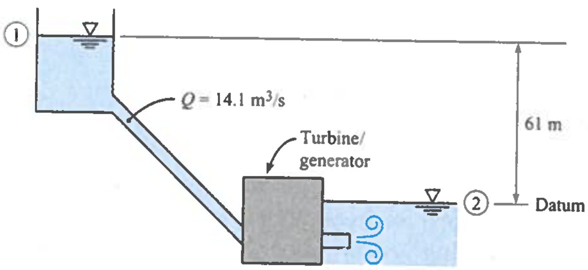
At the maximum rate of power generation, a small hydroelectric power plant takes a discharge of 14.1 m^3/s through an elevation drop of 61 m. The head loss through the intakes, penstock, and outlet works is 1.5 m. The combined efficiency of the turbine and electrical generator is 87%. What is the rate of power generation?



Define the Situation

A small hydroelectric plant is producing electrical power

- Combined head loss: $h_L = 1.5$ m
- Combined efficiency (turbine/generator): $\eta = 0.87$
- **Water** (10°C, 1 atm, Table A.5): $\gamma = 9810\ \text{N/m}^3$



State the Goal

$P_{\text{output from generator}}$ (MW) ⬅ Power produced by generator

Generate Ideas and Make a Plan

Because this problem involves a fluid system for producing power, select the energy equation. Because power is the goal, also select the power equation. The plan is

1. Write the energy equation (7.29) between section 1 and section 2.
2. Analyze each term in the energy equation.
3. Solve for the head of the turbine h_t.
4. Find the input power to the turbine using the power equation (7.30b).
5. Find the output power from the generator by using the efficiency equation (7.33b).

Take Action (Execute the Plan)

1. Energy equation (general form)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\overline{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\overline{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term-by-term analysis
   - Velocity heads are negligible because $V_1 \approx 0$ and $V_2 \approx$
   - Pressure heads are zero because $p_1 = p_2 = 0$ gage.
   - $h_p = 0$ because there is no pump in the system.
   - Elevation head terms are given.
3. Combine steps 1 and 2:

$$h_t = (z_1 - z_2) - h_L$$
$$= (61\ \text{m}) - (1.5\ \text{m}) = 59.5\ \text{m}$$

Physics: Head supplied to the turbine (59.5 m) is equal to the net elevation change of the dam (61 m) minus the head loss (1.5 m).

4. Power equation

$$P_{\text{input to turbine}} = \gamma Q h_t = (9810\ \text{N/m}^3)(14.1\ \text{m}^3/\text{s})(59.5\ \text{m})$$
$$= 8.23\ \text{MW}$$

5. Efficiency equation

$$P_{\text{output from generator}} = \eta P_{\text{input to turbine}} = 0.87(8.23\ \text{MW})$$
$$= \boxed{7.16\ \text{MW}}$$

Review the Solution and the Process

1. *Knowledge*. Notice that sections 1 and 2 were located on the free surfaces. This is because information is known at these locations.
2. *Discussion*. The maximum power that can be generated is a function of the elevation head and the flow rate. This maximum power is decreased by head loss and by energy losses in the turbine and the generator.

7.6 Contrasting the Bernoulli Equation and the Energy Equation

Although the Bernoulli equation (Eq. 4.21b) and the energy equation (Eq. 7.29) have a simila form and several terms in common, they are not the same equation. This section explains th differences between these two equations. This information is important for conceptual under standing of these two important equations.

The Bernoulli equation and the energy equation are derived in different ways. The Be noulli equation was derived by applying Newton's second law to a particle and then integratin the resulting equation along a streamline. The energy equation was derived by starting with th first law of thermodynamics and then using the Reynolds transport theorem. Consequentl the Bernoulli equation involves only mechanical energy, whereas the energy equation includ both mechanical and thermal energy.

The two equations have different methods of application. The Bernoulli equation applied by selecting two points on a streamline and then equating terms at these points:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

In addition, these two points can be anywhere in the flow field for the special case of irrot tional flow. The energy equation is applied by selecting an inlet section and an outlet sectic and then equating terms as they apply to a control volume located between the inlet and outle

$$\left(\frac{p_1}{\gamma} + \alpha_1\frac{\overline{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2\frac{\overline{V}_2^2}{2g} + z_2\right) + h_t + h_L$$

The two equations have different assumptions. The Bernoulli equation applies to stead incompressible, and inviscid flow. The energy equation applies to steady, viscous, incompres ible flow in a pipe with additional energy being added through a pump or extracted through turbine.

Under special circumstances the energy equation can be reduced to the Bernoulli equ tion. If the flow is inviscid, there is no head loss; that is, $h_L = 0$. If the "pipe" is regarded as small stream tube enclosing a streamline, then $\alpha = 1$. There is no pump or turbine along streamline, so $h_p = h_t = 0$. In this case the energy equation is identical to the Bernoulli equ tion. Note that the energy equation cannot be derived by starting with the Bernoulli equatio

Summary The energy equation is *not* the Bernoulli equation. However, both equations can related to the law of conservation of energy. Thus, similar terms appear in each equation.

7.7 Transitions

The purpose of this section is to illustrate how the energy, momentum, and continuity equ tions can be used together to analyze (a) head loss for an abrupt expansion and (b) forces transitions. These results are useful for designing systems, especially those with large pip such as the penstock in a dam.

Abrupt Expansion

An **abrupt or sudden expansion** in a pipe or duct is a change from a smaller section area t larger section area as shown in Fig. 7.10. Notice that a confined jet of fluid from the smal

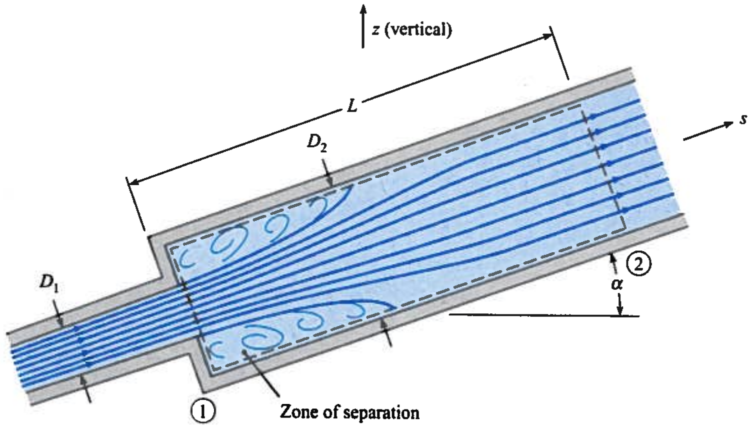


FIGURE 7.10
Flow through an abrupt expansion.

pipe discharges into the larger pipe and creates a zone of separated flow. Because the streamlines in the jet are initially straight and parallel, the piezometric pressure distribution across the jet at section 1 will be uniform.

To analyze the transition, apply the energy equation between sections 1 and 2:

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L \quad (7.34)$$

Assume turbulent flow conditions so $\alpha_1 = \alpha_2 \approx 1$. The momentum equation is

$$\sum F_s = \dot{m}V_2 - \dot{m}V_1$$

Next, let $\dot{m} = \rho AV$ and then identify the forces. Note that the shear force can be neglected because it is small relative to the pressure force. The momentum equation becomes

$$p_1A_2 - p_2A_2 - \gamma A_2L\sin\alpha = \rho V_2^2A_2 - \rho V_1^2A_1$$

or

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) = \frac{V_2^2}{g} - \frac{V_1^2}{g}\frac{A_1}{A_2} \quad (7.35)$$

The continuity equation simplifies to

$$V_1A_1 = V_2A_2 \quad (7.36)$$

Combining Eqs. (7.34) to (7.36) gives an equation for the head loss h_L caused by a sudden expansion:

$$h_L = \frac{(V_1 - V_2)^2}{2g} \quad (7.37)$$

If a pipe discharges fluid into a reservoir, then $V_2 = 0$, and the head loss simplifies to

$$h_L = \frac{V^2}{2g}$$

which is the velocity head in the pipe. This energy is dissipated by the viscous action of the fluid in the reservoir.

Forces on Transitions

To find forces on transitions in pipes, apply the momentum equation in combination with tl energy equation, the flow rate equation, and the head loss equation. This approach is illustrat by Example 7.5.

EXAMPLE 7.5

Applying the Energy and Momentum Equations to Find Force on a Pipe Contraction

Problem Statement

A pipe 30 cm in diameter carries water (10°C, 250 kPa) at a rate of 0.707 m^3/s. The pipe contracts to a diameter of 20 cm. The head loss through the contraction is given by

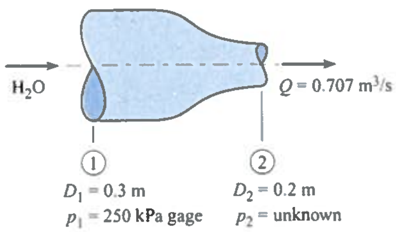
$$h_L = 0.1\frac{V_2^2}{2g}$$

where V_2 is the velocity in the 20 cm pipe. What horizontal force is required to hold the transition in place? Assume the kinetic energy correction factor is 1.0 at both the inlet and exit

Define the Situation

Water flows through a contraction.

- $\alpha_1 = \alpha_2 = 1.0$
- $h_L = 0.1\,(V_2^2/(2g))$



Properties: Water (10°C, 1 atm., Table A.5): $\gamma = 9810\ \text{N/m}^3$

State the Goal

F_x(N) ⬅ Horizontal force acting on the contraction

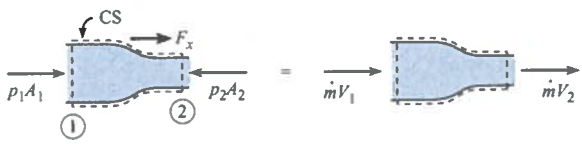
Generate Ideas and Make a Plan

Because force is the goal, start with the momentum equation. To solve the momentum equation, we need p_2. Find this with the energy equation. The step-by-step plan is

1. Derive an equation for F_x by applying the momentum eqn.
2. Derive an equation for p_2 by applying the energy eqn.
3. Calculate p_2.
4. Calculate F_x.

Take Action (Execute the Plan)

1. Momentum equation
   - Sketch a force diagram and a momentum diagram



   - Write the x-direction momentum equation.

$$p_1A_1 - p_2A_2 + F_x = \dot{m}V_2 - \dot{m}V_1$$

   - Rearrange to give

$$F_x = \rho Q(V_2 - V_1) + p_2A_2 - p_1A_1$$

2. Energy equation (from section 1 to section 2)
   - Let $\alpha_1 = \alpha_2 = 1, z_1 = z_2$, and $h_p = h_t = 0$
   - Eq. (7.29) simplifies to

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

   - Rearrange to give

$$p_2 = p_1 - \gamma\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L\right)$$

3. Pressure at section 2.
   - Find velocities using the flow rate equation.

$$V_1 = \frac{Q}{A_1} = \frac{0.707\ \text{m}^3/\text{s}}{(\pi/4) \times (0.3\ \text{m})^2} = 10\ \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.707\ \text{m}^3/\text{s}}{(\pi/4) \times (0.2\ \text{m})^2} = 22.5\ \text{m/s}$$

   - Calculate head loss.

$$h_L = \frac{0.1\,V_2^2}{2g} = \frac{0.1 \times (22.5\ \text{m/s})^2}{2 \times (9.81\ \text{m/s}^2)} = 2.58\ \text{m}$$

- Calculate pressure.

$$p_2 = p_1 - \gamma\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L\right)$$
$$= 250\text{ kPa} - 9.81\text{ kN/m}^3 \times \left(\frac{(22.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)} - \frac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 2.58\text{ m}\right)$$
$$= 21.6\text{ kPa}$$

4. Calculate F_x.

$$F_x = \rho Q(V_2 - V_1) + p_2A_2 - p_1A_1$$
$$= (1000\text{ kg/m}^3)(0.707\text{ m}^3\text{/s})(22.5 - 10)(\text{m/s}) + (21{,}600\text{ Pa})\left(\frac{\pi(0.2\text{ m})^2}{4}\right) - (250{,}000\text{ Pa}) \times \left(\frac{\pi(0.3\text{ m})^2}{4}\right)$$
$$= (8837 + 677 - 17{,}670)\text{N} = -8.16\text{ kN}$$

F_x = 8.16 kN acting to the left

7.8 Hydraulic and Energy Grade Lines

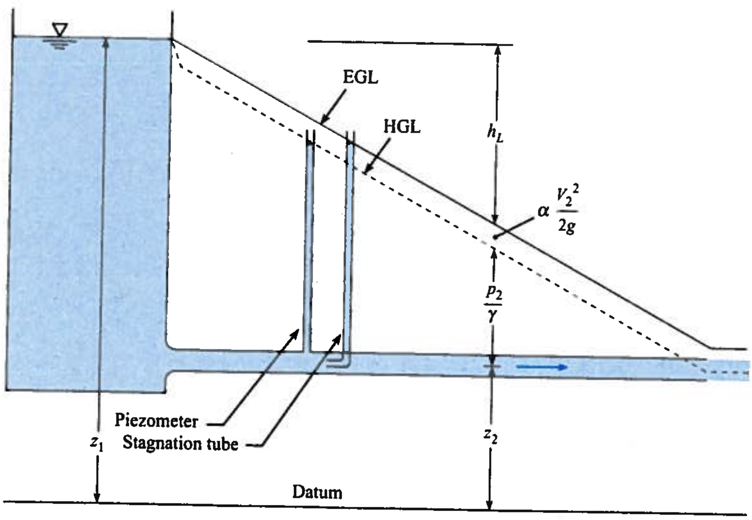
This section introduces the hydraulic grade line (HGL) and the energy grade line (EGL), which are graphical representations that show head in a system. This visual approach provides insights and helps one locate and correct trouble spots in the system (usually points of low pressure).

The **EGL**, shown in Fig. 7.11, is a line that indicates the total head at each location in a system. The *EGL* is related to terms in the energy equation by

$$\text{EGL} = \begin{pmatrix}\text{velocity}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix} = \alpha\frac{V^2}{2g} + \frac{p}{\gamma} + z = \begin{pmatrix}\text{total}\\ \text{head}\end{pmatrix} \quad (7.38)$$

Notice that **total head**, which characterizes the energy that is carried by a flowing fluid, is the sum of velocity head, the pressure head, and the elevation head.

FIGURE 7.11
EGL and HGL in a straight pipe.



The HGL, shown in Fig. 7.11, is a line that indicates the piezometric head at each location in a system:

$$\text{HGL} = \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix} = \frac{p}{\gamma} + z = \begin{pmatrix}\text{piezometric}\\ \text{head}\end{pmatrix} \quad (7.39)$$

Because the HGL gives piezometric head, the HGL will be coincident with the liquid surface in a piezometer as shown in Fig. 7.11. Similarly, the EGL will be coincident with the liquid surface in a stagnation tube.

Tips for Drawing HGLs and EGLs

1. In a lake or reservoir, the HGL and EGL will coincide with the liquid surface. Also, both the HGL and EGL will indicate piezometric head.
2. A pump causes an abrupt rise in the EGL and HGL by adding energy to the flow. For example, see Fig. 7.12.
3. For steady flow in a pipe of constant diameter and wall roughness, the slope $(\Delta h_L/\Delta L)$ of the EGL and the HGL will be constant. For example, see Fig. 7.11.

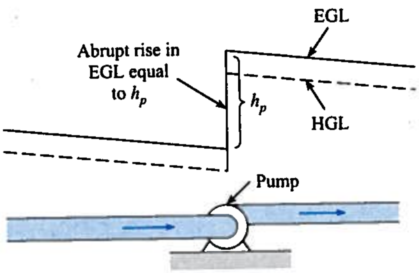


FIGURE 7.12
Rise in EGL and HGL due to pump.

4. Locate the HGL below the EGL by a distance of the velocity head $(\alpha V^2/2g)$.
5. Height of the EGL decreases in the flow direction unless a pump is present.
6. A turbine causes an abrupt drop in the EGL and HGL by removing energy from the flow. For example, see Fig. 7.13.

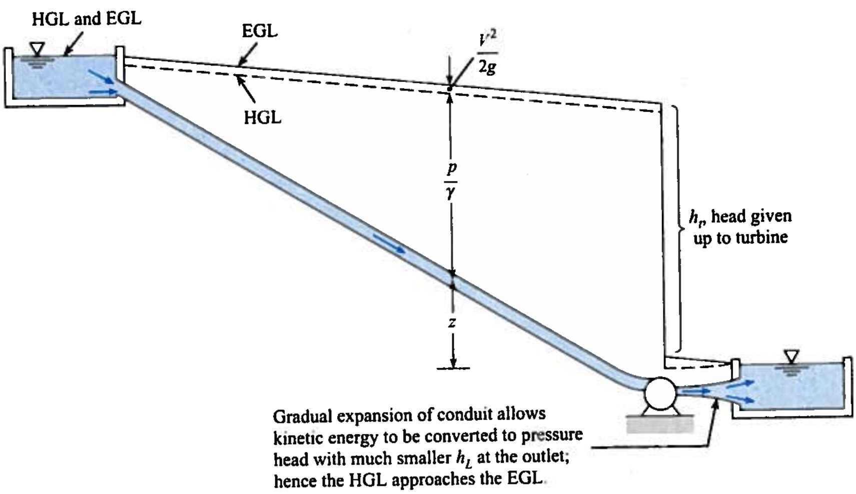


FIGURE 7.13
Drop in EGL and HGL due to turbine.

7. Power generated by a turbine can be increased by using a gradual expansion at the turbine outlet. As shown in Fig. 7.13, the expansion converts kinetic energy to pressure. If the outlet to a reservoir is an abrupt expansion, as in Fig. 7.15, this kinetic energy is lost.
8. When a pipe discharges into the atmosphere the HGL is coincident with the system because $p/\gamma = 0$ at these points. For example, in Figures 7.14 and 7.16, the HGL in the liquid jet is drawn through the centerline of the jet.

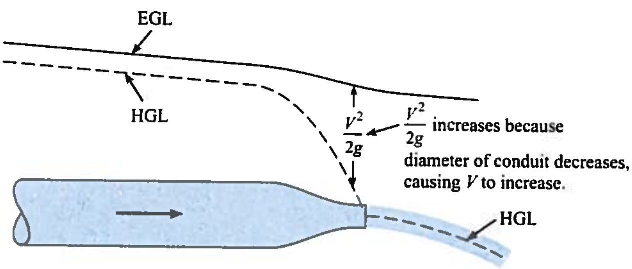


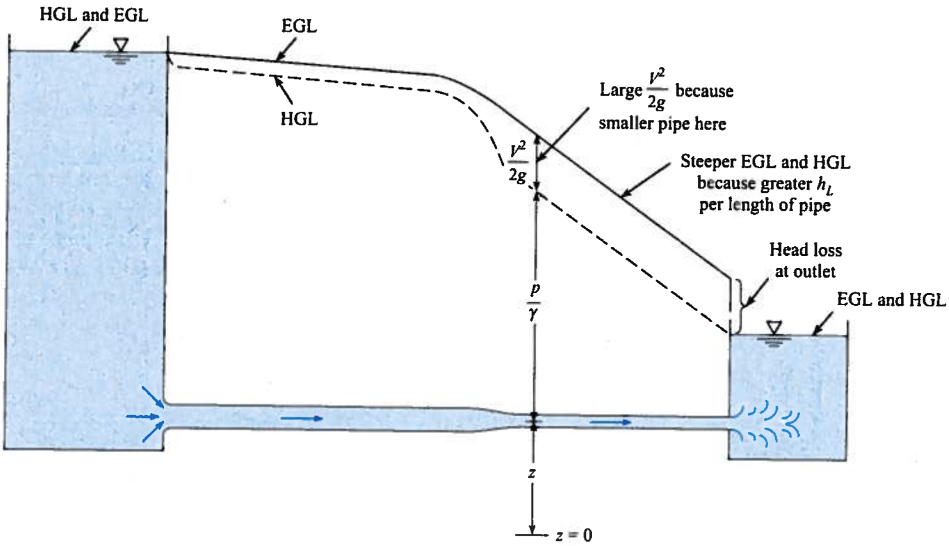
FIGURE 7.14

Change in HGL and EGL due to flow through a nozzle.

9. When a flow passage changes diameter, the distance between the EGL and the HGL will change (see Fig. 7.14 and Fig. 7.15) because velocity changes. In addition, the slope on the EGL will change because the head loss per length will be larger in the conduit with the larger velocity (see Fig. 7.15).

FIGURE 7.15

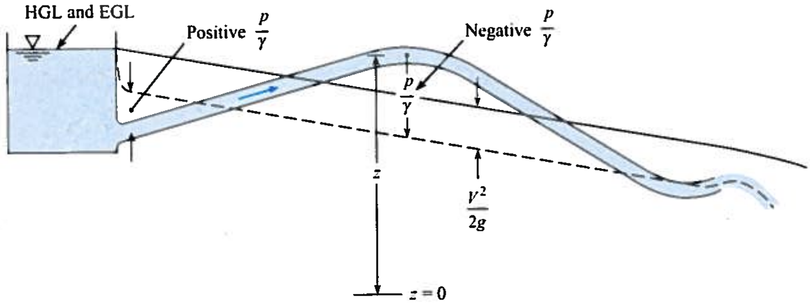
Change in EGL and HGL due to change in diameter of pipe.



10. If the HGL falls below the pipe, then p/γ is negative, indicating subatmospheric pressure (see Fig. 7.16) and a potential location of cavitation.

FIGURE 7.16

Subatmospheric pressure when pipe is above HGL.



The recommended procedure for drawing an EGL and HGL is shown in Example 7.6. Notice how the tips from pp. 274–275 are applied.

EXAMPLE 7.6

Sketching the EGL and HGL for a Piping System

Problem Statement

A pump draws water (50°F) from a reservoir, where the water-surface elevation is 520 ft, and forces the water through a pipe 5000 ft long and 1 ft in diameter. This pipe then discharges the water into a reservoir with water-surface elevation of 620 ft. The flow rate is 7.85 cfs, and the head loss in the pipe is given by

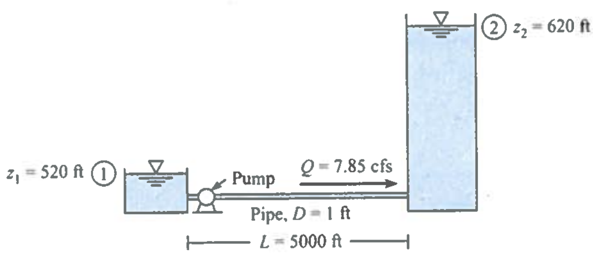
$$h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

Determine the head supplied by the pump, h_p, and the power supplied to the flow, and draw the HGL and EGL for the system. Assume that the pipe is horizontal and is 510 ft in elevation.

Define the Situation

Water is pumped from a lower reservoir to a higher reservoir.

- $h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$
- Water (50°F, 1 atm, Table A.5): $\gamma = 62.4$ lbf/ft³.



State the Goals

1. h_p(ft) ⬅ pump head
2. P(hp) ⬅ power supplied by the pump
3. Draw the HGL and the EGL.

Generate Ideas and Make a Plan

Because pump head and power are goals, apply the energy equation and the power equation, respectively. The step-by-step plan is

1. Locate section 1 and section 2 at top of the reservoirs (see sketch). Then, apply the energy equation (7.29).
2. Calculate terms in the energy equation.
3. Calculate power using the power equation (7.30a).
4. Draw the HGL and EGL.

Take Action (Execute the Plan)

1. Energy equation (general form)

$$\frac{p_1}{\gamma} + \alpha_1\frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$$

- Velocity heads are negligible because $V_1 \approx 0$ and $V_2 \approx 0$.
- Pressure heads are zero because $p_1 = p_2 = 0$ gage.
- $h_t = 0$ because there are no turbines in the system.

$$h_p = (z_2 - z_1) + h_L$$

Interpretation: Head supplied by the pump provides the energy to lift the fluid to a higher elevation plus the energy to overcome head loss.

2. Calculations.
   - Calculate V using the flow rate equation.

$$V = \frac{Q}{A} = \frac{7.85 \text{ ft}^3/\text{s}}{(\pi/4)(1 \text{ ft})^2} = 10 \text{ ft/s}$$

   - Calculate head loss.

$$h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.01\left(\frac{5000 \text{ ft}}{1.0 \text{ ft}}\right)\left(\frac{(10 \text{ ft/s})^2}{2 \times (32.2 \text{ ft/s}^2)}\right)$$
$$= 77.6 \text{ ft}$$

   - Calculate h_p.

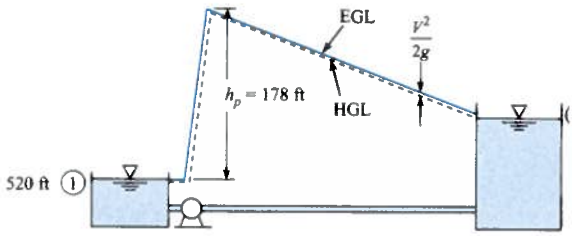
$$h_p = (z_2 - z_1) + h_L = (620 \text{ ft} - 520 \text{ ft}) + 77.6 \text{ ft} = \boxed{178 \text{ ft}}$$

3. Power

$$\dot{W}_p = \gamma Q h_p = \left(\frac{62.4 \text{ lbf}}{\text{ft}^3}\right)\left(\frac{7.85 \text{ ft}^3}{\text{s}}\right)(178 \text{ ft})\left(\frac{\text{hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}}\right)$$
$$= \boxed{159 \text{ hp}}$$

4. HGL and EGL
   - From Tip 1 on p. 274, locate the HGL and EGL along the reservoir surfaces.
   - From Tip 2, sketch in a head rise of 178 ft correspondi to the pump.
   - From Tip 3, sketch the EGL from the pump outlet to tl reservoir surface. Use the fact that the head loss is 77.6 Also, sketch EGL from the reservoir on the left to the pump inlet. Show a small head loss.
   - From Tip 4, sketch the HGL below the EGL by a distar of $V^2/2g \approx 1.6$ ft.
   - From Tip 5, check the sketches to ensure that EGL and HGL are decreasing in the direction of flow (except at pump).

HGL (dashed black line) and EGL (solid blue line)



7.9 Summarizing Key Knowledge

Foundational Concepts

- *Energy* is a property of a system that allows the system to do work on its surroundings. Energy can be classified into five categories: mechanical energy, thermal energy, chemical energy, electrical energy, and nuclear energy.
- *Mechanical work* is done by a force that acts through a distance. A more general definition of work is that *work* is an interaction of a system with the surroundings in such a way that the sole effect on the surroundings could have been the lifting of a weight.
- *Power* is the ratio of work to time or energy to time at an instant in time. Note the key difference between energy and power
  - Energy (and work) describe an *amount* (e.g., how many joules).
  - Power describes an *amount/time* or *rate* (e.g., how many joules/second or watts).
- Machines can be classified into two categories:
  - A *pump* is any machine that adds energy to a flowing fluid.
  - A *turbine* is any machine that extracts energy from a flowing fluid.

Conservation of Energy and Derivation of the Energy Equation

- The law of conservation of energy asserts that work and energy balance.
  - The balance for a closed system is (Energy changes of the system) = (Energy increases due to heat transfer) − (Energy decreases due to the system doing work).

- The balance for a CV is (Energy changes in the CV) = (Energy increases in the CV du to heat transfer) − (Energy out of CV via work done on the surrounding) + (Energy transported into the CV by fluid flow)

- Work can be classified into two categories
  - *Flow work* is work that is done by the pressure force in a flowing fluid
  - *Shaft work* is any work that is not flow work.

The Energy Equation

- The energy equation is the law of conservation of energy simplified so that it applies to common situations that occur in fluid mechanics. Some of the most important assumptions are steady state, one inflow and one outflow port to the CV, constant density, and all thermal energy terms (except for head loss) are neglected.
- The energy equation describes an energy balance for a control volume (CV).

$$(\text{energy into CV}) = (\text{energy out of CV})$$

$$(\text{energy into CV by flow and pumps}) = (\text{energy out by flow, turbines, and head loss})$$

- The energy equation, using math symbols, is

$$\left(\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2\right) + h_t + h_L$$

$$\begin{pmatrix}\text{pressure head}\\ \text{velocity head}\\ \text{elevation head}\end{pmatrix}_1 + \begin{pmatrix}\text{pump}\\ \text{head}\end{pmatrix} = \begin{pmatrix}\text{pressure head}\\ \text{velocity head}\\ \text{elevation head}\end{pmatrix}_2 + \begin{pmatrix}\text{turbine}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{head}\\ \text{loss}\end{pmatrix}$$

- Regarding head
  - Head can be thought of as the ratio of energy to weight for a fluid particle.
  - Head can also describe the energy per time that is passing across a section because hea and power are related by $P = \dot{m}gh$
- Regarding head loss (h_L)
  - Head loss represents an irreversible conversion of mechanical energy to thermal energ through the action of viscosity.
  - Head loss is always positive and is analogous to frictional heating.
  - Head loss for a sudden expansion is given by

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

- Regarding the kinetic energy correction factor α
  - This factor accounts for the distribution of kinetic energy in a flowing fluid. It is define as the ratio of (actual KE/time that crosses a surface) to (KE/time that would cross if tl velocity was uniform).
  - For most situations, engineers set $\alpha = 1$. If the flow is known to be fully developed an laminar, then engineers use $\alpha = 2$. In other cases, one can go back to the mathematica definition and calculate a value of α.

Power and Mechanical Efficiency

- Mechanical efficiency is the ratio of (power output) to (power input) for a machine or system.
- There are several equations that engineers use to calculate power.
  - For translational motion such as a car or an airplane $P = FV$
  - For rotational motion such as the shaft on a pump $P = T\omega$
  - For the pump, the power added to the flow is: $P = \gamma Q h_p$
  - For a turbine, the power extracted from the flow is $P = \gamma Q h_t$

The HGL and EGL

- The hydraulic grade line (HGL) is a profile of the piezometric head, $p/\gamma + z$, along a pipe.
- The energy grade line (EGL) is a profile of the total head, $V^2/2g + p/\gamma + z$, along a pipe.
- If the hydraulic grade line falls below the elevation of a pipe, subatmospheric pressure exists in the pipe at that location, giving rise to the possibility of cavitation.

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PROBLEMS

WileyPLUS: Problem available in *WileyPLUS* at instructor's discretion.

GO: Guided Online (GO) Problem, available in *WileyPLUS* instructor's discretion.

Energy Concepts (§7.1)

7.1 (PLUS) Fill in the blank. Show your work.

a. 1000 J = ________ Cal.

b. ________ ft-lbf = energy to lift a 10 N weight through an elevation difference of 125 m.

c. 12000 Btu = ________ kWh.

d. 32 ft-lbf/s = ________ hp.

e. $[E]$ = [energy] = ________

7.2 From the list below, select one topic that is interesting to you. Then, use references such as the Internet to research your topic and prepare one page of documentation that you could use to present your topic to your peers.

a. Explain how hydroelectric power is produced.

b. Explain how a Kaplan turbine works, how a Francis turbine works, and the differences between these two types of turbines.

c. Explain how a horizontal-axis wind turbine is used tc produce electrical power.

d. Explain how a steam turbine is used to produce elect power.

7.3 (PLUS) Using Section 7.1 and other resources, answer the following questions. Strive for depth, clarity, and accurac Also, strive for effective use of sketches, words, and equations.

a. What are the common forms of energy? Which of the forms are relevant to fluid mechanics?

b. What is work? Describe three example of work that a relevant to fluid mechanics.

c. What are the most common units of power?

d. List three significant differences between power and energy.

7.4 WILEY PLUS Apply the grid method to each situation.

a. Calculate the energy in joules used by a 1 hp pump that is operating for 6 hours. Also, calculate the cost of electricity for this time period. Assume that electricity costs \$0.15 per kW-hr.

b. A motor is being to used to turn the shaft of a centrifugal pump. Apply Eq. (7.3b) on p. 255 of §7.2 to calculate the power in watts corresponding to a torque of 100 lbf-in and a rotation speed of 850 rpm.

c. A turbine produces a power of 7500 ft-lbf/s. Calculate the power in hp and in watts.

7.5 WILEY PLUS Energy (select all that are correct):

a. has same units as work

b. has same units as power

c. has same units work/time

d. can have units of Joule

e. can have units of Watt

f. can have units of ft-lbf

g. can have units of calories

7.6 WILEY PLUS Power (select all that are correct)

a. has same units as energy

b. has same units as energy/time

c. has same units as work/time

d. can have units of Joule

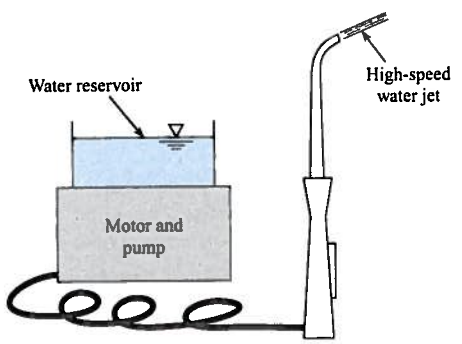
e. can have units of Watt

f. can have units of horsepower

g. can have units of ft-lbf

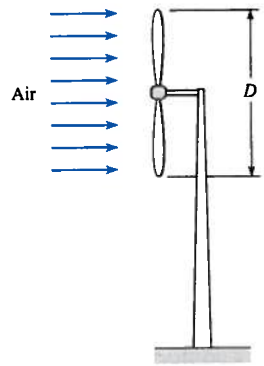
7.7 Estimate the power required to spray water out of the spray bottle that is pictured in Fig. 7.2a on p. 254 of §7.2. *Hint:* Make appropriate assumptions about the number of sprays per unit time and the force exerted by the finger.

7.8 WILEY PLUS The sketch shows a common consumer product called the Water Pik. This device uses a motor to drive a piston pump that produces a jet of water ($d = 1$ mm, $T = 10°C$) with a speed of 27 m/s. Estimate the minimum electrical power in watts that is required by the device. *Hints:* (a) Assume that the power is used only to produce the kinetic energy of the water in the jet; and (b) in a time interval Δt, the amount of mass that flows out the nozzle is Δm, and the corresponding amount of kinetic energy is $(\Delta m V^2/2)$.



PROBLEM 7.8

7.9 An engineer is considering the development of a small wind turbine ($D = 1.25$ m) for home applications. The design wind speed is 15 mph at $T = 10°C$ and $p = 0.9$ bar. The efficiency of the turbine is $\eta = 20\%$, meaning that 20% of the kinetic energy in the wind can be extracted. Estimate the power in watts that can be produced by the turbine. *Hint:* In a time interval Δt, the amount of mass that flows through the rotor is $\Delta m = \dot{m}\Delta t$, and the corresponding amount of kinetic energy in this flow is $(\Delta m V^2/2)$.



PROBLEM 7.9

Conservation of Energy (§7.2)

7.10 WILEY PLUS The first law of thermodynamics for a closed system can be characterized in words as

a. (change in energy in a system) = (thermal energy in) - (work done on surroundings)

b. (change in energy in a system) = (thermal energy out) - (work done by surroundings)

c. either of the above

7.11 WILEY PLUS The application of Reynolds transport theorem to the first law of thermodynamics (select all that are correct)

a. refers to the increase of energy stored in a closed system

b. extends the applicability of the first law from a closed system to an open system (control volume)

c. refers only to heat transfer, and not to work

The Kinetic Energy Correction Factor (§7.3)

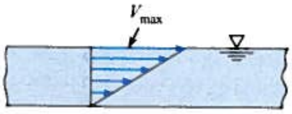
7.12 WILEY PLUS Using Section 7.3 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

a. What is the kinetic-energy correction factor? Why do engineers use this term?

b. What is the meaning of each variable (α, A, V, $\overline{V}$) that appears in Eq. (7.21) on p. 260 of §7.3?

c. What values of α are commonly used?

7.13 For this hypothetical velocity distribution in a wide rectangular channel, evaluate the kinetic-energy correction factor α.

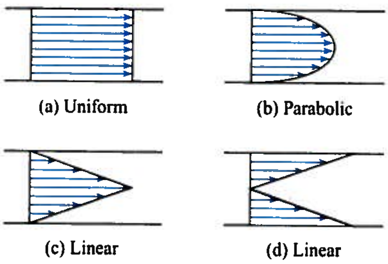


PROBLEM 7.13

7.14 PLUS For these velocity distributions in a round pipe, indicate whether the kinetic-energy correction factor α is greater than, equal to, or less than unity.

7.15 Calculate α for case (*c*).

7.16 Calculate α for case (*d*).



PROBLEMS 7.14, 7.15, 7.16

7.17 An approximate equation for the velocity distribution in a pipe with turbulent flow is

$$\frac{V}{V_{max}} = \left(\frac{y}{r_0}\right)^n$$

where V_{max} is the centerline velocity, y is the distance from the wall of the pipe, r_0 is the radius of the pipe, and n is an exponent that depends on the Reynolds number and varies between 1/6 and 1/8 for most applications. Derive a formula for α as a function of n. What is α if $n = 1/7$?

7.18 An approximate equation for the velocity distribution in a rectangular channel with turbulent flow is

$$\frac{u}{u_{max}} = \left(\frac{y}{d}\right)^n$$

where u_{max} is the velocity at the surface, y is the distance from the floor of the channel, d is the depth of flow, and n is an exponent that varies from about 1/6 to 1/8 depending on the Reynolds number. Derive a formula for α as a function of n. What is the value of α for $n = 1/7$?

7.19 The following data were taken for turbulent flow in a circular pipe with a radius of 3.5 cm. Evaluate the kinetic en correction factor. The velocity at the pipe wall is zero.

r (cm)	V (m/s)	r (cm)	V (m/s)
0.0	32.5	2.8	22.03
0.5	32.44	2.9	21.24
1.0	32.27	3.0	20.49
1.5	31.22	3.1	19.6
2.0	28.21	3.2	18.69
2.25	26.51	3.25	18.16
2.5	24.38	3.3	17.54
2.6	23.7	3.35	17.02
2.7	22.88	3.4	16.14

The Energy Equation (§7.3)

7.20 Using Section 7.3 and other resources, answer the questions below. Strive for depth, clarity, and accuracy. Also, strive for effective use of sketches, words, and equations.

a. What is conceptual meaning of the first law of thermodynamics for a system?

b. What is flow work? How is the equation for flow wo (Eq. 7.16) on p. 259 of §7.3 derived?

c. What is shaft work? How is shaft work different thar flow work?

7.21 Using Section 7.3 and other resources, answer the questions below. Strive for depth, clarity, and accuracy. Also, strive for effective use of sketches, words, and equations.

a. What is head? How is head related to energy? To pow

b. What is head of a turbine?

c. How is head of a pump related to power? To energy?

d. What is head loss?

7.22 PLUS **part (a) only** Using Sections 7.3 and 7.7 and usin other resources, answer the following questions. Strive for de clarity, and accuracy. Also, strive for effective use of sketches, words and equations.

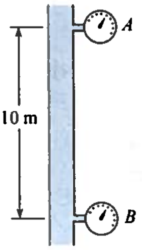
a. What are the five main terms in the energy equation (7 on p. 262 of §7.3? What does each term mean?

b. How are terms in the energy equation related to ene To power?

c. What assumptions are required for using the energy equation (7.29) on p. 262 of §7.3?

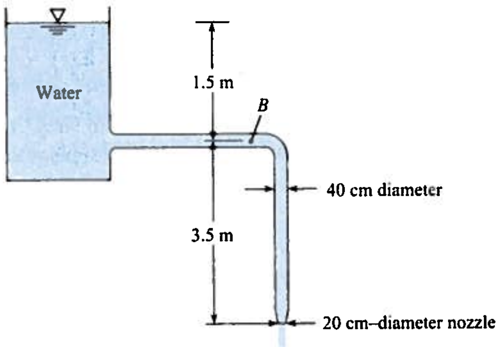
7.23 Using the energy equation (7.29 on p. 262 of §7.3), prove that fluid in a pipe will flow from a location with high piezom head to a location with low piezometric head. Assume there ar no pumps or turbines and that the pipe has a constant diamete

7.24 PLUS Water flows at a steady rate in this vertical pipe. Th pressure at A is 10 kPa, and at B it is 98.1 kPa. Then the flow i the pipe is (a) upward, (b) downward, or (c) no flow. (*Hint:* Se problem 7.23.)



PROBLEM 7.24

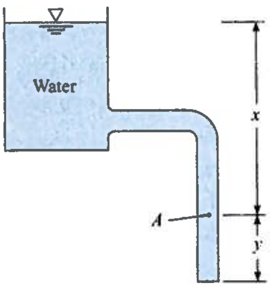
7.25 Determine the discharge in the pipe and the pressure at point *B*. Neglect head losses. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.25

7.26 A pipe drains a tank as shown. If $x = 14$ ft, $y = 4$ ft, and head losses are neglected, what is the pressure at point *A* and what is the velocity at the exit? Assume $\alpha = 1.0$ at all locations.

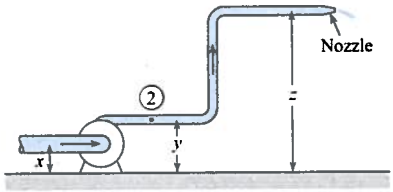
7.27 A pipe drains a tank as shown. If $x = 6$ m, $y = 4$ m, and head losses are neglected, what is the pressure at point *A* and what is the velocity at the exit? Assume $\alpha = 1.0$ at all locations.



PROBLEMS 7.26, 7.27

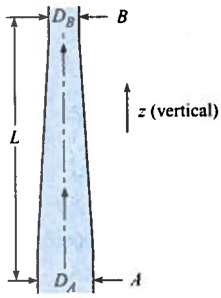
7.28 For this system, the discharge of water is 3.5 ft^3/s, $x = 1.0$ m, $y = 1.5$ m, $z = 6.0$ m, and the pipe diameter is 30 cm. Assuming a head loss of 0.5 m, what is the pressure head at point 2 if the jet from the nozzle is 10 cm in diameter? Assume $\alpha = 1.0$ at all locations.

7.29 For this diagram of an industrial pressure washer system, $x = 1$ ft, $y = 3$ ft, $z = 10$ ft, $Q = 3.5$ ft^3/s, and the hose diameter is 4 in. Assuming a head loss of 1 ft is derived over the distance from point 2 to the jet, what is the pressure at point 2 if the jet from the nozzle is 1-in in diameter? Assume $\alpha = 1.0$ throughou



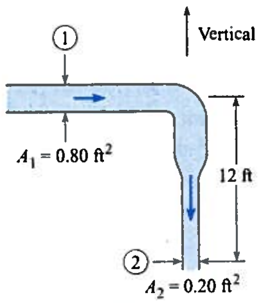
PROBLEMS 7.28, 7.29

7.30 For this refinery pipe, $D_A = 20$ cm, $D_B = 14$ cm, and $L = 1$ m. If crude oil ($S = 0.90$) is flowing at a rate of 0.05 m^3/s, determine the difference in pressure between sections *A* and *B*. Neglect head losses.



PROBLEM 7.30

7.31 Gasoline having a specific gravity of 0.8 is flowing in the pipe shown at a rate of 5 cfs. What is the pressure at section 2 when the pressure at section 1 is 18 psig and the head loss is 6 ft between the two sections? Assume $\alpha = 1.0$ at all locations.

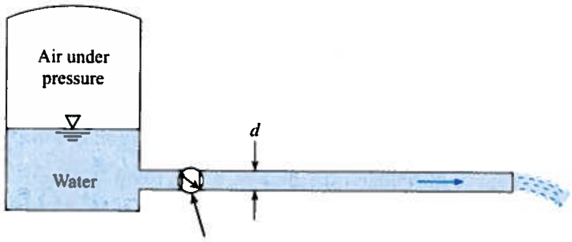


PROBLEM 7.31

7.32 Water flows from a pressurized tank as shown. The pressure in the tank above the water surface is 100 kPa gage, and the water surface level is 8 m above the outlet. The water exit velocity is 10 m/s. The head loss in the system varies as $h_L = K_L V^2/2g$, where K_L is the minor-loss coefficient. Find the value for K_L. Assume $\alpha = 1.0$ at all locations.

7.33 PLUS A reservoir with water is pressurized as shown. The pipe diameter is 1 in. The head loss in the system is given by $h_L = 5V^2/2g$. The height between the water surface and the pipe outlet is 10 ft. A discharge of 0.10 ft^3/s is needed. What must the pressure in the tank be to achieve such a flow rate? Assume $\alpha = 1.0$ at all locations.

7.34 In the figure shown, suppose that the reservoir is open to the atmosphere at the top. The valve is used to control the flow rate from the reservoir. The head loss across the valve is given as $h_L = 4V^2/2g$, where V is the velocity in the pipe. The cross-sectional area of the pipe is 8 cm^2. The head loss due to friction in the pipe is negligible. The elevation of the water level in the reservoir above the pipe outlet is 9 m. Find the discharge in the pipe. Assume $\alpha = 1.0$ at all locations.



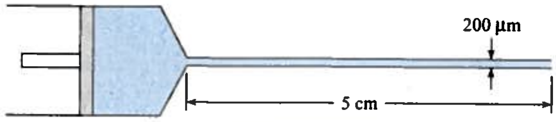
PROBLEMS 7.32, 7.33, 7.34

7.35 PLUS A minor artery in the human arm, diameter $D =$ 3 mm, tapers gradually over a distance of 20 cm to a diameter of $d = 1.6$ mm. The blood pressure at D is 110 mm Hg, and at d is 85 mm Hg. What is the head loss (m) that occurs over this 20-cm distance if the blood ($S = 1.06$) is moving with a flowrate of 300 milliliters/min, and the arm is being held horizontally? Idealize the flow in the artery as steady, the fluid as Newtonian, and the walls of the artery as rigid.

7.36 PLUS As shown, a microchannel is being designed to transfer fluid in a MEMS (microelectrical mechanical system) application. The channel is 200 micrometers in diameter and is 5 cm long. Ethyl alcohol is driven through the system at the rate of 0.1 microliters/s (μL/s) with a syringe pump, which is essentially a moving piston. The pressure at the exit of the channel is atmospheric. The flow is laminar, so $\alpha = 2$. The head loss in the channel is given by

$$h_L = \frac{32\mu LV}{\gamma D^2}$$

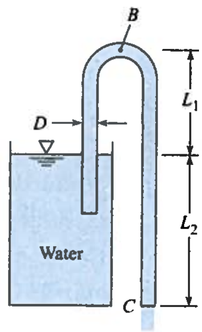
where L is the channel length, D the diameter, V the mean velocity, μ the viscosity of the fluid, and γ the specific weight of the fluid. Find the pressure in the syringe pump. The velocity head associated with the motion of the piston in the syringe pump is negligible.



PROBLEM 7.36

7.37 Firefighting equipment requires that the exit velocity of the firehose be 30 m/s at an elevation of 45 m above the hyd The nozzle at the end of the hose has a contraction ratio of 4 ($A_e/A_{hose} = 1/4$). The head loss in the hose is $8V^2/2g$, where is the velocity in the hose. What must the pressure be at the hydrant to meet this requirement? The pipe supplying the hydrant is much larger than the firehose.

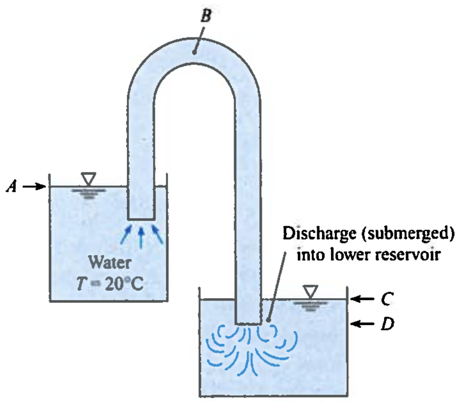
7.38 GO The discharge in the siphon is 2.80 cfs, $D = 8$ in., $L_1 = 3$ ft, and $L_2 = 3$ ft. Determine the head loss between the reservoir surface and point C. Determine the pressure at poin if three-quarters of the head loss (as found above) occurs between the reservoir surface and point B. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.38

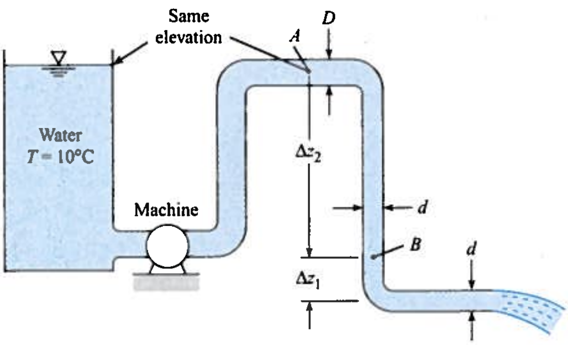
7.39 GO For this siphon the elevations at A, B, C, and D are 30 m, 32 m, 27 m, and 26 m, respectively. The head loss between the inlet and point B is three-quarters of the veloci head, and the head loss in the pipe itself between point B an the end of the pipe is one-quarter of the velocity head. For t conditions, what is the discharge and what is the pressure at point B? The pipe diameter = 25 cm. Assume $\alpha = 1.0$ at all locations.

7.40 PLUS For this system, point B is 10 m above the bottom of the upper reservoir. The head loss from A to B is $1.1V^2/2g$, and the pipe area is 8×10^{-4} m^2. Assume a constant discharge of 8×10^{-4} m^3/s. For these conditions, what will be the depth of water in the upper reservoir for whic cavitation will begin at point B? Vapor pressure = 1.23 kI and atmospheric pressure = 100 kPa. Assume $\alpha = 1.0$ at locations.



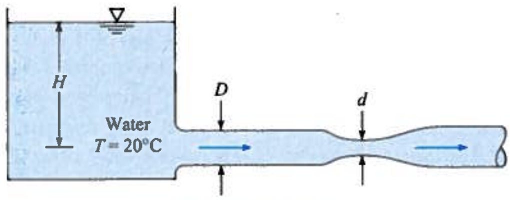
PROBLEMS 7.39, 7.40

7.41 In this system, $d = 6$ in., $D = 12$ in., $\Delta z_1 = 6$ ft, and $\Delta z_2 = 12$ ft. The discharge of water in the system is 10 cfs. Is the machine a pump or a turbine? What are the pressures at points A and B? Neglect head losses. Assume $\alpha = 1.0$ at all locations.



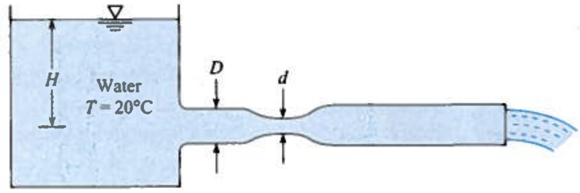
PROBLEM 7.41

7.42 The pipe diameter D is 30 cm, d is 15 cm, and the atmospheric pressure is 100 kPa. What is the maximum allowable discharge before cavitation occurs at the throat of the venturi meter if $H = 5$ m? Assume $\alpha = 1.0$ at all locations.



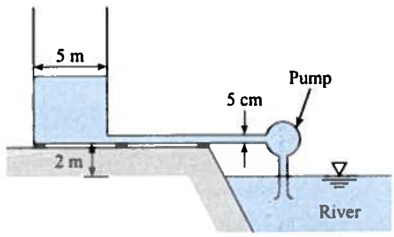
PROBLEM 7.42

7.43 In this system $d = 15$ cm, $D = 35$ cm, and the head loss from the venturi meter to the end of the pipe is given by $h_L = 1.5\ V^2/2g$, where V is the velocity in the pipe. Neglecting all other head losses, determine what head H will first initiate cavitation if the atmospheric pressure is 100 kPa absolute. What will be the discharge at incipient cavitation? Assume $\alpha = 1.0$ at all locations.



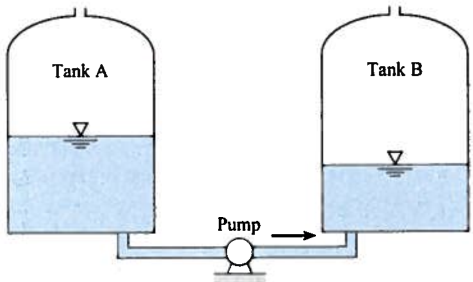
PROBLEM 7.43

7.44 A pump is used to fill a tank 5 m in diameter from a river as shown. The water surface in the river is 2 m below the bottom of the tank. The pipe diameter is 5 cm, and the head loss in the pipe is given by $h_L = 10\ V^2/2g$, where V is the mean velocity in the pipe. The flow in the pipe is turbulent, so $\alpha = 1$. The head provided by the pump varies with discharge through the pump as $h_p = 20 - 4 \times 10^4\ Q^2$, where the discharge is given in cubic meters per second (m^3/s) and h_p is in meters. How long will it take to fill the tank to a depth of 10 m?



PROBLEM 7.44

7.45 A pump is used to transfer SAE-30 oil from tank A to tank B as shown. The tanks have a diameter of 12 m. The initial depth of the oil in tank A is 20 m, and in tank B the depth is 1 m. The pump delivers a constant head of 60 m. The connecting pipe has a diameter of 20 cm, and the head loss due to friction in the pipe is $20\ V^2/2g$. Find the time required to transfer the oil from tank A to B; that is, the time required to fill tank B to 20 m depth.



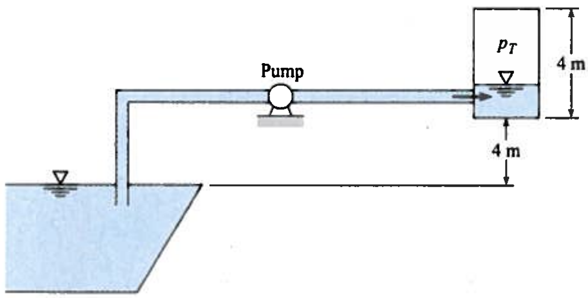
PROBLEM 7.45

7.46 A pump is used to pressurize a tank to 300 kPa abs. The tank has a diameter of 2 m and a height of 4 m. The initial level of water in the tank is 1 m, and the pressure at the water surface is 0 kPa gage. The atmospheric pressure is 100 kPa. The pump operates with a constant head of 50 m. The water is drawn from a source that is 4 m below the tank bottom. The pipe connecting the source and the tank is 4 cm in diameter and the head loss, including the expansion loss at the tank, is 10 $V^2/2g$. The flow is turbulent.

Assume the compression of the air in the tank takes place isothermally, so the tank pressure is given by

$$p_T = \frac{3}{4 - z_t} p_0$$

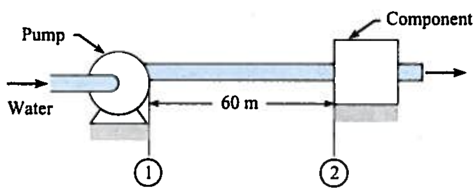
where z_t is the depth of fluid in the tank in meters. Write a computer program that will show how the pressure varies in the tank with time, and find the time to pressurize the tank to 300 kPa abs.



PROBLEM 7.46

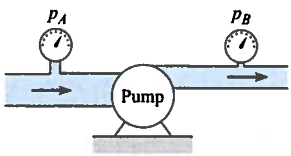
The Power Equation (§7.4)

7.47 As shown, water at 15°C is flowing in a 15-cm-diameter by 60-m-long run of pipe that is situated horizontally. The mean velocity is 2 m/s, and the head loss is 2 m. Determine the pressure drop and the required pumping power to overcome head loss in the pipe.



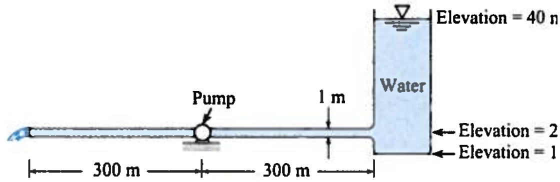
PROBLEM 7.47

7.48 The pump shown in the figure supplies energy to the flow such that the upstream pressure (12 in. pipe) is 5 psi and the downstream pressure (6 in. pipe) is 55 psi when the flow of water is 3.0 cfs. What horsepower is delivered by the pump to the flow? Assume α = 1.0 at all locations.



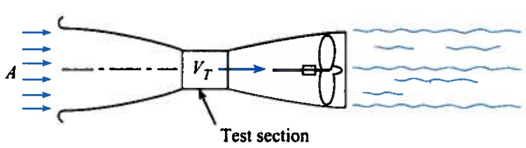
PROBLEM 7.48

7.49 A water discharge of 8 m³/s is to flow through thi horizontal pipe, which is 1 m in diameter. If the head loss is given as 7 $V^2/2g$ (V is velocity in the pipe), how much powe will have to be supplied to the flow by the pump to produce this discharge? Assume α = 1.0 at all locations.



PROBLEM 7.49

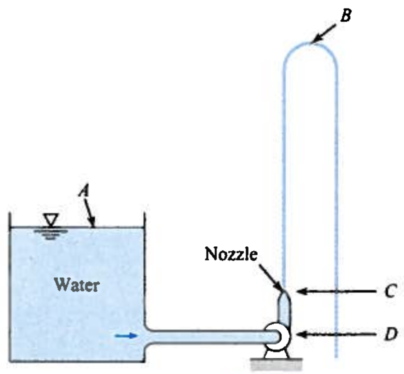
7.50 An engineer is designing a subsonic wind tunnel. The t section is to have a cross-sectional area of 4 m² and an airspe of 60 m/s. The air density is 1.2 kg/m³. The area of the tunne exit is 10 m². The head loss through the tunnel is given by $h_L = (0.025)(V_T^2/2g)$, where V_T is the airspeed in the test sect Calculate the power needed to operate the wind tunnel. *Hint* Assume negligible energy loss for the flow approaching the tunnel in region A, and assume atmospheric pressure at the outlet section of the tunnel. Assume α = 1.0 at all locations.



PROBLEM 7.50

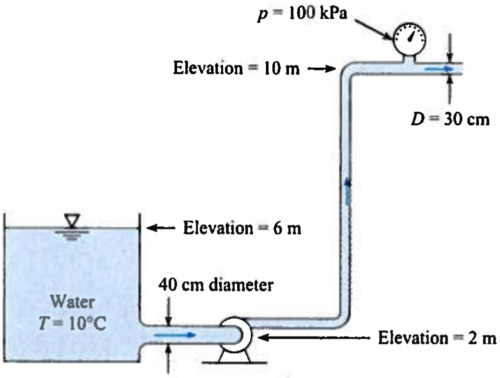
7.51 Neglecting head losses, determine what horsepow the pump must deliver to produce the flow as shown. Here tl elevations at points A, B, C, and D are 117 ft, 154 ft, 110 ft, an 90 ft, respectively. The nozzle area is 0.10 ft².

7.52 Neglecting head losses, determine what power th pump must deliver to produce the flow as shown. Here the elevations at points A, B, C, and D are 40 m, 65 m, 35 m, and 30 m, respectively. The nozzle area is 25 cm².



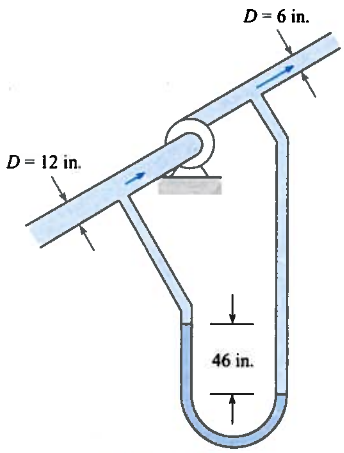
PROBLEMS 7.51, 7.52

7.53 Water (10°C) is flowing at a rate of 0.35 m^3/s, and it is assumed that $h_L = 2\ V^2/2g$ from the reservoir to the gage, where V is the velocity in the 30-cm pipe. What power must the pump supply? Assume $\alpha = 1.0$ at all locations.



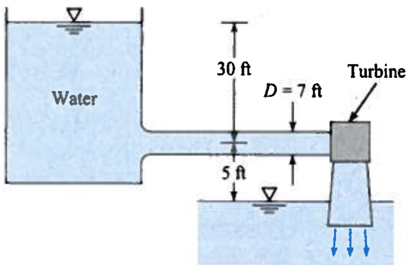
PROBLEM 7.53

7.54 In the pump test shown, the rate of flow is 6 cfs of oil (S = 0.88). Calculate the horsepower that the pump supplies to the oil if there is a differential reading of 46 in. of mercury in the U-tube manometer. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.54

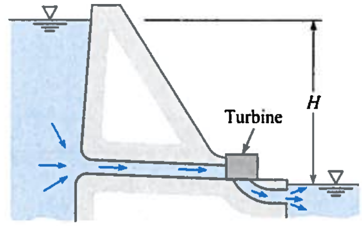
7.55 If the discharge is 500 cfs, what power output may be expected from the turbine? Assume that the turbine efficiency is 90% and that the overall head loss is $1.5\ V^2/2g$,



PROBLEM 7.55

where V is the velocity in the 7 ft penstock Assume $\alpha = 1.0$ at all locations.

7.56 A small-scale hydraulic power system is shown. The elevation difference between the reservoir water surface and the pond water surface downstream of the reservoir, H, is 24 m. The velocity of the water exhausting into the pond is 7 m/s, and the discharge through the system is 4 m^3/s. The head loss due to friction in the penstock (inlet pipe to turbine, under very high pressure) is negligible. Find the power produced by the turbine in kilowatts.



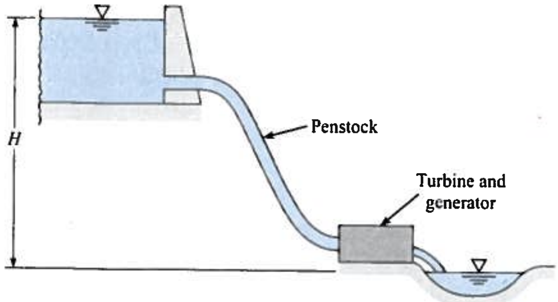
PROBLEM 7.56

Mechanical Efficiency (§7.5)

7.57 A fan produces a pressure rise of 6 mm of water to move air through a hair dryer. The mean velocity of the air at the exit is 10 m/s, and the exit diameter is 44 mm. Estimate the electrical power in watts that needs to be supplied to operate the fan. Assume that the fan/motor combination has an efficiency of 60%.

PROBLEM 7.57 (Photo by Donald Elger)

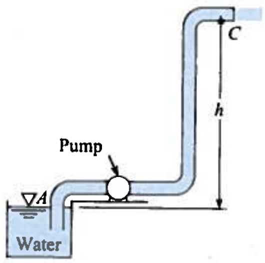
7.58 An engineer is making an estimate for a home owner. This owner has a small stream (Q = 1.4 cfs, T = 40°F) that is located at an elevation H = 34 ft above the owner's residence. The owner is proposing to dam the stream, diverting the flow through a pipe (penstock). This flow will spin a hydraulic turbine, which in turn will drive a generator to produce electrical power. Estimate the maximum power in kilowatts that can be generated if there is no head loss and both the turbine and generator are 100% efficient. Also, estimate the power if the head loss is 5.5 ft, the turbine is 70% efficient, and the generator is 90% efficient.



PROBLEM 7.58

7.59 PLUS The pump shown draws water through an 8 in. suction pipe and discharges it through a 6 in. pipe in which the velocity is 12 ft/s. The 6 in. pipe discharges horizontally into air at *C*. To what height *h* above the water surface at *A* can the water be raised if 17 hp is used by the pump? The pump operates at 60% efficiency and that the head loss in the pipe between *A* and *C* is equal to 2 $V_C^2/2g$. Assume $\alpha = 1.0$ throughout.

7.60 PLUS The pump shown draws water (20°C) through a 20 cm suction pipe and discharges it through a 10 cm pipe in which the velocity is 3 m/s. The 10 cm pipe discharges horizontally into air at point *C*. To what height *h* above the water surface at *A* can the water be raised if 26 kW is delivered to the pump? Assume that the pump operates at 60% efficiency and that the head loss in the pipe between *A* and *C* is equal to 2 $V_C^2/2g$. Assume $\alpha = 1.0$ throughout.



PROBLEMS 7.59, 7.60

7.61 PLUS A pumping system is to be designed to pump crude oil a distance of 1 mile in a 1 foot-diameter pipe at a rate of 3500 gpm. The pressures at the entrance and exit of the pipe are atmospheric, and the exit of the pipe is 200 feet higher than the entrance. The pressure loss in the system due to pipe friction is 60 psi. The specific weight of the oil is 53 lbf/ft^3. Find the power, in horsepower, required for the pump.

Contrasting Bernoulli Eqn. to Energy Eqn. (§7.6)

7.62 How is the energy equation (7.29) on p. 262 of §7.3 similar to the Bernoulli equation? How is it different? Give three important similarities and three important differences.

Transitions (§7.7)

7.63 PLUS What is the head loss at the outlet of the pipe tha discharges water into the reservoir at a rate of 10 cfs if the diameter of the pipe is 12 in.?

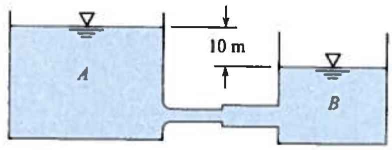
7.64 PLUS What is the head loss at the outlet of the pipe tha discharges water into the reservoir at a rate of 0.5 m^3/s if the diameter of the pipe is 50 cm?

PROBLEMS 7.63, 7.64

7.65 PLUS A 7 cm pipe carries water with a mean velocity of 2 If this pipe abruptly expands to a 15 cm pipe, what will be th head loss due to the abrupt expansion?

7.66 A 6 in. pipe abruptly expands to a 12 in. size. If the discharge of water in the pipes is 5 cfs, what is the head loss to abrupt expansion?

7.67 PLUS Water is draining from tank *A* to tank *B*. The eleva difference between the two tanks is 10 m. The pipe connecti the two tanks has a sudden-expansion section as shown. The cross-sectional area of the pipe from *A* is 8 cm^2, and the area the pipe into *B* is 25 cm^2. Assume the head loss in the system consists only of that due to the sudden-expansion section an the loss due to flow into tank *B*. Find the discharge between two tanks.



PROBLEM 7.67

7.68 GO A 40 cm pipe abruptly expands to a 60 cm size. Th pipes are horizontal, and the discharge of water from the sma size to the larger is 1.0 m^3/s. What horizontal force is require hold the transition in place if the pressure in the 40 cm pipe i 70 kPa gage? Also, what is the head loss? Assume $\alpha = 1.0$ at a locations.

7.69 GO Water ($\gamma = 62.4$ lbf/ft^3) flows through a horizontal constant diameter pipe with a cross-sectional area of 9 in^2. Th velocity in the pipe is 15 ft/s, and the water discharges to the atmosphere. The head loss between the pipe joint and the end o the pipe is 3 ft. Find the force on the joint to hold the pipe. The p is mounted on frictionless rollers. Assume $\alpha = 1.0$ at all locatio

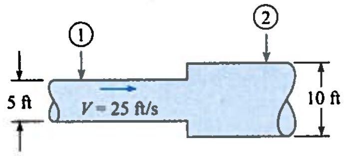
PROBLEM 7.69

7.70 This abrupt expansion is to be used to dissipate the high-energy flow of water in the 5 ft–diameter penstock. Assume $\alpha = 1.0$ at all locations.

a. What power (in horsepower) is lost through the expansion?

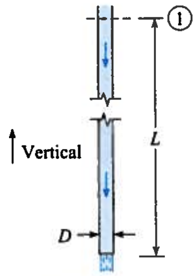
b. If the pressure at section 1 is 5 psig, what is the pressure at section 2?

c. What force is needed to hold the expansion in place?



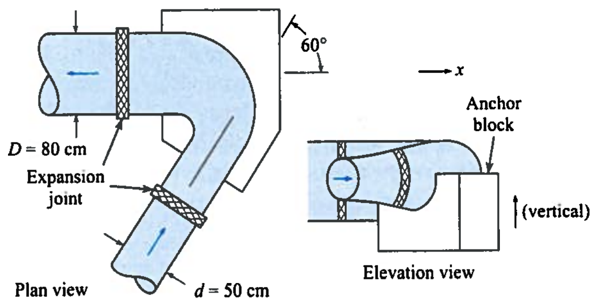
PROBLEM 7.70

7.71 This rough aluminum pipe is 6 in. in diameter. It weighs 1.5 lb per foot of length, and the length L is 50 ft. If the discharge of water is 6 cfs and the head loss due to friction from section 1 to the end of the pipe is 10 ft, what is the longitudinal force transmitted across section 1 through the pipe wall?



PROBLEM 7.71

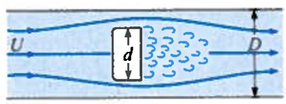
7.72 Water flows in this bend at a rate of 5 m^3/s, and the pressure at the inlet is 650 kPa. If the head loss in the bend is 10 m, what will the pressure be at the outlet of the bend? Also estimate the force of the anchor block on the bend in the x direction required to hold the bend in place. Assume $\alpha = 1.0$ at all locations.



PROBLEMS 7.72, 7.73

7.73 WILEY PLUS In a local water treatment plant, water flows in this bend at a rate of 7 m^3/s, and the pressure at the inlet is 800 kPa. If the head loss in the bend is 13 m, what will the pressure be at the outlet of the bend? Also estimate the force of the anchor block on the bend in the x direction required to hold the bend in place. Assume $\alpha = 1.0$ at all locations.

7.74 Fluid flowing along a pipe of diameter D accelerates aroun a disk of diameter d as shown in the figure. The velocity far upstream of the disk is U, and the fluid density is ρ. Assuming incompressible flow and that the pressure downstream of the disk is the same as that at the plane of separation, develop an expression for the force required to hold the disk in place in terms of U, D, d, and ρ. Using the expression you developed, determine the force when $U = 10$ m/s, $D = 5$ cm, $d = 4$ cm, and $\rho = 1.2$ kg/m^3. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.74

EGL and HGL (§7.8)

7.75 WILEY PLUS **part (b) only** Using Section 7.8 and other resources, answer the following questions. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations i ways that enhance the effectiveness of your communication.

a. What are three important reasons that engineers use th HGL and the EGL?

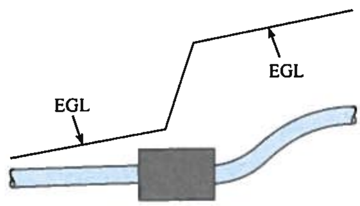
b. What factors influence the magnitude of the HGL? What factors influence the magnitude of the EGL?

c. How are the EGL and HGL related to the piezometer? To the stagnation tube?

d. How is the EGL related to the energy equation?

e. How can you use an HGL or an EGL to determine the direction of flow?

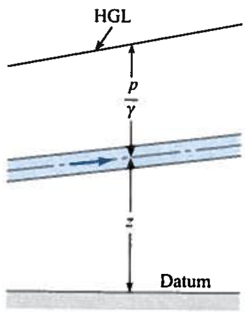
7.76 WILEY PLUS The energy grade line for steady flow in a uniform-diameter pipe is shown. Which of the following could be in the "black box"? (a) a pump, (b) a partially closed valve, (c) an abrup



PROBLEM 7.76

expansion, or (d) a turbine. Choose all valid answer(s) and state your rationale.

7.77 If the pipe shown has constant diameter, is this type of HGL possible? If so, under what additional conditions? If not, why not?



PROBLEM 7.77

7.78 WILEY PLUS For the system shown,

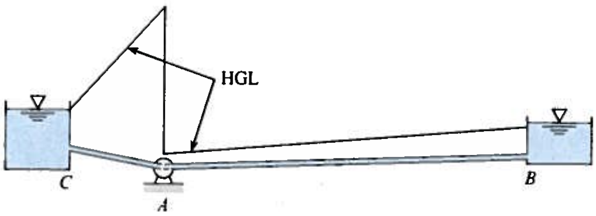
a. What is the flow direction?

b. What kind of machine is at A?

c. Do you think both pipes, AB and CA, are the same diameter?

d. Sketch in the EGL for the system.

e. Is there a vacuum at any point or region of the pipes? If so, identify the location.



PROBLEM 7.78

7.79 The HGL and the EGL are as shown for a certain flow system.

a. Is flow from A to E or from E to A?

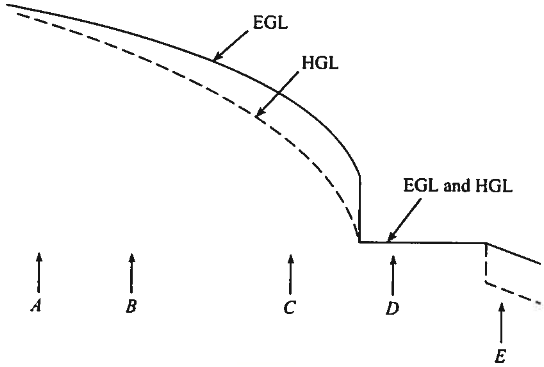
b. Does it appear that a reservoir exists in the system?

c. Does the pipe at E have a uniform or a variable diameter?

d. Is there a pump in the system?

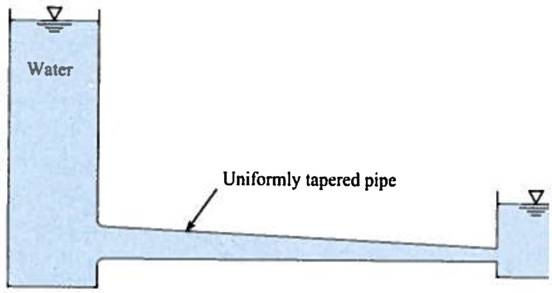
e. Sketch the physical setup that could yield the conditions shown between C and D.

f. Is anything else revealed by the sketch?



PROBLEM 7.79

7.80 Sketch the HGL and the EGL for this conduit, which t uniformly from the left end to the right end.



PROBLEM 7.80

7.81 WILEY PLUS The HGL and the EGL for a pipeline are shown i figure.

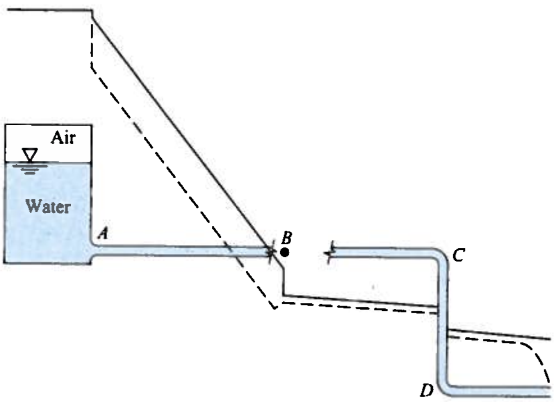
a. Indicate which is the HGL and which is the EGL.

b. Are all pipes the same size? If not, which is the smal

c. Is there any region in the pipes where the pressure i below atmospheric pressure? If so, where?

d. Where is the point of maximum pressure in the syst

e. Where is the point of minimum pressure in the syst



PROBLEM 7.81

f. What do you think is located at the end of the pipe at point *E*?

g. Is the pressure in the air in the tank above or below atmospheric pressure?

h. What do you think is located at point *B*?

7.82 GO Assume that the head loss in the pipe is given by $h_L = 0.014(L/D)(V^2/2g)$, where *L* is the length of pipe and *D* is the pipe diameter. Assume $\alpha = 1.0$ at all locations.

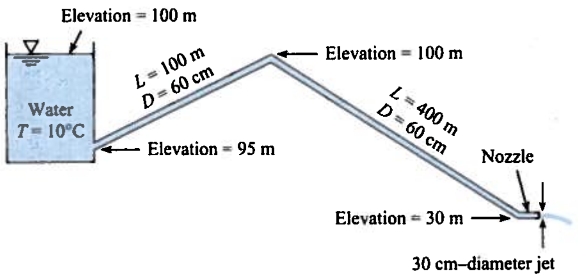
a. Determine the discharge of water through this system.

b. Draw the HGL and the EGL for the system.

c. Locate the point of maximum pressure.

d. Locate the point of minimum pressure.

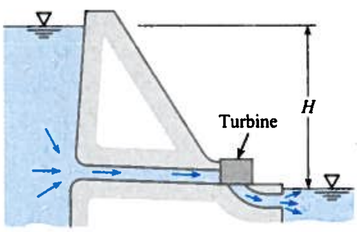
e. Calculate the maximum and minimum pressures in the system.



PROBLEM 7.82

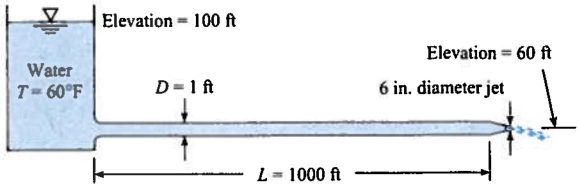
7.83 Sketch the HGL and the EGL for the reservoir and pipe of Example 7.2.

7.84 The discharge of water through this turbine is 1000 cfs. What power is generated if the turbine efficiency is 85% and the total head loss is 4 ft? $H = 100$ ft. Also, carefully sketch the EGL and the HGL.



PROBLEM 7.84

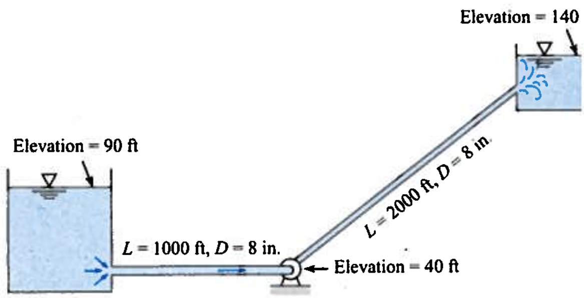
7.85 Water flows from the reservoir through a pipe and then discharges from a nozzle as shown. The head loss in the pipe itself is given as $h_L = 0.025(L/D)(V^2/2g)$, where *L* and *D* are the length and diameter of the pipe and *V* is the velocity in the pipe. What is the discharge of water? Also draw the HGL and EGL for the system. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.85

7.86 WILEY PLUS Refer to Fig. 7.15 on p. 275 of §7.8. Assume that the head loss in the pipes is given by $h_L = 0.02(L/D)(V^2/2g)$, where *V* is the mean velocity in the pipe, *D* is the pipe diameter, and *L* i the pipe length. The water surface elevations of the upper and lower reservoirs are 100 m and 70 m, respectively. The respective dimensions for upstream and downstream pipes are $D_u = 30$ cm, and $L_u = 200$ m, and $D_d = 15$ cm, and $L_d = 100$ m. Determine the discharge of water in the system.

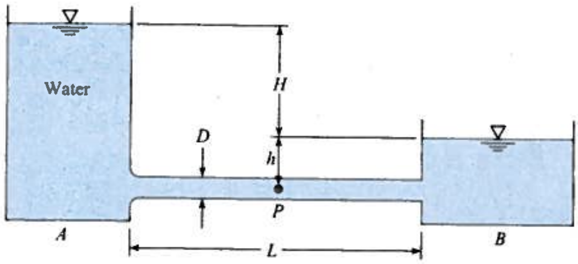
7.87 What horsepower must be supplied to the water to pump 3.0 cfs at 68°F from the lower to the upper reservoir? Assume tha the head loss in the pipes is given by $h_L = 0.018(L/D)(V^2/2g)$, where *L* is the length of the pipe in feet and *D* is the pipe diameter in feet. Sketch the HGL and the EGL.



PROBLEM 7.87

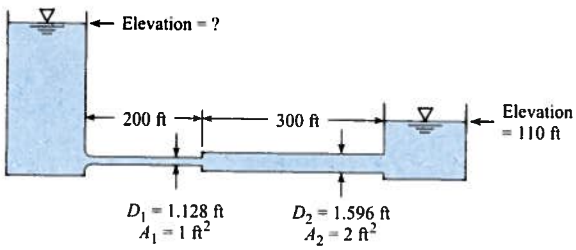
7.88 Water flows from reservoir *A* to reservoir *B*. The water temperature in the system is 10°C, the pipe diameter *D* is 1 m, and the pipe length *L* is 300 m. If $H = 16$ m, $h = 2$ m, and the pipe head loss is given by $h_L = 0.01(L/D)(V^2/2g)$, where *V* is the velocity in the pipe, what will be the discharge in the pipe? In your solution, include the head loss at the pipe outlet, and sketcl the HGL and the EGL. What will be the pressure at point *P* halfway between the two reservoirs? Assume $\alpha = 1.0$ at all locations.

7.89 GO Water flows from reservoir *A* to reservoir *B* in a deser retirement community. The water temperature in the system is 100°F, the pipe diameter *D* is 4 ft, and the pipe length *L* is 200 ft. If $H = 35$ ft, $h = 10$ ft, and the pipe head loss is given by $h_L = 0.01(L/D)(V^2/2g)$, where *V* is the velocity in the pipe, what will be the discharge in the pipe? In your solution, include the head loss at the pipe outlet. What will be the pressure at point *P* halfway between the two reservoirs? Assume $\alpha = 1.0$ at all locations.



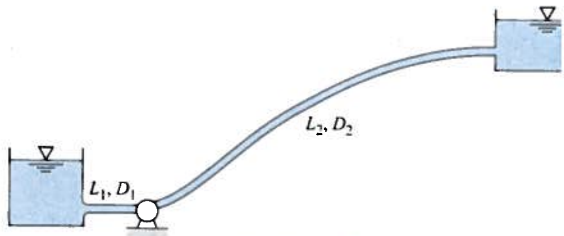
PROBLEMS 7.88, 7.89

7.90 Water flows from the reservoir on the left to the reservoir on the right at a rate of 16 cfs. The formula for the head losses in the pipes is $h_L = 0.02(L/D)(V^2/2g)$. What elevation in the left reservoir is required to produce this flow? Also carefully sketch the HGL and the EGL for the system. *Note:* Assume the head-loss formula can be used for the smaller pipe as well as for the larger pipe. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.90

7.91 What power is required to pump water at a rate of 3 m^3/s from the lower to the upper reservoir? Assume the pipe head loss is given by $h_L = 0.018(L/D)(V^2/2g)$, where L is the length of [cut off] D is the pipe diameter, and V is the velocity in the pipe. The water temperature is 10°C, the water surface elevation in the lower reservoir is 150 m, and the surface elevation in the up[per] reservoir is 250 m. The pump elevation is 100 m, $L_1 = 100$ m, $L_2 = 1000$ m, $D_1 = 1$ m, and $D_2 = 50$ cm. Assume the pump motor efficiency is 74%. In your solution, include the head l[oss] at the pipe outlet and sketch the HGL and the EGL. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.91

7.92 Refer to Fig. 7.16 on p. 276 of §7.8. Assume that the he[ad] loss in the pipe is given by $h_L = 0.02(L/D)(V^2/2g)$, where V i[s the] mean velocity in the pipe, D is the pipe diameter, and L is the pipe length. The elevations of the reservoir water surface, the highest point in the pipe, and the pipe outlet are 250 m, 250 and 210 m, respectively. The pipe diameter is 30 cm, and the length is 200 m. Determine the water discharge in the pipe, a[nd] assuming that the highest point in the pipe is halfway along pipe, determine the pressure in the pipe at that point. Assum[e] $\alpha = 1.0$ at all locations.