BUILDING A SOLID FOUNDATION

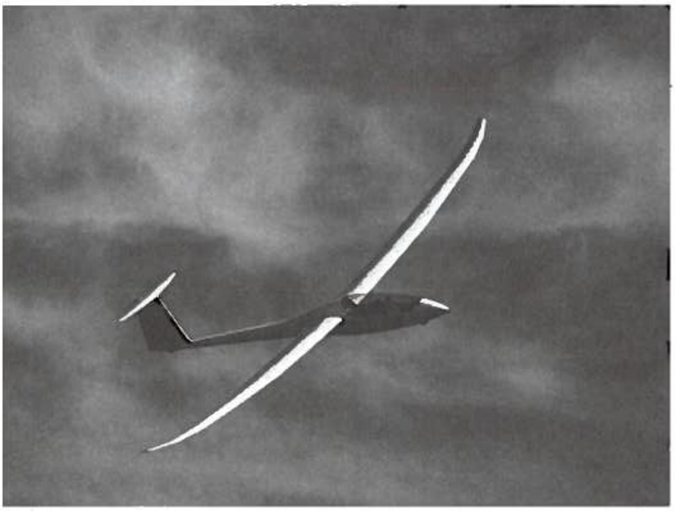


FIGURE 1.1

As engineers, we get to design cool systems like this glider. This is exciting! (© Ben Blankenburg/Corbis RF/Age Fotostock America, Inc.)

Chapter Road Map

The purpose of this chapter is to help students buil foundation for learning fluid mechanics. The chap has three main objectives: to define engineering, describe fluids, and to introduce skills that are use for solving engineering problems.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define engineering, fluid mechanics, and learning. (§1
- Define fluid, liquid, and gas. (§1.2)
- Describe the characteristics of liquids and gases. (§1.2
- Explain macroscopic and microscopic descriptions. (§1
- Explain the continuum assumption. (§1.3)
- Define a fluid particle. (§1.3)
- Describe units and dimensions. (§1.4)
- Determine if a set of units are consistent. (§1.4)
- Apply the grid method to carry and cancel units. (§1.5
- Apply the ideal gas law, or IGL. (§1.6)
- Describe the Wales-Woods model, or WWM. (§1.7)
- Check for dimensional homogeneity, or DH. (§1.8)
- Define a π-group. Define the derivative and the integral. (§

"Begin difficult things while they are easy. Do great things when they are small. The difficult things of the world must have once been easy. The great things must have once been small. A thousand-mile journey begins with one step."

-Lao-tzu (Chinese philosopher who founded Taoism in about 600 BC)

In this chapter, we invite you to take the first steps of your journey in learning fluid mechanics. We have been on this journey most of our lives, and we love to share our passion and our knowledge with you as you *walk your path.*

1.1 Defining Engineering Fluid Mechanics

As engineers we ought to be able to explain to a layperson what our discipline is about. Thu this section defines engineering fluid mechanics and defines learning.

Engineering

Engineers design systems that benefit people. For example, Fig. 1.1 shows a glider and Fig. 1.2 shows wind turbines being used to generate electrical power for a community. Fig. 1.2b show people working on slow-sand filter technology. This technology is used to produce safe drinl ing water for families. The person in the center of the photo is a mechanical engineering stu dent who worked on this project during his senior year. We also design hydroelectric powe systems such as Hoover Dam. We design oil pipelines, artificial hearts, jet engines, and coolin systems for buildings. *Engineers design the technology of the world.*

(a) (b)

FIGURE 1.2
(a) Commercial wind turbines in Oregon.
(b) Engineering slow sand filter technology near Nairobi Kenya. (Photos by Donald Elger)

The National Research Council (1) states that "*engineering is the process of designing th human made world.*" They assert that science involves study of the natural world, wherea engineering involves modifying the world to meet human needs. Of course, science, math, an engineering are interwoven. Thus, the central purpose of engineering education is to teac engineering students how to design the human-made world in ways that integrate and capital ize on math and science while considering foremost the needs of people.

Regarding **math**, this can be defined as the abstract and logical study of numbers, quanti ties, and space. **Science** is the systematic study of the physical world through observation an experiment. Science differs from math in that math is about abstraction and symbols, wherea science is about understanding the physical world. Science is the music and math is means c writing the music down. **Technology** is the collection of machinery, equipment, and tool developed from scientific knowledge. By applying existing technology, engineers leverage th progress of those who have come before.

In addition to math, science, and technology, engineers apply knowledge from other field such as economics, sociology, and psychology. Although these fields are applied to a lesse degree than math and science, they are still important. Thus, we say that engineers apply th knowledge (i.e., collective wisdom) of humankind.

When Cegnar (2), a practicing engineer, saw Katehi et al.'s description (1), he suggeste that engineering requires more that just math, science, and technology. Cegnar stated tha solutions also involve creativity and innovation. Solutions involve persistence and struggle in the face of challenges. Solutions involve constraints such as time and money. Solutions involve the ability to simplify and idealize that which is complex. The skills that professionals use to be creative, to handle adversity, to manage constraints, and to idealize are called the *art* of engineering.

Fig. 1.3 summarizes ideas about engineering. The upper row summarizes what engineering is. The lower row summarizes how engineering is done and why engineering is done. The term **process** means a systematic and effective method for getting results.

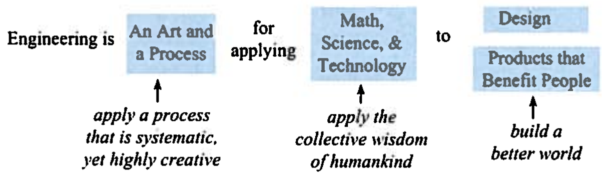


FIGURE 1.3
A summary of ideas ab engineering.

Definition of Fluid Mechanics

Mechanics is the field of science focused on the motion of material bodies. Mechanics involves force, energy, motion, deformation, and material properties. When mechanics applies to material bodies in the solid phase, the discipline is called **solid mechanics**. When the material body is in the gas or liquid phase, the discipline is called fluid mechanics.

In summary, **fluid mechanics** is the science of energy, motion, deformation, and properties when the material is in the gas or liquid phase.

Definition of Learning

Researchers at the Harvard Graduate School of Education (3, 4) define **understanding** as the ability to carry out performances that show one's grasp of a subject and advance it at the same time. Understanding is about being able to apply knowledge in new ways. Based on these ideas, we define **learning** as the process of developing (or improving) one's abilities to do something useful while also advancing one's ability to learn in the future.

Summary To learn engineering fluid mechanics means to develop the ability to design systems that involve fluids while also advancing one's abilities to learn in the future.

1.2 Describing Liquids and Gases

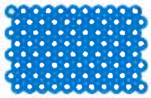
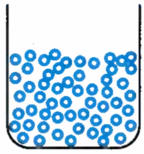
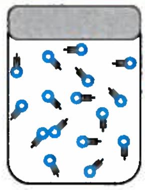
Designers need to understand the nature of the materials they work with. Thus, this section describes fluids. A wonderful starting point is the atomic hypothesis as stated by the Nobel-prize-winning physicist Richard Feynman (5):

> *If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or atomic fact, or whatever you wish to call it) that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.*

A **fluid** is a substance whose molecules move freely past each other. More specifically fluid is a substance that will continuously deform (i.e., flow) under the action of a shear stre Alternatively, a solid will deform under the action of a shear stress but will not flow like a flu Both liquids and gases are classified as fluids.

Because of differences in the forces between molecules, liquids and gases behave diffe ently. As shown in the first row of Table 1.1, a liquid will take the shape of a contain whereas a gas will expand to fill a closed container. The behavior of the liquid is produced strong attractive force between the molecules. This strong attractive force also explains w the density of a liquid is much higher than the density of gas (see the fourth row). T attributes in Table 1.1 can be generalized by defining a gas and liquid based on the diffe ences in the attractive forces between molecules. A **gas** is a phase of material in which mo ecules are widely spaced, molecules move about freely, and forces between molecules a minuscule, except during collisions. Alternatively, a **liquid** is a phase of material in whi molecules are closely spaced, molecules move about, and there are strong attractive forc between molecules.

TABLE 1.1 Comparison of Solids, Liquids, and Gases

Attribute	Solid	Liquid	Gas
Typical Visualization			
Description	Solids hold their shape; no need for a container	Liquids take the shape of the container and will stay in open container	Gases expand to fill a closed container
Mobility of Molecules	Molecules have low mobility because they are bound in a structure by strong intermolecular forces	Molecules move around freely even though there are strong intermolecular forces between molecules	Molecules move around freely with little interaction except during collisions; this is why gases expand to fill their container
Typical Density	Often high; e.g., density of steel is 7700 kg/m^3	Medium; e.g., density of water is 1000 kg/m^3	Small; e.g., density of air at sea level is 1.2 kg/m^3
Molecular Spacing	Small—molecules are close together	Small—molecules are held close together by intermolecular forces	Large—on average, molecules are far apart
Effect of Shear Stress	Produces deformation	Produces flow	Produces flow
Effect of Normal Stress	Produces deformation that may associate with volume change; can cause failure	Produces deformation associated with volume change	Produces deformation associated with volume change
Viscosity	NA	High; decreases as temperature increases	Low; increases as temperature increases
Compressibility	Difficult to compress; bulk modulus of steel is 160×10^9 Pa	Difficult to compress; bulk modulus of liquid water is 2.2×10^9 Pa	Easy to compress; bulk modulus of a gas at room conditions is about 1.0×10^5 Pa

1.3 Idealizing Matter

Engineers apply idealized* models to characterize material behavior. Thus, this section presents ideas for understanding materials and their behaviors.

The Microscopic and Macroscopic Viewpoints

A **microscopic viewpoint** describes material behavior by characterizing the behavior of atoms and molecules, often using statistical methods to characterize average molecular behavior. Alternatively, a **macroscopic viewpoint** describes material behavior without resulting to models at the atomic level. The macroscopic viewpoint is simpler, so it is used more often.

Matter can be studied from a macroscopic viewpoint or a microscopic viewpoint. Most engineering models are based on a macroscopic viewpoint. However, in selected cases such as the kinetic theory of gases, the microscopic viewpoint is useful. In addition, the microscopic model is useful for understanding phenomena such as surface tension and viscosity.

The Continuum Assumption

Because a body of fluid is comprised of molecules, properties are due to average molecular behavior. That is, a fluid usually behaves as if it were comprised of continuous matter that is infinitely divisible into smaller and smaller parts. This idea is called the **continuum assumption**.

When the continuum assumption is valid, engineers can apply limit concepts from differential calculus. A limit concept typically involves letting a length, an area, or a volume approach zero. Because of the continuum assumption, fluid properties such as density and velocity can be considered continuous functions of position with a value at each point in space.

To gain insight into the validity of the continuum assumption, consider a hypothetical experiment to find density. Fig. 1.4a shows a container of gas in which a volume ΔV has been identified. The idea is to find the mass of the molecules Δm inside the volume and then to calculate density by

$$\rho = \frac{\Delta m}{\Delta V}$$

The calculated density is plotted in Fig. 1.4b. When the measuring volume ΔV is very small (approaching zero), the number of molecules in the volume will vary with time because of the random nature of molecular motion. Thus, the density will vary as shown by the wiggles in the blue line. As volume increases, the variations in calculated density will decrease until the calculated density is independent of the measuring volume. This condition corresponds to the

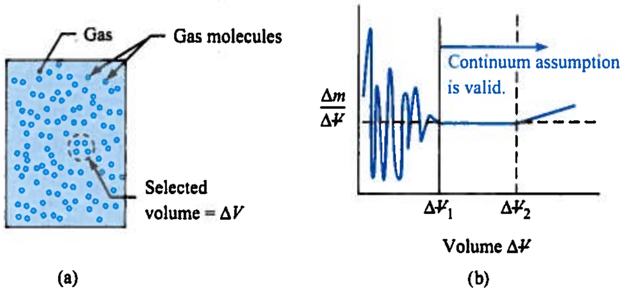


FIGURE 1.4
When a measuring volu ΔV is large enough for random molecular effec to average out, the continuum assumption i valid.

*Engineers idealize because this makes things easier and faster. To *idealize* means to simplify an entity (an idea, a physical system, a mathematical model, etc.) by removing extraneous details that have little impact on utility.

vertical line at ΔV_1. If the volume is too large, as shown by ΔV_2, then the value of density m change due to spatial variations.

In most applications, the continuum assumption is valid as shown by the next example.

EXAMPLE. Probability theory shows that including 10^6 molecules in a volume will allc the determination of density to within 1%. Thus, a cube that contains 10^6 molecules shou be large enough to accurately estimate macroscopic properties such as density and veloci Find the length of a cube that contains 10^6 molecules. Assume room conditions. Do calcul tions for (a) water, and (b) air.

Solution. (a) The number of moles of water is $10^6/6.02 \times 10^{23} = 1.66 \times 10^{-18}$ mol. The ma of the water is $(1.66 \times 10^{-18}\ \text{mol})(0.0180\ \text{kg/mol}) = 2.99 \times 10^{-20}$ kg. The volume of the cu is $(2.99 \times 10^{-20}\ \text{kg})/(999\ \text{kg/m}^3) = 2.99 \times 10^{-23}\ \text{m}^3$. Thus, the length of the side of a cube 3.1×10^{-8} m. (b) Repeating this calculation with air gives a length of 3.5×10^{-7} m.

Review. For the continuum assumption to apply, the object being analyzed would ne to be larger than the lengths calculated in the solution. If we select 100 times larger as o criteria, then the *continuum assumption applies* to objects with:

- Length $(L) > 3.1 \times 10^{-6}$ m (for liquid water at room conditions)
- Length $(L) > 3.5 \times 10^{-5}$ m (for air at room conditions)

Given the two length scales just calculated, it is apparent that the *continuum assumpti applies to most problems of engineering importance*. However, there are a few situations whe the problem length scales are too small.

EXAMPLE. When air is in motion at a very low density, such as when a spacecraft ente the earth's atmosphere, then the spacing between molecules is significant in comparison the size of the spacecraft.

EXAMPLE. When a fluid flows through the tiny passages in nanotechnology devices, th the spacing between molecules is significant compared to the size of these passageways.

The Fluid Particle

When developing equations or visualizing the flow of a fluid, it is useful to visualize a sma unit of fluid that is part of a larger body. A **fluid particle** is defined as a small quantity of flu with fixed identity. *Small* means that the lengths of the particle are much smaller that the cha acteristic length(s) of the problem under study. The words *fixed identity* mean that the partic is always comprised of the same matter. Typically, a fluid particle in a flow will change sha (i.e., deform) and change orientation in response to forces. However, the fluid particle w always be comprised of the same matter.

In the development of equations, it is common to let the dimensions of a fluid partic approach zero in sense of the limit from calculus. In this case, we say that the fluid particle *infinitesimal* in size. Because the fluid particle is a macroscopic concept (i.e., assume the co tinuum assumption applies), the idea of an infinitesimal particle is valid.

1.4 Dimensions and Units

As engineers we record data; we measure things. The foundation of measurement is the *dime sion* and the *unit*. Thus, this section introduces these topics.

Dimensions

A **dimension** is a category for measurement. For example, engineers measure power, so power is a dimension. Dimensions can be identified by asking the question: *what are we interested in measuring?* Answers to this question can include force, length, volume, work, and viscosity. Thus, these variables are dimensions.

Dimensions can be related by using equations. For example, Newton's second law, $F = ma$, relates the dimensions of force, mass, and acceleration. Because dimensions can be related, engineers and scientists can express dimensions using a limited set that are called *primary dimensions*. Table 1.2 lists one set of primary dimensions.

TABLE 1.2 Primary Dimensions

Dimension	Symbol	Unit (SI)
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	T	second (s)
Temperature	θ	kelvin (K)
Electric current	i	ampere (A)
Amount of light	C	candela (cd)
Amount of matter	N	mole (mol)

A *secondary dimension* is any dimension that can be expressed using primary dimensions. For example, the secondary dimension "force" is expressed in primary dimensions by using $F = ma$. The primary dimensions of acceleration are L/T^2, so

$$[F] = [ma] = M\frac{L}{T^2} = \frac{ML}{T^2} \tag{1.1}$$

In Eq. (1.1), the square brackets means "dimensions of." Thus $[F]$ means "the dimension of force. Similarly, $[ma]$ means the dimensions of mass times acceleration. This equation reads "the primary dimensions of force are mass times length divided by time squared." Notice that primary dimensions are not enclosed in brackets. For example, ML/T^2 is not enclosed in brackets.

One can find primary dimensions by applying a known equation.

EXAMPLE. Suppose the goal is to find the primary dimensions of work.

Step 1: Find an equation.

$$(\text{work}) = (\text{force})(\text{distance})$$

$$W = Fd$$

Step 2: Use the equation to relate the secondary dimensions:

$$[W] = [Fd] = [F][d]$$

Step 3: Insert primary dimensions and do algebra.

$$[W] = [F][d] = \frac{ML}{T^2} \times L = \frac{ML^2}{T^2}$$

One can also find primary dimensions by looking them up. For example, Table F.1 (inside fro cover) shows that, the primary dimensions of viscosity are *M/LT.* Similarly, Table A.6 (insi back cover) lists primary dimensions for symbols used in this text.

Units

A **unit** is a standard for measurement so that size or magnitude can be characterized. Un allow quantification. For example, to quantify how much volume (a dimension), one selec from a variety of units: liters, cubic meters, cubic feet, etc. For example, one might state tha tank has a volume of 42 liters. The dimension describes what (i.e., the volume) and the ur describes how much (42 liters). Similarly, measurement of energy (a dimension) can be e pressed using units of joules or units of calories. The relationship between units and dime sions is illustrated in Fig. 1.5. As shown, a dimension can be visualized as a number line, anc unit is a way to increment a dimension so that magnitude can be measured.

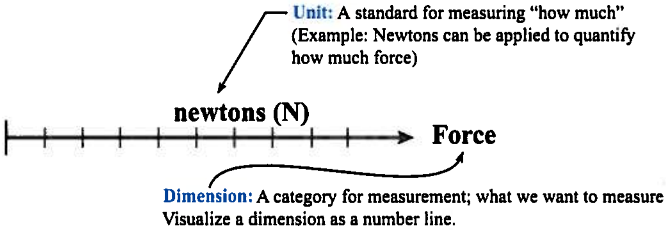


FIGURE 1.5

The relationship between units and a dimension.

✔CHECKPOINT PROBLEM 1.1

Weightwatchers, Inc. has developed "Points"™, which are used to track food intake. Points ar calculated as a function of calories, grams of fat, and grams of fiber. You're only supposed to eat certain number of Points™ in a day. Is the Point™ a dimension or a unit?

Unit Systems

This text uses two units systems:

- The International System of Units (abbreviated SI from the French "Le Système International d'Unités") is based on the meter, kilogram, and second. The SI system is the international standard for measurement.
- The "traditional unit system" employs English units such as the slug for mass, the foot (ft for length, the pound-force (lbf) for force, and the second (s) for time.

Consistent Units

Consistent units are defined as a set of units for which the conversion factors only contain tl number 1.0. This means, for example, that:

$$(1 \text{ unit of force}) = (1 \text{ unit of mass})(1 \text{ unit of acceleration})$$

$$(1 \text{ unit of power}) = (1 \text{ unit of work})/(1 \text{ unit of time}) \quad (1.$$

$$(1 \text{ unit of speed}) = (1 \text{ unit of distance})/(1 \text{ unit of time})$$

Table 1.3 lists consistent units in the SI system and in the traditional system.

TABLE 1.3 Consistent Units

Dimension	SI system	Traditional System
length	meter (m)	foot (ft)
mass	kilogram (kg)	slug (slug)
time	second (s)	second (s)
force	newton (N)	pound force (lbf)
pressure	pascal (Pa)	pound force per square foot (psf)
density	kilogram per meter cubed (kg/m^3)	slug per foot cubed ($slug/ft^3$)
volume	cubic meters (m^3)	cubic feet (ft^3)
power	watt (W)	foot-pound force per second (ft-lbf/s)

Regarding unit practice, three recommendations are

- Use consistent units because this eliminates extraneous unit conversions.
- Use the SI system whenever possible because this system is the international standard, and this system is simpler and leads to more accurate work for most people.
- Become proficient with traditional units because these units are still commonly used.

Organizing Units and Dimensions

Table F.1 (inside front cover) shows how units and dimensions fit together in fluid mechanics. Four primary dimensions (M, L, T, θ) are used to build approximately 12 secondary dimensions (flow rate, pressure, power, etc.). Each of these dimensions can be quantified with many different units.

1.5 Carrying and Canceling Units

Carrying and canceling units in engineering is beneficial, if not essential. Thus, this section introduces a method called the grid method, developed by Wales and Stager (6). Although other methods are available, the grid method is presented here because it is simple and clear.

Example of the Grid Method

The grid method is illustrated in Fig. 1.6. As shown, this calculation is an estimate of the power P required to ride a bicycle at a speed of $V = 20$ mph. The engineer estimated that the required force to move against wind drag is $F = 4.0$ lbf and applied the equation $P = FV$. As shown, the calculation reveals that the power is 159 watts.

FIGURE 1.6
Grid method.

$$P = F \times V = \frac{4\ \text{lbf}}{} \,\Big|\, \frac{20\ \text{mph}}{} \,\Big|\, \frac{1.0\ \text{m/s}}{2.237\ \text{mph}} \,\Big|\, \frac{1.0\ \text{N}}{0.2248\ \text{lbf}} \,\Big|\, \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}}$$

$\boxed{P = 159\ \text{W}}$

The idea of the grid method is to keep multiplying the right side of the equation by t number 1.0 until the units are the desired units. For example in Fig. 1.6, the engineer mul plied the right side of the equation by 1.0 three times.

$$1.0 = \frac{1\ \text{m/s}}{2.237\ \text{mph}} \quad \text{(first time)}$$

$$1.0 = \frac{1.0\ \text{N}}{0.2249\ \text{lbf}} \quad \text{(second time)}$$

$$1.0 = \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} \quad \text{(third time)}$$

Finding Unity Conversion Ratios

Each equation listed above is called a *unity coversion ratio* (conversion ratio for short) becau the pure number 1.0 without units appears on the left side. There are three methods for findi unity conversion ratios. The first method is to derive a formula.

Step 1. Start with a definition:

$$\text{power} = \frac{\text{work}}{\text{time}}$$

Step 2. List the units of each variable.

$$1.0\ \text{W} = 1.0\ \text{watt} = \frac{1.0\ \text{joule}}{1.0\ \text{second}} = \frac{1.0\ \text{newton-meter}}{\text{second}} = \frac{1.0\ \text{N-m}}{\text{s}}$$

Step 3. Do algebra.

$$1.0 = \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}}$$

The second method is look up a formula in the inside front cover of this book.

EXAMPLE. Find the row labeled "speed" in Table F.1 and note that 1.0 m/s = 2.237 mp This formula can be rearranged to give

$$1.0 = \frac{1\ \text{m/s}}{2.237\ \text{mph}}$$

The third method is use a memorized fact. For example, if one can remember that 1.00 in is equal to 2.54 centimeters, one can write

$$1.0 = \frac{1\ \text{inch}}{2.54\ \text{cm}}$$

✔CHECKPOINT PROBLEM 1.2

Which conversion ratio is correct?

(a) 1.0 = (3.785 US gallons)/(1.0 L), (b) 1.0 = (1.0 cm)/(2.54 in), (c) 1.0 = (1.0 lbm)/(2.205 kg)
(d) 1.0 = (3.281 yd)/(1.0 m), (e) 1.0 = (14.7 psi)/(101.3 kPa)

Examples of the Grid Method

The steps of the grid method are listed in the first column of Table 1.4. Examples showing how to apply the steps are presented in the second and third columns.

TABLE 1.4 Applying the Grid Method (Two Examples)

Step	Example 1	Example 2
Problem Statement =>	**Situation:** Convert a pressure of 2.00 psi to pascals.	**Situation:** Find the force in newtons that is needed to accelerate a mass of 10 g at a rate 15 ft/s².
Step 1. Write the equation down	not applicable	$F = ma$
Step 2. Insert numbers and units	$p = 2.00\text{ psi}$	$F = ma = (0.01\text{ kg})(15\text{ ft/s}^2)$
Step 3. Look up conversion ratios (see Table F.1)	$1.0 = \frac{1\text{ Pa}}{1.45 \times 10^{-4}\text{ psi}}$	$1.0 = \frac{1.0\text{ m}}{3.281\text{ ft}}$ $1.0 = \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$
Step 4. Multiply terms and cancel units.	$p = [2.00\,\cancel{\text{psi}}]\left[\frac{1\text{ Pa}}{1.45 \times 10^{-4}\,\cancel{\text{psi}}}\right]$	$F = [0.01\,\cancel{\text{kg}}]\left[\frac{15\,\cancel{\text{ft}}}{\cancel{\text{s}^2}}\right]\left[\frac{1.0\,\cancel{\text{m}}}{3.281\,\cancel{\text{ft}}}\right]\left[\frac{\text{N} \cdot \cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}}\right]$
Step 5. Do calculations.	$p = 13.8\text{ kPa}$	$F = 0.0457\text{ N}$

Using Pounds-Mass and Slugs

Engineers often use pounds-mass (lbm) and slugs in calculations. Thus, this subsection shows how to use these units.

Table F.1 shows how mass units are related. One kilogram of mass is equivalent to 2.2 pounds mass (1 kg = 2.2 lbm). One pound of mass is equivalent to 454 grams (1.0 lbm = 453.6 g). One slug of mass is equivalent to 32.2 pounds mass or 14.6 kilograms (1.0 slug = 32.17 lbm = 14.59 kg).

Mass units can be related to force units by application of $F = ma$. In the SI unit system, a force of 1.0 N is defined as the magnitude of force that will accelerate a mass of 1.0 kg at a rate of 1.0 m/s². Thus,

$$(1.0\text{ N}) \equiv (1.0\text{ kg})(1.0\text{ m/s}^2)$$

Rewriting this expression gives a conversion ratio

$$1.0 = \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} \tag{1.3}$$

When the mass unit is the slug, a force of 1.0 pound-force (lbf) is defined as the force that will accelerate a mass of 1.0 slugs at a rate of 1 ft/s². Thus,

$$(1.0\text{ lbf}) \equiv (1.0\text{ slug})(1.0\text{ ft/s}^2)$$

Rewriting this expression gives the conversion factor

$$1.0 = \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.4}$$

When the mass unit is lbm, a force of 1.0 lbf is defined as the magnitude of force that w accelerate a mass of 1.0 lbm at a rate of 32.2 ft/s^2. So,

$$(1.0 \text{ lbf}) \equiv (1.0 \text{ lbm})(32.2 \text{ ft/s}^2)$$

Thus, the conversion ratio relating force and mass units becomes

$$1.0 = \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \qquad \text{(1.}$$

✔CHECKPOINT PROBLEM 1.3

A force of $F = 10$ lbf accelerates a block at a rate of $a = 5$ ft/s^2. Using $F = ma$, calculate the mass o the block in units of pounds-mass.

Example 1.1 shows how to use the grid method.

EXAMPLE 1.1

Grid Method Applied to Calculating Thrust from a Rocket

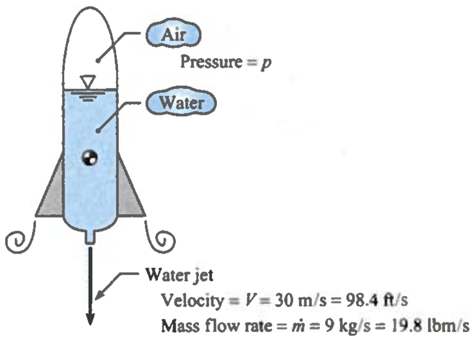
Problem Statement

A water rocket is fabricated by attaching fins to a 1-liter plastic bottle. The rocket is partially filled with water, and the air space above the water is pressurized, causing water to jet out of the rocket and propel the rocket upward. The thrust force T from the water jet is given by $T = \dot{m}V$, where $\dot{m}$ is the rate at which the water flows out of the rocket in units of mass per time and V is the speed of the water jet.
(a) Estimate the thrust force in newtons for a jet velocity of $V = 30$ m/s (98.4 ft/s) where the mass flow rate is $\dot{m} = 9$ kg/s (19.8 lbm/s). (b) Estimate the thrust force in units of pounds-force (lbf). Apply the grid method during your calculations.

Define the Situation

A rocket is propelled by a water jet.

Thrust Force $= T = \dot{m}V$



State the goal

T(N), T(lbf) ⬅ thrust force in newtons and pounds-force

Generate Ideas and Make a Plan

Apply the process given in Table 1.4. When traditional units are used, apply Eq. (1.5).

Take Action (Execute the Plan)

1. Thrust force (SI units)

$$T = \dot{m}V$$

- Insert numbers and units:

$$T\text{ (N)} = \dot{m}V = (9 \text{ kg/s})(30 \text{ m/s})$$

- Insert conversion ratios and cancel units:

$$T\text{ (N)} = \left[\frac{9 \cancel{\text{kg}}}{\cancel{\text{s}}}\right]\left[\frac{30 \cancel{\text{m}}}{\cancel{\text{s}}}\right]\left[\frac{\text{N} \cdot \cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}}\right]$$

$$\boxed{T = 270 \text{ N}}$$

2. Thrust force (traditional units)

$$T = \dot{m}V$$

- Insert numbers and units:

$$T\text{ (lbf)} = \dot{m}V = (19.8 \text{ lbm/s})(98.4 \text{ ft/s})$$

- Insert conversion ratios and cancel units:

$$T\text{ (lbf)} = \left[\frac{19.8 \cancel{\text{lbm}}}{\cancel{\text{s}}}\right]\left[\frac{98.4 \cancel{\text{ft}}}{\cancel{\text{s}}}\right]\left[\frac{\text{lbf} \cdot \cancel{\text{s}^2}}{32.2 \cancel{\text{lbm}} \cdot \cancel{\text{ft}}}\right]$$

$$\boxed{T = 60.5 \text{ lbf}}$$

Review

1. *Validate.* Because 270 N ≃ 60.5 lbf, the answers are the same.
2. *Tip.* To validate calculations in traditional units, one can repeat the calculation in SI units.

1.6 Applying the Ideal Gas Law (IGL)

The design of systems that involve gases (e.g., airbags, shock absorbers, combustion systems, aircraft) often involve application of the IGL. Thus, this section presents this topic.

Theoretical Development of the IGL

Brown et al. (7) states that the IGL was developed empirically. An *empirical* equation is one that was developed by the logical process called induction. *Induction* is the process of making many experimental observations and then concluding that something is always true because every experiment indicates this truth. For example, if a person concludes that the sun will rise tomorrow because it has risen every day in the past, this is an example of inductive reasoning.

The IGL was developed by combining three empirical equations that had been discovered previously. The first of these equation, called Boyle's law, states that when temperature T is held constant, the pressure p and volume $\mathcal{V}$ of a fixed quantity of gas are related by:

$$p\mathcal{V} = \text{constant} \qquad \text{(Boyle's law)}$$

The second equation, Charles's law, states that when pressure is held constant, the temperature and volume $\mathcal{V}$ of a fixed quantity of gas are related by:

$$\frac{\mathcal{V}}{T} = \text{constant} \qquad \text{(Charles's law)}$$

The third equation was derived by a hypothesis formulated by Avogadro: *Equal volumes of gases at the same temperature and pressure contain equal number of molecules.* When Boyle's law, Charles's law, and Avogadro's law are combined, the result is the ideal gas equation in this form:

$$p\mathcal{V} = nR_uT \tag{1.6}$$

where n is the amount of gas measured in units of moles. A **mole** is defined as the amount of matter that contains as many particles as there are atoms in 12 g of carbon-12. This means that a mole of gas will contain 6.02214×10^{23} particles. In Eq. (1.6), R_u is a constant called the universal gas constant; some useful values are

$$R_u = 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} = 1545\frac{\text{ft}\cdot\text{lbf}}{\text{lbmole}\cdot{}^\circ\text{R}}$$

To make the ideal gas law more useful, it can be rearranged to use mass units instead of mole units. To relate moles and mass, let

$$n(\text{moles}) \times \mathcal{M}\frac{(\text{grams})}{(\text{mole})} = m(\text{grams}) \tag{1.7}$$

where $\mathcal{M}$ is the molar mass of the gas and m is mass of the gas. To develop the mass form of the ideal gas equation, substitute Eq. (1.7) into Eq. (1.6).

$$p\mathcal{V} = \frac{m}{\mathcal{M}}R_uT = m\left(\frac{R_u}{\mathcal{M}}\right)T = mRT$$
$$p\mathcal{V} = mRT \tag{1.8}$$

where the specific gas constant R is given by

$$R = (\text{specific gas constant}) = \frac{R_u}{\mathcal{M}} = \frac{\text{ideal gas constant}}{\text{molar mass}} \tag{1.9}$$

To introduce density into the IGL, rewrite Eq. (1.8) and then introduce the definition density ρ:

$$p = \left(\frac{m}{V\!\!\!\!-}\right)RT = \rho RT \quad (1.1$$

✔CHECKPOINT PROBLEM 1.4

If the molar mass of a gas is 35 grams per mole, what is the specific gas constant for this gas i SI units?

Validity of the IGL

An equation is *valid* when calculated values closely match (say within 5%) values that wou be measured if an experiment was done. Regarding the validity of the IGL, some useful tips a presented here.

- For gases near atmospheric conditions, the IGL is a good approximation.
- When both the liquid phase and the gas phase are present (e.g., propane in a tank used for a barbecue), one can consult thermodynamic tables (8) to find the density of the gas phase.
- When a gas is very hot such as the exhaust stream of a rocket, then the gas can ionize or disassociate. Both of these effects can invalidate the ideal gas law.
- To determine if a gas can be characterized with the IGL, one can calculate the compressibility factor, which is commonly given the symbol Z and presented in thermodynamics texts (8).

Working Equations

An equation that is used for applications is called a *working equation.* Working equations fluid mechanics are presented in Table F.2 in the front of the book. In addition, many of th working equations are described in more detail; see, for example, Table 1.5 for the IGL.

Table 1.5 lists the most useful forms of the IGL and lists the variables. Notice the tips in t last column of the table. Tips are identified by parenthesis.

TABLE 1.5 Summary of the Ideal Gas Law Equations

Description	Equation	Variables
Density form of the IGL	$p = \rho RT$ (1.10)	p = pressure (Pa) (use absolute pressure, not gage or vacuum pressure) ρ = density (kg/m^3) R = specific gas constant (J/(kg · K)) (look up R in Table A. T = temperature (K) (use absolute temperature)
Mass form of the IGL	$pV\!\!\!\!- = mRT$ (1.8)	$V\!\!\!\!-$ = volume (m^3) m = mass (kg)

TABLE 1.5 Summary of the Ideal Gas Law Equations (*Continued*)

Description	Equation	Variables
Mole form of the IGL	$pV = nR_uT$ (1.6)	n = number of moles R_u = universal gas constant (R_u = 8.314 J/(mol · K) = 1545 (ft · lbf)/(lbmol · °R))
This equation is used to relate gas constants	$R = \frac{R_u}{\mathcal{M}}$ (1.9)	$\mathcal{M}$ = molar mass (kg/mol)

1.7 The Wales-Woods Model

Engineers use calculations to figure things out. Thus, this section presents a model, called the Wales-Woods model, that reveals how professionals do calculations.

Rationale for the Wales-Woods Model (WWM)

An expert is a person who does things well with minimal effort. For example, an expert golf player hits a golf ball far with little effort. It is human nature to desire the ability to create *great results with minimal effort.*

Learning to do something well is facilitated by *deliberate practice* according to Dr. Anders Ericsson and his colleagues (9). Dr. Ericsson is the Conradi Eminent Scholar of Psychology at Florida State University and an international authority on the development of expertise. He asserts that it is deliberate practice, not innate talent, that leads to expertise. Deliberate practice involves understanding how experts do things and then practicing these fundamentals over a long period of time. Thus, the rationale for the WWM is to reveal how experts solve technical problems so that *students can practice these skills and develop themselves over time into professionals who solve difficult problems with minimal effort.*

The WWM is based on the research of Professors Charles Wales, Anni Nardi, Robert Stager, and Donald Woods (6, 10-17). These researchers studied how experts solved problems, and then they figured out how to teach these patterns to students.

The WWM is effective. Based on 5 years of data, Wales (11) reports that when students were taught problem solving as freshman, the graduation rate increased by 32% and the average grade point average increased by 25%, as compared to the control group, who were not taught these skills. Based on 20 years of data, Woods (17) reports that students taught problem-solving skills, as compared to control groups, showed significant gains in confidence, problem-solving ability, attitude toward lifetime learning, self-assessment, and recruiter response.

Introduction to the WWM

Experts have a method or process that they apply to solve problems. Thus, this subsection introduces this process in the context of solving a textbook problem.

Example 1.2 shows the WWM applied to a textbook problem. The left column shows the problem and the solution. The right column explains how to apply the WWM and lists skills (i.e., actions) that are used in the model.

EXAMPLE 1.2

Applying the IGL to Predict Weight

PROBLEM AND THE SOLUTION

Problem Statement

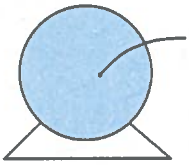
Find the total weight of a 17 ft^3 tank of nitrogen if the nitrogen is pressurized to 500 psia, the tank itself weighs 50 lbf, and the temperature is 20°C. Work in SI units.

EXPLANATION OF THE WWM

To the left is a typical problem statement from a textbook. Experts read and interpret the problem statement. Experts present their own interpretation of the problem.

Define the Situation

A tank contains nitrogen. $W_{tank} = 50\text{ lbf} = 222\text{ N}$



Assumption: The IGL applies.

Nitrogen: (Table A.2) $R_{N_2} = 297\ (\text{J/kg} \cdot \text{K})$.

To **define the situation** is to summarize the problem in a way that shows *how you are idealizing the problem*. Actions:

- Visualize the problem as if it exists in the real world. A useful question to ask is, *what am I looking at?*
- Identify scientific concepts that may be useful. A useful question to ask is, *what are the physics?*
- Summarize the physical situation (write down 1 to 2 sentences).
- List known values of variables.
- Sketch the situation; this sketch is called a *situation diagram*. Use engineering conventions on this diagram.
- Convert units to *consistent units*.
- State main assumptions.
- List fluid properties (see Section 2.4)

State the Goal

W_T(N) ⬅ Weight total (nitrogen + tank)

To **state the goal** is to summarize the results you intend to create. Actions:

- List the variable(s) to be solved for.
- List the units on these variables.
- Describe each variable(s) with a short statement.

Generate Ideas and Make a Plan

Because weight is the goal, let

$$W_T = W_{tank} + W_{N_2} \quad \text{(a)}$$

In Eq. (a), W is known and W_{N_2} is unknown, so it becomes the new goal. Select Newton's law of gravitation because this equation has the new goal in it.

$$W_{N_2} = m_{N_2} g \quad \text{(b)}$$

In Eq. (b), identify that m_{N_2} is unknown. This parameter can be found by applying the ideal gas law.

$$pV = mRT \quad \text{(c)}$$

In Eq. (c), all new variables are known. Thus, the problem is cracked. There are three equations (a, b, and c) and three unknown variables (weight of nitrogen, mass of nitrogen, and total weight of the tank). The step-by-step plan is

1. Calculate mass of nitrogen using Eq. (c).
2. Calculate weight of nitrogen using Eq. (b).
3. Calculate the total weight using Eq. (a).

To **generate ideas** is to consider alternative approaches to reach your goal(s) and to select the best ideas.

The actions that work on most problems are listed here. These steps from Wales et al. (6) can be remembered with the acronym GENI.

- Step 1. Start with Goal
- Step 2. Identify an Equation that contains the goal
- Step 3. In this equation, identify the unknowns (Needs)
- Step 4. In this equation, identify the knowns (Information)
- Step 5. Repeat steps 1 to 4 until the number of equations is equal to the number of unknowns. At this point the problem is figured out (we say *the problem is cracked*)

To **make a plan** is to figure out the steps to reach your goals.

- Identify the easiest and fastest way to get to your goal.
- List the steps.

Note: Most of the time, the steps of the plan are in reverse order of the steps of the reasoning process.

Take Action (Execute the Plan)

1. Ideal gas law (mass form)

$$m_{N_2} = \frac{p\cancel{V}}{RT}$$
$$= \left(\frac{3.45 \times 10^6\ \text{N}}{\text{m}^2}\right)\left(\frac{0.481\ \text{m}^3}{}\right)\left(\frac{\text{kg} \cdot \text{K}}{297\ \text{N} \cdot \text{m}}\right)\left(\frac{1}{293\ \text{K}}\right)$$
$$= 19.1\ \text{kg}$$

2. Newton's law of gravity

$$W_{N_2} = mg = (19.1\ \text{kg})(9.81\ \text{m/s}^2) = 187\ \text{N}$$

3. Total weight

$$W_T = W_{\text{tank}} + W_{N_2} = (222\ \text{N}) + (187\ \text{N}) = \boxed{409\ \text{N}}$$

To **take action** is to execute the steps of the plan. Actions:

- On each step, list the name of the main equation or give another descriptive label.
- Carry and cancel units with the grid method. (Note: Unit cancellations are not shown in the text or solution manual because we have not yet found a simple way to do this.)
- Box the final answer(s).

Review the Solution and the Process

1. *Knowledge.* Use the mass form of the IGL when mass is the goal.
2. *Knowledge.* $W = mg$ can be derived from Newton's law of gravity. Thus, this equation is a special case of this law.
3. *Validate.* To check the IGL assumption, we calculated the compressibility factor and found that the IGL was accurate to within about 98%.
4. *Implications.* For this problem, the weight of the gas is significant as compared to the weight of the tank.
5. *Skill.* To save time, add problem information to the situation diagram.

To **review the solution and the process** is to think critically and then to write one to three useful or insightful thoughts. A[n] effective approach is to ask questions. Examples:

- *Validate.* How can I check (validate) my solution? Does my solution make sense? Why?
- *Implications.* What did I learn? What might my result mean in the real world?
- *Skill(s).* What skills helped me solve this problem? What skills will help me solve problems in the future?
- *Knowledge.* What knowledge was useful for solving this problem? What new ideas did I gain?
- *Discussion.* What aspects of the solution are worthwhile to point out?

Structure of the Wales-Woods Model (WWM)

As shown by Example 1.2, the WWM is comprised of six thinking operations. A *thinking operation* (Table 1.6) is a collection of skills for achieving a certain outcome. Notice that each thinking operation has an outcome and a rationale.

TABLE 1.6 Structure of the WWM (Thinking Operations)

Thinking Operation	Outcome of This Thinking Operation	Why Do This Thinking Operation? (Rationale)
Define the situation	The model (idealization) used to solve the problem is clear, specific, and organized.	So you know how you are idealizing the problem.
State the goal	The goal is specific and actionable (not vague).	So you know where you are at (i.e., the situation) and where you need to go (i.e., the goal).
Generate ideas	The ideas for solving the problem are clear and specific. In addition, there is a logical process that shows how the problem solver was able to find a path to the solution.	Because the reasoning process reveals how the problem can be solved. This gives one the ability to solve unfamiliar problems and reduces or eliminates the need to memorize solutions. In addition, this gives one the satisfaction that *I cracked the problem!*

(Continue[d])

TABLE 1.6 Structure of the WWM (Thinking Operations) (*Continued*)

Thinking Operation	Outcome of This Thinking Operation	Why Do This Thinking Operation? (Rationale)
Make a plan	There is a list of steps for reaching the goal.	To find a simple and effective solution method. To create an organized plan of attack.
Take action (Execution)	The steps for reaching the goal have been executed, and the goal has been attained.	To reach the goals and enjoy the satisfaction of completing the problem.
Review the solution and the process	One to three insightful statements are written down.	To grow. This growth can take multiple forms. Examples: to become better at problem solving, to increase knowledge, to increase abilities to validate, to increase abilities to think critically, and to increase self-awareness of problem solving.

Applying the WWM to a Design Problem

The WWM can be applied to a design problem, for example, redesigning a bike pump (see Fig. 1.7). Suppose that a conventional bike pump take too many strokes to inflate a tire, and a designer wishes to redesign the pump to solve this problem. Example 1.3 illustrates how to apply the WWM to this task.

FIGURE 1.7
A bike pump being used to inflate a mountain bike tire. (Photo by Donald Elger)

EXAMPLE 1.3

The Wales-Woods Model Applied to a Design Problem

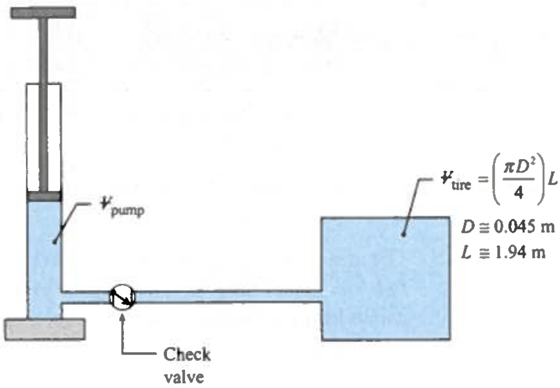
Problem Statement

Size a bike pump that will inflate a typical mountain bike tire in 20 strokes.

Define the Situation

Redesign a bike pump to inflate a bike tire in 20 strokes. Idealize the bike tire as a volumetric region.

Air: (Table A.2) $R_{air} = 287$ (J/kg · K)



Assumptions:

- Idealize the tire as a cylinder of length $L = 1.94$ m and diameter $D = 0.045$ m.
- Assume that $p_{\text{inflate}} = 50$ psig ≈ 450 kPa absolute.
- Isothermal compression: $T = 20\ °\text{C} = 293$ K

State the Goal

V_{pump}(L) ⬅ Volume of pump cylinder in liters. (Note: Using this volume, a designer can select a pump diameter and then calculate a stroke length.)

Generate Ideas and Make a Plan

Because the goal is V, apply the IGL to the pump.

$$V_{\text{pump}} = \frac{m_{\text{pump}} R_{\text{air}} T}{p_{\text{pump}}} \quad \text{(a)}$$

In Eq. (a), all parameters are known except for the mass of air inside the pump = (m_{pump}). To find this variable, apply conservation of mass:

$$(\text{mass of air in tire}) = \left(\frac{\text{mass of air}}{\text{stroke}}\right)(\text{number of strokes}) \quad \text{(b)}$$

$$m_{\text{tire}} = m_{\text{pump}} N$$

In Eq. (b), the unknown, (m_{tire}), can be found using the IGL. Thus, the problem is cracked! The steps for doing calculations are

1. Calculate the mass of air inside the tire using the IGL.
2. Relate masses using: $m_{\text{tire}} = (m_{\text{pump}})(20 \text{ strokes})$.
3. Calculate the volume of the pump using the IGL.

Take Action (Execute the Plan)

1. IGL (apply to tire)

$$V_{\text{tire}} = \left(\frac{\pi D^2}{4}\right)L = \frac{\pi(0.45 \text{ m})^2}{4}(1.94 \text{ m}) = 3.085 \times 10^{-3} \text{ m}^3$$

$$m_{\text{tire}} = \frac{p_{\text{tire}} V_{\text{tire}}}{R_{\text{air}} T}$$

$$= \left(\frac{450 \times 10^3 \text{ N}}{\text{m}^2}\right)\left(\frac{0.003085 \text{ m}^3}{}\right)\left(\frac{\text{kg} \cdot \text{K}}{287 \text{ N} \cdot \text{m}}\right)\left(\frac{1}{293 \text{ K}}\right)$$

$$= 0.0165 \text{ kg}$$

2. Conservation of mass (Eq. b)

$$m_{\text{pump}} = \frac{m_{\text{tire}}}{N} = \frac{0.0165 \text{ kg}}{20} = 0.000825 \text{ kg}$$

3. IGL (apply to pump cylinder):

$$V_{\text{pump}} = \frac{m_{\text{pump}} R_{\text{air}} T}{p_{\text{pump}}}$$

$$= \frac{(0.000825 \text{ kg})}{(101 \times 10^3 \text{ Pa})}\left(\frac{287 \text{ J}}{\text{kg} \cdot \text{K}}\right)\left(\frac{293 \text{ K}}{1}\right)$$

$$= 0.687 \text{ L}$$

Review the Solution and the Process

1. *Skills.* Notice how the system was idealized: a piston/cylinder, a check valve, and a volume to hold air.
2. *Discussion.* The calculated volume is slightly less than the volume of a typical wine bottle (750 mL).
3. *Knowledge.* The specific gas constant R was found in Table A. Note that R is different than the universal gas constant R_u.
4. *Discussion.* To estimate the size of bike pump, assume the typical user can comfortably apply a downward force of about 125 N (28 lbf). Thus, the area of the piston (using gage pressure) is about

   $$A = F/p \approx (125 \text{ N})/(350 \times 10^3 \text{ Pa}) = 0.00036 \text{ m}^2.$$

   The corresponding length of the pump is

   $$L_{\text{pump}} = V/A$$
   $$= (0.000687 \text{ m}^3)/(0.00036 \text{ m}^2) = 1.92 \text{ m}$$

A pump that is nearly 2 meters tall is not practical, so we would not recommend this solution.

Learning the Wales-Woods Model

Learning the WWM is straightforward. Practice the six thinking operations and the embedded skills. Get feedback from teachers or coaches. Recognize that learning the WWM requires a lot of time and patience. It is much like learning the golf swing. Understanding the golf swing is easy, but learning to swing the golf club consistently requires practice over a long period of time.

1.8 Checking for Dimensional Homogeneity (DH)

Checking for DH is a simple and effective approach for checking an equation. Because engineers frequently check for validity, this topic is presented next.

Dimensional Homogeneity (DH)

When the primary dimensions of each term of an equation are the same, the equation **Dimensionally Homogeneous**, or DH for short. Example 1.4 shows how to check an equati for dimensional homogeneity.

EXAMPLE 1.4

Applying Dimensional Homogeneity to the Ideal Gas Law

Problem Statement

Show that the ideal gas law (density form) is dimensionally homogeneous.

Define the Situation

The ideal gas law (density form) is $p = \rho RT$.

State the Goal

Prove that the ideal gas law is DH.

Generate Ideas and Make a Plan

To check for DH, show that the primary dimensions of each term are the same. The steps are

1. Find the primary dimensions of the first term.
2. Find the primary dimensions of the second term.
3. Prove dimensional homogeneity by comparing the terms.

Take Action (Execute the Plan)

1. Primary dimensions (first term)
   - From Table A.6, the primary dimensions are

$$[p] = \frac{M}{LT^2}$$

2. Primary dimensions (second term)
   - From Table A.6, the primary dimensions are

$$[\rho] = M/L^3$$
$$[R] = L^2/\theta T^2$$
$$[T] = \theta$$

   - Thus

$$[\rho RT] = \left(\frac{M}{L^3}\right)\left(\frac{L^2}{\theta T^2}\right)(\theta) = \frac{M}{LT^2}$$

3. Conclusion: The ideal gas law is dimensionally homogeneous because the primary dimensions of each term are the same.

Dimensionless Groups

Engineers often arrange variables so that primary dimensions cancel out. For example, co sider a pipe with an inside diameter D and length L. These variables can be grouped to form new variable L/D, which is an example of a dimensionless group. A *dimensionless group* is a arrangement of variables in which the primary dimensions cancel.

EXAMPLE. The Mach number M, which relates fluid speed V to the speed of sound c, is common dimensionless group.

$$\mathrm{M} = \frac{V}{c}$$

EXAMPLE. Another common dimensionless group is named the Reynolds number ar given the symbol Re. The Reynolds number involves density, velocity, length, and viscosity

$$\mathrm{Re} = \frac{\rho VL}{\mu} \quad \text{(1.1}$$

The convention in this text is to use the symbol [-] to indicate that the primary dimensions a dimensionless group cancel out. For example,

$$[\mathrm{Re}] = \left[\frac{\rho VL}{\mu}\right] = [\text{-}] \quad \text{(1.1}$$

Dimensionless groups are also called π-*groups*.

Primary Dimensions of Derivative and Integral Terms

Because many equations in fluid mechanics involve derivatives or integrals, this subsection shows how to analyze these terms and introduces the definition of the derivative and integral.

Let's start with the derivative. In calculus, the *derivative* is defined as a ratio:

$$\frac{df}{dy} \equiv \lim_{\Delta y \to 0} \frac{\Delta f}{\Delta y}$$

where Δf is an amount or change in a dependent variable and Δy is an amount or change in a independent variable. Thus, the primary dimensions of a first-order derivative can be found by using a ratio:

$$\left[\frac{df}{dy}\right] = \left[\frac{f}{y}\right] = \frac{[f]}{[y]} \tag{1.13}$$

The primary dimensions for a higher-order derivative can also be found by using the basic definition of the derivative. The resulting formula for a second-order derivative is

$$\left[\frac{d^2 f}{dy^2}\right] = \lim_{\Delta y \to 0} \frac{\Delta(df/dy)}{\Delta y} = \left[\frac{f}{y^2}\right] = \frac{[f]}{[y^2]} \tag{1.14}$$

For example, applying Eq. (1.14) to acceleration shows that

$$\left[\frac{d^2 y}{dt^2}\right] = \left[\frac{y}{t^2}\right] = \frac{L}{T^2}$$

To find primary dimensions of an integral, recall from calculus that an *integral* is defined as an infinite sum of terms that are very small (i.e., *infinitesimal*).

$$\int f\,dy \equiv \lim_{N \to \infty} \sum_{i=1}^{N} f\,\Delta y_i$$

Thus,

$$\left[\int f\,dy\right] = [f][y] \tag{1.15}$$

For example, position is given by the integral of velocity with respect to time. Checking primary dimensions for this integral gives

$$\left[\int V\,dt\right] = [V][t] = \frac{L}{T} \cdot T = L$$

Summary One can find primary dimensions on derivative and integral terms by applying fundamental definitions from calculus. This process is illustrated by Example 1.5.

EXAMPLE 1.5

Finding the Primary Dimensions for a Derivative and an Integral

Problem Statement

Find the primary dimensions of $\mu \frac{d^2 u}{dy^2}$, where μ is viscosity, u is fluid velocity, and y is distance. Repeat for $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V\!\!\!-} \rho\, dV\!\!\!-$ where t is time, $V\!\!\!-$ is volume, and ρ is density.

Define the Situation

Term 1 is $\mu \frac{d^2 u}{dy^2}$. Term 2 is $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V\!\!\!-} \rho\, dV\!\!\!-$.

State the Goal

Find the primary dimensions on term 1 and term 2.

Generate Ideas and Make a Plan

1. Because a second-order derivative is involved in term 1, apply Eq. (1.14).
2. Because a first-order derivative and an integral is involved in term 2, apply Eqs. (1.13) and (1.15).

Take Action (Execute the Plan)

1. Primary dimensions of $\mu \frac{d^2u}{dy^2}$
   - From Table A.6:

$$[\mu] = M/LT$$
$$[u] = L/T$$
$$[y] = L$$

   - Apply Eq. (1.14):

$$\left[\frac{d^2u}{dy^2}\right] = \left[\frac{u}{y^2}\right] = \frac{L/T}{L^2}$$

   - Combine the previous two steps:

$$\left[\mu \frac{d^2u}{dy^2}\right] = [\mu]\left[\frac{d^2u}{dy^2}\right] = \left(\frac{M}{LT}\right)\left(\frac{L/T}{L^2}\right) = \boxed{\frac{M}{L^2T^2}}$$

2. Primary dimensions of $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V} \rho \, dV$
   - Find primary dimensions from Table A.6:

$$[t] = T$$
$$[\rho] = M/L^3$$
$$[V] = L^3$$

   - Apply Eqs. (1.13) and (1.15) together:

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}\int_{V} \rho \, dV\right] = \left[\frac{\rho V}{t}\right] = \left(\frac{M}{L^3}\right)\left(\frac{L^3}{T}\right) = \boxed{\frac{M}{T}}$$

Primary Dimensions of a Constant

Some equations have constants, so this subsection shows how to analyze these terms. Th method is illustrated by the next two examples.

EXAMPLE. The hydrostatic equation (below) relates pressure p, density ρ, the gravitation constant g, and elevation z. Find the primary dimensions on the constant C.

$$p + \rho g z = \text{constant} = C$$

Solution. For DH, the constant C needs to have the same primary dimensions as p and ρg Thus the dimensions of C are $[C] = M/LT^2$.

EXAMPLE. Suppose velocity V is given as a function of distance y using two constants and b (below). Find the primary dimensions of the constants.

$$V(y) = ay(b - y)$$

Solution. For dimensional homogeneity both sides of this equation need to have primary d mensions of velocity: $[L/T]$. By inspection, one can conclude that $[b] = L$ and $[a] = L^{-1}T^{-}$ To validate this solution, check the primary dimensions on the right side of the given equatio

$$[ay(b - y)] = [a][y][b - y] = \left(\frac{1}{L \cdot T}\right)(L)(L) = \frac{L}{T}$$

Because these dimensions match the dimensions on velocity, the equation is DH.

1.9 Summarizing Key Knowledge

Definition of Engineering Fluid Mechanics and Learning

- *Engineering* is an art and a process for applying math, science, and technology to design products that benefit humankind.

- *Fluid Mechanics* is the branch of physics that is concerned with forces, motion, and energy as these ideas apply to materials that are in the liquid or gas phases.
- *Learning* is the process of (a) developing (or improving) one's abilities to do something useful, while also (b) increasing one's capacity for future learning.

Fluids, Liquids, and Gases

- Both liquids and gasses are classified as fluids. A *fluid* is defined as a material that deforms continuously under the action of a shear stress.
- A significant difference between gases and liquids is that the molecules in liquids experience strong intermolecular forces, whereas the molecules in gases move about freely with little or no interactions except during collisions.
- Liquids and gases differ in many important respects. Gases expand to fill their containers, whereas liquids will occupy a fixed volume. Gases have much smaller values of density than liquids. For other differences, see Table 1.1 (p. 4).

Ideas for Idealizing Material Behavior

- A *microscopic viewpoint* involves understanding material behavior by understanding what the molecules are doing.
- A *macroscopic viewpoint* involves understanding material behavior without the need to consider what the molecules are doing.
- In the *continuum assumption*, matter is idealized as consisting of continuous material that can be broken into smaller and smaller parts.
- The continuum assumptions applies to most fluid flows.
- A *fluid particle* is a small quantity of fluid with fixed identity and with length dimensions that are very small (e.g., 1/100th) as compared to problem dimensions.

Units and Dimensions

- Dimensions and units are the basis for measurement.
- A *dimension* is a category for measurement. Examples include mass, force, and energy.
- *Units* are the divisions by which a dimension is measured.
- Each dimension can be quantified using a variety of different units. For example, energy can be quantified using joules, calories, ft-lbf, and N-m.
- All dimensions can be expressed using a limited set of *primary dimensions*. Dimensions that are not primary dimensions are called secondary dimensions.
- Fluid mechanics uses four primary dimensions: mass (M), length (L), time (T), and temperature (θ).

The Grid Method

- The *grid method* is a systematic way to carry and cancel units.
- The main idea of the grid method is to multiply terms in equations by the pure number 1.0 (called a conversion ratio).
- A *conversion ratio* is an equality relationship between units such that the pure number 1.0 appears on one side of an equation. Examples of conversion ratios are 1.0 = (1.0 kg)/(2.2 lbm) and 1.0 = (1.0 lbf)/(4.45 N).

The Ideal Gas Law (IGL)

- Many real gases can be idealized as an ideal gas.
- In the IGL, temperature must be in absolute temperature units (Kelvin or Rankine).
- In the IGL, pressure must be absolute pressure, not gage or vacuum pressure.
- Three useful and equivalent ways to express the IGL are given here.

$$p = \rho RT \quad \text{(density form)}$$
$$p = mRT \quad \text{(mass form)}$$
$$p\forall = nR_uT \quad \text{(mole form)}$$

- The universal gas constant R_u and the specific gas constant R are related by $R = R_u/\mathcal{M}$ where $\mathcal{M}$ is the molar mass (kg/mol) of the gas.
- For a summary of the equations of the IGL, see Table 1.5 (p. 14).

The Wales-Woods Model (WWM)

- The WWM is an idealization of what experts do when they solve problems.
- The WWM is comprised of six *thinking operations*: define the situation, state the goal, generate ideas, make a plan, take action, and review the process and the results. Table 1.6 on page 17 summarizes the thinking operations.
- Each thinking operation can be broken down into specific actions (skills); see Example 1 (p. 16) for a listing of relevant skills.

Dimensional Homogeneity (DH)

- *Dimension homogeneity* means that each term in an equation has the same primary dimensions. This means that each term will also have the same units.
- To check to see if an equation is DH, calculate the primary dimensions on each term.
- A *dimensionless group* (also known as a π-*group*) is a group of variables arranged so that the primary dimensions cancel out.
- From calculus
  - The derivative is defined as a ratio:

$$\frac{df}{dy} = \lim_{\Delta y \to 0} \frac{\Delta f}{\Delta y}$$

  - The integral is defined as a infinite sum of small terms:

$$\int f\,dy = \lim_{N \to \infty} \sum_{i=1}^{N} f\,\Delta y_i$$

- To find the primary dimensions on a derivative or integral term, apply the definitions of these operations.

REFERENCES

1. Katehi, L., Pearson, G., and Feder, M., Eds., *Understanding the Status and Improving the Prospects: Committee on K–12 Engineering Education*, National Academies Press, Washington D.C., 2010, 27.

2. Personal communication with Erik Cegnar, practicing engineering at IVUS Innovations Technology, 2010.

3. Blythe, T. *The Teaching for Understanding Guide*. Jossey-Bass, San Francisco, 1998, 13.

4. Wiske, M.S., Ed. *Teaching for Understanding: Linking Research with Practice*. San Francisco: Jossey-Bass, 1998.

5. Feynman, R.P., Leighton, R.B., and Sands, M. *The Feynman Lectures on Physics*, Vol. 1, Reading, MA: Addison-Wesley, p. 1–2.

6. Wales, C.E., and Stager, R.A. *Thinking with Equations*, Center for Guided Design, West Virginia University, Morgantown, WV., 1990.

7. Brown, T.L., LeMay, H.E., and Bursten, B.E. *Chemistry: The Central Science*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

8. Moran, M.J., Shapiro, H.N., Boettner, D.D., and Bailey M.B. *Fundamentals of Engineering Thermodynamics*. Hoboken, NJ: Wiley, 2011, p. 122.

9. Ericsson, K.A., Prietula, M.J., and Cokely, E.T. "The Making of an Expert." *Harvard Business Review* (July–August, 2007).

10. Wales, C.E. "Guided Design: Why & How You Should Use It," *Engineering Education*, vol. 62, no. 8 (1972).

11. Wales, C.E. "Does How You Teach Make a Difference?" *Engineering Education*, vol. 69, no. 5, 81–85 (1979).

12. Wales, C.E., Nardi, A.H., and Stager, R.A. *Thinking Skills: Making a Choice*. Center for Guided Design, West Virginia University, Morgantown, WV., 1987.

13. Wales, C.E., Nardi, A.H., and Stager, R.A. *Professional-Decision-Making*, Center for Guided Design, West Virginia University, Morgantown, WV., 1986.

14. Wales, C.E., and Stager, R.A., (1972b). "The Design of an Educational System." *Engineering Education*, vol. 62, no. 5 (19

15. Wales, C.E., and Stager, R.A. *Thinking with Equations*. Center for Guided Design, West Virginia University, Morgantown, WV., 1990.

16. Woods, D.R. "How I Might Teach Problem-Solving?" in J. Stice, (ed.) *Developing Critical Thinking and Problem-Solving Abilities*. New Directions for Teaching and Learning, no. 30, San Francisco: Jossey-Bass, 1987.

17. Woods, D.R. "An Evidence-Based Strategy for Problem Solving." *Engineering Education*, vol. 89. no. 4, 443–459 (2000

PROBLEMS

Problem available in *WileyPLUS* at instructor's discretion.

Guided Online (GO) Problem, available in *WileyPLUS* instructor's discretion.

Defining Engineering Fluid Mechanics (§1.1)

1.1 Read the definition of engineering in §1.1. How does this compare with your ideas of what engineering is? What is similar? What is different?

1.2 Given the definition of engineering in §1.1, what do you think that you should be learning? How do you know if you have learned it?

1.3 Should the definition of engineering in §1.1 include the idea that engineers also need to be very good with humanities and social sciences? What do you believe? Why?

1.4 Select an engineered design (e.g., hydroelectric power as in a dam, an artificial heart) that involves fluid mechanics and is also highly motivating to you. Write a one-page essay that addresses the following questions. Why is this application motivating to you? How does the system you selected work? What role did engineers play in the design and development of this system?

1.5 Many engineering students believe that fixing a washing machine is an example of engineering because it involves solving a problem. Write a brief essay in which you address the following questions: Is fixing a washing machine an example of engineering? Why or why not? How do your ideas align or misalign with the definition of engineering given in §1.1?

Describing Liquids and Gases (§1.2)

1.6 Propose three new rows for Table 1.1, (p. 4, §1.2) and fill them in.

1.7 Based on molecular mechanisms, explain why aluminum melts at 660°C, whereas ice will melt at 0°C.

Idealizing Matter (§1.3)

1.8 The continuum assumption (select all that apply)

- **a.** applies in a vacuum such as in outer space
- **b.** assumes that fluids are infinitely divisible into smalle and smaller parts
- **c.** is a bad assumption when the length scale of the prob or design is similar to the spacing of the molecules
- **d.** means that density can idealized as a continuous fun of position
- **e.** only applies to gases

1.9 A fluid particle

- **a.** is defined as one molecule
- **b.** is small given the scale of the problem being conside
- **c.** is so small that the continuum assumption does not a

Dimensions and Units (§1.4)

1.10 For each variable given, list three common units.

- **a.** Volume flow rate (Q), mass flow rate ($\dot{m}$), and pressur
- **b.** Force, energy, power
- **c.** Viscosity

1.11 In Table F.2 (front of book), find the hydrostatic equation. For each form of the equation that appears, list th name, symbol, and primary dimensions of each variable.

1.12 For each of the following units in Table F.1 (front book), present in terms of its primary dimensions: kWh, poi slug, cfm, cSt.

1.13 In the context of measurement, a dimension is:

a. a category for measurement

b. a standard of measurement for size or magnitude

c. an increment for measuring "how much"

1.14 What is the approximate mass in units of slugs for

a. a 2-liter bottle of water?

b. a typical adult male?

c. a typical automobile?

1.15 In the following list, identify which parameters are dimensions and which paramenters are units: slug, mass, kg, energy/time, meters, horsepower, pressure, and pascals.

1.16 Of the three lists below, which sets of units are consistent? Select all that apply.

a. pounds-mass, pounds-force, feet, and seconds.

b. slugs, pounds-force, feet, and seconds

c. kilograms, newtons, meters, and seconds.

Carrying/Canceling Units: Grid Method (§1.5)

1.17 In your own words, describe what actions need to be taken in each step of the grid method.

1.18 Which of these is a correct conversion ratio? Select all that apply.

a. 1 = 1 hp/(550 ft-lbf/s)

b. 1 = 101.3 kPa/(14.7 lbf/in^2)

c. 1 = 3.785 U.S. gal/(1.0 L)

1.19 If the local atmospheric pressure is 93 kPa, use the grid method to find the pressure in units of

a. psia

b. psf

c. bar

d. atmospheres

e. feet of water

f. inches of mercury

1.20 Apply the grid method to calculate the density of an ideal gas using the formula $\rho = p/RT$. Express your answer in lbm/ft^3. Use the following data: absolute pressure is $p = 60$ psi, the gas constant is $R = 1716$ ft-lbf/slug-°R, and the temperature is $T = 180$ °F.

1.21 The pressure rise Δp associated with wind hitting a window of a building can be estimated using the formula $\Delta p = \rho(V^2/2)$, where ρ is density of air and V is the speed of the wind. Apply the grid method to calculate pressure rise for $\rho = 1.2$ kg/m^3 and $V = 60$ mph.

a. Express your answer in pascals.

b. Express your answer in pounds-force per square inch (psi).

c. Express your answer in inches of water column (in H_2O).

1.22 Apply the grid method to calculate force using $F = ma$.

a. Find force in newtons for $m = 10$ kg and $a = 10$ m/s^2.

b. Find force in pounds-force for $m = 10$ lbm and $a = 10$ ft/s^2.

c. Find force in newtons for $m = 10$ slug and $a = 10$ ft/

1.23 When a bicycle rider is traveling at a speed of $V = 24$ mph, the power P she needs to supply is given by $P = FV$, where $F = 5$ lbf is the force necessary to overcome aerodynamic drag. Apply the grid method to calculate:

a. power in watts.

b. energy in food calories to ride for 1 hour.

1.24 Apply the grid method to calculate the cost in U.S. dollars to operate a pump for one year. The pump power is 20 hp. The pump operates for 20 hr/day, and electricity costs $0.10 per kWh.

Ideal Gas Law (IGL) (§1.6)

1.25 Start with the ideal gas law and prove that

a. Boyle's law is true.

b. Charles's law is true.

1.26 Calculate the number of molecules in

a. one cubic centimeter of liquid water at room conditio

b. one cubic centimeter of air at room conditions

1.27 Start with the mole-form of the ideal gas law and show th steps to prove that the mass form is correct.

1.28 Start with the universal gas constant and show that $R_{N_2} = 297$ J/(kg · K).

1.29 A spherical tank holds CO_2 at a pressure of 3 atmospher and a temperature of 20°C. During a fire, the temperature is increased by a factor of 4 to 80°C. Does the pressure also increase by a factor of 4? Justify your answer using equations.

1.30 An engineer living at an elevation of 2500 ft is conducting experiments to verify predictions of glider performance. To process data, density of ambient air is needed. The engineer measures temperature (74.3°F) and atmospheric pressure (27.3 inches of mercury). Calculate density in units of kg/m^3. Compare the calculated value with data from Table A.2 and make a recommendation about the effects of elevation on density; that is, are the effects of elevation significant?

1.31 Calculate the density and specific weight of carbon dioxide at a pressure of 300 kN/m^2 absolute and 60°C.

1.32 Determine the density of methane gas at a pressure of 300 kN/m^2 absolute and 60°C.

1.33 A spherical tank is being designed to hold 10 moles c methane gas at a pressure of 2 bar and a temperature of 70°F. What diameter spherical tank should be used?

1.34 Natural gas is stored in a spherical tank at a temperature of 10°C. At a given initial time, the pressure in the tank is 100 kPa gage, and the atmospheric pressure is 100 kPa absolute. Some time later, after considerably more gas is pumpe into the tank, the pressure in the tank is 200 kPa gage, and the temperature is still 10°C. What will be the ratio of the mass of natural gas in the tank when $p = 200$ kPa gage to that when the pressure was 100 kPa gage?

1.35 WILEY PLUS At a temperature of 100°C and an absolute pressure of 5 atmospheres, what is the ratio of the density of water to the density of air, ρ_w/ρ_a?

1.36 WILEY GO Find the total weight of a 6 ft^3 tank of oxygen if the oxygen is pressurized to 400 psia, the tank itself weighs 90 lbf, and the temperature is 70°F.

1.37 WILEY GO A 4 m^3 oxygen tank is at 20°C and 700 kPa. The valve is opened, and some oxygen is released until the pressure in the tank drops to 500 kPa. Calculate the mass of oxygen that has been released from the tank if the temperature in the tank does not change during the process.

1.38 WILEY PLUS What is the (a) specific weight, and (b) density of air at an absolute pressure of 600 kPa and a temperature of 50°C?

1.39 WILEY PLUS Meteorologists often refer to air masses in forecasting the weather. Estimate the mass of 1 mi^3 of air in slugs and kilograms. Make your own reasonable assumptions with respect to the conditions of the atmosphere.

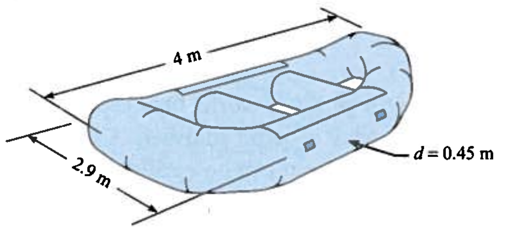
1.40 A bicycle rider has several reasons to be interested in the effects of temperature on air density. The aerodynamic drag force decreases linearly with density. Also, a change in temperature will affect the tire pressure.

a. To visualize the effects of temperature on air density, write a computer program that calculates the air density at atmospheric pressure for temperatures from −10°C to 50°C.

b. Also assume that a bicycle tire was inflated to an absolute pressure of 450 kPa at 20°C. Assume the volume of the tire does not change with temperature. Write a program to show how the tire pressure changes with temperature in the same temperature range, −10°C to 50°C.

Prepare a table or graph of your results for both problems. What engineering insights do you gain from these calculations?

1.41 A design team is developing a prototype CO_2 cartridge for a manufacturer of rubber rafts. This cartridge will allow a user to quickly inflate a raft. A typical raft is shown in the sketch.



PROBLEM 1.41

Assume a raft inflation pressure of 3 psi (this means that the absolute pressure is 3 psi greater than local atmospheric pressure). Estimate the volume of the raft and the mass of C in grams in the prototype cartridge.

1.42 A team is designing a helium-filled balloon that will fly an altitude of 80,000 ft. As the balloon ascends, the upward fo (buoyant force) will need to exceed the total weight. Thus, we is critical. Estimate the weight (in newtons) of the helium insi the balloon. The balloon is inflated at a site where the atmosp pressure is 0.89 bar and the temperature is 22°C. When inflat prior to launch, the balloon is spherical (radius 1.3 m) and th inflation pressure equals the local atmospheric pressure.

Engineering Calculations and the WWM (§1.7)

1.43 Apply the WWM and the grid method to find the acceleration for a force of 2 N acting on an object of mass 7 ounces. The relevant equation is Newton's second law of motion, $F = ma$. Work in SI units, and provide the answer in meters per second squared (m/s^2).

1.44 In Example 1.2 (p. 16, §1.7), what are the three steps th engineer takes to "State the Goal"?

1.45 For Problem 1.37 above, complete the "Define the Situa "State the Goal," and "Generate Ideas and Make a Plan" operations of the WWM.

Dimensional Homogeneity (DH) (§1.8)

1.46 The hydrostatic equation is $p/\gamma + z = C$, where p is pres γ is specific weight, z is elevation, and C is a constant. Prove th the hydrostatic equation is dimensionally homogeneous.

1.47 WILEY PLUS Find the primary dimensions of each of the follo terms.

a. $(\rho V^2)/2$ (kinetic pressure), where ρ is fluid density a V is velocity

b. T (torque)

c. P (power)

d. $(\rho V^2 L)/\sigma$ (Weber number), where ρ is fluid density, velocity, L is length, and σ is surface tension

1.48 The power provided by a centrifugal pump is given by $P = \dot{m}gh$, where $\dot{m}$ is mass flow rate, g is the gravitational constant, and h is pump head. Prove that this equation is dimensionally homogeneous.

1.49 WILEY PLUS Find the primary dimensions of each of the follo terms.

a. $\int_A \rho V^2 dA$, where ρ is fluid density, V is velocity, and A is

b. $\frac{d}{dt}\int_{\forall} \rho V\, d\forall$, where $\frac{d}{dt}$ is the derivative with respect time, ρ is density, and $\forall$ is volume.