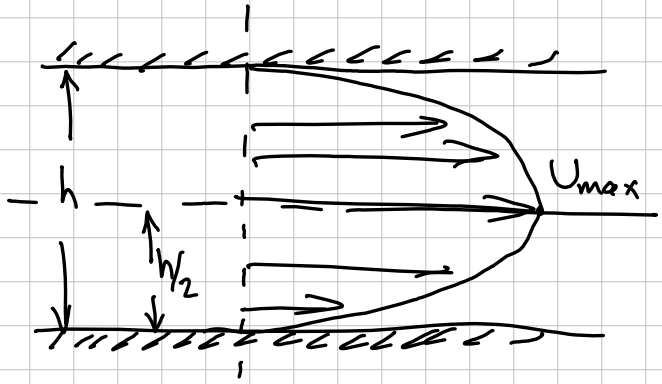
ES-8 SPRING 2024

P1. FLOW BETWEEN PARALLEL PLATES VELOCITY PROFILE:

$$u(y) = \frac{4U_{max}}{h^2}(hy - y^2)$$



KNOWN

GEOMETRY

$u(y)$

UNKNOWN

$\bar{U}$ (mean section velocity)

Q for depth W

GOVERNING PRINCIPLES

$$\bar{U} = \frac{\int_0^h u(y)\,dy}{\int_0^h dy}$$

$$Q = \bar{U}A$$

SOLUTION 5.

$$U = \frac{\int_0^h \frac{4U_{max}}{h^2}(hy - y^2)\,dy}{\int_0^h dy}$$

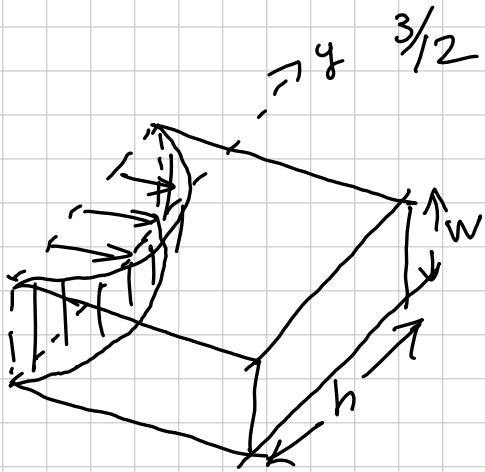
$$= \frac{\frac{4U_{max}}{h^2}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h}{y\Big|_0^h}$$

$$= \frac{\frac{4U_{max}}{h^2}\left[\frac{h^3}{2} - \frac{h^3}{3}\right]}{h} = 4U_{max}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 4U_{max}\left[\frac{3}{6} - \frac{2}{6}\right] = 4U_{max}\left[\frac{1}{6}\right]$$

$$= \frac{2 \cdot 2\, U_{max}}{3 \cdot 2} = \frac{2}{3} U_{max}$$ ← MEAN SECTION VELOCITY

$$Q = \bar{U} A$$



$$\bar{U} = \frac{2}{3} U_{max}$$

$$A = hw$$

$$Q = \frac{2}{3} U_{max} h w$$ ← DISCHARGE.

DISCUSSION

APPLICATION OF DEFN. MEAN SECTION VELOCITY (EQ. 4-3) IN DA BOOK!

PROBLEM 2

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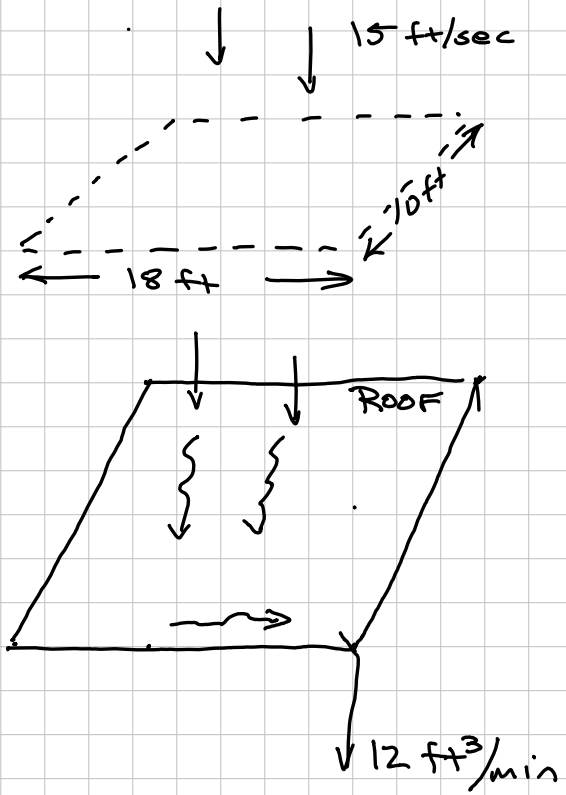


RAIN FALLS VERTICALLY ONTO ROOF WITH SPEED 15 ft/s. ROOF CATCH & ACCUMULATES 12 ft³/min

FIND

1) AMOUNT RAINWATER IN 1 ft³/AIR

2) DROPS PER ft³/air

KNOWN

DIMENSIONS ROOF, V_{fall}

V_{drop}

UNKNOWN

Q_{RAIN}/ft^3 AIR

$\#_{DROPS}/ft^3$ AIR

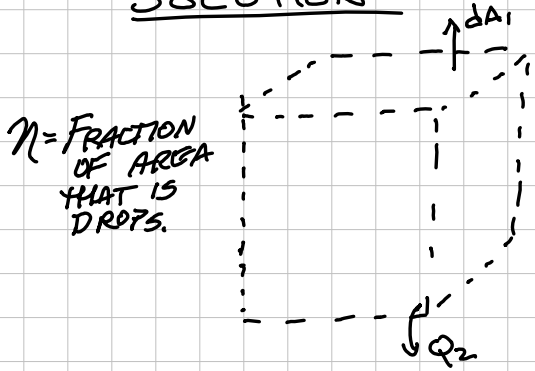
GOVERNING EQN

CONTINUITY

$Q = VA$

$\forall_{drop} = 4/3\,\pi r^3 = \frac{\pi d^3}{6}$

SOLUTION



η = FRACTION OF AREA THAT IS DROPS.

CONTINUITY

$$0 = \frac{d}{dt}\int_{CV} \rho\, d\forall + \int_{C.S.} \rho\, V \cdot dA$$

$$\eta \cancel{\emptyset}\,(15 ft/s)(10 ft)(18 ft) - \cancel{\emptyset}\; 12 ft^3/min = 0$$

$$\eta\,(15 ft/s)(10 ft)(18 ft)\left(\frac{60 s}{min}\right) = 12 ft^3/min$$

SOLVE FOR η

$\eta = 0.0000741$ ← FRACTION OF AREA THAT IS WATER

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$\forall_{WATER}$ per $1\ ft^3$/air is

$0.0000741\ ft^3$ water/ft^3 air ←

(so MOSTLY AIR)

DROPS

$$\forall_{drop} = \frac{\pi d^3}{6} = \pi \frac{\left(\frac{0.18}{12}\right)^3}{6}$$

$$= 0.0000018\ ft^3/drop$$

$$\therefore \quad \frac{0.0000741\ ft^3}{0.0000018 \frac{ft^3}{drop}} = 41\ drops/ft^3$$

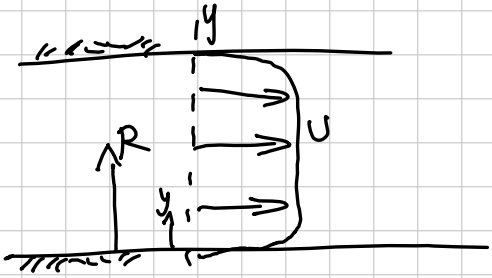
DISCUSSION

- REALLY JUST APPLICATION OF CONTINUNITY. THE RELATIVELY SMALL NUMBERS COMPLICATES THINGS A LITTLE
- FIND $\forall/ft^3$; then how many drops make up volume

PROBLEM 3

CIRCULAR PIPE

$$u(y) = U\left(\frac{y}{R}\right)^{1/7}$$



FIND $\bar{U}$, Q

KNOWN

$u(y)$, GEOMETRY

UNKNOWN

$\bar{U}$, Q

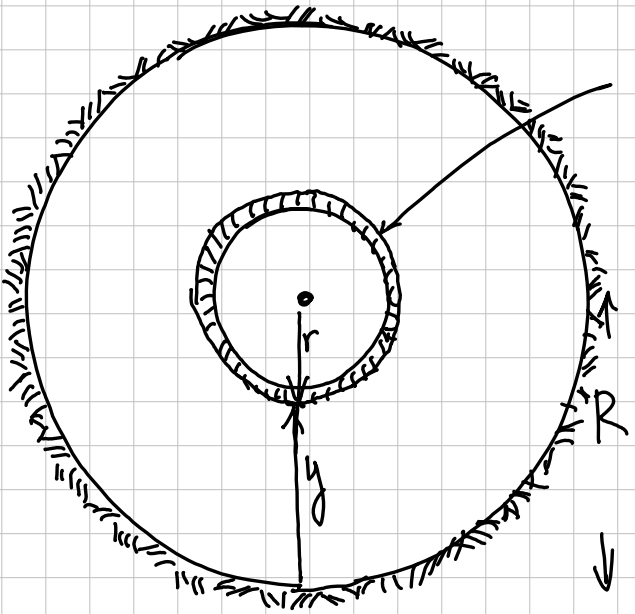
GOVERNING PRINCIPLES

$$\bar{U} = \frac{\int u \cdot dA}{\int dA}, \quad Q = \bar{U} A$$

SOLUTION

RECOGNIZE CIRCULAR CROSS-SECTION.

KEEP $U(y)$ FORM, BUT EXPRESS dA IN TERMS OF R AND y.



$$r = R - y; \qquad dA = 2\pi (R-y)\, dy$$

$$\bar{U} = \frac{\int_0^R 2\pi (R-y)\, U \left(\frac{y}{R}\right)^{1/7} dy}{\int_0^R 2\pi (R-y)\, dy}$$

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$$\bar{U} = \frac{\int_0^R 2\pi (R-y)\, U \left(\frac{y}{R}\right)^{1/7} dy}{\int_0^R 2\pi (R-y)\, dy}$$

$$= \frac{2\pi U \int_0^R \frac{R\, y^{1/7}}{R^{1/7}} - \frac{y^{8/7}}{R^{1/7}}\, dy}{2\pi \int_0^R R - y\, dy}$$

$$= \frac{\frac{2\pi U}{R^{1/7}} \left[\frac{R\, y^{8/7}}{8/7} - \frac{y^{15/7}}{15/7} \right]_0^R}{2\pi R y - \frac{y^2}{2} \Big|_0^R}$$

$$= \frac{\frac{2\pi U}{R^{1/7}} \left(\frac{R \cdot R^{8/7}}{8/7} - \frac{R^{15/7}}{15/7} \right)}{2\pi R^2 - \frac{2\pi R^2}{2}}$$

$$= \frac{2\pi U \left(\frac{R \cdot R^{7/7}}{8/7} - \frac{R^{14/7}}{15/7} \right)}{2\pi R^2 - \pi R^2}$$

$$= \frac{2\pi U\left(\frac{R - R^{7/7}}{8/7} - \frac{R^{14/7}}{15/7}\right)}{2\pi R^2 - \pi R^2}$$

$$= \frac{2U\left(\frac{R^2}{8/7} - \frac{R^2}{15/7}\right)}{R^2}$$

$$= \frac{2U R^2\left(\frac{7}{8} - \frac{7}{15}\right)}{R^2}$$

$$= 2U\left(\frac{7(15) - 8(7)}{(8)(15)}\right)$$

$$= 2U\left(\frac{49}{120}\right) = \frac{49}{60}U$$

TA DA!

VOLUMETRIC FLOW RATE

DEFN: $\bar{U} \cdot A$

$\overline{U} \cdot A$ $\quad A = \pi R^2$ $\quad 11/12$

$$\therefore \quad Q = \frac{49}{60} U \pi R^2$$

Discussion

- Direct application of defn. $\overline{U}$.
- Complexity in that want to perform integration across pipe in $+y$ coordinates because the $1/7$ power otherwise complicates integral

PIPE SYSTEM

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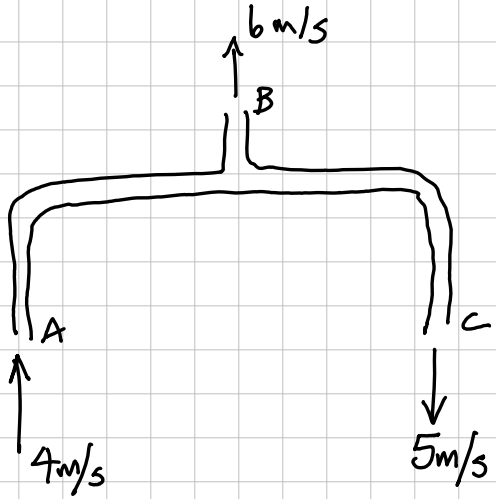
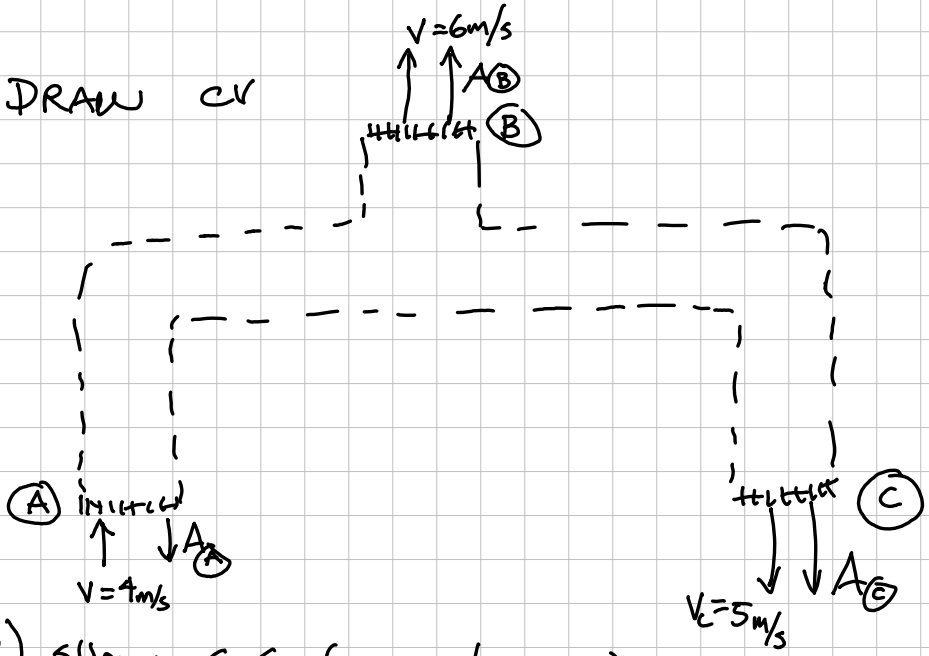


1) DRAW CV



2) SHOW C.S. (INLETS/OUTLETS)

3) SHOW VELOCTIES & AREA VECTORS