CE 3105 - Mechanics of Fluids Laboratory

Experiment 2: Forces on Plane Surfaces, Archimedes' Principle

http://bit.ly/CE3105Statics

1 Objectives

- Understand the fluid statics and forces on a body
- Measure the buoyancy force on a number of objects
- Determine the hydrostatic thrust acting on a plane surface immersed in water when the surface is fully submerged

2 Theory

2.1 Archimedes Principle

The Archimedes' volume discovery states that when a solid object is completely immersed into a fluid (say water), the volume of the fluid displaced is equal to the volume of the object. When an object is partially displaced then the volume of the water displaced is equal to the volume of the object that is immersed. Archimedes discovery is useful to find the volume of irregularly shaped objects - Simply submerge them into a fluid and measure the volume displaced.

The Archimedes' volume discovery can be generalized into the Archimedes principle. Archimedes principle states that when a solid object is fully or partially submerged into a fluid then the upward buoyant force exerted by the fluid on the object is equal to the weight of the fluid displaced by the object. Mathematically, this can be stated as:

$$B = w_f = \rho_f \times V_f \times g \tag{1}$$

Where, V_f is the volume of the fluid displaced m^3, ρ_f is the density of the fluid (g/m^3), g is the acceleration due to gravity m/s^2 and B is the buoyant force (N) and w_f is the weight of the displaced fluid (N).

Consider a submerged object shown in Figure 1. The weight of the object causes the body to sink into the fluid, while the upward buoyant force tries to push it up. If the object does not sink (or raise) then these forces are balanced. Also, for a submerged fluid, the volume of the object equals the

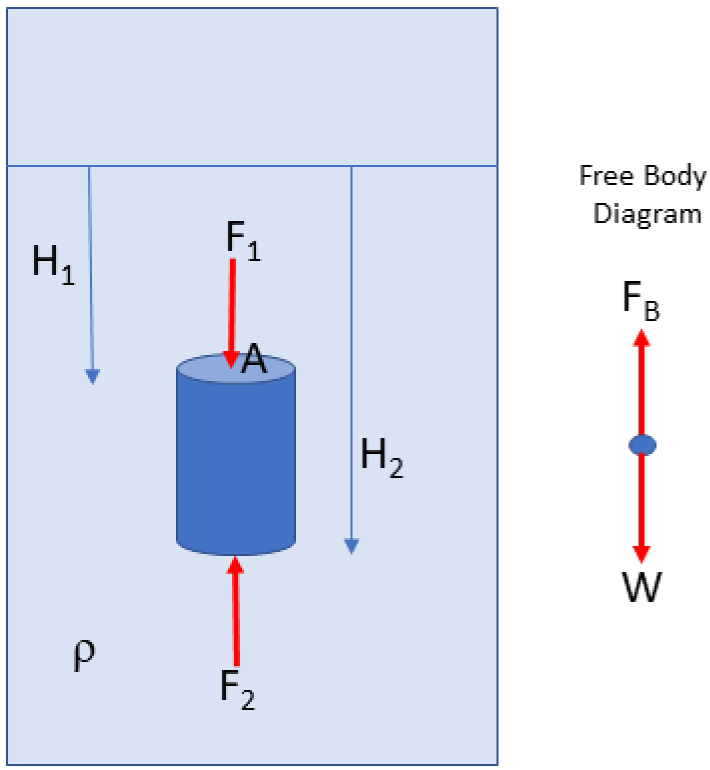

Figure 1: Forces on a Submerged Object

volume of the fluid displaced. Therefore, the resultant force can be written as:

$$B - w_o = (\rho_f - \rho_o)\, V_o g \tag{2}$$

Where, B is the buoyant force (N), w_o is the weight of the object (N), ρ_f is the density of the fluid g/m^3,ρ_o is the density of the object g/m^3, g is the acceleration due to gravity m/s^2 and V_o is the volume of the object m^3. From, Archimedes Principle, the Buoyant force (B) is equal to the weight of the fluid displaced. From Equation 2, we can say an object will be pushed upward if ρ_f is greater ρ_o or sink (pushed downward) when ρ_f less than ρ_o.

It is important to remember that when an object sinks to the bottom of the fluid in a container, there is still a buoyant force acting on it. This buoyant force is still equal to the weight of the fluid that is displaced. However, the upward buoyant force exerted on the object is insufficient to overcome the downward gravitational force which causes the object to sink. In this situation, as the object is fully submerged, the volume of the object is equal to the volume of the fluid displaced. The object will feel lighter than in air due to buoyancy.

When an object is floating on a surface (neither raising or sinking) then the volume of the fluid displaced by the object is not equal to the volume of the object. It is only equal to the volume of the object that is submerged in the fluid. However, the upward buoyant force of the fluid exactly balances the weight of the object. Also, from Archimedes principle, the buoyant force is equal to the weight of the fluid that is displaced. Mathematically,

$$B - W_o = 0 \tag{3}$$

$$\rho_f \times V_f \times g = \rho_o \times V_o \times g \tag{4}$$

$$\frac{V_o}{V_f} = \frac{\rho_f}{\rho_o} \tag{5}$$

We can therefore find the volume of the object V_o is we know the density of the substance.

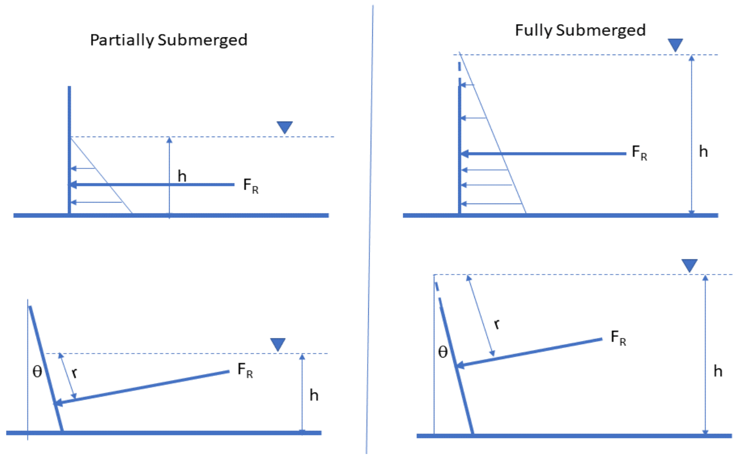

Figure 2: Center of Pressure on a Plane Surface

2.2 Forces on Plane Surfaces

The force exerted by fluids on surfaces they come to contact with is important when determining engineering structures to hold fluids (e.g., tanks, reservoirs). If water is our fluid of interest, then the hydrostatic pressure exerted by the fluid varies linearly with depth. It is equal to zero gage pressure when the water is open to atmosphere and increases downward (see Figure 2). For engineering design, we replace the pressure distribution with a resultant hydrostatic pressure acting at the centroid. This resultant force is obtained by multiplying the hydrostatic pressure with the cross-sectional area over which the fluid exerts the pressure. The pressure is assumed to act normal to this planar area. The point where the resultant pressure (force) is assumed to act is also referred to the **center of pressure** (see Figure 2).

The center of the pressure is defined as "the point in a plane at which the total fluid thrust can be said to be acting normal on that plane". Figure 3 show the apparatus which permits the moment due to to the total fluid thrust on a wholly or partially submerged plane surface to be measured directly and compared with theoretical analysis. Figure 4 illustrates the apparatus set up in laboratory.

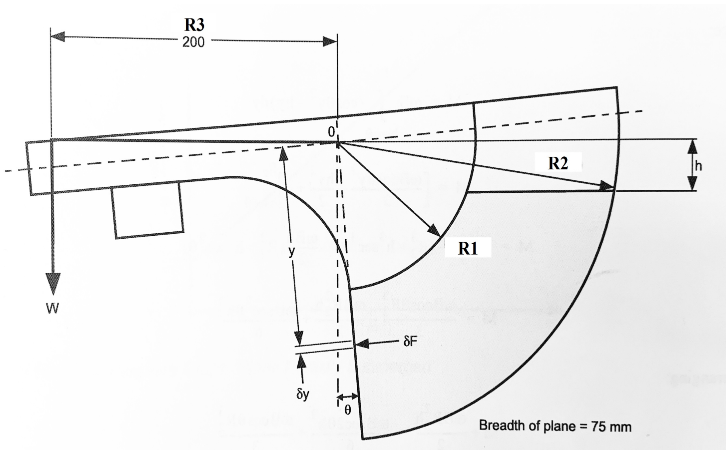

Figure 3: Pressure Forces on a Plane Surface

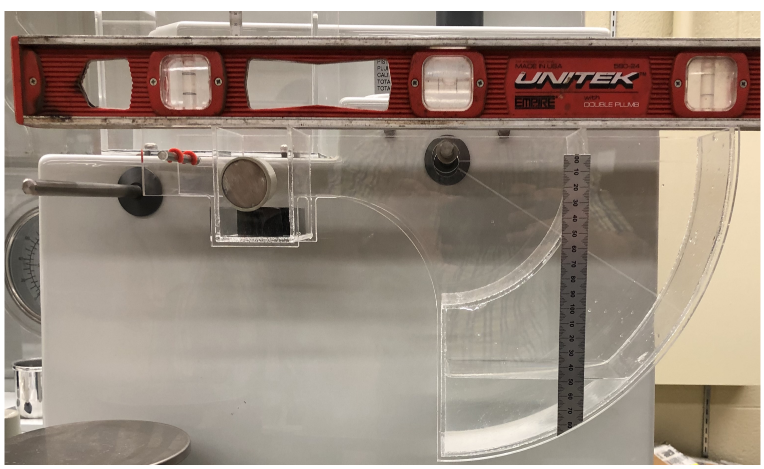

Figure 4: Apparatus set up in Laboratory

The following analysis is applied to the general condition of plane surface at various angles when it is wholly or partially submerged in a fluid.

Consider an element at an inclined depth y and height δy. The force on this infinitesimal element can be written as:

$$\delta F = \gamma_w (y cos\theta - h) W \delta y \tag{6}$$

Where, γ_w is the weight per unit volume of the fluid N/m^3, W is the width of the plane normal to the direction of the force and h is the height to the water surface measured from the pivot point O (see Figure 3). The moment of force on element about the point O δM can be written as:

$$\delta M = \gamma_w W (y cos\theta - h) y \delta y \tag{7}$$

The total moment (M) over the entire submerged surface can be obtained by integrating equation 7 as follows:

$$M = \gamma_w W \int (cos\theta y^2 - hy) dy \tag{8}$$

The limits of integration in 8 depends upon whether the plane is fully submerged or partially submerged. For the fully submerged case, the limits are from R_1 to R_2 and the equation for the moment around point O can be written as:

$$M = \frac{\gamma_w W cos\theta}{3}(R_2^3 - R_1^3) - \frac{\gamma_w W}{2}(R_2^2 - R_1^2)h \tag{9}$$

This equation is of the form of $y = mx + c$,therefore, a plot of M against h will yield a straight line of gradient $-\frac{\gamma_w W}{2}(R_2^2 - R_1^2)$. The slope can be used to calculate the unknown specific weight γ_w.

For a partially submerged plane, the limits of the integration are from R_2 to $hsec\theta$. The moment in this case is given as:

$$M = \frac{\gamma_w W sec^2\theta h^3}{6} + \frac{\gamma_w W cos\theta R_2^3}{3} - \frac{\gamma_w W R_2^2 h}{2} \tag{10}$$

A plot of $M + \frac{\gamma_w W R_2^2 h}{2}$ versus h^3 will result in a straight line plot.

3 Procedure

PART I

1. Measure the temperature of the water
2. Fill a graduated cylinder with water, record initial volume level
3. Weigh the object #1
4. Immerse object #1 in the water, record the new volume level.
5. remove the object and repeat the above steps to record the volume displaced
6. Weigh the object #2.
7. Immerse object #2 completely in the water, record the volume displaced
8. remove the object and repeat the above steps
9. Weigh the object #3.
10. Immerse object #3 completely in the water, record the volume displaced.
11. remove the object and repeat the above steps

PART II

1. Measure the temperature of the water
2. Before each experiment make sure both tanks are empty and trim the assembly to bring the submerged plane to the vertical (0° position)
3. Pour water into the trim tank until the balance reaches the 0° position.You may need to add one of the weight hangers and a few masses to help.

4. Add the second weight hanger and additional weights.
5. Pour water into the quadrant tank until the apparatus is level again. Note the additional weights and the level of the water (h).
6. Use a ruler to measure the distance to the outer edge of the water surface from the planar surface (b).
7. Repeat the procedure for the full range of weights (at least 3 measurements for partially submerged and fully submerged cases)

4 Calculations

PART I

1. Obtain the density of water and other materials used in the lab for the measured temperature from a reputable source (textbook, web). Note the citation
2. Calculate the volume of displaced water, ΔV, for all objects
3. Using Equation 1, find the buoyancy force, F_B, for each object
4. For fully submerged objects, calculate the volume of the object based on Archimedes volume discovery.
5. Use the measured mass and the density of the object to calculate the volume of the objects. How do they compare with volumes calculated in the above step?
6. For the floating object - Calculate the mass of the object using Archimedes principle. How does this mass compare to the measured value?

PART II

1. Calculate moment (M) for each weight.
2. Using the $\sum M = 0$ principle calculate the approximate vertical force for each set of measurements. This force acts at a distance of 3b/8 from

the planar surface. This is strictly true for partially submerged conditions but is taken as a first-order approximation for the fully submerged case as well.

3. Plot moment M (Nm) vs. height of depth of fluid, h (m) for the fully submerged dataset. Fit a straight line and compute the coefficient of determination R^2.

4. Use the slope of the line from previous step - which is equal to $-\frac{\gamma_w W}{2}(R_2^2 - R_1^2)$. Using R_1 = 100 mm and R_2= 200 mm, calculate the weight of water per unit volume (γ_w). Compare this value to the specific weight reported in the literature for the measured temperature.

5. For the partially submerged dataset plot - $M + \frac{\gamma_w W R_2^2 h}{2}$ versus h^3. Fit a straight line and evaluate the adequacy of the fit using the coefficient of determination R^2.

5 Interpretation Questions (for Report)

1. What is Archimedes' Principle?

2. A rock is thrown in a beaker of water and it sinks to the bottom. Is the buoyant force on the rock greater than, less than, or equal to the weight of the rock? Explain your answer

3. An embankment that is 50 m high x 20 m wide is to be constructed to hold water. Assuming the embankment is to be constructed using concrete of density 2400 kg/m^3, what is the minimum thickness necessary to withhold the water when full such that there is no over-turning. Assume the embankment is a cuboid. Hint: the moment exerted by the embankment at the bottom must balance the moment exerted by the water.

6 Datasheet

A datasheet will be provided to you in the lab.