EXAMPLE 6.11 PROPELLANT MASS RATIO FOR ACHIEVING ORBITAL VELOCITY

A single-stage rocket utilizing a liquid oxygen/kerosene propellant has a specific impulse of 3200 m/s. The orbital velocity for an earth satellite is 7600 m/s. What would be the ratio of propellant mass to total initial mass to achieve orbital velocity?

Problem Definition

Situation: Rocket launch to achieve orbital velocity.

Find: Ratio of propellent mass to initial mass.

Plan

1. Use Eq. (6.18) to calculate initial/final mass ratio.
2. Calculate the propellant/initial mass ratio using $m_p = m_i - m_f$.

Solution

1. From Eq. (6.18)

$$v_{bo} = I_{sp} \ln \frac{m_i}{m_f}$$

$$\frac{m_i}{m_f} = \exp\left(\frac{v_{bo}}{I_{sp}}\right) = \exp\left(\frac{7600}{3200}\right) = 10.7$$

2. Solve for propellant/initial mass ratio:

$$\frac{m_p}{m_i} = 1 - \frac{m_f}{m_i}$$

$$= 1 - \frac{1}{10.7} = \boxed{0.906}$$

Review

For single-stage rockets, a very large fraction of the initial mass must be propellant to achieve orbital speeds. For this reason, multi-stage rockets are used in space applications.

Example 6.11 illustrates the use of the equation for rocket burnout velocity for conditions necessary to achieve orbital velocity for a earth satellite.

Water Hammer: Physical Description

Whenever a valve is closed in a pipe, a positive pressure wave is created upstream of the valve and travels up the pipe at the speed of sound. In this context a positive pressure wave is defined as one for which the pressure is greater than the existing steady-state pressure. This pressure wave may be great enough to cause pipe failure. Therefore, a basic understanding of this process, which is called *water hammer,* is necessary for the proper design and operation of such systems. The simplest case of water hammer will be considered here. For a more comprehensive treatment of the subject, the reader is referred to Chaudhry (1) and Streeter and Wylie (2).

Consider flow in the pipe shown in Fig. 6.7. Initially the valve at the end of the pipe is only partially open (Fig. 6.7*a*); consequently, an initial velocity V and initial pressure p_0 exist in the pipe. At time $t = 0$ it is assumed that the valve is instantaneously closed, thus creating a pressure increase behind the valve and a pressure wave that travels from the valve toward the reservoir at the speed of sound, *c*. All the water between the pressure wave and the upper end of the pipe will have the initial velocity V, but all the water on the other side of the pressure wave (between the wave and the valve) will be at rest. This condition is shown in Fig. 6.7*b*. Once the pressure wave reaches the upper end of the pipe (after time $t = L/c$), it can be visualized that all of the water in the pipe will be under a pressure $p_0 + \Delta p$; however, the pressure in the reservoir at the end of the pipe is only p_0. This imbalance of pressure at the reservoir end causes the water to flow from the pipe back into the reservoir with a velocity V. Thus a new pressure wave is formed that travels toward the valve end of the pipe (Fig. 6.7*c*), and the pressure on the reservoir side of the wave is reduced to p_0. When this wave finally reaches the valve, all the water in the pipe is flowing toward the reservoir with a velocity V. This condition is only momentary, however, because the closed valve prevents any sustained flow.

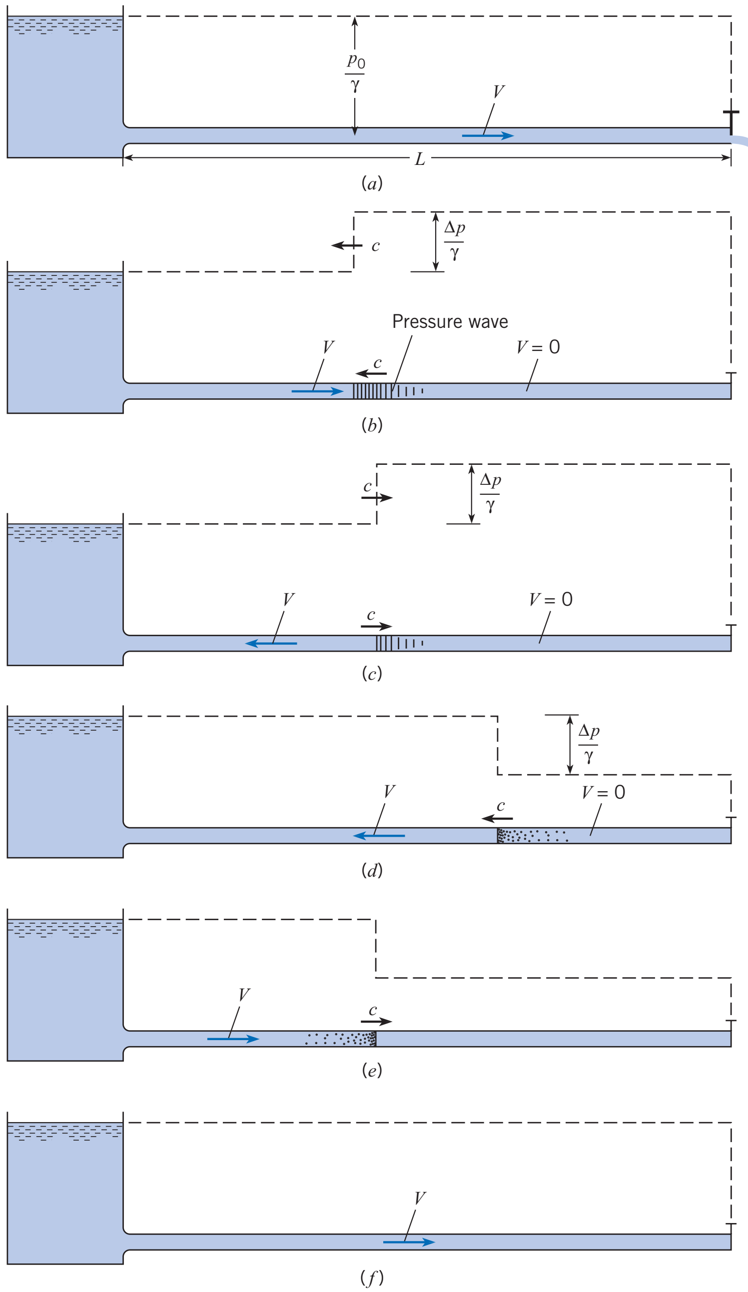


Figure 6.7

Water hammer process. (a) Initial condition. (b) Condition during time $0 < t < L/c$. (c) Condition during time $L/c < t < 2L/c$. (d) Condition during time $2L/c < t < 3L/c$. (e) Condition during time $3L/c < t < 4L/c$. (f) Condition at time $t = 4L/c$.

Next, during time $2L/c < t < 3L/c$, a rarefied wave of pressure $(p < p_0)$ travels up to the reservoir, as shown in Fig. 6.7*d*. When the wave reaches the reservoir, all the water in the pipe has a pressure less than that in the reservoir. This imbalance of pressure causes flow to be established again in the entire pipe, as shown in Fig. 6.7*f*, and the condition is exactly the same as in the initial condition (Fig. 6.7*a*). Hence the process will repeat itself in a periodic manner.

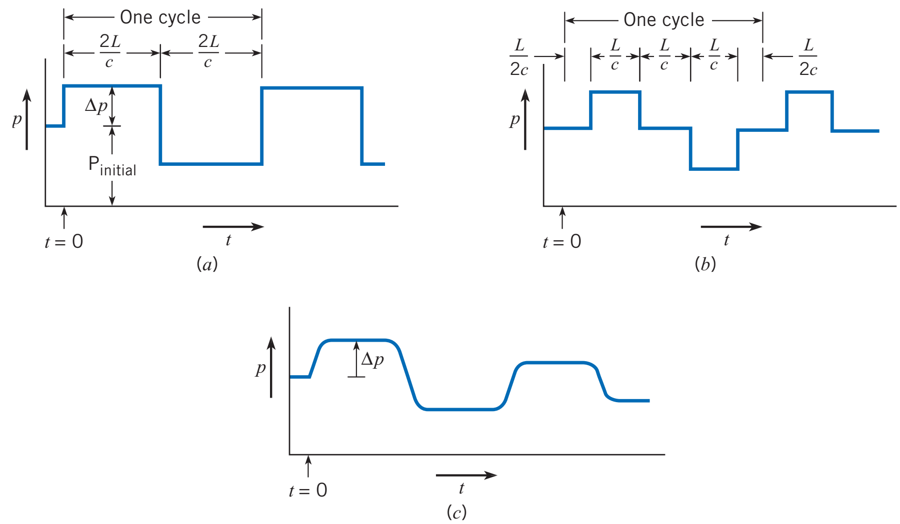


Figure 6.8

Variation of water hammer pressure with time at two points in a pipe. (a) Location: adjacent to valve. (b) Location: at midpoint of pipe. (c) Actual variation of pressure near valve.

From this description, it may be seen that the pressure in the pipe immediately upstream of the valve will be alternately high and low, as shown in Fig. 6.7*a.* A similar observation for the pressure at the midpoint of the pipe reveals a more complex variation of pressure with time, as shown in Fig. 6.8*b*. Obviously, a valve cannot be closed instantaneously, and viscous effects, which were neglected here, will have a damping effect on the process. Therefore, a more realistic pressure–time trace for the point just upstream of the valve is given in Fig. 6.8*c*. The finite time of closure erases the sharp discontinuities in the pressure trace that were present in Fig. 6.8*a.* However, it should be noted that the maximum pressure developed at the valve will be virtually the same as for instantaneous closure if the time of closure is less than $2L/c$. That is, the change in pressure will be the same for a given change in velocity unless the negative wave from the reservoir mitigates the positive pressure, and it takes a time $2L/c$ before this negative wave can reach the valve. The value $2L/c$ is called the *critical time of closure* and is given the symbol t_c.

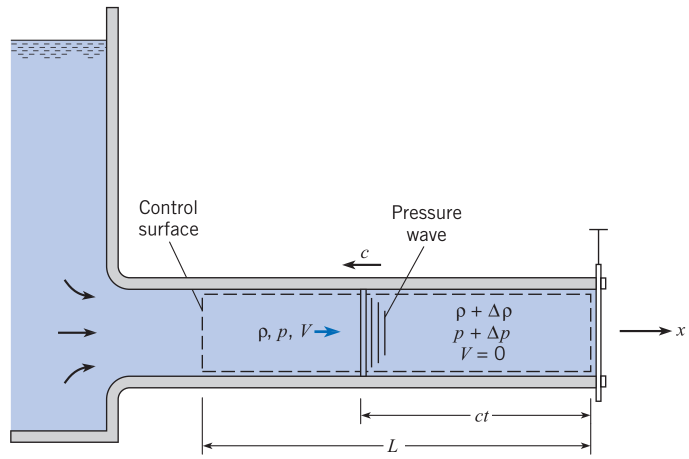
Magnitude of Water Hammer Pressure and Speed of Pressure Wave

The quantitative relations for water hammer can be analyzed with the momentum equation by letting the control volume either move with the pressure wave, thus creating steady motion, or be fixed, thus retaining the inherently unsteady character of the process. To illustrate the use of the momentum equation with unsteady motion, the latter approach will be taken. Consider a pressure wave in a rigid pipe, as shown in Fig. 6.9. The density, pressure, and velocity of the fluid on the reservoir side of the pressure wave are ρ, p, and V, respectively, and the similar quantities on the valve side of the wave are $\rho + \Delta\rho$, $p + \Delta p$, and 0. Because the wave in this case is traveling from the valve to the reservoir, its distance from the valve at any time t is given as ct. The momentum equation can now be applied to the flow in the control volume. Let the x-direction be along the pipe. The equation for x-momentum, Eq. 6.7a, simplifies to

$$\sum F_x = \frac{d}{dt}\int_{\text{cv}} v_x \rho \, d\forall - \dot{m} v_i$$

Figure 6.9

Pressure wave in a pipe.



The force terms are given by

$$\sum F_x = pA - (p + \Delta p)A$$

The inlet momentum flow is given by $\dot{m}v_i = \rho A V^2$. The momentum within the control volume decreases with time because fluid that is in motion stops as the pressure wave passes by. Evaluation of the momentum accumulation term gives

$$\frac{d}{dt}\int_{\text{cv}} v_x \rho \, d\mathcal{V} = \frac{d}{dt}[V\rho(L - ct)A]$$
$$= -V\rho c A$$

When force and momentum terms are substituted into the momentum equation, one obtains

$$pA - (p + \Delta p)A = -\rho V^2 A - \rho V c A$$

This reduces to

$$\Delta p = \rho V^2 + \rho V c$$

In this equation the first term on the right-hand side is usually negligible with respect to the second term on the right, because for liquids c is much greater than V. Consequently, the equation simplifies to

$$\Delta p = \rho V c \tag{6.19}$$

The speed of the pressure wave can be obtained by applying the continuity equation to the control volume in Fig. 6.9. The continuity equation is

$$0 = \sum \dot{m}_o - \sum \dot{m}_i + \frac{d}{dt}\int_{\text{cv}} \rho \, d\mathcal{V}$$

and when applied to Fig. 6.9 results in

$$0 = \dot{m}_i + \frac{d}{dt}[\rho(L - ct)A + (\rho + \Delta\rho)ctA]$$

because there is no mass flow out of the control volume. The mass flow rate is given by $\dot{m}_i = \rho V A$, so the continuity equation reduces to

$$\frac{\Delta\rho}{\rho} = \frac{V}{c}$$

or

$$c = \frac{V}{\Delta\rho/\rho} \tag{6.20}$$

However, by definition $E_v = \Delta p/(\Delta\rho/\rho)$. Therefore,

$$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{E_v} \tag{6.21}$$

Now when $\Delta\rho/\rho$ is eliminated between Eqs. (6.20) and (6.21), the result is

$$c = \frac{VE_v}{\Delta p} \tag{6.22}$$

From Eq. (6.19), $\Delta p = \rho Vc$. Therefore, Eq. (6.22) becomes

$$c = \sqrt{\frac{E_v}{\rho}} \tag{6.23}$$

Thus, by application of the momentum and continuity equations, expressions for both Δp and c have been derived.

Example 6.12 illustrates how to calculate the pressure rise due to the water hammer effect.

EXAMPLE 6.12 PRESSURE RISE DUE TO WATER HAMMER EFFECT

A rigid pipe leading from a reservoir is 3000 ft long, and water is flowing through it with a velocity of 4 ft/s. If the initial pressure at the downstream end is 40 psig, what maximum pressure will develop at the downstream end when a rapid-acting valve at that end is closed in 1 s?

Problem Definition

Situation: Water flowing in pipe and valve closed quickly.

Find: Maximum pressure (psig) at downstream end.

Assumptions: Water temperature is 60°F.

Properties: From Table A.5, $E_v = 3.2 \times 10^5$ lbf/in^2, and $\rho = 1.94$ slugs/ft^3.

Plan

1. Calculate the speed of sound in the water from Eq. (6.23).
2. Calculate the critical closure time, t_c.
3. Check to ensure that valve closure time is less than t_c.
4. Calculate pressure rise using Eq. (6.19) and add initial pipe pressure.

Solution

1. Calculation for sound speed:

$$c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{320{,}000 \text{ lbf/in}^2 \times 144 \text{ in}^2/\text{ft}^2}{1.94 \text{ slugs/ft}^3}} = 4874 \text{ ft/s}$$

2. Calculation for critical closure time:

$$t_c = 2L/c$$
$$= 2(3000 \text{ ft}/4874 \text{ ft/s}) = 1.23 \text{ s}$$

3. Closure time of 1 s is less than 1.23 s.
4. Pressure rise calculation:

$$\Delta p = \rho Vc$$
$$= 1.94 \text{ slugs/ft}^3 \times 4 \text{ ft/s} \times 4874 \text{ ft/s}$$
$$= 37{,}820 \text{ lbf/ft}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 263 \text{ psi}$$

Maximum pressure is

$$p_{\text{max}} = 40 + 263 = \boxed{303 \text{ psig}}$$

As indicated by Example 6.12, water hammer pressures can be quite large. Therefore, engineers must design piping systems to keep the pressure within acceptable limits. This is done by installing an accumulator near the valve and/or operating the valve in such a way that rapid closure is prevented. Accumulators may be in the form of air chambers for relatively small systems, or surge tanks (a surge tank is a large open tank connected by a branch pipe to the main pipe) for large systems. Another way to eliminate excessive water-hammer pressures is to install pressure-relief valves at critical points in the pipe system. These valves are pressure-activated so that water is automatically diverted out of the system when the water-hammer pressure reaches excessive levels.

6.5

Moment-of-Momentum Equation

The moment-of-momentum equation is very useful for situations that involve torques. Examples include analyses of rotating machinery such as pumps, turbines, fans, and blowers.

Torques acting on a control volume are related to changes in angular momentum through the moment-of-momentum equation. Development of this equation parallels the development of the momentum equation as presented in Section 6.1. When forces act on a system of particles, used to represent a fluid system, Newton's second law of motion can be used to derive an equation for rotational motion:

$$\sum \boldsymbol{M} = \frac{d(\boldsymbol{H}_{\text{sys}})}{dt} \tag{6.24}$$

where $\boldsymbol{M}$ is a moment and $\boldsymbol{H}_{\text{sys}}$ is the total angular momentum of all mass forming the system.

Equation (6.24) is a Lagrangian equation, which can be converted to an Eulerian form using the Reynolds transport theorem from Eq. (5.21). The extensive property B_{sys} becomes the angular momentum of the system: $B_{\text{sys}} = \boldsymbol{H}_{\text{sys}}$. The intensive property b becomes the angular momentum per unit mass. The angular momentum of an element is $\boldsymbol{r} \times m\boldsymbol{v}$, and so $b = \boldsymbol{r} \times \boldsymbol{v}$. Substituting for B_{sys} and b into Eq. (5.21) gives

$$\frac{d(\boldsymbol{H}_{\text{sys}})}{dt} = \frac{d}{dt}\int_{\text{cv}} (\boldsymbol{r} \times \boldsymbol{v})\rho \, d\forall + \int_{\text{cs}} (\boldsymbol{r} \times \boldsymbol{v})\rho \boldsymbol{V} \cdot d\boldsymbol{A} \tag{6.25}$$

Combining Eqs. (6.24) and (6.25) gives the integral form of the *moment-of-momentum equation*:

$$\sum \boldsymbol{M} = \frac{d}{dt}\int_{\text{cv}} (\boldsymbol{r} \times \boldsymbol{v})\rho \, d\forall + \int_{\text{cs}} (\boldsymbol{r} \times \boldsymbol{v})\rho \boldsymbol{V} \cdot d\boldsymbol{A} \tag{6.26}$$

where $\boldsymbol{r}$ is a position vector that extends from the moment center, $\boldsymbol{V}$ is flow velocity relative to the control surface, and $\boldsymbol{v}$ is flow velocity relative to the inertial reference frame selected.

The moment-of-momentum equation has the following physical interpretation: The sum of moments acting on the material within the control volume equals the rate of change of angular momentum within the control volume plus the net rate at which angular momentum flows out of the control volume.