3 FLUID STATICS

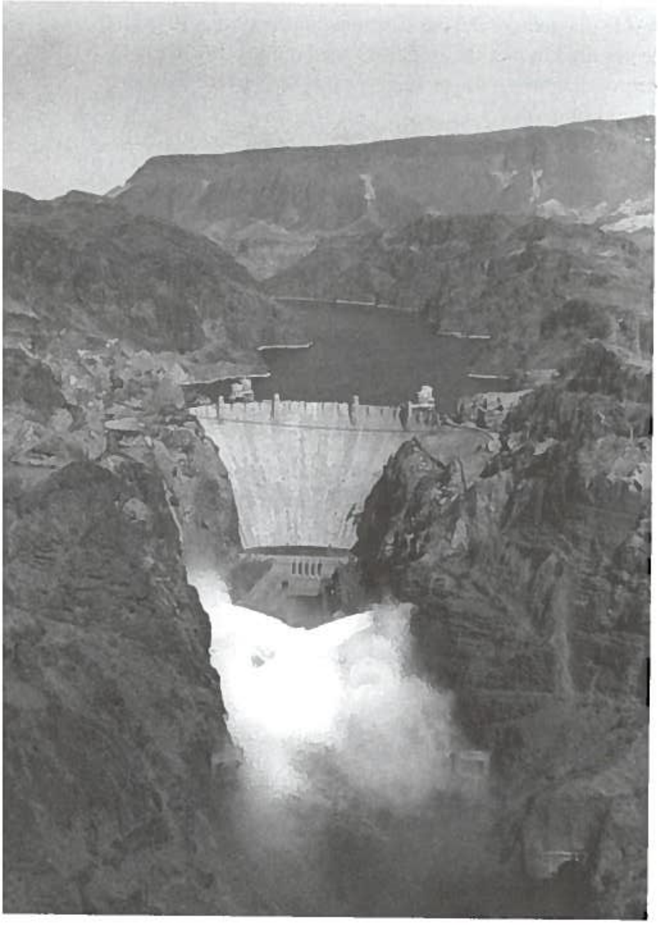


FIGURE 3.1

The first man-made structure to exceed the masonry mass of the Great Pyramid of Giza was the Hoover Dam. Design of dams involves calculations of hydrostatic forces. (Photo courtesy of U.S. Bureau of Reclamation, Lower Colorado Region)

Chapter Road Map

This chapter introduces concepts related to pressur and describes how to calculate forces associated v distributions of pressure. The emphasis is on fluids hydrostatic equilibrium.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define hydrostatic equilibrium. Define pressure. (§3.1)
- Convert between gage, absolute, and vacuum pressure. (§3
- Convert pressure units. (§3.1)
- List the steps to derive the hydrostatic differential equatior (§3.2)
- Describe the physics of the hydrostatics equation and the meaning of the variables that appear in the equation. Ap the hydrostatic equation. (§3.2)
- Explain how these instruments work: mercury barometer, piezometer, manometer, and Bourdon tube gage. (§3.3)
- Apply the manometer equations. (§3.3)
- Explain center-of-pressure and hydrostatically equivalent for Describe how pressure is related to pressure force. (§3.4)
- Apply the panel equations to predict forces and moments (§3.4)
- Solve problems that involve curved surfaces. (§3.5)
- Describe the physics of the buoyancy equation and the meaning of the variables that appear in the equation. Ap the buoyancy equation. (§3.6)
- Determine if floating objects are stable or unstable. (§3.7

As shown in Fig. 3.2, the hydrostatic condition involves equilibrium of a fluid particle. **Hydrostatic equilibrium** means that each fluid particle is in force equilibrium with the net force due to pressure balancing the weight of the fluid particle. Equations in this chapter are based on an assumption of hydrostatic equilibrium.

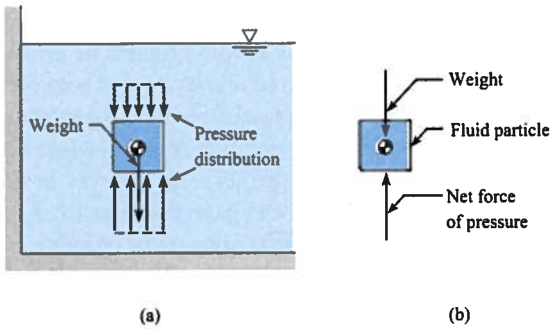


FIGURE 3.2
The hydrostatic conditi
(a) A fluid particle in a
body of fluid.
(b) Forces acting on th
fluid particle.

3.1 Describing Pressure

Because engineers use pressure in the solution of nearly all fluid mechanics problems, this section introduces fundamental ideas about pressure.

Pressure

Pressure is the ratio of normal force to area at a point.

$$p = \frac{\text{magnitude of normal force}}{\text{unit area}}\Bigg|_{\substack{\text{at a point}\\ \text{due to a fluid}}} = \lim_{\Delta A \to 0} \frac{|\Delta \vec{F}_{\text{normal}}|}{\Delta A} \tag{3.1}$$

Pressure is defined at a point because pressure typically varies with each (x, y, z) location in a flowing fluid.

Pressure is a scalar that produces a resultant force by its action on an area. The resultant force is normal to the area and acts in a direction toward the surface (compressive).

Pressure is caused by the molecules of the fluid interacting with the surface. For example, when a soccer ball is inflated, the internal pressure on the skin of the ball is caused by air molecules striking the wall.

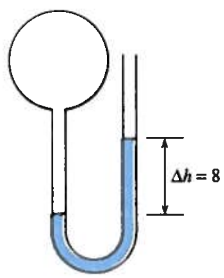
Units of pressure can be organized into three categories:

- *Force per area.* The SI unit is the newtons per square meter or pascals (Pa). The traditional units include psi, which is pounds-force per square inch, and psf, which is pounds-force per square foot.
- *Liquid column height.* Sometimes pressure units give an equivalent height of a column of liquid. For example, pressure in a balloon will push a water column upward about 8 inches as shown in Fig. 3.3. Engineers state that the pressure in the balloon is 8 inches of water: $p =$ 8 in-H_2O. When pressure is given in units of "height of a fluid column," the pressure value can be directly converted to other units using Table F.1. For example, the pressure in the balloon is

$$p = (8\ \text{in-H}_2\text{O})(248.8\ \text{Pa/in-H}_2\text{O}) = 1.99\ \text{kPa}$$

- *Atmospheres.* Sometimes pressure units are stated in terms of atmospheres where 1.0 atm is the air pressure at sea level at standard conditions. Another common unit is the bar, which is very nearly equal to 1.0 atm. (1.0 bar $= 10^5$ kPa)

FIGURE 3.3
Pressure in a balloon
causing a column of w
to rise 8 inches.



Standard atmospheric pressure in various units is

$$1.0\text{ atm} = 101.3\text{ kPa} = 14.70\text{ psi} = 33.9\text{ ft-H}_2\text{O} = 760\text{ mm-Hg} = 29.92\text{ in-Hg} = 1.013\text{ b}$$

Absolute Pressure, Gage Pressure, and Vacuum Pressure

Absolute pressure is referenced to regions such as outer space, where the pressure is essential zero because the region is devoid of gas. The pressure in a perfect vacuum is called absolu zero, and pressure measured relative to this zero pressure is termed **absolute pressure**.

When pressure is measured relative to prevailing local atmospheric pressure, the pressu value is called **gage pressure**. For example, when a tire pressure gage gives a value of 300 kl (44 psi), this means that the absolute pressure in the tire is 300 kPa greater than local atm spheric pressure. To convert gage pressure to absolute pressure, add the local atmospher pressure. For example, a gage pressure of 50 kPa recorded in a location where the atmospher pressure is 100 kPa is expressed as either

$$p = 50\text{ kPa gage} \quad \text{or} \quad p = 150\text{ kPa abs} \qquad (3.$$

In SI units, gage and absolute pressures are identified after the unit as shown in Eq. (3.2). I tradtional units, gage pressure is identifed by adding the letter g to the unit abbreviation. Fo example, a gage pressure of 10 pounds per square foot is designated as 10 psfg. Similarly, th letter a is used to denote absolute pressure. For example, an absolute pressure of 20 pounc force per square inch is designated as 20 psia.

When pressure is less than atmospheric, the pressure can be described using vacuum pre sure. **Vacuum pressure** is defined as the difference between atmospheric pressure and actu pressure. Vacuum pressure is a positive number and equals the absolute value of gage pressu (which will be negative). For example, if a gage connected to a tank indicates a vacuum pre sure of 31.0 kPa, this can also be stated as 70.0 kPa absolute, or −31.0 kPa gage.

Figure 3.4 provides a visual description of the three pressure scales. Notice that $p_B =$ 7.45 psia is equivalent to −7.25 psig and +7.25 psi vacuum. Notice that $p_A =$ of 301 kPa abs equivalent to 200 kPa gage. Gage, absolute, and vacuum pressure can be related using equa tions labeled as the "pressure equations."

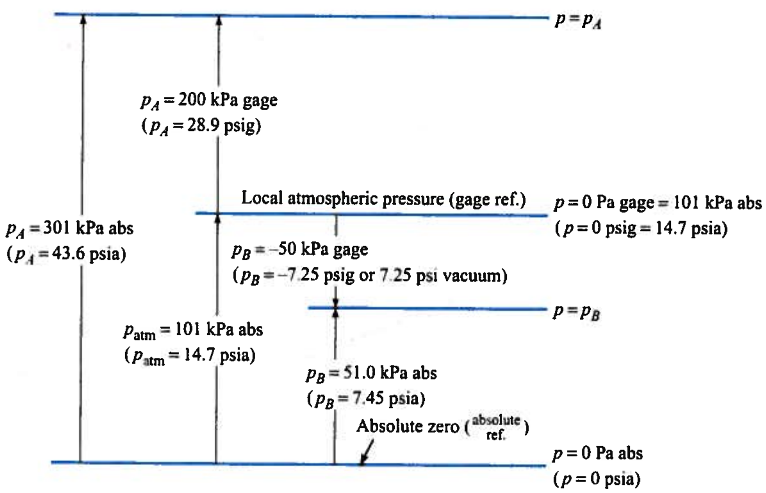
$$p_{\text{gage}} = p_{\text{abs}} - p_{\text{atm}} \qquad (3.3a$$

$$p_{\text{vacuum}} = p_{\text{atm}} - p_{\text{abs}} \qquad (3.3b$$

$$p_{\text{vacuum}} = -p_{\text{gage}} \qquad (3.3c$$

FIGURE 3.4

Example of pressure relations.



EXAMPLE. Convert 5 psi vacuum to absolute pressure in SI units.

Solution. First, convert vacuum pressure to absolute pressure.

$$p_{abs} = p_{atm} - p_{vacuum} = 14.7 \text{ psi} - 5 \text{ psi} = 9.7 \text{ psia}.$$

Second, convert units by applying a conversion ratio from Table F.1.

$$p = (9.7 \text{ psi})\left(\frac{101.3 \text{ kPa}}{14.7 \text{ psi}}\right) = 66{,}900 \text{ Pa absolute}.$$

Review. It is good practice, when writing pressure units, to specify whether the pressure is absolute, gage, or vacuum.

EXAMPLE. Suppose the pressure in a car tire is specified as 3 bar. Find the absolute pressure in units of kPa.

Solution. Recognize that tire pressure is commonly specified in gage pressure. Thus, convert the gage pressure to absolute pressure.

$$p_{abs} = p_{atm} + p_{gage} = (101.3 \text{ kPa}) + (3 \text{ bar})\frac{(101.3 \text{ kPa})}{(1.013 \text{ bar})} = 401 \text{ kPa absolute}$$

Hydraulic Machines

A **hydraulic machine** uses a fluid to transmit forces or energy to assist in the performance of a human task. An example of a hydraulic machine is a hydraulic car jack in which a user can supply a small force to a handle and lift an automobile. Other examples of hydraulic machines include braking systems in cars, forklift trucks, power steering systems in cars, and airplane control systems (3).

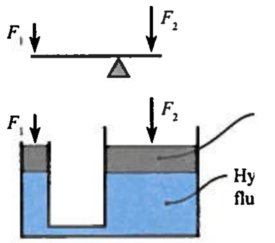
The hydraulic machine provides a mechanical advantage (Fig. 3.5). **Mechanical advantage** is defined as the ratio of output force to input force:

$$(\text{mechanical advantage}) \equiv \frac{(\text{output force})}{(\text{input force})} \tag{3.4}$$

Mechanical advantage of a lever (Fig. 3.5) is found by summing moments about the fulcrum to give $F_1L_1 = F_2L_2$, where L denotes the length of the lever arm.

$$(\text{mechanical advantage; lever}) \equiv \frac{(\text{output force})}{(\text{input force})} = \frac{F_2}{F_1} = \frac{L_1}{L_2} \tag{3.5}$$

FIGURE 3.5
Both the lever and hydraulic machine prov a mechanical advantag



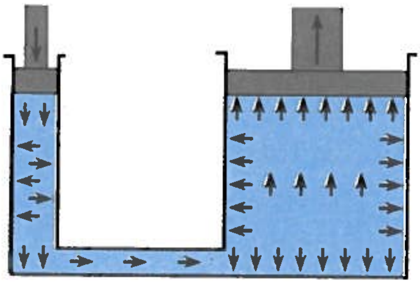
To find mechanical advantage of the hydraulic machine, apply force equilibrium to each piston (Fig. 3.5) to give $F_1 = p_1A_1$ and $F_2 = p_2A_2$, where p is pressure in the cylinder and A is face area of the piston. Next, let $p_1 = p_2$ and solve for the mechanical advantage

$$(\text{mechanical advantage; hydraulic machine}) \equiv \frac{(\text{output force})}{(\text{input force})} = \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{D_2^2}{D_1^2} \tag{3.6}$$

The hydraulic machine is often used to illustrate Pascal's principle. This principle states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase at every other point in the container. This principle is evident when a balloon is inflated because the balloon expands evenly in all directions. The principle is also evident in the hydraulic machine (Fig. 3.6).

FIGURE 3.6

The figures show how the hydraulic machine can be used to illustrate Pascal's principle.



✔ CHECKPOINT PROBLEM 3.1

What is the mechanical advantage of this hydraulic machine? (neglect pressure changes due to elevation changes)

$W = 2$ tons, $S = 0.9$

$h = 3$ inch, $D_2 = 6$ inch, $D_1 = 1$ inch

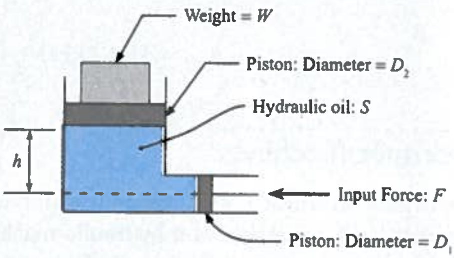
a. 2:1

b. 4:1

c. 6:1

d. 16:1

e. 36:1

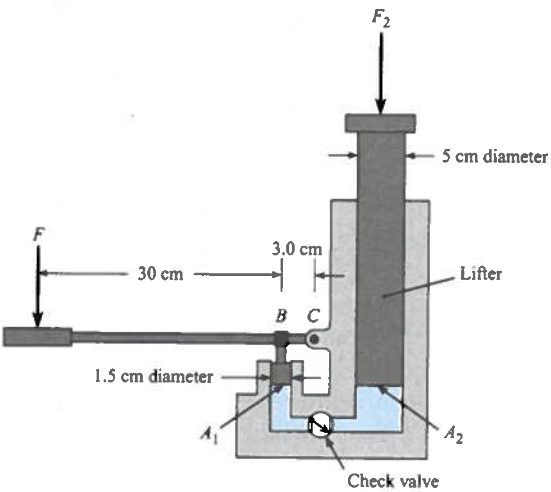


EXAMPLE 3.1

Applying Force Equilibrium to a Hydraulic Jack

Problem Statement

A hydraulic jack has the dimensions shown. If one exerts a force F of 100 N on the handle of the jack, what load, F_2, can the jack support? Neglect lifter weight.



Define the Situation

A force of $F = 100$ N is applied to the handle of a jack.

Assumption: Weight of the lifter (see sketch) is negligible.

State the Goal

F_2(N) ⬅ Load that the jack can lift

Generate Ideas and Make a Plan

Because the goal is F_2, apply force equilibrium to the lifter. Then, analyze the small piston and the handle. The plan is

1. Calculate force acting on the small piston by applying moment equilibrium.
2. Calculate pressure p_1 in the hydraulic fluid by applying force equilibrium.
3. Calculate the load F_2 by applying force equilibrium.

Take Action (Execute the Plan)

1. Moment equilibrium (handle)

$$\sum M_c = 0$$

$$(0.33 \text{ m}) \times (100 \text{ N}) - (0.03 \text{ m})F_1 = 0$$

$$F_1 = \frac{0.33 \text{ m} \times 100 \text{ N}}{0.03 \text{ m}} = 1100 \text{ N}$$

2. Force equilibrium (small piston)

$$\sum F_{\text{small piston}} = p_1A_1 - F_1 = 0$$
$$p_1A_1 = F_1 = 1100\text{ N}$$

Thus

$$p_1 = \frac{F_1}{A_1} = \frac{1100\text{ N}}{\pi d^2/4} = 6.22 \times 10^6\text{ N/m}^2$$

3. Force equilibrium (lifter)

Note that $p_1 = p_2$ because they are at the same elevation (this fact will be established in the next section).

$$\sum F_{\text{lifter}} = F_2 - p_1A_2 = 0$$

$$F_2 = p_1A_2 = \left(6.22 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4} \times (0.05\text{ m})^2\right) = \boxed{12.2\text{ kN}}$$

Review the Results and the Process

1. *Discussion.* The jack in this example, which combines a lever and a hydraulic machine, provides an output force of 12,200 N from an input force of 100 N. Thus, this jack provides a mechanical advantage of 122 to 1.
2. *Knowledge.* Hydraulic machines are analyzed by applying force and moment equilibrium. The force of pressure is typical given by $F = pA$.

3.2 Calculating Pressure Changes Associated with Elevation Changes

Pressure changes when elevation changes. For example, as a submarine dives to deeper depth, water pressure increases. Conversely, as an airplane gains elevation, air pressure decreases. Because engineers predict pressure changes associated with elevation change, this section introduces the relevant equations.

Theory: The Hydrostatic Differential Equation

All equations in fluid statics are based on the hydrostatic differential equation, which is derived in this subsection. To begin the derivation, visualize any region of static fluid (e.g., water behind a dam), isolate a cylindrical body, and then sketch a free-body diagram (FBD) as shown in Fig. 3.7. Notice that the cylindrical body is oriented so that its longitudinal axis is parallel to an arbitrary ℓ direction. The body is $\Delta\ell$ long, ΔA in cross-sectional area, and inclined at an angle α with the horizontal. Apply force equilibrium in the ℓ direction:

$$\sum F_\ell = 0$$
$$F_{\text{Pressure}} - F_{\text{Weight}} = 0$$
$$p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta\ell \sin\alpha = 0$$

Simplify and divide by the volume of the body $\Delta\ell\Delta A$ to give

$$\frac{\Delta p}{\Delta \ell} = -\gamma \sin\alpha$$

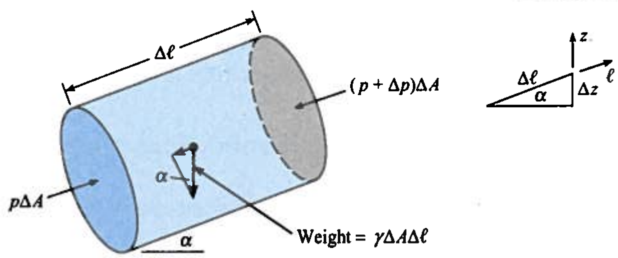


FIGURE 3.7
The system used to der the hydrostatic differen equation.

From Fig. 3.7, the sine of the angle is given by

$$\sin\alpha = \frac{\Delta z}{\Delta \ell}$$

Combining the previous two equations and letting Δz approach zero gives

$$\lim_{\Delta z \to 0} \frac{\Delta p}{\Delta z} = -\gamma$$

The final result is

$$\frac{dp}{dz} = -\gamma \qquad \text{(hydrostatic differential equation)}$$ (3

Equation (3.7) is valid in a body of fluid when the force balance shown in Fig. 3.2 is satisfie

Equation (3.7) means that changes in pressure correspond to changes in elevation. If o travels upward in the fluid (positive z direction), the pressure decreases; if one goes downwa (negative z), the pressure increases; if one moves along a horizontal plane, the pressure remai constant. Of course, these pressure variations are exactly what a diver experiences when a cending or descending in a lake or pool.

Derivation of the Hydrostatic Equation

This subsection shows how to derive the hydrostatic equation, which is used to calculate pressu variations in a fluid with constant density. To begin, assume that specific weight γ is constant a integrate Eq. (3.7) to give

$$p + \gamma z = p_z = \text{constant}$$ (3

where the term z is the elevation (vertical distance) above a fixed horizontal reference pla called a datum, and p_z is **piezometric pressure**. Dividing Eq. (3.8) by γ gives

$$\frac{p_z}{\gamma} = \left(\frac{p}{\gamma} + z\right) = h = \text{constant}$$ (3.

where h is the **piezometric head**. Because h is constant Eq. (3.9) can be written as:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$ (3.10

where the subscripts 1 and 2 identify any two points in a static fluid of constant density. Mu tiplying Eq. (3.10a) by γ gives

$$p_1 + \gamma z_1 = p_2 + \gamma z_2$$ (3.10

In Eq. (3.10b), letting $\Delta p = p_2 - p_1$ and letting $\Delta z = z_2 - z_1$ gives

$$\Delta p = -\gamma \Delta z$$ (3.10

The hydrostatic equation is given by either Eq. (3.10a), (3.10b), or (3.10c). These thr equations are equivalent because any one of the equations can be used to derive the other tw The hydrostatic equation is valid for any constant density fluid in hydrostatic equilibrium.

Notice that the hydrostatic equation involves

$$\text{piezometric head} = h \equiv \left(\frac{p}{\gamma} + z\right)$$ (3.1

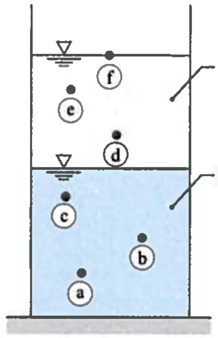
$$\text{piezometric pressure} = p_z \equiv (p + \gamma z)$$ (3.1

To calculate piezometric head or piezometric pressure, an engineer identifies a specific location in a body of fluid and then uses the value of pressure and elevation at that location. Piezometric pressure and head are related by

$$p_z = h\gamma \tag{3.13}$$

Piezometric head, h, a property that is widely used in fluid mechanics, characterizes hydrostatic equilibrium. When hydrostatic equilibrium prevails in a body of fluid of constant density, then h will be constant at all locations. For example, Fig. 3.8 shows a container with oil floating on water. Because piezometric head is constant in the water, $h_a = h_b = h_c$. Similarly the piezometric head is constant in the oil: $h_d = h_e = h_f$. Notice that piezometric head is not constant when density changes. For example, $h_c \neq h_d$ because points c and d are in different fluids with different values of density.

FIGURE 3.8 Oil floating on water.



✔ CHECKPOINT PROBLEM 3.2

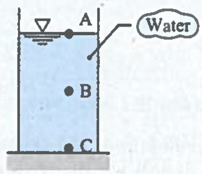
In the glass of water shown, which location has the highest value of piezeometric head? Which location has the highest value of the piezometric pressure?

a. A

b. B

c. C

d. None of the above



Hydrostatic Equation: Working Equations and Examples

The hydrostatic equation is summarized in Table 3.1.

TABLE 3.1 Summary of the Hydrostatic Equation

Name and Description	Equation	Terms
Head Form: Physics: (pressure head + elevation head at point 1) = (pressure head + elevation head at point 2). Another way to state the physics: The piezometric head in a static fluid with uniform density is constant at every point.	$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$ (3.10)	p = pressure (N/m^2) (use absolute or gage pressure; n vacuum pressure) (p/γ is also called pressure head z = elevation (m) (sketch a datum and measure z f this datum) (z is also called elevation head) γ = specific weight (N/m^3) $p/\gamma + z$ = piezometric head (m)
Pressure Change (Δp) Form: Physics: For an elevation change of Δz, the pressure in a static fluid with uniform density will change by $\gamma \Delta z$.	$\Delta p = -\gamma \Delta z = -\rho g \Delta z$ (3.10)	Δp = change in pressure between points 1 & 2 (Pa) Δz = change in elevation between points 1 & 2 (m) ρ = density (kg/m^3) g = gravitational constant (9.81 m/s^2

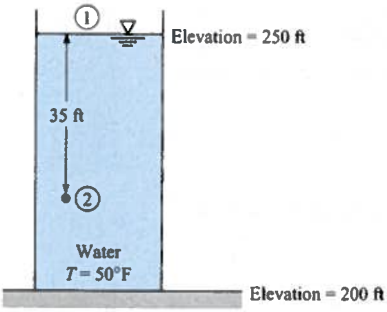
Example 3.2 shows the process for applying the hydrostatic equation.

EXAMPLE 3.2

Applying the Hydrostatic Equation to Find Pressure in a Tank

Problem Statement

What is the water pressure at a depth of 35 ft in the tank shown?



Define the Situation

Water is contained in a tank that is 50 ft deep.

Properties. Water (50 °F, 1 atm, Table A.5): $\gamma = 62.4\ \text{lbf/ft}^3$.

State the Goal

p_2 (psig) ⬅ Water pressure at point 2.

Generate Ideas and Make a Plan

Apply the idea that piezometric head is constant. Steps:

1. Equate piezometric head at elevation 1 with piezometric head at elevation 2 (i.e., apply Eq. 3.10a).
2. Analyze each term in Eq. (3.10a).
3. Solve for the pressure at elevation 2.

Take Action (Execute the Plan)

1. Hydrostatic equation (Eq. 3.10a)

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

2. Term-by-term analysis of Eq. (3.10a) yields:
   - $p_1 = p_{atm} = 0$ psig
   - $z_1 = 250$ ft
   - $z_2 = 215$ ft
3. Combine steps 1 and 2; solve for p_2

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$0 + 250\ \text{ft} = \frac{p_2}{62.4\ \text{lbf/ft}^3} + 215\ \text{ft}$$

$$p_2 = 2180\ \text{psfg} = \boxed{15.2\ \text{psig}}$$

Review the Solution and the Process

1. *Validation.* The calculated pressure change (15 psig) is slightly greater than 1 atm (14.7 psi). Because one atmosphere corresponds to a water column of 33.9 ft and this problem involves 35 ft of water column, the solution appears correct.
2. *Skill.* This example shows how to write down a governing equation and then analyze each term. This skill is called *term-by-term analysis*.
3. *Knowledge.* The gage pressure at the free surface of a liquid in contact with the atmosphere is zero ($p_1 = 0$ in this example).
4. *Skill.* Label a pressure as absolute or gage or vacuum. For this example, the pressure unit (psig) denotes a gage pressure.
5. *Knowledge.* The hydrostatic equation is valid when density is constant. This condition is met on this problem.

Example 3.3 shows how to find pressure by applying the idea of "constant piezometric head" to a problem involving several fluids. Notice the continuity of pressure across a planar interface.

EXAMPLE 3.3

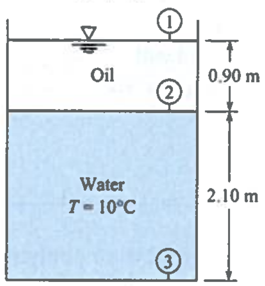
Applying the Hydrostatic Equation to Oil and Water in a Tank

Problem Statement

Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water (10°C). The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?

Problem Definition

Oil and water are contained in a tank.



Water (10°C, 1 atm, Table A.5) $\gamma_{\text{water}} = 9810\ \text{N/m}^3$.

Oil. $\gamma_{\text{oil}} = S\gamma_{\text{water, 4°C}} = 0.8(9810\ \text{N/m}^3) = 7850\ \text{N/m}^3$.

State the Goal

p_3 (kPa gage) ⬅ pressure at bottom of the tank

Generate Ideas and Make a Plan

Because the goal is p_3, apply the hydrostatic equation to the water. Then, analyze the oil. The plan steps are

1. Find p_2 by applying the hydrostatic equation (3.10a).
2. Equate pressures across the oil–water interface.
3. Find p_3 by applying the hydrostatic equation given in Eq. (3.10a).

Solution

1. Hydrostatic equation (oil)

$$\frac{p_1}{\gamma_{\text{oil}}} + z_1 = \frac{p_2}{\gamma_{\text{oil}}} + z_2$$

$$\frac{0\ \text{Pa}}{\gamma_{\text{oil}}} + 3\ \text{m} = \frac{p_2}{0.8 \times 9810\ \text{N/m}^3} + 2.1\ \text{m}$$

$$p_2 = 7.063\ \text{kPa}$$

2. Oil–water interface

$$p_2|_{\text{oil}} = p_2|_{\text{water}} = 7.063\ \text{kPa}$$

3. Hydrostatic equation (water)

$$\frac{p_2}{\gamma_{\text{water}}} + z_2 = \frac{p_3}{\gamma_{\text{water}}} + z_3$$

$$\frac{7.063 \times 10^3\ \text{Pa}}{9810\ \text{N/m}^3} + 2.1\ \text{m} = \frac{p_3}{9810\ \text{N/m}^3} + 0\ \text{m}$$

$$\boxed{p_3 = 27.7\ \text{kPa gage}}$$

Review

Validation: Because oil is less dense than water, the answer should be slightly smaller than the pressure corresponding to a water column of 3 m. From Table F.1, a water column of 10 m ≈ 1 atm. Thus, a 3 m water column should produce a pressure of about 0.3 atm = 30 kPa. The calculated value appears correct.

Pressure Variation in the Atmosphere

This subsection describes how to calculate pressure, density and temperature in the atmosphere for applications such as modeling of atmospheric dynamics and the design of gliders, airplanes, balloons, and rockets.

Equations for pressure variation in the earth's atmosphere are derived by integrating the hydrostatic differential equation (3.7). To begin the derivation, write the ideal gas law (2.5):

$$\rho = \frac{p}{RT} \tag{3.14}$$

Multiply by g:

$$\gamma = \frac{pg}{RT} \tag{3.15}$$

Equation (3.15) requires temperature-versus-elevation data for the atmosphere. It is common practice to use the U.S. Standard Atmosphere (1). The **U.S. Standard Atmosphere** defines values for atmospheric temperature, density, and pressure over a wide range of altitudes. The first model was published in 1958; this was updated in 1962, 1966, and 1976. The U.S. Standard Atmosphere gives average conditions over the United States at 45° N latitude in July.

The U.S. Standard Atmosphere also gives average conditions at sea level. The sea level temperature is 15°C (59°F), the pressure is 101.33 kPa abs (14.696 psia), and the density is 1.225 kg/m³ (0.002377 slugs/ft³).

Temperature data for the U.S. Standard Atmosphere are given in Fig. 3.9 for the lower 30 km of the atmosphere. The atmosphere is about 1000 km thick and is divided into five layers, so Fig. 3.9 only gives data near the earth's surface. In the **troposphere**, defined as the

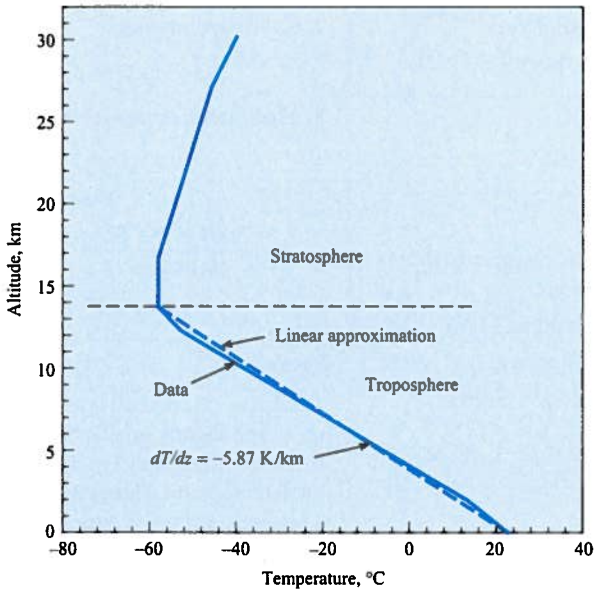


FIGURE 3.9
Temperature variation with altitude for the U.S. standard atmosphere in July (1).

layer between sea level and 13.7 km (45,000 ft), the temperature decreases nearly linearly witl increasing elevation at a lapse rate of 5.87 K/km. The **stratosphere** is the layer that begins at th top of the troposphere and extends up to about 50 km. In the lower regions of the stratospher the temperature is constant at $-57.5°C$, to an altitude of 16.8 km (55,000 ft), and then th temperature increases monotonically to $-38.5°C$ at 30.5 km (100,000 ft).

Pressure Variation in the Troposphere

Let the temperature T be given by

$$T = T_0 - \alpha(z - z_0) \quad (3.1\epsilon$$

In this equation T_0 is the temperature at a reference level where the pressure is known, and α i the lapse rate. Combine Eq. (3.15) with the hydrostatic differential equation (3.7) to give

$$\frac{dp}{dz} = -\frac{pg}{RT} \quad (3.17$$

Substituting Eq. (3.16) into Eq. (3.17) gives

$$\frac{dp}{dz} = -\frac{pg}{R[T_0 - \alpha(z - z_0)]}$$

Separate the variables and integrate to obtain

$$\frac{p}{p_0} = \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

Thus, the atmospheric pressure variation in the troposphere is

$$p = p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} \quad (3.18$$

Example 3.7 shows how to apply Eq. (3.18) to find pressure at a specified elevation in th troposphere.

Pressure Variation in the Lower Stratosphere

In the lower part of the stratosphere (13.7 to 16.8 km above the earth's surface as shown in Fig. 3.9), the temperature is approximately constant. In this region, integration of Eq. (3.17) gives

$$\ln p = \frac{zg}{RT} + C$$

At $z = z_0, p = p_0$, so the preceding equation reduces to

$$\frac{p}{p_0} = e^{-(z-z_0)g/RT}$$

so the atmospheric pressure variation in the stratosphere takes the form

$$p = p_0 e^{-(z-z_0)g/RT} \tag{3.19}$$

where p_0 is pressure at the interface between the troposphere and stratosphere, z_0 is the elevation of the interface, and T is the temperature of the stratosphere. Example 3.5 shows how to apply Eq. (3.19) to find pressure at a specified elevation in the troposphere.

EXAMPLE 3.4

Predicting Pressure in the Troposphere

Problem Statement

If the sea level pressure and temperature are 101.3 kPa and 23°C, what is the pressure at an elevation of 2000 m, assuming that standard atmospheric conditions prevail?

Situation

Standard atmospheric conditions prevail at an elevation of 2000 m.

Goal

p(kPa absolute) ⬅ atmospheric pressure at $z = 2000$ m

Plan

Calculate pressure using Eq. (3.18).

Action

$$p = p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

where $p_0 = 101{,}300\ \text{N/m}^2$, $T_0 = 273 + 23 = 296$ K, $\alpha = 5.87 \times 10^{-3}$ K/m, $z - z_0 = 2000$ m, and $g/\alpha R = 5.823$. Then

$$p = 101.3\left(\frac{296 - 5.87 \times 10^{-3} \times 2000}{296}\right)^{5.823}$$

$$= \boxed{80.0 \text{ kPa absolute}}$$

EXAMPLE 3.5

Calculating Pressure in the Lower Stratosphere

Problem Statement

If the pressure and temperature are 2.31 psia ($p = 15.9$ kPa absolute) and −71.5°F (−57.5°C) at an elevation of 45,000 ft (13.72 km), what is the pressure at 55,000 ft (16.77 km), assuming isothermal conditions over this range of elevation?

Situation

Standard atmospheric conditions prevail at an elevation of 55,000 ft (16.77 km).

Goal

p ⬅ Atmospheric pressure (psia and kPa absolute) at an elevation of 55,000 ft (16.77 km)

Plan

Calculate pressure using Eq. (3.19).

Action

For isothermal conditions,

$$T = -71.5 + 460 = 388.5°\text{R}$$

$$p = p_0 e^{-(z-z_0)g/RT} = 2.31 e^{-(10{,}000)(32.2)/(1716 \times 388.5)}$$

$$= 2.31 e^{-0.483}$$

Therefore the pressure at 55,000 ft is

$$\boxed{p = 1.43 \text{ psia}}$$

SI units

$$\boxed{p = 9.83 \text{ kPa absolute}}$$

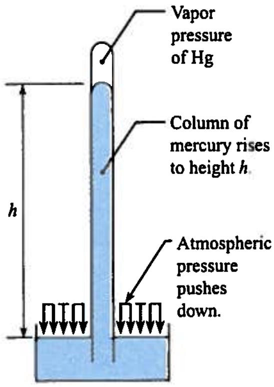
3.3 Measuring Pressure

When engineers design and conduct experiments, pressure nearly always needs to be me sured. Thus, this section describes five scientific instruments for measuring pressure.

Barometer

FIGURE 3.10

A mercury barometer.



An instrument that is used to measure atmospheric pressure is called a **barometer**. The mo common types are the mercury barometer and the aneroid barometer. A mercury baromet is made by inverting a mercury-filled tube in a container of mercury as shown in Fig. 3.1 The pressure at the top of the mercury barometer will be the vapor pressure of mercu which is very small: $p_v = 2.4 \times 10^{-6}$ atm at 20°C. Thus, atmospheric pressure will push tl mercury up the tube to a height h. The mercury barometer is analyzed by applying the hydr static equation:

$$p_{atm} = \gamma_{Hg} h + p_v \approx \gamma_{Hg} h \quad (3.2$$

Thus, by measuring h, local atmospheric pressure can be determined using Eq. (3.20).

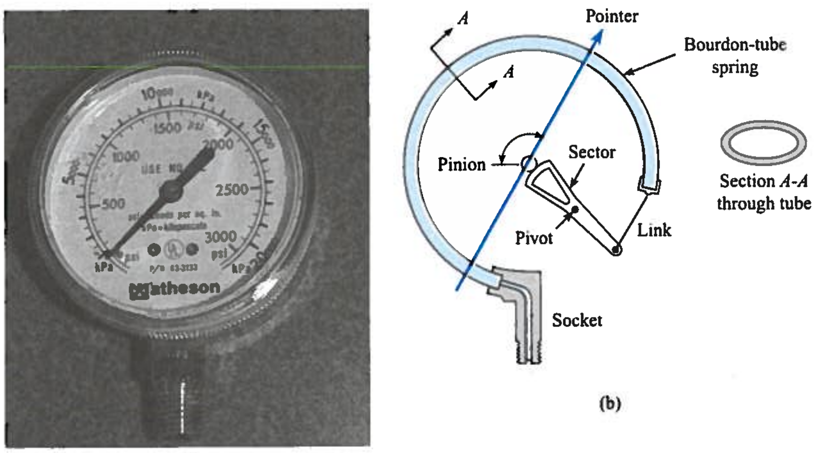
An aneroid barometer works mechanically. An aneroid is an elastic bellows that h been tightly sealed after some air was removed. When atmospheric pressure changes, tl causes the aneroid to change size, and this mechanical change can be used to deflect a need to indicate local atmospheric pressure on a scale. An aneroid barometer has some adva tages over a mercury barometer because it is smaller and allows data recording over time.

Bourdon-Tube Gage

A **Bourdon-tube** gage, Fig. 3.11, measures pressure by sensing the deflection of a coiled tul The tube has an elliptical cross section and is bent into a circular arc, as shown in Fig. 3.11 When atmospheric pressure (zero gage pressure) prevails, the tube is undeflected, and for tl

FIGURE 3.11

Bourdon-tube gage. (a) View of typical gage. (Photo by Donald Elger) (b) Internal mechanism (schematic).



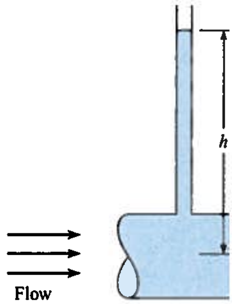
(a) (b)

condition the gage pointer is calibrated to read zero pressure. When pressure is applied to the gage, the curved tube tends to straighten (much like blowing into a party favor to straighten it out), thereby actuating the pointer to read a positive gage pressure. The Bourdon-tube gage is common because it is low cost, reliable, easy to install, and available in many different pressure ranges. There are disadvantages: dynamic pressures are difficult to read accurately; accuracy of the gage can be lower than other instruments; and the gage can be damaged by excessive pressure pulsations.

Piezometer

A **piezometer** is a vertical tube, usually transparent, in which a liquid rises in response to a positive gage pressure. For example, Fig. 3.12 shows a piezometer attached to a pipe. Pressure in the pipe pushes the water column to a height h, and the gage pressure at the center of the pipe is $p = \gamma h$, which follows directly from the hydrostatic equation (3.10c). The piezometer has several advantages: simplicity, direct measurement (no need for calibration), and accuracy. However, a piezometer cannot easily be used for measuring pressure in a gas, and a piezometer is limited to low pressures because the column height becomes too large at high pressures.

FIGURE 3.12
Piezometer attached to a pipe.



Manometer

A **manometer**, often shaped like the letter "U," is a device for measuring pressure by raising or lowering a column of liquid. For example, Fig. 3.13 shows a U-tube manometer that is being used to measure pressure in a flowing fluid. In the case shown, positive gage pressure in the pipe pushes the manometer liquid up a height Δh. To use a manometer, engineers relate the height of the liquid in the manometer to pressure as illustrated in Example 3.6.

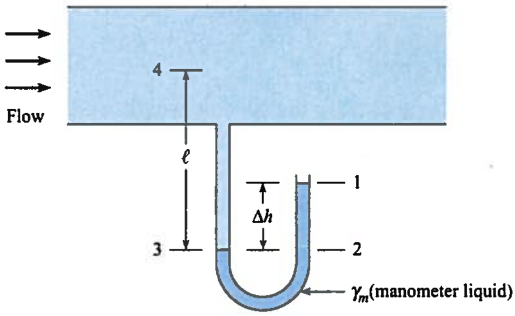


FIGURE 3.13
U-tube manometer.

EXAMPLE 3.6

Pressure Measurement (U-Tube Manometer)

Problem Statement

Water at 10°C is the fluid in the pipe of Fig. 3.13, and mercury is the manometer fluid. If the deflection Δh is 60 cm and ℓ is 180 cm, what is the gage pressure at the center of the pipe?

Define the Situation

Pressure in a pipe is being measured using a U-tube manometer.

Properties:

Water (10°C), Table A.5, $\gamma = 9810\ \text{N/m}^3$.

Mercury, Table A.4: $\gamma = 133{,}000\ \text{N/m}^3$.

State the Goal

Calculate gage pressure (kPa) in the center of the pipe.

Generate Ideas and Make a Plan

Start at point 1 and work to point 4 using ideas from Eq. (3.10c). When fluid depth increases, add a pressure change. When fluid depth decreases, subtract a pressure change.

Take Action (Execute the Plan)

1. Calculate the pressure at point 2 using the hydrostatic equation (3.10c).

$$p_2 = p_1 + \text{pressure increase between 1 and 2} = 0 + \gamma_m \Delta h_{12}$$
$$= \gamma_m(0.6\text{ m}) = (133{,}000\text{ N/m}^3)(0.6\text{ m})$$
$$= 79.8\text{ kPa}$$

2. Find the pressure at point 3.
   - The hydrostatic equation with $z_3 = z_2$ gives

$$p_3|_{\text{water}} = p_2|_{\text{water}} = 79.8\text{ kPa}$$

   - When a fluid-fluid interface is flat, pressure is constant across the interface. Thus, at the oil–water interface

$$p_3|_{\text{mercury}} = p_3|_{\text{water}} = 79.8\text{ kPa}$$

3. Find the pressure at point 4 using the hydrostatic equation given in Eq. (3.10c).

$$p_4 = p_3 - \text{pressure decrease between 3 and 4} = p_3 - \gamma_w \ell$$
$$= 79{,}800\text{ Pa} - (9810\text{ N/m}^3)(1.8\text{ m})$$
$$= 62.1\text{ kPa gage}$$

Once one is familiar with the basic principle of manometry, it is straightforward to wr a single equation rather than separate equations as was done in Example 3.6. The single equ tion for evaluation of the pressure in the pipe of Fig 3.13 is

$$0 + \gamma_m \Delta h - \gamma \ell = p_4$$

One can read the equation in this way: Zero pressure at the open end, plus the change in pre sure from point 1 to 2, minus the change in pressure from point 3 to 4, equals the pressure the pipe. The main concept is that pressure increases as depth increases and decreases as dep decreases.

The general equation for the pressure difference measured by the manometer is:

$$p_2 = p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i \qquad \text{(3.2}$$

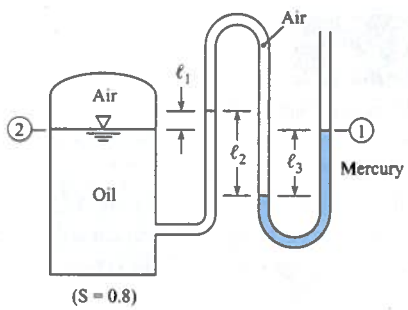
where γ_i and h_i are the specific weight and deflection in each leg of the manometer. It does n matter where one starts; that is, where one defines the initial point 1 and final point 2. Wh liquids and gases are both involved in a manometer problem, it is well within engineering a curacy to neglect the pressure changes due to the columns of gas. This is because $\gamma_{\text{liquid}} \gg \gamma_{\text{g}}$ Example 3.7 shows how to apply Eq. (3.21) to perform an analysis of a manometer that us multiple fluids.

EXAMPLE 3.7

Manometer Analysis

Problem Statement

What is the pressure of the air in the tank if $\ell_1 = 40$ cm, $\ell_2 = 100$ cm, and $\ell_3 = 80$ cm?



Define the Situation

A tank is pressurized with air.

Assumptions: Neglect the pressure change in the air column.

Properties:

- Oil: $\gamma_{oil} = S\gamma_{water} = 0.8 \times 9810\ \text{N/m}^3 = 7850\ \text{N/m}^3$.
- Mercury, Table A.4: $\gamma = 133{,}000\ \text{N/m}^3$.

State the Goal

Find the pressure (kPa gage) in the air.

Generate Ideas and Make a Plan

Apply the manometer equation (3.21) from location 1 to locatio

Take Action (Execute the Plan)

Manometer equation

$$p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i = p_2$$

$$p_1 + \gamma_{\text{mercury}}\ell_3 - \gamma_{\text{air}}\ell_2 + \gamma_{\text{oil}}\ell_1 = p_2$$

$$0 + (133{,}000\ \text{N/m}^3)(0.8\ \text{m}) - 0 + (7850\ \text{N/m}^3)(0.4\ \text{m}) =$$

$$\boxed{p_2 = p_{\text{air}} = 110\ \text{kPa gage}}$$

Because the manometer configuration shown in Fig. 3.14 is common, it is useful to derive an equation specific to this application. To begin, apply the manometer equation (3.21) between points 1 and 2:

$$p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i = p_2$$

$$p_1 + \gamma_A(\Delta y - \Delta h) - \gamma_B \Delta h - \gamma_A(\Delta y + z_2 - z_1) = p_2$$

Simplifying gives

$$(p_1 + \gamma_A z_1) - (p_2 + \gamma_A z_2) = \Delta h(\gamma_B - \gamma_A)$$

Dividing through by γ_A gives

$$\left(\frac{p_1}{\gamma_A} + z_1\right) - \left(\frac{p_2}{\gamma_A} + z_2\right) = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right)$$

Recognize that the terms on the left side of the equation are piezometric head and rewrite to give the final result:

$$h_1 - h_2 = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right) \tag{3.22}$$

Equation (3.22) is valid when a manometer is used as shown in Fig. 3.14. Example 3.8 shows how this equation is used.

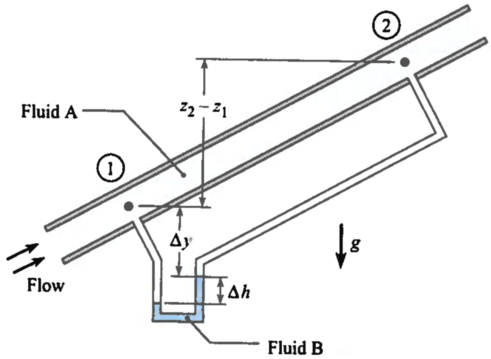


FIGURE 3.14 Apparatus for determini change in piezometric head corresponding to flow in a pipe.

EXAMPLE 3.8

Change in Piezometric Head for Pipe Flow

Problem Statement

A differential mercury manometer is connected to two pressure taps in an inclined pipe as shown in Fig. 3.14. Water at 50°F is flowing through the pipe. The deflection of mercury in the manometer is 1 inch. Find the change in piezometric pressure and piezometric head between points 1 and 2.

Define the Situation

Water is flowing in a pipe.

Properties:

1. Water (50 °F), Table A.5, $\gamma_{\text{water}} = 62.4\ \text{lbf/ft}^3$.
2. Mercury, Table A.4, $\gamma_{\text{Hg}} = 847\ \text{lbf/ft}^3$.

State the Goal

Find the

- Change in piezometric head (ft) between points 1 and 2.
- Change in piezometric pressure (psfg) between 1 and 2.

Generate Ideas and Make a Plan

1. Find difference in the piezometric head using Eq. (3.22).
2. Relate piezometric head to piezometric pressure using Eq. (3.13).

Take Action (Execute the Plan)

1. Difference in piezometric head

$$h_1 - h_2 = \Delta h\left(\frac{\gamma_{\text{Hg}}}{\gamma_{\text{water}}} - 1\right) = \left(\frac{1}{12}\ \text{ft}\right)\left(\frac{847\ \text{lbf/ft}^3}{62.4\ \text{lbf/ft}^3} - 1\right)$$

$$= \boxed{1.05\ \text{ft}}$$

2. Piezometric pressure

$$p_z = h\gamma_{\text{water}}$$

$$= (1.05\ \text{ft})(62.4\ \text{lbf/ft}^3) = \boxed{65.5\ \text{psf}}$$

Summary of the Manometer Equations

These manometer equations are summarized in Table 3.2. Because the equations were deriv from the hydrostatic equation, they have the same assumptions: constant fluid density a hydrostatic conditions.

The process for applying the manometer equations is

Step 1. For measurement of pressure at a point, select Eq. (3.21). For measurement of pr sure or head change between two points in a pipe, select Eq. (3.22).

Step 2. Select points 1 and 2 where you know information or where you want to fi information.

Step 3. Write the general form of the manometer equation.

Step 4. Perform a "term-by-term analysis."

TABLE 3.2 Summary of the Manometer Equations

Description	Equation	Terms
Use this equation for a manometer that has an open end (for an example of this type of manometer, see Fig. 3.13 on page 73).	$p_2 = p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$ (3.21)	p_1 = pressure at point 1 (N/m^2) p_2 = pressure at point 2 (N/m^2) γ_i = specific weight of fluid i (N/m^3) h_i = deflection of fluid in leg i (m)
Use this equation for a manometer that is being used to measure differential pressure in a pipe with a flowing fluid (for an example of this type of manometer, see Fig. 3.14 on page 75).	$h_1 - h_2 = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right)$ (3.22)	$h_1 = p_1/\gamma_A + z_1$ = piezometric head at point 1 (m) $h_2 = p_2/\gamma_A + z_2$ = piezometric head at point 2 (| Δh = deflection of the manometer fluid (m) γ_A = specific weight of the flowing fluid (N/m^3) γ_B = specific weight of the manometer fluid (N/r

Pressure Transducers

A **pressure transducer** is a device that converts pressure to an electrical signal. Modern factories and systems that involve flow processes are controlled automatically, and much of their operation involves sensing of pressure at critical points of the system. Therefore, pressure-sensing devices, such as pressure transducers, are designed to produce electronic signals that can be transmitted to oscillographs or digital devices for record-keeping or to control other devices for process operation. Basically, most transducers are tapped into the system with one side of a small diaphragm exposed to the active pressure of the system. When the pressure changes, the diaphragm flexes, and a sensing element connected to the other side of the diaphragm produces a signal that is usually linear with the change in pressure in the system. There are many types of sensing elements; one common type is the resistance-wire strain gage attached to a flexible diaphragm as shown in Fig. 3.15. As the diaphragm flexes, the wires of the strain gage change length, thereby changing the resistance of the wire. This change in resistance is converted into a voltage change that can then be used in various ways.

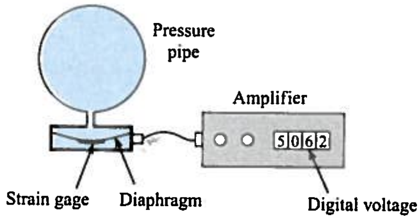


FIGURE 3.15
Schematic diagram of strain-gage pressure transducer.

Another type of pressure transducer used for measuring rapidly changing high pressures, such as the pressure in the cylinder head of an internal combustion engine, is the piezoelectric transducer (2). These transducers operate with a quartz crystal that generates a charge when subjected to a pressure. Sensitive electronic circuitry is required to convert the charge to a measurable voltage signal.

Computer data acquisition systems are used widely with pressure transducers. The analog signal from the transducer is converted (through an A/D converter) to a digital signal that can be processed by a computer. This expedites the data acquisition process and facilitates storing data.

3.4 Predicting Forces on Plane Surfaces (Panels)

Engineers predict hydrostatic forces on large structures such as dams. Thus, this section explains how to relate pressure to force. Next, this section describes how to calculate hydrostatic forces on panels, where a panel is a flat surface.

The Pressure Distribution

A **pressure distribution** (Fig. 3.16) is a visual or mathematical description that shows how pressure varies from point to point along a surface. For example, in the figure the pressure will be high in the front of the cylinder and low in the back of the cylinder. Notice that the pressure distribution is *always compressive* and that pressure is *always normal to the surface.*

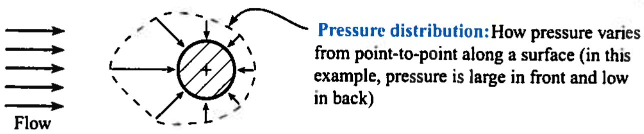


FIGURE 3.16
The pressure distribution caused by a fluid flowir over a circular cylinder.

Relating Pressure to Force

To relate pressure to force, select a small area dA (Fig. 3.17) on a surface. Then, define a norn vector **n** that is positive in a direction outward from the surface. The magnitude of the force $dF = pdA$, and the direction of the force is inward toward the surface. Thus, the force $d\mathbf{F}$ is

$$d\mathbf{F} = (-p)\,\mathbf{n}dA$$

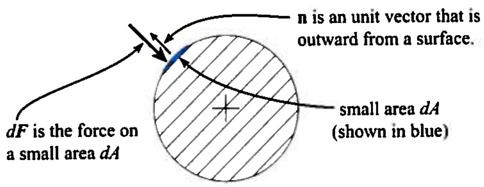


FIGURE 3.17
Terms used to define the pressure force.

where the negative sign is used because the force acts inward. To obtain the total force, add the forces acting on each small area:

$$\text{Net force due to a pressure distribution} = \mathbf{F}_p = \sum d\mathbf{F} = \sum(-p)\mathbf{n}dA$$

Because an integral is defined as an infinite sum, this equation can be written as

$$\text{Net force due to a pressure distribution} \equiv \mathbf{F}_p = \int_{\text{Area}} (-p)\mathbf{n}dA \qquad (3.$$

In summary, the net force due to pressure can be found by integrating pressure over a while using a normal vector to keep track of the direction of incremental force on each u of area.

Force of a Uniform Pressure Distribution

When pressure is the same at every point, as shown in Fig. 3.18a, the pressure distribution is cal a **uniform pressure distribution**. For a uniform pressure distribution, Eq. (3.23) reduces to

$$F_p = \int_A pdA = pA$$

The resultant force of pressure F_p passes through a point called the **center of pressure (C** Notice that the CP is represented using a circle with a "plus symbol" inside. For a uniform pı sure distribution on a panel, the CP is located at the centroid of area.

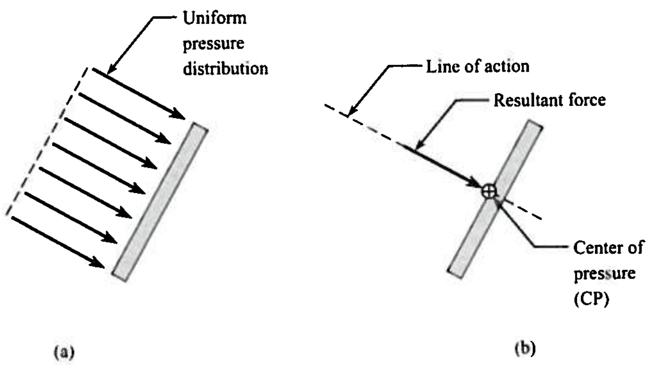


FIGURE 3.18
(a) Uniform pressure distribution, and (b) equivalent force.

Hydrostatic Pressure Distribution

When a pressure distribution is produced by a fluid in hydrostatic equilibrium (Fig. 3.19a), then the pressure distribution is called a **hydrostatic pressure distribution**. Notice that a hydrostatic pressure distribution is linear with depth. In Fig. 3.19b, the pressure distribution is represented by a resultant force that acts at the CP. Notice that the CP is located below the centroid of area.

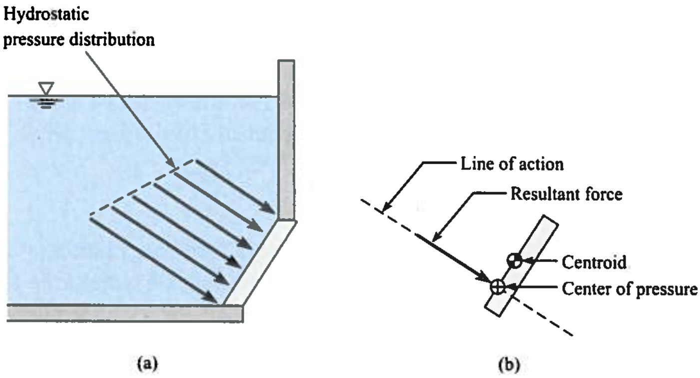


FIGURE 3.19
(a) Hydrostatic pressur
distribution, and
(b) Resultant force **F** ac
at the center of pressu

Force on a Panel (Magnitude)

Next, we will show how to find the force on one face of a panel (e.g., a gate, a wall, a dam) that is acted on by a hydrostatic pressure distribution. To begin, sketch a panel of arbitrary shape submerged in a liquid (Fig. 3.20). Line AB is the edge view of a panel. The plane of the panel intersects the horizontal liquid surface at axis 0-0 with an angle α. The distance from the axis 0-0 to the horizontal axis through the centroid of the area is given by $\bar{y}$. The distance from 0-0 to the differential area dA is y.

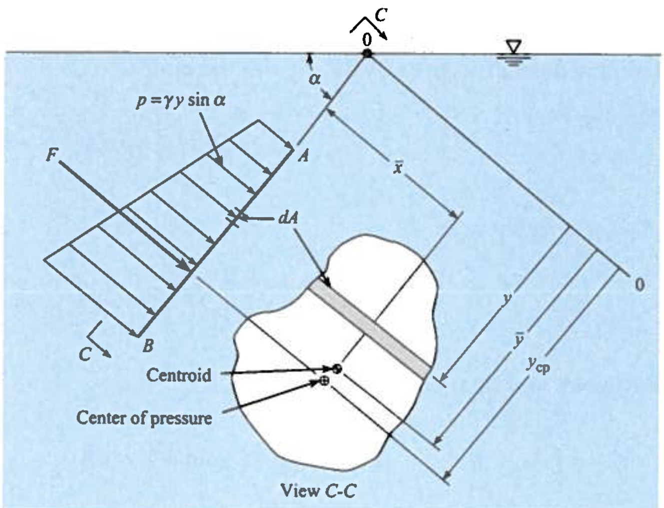


FIGURE 3.20
Distribution of hydrostc
pressure on a plane
surface.

The force due to pressure is given by Eq. (3.23), which reduces to

$$F_p = \int_A p dA \tag{3.24}$$

In Eq. (3.24), the pressure can be found with the hydrostatic equation:

$$p = \gamma \Delta z = \gamma y \sin \alpha \tag{3.25}$$

Combine Eqs (3.24) and (3.25) to give

$$F_p = \int_A p dA = \int_A \gamma y \sin\alpha \, dA = \gamma \sin\alpha \int_A y dA \qquad \text{(3.2}$$

Because the integral on the right side of Eq. (3.24) is the first moment of the area, replace tl integral by its equivalent, $\bar{y}A$. Therefore

$$F_p = \gamma \bar{y} A \sin\alpha = (\gamma \bar{y} \sin\alpha)A \qquad \text{(3.2}$$

Apply the hydrostatic equation to show that the variables within the parentheses on the rig side of Eq. (3.27) is the pressure at the centroid of the area. Thus,

$$F_p = \bar{p}A \qquad \text{(3.2}$$

Equation (3.28) shows that the hydrostatic force on a panel of arbitrary shape (e.g., recta gular, round, elliptical) is given by the product of panel area and pressure at the centro of area.

Finding the Location of the Force on Panel (Center of Pressure)

This subsection shows how to derive an equation for the vertical location of the center of pre sure (CP). For the panel shown in Fig. 3.20 to be in moment equilibrium, the torque due to tl resultant force F_p must balance the torque due to each differential force.

$$y_{cp}F_p = \int y \, dF$$

Note that y_{cp} is "slant" distance from the center of pressure to the surface of the liquid. The lat "slant" denotes that the distance is measured in the plane that runs through the panel. The d ferential force dF is given by $dF = p \, dA$; therefore,

$$y_{cp}F = \int_A yp \, dA$$

Also, $p = \gamma y \sin\alpha$, so

$$y_{cp}F = \int_A \gamma y^2 \sin\alpha \, dA \qquad \text{(3.2}$$

Because γ and $\sin\alpha$ are constants,

$$y_{cp}F = \gamma \sin\alpha \int_A y^2 \, dA \qquad \text{(3.3}$$

The integral on the right-hand side of Eq. (3.30) is the second moment of the area (often call the area moment of inertia). This shall be identified as I_0. However, for engineering applic tions it is convenient to express the second moment with respect to the horizontal centroic axis of the area. Hence by the parallel-axis theorem,

$$I_0 = \bar{I} + \bar{y}^2 A \qquad \text{(3.3}$$

Substitute Eq. (3.31) into Eq. (3.30) to give

$$y_{cp}F = \gamma \sin \alpha(\bar{I} + \bar{y}^2 A)$$

However, from Eq. (3.25), $F = \gamma\bar{y} \sin \alpha A$. Therefore,

$$y_{cp}(\gamma\bar{y} \sin\alpha\, A) = \gamma \sin \alpha(\bar{I} + \bar{y}^2A) \quad (3.32)$$

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} \quad (3.33)$$

In Eq. (3.33), the area moment of inertia $\bar{I}$ is taken about a horizontal axis that passes through the centroid of area. Formulas for $\bar{I}$ are presented in Fig. A.1. The slant distance $\bar{y}$ measures the length from the surface of the liquid to the centroid of the panel along an axis that is aligned with the "slant of the panel" as shown in Fig. 3.20.

Equation (3.33) shows that the Center of Pressure (CP) will be situated below the centroid. The distance between the CP and the centroid depends on the depth of submersion, which is characterized by $\bar{y}$, and on the panel geometry, which is characterized by $\bar{I}/A$.

Due to assumptions in the derivations, Eqs. (3.28) and (3.33) have several limitations. First, they only apply to a single fluid of constant density. Second, the pressure at the liquid surface needs to be $p = 0$ gage to correctly locate the CP. Third, Eq. (3.33) gives only the vertical location of the CP, not the lateral location.

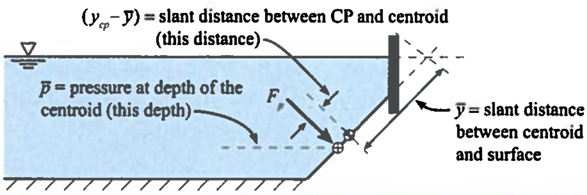
Summary of the Panel Equations

The panel equations (Table 3.3) are used to calculate the force on a flat plate that is subjected to a hydrostatic pressure distribution.

TABLE 3.3 Summary of the Panel Equations

Description	Equation	Terms
Apply this equation to predict the magnitude of the hydrostatic force.	$F_p = \bar{p}A$ (3.28)	F_p = resultant force due to pressure distribution (N) $\bar{p}$ = pressure at the depth of the centroid (Pa) A = area of the surface of the plate (m^2)
Apply this equation to locate the center of pressure (CP).	$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$ (3.33)	$(y_{cp} - \bar{y})$ = slant distance from the centroid to the center of pressure (m) $\bar{I}$ = area moment of inertia of panel about centroidal axis (m^4) (for formulas, see Fig. A.1 on page A-1) $\bar{y}$ = slant distance from centroid to liquid surface (m)

This figure defines terms.



EXAMPLE 3.9

Hydrostatic Force Due to Concrete

Problem Statement

Determine the force acting on one side of a concrete form 2.44 m high and 1.22 m wide (8 ft by 4 ft) that is used for pouring a basement wall. The specific weight of concrete is 23.6 kN/m^3 (150 lbf/ft^3).

Define the Situation

Concrete in a liquid state acts on a vertical surface.

The vertical wall is 2.44 m high and 1.22 m wide

Assumptions: Freshly poured concrete can be represented as a liquid.

Properties: Concrete: $\gamma = 23.6$ kN/m^3.

State the Goal

Find the resultant force (kN) acting on the wall.

Plan

Apply the panel equation (3.28).

Solution

1. Panel equation

$$F = \bar{p}A$$

2. Term-by-term analysis

- $\bar{p}$ = pressure at depth of the centroid

$$\bar{p} = (\gamma_{\text{concrete}})(z_{\text{centroid}}) = (23.6\ \text{kN/m}^3)(2.44/2\ \text{m}) = 28.79\ \text{kPa}$$

- A = area of panel

$$A = (2.44\ \text{m})(1.22\ \text{m}) = 2.977\ \text{m}^2$$

3. Resultant force

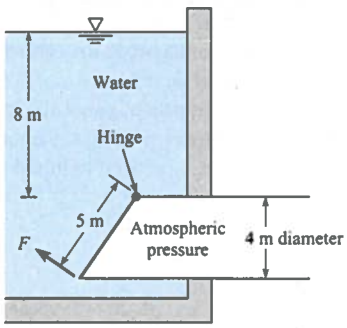
$$F = \bar{p}A = (28.79\ \text{kPa})(2.977\ \text{m}^2) = \boxed{85.7\ \text{kN}}$$

EXAMPLE 3.10

Force to Open an Elliptical Gate

Problem Statement

An elliptical gate covers the end of a pipe 4 m in diameter. If the gate is hinged at the top, what normal force F is required to open the gate when water is 8 m deep above the top of the pipe and the pipe is open to the atmosphere on the other side? Neglect the weight of the gate.



Define the Situation

Water pressure is acting on an elliptical gate.

Properties: Water (10°C), Table A.5: $\gamma = 9810$ N/m^3.

Assumptions:

1. Neglect the weight of the gate.
2. Neglect friction between the bottom on the gate and the pipe wall.

State the Goal

F(N) ⬅ Force needed to open gate.

Generate Ideas and Make a Plan

1. Calculate resultant hydrostatic force using $F = \bar{p}A$.
2. Find the location of the center of pressure using Eq. (3.33).
3. Draw an FBD of the gate.
4. Apply moment equilibrium about the hinge.

Take Action (Execute the Plan)

1. Hydrostatic (resultant) force

- $\bar{p}$ = pressure at depth of the centroid

$$\bar{p} = (\gamma_{\text{water}})(z_{\text{centroid}}) = (9810\ \text{N/m}^3)(10\ \text{m}) = 98.1\ \text{kPa}$$

- A = area of elliptical panel (using Fig. A.1 to find formula)

$$A = \pi ab = \pi(2.5\ \text{m})(2\ \text{m}) = 15.71\ \text{m}^2$$

- Calculate resultant force

$$F_p = \bar{p}A = (98.1\text{ kPa})(15.71\text{ m}^2) = \boxed{1.54\text{ MN}}$$

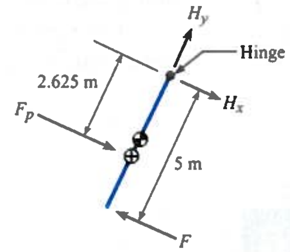
2. Center of pressure
   - $\bar{y} = 12.5$ m, where $\bar{y}$ is the slant distance from the water surface to the centroid.
   - Area moment of inertia $\bar{I}$ of an elliptical panel using a formula from Fig. A.1

$$\bar{I} = \frac{\pi a^3 b}{4} = \frac{\pi(2.5\text{ m})^3(2\text{ m})}{4} = 24.54\text{ m}^4$$

   - Finding center of pressure

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{25.54\text{ m}^4}{(12.5\text{ m})(15.71\text{ m}^2)} = 0.125\text{ m}$$

3. FBD of the gate:



4. Moment equilibrium

$$\sum M_{\text{hinge}} = 0$$

$$1.541 \times 10^6\text{ N} \times 2.625\text{ m} - F \times 5\text{ m} = 0$$

$$F = \boxed{809\text{ kN}}$$

3.5 Calculating Forces on Curved Surfaces

As engineers, we calculate forces on curved surfaces when we are designing components such as tanks, pipes, and curved gates. Thus, this topic is described in this section.

Consider the curved surface *AB* in Fig. 3.21a. The goal is to represent the pressure distribution with a resultant force that passes through the center of pressure. One approach is to integrate the pressure force along the curved surface and find the equivalent force. However, it is easier to sum forces for the free body shown in the upper part of Fig. 3.21b. The lower sketch in Fig. 3.21b shows how the force acting on the curved surface relates to the force *F* acting on the free body. Using the FBD and summing forces in the horizontal direction shows that

$$F_x = F_{AC} \tag{3.34}$$

The line of action for the force F_{AC} is through the center of pressure for side *AC*.

The vertical component of the equivalent force is

$$F_y = W + F_{CB} \tag{3.35}$$

where *W* is the weight of the fluid in the free body and F_{CB} is the force on the side *CB*.

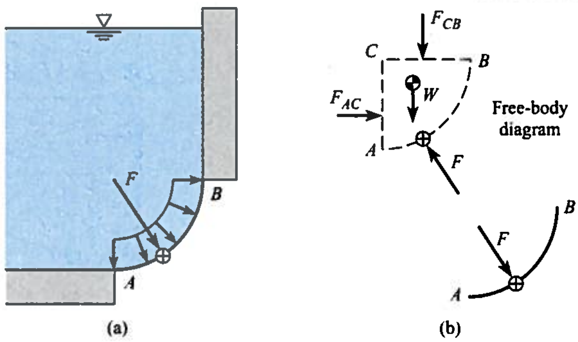


FIGURE 3.21
(a) Pressure distribution equivalent force.
(b) Free-body diagram action-reaction force p

The force F_{CB} acts through the centroid of surface CB, and the weight acts through t center of gravity of the free body. The line of action for the vertical force may be found by sur ming the moments about any convenient axis.

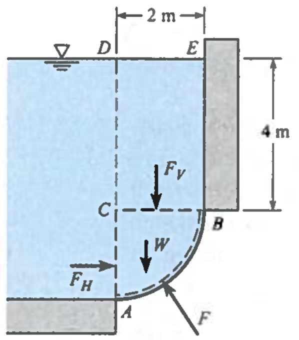
Example 3.11 illustrates how curved surface problems can be solved by applying equili rium concepts together with the panel force equations.

EXAMPLE 3.11

Hydrostatic Force on a Curved Surface

Problem Statement

Surface AB is a circular arc with a radius of 2 m and a width of 1 m into the paper. The distance EB is 4 m. The fluid above surface AB is water, and atmospheric pressure prevails on the free surface of the water and on the bottom side of surface AB. Find the magnitude and line of action of the hydrostatic force acting on surface AB.



Define the Situation

Situation: A body of water is contained by a curved surface.

Properties: Water (10°C), Table A.5: $\gamma = 9810\ \text{N/m}^3$.

State the Goal

Find:

1. Hydrostatic force (in newtons) on the curved surface AB.
2. Line of action of the hydrostatic force.

Generate Ideas and Make a Plan

Apply equilibrium concepts to the body of fluid ABC.

1. Find the horizontal component of F by applying Eq. (3.34).
2. Find the vertical component of F by applying Eq. (3.35).
3. Find the line of action of F by finding the lines of action of components and then using a graphical solution.

Take Action (Execute the Plan)

1. Force in the horizontal direction

$$F_x = F_H = \bar{p}A = (5\ \text{m})(9810\ \text{N/m}^3)(2 \times 1\ \text{m}^2)$$
$$= 98.1\ \text{kN}$$

2. Force in the vertical direction
   - Vertical force on side CB

$$F_V = \bar{p}_0 A = 9.81\ \text{kN/m}^3 \times 4\ \text{m} \times 2\ \text{m} \times 1\ \text{m} = 78.5\ \text{kN}$$

   - Weight of the water in volume ABC

$$W = \gamma \forall_{ABC} = (\gamma)(\tfrac{1}{4}\pi r^2)(w)$$
$$= (9.81\ \text{kN/m}^3) \times (0.25 \times \pi \times 4\ \text{m}^2)(1\ \text{m}) = 30.8\ \text{kN}$$

   - Summing forces

$$F_y = W + F_V = 109.3\ \text{kN}$$

3. Line of action (horizontal force)

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A} = (5\ \text{m}) + \left(\frac{1 \times 2^3/12}{5 \times 2 \times 1}\text{m}\right)$$
$$y_{cp} = 5.067\ \text{m}$$

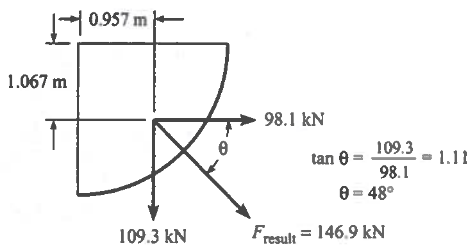
4. The line of action (x_{cp}) for the vertical force is found by summing moments about point C:

$$x_{cp}F_y = F_V \times 1\ \text{m} + W \times \bar{x}_w$$

The horizontal distance from point C to the centroid of the area ABC is found using Fig. A.1: $\bar{x}_W = 4r/3\pi = 0.849$ m. Thus,

$$x_{cp} = \frac{78.5\ \text{kN} \times 1\ \text{m} + 30.8\ \text{kN} \times 0.849\ \text{m}}{109.3\ \text{kN}} = 0.957\ \text{m}$$

5. The resultant force that acts on the curved surface is shown in the following figure.



The central idea of this section is that *forces on curved surfaces may be found by applying equilibrium concepts to systems comprised of the fluid in contact with the curved surface.* Notice how equilibrium concepts are used in each of the following situations.

Consider a sphere holding a gas pressurized to a gage pressure p_i as shown in Fig. 3.22. The indicated forces act on the fluid in volume *ABC*. Applying equilibrium in the vertical direction gives

$$F = p_i A_{AC} + W$$

Because the specific weight for a gas is quite small, engineers usually neglect the weight of the gas:

$$F = p_i A_{AC} \quad (3.36)$$

Another example is finding the force on a curved surface submerged in a reservoir of liquid as shown in Fig. 3.23a. If atmospheric pressure prevails above the free surface and on the outside of surface *AB*, then force caused by atmospheric pressure cancels out, and equilibrium gives

$$F = \gamma \forall_{ABCD} = W\downarrow \quad (3.37)$$

Hence the force on surface *AB* equals the weight of liquid above the surface, and the arrow indicates that the force acts downward.

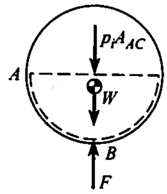
Now consider the situation where the pressure distribution on a thin curved surface comes from the liquid underneath, as shown in Fig. 3.23b. If the region above the surface, volume *abcd*, were filled with the same liquid, the pressure acting at each point on the upper surface of *ab* would equal the pressure acting at each point on the lower surface. In other words, there would be no net force on the surface. Thus, the equivalent force on surface *ab* is given by

$$F = \gamma \forall_{abcd} = W\downarrow \quad (3.38)$$

where *W* is the weight of liquid needed to fill a volume that extends from the curved surface to the free surface of the liquid.

FIGURE 3.22

Pressurized spherical showing forces that a the fluid inside the mc region.



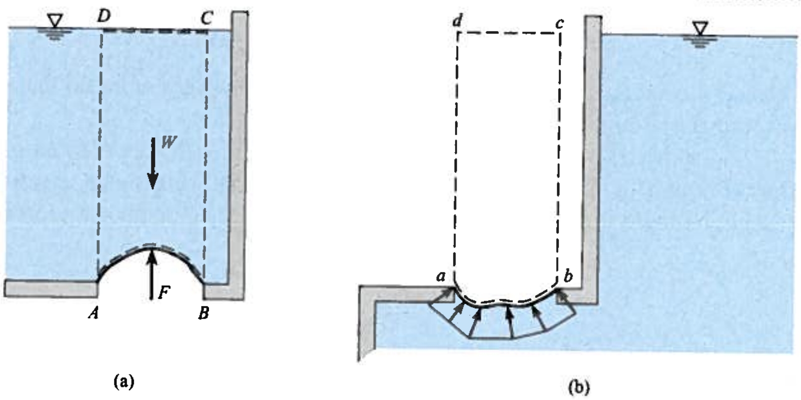


FIGURE 3.23

Curved surface with (c liquid above and (b) li below. In (a), arrows represent forces acting the liquid. In (b), arrow represent the pressure distribution on surface

3.6 Calculating Buoyant Forces

Engineers calculate buoyant forces for applications such as the design of ships, sediment transport in rivers, and fish migration. Buoyant forces are sometimes significant in problems involving gases, for example, a weather balloon. Thus, this section describes how to calculate the buoyant force on an object.

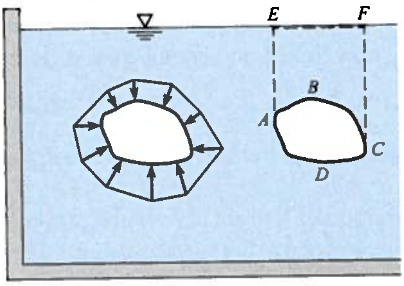
A **buoyant force** is defined as an upward force (with respect to gravity) on a body that totally or partially submerged in a fluid, either a liquid or gas. Buoyant forces are caused by tl hydrostatic pressure distribution.

The Buoyant Force Equation

To derive an equation, consider a body *ABCD* submerged in a liquid of specific weight (Fig. 3.24). The sketch on the left shows the pressure distribution acting on the body. As shov by Eq. (3.38), pressures acting on the lower portion of the body create an upward force eqı to the weight of liquid needed to fill the volume above surface *ADC*. The upward force is

$$F_{\text{up}} = \gamma(V_b + V_a)$$

FIGURE 3.24
Two views of a body immersed in a liquid.



where V_b is the volume of the body (i.e., volume *ABCD*) and V_a is the volume of liquid abc the body (i.e., volume *ABCFE*). As shown by Eq. (3.37), pressures acting on the top surface the body create a downward force equal to the weight of the liquid above the body:

$$F_{\text{down}} = \gamma V_a$$

Subtracting the downward force from the upward force gives the net or buoyant force F_B act on the body:

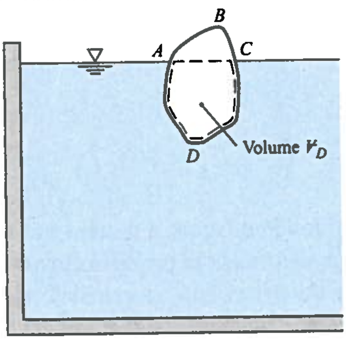
$$F_B = F_{\text{up}} - F_{\text{down}} = \gamma V_b \quad (3.$$

Hence, the net force or buoyant force (F_B) equals the weight of liquid that would be neede occupy the volume of the body.

Consider a body that is floating as shown in Fig. 3.25. The marked portion of the ob has a volume V_D. Pressure acts on curved surface *ADC* causing an upward force equal to weight of liquid that would be needed to fill volume V_D. The buoyant force is given by

$$F_B = F_{\text{up}} = \gamma V_D \quad (3.$$

FIGURE 3.25
A body partially submerged in a liquid.



Hence, the buoyant force equals the weight of liquid that would be needed to occupy the volume $\forall_D$. This volume is called the displaced volume. Comparison of Eqs. (3.39) and (3.40) shows that one can write a single equation for the buoyant force:

$$F_B = \gamma \forall_D \qquad (3.41a)$$

In Eq. (3.41a), $\forall_D$ is the volume that is displaced by the body. If the body is totally submerged, the displaced volume is the volume of the body. If a body is partially submerged, the displaced volume is the portion of the volume that is submerged.

Eq. (3.41b) is only valid for a single fluid of uniform density. The general principle of buoyancy is called **Archimedes' principle**:

$$\text{(buoyant force)} = F_B = \text{(weight of the displaced fluid)} \qquad (3.41b)$$

The buoyant force acts at a point called the center of buoyancy, which is located at the center of gravity of the displaced fluid.

✔CHECKPOINT PROBLEM 3.3

Consider a balloon filled with helium (case A) and a balloon filled with air (case B). Which statement is correct?

a. Buoyant force (case A) > Buoyant force (case B)
b. Buoyant force (case A) < Buoyant force (case B)
c. Buoyant force (case A) = Buoyant force (case B)

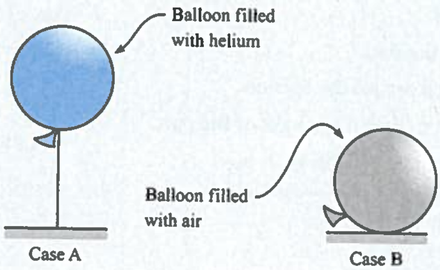
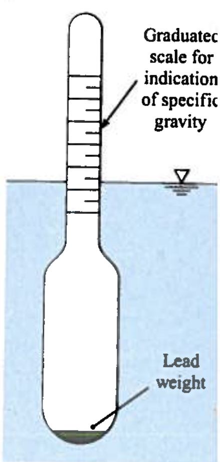


FIGURE 3.26
Hydrometer



The Hydrometer

A **hydrometer** (Fig. 3.26) is an instrument for measuring the specific gravity of liquids. It is typically made of a glass bulb that is weighted on one end so the hydrometer floats in an upright position. A stem of constant diameter is marked with a scale, and the specific weight of the liquid is determined by the depth at which the hydrometer floats. The operating principle of the hydrometer is buoyancy. In a heavy liquid (i.e., high γ), the hydrometer will float shallower because a lesser volume of the liquid must be displaced to balance the weight of the hydrometer. In a light liquid, the hydrometer will float deeper.

EXAMPLE 3.12

Buoyant Force on a Metal Part

Problem Statement

A metal part (object 2) is hanging by a thin cord from a floating wood block (object 1). The wood block has a specific gravity $S_1 = 0.3$ and dimensions of $50 \times 50 \times 10$ mm. The metal part has a volume of 6600 mm^3. Find the mass m_2 of the metal part and the tension T in the cord.

Define the Situation

A metal part is suspended from a floating block of wood.

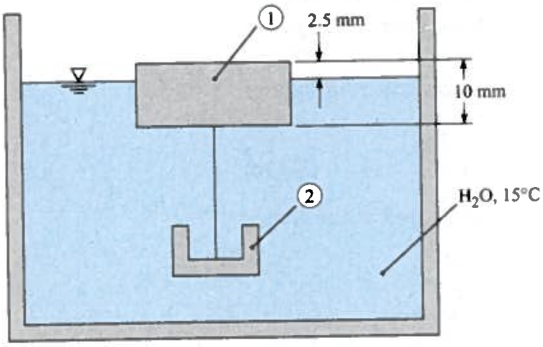
Properties:

Water (15°C), Table A.5: $\gamma = 9800$ N/m^3.

Wood: $S_1 = 0.3$.

State the Goal

- Find the mass (in grams) of the metal part.
- Calculate the tension (in newtons) in the cord.

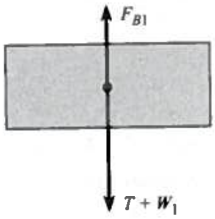


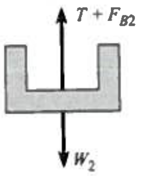
Generate Ideas and Make a Plan

1. Draw FBDs of the block and the part.
2. Apply equilibrium to the block to find the tension.
3. Apply equilibrium to the part to find the weight of the part.
4. Calculate the mass of the metal part using $W = mg$.

Take Action (Execute the Plan)

1. FBDs





2. Force equilibrium (vertical direction) applied to block

$$T = F_{B1} - W_1$$

- Buoyant force $F_{B1} = \gamma V_{D1}$, where V_{D1} is the submerged volume

$$F_{B1} = \gamma V_{D1}$$
$$= (9800\ \text{N/m}^3)(50 \times 50 \times 7.5\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.184\ \text{N}$$

- Weight of the block

$$W_1 = \gamma S_1 V_1$$
$$= (9800\ \text{N/m}^3)(0.3)(50 \times 50 \times 10\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.0735\ \text{N}$$

- Tension in the cord

$$T = (0.184 - 0.0735) = \boxed{0.110\ \text{N}}$$

3. Force equilibrium (vertical direction) applied to metal part

- Buoyant force

$$F_{B2} = \gamma V_2 = (9800\ \text{N/m}^3)(6600\ \text{mm}^3)(10^{-9}) = 0.0647\ \text{N}$$

- Equilibrium equation

$$W_2 = T + F_{B2} = (0.110\ \text{N}) + (0.0647\ \text{N})$$

4. Mass of metal part

$$m_2 = W_2/g = \boxed{17.8\ \text{g}}$$

Review the Solution and the Process

Discussion. Notice that tension in the cord (0.11 N) is less than the weight of the metal part (0.18 N). This result is consistent with the common observation that an object will "weigh less in water than in air."

Tip. When solving problems that involve buoyancy, draw an FBD.

3.7 Predicting Stability of Immersed and Floating Bodies

Engineers calcuate whether an object will tip over or remain in an upright position wl placed in a liquid, for example for the design of ships and buoys. Thus, stability is presentec this section.

Immersed Bodies

When a body is completely immersed in a liquid, its stability depends on the relative positions of the center of gravity of the body and the centroid of the displaced volume of fluid, which is called the **center of buoyancy**. If the center of buoyancy is above the center of gravity (see Fig. 3.27a) any tipping of the body produces a righting couple, and consequently, the body is stable. Alternatively, if the center of gravity is above the center of buoyancy, any tipping produces an overturning moment, thus causing the body to rotate through 180° (see Fig. 3.27c). If the center of buoyancy and center of gravity are coincident, the body is neutrally stable—that is, it lacks a tendency for righting itself or for overturning (see Fig. 3.27b).

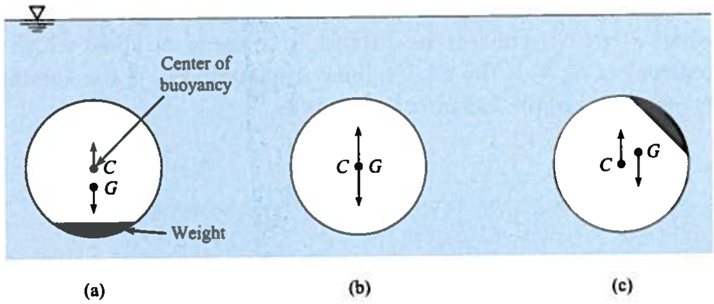


FIGURE 3.27
Conditions of stability fo immersed bodies. (a) Stable. (b) Neutral. (c) Unstable.

Floating Bodies

The question of stability is more involved for floating bodies than for immersed bodies because the center of buoyancy may take different positions with respect to the center of gravity, depending on the shape of the body and the position in which it is floating. For example, consider the cross section of a ship shown in Fig. 3.28a. Here the center of gravity G is above the center of buoyancy C. Therefore, at first glance it would appear that the ship is unstable and could flip over. However, notice the position of C and G after the ship has taken a small angle of heel. As shown in Fig. 3.28b, the center of gravity is in the same position, but the center of buoyancy has moved outward of the center of gravity, thus producing a righting moment. A ship having such characteristics is stable.

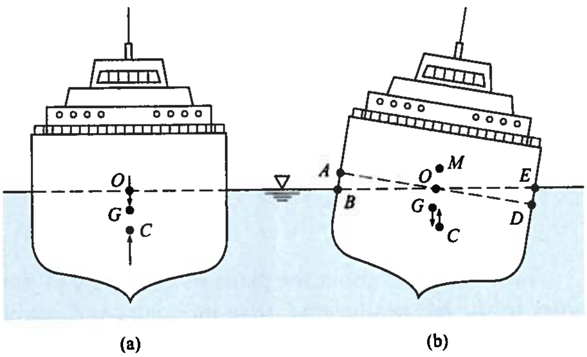


FIGURE 3.28
Ship stability relations.

The reason for the change in the center of buoyancy for the ship is that part of the original buoyant volume, as shown by the wedge shape AOB, is transferred to a new buoyant volume EOD. Because the buoyant center is at the centroid of the displaced volume, it follows that for this case the buoyant center must move laterally to the right. The point of intersection of the lines of action of the buoyant force before and after heel is called the *metacenter M*, and distance *GM* is called the *metacentric height*. If *GM* is positive—that is, if *M* is above G— ship is stable; however, if *GM* is negative, the ship is unstable. Quantitative relations involv these basic principles of stability are presented in the next paragraph.

Consider the ship shown in Fig. 3.29, which has taken a small angle of heel α. First eva ate the lateral displacement of the center of buoyancy, CC'; then it will be easy by simple tri nometry to solve for the metacentric height *GM* or to evaluate the righting moment. Re that the center of buoyancy is at the centroid of the displaced volume. Therefore, resort to fundamentals of centroids to evaluate the displacement CC'. From the definition of the c troid of a volume,

$$\bar{x}\forall = \sum x_i \Delta\forall_i \qquad (3.$$

where $\bar{x} = CC'$, which is the distance from the plane about which moments are taken to centroid of $\forall$; $\forall$ is the total volume displaced; $\Delta\forall_i$ is the volume increment; and x_i is moment arm of the increment of volume.

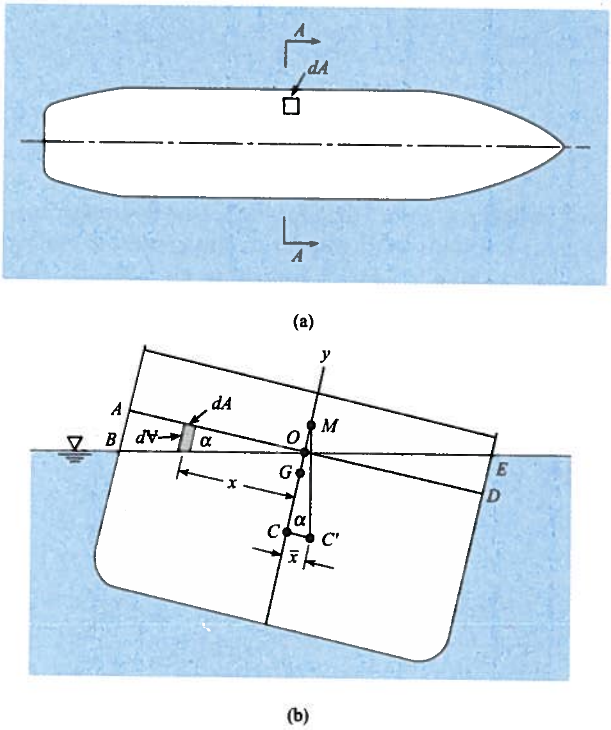


FIGURE 3.29
(a) Plan view of ship at waterline.
(b) Section A-A of ship.

Take moments about the plane of symmetry of the ship. Recall from mechanics that vo umes to the left produce negative moments and volumes to the right produce positive m ments. For the right side of Eq. (3.42) write terms for the moment of the submerged volur about the plane of symmetry. A convenient way to do this is to consider the moment of t volume before heel, subtract the moment of the volume represented by the wedge *AOB*, ai add the moment represented by the wedge *EOD*. In a general way this is given by the followi equation:

$$\bar{x}\forall = \text{moment of } \forall \text{ before heel} - \text{moment of } \forall_{AOB} + \text{moment of } \forall_{EOD} \qquad (3.4$$

Because the original buoyant volume is symmetrical with y-y, the moment for the first term on the right is zero. Also, the sign of the moment of $\forall_{AOB}$ is negative; therefore, when this negative moment is subtracted from the right-hand side of Eq. (3.43), the result is

$$\bar{x}\forall = \sum x_i \Delta\forall_{iAOB} + \sum x_i \Delta\forall_{iEOD} \tag{3.44}$$

Now, express Eq. (3.44) in integral form:

$$\bar{x}\forall = \int_{AOB} x\, d\forall + \int_{EOD} x\, d\forall \tag{3.45}$$

But it may be seen from Fig. 3.29b that $d\forall$ can be given as the product of the length of the differential volume, $x \tan \alpha$, and the differential area, dA. Consequently, Eq. (3.45) can be written as

$$\bar{x}\forall = \int_{AOB} x^2 \tan \alpha\, dA + \int_{EOD} x^2 \tan \alpha\, dA$$

Here $\tan \alpha$ is a constant with respect to the integration. Also, because the two terms on the right-hand side are identical except for the area over which integration is to be performed, combine them as follows:

$$\bar{x}\forall = \tan \alpha \int_{A_{\text{waterline}}} x^2\, dA \tag{3.46}$$

The second moment, or moment of inertia of the area defined by the waterline, is given the symbol I_{00}, and the following is obtained:

$$\bar{x}\forall = I_{00} \tan \alpha$$

Next, replace $\bar{x}$ by CC' and solve for CC':

$$CC' = \frac{I_{00} \tan \alpha}{\forall}$$

From Fig. 3.29b,

$$CC' = CM \tan \alpha$$

Thus eliminating CC' and $\tan \alpha$ yields

$$CM = \frac{I_{00}}{\forall}$$

However,

$$GM = CM - CG$$

Therefore the *metacentric height* is

$$GM = \frac{I_{00}}{\forall} - CG \tag{3.47}$$

Equation (3.47) is used to determine the stability of floating bodies. As already noted, if GM is positive, the body is stable; if GM is negative, it is unstable.

Note that for small angles of heel α, the righting moment or overturning moment is given as follows:

$$\text{RM} = \gamma \forall GM\alpha \tag{3.48}$$

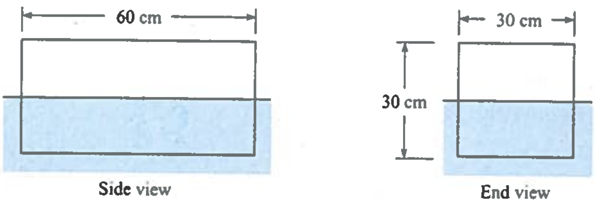
However, for large angles of heel, direct methods of calculation based on these same principles would have to be employed to evaluate the righting or overturning moment.

EXAMPLE 3.13

Stability of a Floating Block

Problem Statement

A block of wood 30 cm square in cross section and 60 cm long weighs 318 N. Will the block float with sides vertical as shown?



Define the Situation

A block of wood is floating in water.

State the Goal

Determine the stable configuration of the block of wood.

Generate Ideas and Make a Plan

1. Apply force equilibrium to find the depth of submergence.
2. Determine if block is stable about the long axis by applying Eq. (3.47).
3. If block is not stable, repeat steps 1 and 2.

Take Action (Execute the Plan)

1. Equilibrium (vertical direction)

$$\sum F_y = 0$$

$$-\text{weight} + \text{buoyant force} = 0$$

$$-318\text{ N} + 9810\text{ N/m}^3 \times 0.30\text{ m} \times 0.60\text{ m} \times d = 0$$

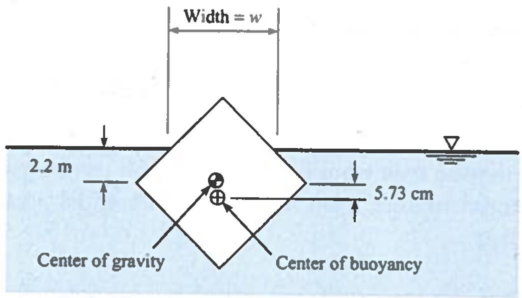
$$d = 0.18\text{ m} = 18\text{ cm}$$

2. Stability (longitudinal axis)

$$GM = \frac{I_{00}}{V} - CG = \frac{\frac{1}{12} \times 60 \times 30^3}{18 \times 60 \times 30} - (15 - 9)$$

$$= 4.167 - 6 = -1.833\text{ cm}$$

Because the metacentric height is negative, the block is not stable about the longitudinal axis. Thus a slight disturbance will make it tip to the orientation shown below.



3. Equilibrium (vertical direction—see preceding figure)

$$-\text{weight} + \text{buoyant force} = 0$$

$$-(318\text{ N}) + (9810\text{ N/m}^3)(V_D) = 0$$

$$V_D = 0.0324\text{ m}^3$$

4. Find the dimension w.

(Displaced volume)
= (Block volume) − (Volume above the waterline).

$$V_D = 0.0324\text{ m}^3 = (0.3^2)(0.6)\text{ m}^3 - \frac{w^2}{4}(0.6\text{ m})$$

$$w = 0.379\text{ m}$$

5. Moment of inertia at the waterline

$$I_{00} = \frac{bh^3}{12} = \frac{(0.6\text{ m})(0.379\text{ m})^3}{12} = 0.00273\text{ m}^4$$

6. Metacentric height

$$GM = \frac{I_{00}}{V} - CG = \frac{0.00273\text{ m}^4}{0.0324\text{ m}^3} - 0.0573\text{ m} = 0.027\text{ m}$$

Because the metacentric height is positive, the block will be stable in this position.

3.8 Summarizing Key Knowledge

Pressure and Hydrostatic Equilibrium

- A *hydrostatic condition* means that the weight of each fluid particle is balanced by the net pressure force.
- *Pressure p* is ratio of (magnitude of normal force due to a fluid) to (area) at a point.

- Pressure always acts to compress the material that is in contact with the fluid exerting the pressure.
- Pressure is a scalar quantity; not a vector.

- Engineers express pressure with gage pressure, absolute pressure, and vacuum pressure.
  - Absolute pressure is measured relative to absolute zero.
  - Gage pressure gives the magnitude of pressure relative to atmospheric pressure.

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}}$$

  - Vacuum pressure gives the magitude of the pressure below atmospheric pressure.

$$p_{\text{vacuum}} = p_{\text{atm}} - p_{\text{abs}}$$

Describing Pressure and Hydrostatic Equilibrium

- The weight of a fluid causes pressure to increase with increasing depth, giving the *hydrostatic differential equation*. The equations that are used in hydrostatics are derived from this equation. The hydrostatic differential equation is

$$\frac{dp}{dz} = -\gamma = -\rho g$$

- If density is constant, the hydrostatic differential equation can be integrated to give the hydrostatic equation. The meaning (i.e., physics) of the hydrostatic equation is that pizeometric head (or piezometric pressure) is everywhere constant in a static body of fluid.

$$\frac{p}{\gamma} + z = \text{constant}$$

Pressure Distributions and Forces Due to Pressure

- A fluid in contact with a surface produces a *pressure distribution*, which is a mathematical or visual description of how the pressure varies along the surface.
- To find the force due to a pressure distribution, integrate the pressure distribution over area using a normal vector to track the direction of the force acting on dA.

$$\text{Net force due to a pressure distribution} = \mathbf{F}_p = \int_A (-p)\mathbf{n}dA$$

- A pressure distribution is often represented as a statically equivalent force $\mathbf{F}_p$ acting at the *center of pressure* (CP)
- A *uniform pressure distribution* means that the pressure is the same at every point on a suface. Pressure distributions due to gases are typically idealized as uniform pressure distributions.
- A *hydrostatic pressure distibution* means that the pressure varies according to $dp/dz = -\gamma$

Force on a Flat Surface (Hydrostatic Pressure Distribution)

- For a panel subjected to a hydrostatic pressure distribution, the hydrostatic force is

$$F_p = \bar{p}A$$

- This hydrostatic force
  - Acts *at* the centroid of area for a uniform pressure distribution
  - Acts *below* the centroid of area for a hydrostatic pressure distibution. The slant distanc between the center of pressure and the centroid of area is given by

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$

Hydrostatic Forces on a Curved Surface

- When a surface is curved, one can find the pressure force by applying force equilibrium t a free body comprised of the fluid in contact with the surface.

The Buoyant Force

- The *buoyant force* is the pressure force on a body that is partially or totally submerged in a flui
- The magnitude of the buoyant force is given by

$$\text{Buoyant force} = F_B = \text{Weight of the displaced fluid}$$

- The center of buoyancy is located at the center of gravity of the displaced fluid. The direction of the buoyant force is opposite the gravity vector.
- When the buoyant force is due to a single fluid with constant density, the magnitude of th buoyant force is:

$$F_B = \gamma V\!\!\!/_D$$

Hydrodynamic Stability

- Hydrodynamic stability means that if an object is displaced from equilibrium then there i a moment that causes the object to return to equilibrium.
- The criteria for stability are
  - *Immersed object.* The body is stable if the center of gravity is below the center of buoyanc
  - *Floating object.* The body is stable if the metacentric height is positive.

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2. Holman, J. P., and W. J. Gajda, Jr. *Experimental Methods for Engineers.* New York: McGraw-Hill, 1984.

3. Wikipedia contributors "Hydraulic machinery," Wikipedia, The Free Encyclopedia, http://en.wikipedia.org/w/index.php?title=Hydraulic_machinery&oldid=161288040 (accessed October 4, 2007).

PROBLEMS

PLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

Describing Pressure (§3.1)

3.1 PLUS Apply the grid method (§1.5 in Ch. 1) to each situation.

a. If pressure is 6 inches of water (vacuum), what is gage pressure in kPa?

b. If the pressure is 180 kPa abs, what is the gage pressure in psi

c. If gage pressure is 0.4 bar, what is absolute pressure in psi?

d. If a person's blood pressure is 96 mm Hg, what is their blood pressure in kPa abs?

3.2 WILEY PLUS A 100 mm diameter sphere contains an ideal gas at 20°C. Apply the grid method (§1.5 in Ch. 1) to calculate the density in units of kg/m³.

a. Gas is helium. Gage pressure is 20 in H_2O.
b. Gas is methane. Vacuum pressure is 3 psi.

3.3 WILEY PLUS For the questions below, assume standard atmospheric pressure.

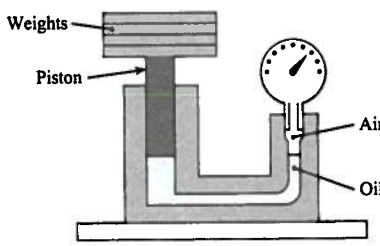
a. For a vacuum pressure of 30 kPa, what is the absolute pressure? Gage pressure?
b. For a pressure of 13.8 psig, what is the pressure in psia?
c. For a pressure of 200 kPa gage, what is the absolute pressure in kPa?
d. Give the pressure 100 psfg in psfa.

3.4 WILEY PLUS The local atmospheric pressure is 99.0 kPa. A gage on an oxygen tank reads a pressure of 300 kPa gage. What is the pressure in the tank in kPa abs?

3.5 Using §3.1 and other resources, answer the following questions. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

a. What are five important facts that engineers need to know about pressure?
b. What are five common instances in which people use gage pressure?
c. What are the most common units for pressure?
d. Why is pressure defined using a derivative?
e. How is pressure similar to shear stress? How does pressure differ from shear stress?

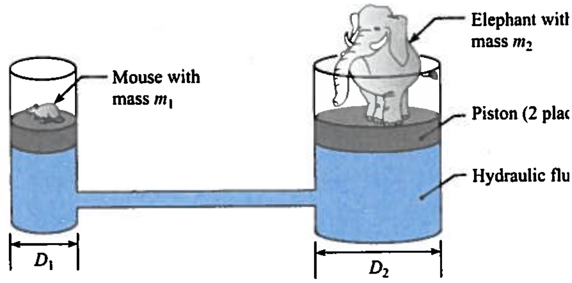
3.6 WILEY GO The Crosby gage tester shown in the figure is used to calibrate or to test pressure gages. When the weights and the piston together weigh 140 N, the gage being tested indicates 200 kPa. If the piston diameter is 30 mm, what percentage of error exists in the gage?



PROBLEM 3.6

3.7 WILEY PLUS As shown, a mouse can use the mechanical advantage provided by a hydraulic machine to lift up an elephant.

a. Derive an algebraic equation that gives the mechanical advantge of the hydraulic machine shown. Assume the pistons are frictionless and massless.
b. A mouse can have a mass of 25 g and an elephant a mass of 7500 kg. Determine a value of D_1 and D_2 so that the mouse can support the elephant.



PROBLEM 3.7

3.8 Find a parked automobile for which you have informati on tire pressure and weight. Measure the area of tire contact with the pavement. Next, using the weight information and t pressure, use engineering principles to calculate the contact area. Compare your measurement with your calculation and discuss.

Deriving and Applying the Hydrostatic Equation (§3.2)

3.9 WILEY PLUS To derive the hydrostatic equation, which of the following must be assumed? (Select all that are correct.)

a. the specific weight is constant
b. the fluid has no charged particles
c. the fluid is at equilibrium

3.10 Imagine two tanks. Tank A is filled to depth h with water. Tank B is filled to depth h with oil. Which tank has largest pressure? Why? Where in the tank does the largest pressure occur?

3.11 Consider Figure 3.8 on p. 67 of §3.2.

a. Which fluid has the larger density?
b. If you graphed pressure as a function of z in these tw layered liquids, in which fluid does the pressure chan more with each incremental change in z?

3.12 WILEY PLUS Apply the grid method (§1.5 in Ch. 1) with the hydrostatic equation ($\Delta p = \gamma \Delta z$) to each of the following cases.

a. Predict the pressure change Δp in kPa for an elevatio change Δz of 10 ft in a fluid with a density of 90 lbm,
b. Predict the pressure change in psf for a fluid with S = and an elevation change of 22 m.
c. Predict pressure change in inches of water for a fluic with a density of 1.2 kg/m³ and an elevation change 1000 ft.
d. Predict the elevation change in millimeters for a flui with S = 13 that corresponds to a change in pressur 1/6 atm.

3.13 WILEY PLUS Using §3.2 and other resources, answer the follo questions. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enha the effectiveness of your communication.

a. What does hydrostatic mean? How do engineers identify whether a fluid is hydrostatic?

b. What are the common forms on the hydrostatic equation? Are the forms equivalent or are they different?

c. What is a datum? How do engineers establish a datum?

d. What are the main ideas of Eq. (3.10) on p. 66 of §3.2? That is, what is the meaning of this equation?

e. What assumptions need to be satisfied to apply the hydrostatic equation?

3.14 Apply the grid method to each situation.

a. What is the change in air pressure in pascals between the floor and the ceiling of a room with walls that are 10 ft tall.

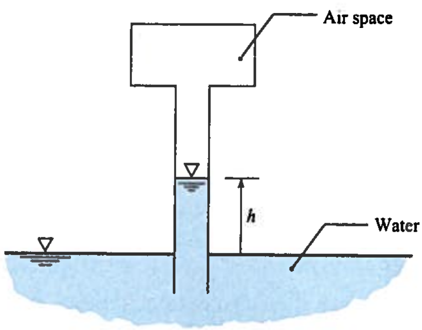
b. A diver in the ocean (S = 1.03) records a pressure of 2.5 atm on her depth gage. How deep is she?

c. A hiker starts a hike at an elevation where the air pressure is 940 mbar, and he ascends 1200 ft to a mountain summit. Assuming the density of air is constant, what is the pressure in mbar at the summit?

d. Lake Pend Oreille, in northern Idaho, is one of the deepest lakes in the world, with a depth of 350 m in some locations. This lake is used as a test facility for submarines. What is the maximum pressure that a submarine could experience in this lake?

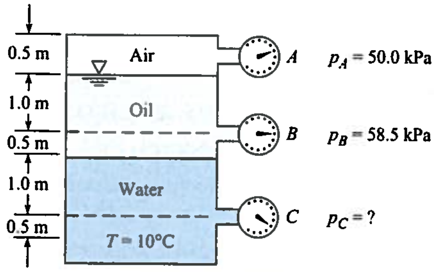
e. A 70 m tall standpipe (a standpipe is vertical pipe that is filled with water and open to the atmosphere) is used to supply water for fire fighting. What is the maximum pressure in the standpipe?

3.15 As shown, an air space above a long tube is pressurized to 50 kPa vacuum. Water (20°C) from a reservoir fills the tube to a height h. If the pressure in the air space is changed to 25 kPa vacuum, will h increase or descrease and by how much? Assume atmospheric pressure is 100 kPa.



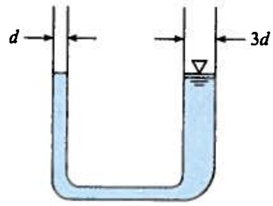
PROBLEM 3.15

3.16 For the closed tank with Bourdon-tube gages tapped into it, what is the specific gravity of the oil and the pressure reading on gage C?



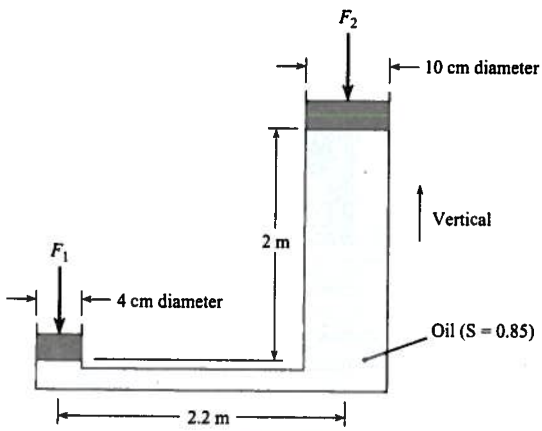
PROBLEM 3.16

3.17 This manometer contains water at room temperature. The glass tube on the left has an inside diameter of 1 mm (d = 1.0 mm). The glass tube on the right is three times as large. For these conditions, the water surface level in the left tube will be (a) higher than the water surface level in the right tube, (b) equal to the water surface level in the right tube, or (c) less than the water surface level in the right tube. State your main reason or assumption for making your choice.



PROBLEM 3.17

3.18 If a 200 N force F_1 is applied to the piston with the 4 cm diameter, what is the magnitude of the force F_2 that can be resisted by the piston with the 10 cm diameter? Neglect the weights of the pistons.



PROBLEM 3.18

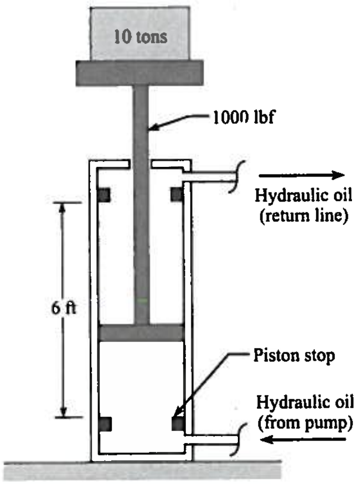
3.19 Regarding the hydraulic jack in Problem 3.18, which ideas were used to analyze the jack? (select all that apply)

a. pressure = (force)(area)
b. pressure increases linearly with depth in a hydrostatic fluid
c. the pressure at the very bottom of the 4-cm chamber is larger than the pressure at the very bottom of the 10-cm chamber
d. when a body is stationary, the sum of forces on the object is zero
e. when a body is stationary, the sum of moments on the object is zero
f. pressure = (weight/volume)(change in elevation)

3.20 Some skin divers go as deep as 50 m. What is the gage pressure at this depth in fresh water, and what is the ratio of the absolute pressure at this depth to normal atmospheric pressure? Assume $T = 20°C$.

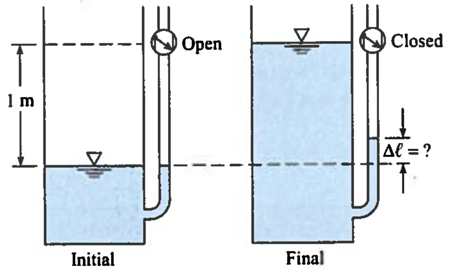
3.21 Water occupies the bottom 0.8 m of a cylindrical tank. On top of the water is 0.3 m of kerosene, which is open to the atmosphere. If the temperature is 20°C, what is the gage pressure at the bottom of the tank?

3.22 An engineer is designing a hydraulic lift with a capacity of 10 tons. The moving parts of this lift weigh 1000 lbf. The lift should raise the load to a height of 6 ft in 20 seconds. This will be accomplished with a hydraulic pump that delivers fluid to a cylinder. Hydraulic cylinders with a stroke of 72 inches are available with bore sizes from 2 to 8 inches. Hydraulic piston pumps with an operating pressure range from 200 to 3000 psig are available with pumping capacities of 5, 10, and 15 gallons per minute. Select a hydraulic pump size and a hydraulic cylinder size that can be used for this application.



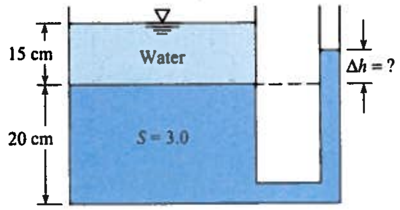
PROBLEM 3.22

3.23 A tank with an attached manometer contains water at 20°C. The atmospheric pressure is 100 kPa. There is a stopcock located 1 m from the surface of the water in the manometer. The stopcock is closed, trapping the air in the manometer, and water is added to the tank to the level of the stopcock. Find the increase in elevation of the water in the manometer assuming the air in the manometer is compressed isothermally.



PROBLEM 3.23

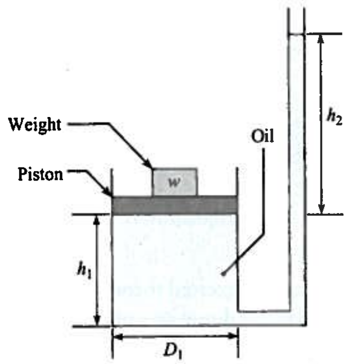
3.24 A tank is fitted with a manometer on the side, as shown. The liquid in the bottom of the tank and in the manc has a specific gravity (S) of 3.0. The depth of this bottom liqu 20 cm. A 15 cm layer of water lies on top of the bottom liqui Find the position of the liquid surface in the manometer.



PROBLEM 3.24

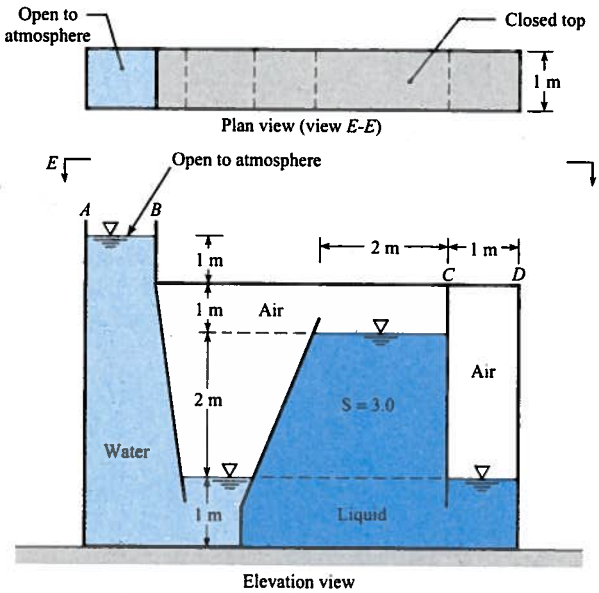
3.25 As shown, a load acts on a piston of diameter D The piston rides on a reservoir of oil of depth h_1 and specifi gravity S. The reservoir is connected to a round tube of dia D_2 and oil rises in the tube to height h_2. The oil in the tube open to atmosphere. Derive an equation for the height h_2 i terms of the weight W of the load and other relevant varial Neglect the weight of the piston.

3.26 As shown, a load of mass 5 kg is situated on a piston diameter $D_1 = 120$ mm. The piston rides on a reservoir of depth $h_1 = 42$ mm and specific gravity S = 0.8. The reserv connected to a round tube of diameter $D_2 = 5$ mm and oil in the tube to height h_2. Find h_2. Assume the oil in the tube open to atmosphere and neglect the weight of the piston.



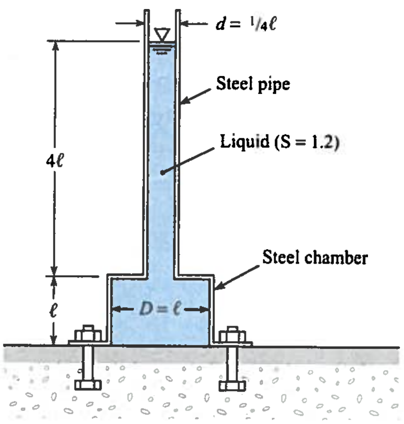
PROBLEMS 3.25, 3.26

3.27 What is the maximum gage pressure in the odd tank shown in the figure? Where will the maximum pressure occur? What is the hydrostatic force acting on the top (CD) of the last chamber on the right-hand side of the tank? Assume $T = 10°C$.



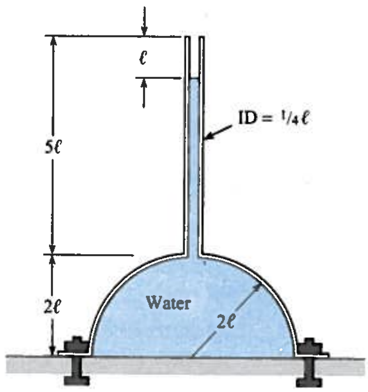
PROBLEM 3.27

3.28 The steel pipe and steel chamber shown in the figure together weigh 600 lbf. What force will have to be exerted on the chamber by all the bolts to hold it in place? The dimension ℓ is equal to 2.5 ft. *Note:* There is no bottom on the chamber—only a flange bolted to the floor.



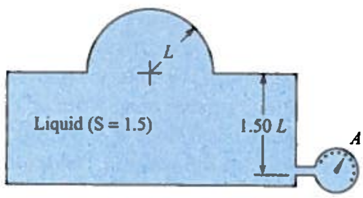
PROBLEM 3.28

3.29 What force must be exerted through the bolts to hold the dome in place? The metal dome and pipe weigh 6 kN. The dome has no bottom. Here $\ell = 80$ cm and the specific weight of the water is $\gamma = 9810\ \text{N/m}^3$.



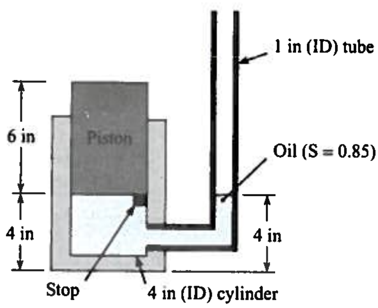
PROBLEM 3.29

3.30 Find the vertical component of force in the metal at the base of the spherical dome shown when gage A reads 5 psig. Indicate whether the metal is in compression or tension. The specific gravity of the enclosed fluid is 1.5. The dimension L is 2 ft. Assume the dome weighs 1000 lbf.



PROBLEM 3.30

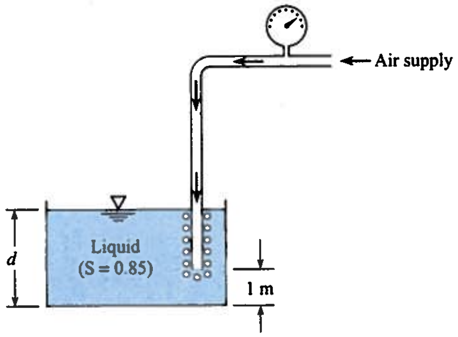
3.31 The piston shown weighs 10 lbf. In its initial position the piston is restrained from moving to the bottom of the cylinder by means of the metal stop. Assuming there is neither friction nor leakage between piston and cylinder, what volume c oil (S = 0.85) would have to be added to the 1 in. tube to cause the piston to rise 1 in. from its initial position?



PROBLEM 3.31

3.32 Consider an air bubble rising from the bottom of a lake. Neglecting surface tension, determine approximately what the ratio of the density of the air in the bubble will be at a depth of 34 ft to its density at a depth of 8 ft.

3.33 One means of determining the surface level of liquid in a tank is by discharging a small amount of air through a small tube, the end of which is submerged in the tank, and reading the pressure on the gage that is tapped into the tube. Then the level of the liquid surface in the tank can be calculated. If the pressure on the gage is 15 kPa, what is the depth d of liquid in the tank?



PROBLEM 3.33

Calculating Pressure in the Atmosphere (§3.2)

3.34 For Fig. 3.9 on p. 70 of §3.2 that describes temperature variation with altitude, answer the following questions.

a. Does the linear approximation relating temperature to altitude apply in the troposphere or the stratosphere?

b. At approximately what altitude in the earth's atmosphere does the linear approximation for temperature variation fail?

3.35 The boiling point of water decreases with elevation because of the pressure change. What is the boiling point of water at an elevation of 2000 m and at an elevation of 4000 m for standard atmospheric conditions?

3.36 From a depth of 10 m in a lake to an elevation of 4000 m in the atmosphere, plot the variation of absolute pressure. Assume that the lake water surface elevation is at mean sea level and assume standard atmospheric conditions.

3.37 WILEY PLUS Assume that a woman must breathe a constant mass rate of air to maintain her metabolic processes. If she inhales and exhales 16 times per minute at sea level, where the temperature is 59°F (15°C) and the pressure is 14.7 psia (101 kPa), what would you expect her rate of breathing at 18,000 ft (5486 m) to be? Use standard atmospheric conditions.

3.38 A pressure gage in an airplane indicates a pressure of 95 kPa at takeoff, where the airport elevation is 1 km and the temperature is 10°C. If the standard lapse rate of 5.87°C/km is assumed, at what elevation is the plane when a pressure of 75 kPa is read? What is the temperature for that condition?

3.39 Denver, Colorado, is called the "mile-high" city. What are the pressure, temperature, and density of the air when standard atmospheric conditions prevail? Give your answer in traditional and SI units.

3.40 WILEY PLUS An airplane is flying at 10 km altitude in a U.S. standard atmosphere. If the internal pressure of the aircraft interior is 100 kPa, what is the outward force on a window The window is flat and has an elliptical shape with lengths of 300 mm along the major axis and 200 mm along the minor axis.

3.41 The mean atmospheric pressure on the surface of M is 7 mbar, and the mean surface temperature is −63°C. T atmosphere consists primarily of CO_2 (95.3%) with small amounts of nitrogen and argon. The acceleration due to gravity on the surface is 3.72 m/s^2. Data from probes ente the Martian atmosphere show that the temperature variat with altitude can be approximated as constant at −63°C f the Martian surface to 14 km, and then a linear decrease a lapse rate of 1.5°C/km up to 34 km. Find the pressure a 8 km and 30 km altitude. Assume the atmosphere is pure carbon dioxide. Note that the temperature distribution in the atmosphere of Mars differs from that of Earth because t region of constant temperature is adjacent to the surface an region of decreasing temperature starts at an altitude of 14 k

3.42 Design a computer program that calculates the pressur and density for the U.S. standard atmosphere from 0 to 30 kı altitude. Assume the temperature profiles are linear and are approximated by the following ranges, where z is the altitude kilometers:

0–13.72 km	$T = 23.1 - 5.87z$ (°C)
13.7–16.8 km	$T = -57.5$°C
16.8–30 km	$T = -57.5 + 1.387(z - 16.8)$°C

Measuring Pressure (§3.3)

3.43 Match the following pressure-measuring devices with t correct name. The device names are: barometer, Bourdon ga piezometer, manometer, and pressure transducer.

a. A vertical or U-shaped tube where changes in pressu are documented by changes in relative elevation of a liquid that is usually denser than the fluid in the syst measured; can be used to measure vacuum.

b. Typically contains a diaphragm, a sensing element, a conversion to an electric signal.

c. A round face with a scale to measure needle deflectic where the needle is deflected by changes in extensior coiled hollow tube.

d. A vertical tube where a liquid rises in response to a positive gage pressure.

e. An instrument used to measure atmospheric pressur various designs.

Applying the Manometer Equations (§3.3)

3.44 WILEY PLUS Which is the more correct way to describe the tw summation (Σ) terms of the manometer equation, Eq (3.21) p. 74 of §3.3?

a. Add the downs and subtract the ups.

b. Subtract the downs and add the ups.

3.45 Using the Internet and other resources, answer the following questions:

a. What are three common types of manometers? For each type, make a sketch and give a brief description.

b. How would you build a manometer from materials that are commonly available? Sketch your design concept.

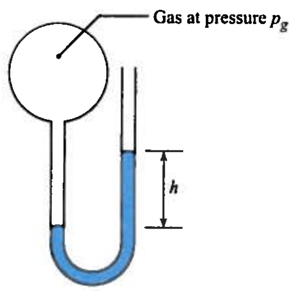
3.46 As shown, gas at pressure p_g raises a column of liquid to a height h. The gage pressure in the gas is given by $p_g = \gamma_{\text{liquid}} h$. Apply the grid method (p. 00) to each situation that follows.

a. The manometer uses a liquid with S = 1.3. Calculate pressure in psia for $h = 1$ ft.

b. The manometer uses mercury. Calculate the column rise in mm for a gage pressure of 0.25 atm.

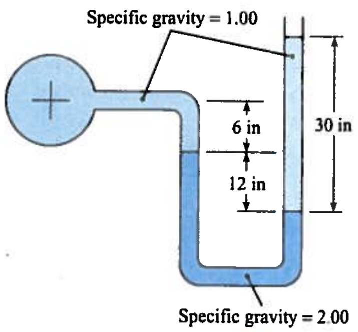
c. The liquid has a density of 30 lbm/ft^3. Calculate pressure in psfg for $h = 4$ inches.

d. The liquid has a density of 800 kg/m^3. Calculate the gage pressure in bar for $h = 3$ m.



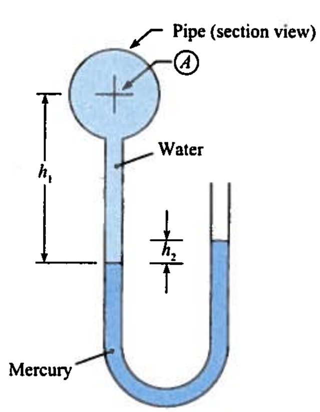
PROBLEM 3.46

3.47 Is the gage pressure at the center of the pipe (a) negative, (b) zero, or (c) positive? Neglect surface tension effects and state your rationale.



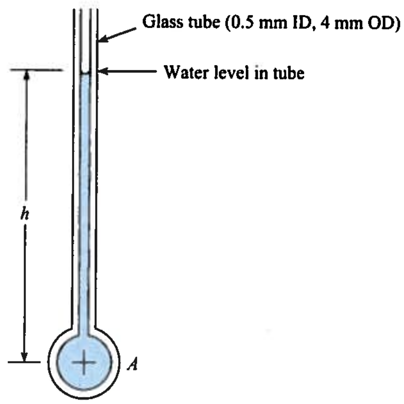
PROBLEM 3.47

3.48 Determine the gage pressure at the center of the pipe (point A) in pounds per square inch when the temperature is 70°F with $h_1 = 16$ in. and $h_2 = 2$ in.



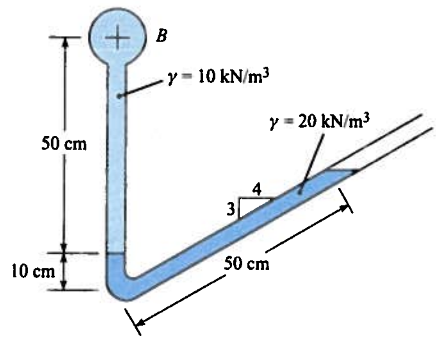
PROBLEM 3.48

3.49 Considering the effects of surface tension, estimate the gage pressure at the center of pipe A for $h = 120$ mm and $T = 20$°C.



PROBLEM 3.49

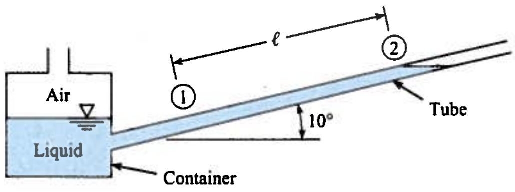
3.50 What is the pressure at the center of pipe B?



PROBLEM 3.50

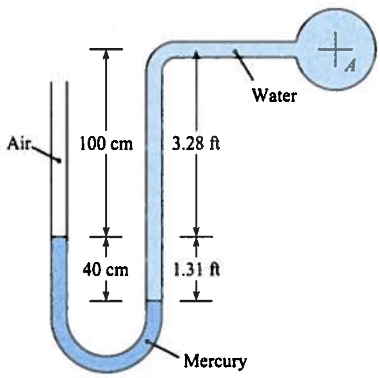
3.51 The ratio of container diameter to tube diameter is 8. Wh air in the container is at atmospheric pressure, the free surface i the tube is at position 1. When the container is pressurized, the liquid in the tube moves 40 cm up the tube from position 1 to position 2. What is the container pressure that causes this deflection? The liquid density is 1200 kg/m^3.

3.52 The ratio of container diameter to tube diameter is 10. When air in the container is at atmospheric pressure, the free surface in the tube is at position 1. When the container is pressurized, the liquid in the tube moves 3 ft up the tube from position 1 to position 2. What is the container pressure that causes this deflection? The specific weight of the liquid is 50 lbf/ft^3.



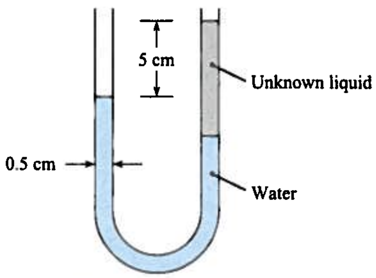
PROBLEMS 3.51, 3.52

3.53 WILEY PLUS Determine the gage pressure at the center of pipe A in pounds per square inch and in kilopascals.



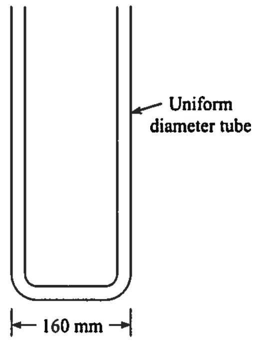
PROBLEM 3.53

3.54 A device for measuring the specific weight of a liquid consists of a U-tube manometer as shown. The manometer tube has an internal diameter of 0.5 cm and originally has water in it. Exactly 2 cm^3 of unknown liquid is then poured into one leg of the manometer, and a displacement of 5 cm is measured between the surfaces as shown. What is the specific weight of the unknown liquid?



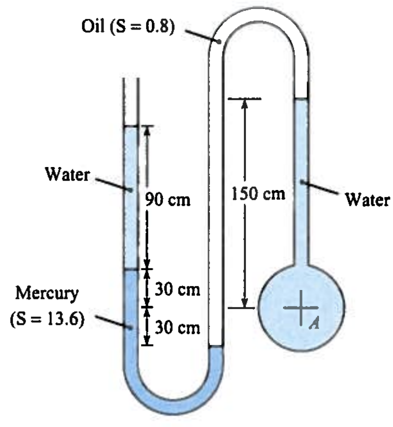
PROBLEM 3.54

3.55 Mercury is poured into the tube in the figure until the mercury occupies 375 mm of the tube's length. An equal volur water is then poured into the left leg. Locate the water and me surfaces. Also determine the maximum pressure in the tube.



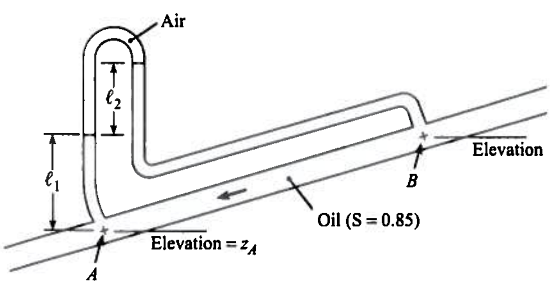
PROBLEM 3.55

3.56 WILEY PLUS Find the pressure at the center of pipe A. $T = 10°C$.



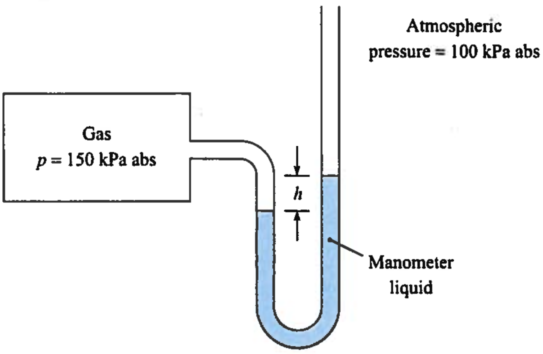
PROBLEM 3.56

3.57 Determine (a) the difference in pressure and (b) the difference in piezometric head between points A and B. The elevations z_A and z_B are 10 m and 11 m, respectively, $\ell_1 = 1$ and the manometer deflection ℓ_2 is 50 cm.



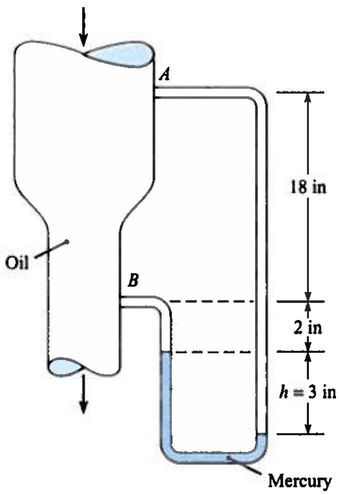
PROBLEM 3.57

3.58 The deflection on the manometer is h meters when the pressure in the tank is 150 kPa absolute. If the absolute pressu the tank is doubled, what will the deflection on the manomet



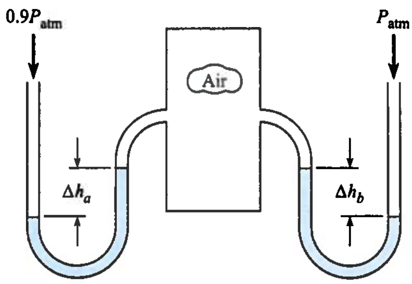
PROBLEM 3.58

3.59 WILEY PLUS A vertical conduit is carrying oil (S = 0.95). A differential mercury manometer is tapped into the conduit at points *A* and *B*. Determine the difference in pressure between *A* and *B* when h = 3 in. What is the difference in piezometric head between *A* and *B*?



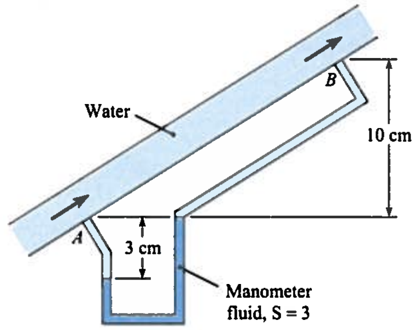
PROBLEM 3.59

3.60 Two water manometers are connected to a tank of air. One leg of the manometer is open to 100 kPa pressure (absolute) while the other leg is subjected to 90 kPa. Find the difference in deflection between both manometers, $\Delta h_a - \Delta h_b$.



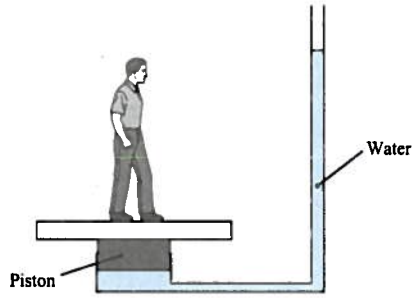
PROBLEM 3.60

3.61 A manometer is used to measure the pressure difference between points *A* and *B* in a pipe as shown. Water flows in the pipe and the specific gravity of the manometer fluid is 2.8. The distances and manometer deflection are indicated on the figure. Find (a) the pressure differences $p_A - p_B$, and (b) the difference in piezometric pressure, $p_{z,A} - p_{z,B}$. Express both answers in kPa.



PROBLEM 3.61

3.62 A novelty scale for measuring a person's weight by having the person stand on a piston connected to a water reservoir and stand pipe is shown in the diagram. The level of the water in the stand pipe is to be calibrated to yield the person's weight in pounds force. When the person stands on the scale, the height of the water in the stand pipe should be near eye level so the person can read it. There is a seal around the piston that prevents leaks but does not cause a significant frictional force. The scale should function for people who weigh between 60 and 250 lbf and are between 4 and 6 feet tall. Choose the piston size and standpipe diameter. Clearly state the design features you considered. Indicate how you would calibrate the scale on the standpipe. Would the scale be linear?



PROBLEM 3.62

Applying the Panel Force Equations (§3.4)

3.63 Using §3.4 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

a. For hydrostatic conditions, what do typical pressure distributions on a panel look like? Sketch three examples that correspond to different situations.

b. What is a center of pressure (CP)? What is a centroid of area?

c. In Eq. (3.28) on p. 80 of §3.4, what does $\bar{p}$ mean? What factors influence the value of $\bar{p}$?

d. What is the relationship between the pressure distribution on a panel and the resultant force?

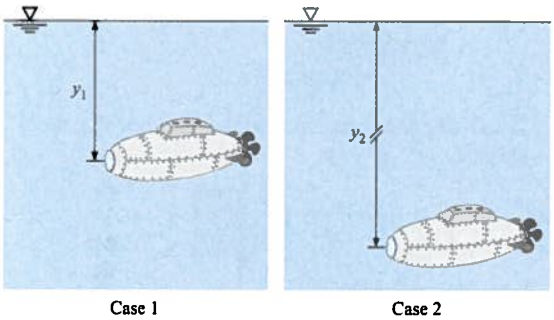
e. How far is the CP from the centroid of area? What factors influence this distance?

3.64 Part 1. Consider the equation for the distance between the CP and the centroid of a submerged panel (Eq. (3.33) on p. 81 of §3.4). In that equation, y_{cp} is

a. the vertical distance from the water surface to the CP.

b. the slant distance from the water surface to the CP.

Part 2. Consider the figure shown. For case 1 as shown, the viewing window on the front of a submersible exploration vehicle is at a depth of y_1. For case 2, the submersible has moved deeper in the ocean, to y_2. As a result of this increased overall depth of the submersible and its window, does the spacing between the CP and centroid (a) get larger, (b) stay the same, or (c) get smaller?



PROBLEM 3.64

3.65 Which of these assumptions and/or limitations must be known when using Eq. (3.33) on p. 81 of §3.4 for a submerged surface or panel to calculate the distance between the centroid of the panel and the center of pressure of the hydrostatic force (select all that apply):

a. The equation only applies to a single fluid of constant density

b. The pressure at the surface must be p = 0 gage

c. The panel must be vertical

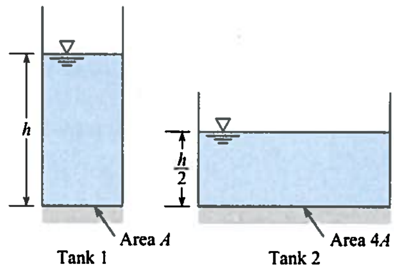
d. The equation gives only the vertical location (as a slant distance) to the CP, not the lateral distance from the edge of the body

3.66 Two cylindrical tanks have bottom areas A and $4A$ respectively, and are filled with water to the depths shown.

a. Which tank has the higher pressure at the bottom of the tank?

b. Which tank has the greater force acting downward on the bottom circular surface?

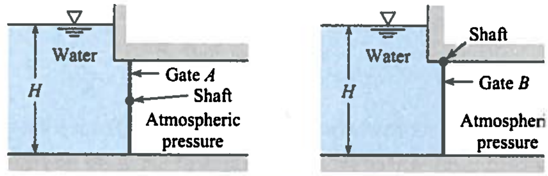
3.67 What is the force acting on the gate of an irrigation ditch if the ditch and gate are 4 ft wide, 4 ft deep, and the ditch is competely full of water? There is no water on the other side of the gate. The weather has been hot for weeks, so the water is 70°F.



PROBLEM 3.66

3.68 Consider the two rectangular gates shown in the figure. They are both the same size, but gate A is held in plac a horizontal shaft through its midpoint and gate B is cantile to a shaft at its top. Now consider the torque T required to hc the gates in place as H is increased. Choose the valid stateme (a) T_A increases with H. (b) T_B increases with H. (c) T_A does change with H. (d) T_B does not change with H.

3.69 For gate A, choose the statements that are valid: (a) The hydrostatic force acting on the gate increases as H increases. (b) The distance between the CP on the gate and t centroid of the gate decreases as H increases. (c) The distanc between the CP on the gate and the centroid of the gate rem constant as H increases. (d) The torque applied to the shaft t prevent the gate from turning must be increased as H increa (e) The torque applied to the shaft to prevent the gate from turning remains constant as H increases.



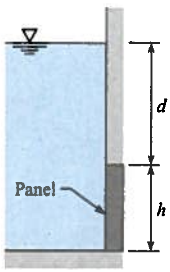
PROBLEMS 3.68, 3.69

3.70 As shown, water (15°C) is in contact with a squar panel; $d = 1$ m and $h = 2$ m.

a. Calculate the depth of the centroid

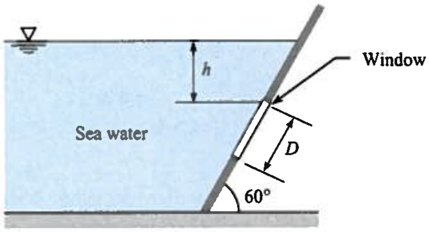
b. Calculate the resultant force on the panel

c. Calculate the distance from the centroid to the CP.



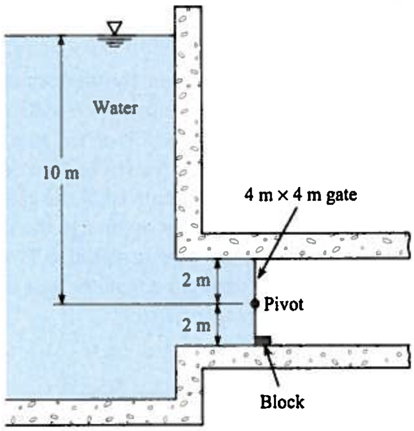
PROBLEM 3.70

3.71 GO As shown, a round viewing window of diameter $D = 0.5$ m is situated in a large tank of seawater ($S = 1.03$). The top of the window is 1.5 m below the water surface, and the window is angled at 60° with respect to the horizontal. Find the hydrostatic force acting on the window and locate the corresponding CP.



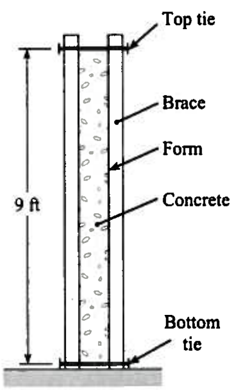
PROBLEM 3.71

3.72 PLUS Find the force of the gate on the block. See sketch.



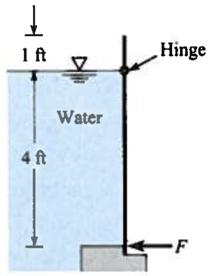
PROBLEM 3.72

3.73 Assume that wet concrete ($\gamma = 150$ lbf/ft^3) behaves as a liquid. Determine the force per unit foot of length exerted on the forms. If the forms are held in place as shown, with ties between vertical braces spaced every 2 ft, what force is exerted on the bottom tie?



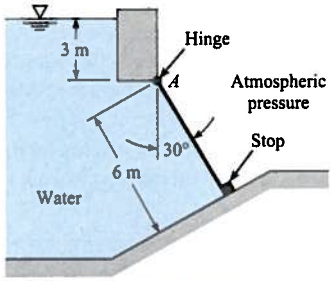
PROBLEM 3.73

3.74 PLUS A rectangular gate is hinged at the water line, as shown. The gate is 4 ft high and 8 ft wide. The specific weight of water is 62.4 lbf/ft^3. Find the necessary force (in lbf) applied at the bottom of the gate to keep it closed.



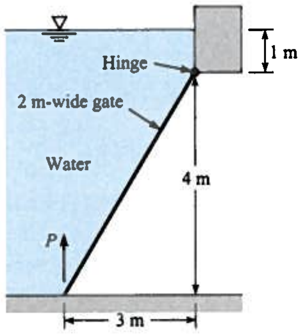
PROBLEM 3.74

3.75 The gate shown is rectangular and has dimensions 6 m by 4 m. What is the reaction at point A? Neglect the weight of the gate.



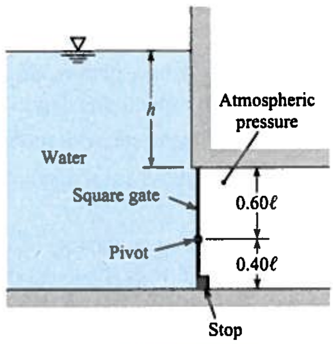
PROBLEM 3.75

3.76 PLUS Determine P necessary to just start opening the 2 m–wide gate.



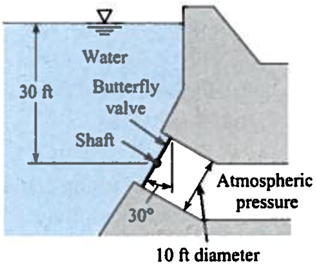
PROBLEM 3.76

3.77 PLUS The square gate shown is eccentrically pivoted so that it automatically opens at a certain value of h. What is that value in terms of ℓ?



PROBLEM 3.77

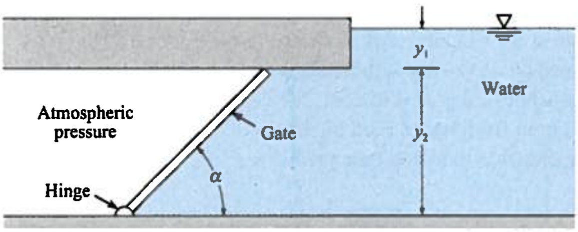
3.78 This 10 ft-diameter butterfly valve is used to control the flow in a 10 ft-diameter outlet pipe in a dam. In the position shown, it is closed. The valve is supported by a horizontal shaft through its center. What torque would have to be applied to the shaft to hold the valve in the position shown?



PROBLEM 3.78

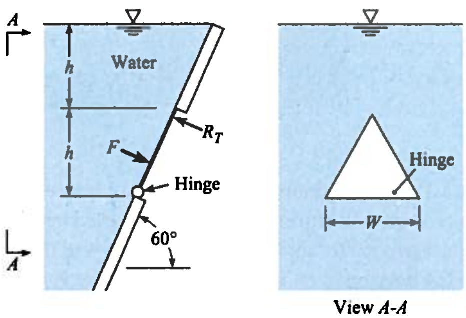
3.79 For the gate shown, $\alpha = 45°$, $y_1 = 1$ m, and $y_2 = 4$ m. Will the gate fall or stay in position under the action of the hydrostatic and gravity forces if the gate itself weighs 150 kN and is 1.0 m wide? Assume $T = 10°$C. Use calculations to justify your answer.

3.80 For this gate, $\alpha = 45°$, $y_1 = 3$ ft, and $y_2 = 6$ ft. Will the gate fall or stay in position under the action of the hydrostatic and gravity forces if the gate itself weighs 18,000 lb and is 3 ft wide? Assume $T = 50°$F. Use calculations to justify your answer.



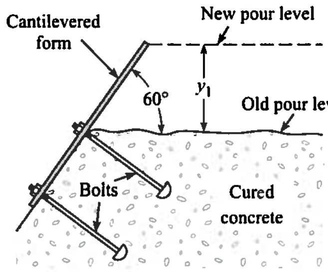
PROBLEMS 3.79, 3.80

3.81 Determine the hydrostatic force F on the triangular gate, which is hinged at the bottom edge and held by the reaction R_T at the upper corner. Express F in terms of γ, h, and W. Also determine the ratio R_T/F. Neglect the weight of the gate.



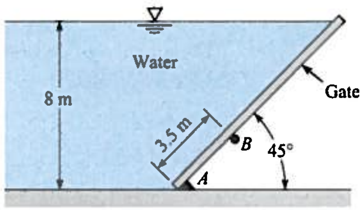
PROBLEM 3.81

3.82 In constructing dams, the concrete is poured in of approximately 1.5 m ($y_1 = 1.5$ m). The forms for the fac the dam are reused from one lift to the next. The figure sho one such form, which is bolted to the already cured concre the new pour, what moment will occur at the base of the fo per meter of length (normal to the page)? Assume that con acts as a liquid when it is first poured and has a specific we 24 kN/m³.



PROBLEM 3.82

3.83 The plane rectangular gate can pivot about the suppo For the conditions given, is it stable or unstable? Neglect th weight of the gate. Justify your answer with calculations.

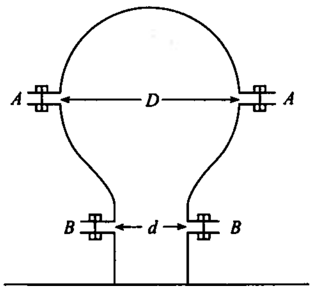


PROBLEM 3.83

Calculating Pressure on Curved Surfaces (§3.5)

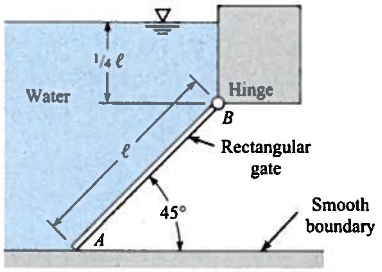
3.84 Two hemispheric shells are perfectly sealed tog and the internal pressure is reduced to 25% of atmospheric pressure. The inner radius is 10.5 cm, and the outer radius 10.75 cm. The seal is located halfway between the inner an outer radius. If the atmospheric pressure is 101.3 kPa, what is required to pull the shells apart?

3.85 If exactly 20 bolts of 2.5 cm diameter are needed to h the air chamber together at A-A as a result of the high pres within, how many bolts will be needed at B-B? Here $D = 4$ and $d = 20$ cm.

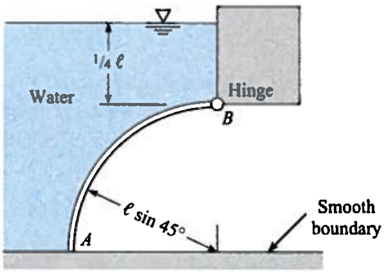


PROBLEM 3.85

3.86 For the plane rectangular gate ($\ell \times w$ in size), figure (*a*), what is the magnitude of the reaction at *A* in terms of γ_w and the dimensions ℓ and w? For the cylindrical gate, figure (*b*), will the magnitude of the reaction at *A* be greater than, less than, or the same as that for the plane gate? Neglect the weight of the gates.



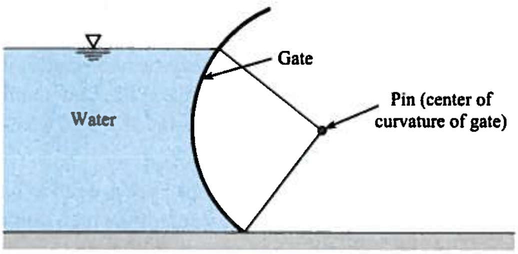
(a) Plane gate



(b) Curved gate

PROBLEM 3.86

3.87 Water is held back by this radial gate. Does the resultant of the pressure forces acting on the gate pass above the pin, through the pin, or below the pin?



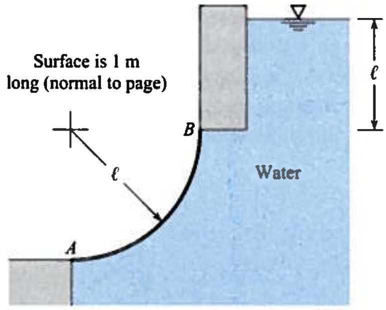
PROBLEM 3.87

3.88 For the curved surface *AB*:

a. Determine the magnitude, direction, and line of action of the vertical component of hydrostatic force acting on the surface. Here $\ell = 1$ m.

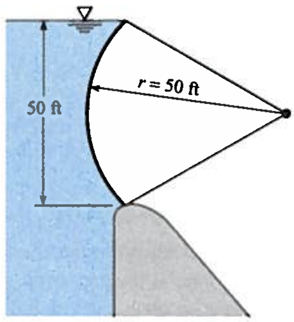
b. Determine the magnitude, direction, and line of action of the horizontal component of hydrostatic force acting on the surface.

c. Determine the resultant hydrostatic force acting on the surface.



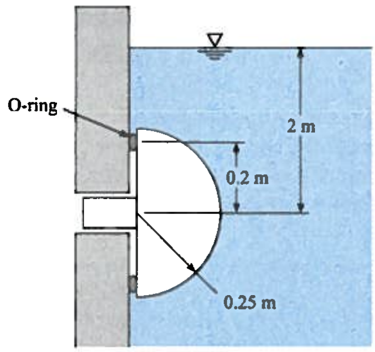
PROBLEM 3.88

3.89 Determine the hydrostatic force acting on the radial gate i the gate is 40 ft long (normal to the page). Show the line of actic of the hydrostatic force acting on the gate.



PROBLEM 3.89

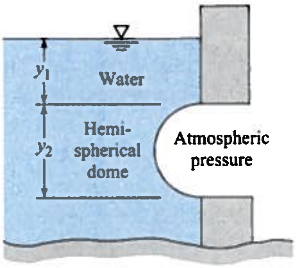
3.90 WILEY PLUS A plug in the shape of a hemisphere is inserted in a hole in the side of a tank as shown in the figure. The plug is sealed by an O-ring with a radius of 0.2 m. The radius of the hemispherical plug is 0.25 m. The depth of the center of the plu is 2 m in fresh water. Find the horizontal and vertical forces on the plug due to hydrostatic pressure.



PROBLEM 3.90

3.91 WILEY PLUS This dome (hemisphere) is located below the water surface as shown. Determine the magnitude and sign of the forc components needed to hold the dome in place and the line of action of the horizontal component of force. Here $y_1 = 1$ m and $y_2 = 2$ m. Assume $T = 10°$C.

3.92 Consider the dome shown. This dome is 10 ft in diameter, but now the dome is not submerged. The water surface is at the level of the center of curvature of the dome. For these conditions, determine the magnitude and direction of the resultant hydrostatic force acting on the dome.



PROBLEM 3.91, 3.92

Calculating Buoyant Forces (§3.6)

3.93 Apply the grid method (§1.5 in Ch. 1) to each situation below.

- **a.** Determine the buoyant force in newtons on a basketball that is floating in a lake (10°C).
- **b.** Determine the buoyant force in newtons on a 1 mm copper sphere that is immersed in kerosene.
- **c.** Determine the buoyant force in newtons on a 12 inch–diameter balloon. The balloon is filled with helium and situated in ambient air (20°C).

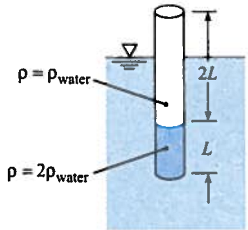
3.94 Using §3.6 and other resources, answer the following questions. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

- **a.** Why learn about buoyancy? That is, what are important technical problems that involve buoyant forces?
- **b.** For a buoyant force, where is the CP? Where is the line of action?
- **c.** What is displaced volume? Why is it important?
- **d.** What is the relationship between pressure distribution and buoyant force?

3.95 Three spheres of the same diameter are submerged in the same body of water. One sphere is steel, one is a spherical balloon filled with water, and one is a spherical balloon filled with air.

- **a.** Which sphere has the largest buoyant force?
- **b.** If you move the steel sphere from a depth of 1 ft to 10 ft, what happens to the magnitude of the buoyant force acting on that sphere?
- **c.** If all 3 spheres are released from a cage at a depth of 1 m, what happens to the 3 spheres, and why?

3.96 As shown, a uniform-diameter rod is weighted at one
and is floating in a liquid. The liquid (a) is lighter than wate
(b) must be water, or (c) is heavier than water. Show your w

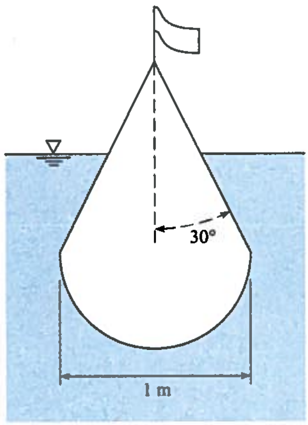


PROBLEM 3.96

3.97 An 800 ft ship has a displacement of 35,000 tons
the area defined by the waterline is 38,000 ft². Will the ship
more or less draft when steaming from salt water to fresh w
How much will it settle or rise?

3.98 A submerged spherical steel buoy that is 1.2 m i
diameter and weighs 1200 N is to be anchored in salt water
20 m below the surface. Find the weight of scrap iron that
should be sealed inside the buoy in order that the force on i
anchor chain will not exceed 4.5 kN.

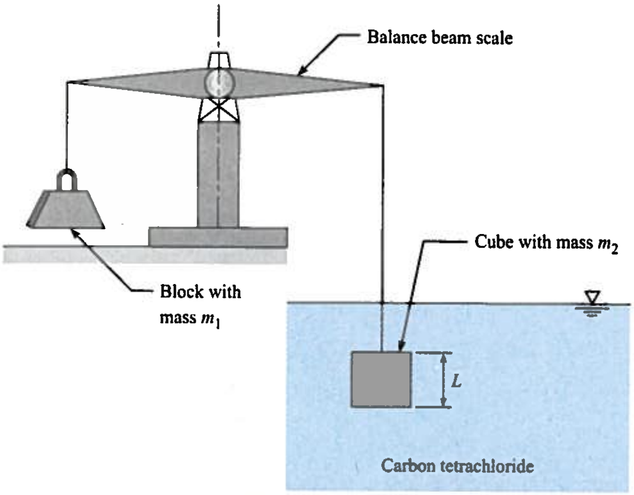
3.99 A buoy is designed with a hemispherical bottom and
conical top as shown in the figure. The diameter of the
hemisphere is 1 m, and the half angle of the cone is 30°. The
has a mass of 460 kg. Find the location of the water line on
buoy floating in sea water ($\rho = 1010$ kg/m³).



PROBLEM 3.99

3.100 A rock weighs 925 N in air and 781 N in water.
its volume.

3.101 As shown, a cube ($L = 60$ mm) suspended in c
tetracloride is exactly balanced by an object of mass m_1 =
Find the mass m_2 of the cube.

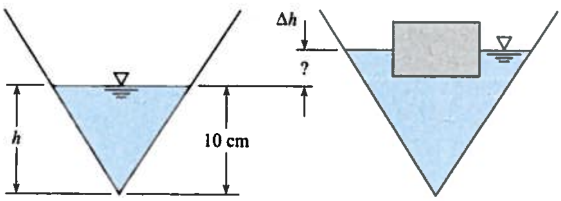


PROBLEM 3.101

3.102 WILEY PLUS A block of material of unknown volume is submerged in water and found to weigh 300 N (in water). The same block weighs 700 N in air. Determine the specific weight and volume of the material.

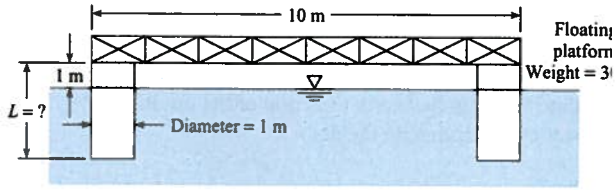
3.103 A 1 ft-diameter cylindrical tank is filled with water to a depth of 2 ft. A cylinder of wood 5 in. in diameter and 2.5 in. long is set afloat on the water. The weight of the wood cylinder is 2 lbf. Determine the change (if any) in the depth of the water in the tank.

3.104 A 90° inverted cone contains water as shown. The volume of the water in the cone is given by $\forall = (\pi/3)h^3$. The original depth of the water is 10 cm. A block with a volume of 200 cm^3 and a specific gravity of 0.6 is floated in the water. What will be the change (in cm) in water surface height in the cone?



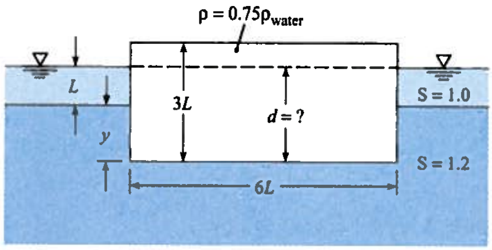
PROBLEM 3.104

3.105 WILEY PLUS The floating platform shown is supported at each corner by a hollow sealed cylinder 1 m in diameter. The platform itself weighs 30 kN in air, and each cylinder weighs 1.0 kN per meter of length. What total cylinder length L is required for the platform to float 1 m above the water surface? Assume that the specific weight of the water (brackish) is 10,000 N/m^3. The platform is square in plan view.



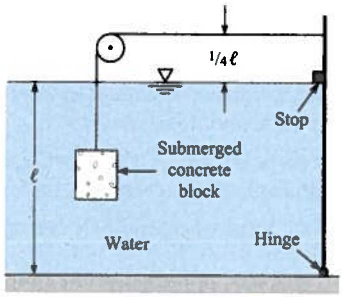
PROBLEM 3.105

3.106 To what depth d will this rectangular block (with density 0.75 times that of water) float in the two-liquid reservoir?



PROBLEM 3.106

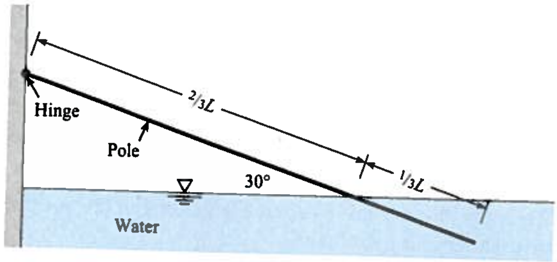
3.107 WILEY PLUS Determine the minimum volume of concrete ($\gamma = 23.6$ kN/m^3) needed to keep the gate (1 m wide) in a closed position, with $\ell = 2$ m. Note the hinge at the bottom of the gate.



PROBLEM 3.107

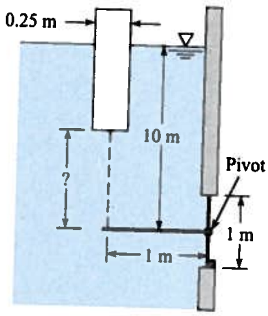
3.108 A cylindrical container 4 ft high and 2 ft in diameter holds water to a depth of 2 ft. How much does the level of the water in the tank change when a 5 lb block of ice is placed in the container? Is there any change in the water level in the tank when the block of ice melts? Does it depend on the specific gravity of the ice? Explain all the processes.

3.109 WILEY PLUS The partially submerged wood pole is attached to the wall by a hinge as shown. The pole is in equilibrium under the action of the weight and buoyant forces. Determine the density of the wood.



PROBLEM 3.109

3.110 A gate with a circular cross section is held closed by a lever 1 m long attached to a buoyant cylinder. The cylinder is 25 cm in diameter and weighs 200 N. The gate is attached to a horizontal shaft so it can pivot about its center. The liquid is water. The chain and lever attached to the gate have negligible weight. Find the length of the chain such that the gate is just on the verge of opening when the water depth above the gate hinge is 10 m.



PROBLEM 3.110

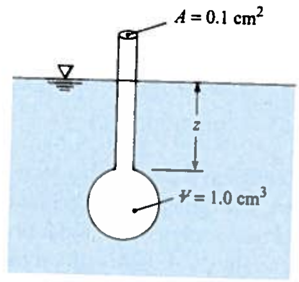
3.111 A balloon is to be used to carry meteorological instruments to an elevation of 15,000 ft where the air pressure is 8.1 psia. The balloon is to be filled with helium, and the material from which it is to be fabricated weighs 0.01 lbf/ft^2. If the instruments weigh 8 lbf, what diameter should the spherical balloon have?

3.112 A weather balloon is constructed of a flexible material such that the internal pressure of the balloon is always 10 kPa higher than the local atmospheric pressure. At sea level the diameter of the balloon is 1 m, and it is filled with helium. The balloon material, structure, and instruments have a mass of 100 g. This does not include the mass of the helium. As the balloon rises, it will expand. The temperature of the helium is always equal to the local atmospheric temperature, so it decreases as the balloon gains altitude. Calculate the maximum altitude of the balloon in a standard atmosphere.

Measuring ρ, γ, and *S* with Hydrometers (§3.6)

3.113 The hydrometer shown weighs 0.015 N. If the stem sinks 6.0 cm in oil (z = 6.0 cm), what is the specific gravity of the oil?

3.114 The hydrometer shown sinks 5.3 cm (z = 5.3 water (15°C). The bulb displaces 1.0 cm^3, and the stem area is Find the weight of the hydrometer.

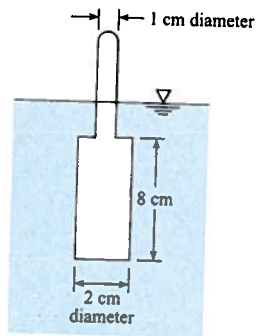


PROBLEMS 3.113, 3.114

3.115 A common commercial hydrometer for measu the amount of antifreeze in the coolant system of an auton engine consists of a chamber with differently colored balls. system is calibrated to give the range of specific gravity by distinguishing between the balls that sink and those that fl The specific gravity of an ethylene glycol-water mixture var from 1.012 to 1.065 for 10% to 50% by weight of ethylene g Assume there are six balls, 1 cm in diameter each, in the chamber. What should the weight of each ball be to provide a range of specific gravities between 1.01 and 1.06 with 0.01 intervals?

3.116 A hydrometer with the configuration shown has a bulb diameter of 2 cm, a bulb length of 8 cm, a stem diameter of 1 cm, a length of 8 cm, and a mass of 40 g. Wh is the range of specific gravities that can be measured with hydrometer?

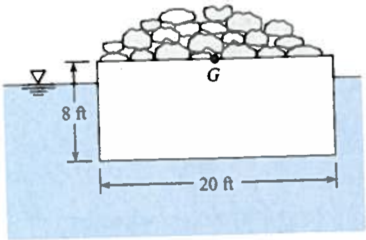
(*Hint:* Liquid levels range between bottom and top of stem.)



PROBLEM 3.116

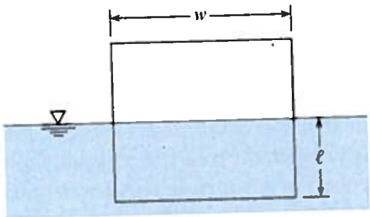
Predicting Stability (§3.7)

3.117 A barge 20 ft wide and 40 ft long is loaded with rocks a shown. Assume that the center of gravity of the rocks and bar is located along the centerline at the top surface of the barge. I the rocks and the barge weigh 400,000 lbf, will the barge float upright or tip over?



PROBLEM 3.117

3.118 A floating body has a square cross section with side w as shown in the figure. The center of gravity is at the centroid of the cross section. Find the location of the water line, ℓ/w, where the body would be neutrally stable ($GM = 0$). If the body is floating in water, what would be the specific gravity of the body material?

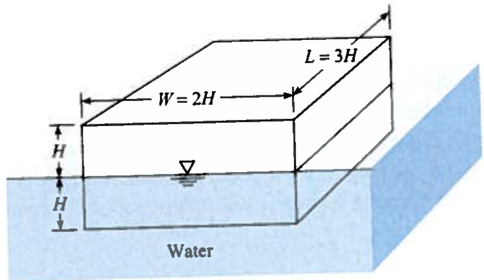


PROBLEM 3.118

3.119 A cylindrical block of wood 1 m in diameter and 1 m long has a specific weight of 7500 N/m^3. Will it float in water with its axis vertical?

3.120 WILEY PLUS A cylindrical block of wood 1 m in diameter and 1 m long has a specific weight of 5000 N/m^3. Will it float in water with the ends horizontal?

3.121 Is the block in this figure stable floating in the position shown? Show your calculations.



PROBLEM 3.121