11 DRAG AND LIFT

FIGURE 11.1

This photo shows the USA Olympic pursuit team being tested so that aerodynamic drag can be reduced. This wind tunnel is located at the General Motors Tech Center in Warren, Michigan. (Andy Sacks/Photodisc/Getty Images)

Chapter Road Map

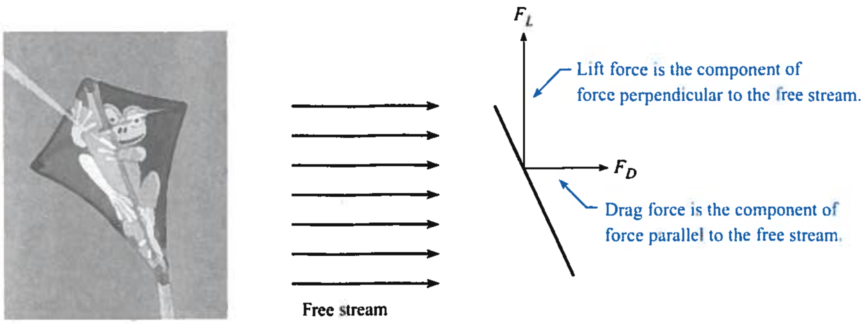
Previous chapters have described the hydrostatic force on a panel, the buoyant force on a submerg object, and the shear force on a flat plate. This chapter expands this list by introducing the lift and drag forces.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define lift and drag. Explain how lift and drag are relate to shear stress and pressure distributions. (§11.1)
- Define form drag. Define friction drag. (§11.1)
- For flow over a circular cylinder, describe the three drag regimes and the drag crisis. (§11.2)
- Define the coefficient of drag and find C_D values. Calcul the drag force. (§11.2, §11.3)
- Describe how to calculate the power required to overcor drag. Solve relevant problems. (§11.3)
- Explain how to calculate terminal velocity. Solve relevant problems. (§11.4)
- Describe vortex shedding. (§11.5)
- Explain what streamlining means. (§11.6)
- Define circulation and describe the circulation theory of li (§11.8)
- Define the coefficient of lift and calculate the lift force. (§11
- Calculate the lift and drag on an airfoil. (§11.9)

When a body moves through a stationary fluid or when a fluid flows past a body, fluid exerts a resultant force. The component of this resultant force that is parallel to the fr stream velocity is called the **drag force**. Similarly, the **lift force** is the component of the res tant force that is perpendicular to the free stream. For example, as air flows over a kite creates a resultant force that can be resolved in lift and drag components as shown in Fig. 11.2. By definition, lift and drag forces are limited to those forces produced by a flowing fluid.



(a) (b)

FIGURE 11.2
(a) A kite. [Photo by Donald Elger]
(b) Forces acting on the kite due to the air flow over the kite.

11.1 Relating Lift and Drag to Stress Distributions

This section explains how lift and drag forces are related to stress distributions. This section also introduces the concepts of form and friction drag. These ideas are fundamental to understanding lift and drag.

Integrating a Stress Distribution to Yield Force

Lift and drag forces are related to the stress distribution on a body through integration. For example, consider the stress acting on the airfoil shown in Fig. 11.3. As shown, there is a pressure distribution and a shear-stress distribution. To relate stress to force, select a differential area as shown in Fig. 11.4. The magnitude of the pressure force is $dF_p = p\,dA$, and the magnitude of the viscous force is $dF_v = \tau dA$.* The differential lift force is normal to the free-stream direction

$$dF_L = -p\,dA\sin\theta - \tau\,dA\cos\theta$$

and the differential drag is parallel to the free-stream direction

$$dF_D = -p\,dA\cos\theta + \tau\,dA\sin\theta$$

Integration over the surface of the airfoil gives lift force (F_L) and drag force (F_D):

$$F_L = \int(-p\sin\theta - \tau\cos\theta)dA \quad (11.1)$$

$$F_D = \int(-p\cos\theta + \tau\sin\theta)dA \quad (11.2)$$

Equations (11.1) and (11.2) show that the lift and drag forces are related to the stress distributions through integration.

*The sign convention on τ is such that a clockwise sense of $\tau\,dA$ on the surface of the foil signifies a positive sign for τ.

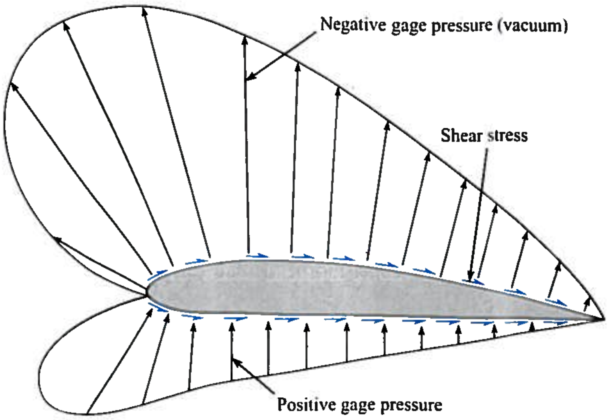


FIGURE 11.3

Pressure and shear stress acting on an airfoil.

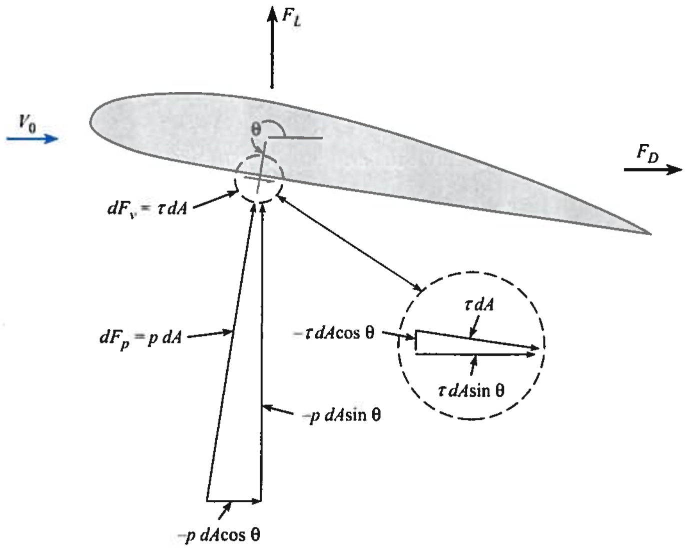


FIGURE 11.4

Pressure and viscous forces acting on a differential element of area.

Form Drag and Friction Drag

Notice that Eq. (11.2) can be written as the sum of two integrals.

$$F_D = \underbrace{\int(-p\cos\theta)dA}_{\text{form drag}} + \underbrace{\int(\tau\sin\theta)dA}_{\text{friction drag}} \quad (11$$

Form drag is the portion of the total drag force that is associated with the pressure distrib tion. **Friction drag** is the portion of the total drag force that is associated with the visco shear-stress distribution. The drag force on any body is the sum of form drag and friction dr In words, Eq. (11.3) can be written as

$$(\text{total drag force}) = (\text{form drag}) + (\text{friction drag}) \quad (11$$

11.2 Calculating Drag Force

This section introduces the drag force equation, the coefficient of drag, and presents data f two-dimensional bodies. This information is used to calculate drag force on objects.

Drag Force Equation

The drag force F_D on a body is found by using the drag force equation:

$$F_D = C_D A\left(\frac{\rho V_0^2}{2}\right) \quad (11.5)$$

where C_D is called the coefficient of drag, A is a reference area of the body, ρ is the fluid density, and V_0 is the free-stream velocity measured relative to the body.

The reference area A depends on the type of body. One common reference area, called **projected area** and given the symbol A_p, is the silhouetted area that would be seen by a person looking at the body from the direction of flow. For example, the projected area of a plate normal to the flow is $b\ell$, and the projected area of a cylinder with its axis normal to the flow is $d\ell$. Other geometries use different reference areas; for example, the reference area for an airplane wing is the planform area, which is the area observed when the wing is viewed from above.

The **coefficient of drag** C_D is a parameter that characterizes the drag force associated with a given body shape. For example, an airplane might have $C_D = 0.03$, and a baseball might have $C_D = 0.4$. The coefficient of drag is a π-group that is defined by

$$C_D \equiv \frac{F_D}{A_{\text{Ref}}(\rho V_0^2/2)} = \frac{(\text{drag force})}{(\text{reference area})(\text{kinetic pressure})} \quad (11.6)$$

Values of the coefficient of drag C_D are usually found by experiment. For example, drag force F_D can be measured using a force balance in a wind tunnel. Then C_D can be calculated using Eq. (11.6). For this calculation, speed of the air in the wind tunnel V_0 can be measured using a Pitot-static tube or similar device, and air density can be calculated by applying the ideal gas law using measured values of temperature and pressure.

Equation (11.5) shows that drag force is related to four variables. Drag is related to the shape of an object because shape is characterized by the value of C_D. Drag is related to the size of the object because size is characterized by the reference area. Drag is related to the density of ambient fluid. Finally, drag is related to the speed of the fluid squared. This means that if the wind velocity doubles and C_D is constant, then the wind load on a building goes up by a factor of four.

✔CHECKPOINT PROBLEM 11.1

Consider a car that is traveling in a straight line at constant speed.

Case A: The car speed is 40 km/h. There is no wind.
Case B: The car speed is 80 km/h. There is no wind.
Case C: The car speed is 65 km/h. There is a 15 km/h steady headwind.

The coefficient of drag is the same in all three cases.

Which statement(s) are true? (Select all that apply).

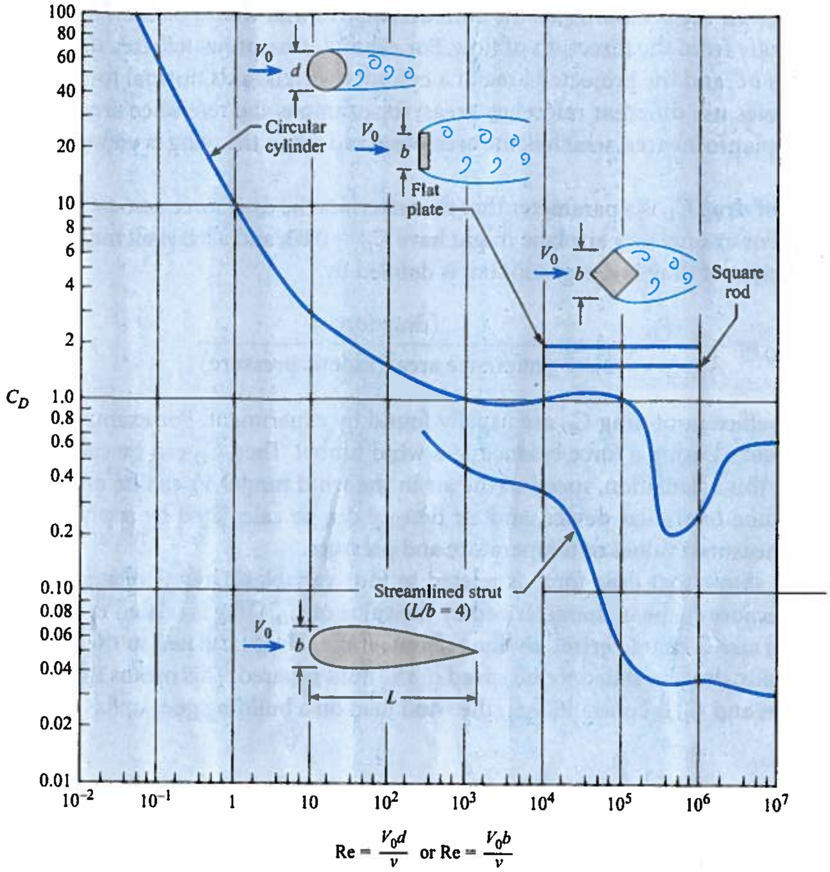
a. (Drag in Case B) = 2(Drag in Case A).
b. (Drag in Case B) = 4(Drag in Case A).
c. (Drag in Case B) = 8(Drag in Case A).
d. (Drag in Case C) < (Drag in Case B).
e. (Drag in Case C) > (Drag in Case B).
f. (Drag in Case C) = (Drag in Case B).

Coefficient of Drag (Two-Dimensional Bodies)

This section presents C_D data and describes how C_D varies with the Reynolds number f objects that can be classified as two dimensional. A **two-dimensional body** is a body with uniform section area and a flow pattern that is independent of the ends of the body. Exampl of two-dimensional bodies are shown in Fig. 11.5. In the aerodynamics literature, C_D values f two-dimensional bodies are called **sectional drag coefficients**. Two-dimensional bodies ca be visualized as objects that are infinitely long in the direction normal to the flow.

FIGURE 11.5

Coefficient of drag versus Reynolds number for two-dimensional bodies. [Data sources: Bullivant (1), DeFoe (2), Goett and Bullivant (3), Jacobs (4), Jones (5), and Lindsey (6).]



The sectional drag coefficient can be used to estimate C_D for real objects. For example, C for a cylinder with a length to diameter ratio of 20 (e.g., $L/D \geq 20$) approaches the section drag coefficient because the end effects have an insignificant contribution to the total dra force. Alternatively, the sectional drag coefficient would be inaccurate for a cylinder with small L/D ratio (e.g., $L/D \approx 1$) because the end effects would be important.

As shown in Fig. 11.5, the Reynolds number sometimes, but not always, influences th sectional drag coefficient. The value of C_D for the flat plate and square rod are independent c Re. The sharp edges of these bodies produce flow separation, and the drag force is due to th pressure distribution (form drag) and not on the shear-stress distribution (friction drag, whic depends on Re). Alternatively, C_D for the cylinder and the streamlined strut show strong R dependence because both form and friction drag are significant.

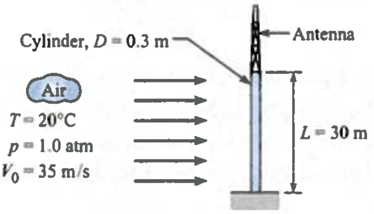
To calculate drag force on an object, find a suitable coefficient of drag, and then apply th drag force equation. This approach is illustrated by Example 11.1.

EXAMPLE 11.1

Drag Force on a Cylinder

Problem Statement

A vertical cylinder that is 30 m high and 30 cm in diameter is being used to support a television transmitting antenna. Find the drag force acting on the cylinder and the bending moment at its base. The wind speed is 35 m/s, the air pressure is 1 atm, and temperature is 20°C.



Define the Situation

Wind is blowing across a tall cylinder.

Assumptions:

- Wind speed is steady.
- Effects associated with the ends of the cylinder are negligible because $L/D = 100$.
- Neglect drag force on the antenna because the frontal area is much less than the frontal area of the cylinder.
- The line of action of the drag force is at an elevation of 15 m.

Properties: Air (20°C), Table A.5: $\rho = 1.2\ \text{kg/m}^3$, and $\mu = 1.81 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2$

State the Goals

Calculate

- Drag force (in N) on the cylinder
- Bending moment (in N · m) at the base of the cylinder

Generate Ideas and Make a Plan

1. Calculate the Reynolds number.
2. Find coefficient of drag using Fig. 11.5
3. Calculate drag force using Eq. (11.5).
4. Calculate bending moment using $M = F_D \cdot L/2$.

Take Action (Execute the Plan)

1. Reynolds number

$$\text{Re}_D = \frac{V_0 D \rho}{\mu} = \frac{35\ \text{m/s} \times 0.30\ \text{m} \times 1.20\ \text{kg/m}^3}{1.81 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2} = 7.0 \times$$

2. From Fig. 11.5, the coefficient of drag is $C_D = 0.20$.
3. Drag force

$$F_D = \frac{C_D A_p \rho V_0^2}{2}$$
$$= \frac{(0.2)(30\ \text{m})(0.3\ \text{m})(1.20\ \text{kg/m}^3)(35^2\ \text{m}^2/\text{s}^2)}{2}$$
$$= \boxed{1323\ \text{N}}$$

4. Moment at the base

$$M = F_D\left(\frac{L}{2}\right) = (1323\ \text{N})\left(\frac{30}{2}\ \text{m}\right) = \boxed{19{,}800\ \text{N} \cdot \text{m}}$$

Discussion of C_D for a Circular Cylinder

Drag Regimes The coefficient of drag C_D, as shown in Fig. 11.4, can be described in terms of three regimes.

Regime I ($\text{Re} < 10^3$). In this regime, C_D depends on both form drag and friction drag. As shown, C_D decreases with increasing Re.

Regime II ($10^3 < \text{Re} < 10^5$). In this regime, C_D has a nearly constant value. The reason is that form drag, which is associated with the pressure distribution, is the dominant cause of drag. Over this range of Reynolds numbers, the flow pattern around the cylinder remains virtually unchanged, thereby producing very similar pressure distributions. This characteristic, the constancy of C_D at high values of Re, is representative of most bodies that have angular form.

Regime III ($10^5 < \text{Re} < 5 \times 10^5$). In this regime, C_D decreases by about 80%, a remarkable change! This change occurs because the boundary layer on the circular cylinder changes. For Reynolds numbers less than 10^5, the boundary layer is laminar, and separation occurs about midway between the upstream side and downstream side of the cylinder (Fig. 11.6). Hence the entire downstream half of the cylinder is exposed to a relatively low pressure, which in turn produces a relatively high value for C_D. When the Reynolds number is increased to about 10^5, the boundary layer becomes turbulent, which causes higher-velocity fluid to be mixed into the

FIGURE 11.6

Flow pattern around a cylinder for $10^3 < \text{Re} < 10^5$.

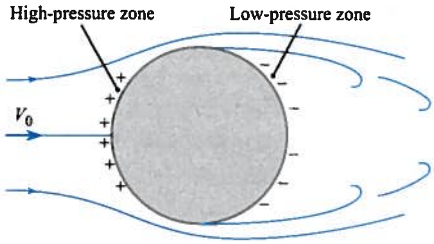
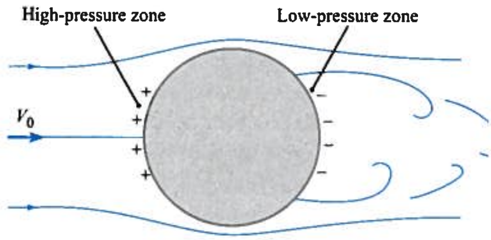


FIGURE 11.7

Flow pattern around a cylinder for $\text{Re} > 5 \times 10^5$.



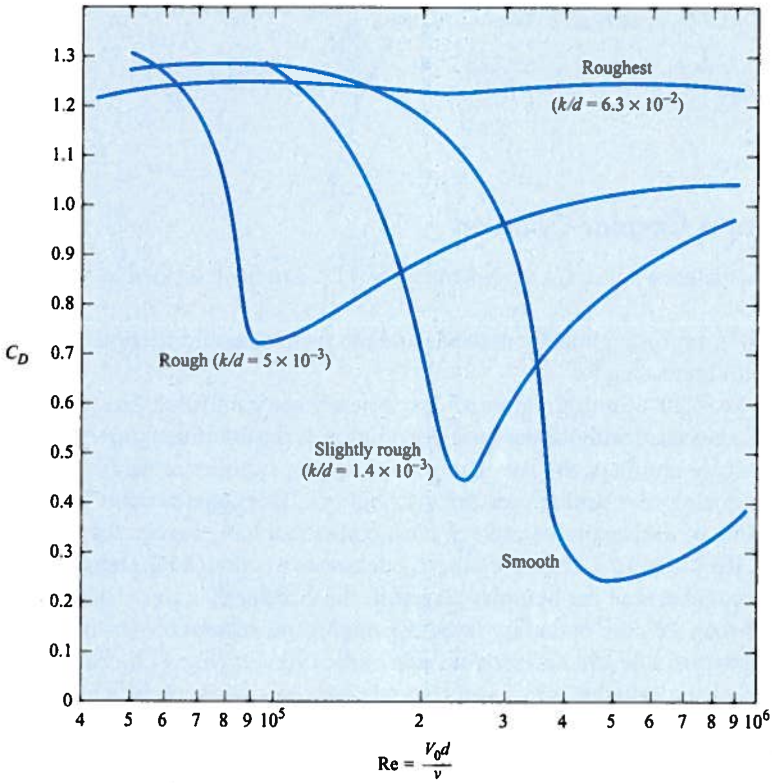
region close to the wall of the cylinder. As a consequence of the presence of this high-velocit high-momentum fluid in the boundary layer, the flow proceeds farther downstream along th surface of the cylinder against the adverse pressure before separation occurs (Fig. 11.7). Thi change in separation produces a much smaller zone of low pressure and the lower value of C_D

Surface Roughness

Surface roughness has a major influence on drag. For example, if the surface of the cylinder i slightly roughened upstream of the midsection, the boundary layer will be forced to become turbu lent at lower Reynolds numbers than those for a smooth cylinder surface. The same trend can als be produced by creating abnormal turbulence in the approach flow. The effects of roughness ar shown in Fig. 11.8 for cylinders that were roughened with sand grains of size k. A small to mediun size of roughness ($10^{-3} < k/d < 10^{-2}$) on a cylinder triggers an early onset of reduction of C_D. How ever, when the relative roughness is quite large ($10^{-2} < k/d$), the characteristic dip in C_D is absent.

FIGURE 11.8

Effects of roughness on C_D for a cylinder. [After Miller et al. (7).]



11.3 Drag of Axisymmetric and 3-D Bodies

Section 11.2 described drag for two-dimensional bodies. Drag on other body shapes is presented in this section. This section also describes power and rolling resistance.

Drag Data

An object is classified as an **axisymmetric body** when the flow direction is parallel to an axis of symmetry of the body and the resulting flow is also symmetric about its axis. Examples of axisymmetric bodies include a sphere, a bullet, and a javelin. When flow is not aligned with an axis of symmetry, the flow field is three dimensional (3D), and the body is classified as a three-dimensional or **3-D body**. Examples of 3-D bodies include a tree, a building, and an automobile.

The principles that apply to two-dimensional flow over a body also apply to axisymmetric flows. For example, at very low values of the Reynolds number, the coefficient of drag is given by exact equations relating C_D and Re. At high values of Re, the coefficient of drag becomes constant for angular bodies, whereas rather abrupt changes in C_D occur for rounded bodies. All these characteristics can be seen in Fig. 11.9, where C_D is plotted against Re for several axisymmetric bodies.

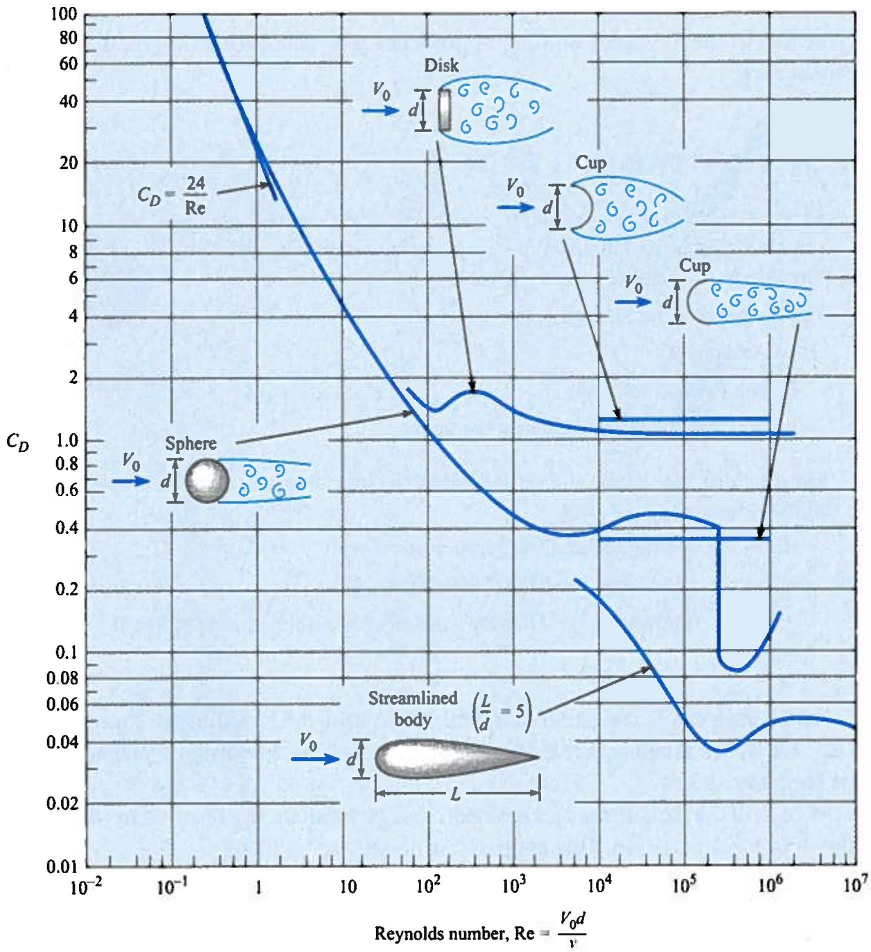


FIGURE 11.9
Coefficient of drag vers
Reynolds number for
axisymmetric bodies.
[Data sources: Abbott (
Brevoort and Joyner (1(
Freeman (11), and Rou
(12).]

The drag coefficient of a sphere is of special interest because many applications involve tl drag of spherical or near-spherical objects, such as particles and droplets. Also, the drag of a sphe is often used as a standard of comparison for other shapes. For Reynolds numbers less than 0.5, tl flow around the sphere is laminar and amenable to analytical solutions. An exact solution l Stokes yielded the following equation, which is called Stokes's equation, for the drag of a sphere:

$$F_D = 3\pi\mu V_0 d \quad \text{(11.}$$

Note that the drag for this laminar flow condition varies directly with the first power of V_0. Th is characteristic of all laminar flow processes. For completely turbulent flow, the drag is a fun tion of the velocity to the second power. When the drag force given by Eq. (11.7) is substitute into Eq. (11.6), the result is the drag coefficient corresponding to Stokes's equation:

$$C_D = \frac{24}{\text{Re}} \quad \text{(11.}$$

Thus for flow past a sphere, when Re $\leq$ 0.5, one may use the direct relation for C_D given i Eq. (11.8).

Several correlations for the drag coefficient of a sphere are available (13). One such co relation has been proposed by Clift and Gauvin (14):

$$C_D = \frac{24}{\text{Re}}(1 + 0.15\text{Re}^{0.687}) + \frac{0.42}{1 + 4.25 \times 10^4 \text{Re}^{-1.16}} \quad \text{(11.}$$

which deviates from the *standard drag curve** by −4% to 6% for Reynolds numbers up to 3×10 Note that as the Reynolds number approaches zero, this correlation reduces to the equation fc Stokes flow.

✔CHECKPOINT PROBLEM 11.2

Suppose you are estimating C_D for an American football oriented so that its long axis is into the wind. You have available Fig. 11.9. Which choice would you make?

(Stockbyte/Getty Images)

I would idealize the football

a. As a sphere

b. As a streamlined body

c. As one of the other bodies on the figure.

Do you think that a spinning football (about its long axis) has a different value of drag than a nonspinning football?

d. Yes, the spinning football will have lower drag.

e. Yes, the spinning football has higher drag.

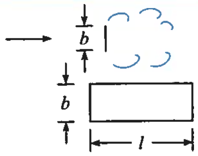
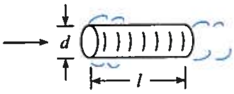
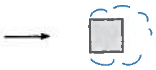
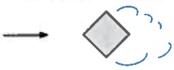
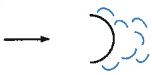
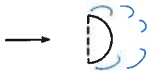
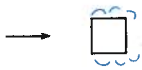
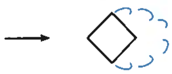
f. No, the spinning football has the same drag as a nonspinning football.

Values for C_D for other axisymmetric and 3-D bodies at high Reynolds number (Re > 10^4) are given in Table 11.1. Extensive data on the drag of various shapes is availabl in Hoerner (15).

To find the drag force on an object, find or estimate the coefficient of drag and then appl the drag force equation. This approach is illustrated by Example 11.2.

*The *standard drag curve* represents the best fit of the cumulative data that have been obtained for drag coefficient c a sphere.

TABLE 11.1 Approximate C_D Values for Various Bodies

	Type of Body	Length Ratio	Re	C_D
	Rectangular plate	$l/b = 1$	$>10^4$	1.18
		$l/b = 5$	$>10^4$	1.20
		$l/b = 10$	$>10^4$	1.30
		$l/b = 20$	$>10^4$	1.50
		$l/b = \infty$	$>10^4$	1.98
	Circular cylinder with axis parallel to flow	$l/d = 0$ (disk)	$>10^4$	1.17
		$l/d = 0.5$	$>10^4$	1.15
		$l/d = 1$	$>10^4$	0.90
		$l/d = 2$	$>10^4$	0.85
		$l/d = 4$	$>10^4$	0.87
		$l/d = 8$	$>10^4$	0.99
	Square rod	∞	$>10^4$	2.00
	Square rod	∞	$>10^4$	1.50
	Triangular cylinder	∞	$>10^4$	1.39
	Semicircular shell	∞	$>10^4$	1.20
	Semicircular shell	∞	$>10^4$	2.30
	Hemispherical shell		$>10^4$	0.39
	Hemispherical shell		$>10^4$	1.40
	Cube		$>10^4$	1.10
	Cube		$>10^4$	0.81
	Cone—60° vertex		$>10^4$	0.49
	Parachute		$\approx 3 \times 10^7$	1.20

Sources: Brevoort and Joyner (10), Lindsey (6), Morrison (16), Roberson et al. (17), Rouse (12), and Scher and Gale (18).

EXAMPLE 11.2

Drag on a Sphere

Problem Statement

What is the drag of a 12-mm sphere that drops at a rate of 8 cm/s in oil ($\mu = 10^{-1}$ N · s/m², S = 0.85)?

Define the Situation

A sphere ($d = 0.012$ m) is falling in oil.

Speed of the sphere is $V = 0.08$ m/s.

Assumptions: Sphere is moving at a steady speed (terminal velocity).

Properties:

Oil: $\mu = 10^{-1}$ N · s/m², S = 0.85, $\rho = 850$ kg/m³

State the Goal

Find: Drag force (in newtons) on the sphere.

Generate Ideas and Make a Plan

1. Calculate the Reynolds number.
2. Find the coefficient of drag using Fig. 11.9.
3. Calculate drag force using Eq. (11.5).

Take Action (Execute the Plan)

1. Reynolds number

$$\text{Re} = \frac{Vd\rho}{\mu} = \frac{(0.08\text{ m/s})(0.012\text{ m})(850\text{ kg/m}^3)}{10^{-1}\text{ N}\cdot\text{s/m}^2} = 8.16$$

2. Coefficient of drag (from Fig. 11.9) is $C_D = 5.3$.
3. Drag force

$$F_D = \frac{C_D A_p \rho V_0^2}{2}$$

$$F_D = \frac{(5.3)(\pi/4)(0.012^2\text{ m}^2)(850\text{ kg/m}^3)(0.08\text{ m/s})^2}{2}$$

$$= \boxed{1.63 \times 10^{-3}\text{ N}}$$

Power and Rolling Resistance

Before reading this section, you can try out your knowledge with the Checkpoint Problem. Th knowledge you need has been previously covered in this text.

✔ CHECKPOINT PROBLEM 11.3

Consider a bicycle racer that is traveling in a straight line at constant speed.

Case A: The speed is 20 km/h. There is no wind.
Case B: The speed is 40 km/h. There is no wind.

For both cases, C_D is the same, and rolling resistance is negligible.

Which statement is true?

a. (Power in Case B) = (Power in Case A).
b. (Power in Case B) = 2(Power in Case A).
c. (Power in Case B) = 4(Power in Case A).
d. (Power in Case B) = 8(Power in Case A).

When power is involved in a problem, the power equation from Chapter 7 is applied. For ex ample, consider a car moving at a steady speed on a level road. Because the car is not accelerat ing, the horizontal forces are balanced as shown in Fig. 11.10. Force equilibrium gives

$$F_{\text{Drive}} = F_{\text{Drag}} + F_{\text{Rolling resistance}}$$

The driving force (F_{Drive}) is the frictional force between the driving wheels and the roac The drag force is the resistance of the air on the car. The rolling resistance is the frictional forc that occurs when an object such as a ball or tire rolls. It is related to the deformation and type of the materials that are in contact. For example, a rubber tire on asphalt will have a larger roll ing resistance than a steel train wheel on a steel rail. The rolling resistance is calculated using

$$F_{\text{Rolling resistance}} = F_r = C_r N \quad (11.10$$

where C_r is the coefficient of rolling resistance and N is the normal force.

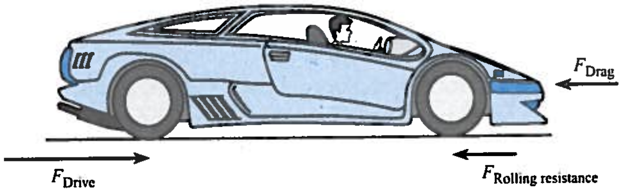


FIGURE 11.10 Horizontal forces acting on car that is moving at a steady speed.

The power required to move the car shown in Fig. 11.10 at a constant speed is given by Eq. (7.2a)

$$P = FV = F_{Drive}V_{Car} = (F_{Drag} + F_{Rolling\ resistance})V_{Car} \quad (11.11)$$

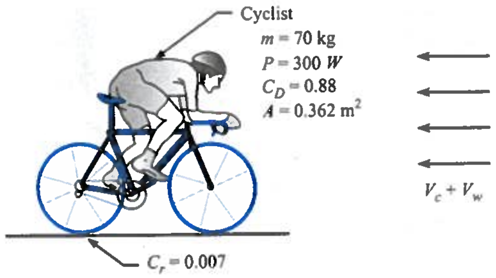
Thus, when power is involved in a problem, apply the equation $P = FV$ while concurrently using a free-body diagram to determine the appropriate force. This approach is illustrated in Example 11.3.

EXAMPLE 11.3

Speed of a Bicycle Rider

Problem Statement

A bicyclist of mass 70 kg supplies 300 watts of power while riding into a 3 m/s headwind. The frontal area of the cyclist and bicycle together is 3.9 ft² = 0.362 m², the drag coefficient is 0.88, and the coefficient of rolling resistance is 0.007. Determine the speed V_c of the cyclist. Express your answer in mph and in m/s.



Define the Situation

A bicycle rider is cycling into a headwind of magnitude $V_w = 3$ m/s.

Assumptions:

1. The path is level, with no hills.
2. Mechanical losses in the bike gear train are zero.

Properties: Air (20°C, 1 atm), Table A.2: $\rho = 1.2$ kg/m³

State the Goal

Find the speed (m/s and mph) of the rider.

Generate Ideas and Make a Plan

1. Relate bike speed (V_c) to power using Eq. (11.11).
2. Calculate rolling resistance.
3. Develop an equation for drag force using Eq. (11.5).
4. Combine steps 1 to 3.
5. Solve for V_c.

Take Action (Execute the Plan)

1. Power equation
   - The power from the bike rider is being used to overcome drag and rolling resistance. Thus,

$$P = (F_D + F_r)V_c$$

2. Rolling resistance

$$F_r = C_rN = C_rmg = 0.007(70\ \text{kg})(9.81\ \text{m/s}^2) = 4.81\ \text{N}$$

3. Drag force
   - V_0 = speed of the air relative to the bike rider

$$V_0 = V_c + 3\ \text{m/s}$$

   - Drag force

$$F_D = C_DA\left(\frac{\rho V_0^2}{2}\right) = \frac{0.88(0.362\ \text{m}^2)(1.2\ \text{kg/m}^3)}{2} \times (V_c + 3\ \text{m/s})^2$$

$$= 0.1911(V_c + 3\ \text{m/s})^2$$

4. Combine results:

$$P = (F_D + F_r)V_c$$

$$300\ \text{W} = (0.1911(V_c + 3)^2 + 4.81)V_c$$

5. Because the equation is cubic, use a spreadsheet program as shown. In this spreadsheet, let V_c vary and then search for the value of V_c that causes the right side of the equation to equal 300. The result is

$$V_c = \boxed{9.12\ \text{m/s} = 20.4\ \text{mph}}$$

V_c (m/s)	RHS (W)
0	0.0
5	85.2
8	223.5
9	291.0
9.1	298.4
9.11	299.1
9.12	299.9
9.13	300.6

11.4 Terminal Velocity

Another common application of the drag force equation is finding the steady-state speed o body that is falling through a fluid. When a body is dropped, it accelerates under the action gravity. As the speed of the falling body increases, the drag increases until the upward fo (drag) equals the net downward force (weight minus buoyant force). Once the forces a balanced, the body moves at a constant speed called the **terminal velocity**, which is identifi as the maximum velocity attained by a falling body.

To find terminal velocity, balance the forces acting on the object, and then solve the resu ing equation. In general this process is iterative, as illustrated by Example 11.4.

EXAMPLE 11.4

Terminal Velocity of a Sphere in Water

Problem Statement

A 20 mm plastic sphere (S = 1.3) is dropped in water. Determine its terminal velocity. Assume $T = 20°C$.

Define the Situation

A smooth sphere ($D = 0.02$ m, S = 1.3) is falling in water.

Properties: Water (20°C), Table A.5, $\nu = 1 \times 10^{-6}$ m²/s, $\rho = 998$ kg/m³, and $\gamma = 9790$ N/m³

State the Goal

Find the terminal velocity (m/s) of the sphere.

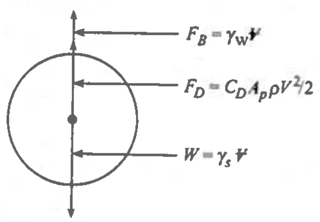
Generate Ideas and Make a Plan

This problem requires an iterative solution because the terminal velocity equation is implicit. The plan steps are

1. Apply force equilibrium.
2. Develop an equation for terminal velocity.
3. To solve the terminal velocity equation, set up a procedure for iteration.
4. To implement the iterative solution, build a table in a spreadsheet program.

Take Action (Execute the Plan)

1. Force equilibrium
   - Sketch a free-body diagram.



   - Apply force equilibrium (vertical direction):

$$F_{\text{Drag}} + F_{\text{Buoyancy}} = W$$

2. Terminal velocity equation
   - Analyze terms in the equilibrium equation:

$$C_D A\left(\frac{\rho V_0^2}{2}\right) + \gamma_w V = \gamma_s V$$

$$C_D\left(\frac{\pi d^2}{4}\right)\left(\frac{\rho V_0^2}{2}\right) + \gamma_w\left(\frac{\pi d^3}{6}\right) = \gamma_s\left(\frac{\pi d^3}{6}\right)$$

   - Solve for V_0

$$V_0 = \left[\frac{(\gamma_s - \gamma_w)(4/3)d}{C_D\rho_w}\right]^{1/2}$$

$$= \left[\frac{(12.7 - 9.79)(10^3\ \text{N/m}^3)(4/3)(0.02\ \text{m})}{C_D \times 998\ \text{kg/m}^3}\right]^{1/2}$$

$$V_0 = \left(\frac{0.0778}{C_D}\right)^{1/2} = \frac{0.279}{C_D^{1/2}}\ \text{m/s}$$

3. Iteration 1
   - Initial guess: $V_0 = 1.0$ m/s
   - Calculate Re:

$$\text{Re} = \frac{Vd}{\nu} = \frac{(1.0\ \text{m/s})(0.02\ \text{m})}{1 \times 10^{-6}\ \text{m}^2/\text{s}} = 20000$$

   - Calculate C_D using Eq. (11.9):

$$C_D = \frac{24}{20000}(1 + 0.15(20000^{0.687})) + \frac{0.42}{1 + 4.25 \times 10^4(20000)^{-1.16}} = 0.456$$

   - Find new value of V_0 (use equation from step 2):

$$V_0 = \left(\frac{0.0778}{C_D}\right)^{1/2} = \frac{0.279}{0.456^{0.5}} = 0.413\ \text{m/s}$$

4. Iterative solution
   - As shown, use a spreadsheet program to build a table. The first row shows the results of iteration 1.
   - The terminal velocity from iteration 1 $V_0 = 0.413$ m/s is used as the initial velocity for iteration 2.
   - The iteration process is repeated until the terminal velocity reaches a constant value of $V_0 = 0.44$ m/s. Notice that convergence is reached in two iterations.

Iteration #	Initial V_0 (m/s)	Re	C_D	New V_0 (m/s)
1	1.000	20000	0.456	0.413
2	0.413	8264	0.406	0.438
3	0.438	8752	0.409	0.436
4	0.436	8721	0.409	0.436
5	0.436	8723	0.409	0.436
6	0.436	8722	0.409	0.436

$V_0 = 0.44$ m/s

11.5 Vortex Shedding

This section introduces vortex shedding, which is important for two reasons: It can be used to enhance heat transfer and mixing, and it can cause unwanted vibrations and failures of structures.

Flow past a bluff body generally produces a series of vortices that are shed alternatively from each side, thereby producing a series of alternating vortices in the wake. This phenomenon is call **vortex shedding**. Vortex shedding for a cylinder occurs for Re $\geqslant$ 50 and gives the flow pattern sketched in Fig. 11.11. In this figure, a vortex is in the process of formation near the top of the cylinder. Below and to the right of the first vortex is another vortex, which was formed and shed a short time before. Thus the flow process in the wake of a cylinder involves the formation and shedding of vortices alternately from one side and then the other. This alternate formation and shedding of vortices creates a cyclic change in pressure with consequent periodicity in side thrust on the cylinder. Vortex shedding was the primary cause of failure of the Tacoma Narrows suspension bridge in the state of Washington in 1940.

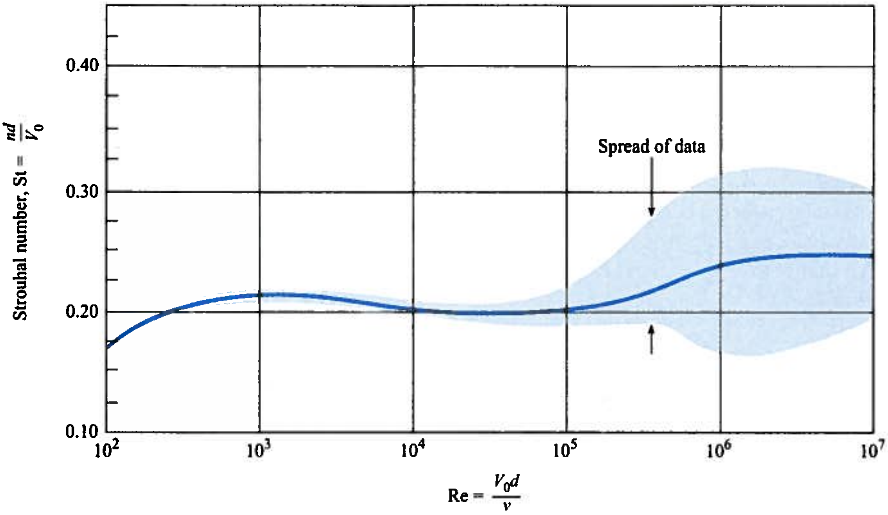
FIGURE 11.11 Formation of a vortex behind a cylinder.

Experiments reveal that the frequency of shedding can be represented by plotting Strouhal number (St) as a function of Reynolds number. The Strouhal number is a π-group defined as

$$\text{St} = \frac{nd}{V_0} \tag{11.12}$$

where n is the frequency of shedding of vortices from one side of the cylinder, in Hz, d is the diameter of the cylinder, and V_0 is the free-stream velocity. The Strouhal number for vortex shedding from a circular cylinder is given in Fig. 11.12. Other cylindrical and two-dimensional bodies also shed vortices. Consequently, the engineer should always be alert to vibration problems when designing structures that are exposed to wind or water flow.

FIGURE 11.12
Strouhal number versus Reynolds number for flow past a circular cylinder. [After Jones (5) and Roshko (8)]



11.6 Reducing Drag by Streamlining

An engineer can design a body shape to minimize the drag force. This process is called **streamlining** and is often focused on reducing form drag. The reason for focusing on form drag is that drag on most bluff objects (e.g., a cylindrical body at Re $>$ 1000) is predominantly due to the pressure variation associated with flow separation. In this case, streamlining involves modifying the body shape to reduce or eliminate separation. The impacts of streamlining can be dramatic. For example, Fig. 11.5 shows that C_D for the streamlined shape is about 1/6 of C_D for the circular cylinder when Re $\approx 5 \times 10^5$.

While streamlining reduces form drag, friction drag is typically increased. This is because there is more surface area on a streamlined body as compared to a nonstreamlined body. Consequently, when a body is streamlined the optimum condition results when the sum of form drag and friction drag is minimum.

Streamlining to produce minimum drag at high Reynolds numbers will probably not produce minimum drag at very low Reynolds numbers. For example, at Re $<$ 1, the majority of the drag of a cylinder is friction drag. Hence, if the cylinder is streamlined, the friction drag will likely be magnified, and C_D will increase.

Another advantage of streamlining at high Reynolds numbers is that vortex shedding is eliminated. Example 11.5 shows how to estimate the impact of streamlining by using a ratio of C_D values.

EXAMPLE 11.5

Comparing Drag on Bluff and Streamlined Shapes

Problem Statement

Compare the drag of the cylinder of Example 11.1 with the drag of the streamlined shape shown in Fig. 11.5. Assume that both shapes have the same projected area.

Define the Situation

The cylinder from Example 11.1 is being compared to a streamlined shape.

Assumptions:

1. The cylinder and the streamlined body have the same projected area.
2. Both objects are two-dimensional bodies (neglect end effects).

State the Goal

Find the ratio of drag force on the streamlined body to drag force on the cylinder.

Generate Ideas and Make a Plan

1. Retrieve Re and C_D from Example 11.1.
2. Find the coefficient of drag for the streamlined shape using Fig. 11.5
3. Calculate the ratio of drag forces using Eq. (11.5).

Take Action (Execute the Plan)

1. From Example 11.1, Re $= 7 \times 10^5$ and C_D(cylinder) = 0.2.
2. Using this Re and Fig. 11.5 gives C_D(streamlined shape) = 0.034.
3. Drag force ratio (derived from Eq. 11.5) is

$$\frac{F_D(\text{streamlined shape})}{F_D(\text{cylinder})} = \frac{C_D(\text{streamlined shape})}{C_D(\text{cylinder})} \times \left(\frac{\cancel{A_p(\rho V_0^2/2)}}{\cancel{A_p(\rho V_0^2/2)}}\right)$$

$$\frac{F_D(\text{streamlined shape})}{F_D(\text{cylinder})} = \frac{0.034}{0.2} = \boxed{0.17}$$

Review the Results and the Process

Discussion. The streamlining provided nearly a sixfold reduction in drag!

11.7 Drag in Compressible Flow

So far, this chapter has described drag for flows with constant density. This section describes drag when the density of a gas is changing due to pressure variations. These types of flow are called *compressible flows*. This information is important for modeling of projectiles such as bullets and rockets.

In steady flow, the influence of compressibility depends on the ratio of fluid velocity to the speed of sound. This ratio is a π-group called the Mach number.

The variation of drag coefficient with Mach number for three axisymmetric bodies is shown in Fig. 11.13. In each case, the drag coefficient increases only slightly with the Mach number at low Mach numbers and then increases sharply as transonic flow (M $\approx$ 1) is approached. Note that the rapid increase in drag coefficient occurs at a higher Mach number (closer to unity) if the body is slender with a pointed nose. The drag coefficient reaches a maximum at a Mach number somewhat larger than unity and then decreases as the Mach number is further increased.

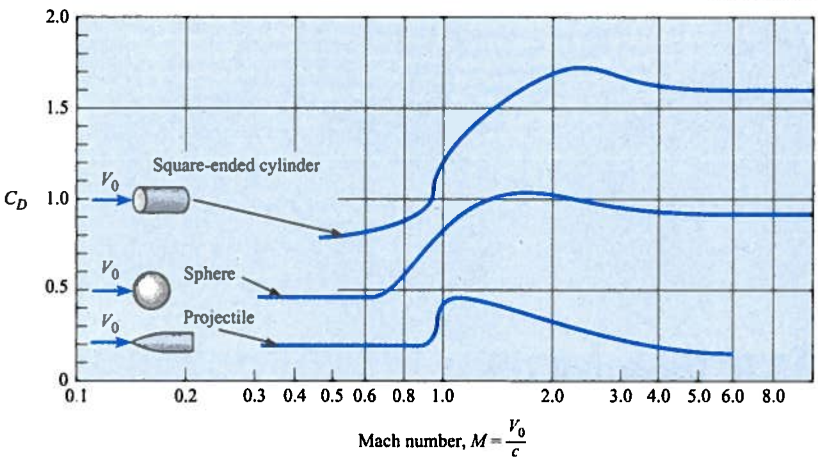


FIGURE 11.13 Drag characteristics of projectile, sphere, and cylinder with compressibility effects. [After Rouse (12)]

The slight increase in drag coefficient with low Mach numbers is attributed to an increase in form drag due to compressibility effects on the pressure distribution. However, as the flow velocity is increased, the maximum velocity on the body finally becomes sonic. The Mach number of the free-stream flow at which sonic flow first appears on the body is called the *critical Mach number*. Further increases in flow velocity result in local regions of supersonic flow (M > 1), which lead to wave drag due to shock wave formation and an appreciab increase in drag coefficient.

The critical Mach number for a sphere is approximately 0.6. Note in Fig. 11.13 that th drag coefficient begins to rise sharply at about this Mach number. The critical Mach numbe for the pointed body is larger, and correspondingly, the rise in drag coefficient occurs at a Mac number closer to unity.

The drag coefficient data for the sphere shown in Fig. 11.13 are for a Reynolds number of th order of 10^4. The data for the sphere shown in Fig. 11.9, on the other hand, are for very low Mac numbers. The question then arises about the general variation of the drag coefficient of a sphere wit both Mach number and Reynolds number. Information of this nature is often needed to predict th trajectory of a body through the upper atmosphere or to model the motion of a nanoparticle.

A contour plot of the drag coefficient of a sphere versus both Reynolds and Mach numbe based on available data (19) is shown in Fig. 11.14. Notice the C_D-versus-Re curve fro Fig. 11.9 in the M = 0 plane. Correspondingly, notice the C_D-versus-M curve from Fig. 11.13 i the Re = 10^4 plane. At low Reynolds numbers C_D decreases with an increasing Mach numbe whereas at high Reynolds numbers the opposite trend is observed. Using this figure, the eng neer can determine the drag coefficient of a sphere at any combination of Re and M. Of cours corresponding C_D contour plots can be generated for any body, provided the data are availabl

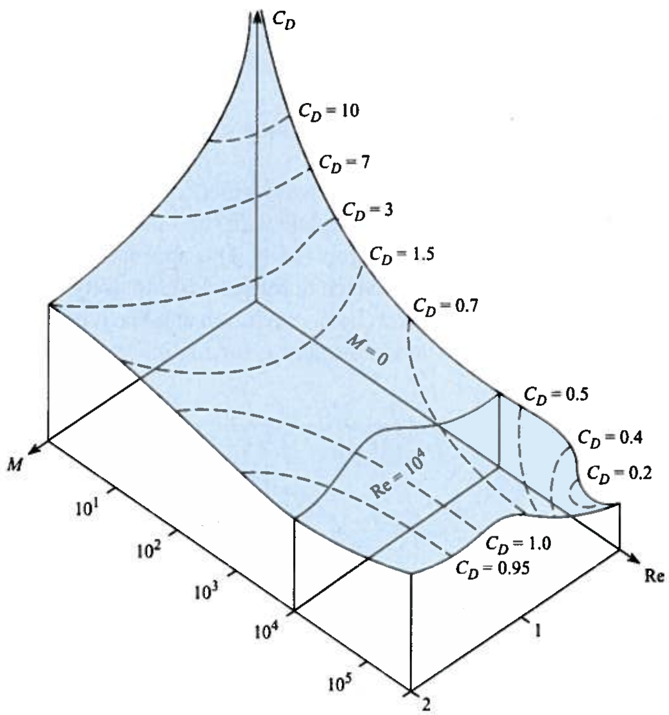


FIGURE 11.14
Contour plot of the drag coefficient of the sphere versus Reynolds and Mach numbers.

11.8 Theory of Lift

This section introduces circulation, the basic cause of lift, as well as the coefficient of lift.

Circulation

Circulation, a characteristic of a flow field, gives a measure of the average rate of rotation o fluid particles that are situated in an area that is bounded by a closed curve. Circulation i

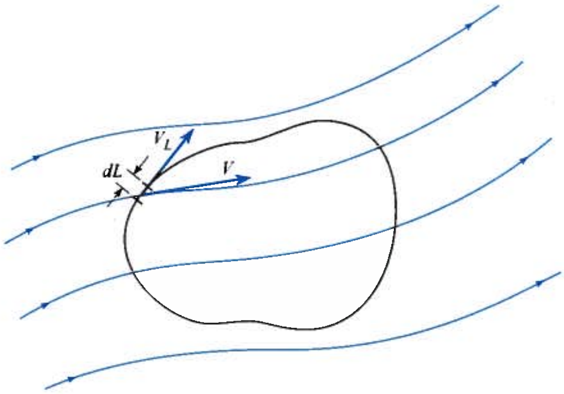


FIGURE 11.15
Concept of circulation.

defined by the path integral as shown in Fig. 11.15. Along any differential segment of the path, the velocity can be resolved into components that are tangent and normal to the path. Signify the tangential component of velocity as V_L. Integrate $V_L\,dL$ around the curve; the resulting quantity is called circulation, which is represented by the Greek letter Γ (capital gamma). Hence

$$\Gamma = \oint V_L\,dL \qquad (11.13)$$

Sign convention dictates that in applying Eq. (11.13), one uses tangential velocity vectors that have a counterclockwise sense around the curve as negative and take those that have a clockwise direction as having a positive contribution.* For example, consider finding the circulation for an irrotational vortex. The tangential velocity at any radius is C/r, where a positive C means a clockwise rotation. Therefore, if circulation is evaluated about a curve with radius r, the differential circulation is

$$d\Gamma = V_L\,dL = \frac{C}{r_1} r_1\,d\theta = C\,d\theta \qquad (11.14)$$

Integrate this around the entire circle:

$$\Gamma = \int_0^{2\pi} C\,d\theta = 2\pi C \qquad (11.15)$$

One way to induce circulation physically is to rotate a cylinder about its axis. Fig. 11.16a shows the flow pattern produced by such action. The velocity of the fluid next to the surface of the cylinder is equal to the velocity of the cylinder surface itself because of the no-slip condition that must prevail between the fluid and solid. At some distance from the cylinder, however, the velocity decreases with r, much like it does for the irrotational vortex. The next section shows how circulation produces lift.

Combination of Circulation and Uniform Flow around a Cylinder

Superpose the velocity field produced for uniform flow around a cylinder, Fig. 11.16b, onto a velocity field with circulation around a cylinder, Fig. 11.16a. Observe that the velocity is reinforced on the top side of the cylinder and reduced on the other side (Fig. 11.16c). Also observe that the stagnation points have both moved toward the low-velocity side of the cylinder. Consistent with the Bernoulli equation (assuming irrotational flow throughout), the pressure on the high-velocity side is lower than the pressure on the low-velocity side. Hence a pressure

*The sign convention is the opposite of that for the mathematical definition of a line integral.

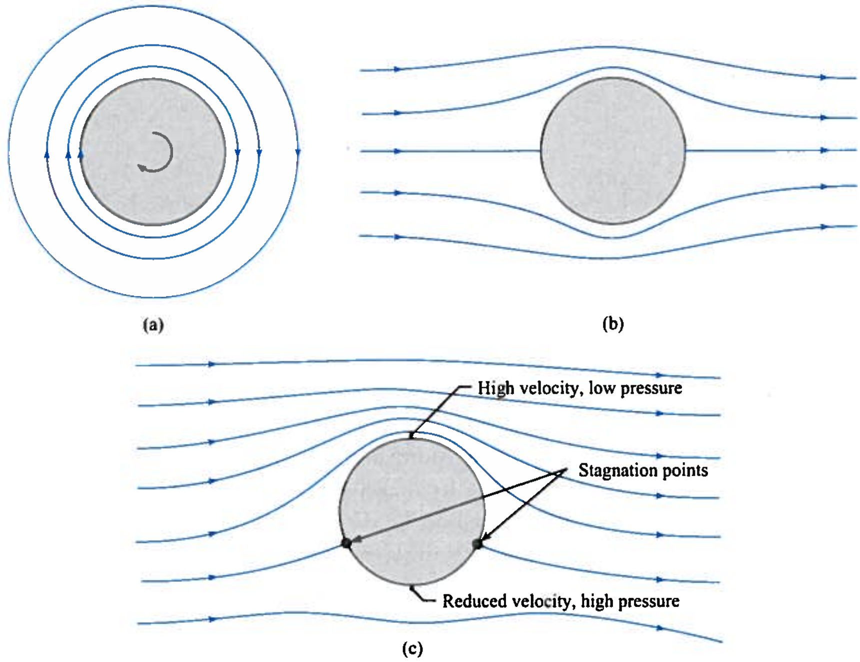


FIGURE 11.16
Ideal flow around a cylinder.
(a) Circulation.
(b) Uniform flow.
(c) Combination of circulation and uniform flow.

differential exists that causes a side thrust, or lift, on the cylinder. According to ideal flow the ory, the lift per unit length of an infinitely long cylinder is given by $F_L/\ell = \rho V_0 \Gamma$, where F_L the lift on the segment of length ℓ. For this ideal irrotational flow there is no drag on the cylir der. For the real-flow case, separation and viscous stresses do produce drag, and the same vi: cous effects will reduce the lift somewhat. Even so, the lift is significant when flow occu past a rotating body or when a body is translating and rotating through a fluid. Hence the rea son for the "curve" on a pitched baseball or the "drop" on a Ping-Pong ball is a fore spin. Th phenomenon of lift produced by rotation of a solid body is called the **Magnus effect** after nineteenth-century German scientist who made early studies of the lift on rotating bodie A paper by Mehta (28) offers an interesting account of the motion of rotating sports balls.

Coefficients of lift and drag for the rotating cylinder with end plates are shown in Fig. 11.1 In this figure, the parameter $r\omega/V_0$ is the ratio of cylinder surface speed to the free-strear velocity, where r is the radius of the cylinder and ω is the angular speed in radians per secon The corresponding curves for the rotating sphere are given in Fig. 11.18.

Coefficient of Lift

The **coefficient of lift** is a parameter that characterizes the lift that is associated with a bod For example, a wing at a high angle of attack will have a high coefficient of lift, and a wing th: has a zero angle of attack will have a low or zero coefficient of lift. The coefficient of lift defined using a π-group:

$$C_L \equiv \frac{F_L}{A(\rho V_0^2/2)} = \frac{\text{lift force}}{\text{(reference area)(dynamic pressure)}} \quad \text{(11.1}$$

To calculate lift force, engineers use the lift equation:

$$F_L = C_L A\left(\frac{\rho V_0^2}{2}\right) \quad \text{(11.1}$$

where the reference area for a rotating cylinder or sphere is the projected area A_p.

FIGURE 11.17

Coefficients of lift and drag for a rotating cylinder. [After Rouse (12).]

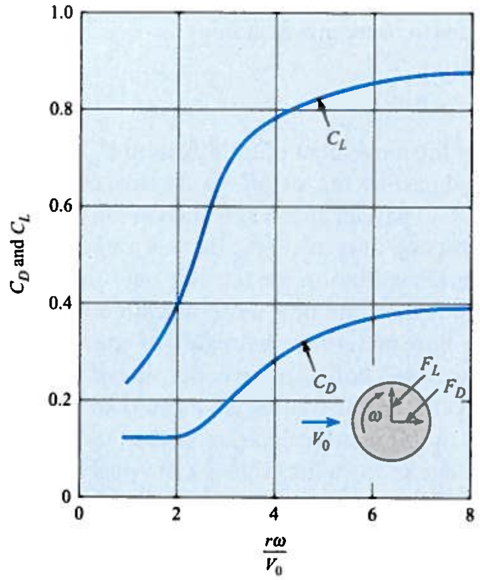
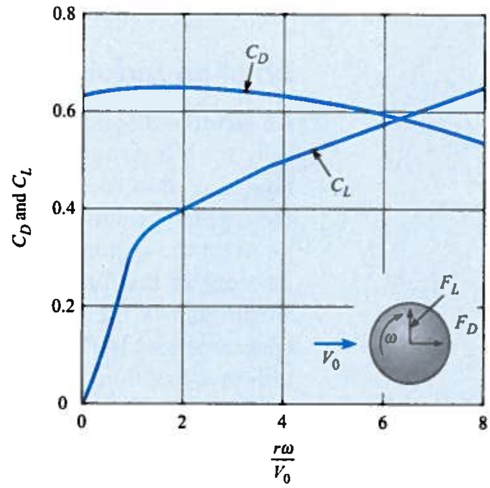


FIGURE 11.18

Coefficients of lift and drag for a rotating sphere. [After Barkla et al. (20). Reprinted with the permission of Cambridge University Press.]



EXAMPLE 11.6

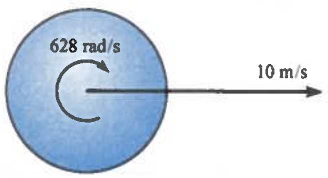
Lift on a Rotating Sphere

Problem Statement

A Ping-Pong ball is moving at 10 m/s in air and is spinning at 100 revolutions per second in the clockwise direction. The diameter of the ball is 3 cm. Calculate the lift and drag force and indicate the direction of the lift (up or down). The density of air is 1.2 kg/m^3.

Define the Situation

A Ping-Pong ball is moving horizontally and rotating.



Properties: Air: $\rho = 1.2$ kg/m^3

State the Goal

Find

1. Drag force (in newtons) on the ball
2. Lift force (in newtons) on the ball
3. The direction of lift (up or down?)

Generate Ideas and Make a Plan

1. Calculate the value of $r\omega/V_0$.
2. Use the value of $r\omega/V_0$ to look up the coefficients of lift and drag on Fig. 11.7.
3. Calculate lift force using Eq. (11.8).
4. Calculate drag force using Eq. (11.5).

Take Action (Execute the Plan)

The rotation rate in rad/s is

$$\omega = (100 \text{ rev/s})(2\pi \text{ rad/rev}) = 628 \text{ rad/s}$$

The rotational parameter is

$$\frac{\omega r}{V_0} = \frac{(628 \text{ rad/s})(0.015 \text{ m})}{10 \text{ m/s}} = 0.942$$

From Fig. 11.18, the lift coefficient is approximately 0.26, and the drag coefficient is 0.64. The lift force is

$$F_L = \frac{1}{2}\rho V_0^2 C_L A_p$$

$$= \frac{1}{2}(1.2 \text{ kg/m}^3)(10 \text{ m/s})^2(0.26)\frac{\pi}{4}(0.03 \text{ m})^2$$

$$= \boxed{1.10 \times 10^{-2} \text{ N}}$$

The lift force is downward. The drag force is

$$F_D = \frac{1}{2}\rho V_0^2 C_D A_p$$

$$= \boxed{27.1 \times 10^{-3} \text{ N}}$$

11.9 Lift and Drag on Airfoils

This section presents information on how to calculate lift and drag on winglike objects. Son typical applications include calculating the takeoff weight of an airplane, determining the si of wings needed, and estimating power requirements to overcome drag force.

Lift of an Airfoil

An **airfoil** is a body designed to produce lift from the movement of fluid around it. Speci cally, lift is a result of circulation in the flow produced by the airfoil. To see this, consid flow of an ideal flow (nonviscous and incompressible) past an airfoil as shown in Fig. 11.19 Here, as for irrotational flow past a cylinder, the lift and drag are zero. There is a stagnatic point on the bottom side near the leading edge, and another on the top side near the tra ing edge of the foil. In the real flow (viscous fluid) case, the flow pattern around the u stream half of the foil is plausible. However, the flow pattern in the region of the trailii edge, as shown in Fig. 11.19a, cannot occur. A stagnation point on the upper side of the fc indicates that fluid must flow from the lower side around the trailing edge and then towa the stagnation point. Such a flow pattern implies an infinite acceleration of the fluid part cles as they turn the corner around the trailing edge of the wing. This is a physical impo sibility, and as we have seen in previous sections of the text, separation occurs at the sha edge. As a consequence of the separation, the downstream stagnation point moves to tl trailing edge. Flow from both the top and bottom sides of the airfoil in the vicinity of the tra ing edge then leaves the airfoil smoothly and essentially parallel to these surfaces at tl trailing edge (Fig. 11.19b).

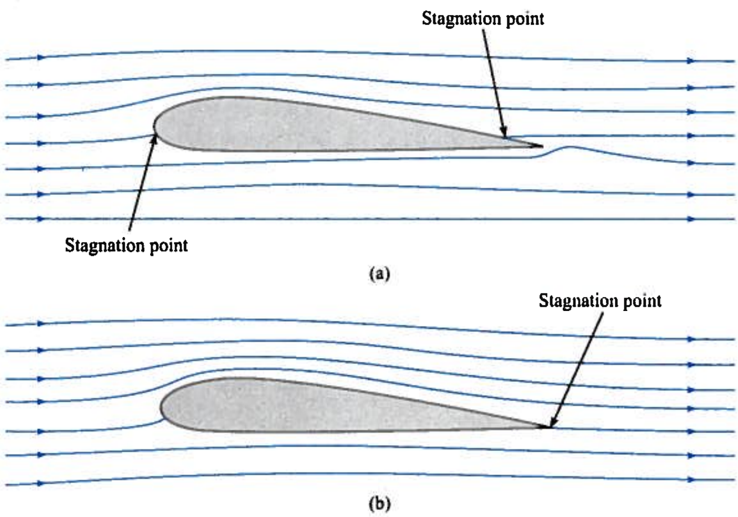


FIGURE 11.19
Patterns of flow around an airfoil.
(a) Ideal flow—no circulation.
(b) Real flow—circulation.

To bring theory into line with the physically observed phenomenon, it was hypothesize that a circulation around the airfoil must be induced in just the right amount so that tl downstream stagnation point is moved all the way back to the trailing edge of the airfoil, th allowing the flow to leave the airfoil smoothly at the trailing edge. This is called the *Kut condition* (21), named after a pioneer in aerodynamic theory. When analyses are made wi this simple assumption concerning the magnitude of the circulation, very good agreement occurs between theory and experiment for the flow pattern and the pressure distribution, as well as for the lift on a two-dimensional airfoil section (no end effects). Ideal flow theory then shows that the magnitude of the circulation required to maintain the rear stagnation point at the trailing edge (the Kutta condition) of a symmetric airfoil with a small angle of attack is given by

$$\Gamma = \pi c V_0 \alpha \tag{11.18}$$

where Γ is the circulation, c is the chord length of the airfoil, and α is the angle of attack of the chord of the airfoil with the free-stream direction (see Fig. 11.20 for a definition sketch).

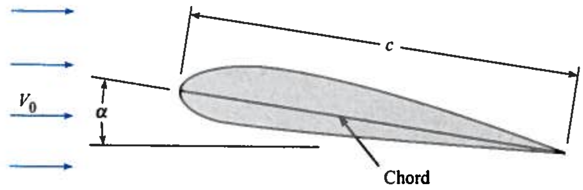


FIGURE 11.20 Definition sketch for an airfoil section.

Like that for the cylinder, the lift per unit length for an infinitely long wing is

$$F_L/\ell = \rho V_0 \Gamma$$

The planform area for the length segment ℓ is ℓc. Hence the lift on segment ℓ is

$$F_L = \rho V_0^2 \pi c \ell \alpha \tag{11.19}$$

For an airfoil the coefficient of lift is

$$C_L = \frac{F_L}{S\rho V_0^2/2} \tag{11.20}$$

where the reference area S is the planform area of the wing—that is, the area seen from the plan view. On combining Eqs. (11.18) and (11.19) and identifying S as the area associated with length segment ℓ, one finds that C_L for irrotational flow past a two-dimensional airfoil is given by

$$C_L = 2\pi\alpha \tag{11.21}$$

Equations (11.19) and (11.21) are the theoretical lift equations for an infinitely long airfoil at a small angle of attack. Flow separation near the leading edge of the airfoil produces deviations (high drag and low lift) from the ideal flow predictions at high angles of attack. Hence experimental wind-tunnel tests are always made to evaluate the performance of a given type of airfoil section. For example, the experimentally determined values of lift coefficient versus α for two NACA airfoils are shown in Fig. 11.21. Note in this figure that the coefficient of lift increases with the angle of attack, α, to a maximum value and then decreases with further increase in α. This condition, where C_L starts to decrease with a further increase in α, is called **stall**. Stall occurs because of the onset of separation over the top of the airfoil, which changes the pressure distribution so that it not only decreases lift but also increases drag. Data for many other airfoil sections are given by Abbott and Von Doenhoff (22).

Airfoils of Finite Length—Effect on Drag and Lift

The drag of a two-dimensional foil at a low angle of attack (no end effects) is primarily viscous drag. However, wings of finite length also have an added drag and a reduced lift associated with

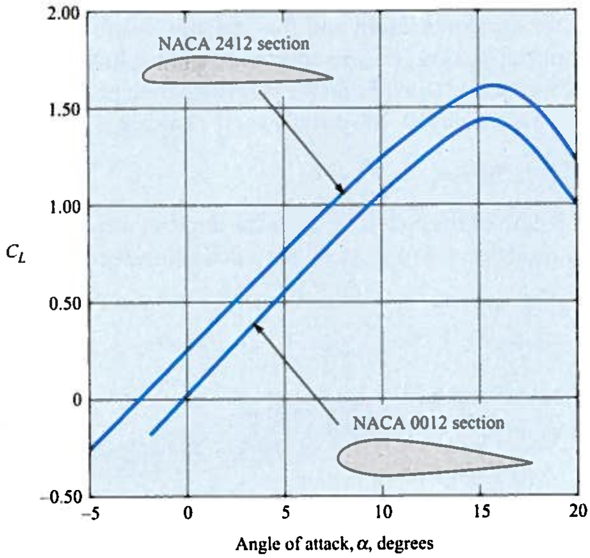


FIGURE 11.21
Values of C_L for two NACA airfoil sections. [After Abbott and Van Doenhoff (22).]

vortices generated at the wing tips. These vortices occur because the high pressure below t wing and the low pressure on top cause fluid to circulate around the end of the wing fro the high-pressure zone to the low-pressure zone, as shown in Fig. 11.22. This induced flc has the effect of adding a downward component of velocity, w, to the approach velocity \ Hence, the "effective" free-stream velocity is now at an angle ($\phi \approx w/V_0$) to the direction of t original free-stream velocity, and the resultant force is tilted back as shown in Fig. 11.23. Th the effective lift is smaller than the lift for the infinitely long wing because the effective angle incidence is smaller. This resultant force has a component parallel to V_0 that is called the *i duced drag* and is given by $F_L\phi$. Prandtl (23) showed that the induced velocity w for an ellipti spanwise lift distribution is given by the following equation:

$$w = \frac{2F_L}{\pi\rho V_0 b^2} \quad \text{(11.2}$$

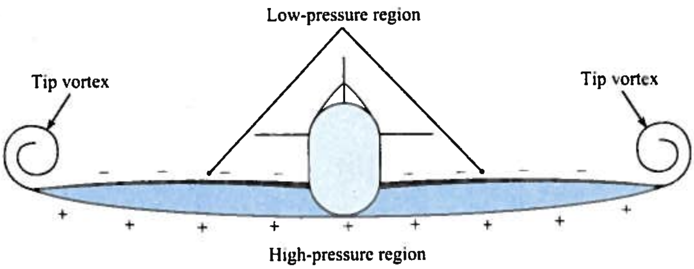
where b is the total length (or span) of the finite wing. Hence

$$F_{Di} = F_L\phi = \frac{2F_L^2}{\pi\rho V_0^2 b^2} = \frac{C_L^2}{\pi}\frac{S^2}{b^2}\frac{\rho V_0^2}{2} \quad \text{(11.2}$$

From Eq. (11.23) it can be easily shown that the coefficient of induced drag, C_{Di}, is given by

$$C_{Di} = \frac{C_L^2}{\pi(b^2/S)} = \frac{C_L^2}{\pi\Lambda} \quad \text{(11.2}$$

FIGURE 11.22
Formation of tip vortices.



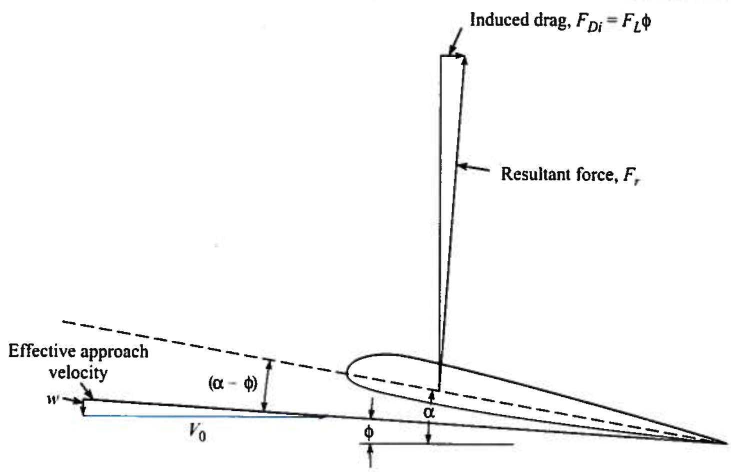


FIGURE 11.23
Definition sketch for induced-drag relations.

which happens to represent the minimum induced drag for any wing planform. Here the ratio b^2/S is called the aspect ratio Λ of the wing, and S is the planform area of the wing. Thus, for a given wing section (constant C_L and constant chord c), longer wings (larger aspect ratios) have smaller induced-drag coefficients. The induced drag is a significant portion of the total drag of an airplane at low velocities and must be given careful consideration in airplane design. Aircraft (such as gliders) and even birds (such as the albatross and gull) that are required to be airborne for long periods of time with minimum energy expenditure are noted for their long, slender wings. Such a wing is more efficient because the induced drag is small. To illustrate the effect of finite span, look at Fig. 11.24, which shows C_L and C_D versus α for wings with several aspect ratios.

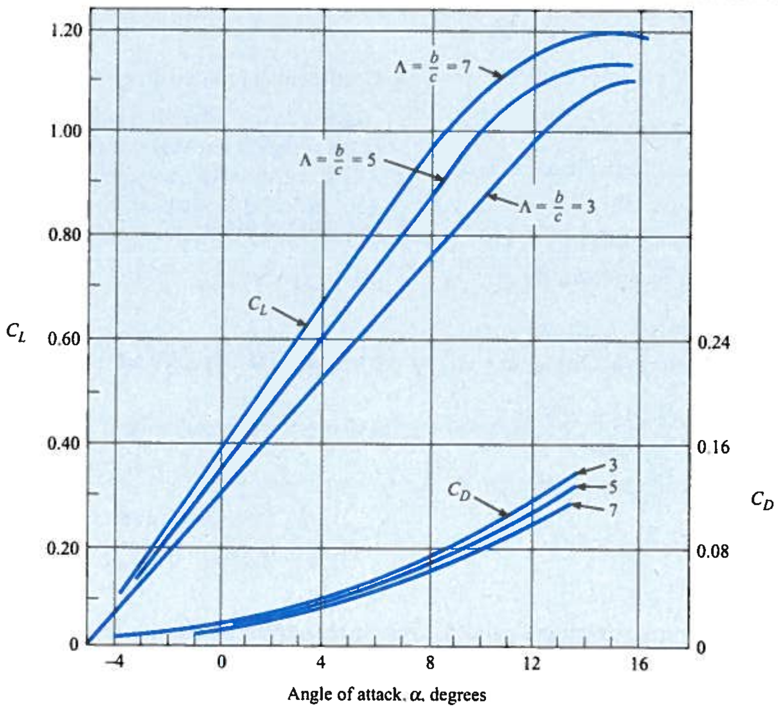


FIGURE 11.24
Coefficients of lift and d
for three wings with asp
ratios of 3, 5, and 7.
[After Prandtl (23).]

The total drag of a rectangular wing is computed by

$$F_D = (C_{D0} + C_{Di})\frac{bc\rho V_0^2}{2} \quad (11.2$$

where C_{D0} is the coefficient of form drag of the wing section and C_{Di} is the coefficient induced drag.

EXAMPLE 11.7

Wing Area for an Airplane

Problem Statement

An airplane with a weight of 10,000 lbf is flying at 600 ft/s at 36,000 ft, where the pressure is 3.3 psia and the temperature is −67°F. The lift coefficient is 0.2. The span of the wing is 54 ft. Calculate the wing area (in ft^2) and the minimum induced drag.

Define the Situation

An airplane (W = 10,000 lbf) is traveling at V_0 = 600 ft/s.

Coefficient of lift is $C_L = 0.2$.

Wing span is b = 54 ft.

Properties: Atmosphere (36,000 ft): $T = -67$°F, p = 3.3 psia

State the Goal

- Calculate the required wing area (in ft^2).
- Find the minimum value of induced drag (in N).

Generate Ideas and Make a Plan

1. Apply the idea gas law to calculate density of air.
2. Apply force equilibrium to derive an equation for the required wing area.
3. Calculate the coefficient of induced drag with Eq. (11.24).
4. Calculate the drag using Eq. (11.25) with $C_{DO} = 0$.

Take Action (Execute the Plan)

1. Ideal gas law

$$\rho = \frac{p}{RT} = \frac{(3.3\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)}{(1716\ \text{ft-lbf/slug-°R})(-67 + 460\text{°R})} = 0.000705\ \text{slug/ft}^3$$

2. Force equilibrium

$$W = F_L = \frac{1}{2}\rho V_0^2 C_L S$$

so

$$S = \frac{2W}{\rho V_0^2 C_L} = \frac{2 \times 10{,}000\ \text{lbf}}{(0.000705\ \text{slug/ft}^3)(600^2\ \text{ft}^2/\text{s}^2)(0.2)} = \boxed{394\ \text{ft}^2}$$

3. Coefficient of induced drag

$$C_{Di} = \frac{C_L^2}{\pi\left(\frac{b^2}{S}\right)} = \frac{0.2^2}{\pi\left(\frac{54^2}{394}\right)} = 0.00172$$

4. The induced drag is

$$D_i = \frac{1}{2}\rho V_0^2 C_{Di} S = \frac{1}{2}(0.000705\ \text{slug/ft}^3)(600\ \text{ft/s})^2(0.00172)(394\ \text{ft}^2) = \boxed{86.0\ \text{lbf}}$$

A graph showing C_L and C_D versus α is given in Fig. 11.25. Note in this graph that C_D separated into the induced-drag coefficient C_{Di} and the form drag coefficient C_{D0}.

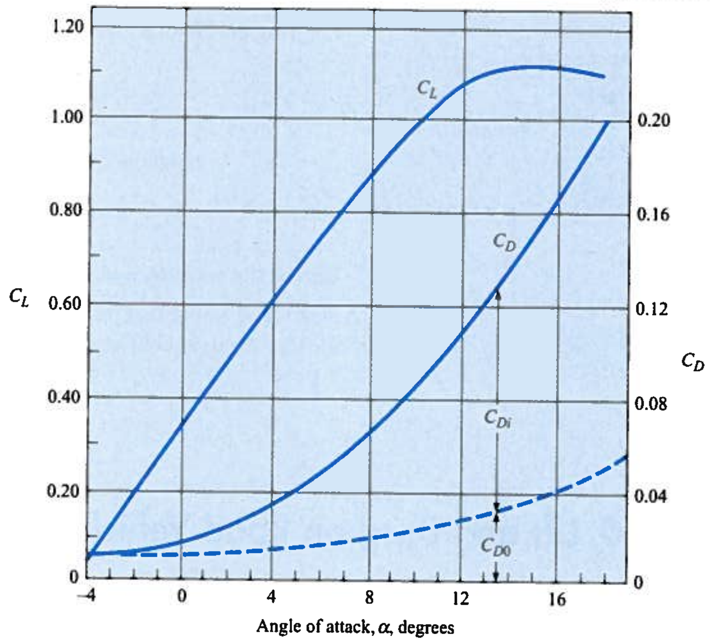


FIGURE 11.25

Coefficients of lift and drag for a wing with an aspect ratio of 5. [After Prandtl (23).]

EXAMPLE 11.8

Takeoff Characteristics of an Airplane

Problem Statement

A light plane (weight = 10 kN) has a wingspan of 10 m and a chord length of 1.5 m. If the lift characteristics of the wing are like those given in Fig. 11.24, what must be the angle of attack for a takeoff speed of 140 km/h? What is the stall speed? Assume two passengers at 800 N each and standard atmospheric conditions.

Define the Situation

- An airplane (W = 10 kN) with two passengers W = 1.6 kN is taking off.
- Wing span is b = 10 m, and chord length is c = 1.5 m.
- Lift coefficient information is given by Fig. 11.24.
- Takeoff speed is V_0 = 140 km/h.

Assumptions:

1. Ground effects can be neglected.
2. Standard atmospheric conditions prevail.

Properties: Air: $\rho = 1.2\ \text{kg/m}^3$

State the Goal

Find

1. Angle of attack (in degrees)
2. Stall speed (in km/h)

Generate Ideas and Make a Plan

1. Find the lift by applying force equilibrium.
2. Calculate the coefficient of lift using Eq. (11.20).
3. Find the angle of attack α from Fig. 11.24.
4. Read the maximum angle of attack from Fig. 11.24, and then calculate the corresponding stall speed using the lift force equation (11.17).

Take Action (Execute the Plan)

1. Force equilibrium (y direction), so lift = weight = 11.6 kN
2. Coefficient of lift

$$C_L = \frac{F_L}{S\rho V_0^2/2}$$

$$= \frac{11{,}600\ \text{N}}{(15\ \text{m}^2)(1.2\ \text{kg/m}^3)[(140{,}000/3600)^2\text{m}^2/\text{s}^2]/2}$$

$$= 0.852$$

3. The aspect ratio is

$$\Lambda = \frac{b}{c} = \frac{10}{1.5} = 6.67$$

4. From Fig. 11.24, the angle of attack is

$$\boxed{\alpha = 7°}$$

From Fig. 11.24, stall will occur when

$$C_L = 1.18$$

Applying the lift force equation gives

$$F_L = C_L A\left(\frac{\rho V_0^2}{2}\right)$$

$$11{,}600 = 1.18(15)\left(\frac{1.2}{2}\right)(V_{\text{stall}})^2$$

$$V_{\text{stall}} = 33.0\text{ m/s} = \boxed{119\text{ km/h}}$$

Review the Solution and the Process

Discussion. Notice that the stall speed (119 km/h) is less than the takeoff speed (140 km/h).

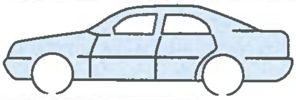
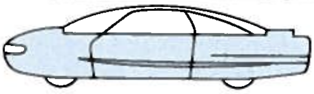
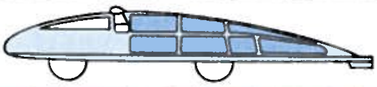
11.10 Lift and Drag on Road Vehicles

Early in the development of cars, aerodynamic drag was a minor factor in performance b cause normal highway speeds were quite low. Thus in the 1920s, coefficients of drag for ca were around 0.80. As highway speeds increased and the science of metal forming became mo advanced, cars took on a less angular shape, so that by the 1940s drag coefficients were 0.70 ar lower. In the 1970s the average C_D for U.S. cars was approximately 0.55. In the early 1980s tl average C_D for American cars dropped to 0.45, and currently auto manufacturers are givi even more attention to reducing drag in designing their cars. All major U.S., Japanese, ar European automobile companies now have models with C_Ds of about 0.33, and some comp nies even report C_Ds as low as 0.29 on new models. European manufacturers were the leade in streamlining cars because European gasoline prices (including tax) have been, for a numb of years, about three times those in the United States. Table 11.2 shows the C_D for a 1932 Fi and for other more contemporary car models.

Great strides have been made in reducing the drag coefficients for passenger cars. Ho ever, significant future progress will be very hard to achieve. One of the most streamlined ca was the "Bluebird," which set a world land-speed record in 1938. Its C_D was 0.16. The min mum C_D of well-streamlined racing cars is about 0.20. Thus, lowering the C_D for passeng cars below 0.30 will require exceptional design and workmanship. For example, the undersi of most cars is aerodynamically very rough (axles, wheels, muffler, fuel tank, shock absorbe and so on). One way to smooth the underside is to add a panel to the bottom of the car. B then clearance may become a problem, and adequate dissipation of heat from the muffler m be hard to achieve. Other basic features of the automobile that contribute to drag but are n very amenable to drag-reduction modifications are interior airflow systems for engine coo ing, wheels, exterior features such as rearview mirrors and antennas, and other surface pr trusions. The reader is directed to two books on road-vehicle aerodynamics, (24) and (2 which address all aspects of the drag and lift of road vehicles in considerably more detail th is possible here.

To produce low-drag vehicles, the basic teardrop shape is an idealized starting point. Tl shape can be altered to accommodate the necessary functional features of the vehicle. F example, the rear end of the teardrop shape must be lopped off to yield an overall vehicle leng that will be manageable in traffic and will fit in our garages. Also, the shape should be wider th its height. Wind-tunnel tests are always helpful in producing the most efficient design. One su test was done on a 3/8 scale model of a typical notchback sedan. Wind-tunnel test results f

TABLE 11.2 Coefficients of Drag for Cars

Make and Model	Profile	C_D
1932 Fiat Balillo		0.60
Volkswagen "Bug"		0.46
Plymouth Voyager		0.36
Toyota Paseo		0.31
Dodge Intrepid		0.31
Ford Taurus		0.30
Mercedes-Benz E320		0.29
Ford Probe V (concept car)		0.14
GM Sunraycer (experimental solar vehicle)		0.12

such a sedan are shown in Fig. 11.26. Here the centerline pressure distribution (distribution of C_P) for the conventional sedan is shown by a solid line, and that for a sedan with a 68 mm rear-deck lip is shown by a dashed line. Clearly the rear-deck lip causes the pressure on the rear of the car to increase (C_P is less negative), thereby reducing the drag on the car itself. It also decreases the lift, thereby improving traction. Of course, the lip itself produces some drag, and these tests show that the optimum lip height for greatest overall drag reduction is about 20 mm.

Research and development programs to reduce the drag of automobiles continue. As an entry in the PNGV (Partnership for a New Generation of Vehicles), General Motors (26) has exhibited a vehicle with a drag coefficient as low as 0.163, which is approximately one-half that of the typical midsize sedan. These automobiles will have a rear engine to eliminate the exhaust system underneath the vehicle and allow a flat underbody. Cooling air for the engine is drawn in through inlets on the rear fenders and exhausted out the rear, reducing the drag due to the wake. The protruding rearview mirrors are also removed to reduce the drag. The cumulative effect of these design modifications is a sizable reduction in aerodynamic drag.

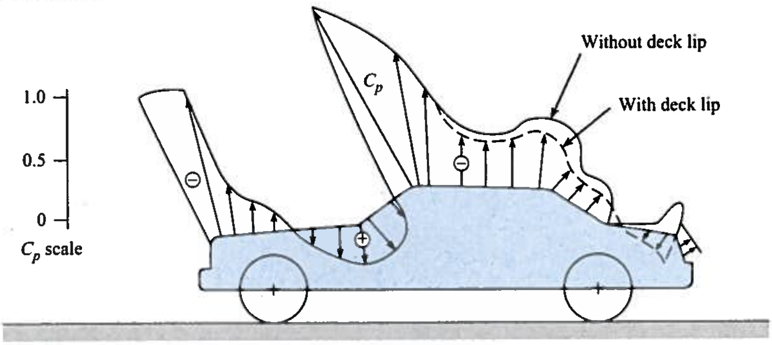


FIGURE 11.26
Effect of rear-deck lip on model surface. Pressure coefficients are plotted normal to the surface. [After Schenkel (25). Reprinted with permission from SAE Paper No. 770389. ©1977 Society of Automotive Engineers, Inc.]

The drag of trucks can be reduced by installing vanes near the corners of the truck bod to deflect the flow of air more sharply around the corner, thereby reducing the degree of sepa ration. This in turn creates a higher pressure on the rear surfaces of the truck, which reduce the drag of the truck.

One of the desired features in racing cars is the generation of negative lift to improve th stability and traction at high speeds. One idea (27) is to generate negative gage pressure unde neath the car by installing a *ground-effect pod*. This is an airfoil section mounted across th bottom of the car that produces a venturi effect in the channel between the airfoil section an the road surface. The design of ground-effect vehicles involves optimizing design paramete to avoid separation and possible increase in drag. Another scheme to generate negative lift the use of vanes as shown in Fig. 11.27. Sometimes "gurneys" are mounted on these vanes t reduce separation effects. Gurneys are small ribs mounted on the upper surface of the van near the trailing edge to induce local separation, reduce the separation on the lower surface c the vane, and increase the magnitude of the negative lift. As the speed of racing cars continu to increase, automobile aerodynamics will play an ever-increasing role in traction, stability, an control.

FIGURE 11.27
Racing car with negative-lift devices.

EXAMPLE 11.9

Calculating Negative Lift on a Race Car

Problem Statement

The rear vane installed on the racing car of Fig. 11.27 is at an angle of attack of 8° and has characteristics like those given in Fig. 11.24. Estimate the downward thrust (negative lift) and drag from the vane that is 1.5 m long and has a chord length of 250 mm. Assume the racing car travels at a speed of 270 km/h on a track where normal atmospheric pressure and a temperature of 30°C prevail.

Define the Situation

- A racing car experiences downward lift from a rear-mounted vane.
- Vane overall length is $\ell = 1.5$ m, and chord length is $c = 0.25$ m.
- Car speed is $V_0 = 270$ km/h $= 75$ m/s.

Properties: Air: $\rho = 1.17$ kg/m^3

State the Goal

Find

- Downward lift force from vane (in newtons)
- Drag force from vane (in newtons)

Generate Ideas and Make a Plan

1. Find the coefficient of lift C_L and the coefficient of drag C_D from Fig. 11.24.
2. Calculate the downward force using the lift force equation (11.17)
3. Calculate the drag using the drag force equation (11.5).

Take Action (Execute the Plan)

1. The aspect ratio is

$$\Lambda = \frac{\ell}{c} = \frac{1.5}{0.25} = 6$$

From Fig. 11.24, the lift and drag coefficients are

$$C_L = 0.93 \text{ and } C_D = 0.070$$

2. Lift force equation

$$F_L = C_L A\left(\frac{\rho V_0^2}{2}\right)$$

$$F_L = 0.93 \times 1.5 \times 0.25 \times 1.17 \times (75)^2/2$$

$$= \boxed{1148 \text{ N}}$$

3. Drag force equation

$$F_D = C_D A\left(\frac{\rho V_0^2}{2}\right) = \left(\frac{C_D}{C_L}\right)F_L$$

$$F_D = (0.070/0.93) \times 1148$$

$$= \boxed{86.4 \text{ N}}$$

11.11 Summarizing Key Knowledge

Relating Lift and Drag to Stress Distributions

- When a body moves relative to a fluid
  - The *drag force* is the component of force that is parallel to the free-stream.
  - The *lift force* is the component of force that is perpendicular to the free stream.
- The lift and drag forces are caused by the stress distributions (pressure and shear stress) acting on the body. Integrating the stress distributions over area gives the lift and drag forces.
- The drag force has two parts:
  - *Form drag* is due to pressure stresses acting on the body.
  - *Friction drag* (also called skin friction) is due to shear stresses acting on the body.

Calculating and Understanding the Drag Force

- Drag force depends on four factors: shape of the body, size, fluid density, and fluid speed squared. These four factors are related through the drag force equation

$$F_D = C_D A\left(\frac{\rho V_0^2}{2}\right)$$

- The *coefficient of drag* (C_D), which characterizes the shape of a body, is a π-group defined by.

$$C_D \equiv \frac{F_D}{A_{\text{Ref}}(\rho V_0^2/2)} = \frac{\text{(drag force)}}{\text{(reference area)(kinetic pressure)}}$$

- (C_D) is typically found by experiment and tabulated in engineering references. Objects are classified into three categories: (a) 2-D bodies, (b) axisymmetric bodies, and (c) 3-D bodies.

- (C_D) for a sphere can be found from charts and equations:
  - Stokes flow (Reynolds numbers < 0.5)

$$C_D = \frac{24}{\text{Re}}$$

  - Clift and Gauvin correlation ($\text{Re} < 3 \times 10^5$)

$$C_D = \frac{24}{\text{Re}}(1 + 0.15\,\text{Re}^{0.687}) + \frac{0.42}{1 + 4.25 \times 10^4\,\text{Re}^{-1.16}}$$

- Drag of bluff bodies and streamlined bodies differs.
  - A *bluff body* is a body with flow separation when the Reynolds number is high enough When flow separation occurs, the drag is mostly form drag.
  - A *streamlined body* does not have separated flow. Consequently, the drag force is mostl friction drag.
- (C_D) for cylinders and spheres drops dramatically at Reynolds numbers near 10^5 because the boundary layer changes from laminar to turbulent, moving the separation point downstream, reducing the wake region, and decreasing the form drag. This effect is callec the *drag crisis.*

Rolling Resistance and Power

- To calculate the power to move a body such as a car or an airplane at a steady speed through a fluid, the usual approach is
  - Step 1. Draw a free body diagram.
  - Step 2. Apply the power equation in the form $P = FV$, where F, the force in the directic of motion, is evaluated from the free body diagram.
- The rolling resistance is the frictional force that occurs when an object such as a ball or tire rolls. The rolling resistance is calculated using

$$F_{\text{Rolling resistance}} = F_r = C_r N \quad (11.2$$

where C_r is the coefficient of rolling resistance and N is the normal force.

Finding Terminal Velocity

- *Terminal velocity* is the steady-state speed of a body that is falling through a fluid.
- When a body has reached terminal velocity, the forces are balanced. These forces typicall are weight, drag, and buoyancy.
- To find terminal velocity, sum the forces in the direction of motion and solve the resultin equation. The solution process often needs to be done using iteration (traditional methoc or using a computer program (modern method).

Vortex Shedding, Streamlining, Compressible Flow

- Vortex shedding can cause beneficial effects (better mixing, better heat transfer) and detrimental effects (unwanted structural vibrations, noise, etc.).
  - *Vortex shedding* is when cylinders and bluff bodies in a cross-flow produce vortices tha are released alternately from each side of the body.
  - The frequency of vortex shedding depends on a π-group called the *Strouhal number.*

- *Streamlining* involves designing a body to minimize the drag force. Usually, streamlining involves designing to reduce or minimize flow separation for a bluff body.
- In high-speed air flows, compressibility effects increase the drag.

The Lift Force

- The lift force on a body depends on four factors: shape, size, density of the flowing fluid, and speed squared. The working equation is.

$$F_L = C_L A\left(\frac{\rho V_0^2}{2}\right)$$

- The *coefficient of lift* (C_L) is a π-group defined by

$$C_L \equiv \frac{F_L}{A_{\text{Ref}}(\rho V_0^2/2)} = \frac{\text{(drag force)}}{\text{(reference area)(kinetic pressure)}}$$

- *Circulation Theory of Lift*. The lift on an airfoil is due to the circulation produced by the airfoil on the surrounding fluid. This circulatory motion causes a change in the momentum of the fluid and a lift on the airfoil.
- The lift coefficient for a symmetric two-dimensional wing (no tip effect) is

$$C_L = 2\pi\alpha$$

  where α is the angle of attack (expressed in radians) and the reference area is the product of the chord and a unit length of wing.
- As the angle of attack increases, the flow separates, the airfoil stalls, and the lift coefficient decreases.
- A wing of finite span produces trailing vortices that reduce the angle of attack and produce an induced drag.
- The drag coefficient corresponding to the minimum induced drag is

$$C_{Di} = \frac{C_L^2}{\pi(b^2/S)} = \frac{C_L^2}{\pi\Lambda}$$

  where b is the wing span and S is the planform area of the wing.

REFERENCES

1. Bullivant, W. K. "Tests of the NACA 0025 and 0035 Airfoils in the Full Scale Wind Tunnel." *NACA Rept.*, 708 (1941).

2. DeFoe, G. L. "Resistance of Streamline Wires." *NACA Tech. Note*, 279 (March 1928).

3. Goett, H. J., and W. K. Bullivant. "Tests of NACA 0009, 0012, and 0018 Airfoils in the Full Scale Tunnel." *NACA Rept.*, 647 (1938).

4. Jacobs, E. N. "The Drag of Streamline Wires." *NACA Tech. Note*, 480 (December 1933).

5. Jones, G. W., Jr. "Unsteady Lift Forces Generated by Vortex Shedding about a Large, Stationary, and Oscillating Cylinder at High Reynolds Numbers." *Symp. Unsteady Flow, ASME* (1968).

6. Lindsey, W. F. "Drag of Cylinders of Simple Shapes." *NACA Rept.*, 619 (1938).

7. Miller, B. L., J. F. Mayberry, and I. J. Salter. "The Drag of Roughened Cylinders at High Reynolds Numbers." *NPL Rept. MAR Sci.*, R132 (April 1975).

8. Roshko, A. "Turbulent Wakes from Vortex Streets." *NACA Rept.*, 1191 (1954).

9. Abbott, I. H. "The Drag of Two Streamline Bodies as Affect by Protuberances and Appendages." *NACA Rept.*, 451 (1932).

10. Brevoort, M. J., and U. T. Joyner. "Experimental Investigation of the Robinson-Type Cup Anemometer." *NAC Rept.*, 513 (1935).

11. Freeman, H. B. "Force Measurements on a 1/40-Scale Model of the U.S. Airship 'Akron.' " *NACA Rept.*, 432 (1932).

12. Rouse, H. *Elementary Mechanics of Fluids.* New York: John Wiley, 1946.

13. Clift, R., J. R. Grace, and M. E. Weber. *Bubbles, Drops and Particles.* San Diego, CA: Academic Press, 1978.

14. Clift, R., and W. H. Gauvin. "The Motion of Particles in Turbulent Gas Streams." *Proc. Chemeca '70*, vol. 1, pp. 14–28 (1970).

15. Hoerner, S. F. *Fluid Dynamic Drag.* Published by the author, 1958.

16. Morrison, R. B. (ed). *Design Data for Aeronautics and Astronautics.* New York: John Wiley, 1962.

17. Roberson, J. A., et al. "Turbulence Effects on Drag of Sharp-Edged Bodies." *J. Hydraulics Div., Am. Soc. Civil Eng.* (July 1972).

18. Scher, S. H., and L. J. Gale. "Wind Tunnel Investigation of the Opening Characteristics, Drag, and Stability of Several Hemispherical Parachutes." *NACA Tech. Note*, 1869 (1949).

19. Crowe, C. T., et al. "Drag Coefficient for Particles in Rarefied, Low Mach-Number Flows." In *Progress in Heat and Mass Transfer*, vol. 6, pp. 419–431. New York: Pergamon Press, 1972.

20. Barkla, H. M., et al. "The Magnus or Robins Effect on Rotating Spheres." *J. Fluid Mech.*, vol. 47, part 3 (1971).

21. Kuethe, A. M., and J. D. Schetzer. *Foundations of Aerodynamics.* New York: John Wiley, 1967.

22. Abbott, H., and A. E. Von Doenhoff. *Theory of Wing Sections.* New York: Dover, 1949.

23. Prandtl, L. "Applications of Modern Hydrodynamics to Aeronautics." *NACA Rept.*, 116 (1921).

24. Hucho, Wolf-Heinrich, ed. *Aerodynamics of Road Vehicles.* London: Butterworth, 1987.

25. Schenkel, Franz K. "The Origins of Drag and Lift Reduction on Automobiles with Front and Rear Spoilers." *SAE Paper*, no. 770389 (February 1977).

26. Sharke, P. "Smooth Body." *Mechanical Engineer*, vol. 121, pp. 74–77 (1999).

27. Smith, C. *Engineer to Win.* Osceola, WI: Motorbooks International, 1984.

28. Mehta, R. D. "Aerodynamics of Sports Balls." *Annual Review of Fluid Mechanics*, 17, p. 151 (March 1985).

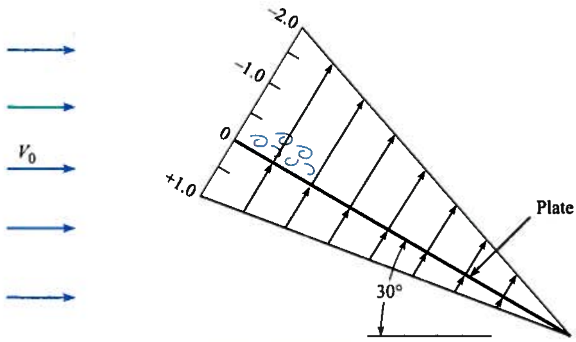
PROBLEMS

WileyPLUS Problem available in *WileyPLUS* at instructor's discretion.

GO Guided Online (GO) Problem, available in *WileyPLUS* at instructor's discretion.

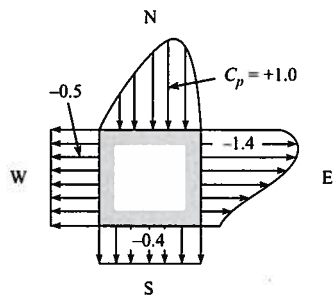
Relating Pressure Distribution and C_D (§11.1)

11.1 WileyPLUS A hypothetical pressure coefficient variation over a long (length normal to the page) plate is shown. What is the coefficient of drag for the plate in this orientation and with the given pressure distribution? Assume that the reference area is the surface area (one side) of the plate.



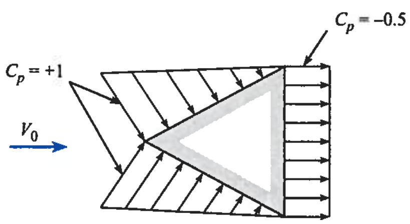
PROBLEM 11.1

11.2 WileyPLUS Flow is occurring past the square rod. The pressure coefficient values are as shown. From which direction do you think the flow is coming? (a) SW direction, (b) SE direction, (c) NW direction, or (d) NE direction.



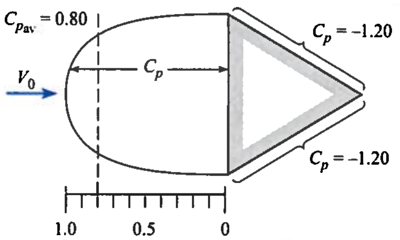
PROBLEM 11.2

11.3 The hypothetical pressure distribution on a rod of triangul (equilateral) cross section is shown, where flow is from left to right. That is, C_p is maximum and equal to $+1.0$ at the leading edge and decreases linearly to zero at the trailing edges. The pressure coefficient on the downstream face is constant with a value of -0.5. Neglecting skin friction drag, find C_D for the rod.



PROBLEM 11.3

11.4 WILEY PLUS The pressure distribution on a rod having a triangular (equilateral) cross section is shown, where flow is from left to right. What is C_D for the rod?



PROBLEM 11.4

11.5 WILEY PLUS Fill in the blanks for the following two statements:

A. ____________ is associated with the viscous shear-stress distribution.

- **a.** Form drag
- **b.** Friction drag

B. ____________ is associated with the pressure distribution

- **a.** Form drag
- **b.** Friction drag.

Calculating Drag Force (§11.2)

11.6 WILEY PLUS The coefficient of drag for a body (select all that apply):

- **a.** is dimensionless
- **b.** is usually determined by experiment
- **c.** depends on thrust
- **d.** depends on the body's shape
- **e.** requires an updraft

11.7 WILEY PLUS Apply the grid method to each situation that follows.

- **a.** Use Eq. (11.5) on p. 409 in §11.2, to predict the drag force in newtons for an automobile that is traveling at $V = 60$ mph on a summer day. Assume that the frontal area is 2 m^2, and the coefficient of drag is $C_D = 0.4$.
- **b.** Apply Eq. (11.5) on p. 409 in §11.2, to predict the speed in mph of a bicycle rider that is subject to a drag force of 5 lbf on a summer's day. Assume the frontal area of the rider is $A = 0.5$ m^2, and the coefficient of drag is $C_D = 0.3$.

11.8 Using the first two sections in this chapter and using other resources, answer the questions that follow. Strive for depth, clarity, and accuracy. Also, strive for effective use of sketches, words, and equations.

- **a.** What are the four most important factors that influence the drag force?
- **b.** How are stress and drag related?
- **c.** What is form drag? What is friction drag?

11.9 Use information in §11.2 and 11.3 to find the coefficient drag for each case described here.

- **a.** A sphere is falling through water, Re = 10,000.
- **b.** Air is blowing normal to a very long circular cylinde and Re = 7,000.
- **c.** Wind is blowing normal to a billboard that is 20 ft wi by 10 ft high.

11.10 Estimate the wind force on a billboard 12 ft high and 3 wide when a 60 mph wind ($T = 60$°F) is blowing normal to it

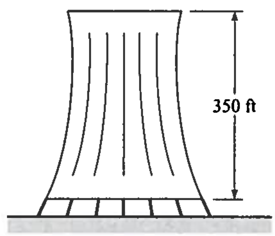
11.11 If Stokes's law is considered valid below a Reynolds number of 0.5, what is the largest raindrop that will fall in accordance with Stokes's law?

11.12 Determine the drag of a 2 ft × 4 ft sheet of plywood he at a right angle to a stream of air (60°F, 1 atm) having a velocit of 35 mph.

11.13 WILEY PLUS Estimate the drag of a thin square plate (3 m by 4 when it is towed through water (10°C). Assume a towing spee of about 5 m/s.

- **a.** The plate is oriented for minimum drag.
- **b.** The plate is oriented for maximum drag.

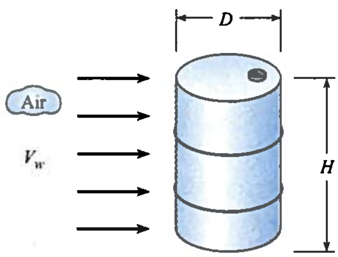
11.14 A cooling tower, used for cooling recirculating water ir modern steam power plant, is 350 ft high and 200 ft average diameter. Estimate the drag on the cooling tower in a 150 mpl wind ($T = 60$°F).



PROBLEM 11.14

11.15 Estimate the wind force that would act on *you* if you w standing on top of a tower in a 30 m/s (115 ft/s) wind on a day when the temperature was 20°C (68°F) and the atmospheric pressure was 96 kPa (14 psia).

11.16 WILEY GO As shown, wind is blowing on a 55-gallon drum. Estimate the wind speed needed to tip the drum over. Work ir units. The mass of the drum is 48 lbm, the diameter is 22.5 in. and the height is 34.5 in.



PROBLEM 11.16

11.17 What drag is produced when a disk 0.75 m in diameter is submerged in water at 10°C and towed behind a boat at a speed of 4 m/s? Assume orientation of the disk so that maximum drag is produced.

11.18 A circular billboard having a diameter of 7 m is mounted so as to be freely exposed to the wind. Estimate the total force exerted on the structure by a wind that has a direction normal to the structure and a speed of 50 m/s. Assume $T = 10°C$ and $p = 101$ kPa absolute.

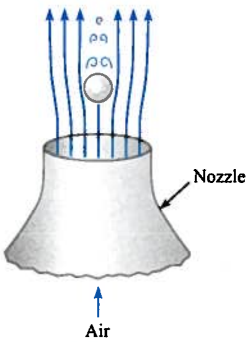
11.19 Consider a large rock situated at the bottom of a river and acted on by a strong current. Estimate a typical speed of the current that will cause the rock to move downstream along the bottom of the river. List and justify all your major assumptions. Shown all calculations and work in SI units.

11.20 Compute the overturning moment exerted by a 35 m/s wind on a smokestack that has a diameter of 2.5 m and a height of 75 m. Assume that the air temperature is 20°C and that p_a is 99 kPa absolute.

11.21 What is the moment at the bottom of a flagpole 20 m high and 8 cm in diameter in a 37.5 m/s wind? The atmospheric pressure is 100 kPa, and the temperature is 20°C.

11.22 A cylindrical anchor (vertical axis) made of concrete ($\gamma = 15$ kN/m^3) is reeled in at a rate of 1.0 m/s by a man in a boat. If the anchor is 30 cm in diameter and 30 cm long, what tension must be applied to the rope to pull it up at this rate? Neglect the weight of the rope.

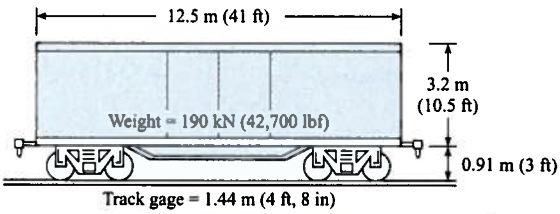
11.23 A Ping-Pong ball of mass 2.6 g and diameter 38 mm is supported by an air jet. The air is at a temperature of 18°C and a pressure of 27 in-Hg. What is the minimum speed of the air jet?



PROBLEM 11.23

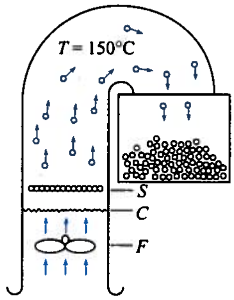
11.24 Estimate the moment at ground level on a signpost supporting a sign measuring 3 m by 2 m if the wind is normal to the surface and has a speed of 35 m/s and the center of the sign is 4 m above the ground. Neglect the wind load on the post itself. Assume $T = 10°C$ and $p = 1$ atm.

11.25 Windstorms sometimes blow empty boxcars off their tracks. The dimensions of one type of boxcar are shown. What minimum wind velocity normal to the side of the car would be required to blow the car over?



PROBLEM 11.25

11.26 A semiautomatic popcorn popper is shown. After the unpopped corn is placed in screen *S*, the fan *F* blows air past th heating coils *C* and then past the popcorn. When the corn pops its projected area increases; thus it is blown up and into a container. Unpopped corn has a mass of about 0.15 g per kerne and an average diameter of approximately 6 mm. When the cor pops, its average diameter is about 18 mm. Within what range of airspeeds in the chamber will the device operate properly?



PROBLEM 11.26

11.27 Hoerner (15) presents data that show that fluttering flag of moderate-weight fabric have a drag coefficient (based on the flag area) of about 0.14. Thus the total drag is about 14 times th skin friction drag alone. Design a flagpole that is 100 ft high an is to fly an American flag 6 ft high. Make your own assumption regarding other required data.

Power, Energy, and Rolling Resistance (§11.2)

11.28 How much power is required to move a spherical-shape submarine of diameter 1.5 m through seawater at a speed of 10 knots? Assume the submarine is fully submerged.

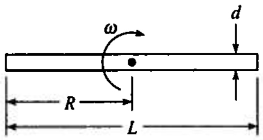
11.29 A blimp flies at 30 ft/s at an altitude where the specific weight of the air is 0.07 lbf/ft^3 and the kinematic viscosity is 1.3×10^{-4} ft^2/s. The blimp has a length-to-diameter ratio of 5 and has a drag coefficient corresponding to the streamlined bo in Fig. 11.9 (on p. 413 in §11.3). The diameter of the blimp is 80 f What is the power required to propel the blimp at this speed?

11.30 Estimate the energy in joules and kcal (food calories) that a runner supplies to overcome aerodynamic drag during a 10 km race. The runner runs a 6:30 pace (i.e., each mil takes 6 minutes and 30 seconds). The product of frontal area an coefficient of drag is $C_DA = 8$ ft^2. (One "food calorie" is equivalent to 4186 J.) Assume an air density of 1.22 kg/m^3.

11.31 A cylindrical rod of diameter d and length L is rotated in still air about its midpoint in a horizontal plane. Assume the drag force at each section of the rod can be calculated assuming a two-dimensional flow with an oncoming velocity equal to the relative velocity component normal to the rod. Assume C_D is constant along the rod.

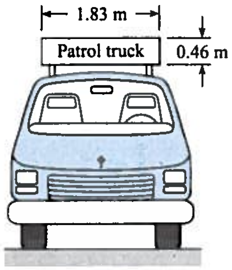
a. Derive an expression for the average power needed to rotate the rod.

b. Calculate the power for $\omega = 50$ rad/s, $d = 2$ cm, $L = 1.5$ m, $\rho = 1.2$ kg/m^3, and $C_D = 1.2$.



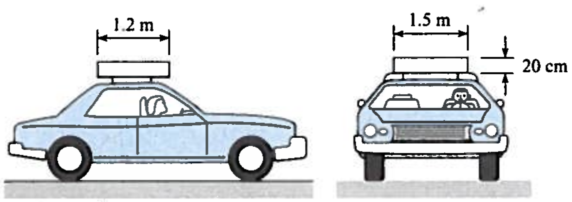
PROBLEM 11.31

11.32 Estimate the additional power (in hp) required for the truck when it is carrying the rectangular sign at a speed of 30 m/s over that required when it is traveling at the same speed but is not carrying the sign.



PROBLEM 11.32

11.33 Estimate the added power (in hp) required for the car when the cartop carrier is used and the car is driven at 100 km/h in a 25 km/h headwind over that required when the carrier is not used in the same conditions.



PROBLEM 11.33

11.34 The resistance to motion of an automobile consists of rolling resistance and aerodynamic drag. The weight of an automobile is 3000 lbf, and it has a frontal area of 20 ft^2. The drag coefficient is 0.30, and the coefficient of rolling friction is 0.02. Determine the percentage savings in gas mileage that one achieves when one drives at 55 mph instead of 65 mph on a l road. Assume an air temperature of 60°F.

11.35 A car coasts down a very long hill. The weight of car is 2000 lbf, and the slope of the grade is 6%. The rolling friction coefficient is 0.01. The frontal area of the car is 18 ft^2, the drag coefficient is 0.29. The density of the air is 0.002 slugs Find the maximum coasting speed of the car in mph.

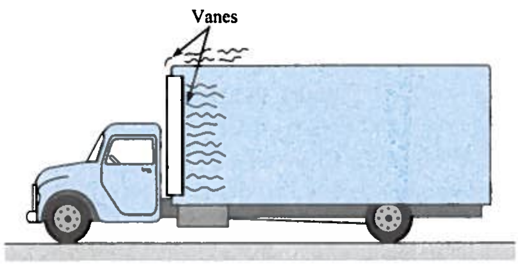
11.36 An automobile with a mass of 1000 kg is driven hill where the slope is 3° (5.2% grade). The automobile is mov at 30 m/s. The coefficient of rolling friction is 0.02, the drag coefficient is 0.4, and the cross-sectional area is 4 m^2. Find th power (in kW) needed for this condition. The air density is 1.2 kg/m^3.

11.37 A bicyclist is coasting down a hill with a slope of into a headwind (measured with respect to the ground) of 7 The mass of the cyclist and bicycle is 80 kg, and the coefficien of rolling friction is 0.02. The drag coefficient is 0.5, and the projected area is 0.5 m^2. The air density is 1.2 kg/m^3. Find the speed of the bicycle in meters per second.

11.38 A bicyclist is capable of delivering 275 W of pow to the wheels. How fast can the bicyclist travel in a 3 m/s headv if his or her projected area is 0.5 m^2, the drag coefficient is 0.3 and the air density is 1.2 kg/m^3? Assume the rolling resistanc is negligible.

11.39 Assume that the horsepower of the engine in the original 1932 Fiat Balillo (see Table 11.2 on p. 433 of §11.10) was 40 bhp (brake horsepower) and that the maximum speed sea level was 60 mph. Also assume that the projected area of t automobile is 30 ft^2. Assume that the automobile is now fitted with a modern 220 bhp motor with a weight equal to the wei of the original motor; thus the rolling resistance is unchanged. What is the maximum speed of the "souped-up" Balillo at sea l

11.40 One way to reduce the drag of a blunt object is to inst vanes to suppress the amount of separation. Such a procedur was used on model trucks in a wind-tunnel study by Kirsch a Bettes. For tests on a van-type truck, they noted that without vanes the C_D was 0.78. However, when vanes were installed around the top and side leading edges of the truck body (see figure), a 25% reduction in C_D was achieved. For a truck with projected area of 8.36 m^2, what reduction in drag force will b effected by installation of the vanes when the truck travels at 100 km/h? Assume standard atmospheric pressure and a temperature of 20°C.



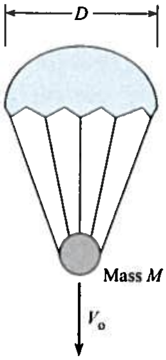
PROBLEM 11.40

11.41 For the truck of Prob. 11.40, assume that the total resistance is given by $R = F_D + C$, where F_D is the air drag and C is the resistance due to bearing friction. If C is constant at 350 N for the given truck, what fuel-savings percentage will be effected by the installation of the vanes when the truck travels at 100 km/h?

Terminal Velocity (§11.4)

11.42 Suppose you are designing an object to fall through seawater with a terminal velocity of exactly 1 m/s. What variables will have the most influence on the terminal velocity? List these variables and justify your decisions.

11.43 As shown, a 35-cm-diameter emergency medicine parachute supporting a mass of 20 g is falling through air (20°C). Assume a coefficient of drag of $C_D = 2.2$, and estimate the terminal velocity V_0. Use a projected area of $(\pi D^2)/4$.

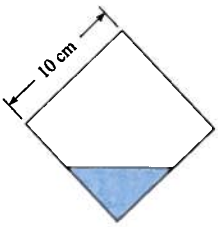


PROBLEM 11.43

11.44 Consider a small air bubble (approximately 4 mm diameter) rising in a very tall column of liquid. Will the bubble accelerate or decelerate as it moves upward in the liquid? Will the drag of the bubble be largely skin friction or form drag? Explain.

11.45 Determine the terminal velocity in water ($T = 10°C$) of a 8-cm ball that weighs 15 N in air.

11.46 This cube is weighted so that it will fall with one edge down as shown. The cube weighs 22.2 N in air. What will be its terminal velocity in water?



PROBLEM 11.46

11.47 A spherical rock weighs 30 N in air and 5 N in water. Estimate its terminal velocity as it falls in water (20°C).

11.48 A spherical balloon 2 m in diameter that is used for meteorological observations is filled with helium at standard conditions. The empty weight of the balloon is 3 N. What velocity of ascent will it attain under standard atmospheric conditions?

11.49 A sphere 2 cm in diameter rises in oil at a velocity of 1.5 cm/s. What is the specific weight of the sphere if the oil density is 900 kg/m^3 and the dynamic viscosity is 0.096 N · s/m

11.50 Estimate the terminal velocity of a 1.5-mm plastic sphere in oil. The oil has a specific gravity of 0.95 and a kinema viscosity of 10^{-4} m^2/s. The plastic has a specific gravity of 1.07. The volume of a sphere is given by $\pi D^3/6$.

11.51 A 120-lbf (534 N) skydiver is free-falling at an altitu of 6500 ft (1980 m). Estimate the terminal velocity in mph for minimum and maximum drag conditions. At maximum drag conditions, the product of frontal area and coefficient of drag is $C_D A = 8$ ft^2 (0.743 m^2). At minimum drag conditions, $C_D A = 1$ f (0.0929 m^2). Assume the pressure and temperature at sea level ar 14.7 psia (101 kPa) and 60°F (15°C). To calculate air properties, use the lapse rate for the U.S. standard atmosphere (see Chapter :

PROBLEM 11.51

11.52 What is the terminal velocity of a 0.5-cm hailstone in air that has an atmospheric pressure of 96 kPa absolute and a temperature of 0°C? Assume that the hailstone has a specific weight of 6 kN/m^3.

11.53 A drag chute is used to decelerate an airplane after touchdown. The chute has a diameter of 12 ft and is deployed when the aircraft is moving at 200 ft/s. The mass of the aircraft is 20,000 lbm, and the density of the air is 0.075 lbm/ft^3. Find th initial deceleration of the aircraft due to the chute.

11.54 A paratrooper and parachute weigh 900 N. What rate of descent will they have if the parachute is 7 m in diameter and tl air has a density of 1.20 kg/m^3?

11.55 If a balloon weighs 0.10 N (empty) and is inflated with helium to a diameter of 60 cm, what will be its terminal velocity in air (standard atmospheric conditions)? The helium is at standard conditions.

11.56 A 2-cm plastic ball with a specific gravity of 1.2 is release from rest in water at 20°C. Find the time and distance needed tc achieve 99% of the terminal velocity. Write out the equation of motion by equating the mass times acceleration to the buoyant force, weight, and drag force and solve by developing a compute program or using available software. Use Eq. (11.9) on p. 414 in §11. for the drag coefficient. [*Hint:* The equation of motion can be expressed in the form

$$\frac{dv}{dt} = -\left(\frac{C_D \mathrm{Re}}{24}\right)\frac{18\mu}{\rho_b d^2}v + \frac{\rho_b - \rho_w}{\rho_b}g$$

where ρ_b is the density of the ball and ρ_w is the density of the water. This form avoids the problem of the drag coefficient approaching infinity when the velocity approaches zero because $C_D \mathrm{Re}/24$ approaches unity as the Reynolds number approaches zero. An "if-statement" is needed to avoid a singularity in Eq. (11.9) when the Reynolds number is zero.]

Theory of Lift (§11.8)

11.57 From the following list, select one topic that is interesting to you. Then, use references such as the Internet to research your topic and prepare one page of written documentation that you could use to present your topic to your peers.

a. Explain how an airplane works.

b. Describe the aerodynamics of a flying bird.

c. Explain how a propeller produces thrust.

d. Explain how a kite flies.

11.58 Apply the grid method to each situation that follows.

a. Use Eq. (11.17), on p. 424 in §11.8, to predict the lift force in newtons for a spinning baseball. Use a coefficient of lift of $C_L = 1.2$. The speed of the baseball is 90 mph. Calculate area using $A = \pi r^2$, where the radius of a baseball is $r = 1.45$ in. Assume a hot summer day.

b. Use Eq. (11.17), on p. 424 in §11.8, to predict the size of wing in mm^2 needed for a model aircraft that has a mass of 570 g. Wing size is specified by giving the wing area (A) as viewed by an observer looking down on the wing. Assume the airplane is traveling at 80 mph on a hot summer day. Use a coefficient of lift of $C_L = 1.2$. Assume straight and level flight so lift force balances weight.

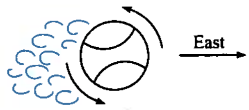
11.59 Using §11.8 and other resources, answer the following questions. Strive for depth, clarity, and accuracy. Also, use effective sketches, words, and equations.

a. What is circulation? Why is it important?

b. What is lift force?

c. What variables influence the magnitude of the lift force?

11.60 WILEY PLUS The baseball is thrown from west to east with a spin about its vertical axis as shown. Under these conditions it will "break" toward the (a) north, (b) south, or (c) neither.

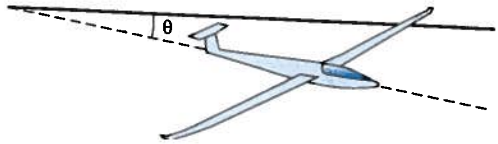


PROBLEM 11.60

11.61 Analyses of pitched baseballs indicate that C_L of a rotating baseball is approximately three times that shown in Fig. 11.18 (on p. 425 in §11.8). This greater C_L is due to the added circulation caused by the seams of the ball. What is the lift of a ball pitched at a speed of 85 mph and with a spin rate of 35 rps? Also, how much will the ball be deflected from its original path by the time it gets to the plate as a result of the lift force? *Note:* The mound-to-plate distance is 60 ft, the weight of the baseball is 5 oz, and the circumference is 9 in. Assume standard atmospheric conditions, and assume that the axis of rotation is vertical.

Lift and Drag on Airfoils (§11.9)

11.62 As shown, a glider traveling at a constant velocity will move along a straight glide path that has an angle θ with respect to the horizontal. The angle θ, also called the glide ratio, is giv by $\theta = (C_D/C_L)$. Use basic principles to prove the preceding statement.



PROBLEM 11.62

11.63 WILEY PLUS A sphere of diameter 100 mm, rotating at a rate of 286 rpm, is situated in a stream of water (15°C) that has a velocity of 1.5 m/s. Determine the lift force (in newtons) on t rotating sphere.

11.64 An airplane wing having the characteristics shown in Fig. 11.24 (on p. 429 in §11.9) is to be designed to lift 1800 lbf when the airplane is cruising at 200 ft/s with an angle of attac of 3°. If the chord length is to be 3.5 ft, what span of wing is required? Assume $\rho = 0.0024$ slugs/ft^3.

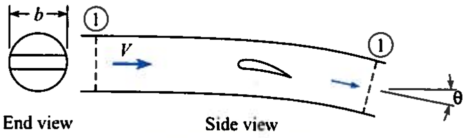
11.65 A boat of the hydrofoil type has a lifting vane with an aspect ratio of 4 that has the characteristics shown in Fig. 11. (on p. 429 in §11.9). If the angle of attack is 4° and the weight the boat is 5 tons, what foil dimensions are needed to support boat at a velocity of 60 fps?

11.66 One wing (wing A) is identical (same cross section) to another wing (wing B) except that wing B is twice as long as wing A. Then for a given wind speed past both wings and wi the same angle of attack, one would expect the total lift of wing B to be (a) the same as that of wing A, (b) less than tha of wing A, (c) double that of wing A, or (d) more than doubl that of wing A.

11.67 What happens to the value of the induced drag coeffic for an aircraft that increases speed in level flight? (a) it increa (b) it decreases, (c) it does not change.

11.68 WILEY PLUS The total drag coefficient for an airplane wing is $C_D = C_{D0} + C_L^2/\pi\Lambda$, where C_{D0} is the form drag coefficient C_L is the lift coefficient and Λ is the aspect ratio of the wing. ' power is given by $P = F_D V = 1/2\, C_D \rho V^3 S$. For level flight the is equal to the weight, so $W/S = 1/2 \rho C_L V^2$, where W/S is call the "wing loading." Find an expression for V for which the pc is a minimum in terms of $V_{MinPower} = f(\rho, \Lambda, W/S, C_{D0})$, and f the V for minimum power when $\rho = 1$ kg/m^3, $\Lambda = 10$, W/S = 600 N/m^2, and $C_D = 0.02$.

11.69 The airstream affected by the wing of an airplane can considered to be a cylinder (stream tube) with a diameter eq to the wingspan, b. Far downstream from the wing, the tube i deflected through an angle θ from the original direction. App the momentum equation to the stream tube between section and 2 and find the lift of the wing as a function of b, ρ, V, anc Relating the lift to the lift coefficient, find θ as a function of b C_L, and wing area, S. Using the relation for induced drag, $F_{Di} = F_L\theta/2$, show that $C_{Di} = C_L^2/\pi\Lambda$, where Λ is the wing aspect ratio.

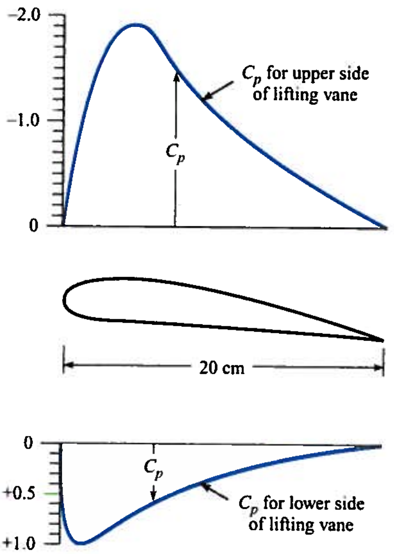


PROBLEM 11.69

11.70 The landing speed of an airplane is 8 m/s faster than its stalling speed. The lift coefficient at landing speed is 1.2, and the maximum lift coefficient (stall condition) is 1.4. Calculate both the landing speed and the stalling speed.

11.71 An airplane has a rectangular-planform wing that has an elliptical spanwise lift distribution. The airplane has a mass of 1000 kg, a wing area of 16 m^2, and a wingspan of 10 m, and it is flying at 50 m/s at 3000 m altitude in a standard atmosphere. If the form drag coefficient is 0.01, calculate the total drag on the wing and the power ($P = F_D V$) necessary to overcome the drag.

11.72 The figure shows the relative pressure distribution for a Göttingen 387-FB lifting vane (19) when the angle of attack is 8°. If such a vane with a 20-cm chord were used as a hydrofoil at a depth of 70 cm, at what speed in 10°C freshwater would cavitation begin? Also, estimate the lift per unit of length of foi at this speed.



PROBLEM 11.72

11.73 Consider the distribution of C_p as given for the wing section in Prob. 11.72. For this distribution of C_p, the lift coefficient C_L will fall within which range of values: (a) $0 < C_L < 1.0$; (b) $1.01 < C_L < 2.0$; (c) $2.01 < C_L < 3.0$; or (d) $3.0 < C_L$?

11.74 The total drag coefficient for a wing with an elliptical lift distribution is $C_D = C_{D_0} + C_L^2/\pi\Lambda$, where Λ is the aspect ratio. Derive an expression for C_L that corresponds to minimum $C_D/($ (maximum C_L/C_D) and the corresponding C_L/C_D.

11.75 WILEY PLUS A glider at 800 m altitude has a mass of 180 kg and a wing area of 20 m^2. The glide angle is 1.7°, and the air density is 1.2 kg/m^3. If the lift coefficient of the glider is 0.83, how many minutes will it take to reach sea level on a calm day?

11.76 The wing loading on an airplane is defined as the aircraf weight divided by the wing area. An airplane with a wing loadir of 2000 N/m^2 has the aerodynamic characteristics given by Fig. 11.25 (on p. 431 in §11.9). Under cruise conditions the lift coefficient is 0.3. If the wing area is 10 m^2, find the drag force.

11.77 An ultralight airplane has a wing with an aspect ratio of and with lift and drag coefficients corresponding to Fig. 11.24 (on p. 429 in §11.9). The planform area of the wing is 200 ft^2. The weight of the airplane and pilot is 400 lbf. The airplane flies at 50 ft/s in air with a density of 0.002 slugs/ft^3. Find the angle o attack and the drag force on the wing.

11.78 Your objective is to design a human-powered aircraft using the characteristics of the wing in Fig. 11.24 (on p. 429 in §11.9). The pilot weighs 130 pounds and is capable of outputtin 1/2 horsepower (225 ft-lbf/s) of continuous power. The aircraft without the wing has a weight of 40 lbf, and the wing can be designed with a weight of 0.12 lbf per square foot of wing area. The drag consists of the drag of the structure plus the drag of th wing. The drag coefficient of the structure, C_{D0} is 0.05, so that th total drag on the craft will be

$$F_D = (C_{D_0} + C_D)\frac{1}{2}\rho V_0^2 S$$

where C_D is the drag coefficient from Fig. 11.24 (on p. 429 in §11.9). The power required is equal to $F_D V_0$. The air density is 0.00238 slugs/ft^3. Assess whether the airfoil is adequate, and if it is, find the optimum design (wing area and aspect ratio).