2 FLUID PROPERTIES

FIGURE 2.1
This photo shows engineers observing a flume. A *flume* is an artificial channel for conveying water. The flume shown is situated in Boise, Idaho, and is used to study sediment transport in rivers. (Photo courtesy of Professor Ralph Budwig of the Center for Ecohydraulics Research, University of Idaho.)

Chapter Road Map

This chapter introduces ideas for idealizing real-wo problems, introduces fluid properties, and presents the viscosity equation.

Learning Objectives

STUDENTS WILL BE ABLE TO

- Define system, boundary, surroundings, state, process, ar property. (§2.1)
- Define density, specific gravity, and specific weight. Rela these properties using calculations. (§2.2)
- Explain the meaning of a constant density flow and discu the relevant issues. (§2.3)
- Look up fluid properties; document the results. (§2.4)
- Define viscosity, shear stress, shear force, velocity gradier velocity profile, the no-slip condition, and kinematic viscosity. (§2.4)
- Apply the shear stress equation to problem solving. (§2.6
- Describe a Newtonian and non-Newtonian fluid. (§2.7)
- Describe surface tension; solve relevant problems. (§2.8)
- Describe vapor pressure; look up data for water. (§2.9)

2.1 Defining the System

To solve real-world problems, engineers idealize the physical world. One aspect of the en neering process is to create a precise definition of what is being analyzed. A **system** is wh ever is being studied or analyzed by the engineer. A system can be a collection of matter, it can be a region in space. Anything that is not part of the system is considered to be part the **surroundings**. The **boundary** is the imaginary surface that separates the system from surroundings.

EXAMPLE. For the flume shown in Fig. 2.1, the water that is situated inside the flume could be defined as the system. For this system, the surroundings would be the flume walls, the air above the flume, etc. Notice that *engineers are specific about what the system is, what the surroundings are, and what boundary is.*

EXAMPLE. Suppose an engineer is analyzing the air flow from a tank being used by a SCUBA diver. As shown in Fig. 2.2, the engineer might select a system comprised of the tank and the regulator. For this system, everything that is external to the tank and regulator is the surroundings. Notice that *the system is defined with a sketch* because this is good professional practice.
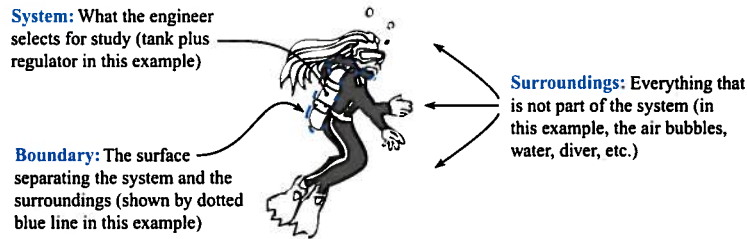

System: What the engineer selects for study (tank plus regulator in this example)

Surroundings: Everything that is not part of the system (in this example, the air bubbles, water, diver, etc.)

Boundary: The surface separating the system and the surroundings (shown by dotted blue line in this example)

FIGURE 2.2

Example of a system, it surroundings, and the boundary.

Engineers select systems in ways that make problem solving the easiest and most correct. Although the choice of system must fit the problem at hand, there are often multiple possibilities for which system to select. This topic will be revisited throughout the text as various kinds of systems are introduced and applied.

Systems are described by specifying numbers that characterize the system. The numbers are called properties. A **property** is a measurable characteristic of a system that depends only on the present conditions within the system.

EXAMPLE. In Fig. 2.2, some examples of properties (i.e., measurable characteristics) are

- The pressure of the air inside the tank
- The density of air inside the tank
- The weight of the system (tank plus air plus regulator)

Some parameters in engineering are measurable, yet they are not properties. For example, work is not a property because the quantity of work depends on how a system interacts with its surroundings. Similarly, neither force nor torque are properties because these parameters depend on the interaction between a system and its surroundings. Heat transfer is not a property. Mass flow rate is not a property.

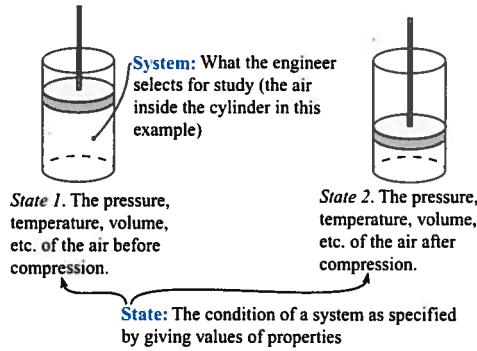The **state** of a system means the condition of the system as defined by specifying its properties.

EXAMPLE. Fig. 2.3 shows air being compressed by a piston in a cylinder. The air inside the cylinder is defined as the system. At state 1, the conditions of the system are defined by specifying properties such as pressure, temperature, and density. Similarly, state 2 is defined by specifying these same properties.

The change of a system from one state to another state is called a **process**.

EXAMPLE. When air is compressed (Fig. 2.3), this is a process because the air (i.e., the system) has changed from one set of conditions (state 1) to another set of conditions (state 2). Engineers label processes that commonly occur. For example, an *isothermal process* is one in which the temperature of the system is held constant. For example, an *adiabatic process* is one in which there is no heat transfer between the system and the surroundings.

FIGURE 2.3

Air in a cylinder being compressed by a piston. State 1 is a label for the conditions of the system prior to compression. State 2 is a label for the conditions of the system after compression.

System: What the engineer selects for study (the air inside the cylinder in this example)

State 1. The pressure, temperature, volume, etc. of the air before compression.

State 2. The pressure, temperature, volume, etc. of the air after compression.

State: The condition of a system as specified by giving values of properties

Properties can be organized into categories. One category is called *material properties.* T purpose of this chapter is to describe the material properties of fluids, which are called **flu properties.** Examples of fluid properties include density, viscosity, and surface tension.

2.2 Characterizing Mass and Weight

Engineers characterize the weight and mass of a fluid with three properties: density, specif weight, and specific gravity.

Mass Density, ρ

Mass Density, ρ (rho), gives the ratio of mass to volume at a point. In particular, select a poi (x, y, z) in space and select a small volume ΔV surrounding the point. The mass of the matt within the volume is Δm, and the density is

$$\rho = \left(\frac{\text{mass}}{\text{volume}}\right)_{\text{point}} = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \qquad (2.$$

For simplicity, the label *mass density* is shortened to *density* for the remainder of this book.

The reason that density is defined at a point is that density can vary with location.

> **EXAMPLE.** In a lake, the temperature of the water varies with depth. Therefore, the densi will also vary with depth because density changes with temperature.

Based on Eq. (2.1), the dimensions of density are

$$[\rho] = \frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3}$$

The SI unit of density is the kilogram per cubic meter (kg/m³). The traditional units of densi are slugs per cubic foot (slug/ft³) or pounds-mass per cubic foot (lbm/ft³).

In general, density varies with both temperature and pressure. However for liquids, tl density is changed very little by changes in pressure, so engineers assume that density depend on temperature only. Fig. 2.4 compares density values for common liquids.
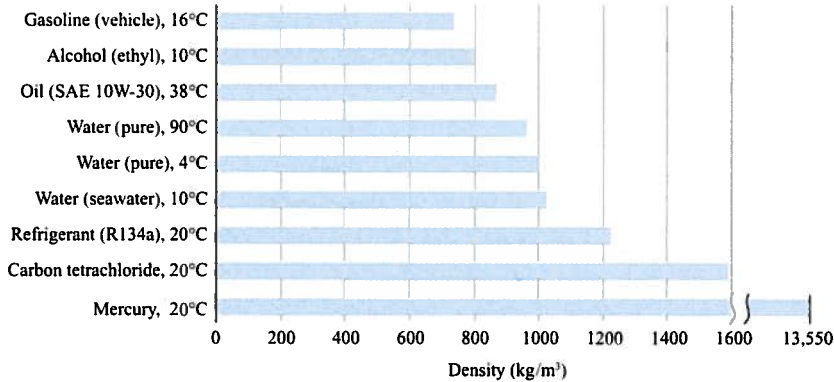
In Fig. 2.4, notice the following:

FIGURE 2.4

Density of common liqu

- When water is heated, the density goes down slightly; in particular, density drops 3.5% for a temperature change of 90°C.
- Most, but not all, liquids have a density within 30% of the density of water.
- Seawater is slightly heavier than freshwater.
- Some liquids, such as oil and gasoline, are lighter than water. When such liquids are immiscible with water, they will float on water.

Because water is common in application, some useful values to memorize are

$$\rho_{water,\ 4°C} = 1000\ \text{kg/m}^3 = 1\ \text{kilogram/liter} = 1\ \text{gram/milliliter}$$

$$\rho_{water,\ 59°F} = 62.4\ \text{lbm/ft}^3 = 1.94\ \text{slug/ft}^3 = 8.345\ \text{lbm/gal (US)}$$

For easy reference, these properties along with similar data for air are presented in the inside front cover of the text in Tables F.3, F.4, and F.5. Additional property data are presented in the appendices in Tables A.2 to A.5.

Specific Weight, γ

Specific weight is represented by the Greek symbol γ (gamma). **Specific weight** is the ratio of weight to volume at a point. In particular, select a point (x, y, z) in space and image a small volume ΔV surrounding the point. The weight of the matter within the volume is ΔW, and the specific weight is:

$$\gamma = \left(\frac{\text{weight}}{\text{volume}}\right)_{\text{point}} = \lim_{\Delta V \to 0} \frac{\Delta W}{\Delta V} \tag{2.2}$$

To relate γ and ρ, recall that weight and mass are related by $W = mg$. Divide this equation by volume to give

$$\gamma = \rho g \tag{2.3}$$

Because water is common in application, some useful values to memorize are

$$\gamma_{water,\ 15°C} = 9800\ \text{N/m}^3$$

$$\gamma_{water,\ 59°F} = 62.4\ \text{lbf/ft}^3$$

Other values of γ are presented in Tables F.4 to F.6 (inside front cover) and in Tables A.2 to A.5 (appendices in back of book).

Specific Gravity, *S* or SG

Specific gravity, which is represented as *S* or SG, is commonly used to characterize liquids and solids. **Specific gravity** is the ratio of the density of a material to the density of water at a reference temperature of 4°C.

$$S = \frac{\rho_{material}}{\rho_{liquid\ water,\ 4°C}} \tag{2.4}$$

Thus, a material with $S < 1$ is less dense than water, and a material with $S > 1$ is more dense than water. Combining Eqs. (2.3) and (2.4) gives

$$S = \frac{\rho_{liquid}}{\rho_{liquid\ water,\ 4°C}} = \frac{\gamma_{liquid}}{\gamma_{liquid\ water,\ 4°C}} \tag{2.5}$$

The properties ρ, γ, and SG are related; if one of these properties is known, the other two can be calculated.

EXAMPLE. Specific weight for mercury is $\gamma_{mercury} = 133$ kN/m³. Calculate the density and specific gravity. Use SI units.

Solution. Applying Eq. (2.3) gives density:

$$\rho_{mercury} = \frac{\gamma_{mercury}}{g} = \frac{(133,000\ N/m^3)}{(9.81\ m/s^2)} = 13,600\ kg/m^3$$

Applying Eq. (2.5) and the reference value for γ_{H_2O} from Table F.6 gives

$$S_{mercury} = \frac{\gamma_{mercury}}{\gamma_{liquid\ water,\ 4°C}} = \frac{(133,000\ N/m^3)}{(9810\ N/m^3)} = 13.6$$

Review. To validate the calculated values of ρ and S, one can consult Table A.4. Note that has no units because it is a ratio.

2.3 Modeling Fluids as Constant Density

Engineers decide if they will idealize a fluid as *constant density* or as *variable density*. This section introduces concepts that are useful for making informed decisions.

The Bulk Modulus of Elasticity

All fluids are compressible. To characterize compressibility, engineers use **bulk modulus of elasticity**, E_v(kPa)

$$E_v = -\frac{dp}{d\mathcal{V}/\mathcal{V}} = -\frac{\text{change in pressure}}{\text{fractional change in volume}} \tag{2.6}$$

where dp is the differential pressure change, $d\mathcal{V}$ is the differential volume change, and $\mathcal{V}$ is the volume of fluid. Because $d\mathcal{V}/\mathcal{V}$ is negative for a positive dp, a negative sign is used in the definition to yield a positive E_v.

The bulk modulus of elasticity of water is approximately 2.2 GN/m², which corresponds to a volume change of about 1/20 of 1% for pressure change of about 10 atmospheres. Thus, water and most liquids are assumed to be incompressible.

Two useful formulas for the bulk modulus of an ideal gas are

$$E_v = p \quad \text{(isothermal process)}$$

$$E_v = kp \quad \text{(adiabatic process)}$$

<div align="right">(2.7)</div>

where p is pressure and $k = c_p/c_v$ is the specific heat ratio.

EXAMPLE. Compare the compressibility of air and water at room conditions.

Solution. Assume the air is at 100 kPa and that the air is being compressed isothermally. With these assumptions, the bulk modulus of air is $E_v(\text{air}) = 1 \times 10^5$ Pa. The bulk modulus of water is $E_v(\text{water}) = 2.2 \times 10^9$ Pa. Thus,

$$\frac{\text{compressibility (air)}}{\text{compressibility (water)}} = \frac{(1/E_v)_{\text{air}}}{(1/E_v)_{\text{water}}} = \frac{2.2 \times 10^9 \text{ Pa}}{1.0 \times 10^5 \text{ Pa}} = 22{,}000$$

Review. This value (22,000) means that the volume change of air will be 22,000 that of water for the same applied pressure change.

The Constant Density Assumption

Constant density means that the density of a flowing fluid can be assumed to be constant spatially and temporally without causing significant changes (say 5%) in numbers that are calculated.

Because liquids have a high value of bulk modulus, they are commonly assumed to be incompressible. Incompressible means that the density of each fluid particle is independent of pressure.

A fluid that is incompressible can still have a variable density, meaning that density differs at various points in space or time.

EXAMPLE. When saline and freshwater are mixing as in estuaries, density variations occur even though the water can assumed to be incompressible.

Regarding gases, it is common to assume that a flowing gas has a constant density. The reason this assumption works is that pressure variations within the flow are not large enough to cause significant density variations.

High-speed flows of gases, such as the flow around a jet airplane, need to be modeled as compressible flows (see Chapter 12). To distinguish *constant density gas flow* from *variable density gas flow*, engineers use the Mach number M. The Mach number is the ratio of the speed of the flowing fluid V to the speed at which sound travels in the fluid c:

$$\text{Mach number} = \text{M} \equiv \frac{V}{c}$$

A *criterion for idealizing a gas as constant density* is:

$$(\text{M} < 0.3) \tag{2.8}$$

<div align="right">(2.8)</div>

When flow is steady and Eq. (2.8) is satisfied, the density variation is less than 5% (2).

EXAMPLE. To gain a feel what Eq. (2.8) means, consider air at 20°C. The speed of sound is $c \approx 340$ m/s. Thus, a flow of air can be assumed to be incompressible for $V < \approx 100$ m/s (220 mph). Because this is quite fast, the majority of gas flows in industrial applications can be idealized as constant density.

Both liquids and gases can have significant density variations when the fluid is bein heated or cooled. Common engineering practice is to assume constant density and then loc up property values at an appropriate average temperature.

EXAMPLE. When liquid water enters a heat exchanger at 20°C and exits at 80°C, commc practice is to assume constant density and look up a value at 50°C. This value, from Table A. is $\rho = 988$ kg/m³.

In summary, it is common to assume constant density when solving fluid mechanics prol lems. Most problems and methods in this text are based on this assumption. An importa exception is the high-speed flow of gases, a topic presented in Chapter 12.

2.4 Finding Fluid Properties

One can look up fluid properties in engineering handbooks, textbooks, or off the Internet. fluid property often depends on the temperature and pressure of the fluid. Thus, it is goc engineering practice to document as shown in Fig. 2.5. Six aspects of good practice are

1. List the name of the fluid.
2. List the temperature and pressure at which the property was reported by the source.
3. Cite the source of the fluid property.
4. List relevant assumptions.
5. List the value and units of the fluid property.
6. Be concise; write down the minimum information required to get the job done.

FIGURE 2.5

Recommended practices for documenting fluid properties.

SITUATION

Air is flowing from a large tank to ambient through a horizontal pipe.
Pipe is 1" Schedule 40. $D = 1.049$ in $= 0.0266$ m.
$V = 10$ m/s, $f = 0.015$, $L = 50$ m.

Assumptions:
Air has constant density (look up properties at 1 atm).
KE correction factor is $\alpha_2 = 1.0$.

Properties:
Air (20°C, 1 atm, Table A.3): $\rho = 1.20$ kg/m³.

GOAL
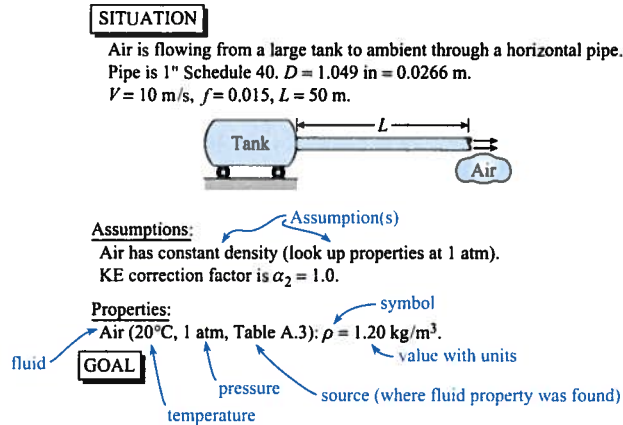
While looking up fluid properties, many details are important. The details are summarized Table 2.2 (page 53). As shown in the next example, Table 2.2 will be used throughout this chapte

EXAMPLE. Where are values of specific gravity (SG) tabulated? Does specific gravi depend on temperature, on pressure, or on both?

Solution. Table 2.2 shows that values of SG are tabulated in Table A.4. Table 2.2 indicat that SG goes down as temperature increases and that SG is constant with pressure.

2.5 Describing Viscous Effects

Viscous effects influence energy loss, drag force, flow separation, and other parameters of interest. Thus, this section introduces concepts that are useful for describing and characterizing viscous effects.

Viscosity

Viscosity, μ (mu), is the fluid property that characterizes resistance to flow.

> **EXAMPLE.** Fluids resist being forced to flow through pipes, so pumps are added to drive the flow through the pipe. For the same flow rate, a fluid with high viscosity (e.g., molasses) will require more power from a pump than a fluid with low viscosity (e.g., water).

> **EXAMPLE.** Fluids resist the motion of immersed objects through them. A small force will easily push a spoon through a bowl of water. This same force will barely move a spoon through a bowl of honey because the viscosity of honey is much higher than the viscosity of water.

Viscosity is also referred to as *absolute viscosity* and *dynamic viscosity*. Viscosity is defined mathematically as the ratio of shear stress to the rate of shear strain at a point.

$$\text{viscosity}(\mu) \equiv \frac{\text{shear stress}}{\text{rate of shear strain}} = \frac{\tau}{\left(\dfrac{dV}{dy}\right)} \qquad (2.9)$$

The symbols on the right side of Eq. (2.9) are described in the next subsections. Eq. (2.9) is called *The Viscosity Equation* in this textbook. Other engineering references call this equation *"Newton's Law of Viscosity."*

Shear Force and Shear Stress

Viscosity leads to forces that are analogous to frictional forces. For example, when fluid flows past a flat plate as shown in Fig. 2.6 the flowing fluid causes a drag force that is called the shear force.
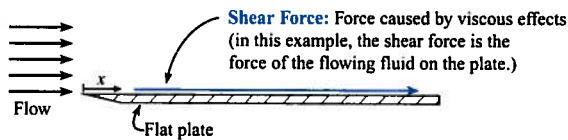

FIGURE 2.6

Viscosity causes a force that is called the shear force.

The shear force is a *distributed force* meaning that the force is spread out over an area. Because the force per unit area is not the same at each x location, a concept called shear stress is used. **Shear stress** is the ratio of tangential force to area at a point on a surface:
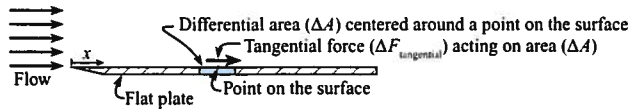
$$\text{shear stress}(\tau) \equiv \left(\frac{\text{tangential force}}{\text{surface area}}\right)_{\text{point on a surface}} = \lim_{\Delta A \to 0} \frac{\Delta F_{\text{tangential}}}{\Delta A} \qquad (2.10)$$

The terms in Eq. (2.10) are illustrated in Fig. 2.7.

Now that τ has been introduced, we can define **shear force** F_s as the net force on a body due to shear stress acting over the body.

FIGURE 2.7

This sketch shows terms
that appear in the
definition of shear stress.

Differential area (ΔA) centered around a point on the surface

Tangential force ($\Delta F_{tangential}$) acting on area (ΔA)
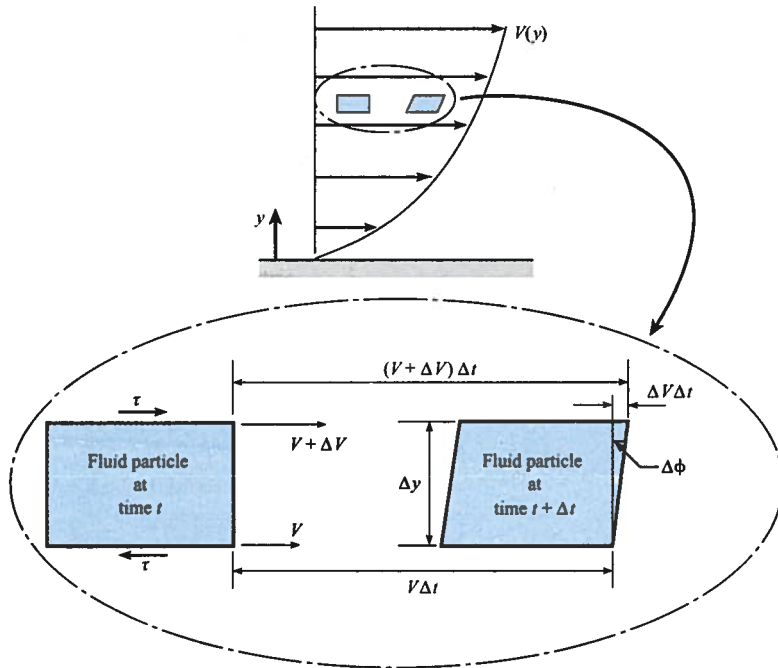
Flow

Flat plate

Point on the surface

Rate of Shear Strain

Viscosity causes a fluid particle to continuously change shape or deform. This deformation
illustrated in Fig. 2.8. As shown, a fluid particle that is rectangular at time t will deform so th
it is nonrectangular at time $t + \Delta t$. The deformation occurs because the fluid at the top of th
particle is moving faster than the fluid at the bottom of the particle. In particular the fluid
the top is moving with speed $V + \Delta V$, and the fluid at the bottom is moving with speed
This change in velocity over distance (called velocity gradient) is linked to deformation of th
fluid particle.

FIGURE 2.8

Depiction of strain caused by a shear stress (force per area) in a fluid. The rate
of strain is the rate of change of the interior angle of the original rectangle.

$V(y)$

$(V + \Delta V)\Delta t$

$\Delta V \Delta t$

$V + \Delta V$

Fluid particle
at
time t

Δy

Fluid particle
at
time $t + \Delta t$

$\Delta \phi$

V

$V \Delta t$

The **rate of shear strain** describes the change in an angle of a particle as a function of time. Th
mathematical definition is

$$\text{rate of shear strain} \equiv \lim_{\Delta t \to 0} \frac{\Delta \phi}{\Delta t} \qquad (2.1$$

where the angle $\Delta\phi$ is defined in the lower sketch of Fig. 2.8. To evaluate Eq. (2.11), use the sketch in Fig. 2.8 to write

$$\tan(\Delta\phi) = \frac{\Delta V \Delta t}{\Delta y} \tag{2.12}$$

In the limit as $\Delta t \to 0$, apply the small angle approximation to write

$$\tan(\Delta\phi) \approx \Delta\phi \tag{2.13}$$

Combine Eqs. (2.11) to (2.13) to give

$$\text{rate of shear strain} \equiv \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta y} = \frac{dV}{dy} \tag{2.14}$$

Eq. (2.14) shows that the rate of shear strain of a fluid particle is equal to the derivative of the velocity with respect to distance. The derivative on the right side of Eq. (2.14) is called the *velocity gradient*, which is the next topic.

The Velocity Profile

Viscous effects cause the velocity of a flowing fluid to vary with distance y as shown in Fig. 2.9. Notice that y measures distance from the wall. The variation of velocity with distance is called a **velocity profile**. The change in velocity is often called a **velocity gradient** because the gradient in mathematics describes a change in a variable with respect to distance.

Viscosity causes fluid near a wall to slow down, thereby creating the no-slip condition and a velocity profile. The velocity profile causes a fluid particle to experience shear stresses as shown.
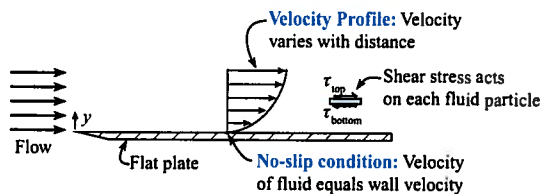
The presence of a velocity gradient indicates that shear stresses act on fluid particles.

EXAMPLE. Fig. 2.9 shows a fluid particle (blue shading) situated in a velocity profile. The fluid above this particle is moving fast relative to the particle, and this causes the particle to experience a force that acts to the right. Similarly, the fluid below the particle is moving slower, and this causes the particle to experience a force acting to the left. These forces can be represented as shear stresses.

The No-Slip Condition

In addition to causing shear stress, viscosity causes the **no-slip condition**, which is labeled in Fig. 2.9. The no-slip condition, originally deduced from experiments, tells us that the *velocity of fluid in contact with a solid will equal the velocity of the solid*. Therefore, in Fig. 2.9 the velocity

of the fluid at the surface of the plate will equal zero: $V(y = 0) = 0$. If a body is moving, for e ample a wing moving through the air, the velocity of the air at a point on the wing surface w equal the velocity of the wing at this same point.

Finding Values of Viscosity

Table 2.2 (page 53) summarizes information for looking up viscosity values. The followir example illustrates how to use this table.

EXAMPLE. Find the *dynamic viscosity* of air at a pressure of 5 atmospheres and a temper ture of 120°C. Assume that air at these conditions is an ideal gas.

Ideas/Plan. Table 2.2 (page 53) indicates that (a) viscosity can be looked up in Table A. and (b) viscosity will vary with temperature, but not pressure. Thus look up viscosity $T = 120°C$ and $p = 1$ atm.

Action. From Table A.3, the viscosity of air at $T = 120°C$ and $p = 1$ atm is $\mu = 2.26 \times 10$ $N \cdot s/m^2$.

✔ CHECKPOINT PROBLEM 2.1

An inventor is considering two lubricants: glycerin and SAE 10W-30 oil.
(a) Which lubricant has a higher viscosity at 150°F?
(b) Which lubricant has a higher viscosity at 230°F?

Kinematic Viscosity, ν

Kinematic viscosity, ν (nu), is a property that combines the viscous and the mass characteristi of a fluid. It is defined mathematically as the ratio of viscosity to density:

$$\nu \equiv \frac{\mu}{\rho} \quad \Rightarrow \quad \frac{N \cdot s/m^2}{kg/m^3} = m^2/s \quad \text{(2.1}$$

The reason that kinematic viscosity is defined as a property is that the ratio μ/ρ occu frequently in equations. Hence, researchers have identified it as a distinct property. shown in Eq. (2.15), the units of ν are m^2/s. The units can be helpful for distinguishir kinematic viscosity from viscosity. Be careful not to mix up μ and ν; they are differe properties!

EXAMPLE. Find the *kinematic viscosity* of water at a pressure of 5 atmospheres and a ter perature of 80°C.

Ideas/Plan. From Table 2.2 (page 53), the kinematic viscosity of water can be found Table A.5. Also, the kinematic viscosity of a liquid is independent of pressure.

Action. From Table A.5, the kinematic viscosity of water at $T = 80°C$ and $p = 1$ atm $\nu = 3.64 \times 10^{-7} m^2/s$.

✔ CHECKPOINT PROBLEM 2.2

What is the kinematic viscosity of nitrogen at 7 atmospheres of pressure (absolute) and a temperatur of 15°C?

2.6 Applying the Viscosity Equation

This section shows how to solve problems when shear stress is a parameter. The working equation is the viscosity equation, Eq. (2.9), which is usually written as

$$\tau = \mu \frac{dV}{dy} \tag{2.16}$$

The equation tells us that shear stress, τ, in a flowing fluid is linearly related to the velocity gradient (dV/dy). The constant of proportionality is the viscosity (μ). Terms in Eq. (2.16) are summarized in Table 2.1.

TABLE 2.1 Summary of the Viscosity Equation

Equation	Terms

$\tau = \mu \dfrac{dV}{dy}$ (2.16)	τ = shear stress (N/m²)
	
	 μ = viscosity (Pa · s)
	    (also called dynamic viscosity or absolute viscosity)
	
	 $\dfrac{dV}{dy}$ = velocity gradient (s⁻¹)
	
	    (also called the rate of shear strain)

One type of problem involves specifying two of the three variables in the viscosity equation and asking for the third variable. This case is illustrated in Example 2.1.

EXAMPLE 2.1

Applying the Viscosity Equation to Calculate Shear Stress in a Poiseuille Flow

Problem Statement

A famous solution in fluid mechanics, called Poiseuille flow, involves laminar flow in a round pipe (See Chapter 10 for details). Consider Poiseuille flow with a velocity profile in the pipe given by

$$V(r) = V_o(1 - (r/r_o)^2)$$

where r is radial position as measured from the centerline, V_o is the velocity at the center of the pipe, and r_o is the pipe radius. Find the shear stress at the center of the pipe, at the wall, and where $r = 1$ cm. The fluid is water (15°C), the pipe diameter is 4 cm, and $V_o = 1$ m/s.

Define the Situation

Water flows in a round pipe (Poiseuille Flow).
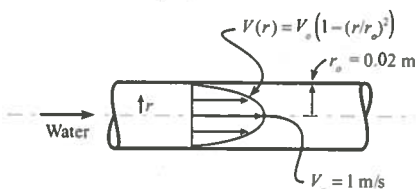
Water (15°C, 1atm, Table A.5): $\mu = 1.14 \times 10^{-3}$ N · s/m².

State the Goal

Calculate the shear stress at three points:

$\tau(r = 0.00$ m) (N/m²) ← pipe centerline
$\tau(r = 0.01$ m) (N/m²) ← middle of the pipe
$\tau(r = 0.02$ m) (N/m²) ← the wall

Generate Ideas and Make a Plan

Because the goal is τ, select the *Viscosity Equation* (Eq. 2.16).

Let the position variable be r instead of y.

$$\tau = -\mu \frac{dV}{dr} \tag{(}$$

Regarding the minus sign in Eq. (a), the y in the viscosity equation is measured from the wall (see Fig. 2.9) The coordinate r is in the opposite direction. The sign change occurs when the variable is changed from y to r.

To find μ, use Table 2.2 on page 53 to identify that

• Viscosity of water at 15°C can be found in Table F.5.
• Viscosity of a liquid is independent of pressure.

To find the velocity gradient in Eq. (a), differentiate the given velocity profile.

$$\frac{dV(r)}{dr} = \frac{d}{dr}(V_o(1 - (r/r_o)^2)) = \frac{-2V_o r}{r_o^2} \tag{(}$$

Now, the goal can be found. **Plan.** Apply Eq. (b) to find the velocity gradient. Then, substitute into Eq. (a).

Take Action (Execute the Plan)

1. Viscosity Equation ($r = 0$ m)

$$\frac{dV(r)}{dr}\bigg|_{r=0\,m} = \frac{-2V_o(0\,m)}{r_o^2} = \frac{-2(1\,m/s)(0\,m)}{(0.02\,m)^2} = 0.0\,s^{-1}$$

$$\tau(r = 0\,m) = -\mu\frac{dV(r)}{dr}\bigg|_{r=0\,m}$$

$$= (1.14 \times 10^{-3}\,N \cdot s/m^2)(0.0\,s^{-1})$$

$$= \boxed{0.0\,N/m^2}$$

2. Viscosity Equation ($r = 0.01$ m)

$$\frac{dV(r)}{dr}\bigg|_{r=0.01\,m} = \frac{-2V_o(0.01\,m)}{r_o^2}$$

$$\frac{-2(1\,m/s)(0.01\,m)}{(0.02\,m)^2} = -50\,s^{-1}$$

Next, calculate shear stress.

$$\tau(r = 0.01\,m) = -\mu\frac{dV(r)}{dr}\bigg|_{r=0.01\,m}$$

$$= (1.14 \times 10^{-3}\,N \cdot s/m^2)(50\,s^{-1})$$

$$= \boxed{0.0570\,N/m^2}$$

3. Viscosity Equation ($r = 0.02$ m)

$$\frac{dV(r)}{dr}\bigg|_{r=0.02\,m} = \frac{-2V_o(0.02\,m)}{r_o^2}$$

$$= \frac{-2(1\,m/s)(0.02\,m)}{(0.02\,m)^2} = -100\,s^{-1}$$

Next, calculate shear stress.

$$\tau(r = 0.02\,m) = -\mu\frac{dV(r)}{dr}\bigg|_{r=0.02\,m}$$

$$= (1.14 \times 10^{-3}\,N \cdot s/m^2)(100\,s^{-1})$$

$$= \boxed{0.114\,N/m^2}$$

Review the Solution and the Process

1. *Tip.* On most problems, including this example, carrying and canceling units is useful, if not critical.

2. *Notice.* Shear stress varies with location. For this example, τ is zero on the centerline of the flow and nonzero everywhere else. The maximum value of shear stress occurs at the wall of the pipe.

3. *Notice.* For flow in a round pipe, the viscosity equation has a minus sign and uses the position coordinate r.
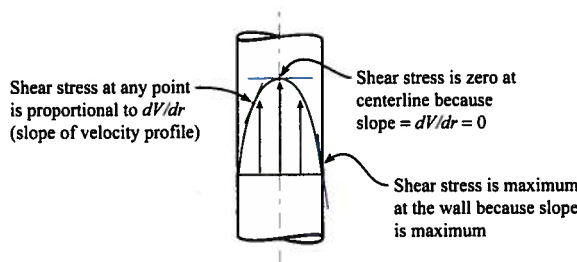
$$\tau = -\mu\frac{dV}{dr}$$

Example 2.1 showed that the *magnitude of shear stress is proportional the velocity gradien* This idea is illustrated in Fig. 2.10. Notice that the figure is drawn vertically so that the variab V is upward and the variable r is horizontal. This was done so that a slope of zero is a horizon tal line and an infinite slope is a vertical line. Another way to show the relationship betwee slope and shear stress is an equation.

$$\tau\uparrow = \mu\frac{dV}{dr}\uparrow \tag{2.1}$$

Eq. (2.17) means that if dV/dy increases, then shear stress τ will increase. Or, that if $dV/$ decreases then τ will decrease.

FIGURE 2.10

The velocity profile from example 2.1. This figure shows that one can make qualitative predictions of the shear stress by examining the slope of the velocity profile.

Shear stress at any point is proportional to dV/dr (slope of velocity profile)

Shear stress is zero at centerline because slope = dV/dr = 0

Shear stress is maximum at the wall because slope is maximum

A second category of problems involves a type of flow, called Couette Flow. In Couette flo as shown in Fig. 2.11, a moving plate causes fluid to flow. Because of the *no-slip condition*, th velocity of the fluid at the top is equal to the velocity of the moving plate. Similarly, the velocity

the fluid at the bottom is zero because the bottom plate is stationary. In the region between the plates, the velocity profile is linear. Additional details about Couette flow are presented in Chapter 9.

When the viscosity equation (Eq. 2.16) is applied to the Couette flow that is shown in Fig. 2.11, the derivative can be replaced with a ratio because the velocity gradient is linear.

$$\tau = \mu\frac{dV}{dy} = \mu\frac{\Delta V}{\Delta y} \tag{2.18}$$

The terms on the right side of Eq. (2.18) can be analyzed as follows

$$\tau = \mu\frac{\Delta V}{\Delta y} = \mu\frac{V_o - 0}{H - 0} = \mu\frac{V_o}{H}$$

Rewrite the equation to give a form of the *Viscosity Equation* that is useful for Couette flow.

$$\tau\Big|_{\text{Couette Flow}} = \text{constant} = \mu\frac{V_o}{H} \tag{2.19}$$

Eq. (2.19) reveals that the shear stress at all points in a Couette flow is constant with a magnitude of $\mu V_o/H$. Example 2.2 presents a typical problem that involves Couette flow.
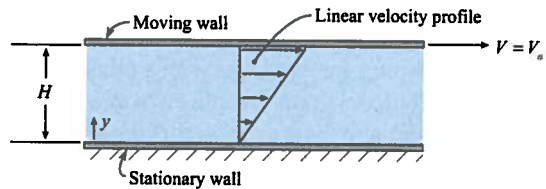
FIGURE 2.11
Couette flow is a flow t is driven by a moving v The velocity profile in th fluid is linear.

EXAMPLE 2.2

Applying the Viscosity Equation to Couette Flow

Problem Statement

A board 1 m by 1 m that weighs 25 N slides down an inclined ramp (slope = 20°) with a constant velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 N · s/m². Assuming that the oil can be modeled as a Couette flow, calculate the space between the board and the ramp.

Define the Situation

A board slides down an oil film on a inclined plane.
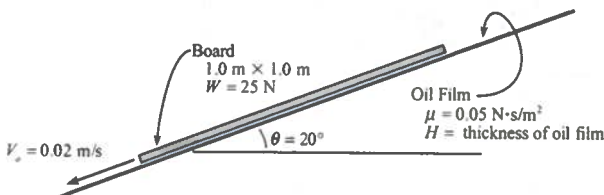
**Assumptions.** (1) Couette flow. (2) Board has constant speed.

State the Goal

H(mm) ⟵ Thickness of the film of oil

Generate Ideas and Make a Plan

Because the goal is H, apply the *Viscosity Equation* (Eq. 2.19):

$$H = \mu\frac{V_o}{\tau} \tag{(}$$

To find the shear stress τ in Eq. (a), draw a *Free Body Diagram* (FBD) of the board. In the FBD, W is the weight, N is the norma force, and F_{shear} is shear force. Because shear stress is constant with x, the shear force can be expressed as $F_{\text{shear}} = \tau A$.
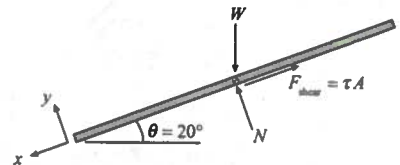
Because the board moves at constant speed, the forces are in balance. Thus, apply *force equilibrium*.

$$\Sigma F_x = 0 = W\sin\theta - \tau A \tag{(}$$

Rewrite Eq. (b) as

$$\tau = (W\sin\theta)/A \tag{(}$$

Eq. (c) can be solved for τ. The plan is

1. Calculate τ using Force Equilibrium (Eq. c).
2. Calculate H using the Shear Stress Equation (Eq. a).

Take Action (Execute the Plan)

1. Force equilibrium

$$\tau = (W\sin\theta)/A = (25\text{ N})(\sin 20°)/(1.0\text{ m}^2) = 8.55\text{ N/m}^2$$

2. Shear stress equation

$$H = \mu\frac{V_o}{\tau} = (0.05\text{ N}\cdot\text{s/m}^2)\frac{(0.02\text{ m/s})}{(8.55\text{ N/m}^2)} = \boxed{0.117\text{ mm}}$$

Review the Solution and the Process

1. H is about 12% of a millimeter; this is quite small.
2. *Tip.* Solving this problem involved drawing an FBD. The FBD is useful for most problems involving Couette flow.

2.7 Characterizing Viscosity

This section presents ideas about developing equations for viscosity as a function of temperature. This section also introduces the non-Newtonian fluid.

Temperature Effects

The viscosity of a gas increases with a temperature rise. In comparison, the viscosity of liquid decreases. To understand the influence of a temperature change on a liquid, it is helpful to rely on an approximate theory (3). In this theory, the molecules in a liquid form a latticelike structure with "holes" where there are no molecules, as shown in Fig. 2.12. Even when the liquid is at rest, the molecules are in constant motion, but confined to cells, or "cages." The cage or lattice structure is caused by attractive forces between the molecules. The cages may be thought of as energy barriers. When the liquid is flowing, there is a shear stress, τ, imposed by one layer on another in the fluid. This shear stress assists a molecule in overcoming the energy barrier that it can more easily move into the next hole. At a higher temperature the size of the energy barrier is smaller, and it is easier for molecules to make the jump, so that the net effect is that the fluid has a a lower viscosity.

FIGURE 2.12

Visualization of molecules in a liquid.
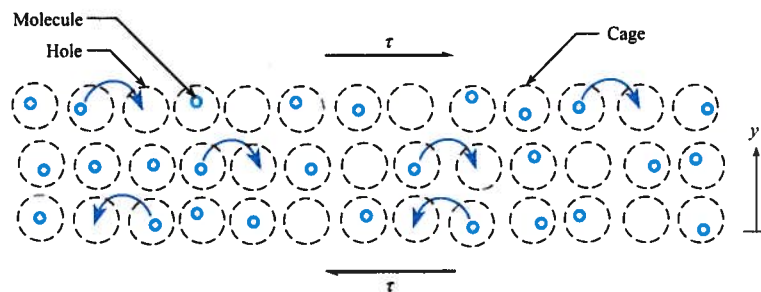
An equation for the variation of liquid viscosity with temperature is

$$\mu = Ce^{b/T}$$

$$(2.\text{?})$$

where C and b are empirical constants that require viscosity data at two temperatures for evaluation. This equation should be used primarily for data interpolation.

The variation of viscosity (dynamic and kinematic) for other fluids is given in Figs. A.2 and A.3. Example 2.3 shows how to find the constants that appear in Eq. (2.20).

EXAMPLE 2.3

Developing an Algebraic Equation for Viscosity of a Liquid as a Function of Temperature

Problem Statement

The dynamic viscosity of water at 20°C is $1.00 \times 10^{-3}\,N \cdot s/m^2$, and the viscosity at 40°C is $6.53 \times 10^{-4}\,N \cdot s/m^2$. Using Eq. (2.20), estimate the viscosity at 30°C.

Define the Situation

An equation for the viscosity of water can be found by using Eq. (2.20) plus the following two data points:
- Water (293 K, 1 atm): $\mu = 1.00 \times 10^{-3}\,N \cdot s/m^2$.
- Water (313 K, 1 atm): $\mu = 6.53 \times 10^{-4}\,N \cdot s/m^2$.

State the Goal

1. Find an equation for $\mu(T)$ for water.
2. Calculate the viscosity of water at $T = 303\ K = 30°C$.

Generate Ideas and Make a Plan

Because Eq. (2.20) has two unknown constants (C and b), and there are two known values for viscosity, the two constants can be found. The plan is

1. Linearize Eq. (2.20) by taking the logarithm.
2. Plug values of μ and T into the linearized equation.
3. Solve for C and b using the linear equations from step 2.
4. Find the equation for $\mu(T)$ and then find $\mu(T = 303\ K)$.

Take Action (Execute the Plan)

1. Take logrithms of both sides of Eq. (2.20)

$$\ln \mu = \ln C + b/T$$

2. Plug data points into Eq. (a)

$$-6.908 = \ln C + 0.00341\,b$$
$$-7.334 = \ln C + 0.00319\,b$$

3. Solve equations in step 2 for C and b

$$\ln C = -13.51 \qquad b = 1936\ K$$
$$C = e^{-13.51} = 1.357 \times 10^{-6}\,(N \cdot s/m^2)$$

4. Substitute C and b into Eq. (2.20)

$$\boxed{\mu(T) = (1.357 \times 10^{-6}\,N \cdot s/m^2)e^{(1936\ K)/T}}$$
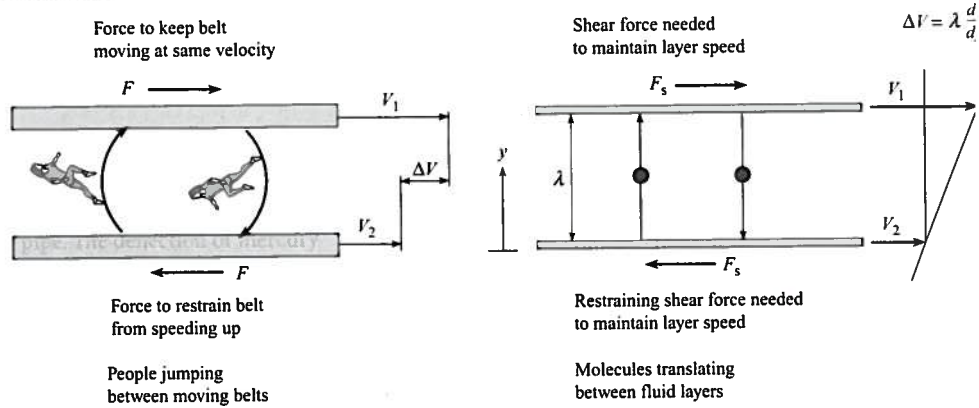
Solve for viscosity at 303 K.

$$\mu = \boxed{8.08 \times 10^{-4}\,N \cdot s/m^2}$$

Review the Solution and the Process

1. *Validation.* The calculated value can be checked by comparing to data in Table A.5. The result differs by 1% from the table value.

2. *Tip.* This solution required absolute temperature units of Kelvin (K). Some problems cannot be solved correctly if one uses temperature units in Celsius (°C).

3. *Tip.* Notice how units were applied in this solution. Using units is good engineering practice.

As compared to liquids, gases do not have zones or cages to which molecules are confined by intermolecular bonding. Gas molecules are always undergoing random motion. If this random motion of molecules is superimposed on two layers of gas, where the top layer is moving faster than the bottom layer, periodically a gas molecule will randomly move from one layer to the other. This behavior of a molecule in a low-density gas is analogous to people jumping back and forth between two conveyor belts moving at different speeds, as shown in Fig. 2.13. When people jump from the high-speed belt to the low-speed belt, a restraining (or braking) force has to be applied to slow the person down (analogous to viscosity). If the people are heavier, or are moving faster, a greater braking force must be applied. This analogy also applies for gas molecules translating between fluid layers where a shear force is needed to maintain the layer speeds. As the gas temperature increases, more of the molecules will be making random jumps. Just as the jumping person causes a braking action on the belt, highly mobile gas molecules have momentum, which must be resisted by the layer to which the molecules jump. Therefore, as the temperature increases, the viscosity, or resistance to shear, also increases.

FIGURE 2.13

Analogy of people
moving between conveyor
belts and gas molecules
translating between fluid
layers.

Force to keep belt
moving at same velocity

Force to restrain belt
from speeding up

People jumping
between moving belts

Shear force needed
to maintain layer speed

Restraining shear force needed
to maintain layer speed

Molecules translating
between fluid layers

An estimate for the variation of gas viscosity with temperature is **Sutherland's equation**

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S}$$

(2.2)

where μ_0 is the viscosity at temperature T_0, and S is Sutherland's constant. All temperatures a
absolute. Sutherland's constant for air is 111 K; values for other gases are given in Table A
Using Sutherland's equation for air yields viscosities with an accuracy of $\pm 2\%$ for temperatu
between 170 K and 1900 K. In general, the effect of pressure on the viscosity of common ga
is minimal for pressures less than 10 atmospheres.

Newtonian versus non-Newtonian Fluids

Fluids for which the shear stress is directly proportional to the rate of strain are called **Ne
tonian fluids**. Because shear stress is directly proportional to the shear strain, dV/dy, a p
relating these variables (see Fig. 2.14) results in a straight line passing through the orig
The slope of this line is the value of the dynamic (absolute) viscosity. For some fluids t

FIGURE 2.14

Shear stress relations for
different types of fluids.
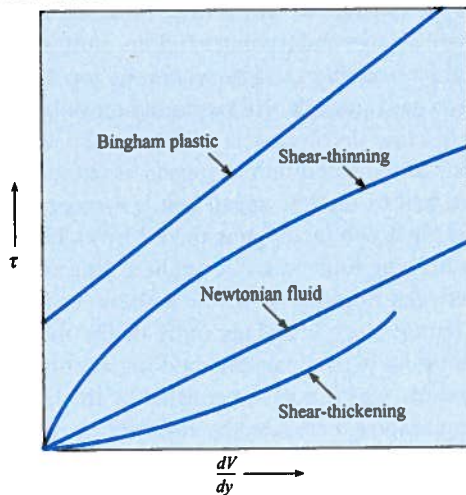
shear stress may not be directly proportional to the rate of strain; these are called non-Newtonian fluids. One class of non-Newtonian fluids, shear-thinning fluids, has the interesting property that the ratio of shear stress to shear strain decreases as the shear strain increases (see Fig. 2.14). Some common shear-thinning fluids are toothpaste, catsup, paints, and printer's ink. Fluids for which the viscosity increases with shear rate are shear-thickening fluids. Some examples of these fluids are mixtures of glass particles in water and gypsum-water mixtures. Another type of non-Newtonian fluid, called a Bingham plastic, acts like a solid for small values of shear stress and then behaves as a fluid at higher shear stress. The shear stress versus shear strain rate for a Bingham plastic is also shown in Fig. 2.14.

In general, non-Newtonian fluids have molecules that are more complex than Newtonian fluids. Thus, if you are working with a fluid that may be non-Newtonian, consider doing some research. The reason is that many of the equations and math models presented in textbooks (including this one) only apply to Newtonian fluids.

To learn more about non-Newtonian fluids, watch the film entitled *Rheological Behavior of Fluids* (4). For more information on the theory of flow of non-Newtonian fluids, see references (5) and (6).

2.8 Characterizing Surface Tension*

Engineers need to be able to predict and characterize surface tension effects because they affect many industrial problems. Some examples of surface tension effects:

- *Wicking.* Water will wick into a paper towel. Ink will wick into paper. Polypropylene, an excellent fiber for cold-weather aerobic activity, wicks perspiration away from the body.
- *Capillary Rise.* A liquid will rise in a small-diameter tube. Water will rise in soil.
- *Capillary Instability.* A liquid jet will break up into drops.
- *Drop and Bubble Formation.* Water on a leaf beads up. A leaky faucet drips. Soap bubbles form.
- *Excess Pressure:* The pressure inside a water drop is higher than ambient pressure. The pressure inside a vapor bubble during boiling is higher than ambient pressure.
- *Walking on Water.* The water strider, an insect, can walk on water. Similarly, a metal paper clip or a metal needle can be positioned to float (through the action of surface tension) on the surface of water.
- *Detergents.* Soaps and detergents improve the cleaning of clothes because they lower the surface tension of water so that the water can more easily wick into the pores of the fabric.

Many experiments have shown that the surface of liquid behaves like a stretched membrane. The material property that characterizes this behavior is surface tension, σ (sigma). Surface tension can be expressed in terms of force:

$$\text{Surface Tension}(\sigma) = \frac{\text{Force along an interface}}{\text{Length of the interface}} \qquad \text{(2.22)}$$

*The authors acknowledge and thank Dr. Eric Aston for his feedback and inputs on this section. Dr. Aston is a Chemical Engineering Professor at the University of Idaho.

Surface tension can also be expressed in terms of the energy:

$$\text{Surface Tension}(\sigma) = \frac{\text{Energy required to increase the surface area of a liquid}}{\text{Unit area}} \tag{2.2}$$

From Eq. (2.22), the units of surface tension are newton per meter (N/m). Surface tensic typically has a magnitude ranging from 1 to 100 mN/m. Units of surface tension can also l joule per meter squared (J/m²) because:

$$\frac{N}{m} = \frac{N \cdot m}{m \cdot m} = \frac{J}{m^2}$$

The physical mechanism of surface tension is based on **cohesive force**, which is the attra tive force between like molecules. Because liquid molecules attract one another, molecules the interior of a liquid (see Fig. 2.15) are attracted equally in all directions. In contrast, mol cules at the surface are pulled toward the center because they have no liquid molecules abo them. This pull on surface molecules draws the surface inward and causes the liquid to seek minimize surface area. This is why a drop of water draws into a spherical shape.

FIGURE 2.15

Forces between molecules in a liquid.

A molecule at the surface is pulled toward the center

A molecule in the interior is pulled equally in all directions
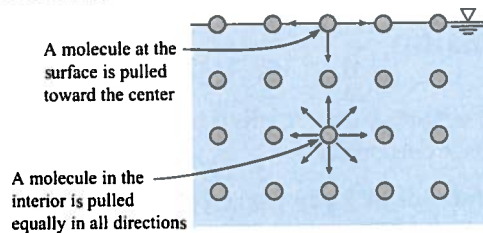
Surface tension of water decreases with temperature (see Fig. 2.16) because thermal expa sion moves the molecules farther apart, and this reduces the average attractive force betwe molecules (i.e., cohesive force goes down). Surface tension is strongly affected by the presence contaminants or impurities. For example, adding soap to water decreases the surface tension. T reason is that impurities concentrate on the surface, and these molecules decrease the avera attractive force between the water molecules. As shown in Fig. 2.16, the surface tension of wa at 20°C is $\sigma = 0.0728 \approx 0.073$ N/m. This value is used in many of the calculations in this tex

FIGURE 2.16

Surface tension of water for a water/air interface. Property values from White (7).

$\sigma = 0.0728$ N/m at $T = 20$°C
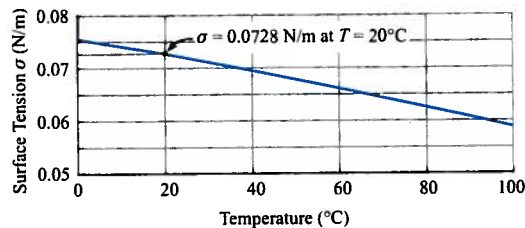
In Fig. 2.16, surface tension is reported for an interface of air and water. It is comm practice to report surface tension data based on the materials that were used during the me surement of the surface tension data.

To learn more about surface tension, we recommend the online film entitled *Surface T sion in Fluid Mechanics* (8) and Shaw's book (9).

Example Problems

Most problems involving surface tension are solved by drawing a free body diagram and applying force equilibrium. The force due to surface tension, from Eq. (2.22), is

$$\text{Force due to surface tension} = F_\sigma = \sigma L \qquad (2.24)$$

where L is the length of a line that lies along the surface of the liquid. The use of force equilibrium to solve problems is illustrated in Examples 2.4 and 2.5.

EXAMPLE 2.4

Applying Force Equilibrium to Calculate the Pressure Rise inside a Water Droplet.

Problem Statement

The pressure inside a water drop is higher than the pressure of the surroundings. Derive a formula for this pressure rise. Then, calculate the pressure rise for a 2-mm-diameter water drop. Use $\sigma = 73$ mN/m.

Define the Situation

Pressure inside a water drop is larger than ambient pressure. $d = 0.002$ m, $\sigma = 73$ mN/m.

Water drop
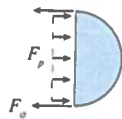p_i = pressure inside the drop
d
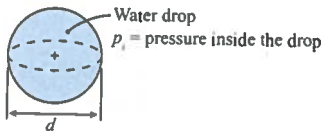
State the Goal

1. Derive an equation for p_i.
2. Calculate p_i in pascals.

Generate Ideas and Make a Plan

Because pressure is involved in a force balance, draw a *Free Body Diagram* (FBD) of the drop.

Force due to pressure = Force due to surface tension
$$F_p = F_\sigma \qquad (a)$$

From Eq. (2.24), the surface tension force is σ times the length of the interface:

$$F_\sigma = \sigma L = \sigma \pi d \qquad ($$

The pressure force is pressure times area:

$$F_p = p_i \frac{\pi d^2}{4} \qquad ($$

Combine Eqs. (a) to (c):

$$p_i \frac{\pi d^2}{4} = \sigma \pi d \qquad ($$

Solve for pressure

$$p_i = \frac{4\sigma}{d} \qquad ($$

The first goal (equation for pressure) has been attained. The next goal (value of pressure) can be found by substituting numbers into Eq. (e).

Take Action (Execute the Plan)

$$p_i = \frac{4\sigma}{d} = \frac{4(0.073 \text{ N/m})}{(0.002 \text{ m})} = \boxed{146 \text{ Pa gage}}$$

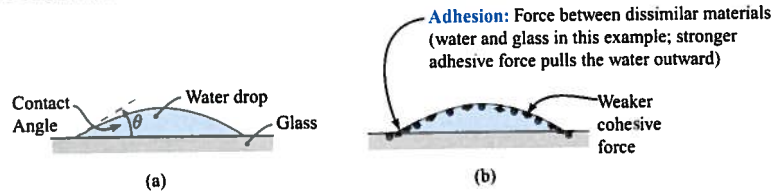#### Review the Results and the Process

1. *Notice.* The answer is expressed as gage pressure. Gage pressure in this context is the pressure rise above ambient.

2. *Physics.* The pressure rise inside a liquid drop is a consequence of the membrane effect of surface tension. One way to visualize this is make an analogy with a balloon filled with air. The pressure inside the balloon pushes outward against the membrane force of the rubber skin. In the same way, the pressure inside a liquid drop pushes outward against the membrane like force of surface tension.

Adhesion and Capillary Action

When a drop of water is placed on glass, the water will wet the glass (see Fig. 2.17) because water is strongly attracted to glass. This attractive force pulls the water outward as shown. The force between dissimilar surfaces is called **adhesion** (see Fig. 2.17b). Water will "wet out" on a surface when *adhesion is greater than cohesion*.
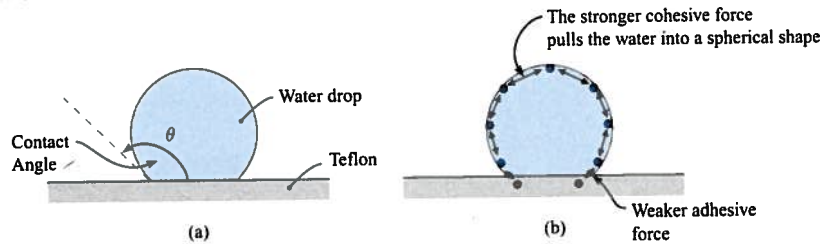
FIGURE 2.17

Water wets glass because adhesion is greater than cohesion. Wetting is associated with a contact angle less than 90°.

Contact Angle — Water drop — Glass

Adhesion: Force between dissimilar materials (water and glass in this example; stronger adhesive force pulls the water outward)

Weaker cohesive force

(a)   (b)

On some surfaces such as Teflon and wax paper, a drop of water will bead up (Fig 2.1 because *adhesion between the water and teflon is less than cohesion of the water*. A surface which water beads up is called *hydrophobic* (water hating). Surface such as glass in which wat drops spread out are called *hydrophillic* (water loving).

FIGURE 2.18

Water beads up a hydrophobic material such as Teflon because adhesion is less than cohesion. A nonwetting surface is associated with a contact angle greater than 90°.

The stronger cohesive force pulls the water into a spherical shape

Contact Angle — Water drop — Teflon

Weaker adhesive force

(a)   (b)

Capillary action describes the tendency of a liquid to rise in narrow tubes or to be drawn in small openings. Capillary action is responsible for water being drawn into the narrow ope ings in soil or into the narrow openings between the fibers of a dry paper towel.

When a capillary tube is placed into a container of water, the water rises up the tu (Fig 2.19) because the adhesive force between the water and the glass pulls the water up tube. This is called capillary rise. Notice how the contact angle for the water is the same in Fi 2.17 and 2.19. Alternatively, when a fluid is nonwetting such as mercury on glass, then liquid will display capillary repulsion.
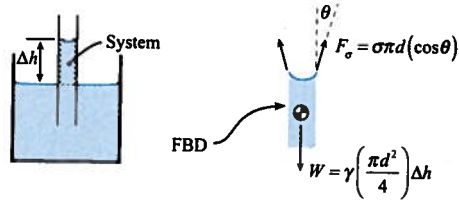
FIGURE 2.19

Water will rise up a glass tube (capillary rise), whereas mercury will move downward (capillary repulsion).

Capillary rise (water and glass) — Glass tube — Capillary repulsion (mercury and glass)
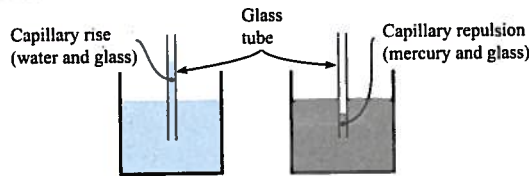
To derive an equation for capillary rise (see Fig. 2.20), define a system comprised of the wa inside the capillary tube. Then, draw a free body diagram (FBD). As shown, the pull of surf tension is lifting the column of water. Applying force equilibrium gives

$$\text{Weight} = \text{Surface Tension Force}$$

$$\gamma\left(\frac{\pi d^2}{4}\right)\Delta h = \sigma \pi d \cos \theta$$

(2.

FIGURE 2.20

Sketches used for deriv
an equation for capillc
rise.

Assume the contact angle is nearly zero so cos θ ≈ 1.0. Note that this is a good assumption for a water/glass interface. Eq. (2.25) simplifies to

$$\Delta h = \frac{4\sigma}{\gamma d} \qquad (2.26)$$

EXAMPLE. Calculate the capillary rise for water (20°C) in a glass tube of diameter $d = 1.6$ mm.

Solution. From Table A.5, $\gamma = 9790$ N/m³. From Fig. 2.16, $\sigma = 0.0728$. Now, calculate capillary rise using Eq. (2.26):

$$\Delta h = \frac{4(0.0728 \text{ N/m})}{(9790 \text{ N/m}^3)(1.6 \times 10^{-3} \text{ m})} = \boxed{18.6 \text{ mm}}$$

✔ **CHECKPOINT PROBLEM 2.3**

Two capillary tubes are placed in a liquid. The diameter of tube A is twice the diameter of tube B. Which statement is true?

a. Capillary rise in both tubes is the same.

b. Capillary rise in tube A is twice that of tube B.

c. Capillary rise in tube B is twice that of tube A.

d. None of the above.

Example 2.5 shows a case involving a non-wetting surface.

EXAMPLE 2.5

Applying Force Equilibrium to Determine the Size of a Sewing Needle That Can Be Supported by Surface Tension

Problem Statement

The Internet shows examples of sewing needles that appear to be "floating" on top of water. This effect is due to surface tension supporting the needle. Determine the largest diameter of sewing needle that can be supported by water. Assume that the needle material is stainless steel with $SG_{ss} = 7.7$.

Define the Situation

A sewing needle is supported by the surface tension of a water surface.

Needle (stainless steel)
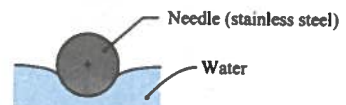
Water

Assumptions

• Assume the sewing needle is a cylinder.
• Neglect end effects.

Properties

• Water (20°C, 1 atm, Fig. 2.16): $\sigma = 0.0728$ N/m
• Water (4°C, 1 atm, Table F.6): $\gamma_{H2O} = 9810$ N/m^3
• SS: $\gamma_{ss} = (7.7)(9810$ N/m$^3) = 75.5$ kN/m^3

State the Goal

d(mm) ◄■ Diameter of the largest needle that can be supported by the water.

Generate Ideas and Make a Plan

Because the weight of the needle is supported by the surface tension force, draw a *Free Body Diagram* (FBD). Select a system comprised of the needle plus the surface layer of the water. The FBD is
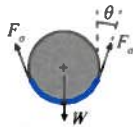
Apply force equilibrium.

Force due to surface tension = Weight of needle

$$F_\sigma = W \tag{a}$$

From Eq. (2.24)

$$F_\sigma = \sigma 2L \cos \theta \tag{b}$$

where L is the length of the needle. The weight of the needle is

$$W = \left(\frac{weight}{volume}\right)[volume] = \gamma_{ss}\left[\left(\frac{\pi d^2}{4}\right)L\right] \tag{c}$$

Combine Eqs. (a), (b), and (c). Also, assume the angle θ is zero because this gives the maximum possible diameter:

$$\sigma 2L = \gamma_{ss}\left(\frac{\pi d^2}{4}\right)L \tag{d}$$

Plan. Solve Eq. (d) for d and then plug numbers in.

Take Action (Execute the Plan)

$$d = \sqrt{\frac{8\sigma}{\pi\gamma_{ss}}} = \sqrt{\frac{8(0.0728 \text{ N/m})}{\pi(75.5 \times 10^3 \text{ N/m}^3)}} = \boxed{1.57 \text{ mm}}$$

Review the Solution and the Process

Notice. When applying specific gravity, look up water properties at the reference temperature of 4°C.

2.9 Predicting Boiling Using Vapor Pressure

A liquid, even at a low temperature, can boil as it flows through a system. This boiling can duce performance and damage equipment. Thus, engineers need to be able to predict wh boiling will occur. This prediction is based on the vapor pressure.

Vapor pressure, p_v(kPa), is the pressure at which the liquid phase and the vapor phase a material will be in thermal equilibrium. Vapor pressure is also called *saturation pressure*, a the corresponding temperature is called *saturation temperature*.

Vapor pressure can be visualized on a *phase diagram*. A phase diagram for water is shown Fig. 2.21. As shown, water will exist in the liquid phase for any combination of temperature a pressure that lies above the blue line. Similarly, the water will exist in the vapor phase for poi below the blue line. Along the blue line, the liquid and vapor phases are in thermal equilibriu When boiling occurs, the pressure and temperature of the water will be given by one of the poi on the blue line. In addition to Fig. 2.21, data for vapor pressure of water are tabulated in Table /

FIGURE 2.21

A phase diagram for water.
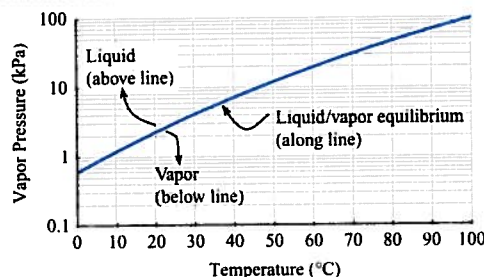
EXAMPLE. Water at 20°C flows through a venturi nozzle and boils. Explain why. Also, give the value of pressure in the nozzle.

Solution. The water is boiling because the pressure has dropped to the vapor pressure. Table 2.2 (page 53) indicates that p_v can be looked up in Table A.5. Thus, the vapor pressure at 20°C (Table A.5) is $p_v = 2.34$ kPa absolute. This value can be validated by using Fig. 2.21.

Review. Vapor pressure is commonly expressed used *absolute pressure*. Absolute pressure is the value of pressure as measured relative to a pressure of absolute zero.

2.10 Characterizing Thermal Energy in Flowing Gases

Engineers characterize thermal energy changes using properties introduced in this section. Thermal energy is the energy associated with molecules in motion. This means that thermal energy is associated with temperature change (sensible energy change) and phase change (latent energy change). For most fluids problems, thermal properties are not important. However, thermal properties are used for compressible flow of gases (Chapter 12).

Specific Heat, c

Specific heat characterizes the amount of thermal energy that must be transferred to a unit mass of substance to raise its temperature by one degree. The dimensions of specific heat are energy per unit mass per degree temperature change, and the corresponding units are J/kg · K.

The magnitude of c depends on the process. If a gas is heated at *constant volume*, less energy is required than if the gas is heated at *constant pressure*. This is because a gas that is heated at constant pressure must do work as it expands against its surroundings.

The *constant volume specific heat*, c_v, applies to a process carried out at constant volume. The *constant pressure specific heat*, c_p, applies to process carried out at constant pressure. The ratio c_p/c_v is called the **specific heat ratio** and given the symbol k. Values for c_p and k for various gases are given in Table A.2.

Internal Energy

Internal energy includes all the energy in matter except for the kinetic energy and potential energy. Thus, internal energy includes multiple forms of energy such as chemical energy, electrical energy, and thermal energy. Specific internal energy, u, has dimensions of energy per unit mass. The units are J/kg.

Enthalpy

When a material is heated at constant pressure, the energy balance is

$$(\text{energy added}) = \left(\begin{array}{c}\text{energy to increase}\\\text{thermal energy}\end{array}\right) + \left(\begin{array}{c}\text{energy to do work}\\\text{as the material expands}\end{array}\right)$$

The work term is needed because the material is exerting a force over a distance as it pushes its surroundings away during the process of thermal expansion.

Enthalpy is a property that characterizes the amount of energy associated with a constant temperature heating or cooling process. Enthalpy per unit mass is defined mathematically by

$$(\text{enthalpy}) = (\text{internal energy}) + (\text{pressure over density})$$
$$h = u + p/\rho$$

Ideal Gas Behavior

For an ideal gas, the properties h, u, c_p, and c_v depend only on temperature, not on pressure

2.11 Summarizing Key Knowledge

Systems and Associated Concepts

- The *system* is the matter that the engineer selects for study.
- The *surroundings* are everything else that is not part of the system.
- The *boundary* is the surface that separates the system from its surroundings.
- The *state* of a system is the condition of the system as specified by values of the propertie of the system.
- A *process* is a change of a system from one state to another.
- A *property* is a measurable characteristic of a system that depends only on the presen state.

Constant Density Assumption

- All fluids, including liquids, are compressible.
- Modeling a fluid as *constant density* means that one assumes the density is constant with position and time. *Variable density* means the density can change with position or time.
- Modeling a fluid as incompressible means one assumes that the density of each fluid particle is constant.
- Most fluid problems are idealized as constant density problems. A notable exception is th high speed flow of gases.
- A gas should be modeled as compressible when the Mach number is greater than 0.3.

Viscosity Concepts

- *Viscosity* μ is also called dynamic viscosity or absolute viscosity.
- Viscosity is related to *kinematic viscosity* by $v = \mu/\rho$. Viscosity and kinematic are differen properties.
- The *velocity profile* is a plot or equation that shows how velocity varies with position.
- The *no-slip condition* means that the velocity of fluid in contact with a solid surface will equal the velocity of the surface.
- The *shear force*, F_s is the net force due to shear stress.
- *Shear stress* τ is the tangential force per area at a point.
- A *Newtonian liquid* is one in which a plot of τ versus dV/dy is a straight line.
- A *non-Newtonian liquid* has a stress-strain relationship that is nonlinear. In general non-Newtonian liquids have more complex molecular structures than Newtonian fluids; examples of non-Newtonian liquids include paint, toothpaste, and molten plastics.
- Equations developed for Newtonian fluids (i.e., many textbook equations) do not apply t non-Newtonian fluids.

The Viscosity Equation

The viscosity equation (Eq. 2.16) relates shear stress to velocity gradient. The equation is

$$\tau = \mu \frac{dV}{dy}$$

Terms in the viscosity equation are summarized in Table 2.1 (page 39). Problems that are solvable with the viscosity equation can be classified into two categories.

- *Direct Calculations Problems.* Problems in this category specify two of the three variables in the equation and ask for the third variable. See Example 2.1 on page 39.
- *Couette Flow Problems.* Problems in this category involve a linear velocity profile in a small gap. See Example 2.2 on page 41.

Miscellaneous Topics

- A liquid flowing in a system can boil when the pressure drops to the vapor pressure. This boiling is typically detrimental to a design.
- To document a fluid property, list the source, temperature, pressure, and main assumptions.
- Surface tension problems are usually solved by drawing an FBD and summing forces.
- The formula for capillary rise of water in a glass tube is $\Delta h = (4\sigma)/(\gamma d)$.

Fluid Properties

Table 2.2 summarizes the most useful fluid properties. Columns 1 and 2 describe the property. Columns 3 and 4 describe how the property varies with temperature and pressure. Blue shading is used to distinguish between gases and liquids. For example, look in the row for viscosity. The viscosity of gases increases with a temperature rise, whereas the viscosity of liquids decreases with a temperature rise. The *Notes* column gives tips and lists the locations in this text where fluid properties can be found.

TABLE 2.2 Summary of Fluid Properties

Property	Units (SI)	Temperature Effects	Pressure Effects (common trends)	Notes
Density (ρ): Ratio of mass to volume at a point	$\frac{kg}{m^3}$	$\rho\downarrow$ as $T\uparrow$ if gas is free to expand	$\rho\uparrow$ as $p\uparrow$ if gas is compressed.	• *Air.* Find ρ in Table F.4 or Table A.3. • *Other Gases.* Find ρ in Table A.2. • *Caution!* Tables for gases are for $p = 1$ atm. For oth pressures, find ρ using the ideal gas law.
		$\rho\downarrow$ as $T\uparrow$ for liquids	ρ of liquids are constant with pressure	• *Water.* Find ρ in Table F.5 or Table A.5. • *Note.* For water, $\rho\uparrow$ as $T\uparrow$ for temperatures from 0 about 4°C. Maximum density of water is at $T \approx 4$ • *Other Liquids.* Find ρ in Table A.4.
Specific Weight (γ): Ratio of weight to volume at a point	$\frac{N}{m^3}$	$\gamma\downarrow$ as $T\uparrow$ if fluid is free to expand	same trends as density	• Use same tables as for density. • ρ and γ can be related using $\gamma = \rho g$. • *Caution!* Tables for gases are for $p = 1$ atm. For oth pressures, find γ using the ideal gas law and $\gamma = \rho$
Specific Gravity (S or SG): Ratio of (density of a liquid) to (density of water at 4°C)	none	SG$\downarrow$ as $T\uparrow$	SG of liquids are constant with pressure	• Find SG data in Table A.4. • SG is used for liquids, not commonly used for gase • Density of water (at 4°C) is listed in Table F.6.

TABLE 2.2 Summary of Fluid Properties (*Continued*)

Property	Units (SI)	Temperature Effects	Pressure Effects (common trends)	Notes
Viscosity (μ): A property that characterizes resistance to shear stress and fluid friction	$\dfrac{N \cdot s}{m^2}$	$\mu\uparrow$ as $T\uparrow$ for gases.	μ of gases is independent of pressure	• *Air:* Find μ in Table F.4, Table A.3, Fig. A.2. • *Other gases:* Find properties in Table A.2, Fig. A.2. • *Hint:* Viscosity is also known as dynamic viscosity and absolute viscosity. • *Caution!* Avoid confusing viscosity and kinematic viscosity; these are different properties.
		$\mu\downarrow$ as $T\uparrow$ for liquids.	μ of liquids is independent of pressure	• *Water:* Find μ in Table F.5, Table A.5, Fig. A.2. • *Other Liquids.* Find μ in Table A.4, Fig. A.2.
Kinematic Viscosity (ν): A property that characterizes the mass and viscous properties of a fluid	$\dfrac{m^2}{s}$	$\nu\uparrow$ as $T\uparrow$ for gases	$\nu\uparrow$ as $p\uparrow$ for gases	• *Air:* Find μ in Table F.4, Table A.3, Fig. A.3. • *Other gases:* Find properties in Table A.2, Fig. A.3. • *Caution!* Avoid confusing viscosity and kinematic viscosity; these are different properties. • *Caution!* Gas tables are for $p = 1$ atm. For other pressures, look up $\mu = \mu(T)$, then find ρ using the ideal gas law, and calculate ν using $\nu = \mu/\rho$.
		$\nu\downarrow$ as $T\uparrow$ for liquids.	ν of liquids is independent of pressure	• *Water:* Find ν in Table F.5, Table A.5, Fig. A.3. • *Other liquids:* Find ν in Table A.4, Fig. A.3.
Surface Tension (σ): A property that characterizes the tendency of a liquid surface to behave as a stretched membrane	$\dfrac{N}{m}, \dfrac{J}{m^2}$	$\sigma\downarrow$ as $T\uparrow$ for liquids.	σ of liquids is independent of pressure	• *Water:* Find σ in Fig. 2.16 (page 46). • *Other liquids:* Find σ in Table A.4. • Surface tension is a property of liquids (not gases). • Surface tension is greatly reduced by contaminates or impurities.
Vapor Pressure p_v: The pressure at which a liquid will boil	Pa	$p_v\uparrow$ as $T\uparrow$ for liquids	not applicable	• *Water:* Find p_v in Fig. 2.21 (page 50) and Table A-5
Bulk Modulus of Elasticity E_v: A property that characterizes the compressibility of a fluid	Pa	not presented here	not presented here	• *Ideal gas (isothermal process):* $E_v = p =$ pressure. • *Ideal gas (adiabatic process):* $E_v = kp$; $k = c_p/c_v$. • *Water:* $E_v \approx 2.2 \times 10^9$ Pa.

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PROBLEMS

Defining the System (§2.1)

2.1 A system is separated from its surrounding by a

    a. border

    b. boundary

    c. dashed line

    d. dividing surface

Characterizing Weight and Mass (§2.2)

2.2 How are density and specific weight related?

2.3 PLUS Density is (select all that apply)

    a. weight/volume

    b. mass/volume

    c. volume/mass

    d. mass/weight

2.4 PLUS Which of these are units of density? (select all that apply)

    a. kg/m^3

    b. mg/cm^3

    c. lbm/ft^3

    d. $slug/ft^3$

2.5 PLUS Specific gravity (select all that apply)

    a. can have units of N/m^3

    b. is dimensionless

    c. increases with temperature

    d. decreases with temperature

2.6 If a liquid has a specific gravity of 1.7, what is the density in slugs per cubic feet? What is the specific weight in lbf per cubic feet?

2.7 What are SG, γ, and ρ for mercury? State your answers in SI units and in traditional units.

2.8 PLUS If a gas has $\gamma = 15$ N/m³, what is its density? State your answers in SI units and in traditional units.

Bulk Modulus of Elasticity (§2.3)

2.9 PLUS If you have a bulk modulus of elasticity that is a very large number, then a small change in pressure would cause

    a. a very large change in volume

    b. a very small change in volume

2.10 PLUS Dimensions of the bulk modulus of elasticity are

    a. the same as the dimensions of pressure/density

    b. the same as the dimensions of pressure/volume

    c. the same as the dimensions of pressure

2.11 The bulk modulus of elasticity of ethyl alcohol is 1.06×10^9 Pa. For water, it is 2.15×10^9 Pa. Which of these liquids is easier to compress?

    a. ethyl alcohol

    b. water

2.12 PLUS A pressure of 2×10^6 N/m² is applied to a mass water that initially filled a 2000 cm³ volume. Estimate its vol after the pressure is applied.

2.13 PLUS Calculate the pressure increase that must be app to water to reduce its volume by 2%.

2.14 PLUS An open vat in a food processing plant contains of water at 20°C and atmospheric pressure. If the water is he to 80°C, what will be the percentage change in its volume? I vat has a diameter of 3 m, how much will the water level rise to this temperature increase?

Finding Fluid Properties (§2.4)

2.15 Where in this text can you find:

    a. density data for such liquids as oil and mercury?

    b. specific weight data for air (at standard atmospheri pressure) at different temperatures?

    c. specific gravity data for sea water and kerosene?

2.16 PLUS Regarding water and seawater:

    a. Which is more dense, seawater or freshwater?

    b. Find (SI units) the density of seawater (10°C, 3.39 salinity).

    c. Find the same in traditional units.

    d. What pressure is specified for the values in (b) and

2.17 PLUS If the density, ρ, of air (in an open system at atmosp pressure) increases by a factor of 1.4x due to a temperature ch

    a. specific weight increases by 1.4x

    b. specific weight increases by 13.7x

    c. specific weight remains the same

Describing Viscous Effects (§2.5)

2.18 The following questions relate to viscosity.

    a. What are the primary dimensions of viscosity? Wha five common units?

    b. What is the viscosity of SAE 10W-30 motor oil at 1 (in traditional units)?

2.19 PLUS Shear stress has dimensions of

    a. force/area

    b. dimensionless

2.20 PLUS The term dV/dy, the velocity gradient

    a. has dimensions of L/T and represents shear strain

    b. has dimensions of T^{-1} and represents the rate of sh strain

2.21 PLUS For the velocity gradient dV/dy

    a. the change in velocity dV is in the direction of flow

    b. the change in velocity dV is perpendicular to flow

2.22 PLUS The no-slip condition

    a. only applies to ideal flow

    b. only applies to rough surfaces

    c. means velocity, V, is zero at the wall

    d. means velocity, V, is the velocity of the wall

2.23 PLUS Kinematic viscosity (select all that apply)

    a. is another name for absolute viscosity

    b. is viscosity/density

    c. is dimensionless because forces are canceled out

    d. has dimensions of L^2/T

    e. is only used with compressible fluids

2.24 What is the change in the viscosity and density of water between 10°C and 70°C? What is the change in the viscosity and density of air between 10°C and 70°C? Assume standard atmospheric pressure ($p = 101$ kN/m^2 absolute).

2.25 PLUS Determine the change in the kinematic viscosity of air that is heated from 10°C to 70°C. Assume standard atmospheric pressure.

2.26 PLUS Find the dynamic and kinematic viscosities of kerosene, SAE 10W-30 motor oil, and water at a temperature of 38°C (100°F).

2.27 What is the ratio of the dynamic viscosity of air to that of water at standard pressure and a temperature of 20°C? What is the ratio of the kinematic viscosity of air to that of water for the same conditions?

Applying the Viscosity Equation (§2.6)

2.28 PLUS At a point in a flowing fluid, the shear stress is 1×10^{-4} psi, and the velocity gradient is 1 s^{-1}.

    a. What is the viscosity in traditional units?

    b. Convert this viscosity to SI units.

    c. Is this fluid more, or less, viscous than water?

2.29 PLUS SAE 10W-30 oil with viscosity 1×10^{-4} lbf · s/ft^2 is used as a lubricant between two parts of a machine that slide past one another with a velocity difference of 6 ft/s. What spacing, in inches, is required if you don't want a shear stress of more than 2 lbf/ft^2?

2.30 The velocity distribution for water (20°C) near a wall is given by $u = a(y/b)^{1/6}$, where $a = 10$ m/s, $b = 2$ mm, and y is the distance from the wall in mm. Determine the shear stress in the water at $y = 1$ mm.

2.31 The velocity distribution for the flow of crude oil at 100°F ($\mu = 8 \times 10^{-5}$ lbf · s/ft^2) between two walls is shown and is given by $u = 100y(0.1 - y)$ ft/s, where y is measured in feet and the space between the walls is 0.1 ft. Plot the velocity distribution and determine the shear stress at the walls.
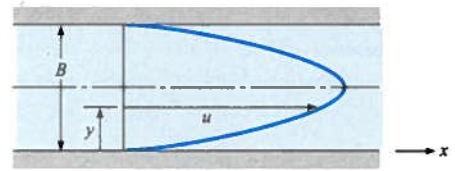
PROBLEMS 2.31, 2.32, 2.33

2.32 PLUS **(part a only)** A liquid flows between parallel boundaries as shown above. The velocity distribution near the lower wall is given in the following table:

y in mm	V in m/S
0.0	0.00
1.0	1.00
2.0	1.99
3.0	2.98

   **a.** If the viscosity of the liquid is 10^{-3} N · s/m^2, what is the maximum shear stress in the liquid?

    b. Where will the minimum shear stress occur?

2.33 GO Suppose that glycerin is flowing ($T = 20°C$) and that the pressure gradient dp/dx is -1.6 kN/m^3. What are the velocity and shear stress at a distance of 12 mm from the wall if the space B between the walls is 5.0 cm? What are the shear stress and velocity at the wall? The velocity distribution for viscous flow between stationary plates is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}(By - y^2)$$

2.34 PLUS Two plates are separated by a 1/8-in. space. The lower plate is stationary; the upper plate moves at a velocity of 25 ft/s. Oil (SAE 10W-30, 150°F), which fills the space between the plates, has the same velocity as the plates at the surface of contact. The variation in velocity of the oil is linear. What is the shear stress in the oil?

2.35 PLUS The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume a linear velocity distribution.
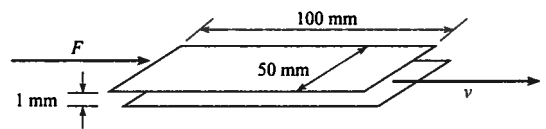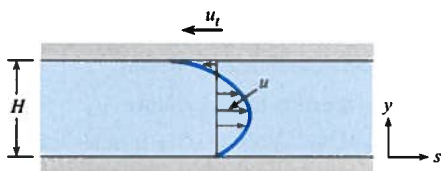

PROBLEM 2.35

2.36 A laminar flow occurs between two horizontal parallel plates under a pressure gradient dp/ds (p decreases in the positive s direction). The upper plate moves left (negative) at velocity u_t. The expression for local velocity u is given as

$$u = -\frac{1}{2\mu}\frac{dp}{ds}(Hy - y^2) + u_t\frac{y}{H}$$

a. Is the magnitude of the shear stress greater at the moving plate ($y = H$) or at the stationary plate ($y = 0$)?

b. Derive an expression for the y position of zero shear stress.

c. Derive an expression for the plate speed u_t required to make the shear stress zero at $y = 0$.

PROBLEM 2.36

2.37 This problem involves a cylinder falling inside a pipe that is filled with oil, as depicted in the figure. The small space between the cylinder and the pipe is lubricated with an oil film that has viscosity μ. Derive a formula for the steady rate of descent of a cylinder with weight W, diameter d, and length ℓ sliding inside a vertical smooth pipe that has inside diameter D. Assume that the cylinder is concentric with the pipe as it falls. Use the general formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200 mm long and weighs 15 N. The lubricant is SAE 20W oil at 10°C.
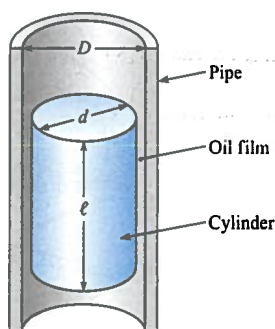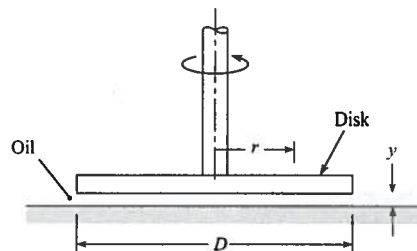

PROBLEM 2.37

2.38 The device shown consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil.

a. If the disk is rotated at a rate of 1 rad/s, what will be the ratio of the shear stress in the oil at $r = 2$ cm to the shear stress at $r = 3$ cm?

b. If the rate of rotation is 2 rad/s, what is the speed of oil in contact with the disk at $r = 3$ cm?

c. If the oil viscosity is 0.01 N · s/m² and the spacing y 2 mm, what is the shear stress for the conditions no in part (b)?
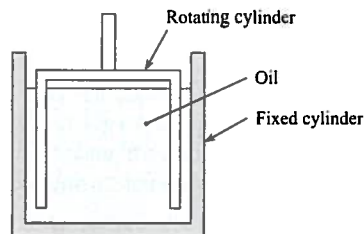

PROBLEM 2.38

2.39 Some instruments having angular motion are damped means of a disk connected to the shaft. The disk, in turn, is immersed in a container of oil, as shown. Derive a formula fo the damping torque as a function of the disk diameter D, spacin rate of rotation ω, and oil viscosity μ.
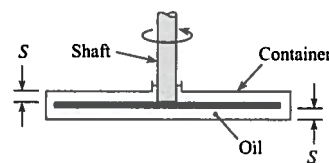
PROBLEM 2.39

2.40 One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder The gap between the cylinder must be very small to achieve a linear velocity distribution in liquid. (Assume the maximum spacing for proper operation 0.05 in.). Design a viscometer that will be used to measure th viscosity of motor oil from 50°F to 200°F.

PROBLEM 2.40

Characterizing Viscosity (§2.7)

2.41 If temperature increases, does the viscosity of water increase or decrease? Why? If temperature increases, does th viscosity of air increase or decrease? Why?

2.42 Sutherland's equation (select all that apply):

  a. relates temperature and viscosity

  b. must be calculated using Kelvin

  c. requires use of a single universal constant for all gases

  d. requires use of a different constant for each gas

2.43 🅿🅻🆄🆂 When looking up values for density, absolute viscosity, and kinematic viscosity, which statement is true for both liquids and gases?

  a. all three of these properties vary with temperature

  b. all three of these properties vary with pressure

  c. all three of these properties vary with temperature and pressure

2.44 Common Newtonian fluids are

  a. toothpaste, catsup, and paint

  b. water, oil, and mercury

  c. all of the above

2.45 Which of these flows (deforms) with even a small shear stress applied?

  a. a Bingham plastic

  b. a Newtonian fluid

2.46 Using Sutherland's equation and the ideal gas law, develop an expression for the kinematic viscosity ratio v/v_0 in terms of pressures p and p_0 and temperatures T and T_0, where the subscript 0 refers to a reference condition.

2.47 🅿🅻🆄🆂 The dynamic viscosity of air at 15°C is 1.78×10^{-5} N · s/m². Using Sutherland's equation, find the viscosity at 100°C.

2.48 The kinematic viscosity of methane at 15°C and atmospheric pressure is 1.59×10^{-5} m²/s. Using Sutherland's equation and the ideal gas law, find the kinematic viscosity at 200°C and 2 atmospheres.

2.49 The dynamic viscosity of nitrogen at 59°F is 3.59×10^{-7} lbf · s/ft². Using Sutherland's equation, find the dynamic viscosity at 200°F.

2.50 🅿🅻🆄🆂 The kinematic viscosity of helium at 59°F and 1 atmosphere is 1.22×10^{-3} ft²/s. Using Sutherland's equation and the ideal gas law, find the kinematic viscosity at 30°F and a pressure of 1.5 atmospheres.

2.51 Ammonia is very volatile, so it may be either a gas or a liquid at room temperature. When it is a gas, its absolute viscosity at 68°F is 2.07×10^{-7} lbf · s/ft² and at 392°F is 3.46×10^{-7} lbf · s/ft². Using these two data points, find Sutherland's constant for ammonia.

2.52 🅿🅻🆄🆂 The viscosity of SAE 10W-30 motor oil at 38°C is 0.067 N · s/m² and at 99°C is 0.011 N · s/m². Using Eq. (2.20) (p. 42, §2.7) for interpolation, find the viscosity at 60°C. Compare this value with that obtained by linear interpolation.

2.53 The viscosity of grade 100 aviation oil at 100°F is 4.43×10^{-3} lbf · s/ft² and at 210°F is 3.9×10^{-4} lbf · s/ft². Using Eq. (2.20) (p. 42, §2.7), find the viscosity at 150°F.

2.54 Find the kinematic and dynamic viscosities of air and wa at a temperature of 40°C (104°F) and an absolute pressure of 170 kPa (25 psia).

2.55 🅿🅻🆄🆂 Consider the ratio μ_{100}/μ_{50}, where μ is the viscosity of oxygen and the subscripts 100 and 50 are the temperatures of the oxygen in degrees Fahrenheit. Does this ratio have a value (a) less than 1, (b) equal to 1, or (c) greater than 1?

Characterizing Surface Tension (§2.8)

2.56 🅿🅻🆄🆂 Surface tension: (select all that apply)

  a. only occurs at an interface, or surface

  b. has dimensions of energy/area

  c. has dimensions of force/area

  d. has dimensions of force/length

  e. depends on adhesion and cohesion

  f. varies as a function of temperature

2.57 🅿🅻🆄🆂 Which of the following is the formula for the gage pressure within a very small spherical droplet of water:

  (a) $p = \sigma/d$, (b) $p = 4\sigma/d$, or (c) $p = 8\sigma/d$?

2.58 A spherical soap bubble has an inside radius R, a film thickness t, and a surface tension σ. Derive a formula for the pressure within the bubble relative to the outside atmospheric pressure. What is the pressure difference for a bubble with a 4 mm radius? Assume σ is the same as for pure water.

2.59 🅿🅻🆄🆂 A water bug is suspended on the surface of a pond b surface tension (water does not wet the legs). The bug has six legs, and each leg is in contact with the water over a length of 5 mm. What is the maximum mass (in grams) of the bug if it is to avoid sinking?
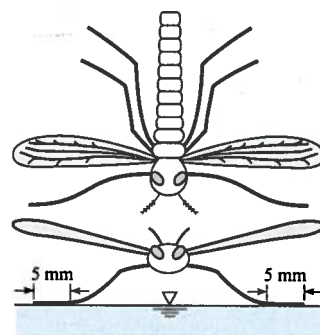
PROBLEM 2.59

2.60 A water column in a glass tube is used to measure the pressure in a pipe. The tube is 1/4 in. (6.35 mm) in diameter. How much of the water column is due to surface-tension effects What would be the surface-tension effects if the tube were 1/8 i (3.2 mm) or 1/32 in. (0.8 mm) in diameter?

2.61 Calculate the maximum capillary rise of water between two vertical glass plates spaced 1 mm apart.

PROBLEM 2.61

2.62 What is the pressure within a 1 mm spherical droplet of water relative to the atmospheric pressure outside?

2.63 By measuring the capillary rise in a tube, one can calculate the surface tension. The surface tension of water varies linearly with temperature from 0.0756 N/m at 0°C to 0.0589 N/m at 100°C. Size a tube (specify diameter and length) that uses capillary rise of water to measure temperature in the range from 0°C to 100°C. Is this design for a thermometer a good idea?

2.64 GO Capillary rise can be used to describe how far water will rise above a water table because the interconnected pores in the soil act like capillary tubes. This means that deep-rooted plants in the desert need only grow to the top of the "capillary fringe" in order to get water; they do not have to extend all the way down to the water table.

    a. Assuming that interconnected pores can be represented as a continuous capillary tube, how high is the capillary rise in a soil consisting of a silty soil, with pore diameter of 10 μm?

    b. Is the capillary rise higher in a soil with fine sand (pore diam. approx. 0.1 mm), or in fine gravel (pore diam. approx. 3 mm)?

    c. Root cells extract water from soil using capillarity. For root cells to extract water from the capillary zone, do the pores in a root need to be smaller than, or greater than, the pores in the soil? Ignore osmotic effects.

2.65 Consider a soap bubble 2 mm in diameter and a droplet of water, also 2 mm in diameter, that are falling in air. If the value of the surface tension for the film of the soap bubble is assumed to be the same as that for water, which has the greater pressure inside it? (a) the bubble, (b) the droplet, (c) neither—the pressure is the same for both.

2.66 A drop of water at 20°C is forming under a solid surface. The configuration just before separating and falling as a drop is shown in the figure. Assume the forming drop has the volume of a hemisphere. What is the diameter of the hemisphere just before separating?
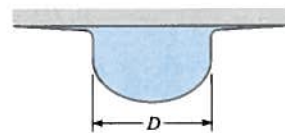
PROBLEM 2.66

2.67 PLUS The surface tension of a liquid is being measured a ring as shown. The ring has an outside diameter of 10 cm a an inside diameter of 9.5 cm. The mass of the ring is 10 g. Th force required to pull the ring from the liquid is the weight corresponding to a mass of 16 g. What is the surface tension the liquid (in N/m)?
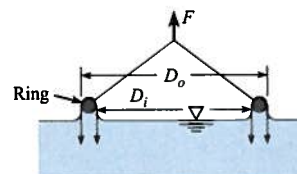
PROBLEM 2.67

Vapor Pressure (§2.9)

2.68 If liquid water at 30°C is flowing in a pipe and the pres drops to the vapor pressure, what happens in the water?

    a. the water begins condensing on the walls of the pip

    b. the water boils

    c. the water flashes to vapor

2.69 PLUS How does vapor pressure change with increasing temperature?

    a. it increases

    b. it decreases

    c. it stays the same

2.70 PLUS Water is at 20°C, and the pressure is lowered unti vapor bubbles are noticed to be forming. What must the magn of the pressure be?

2.71 A student in the laboratory plans to exert a vacuum in head space above a surface of water in a closed tank. She pla for the absolute pressure in the tank to be 10,400 Pa. The temperature in the lab is 20°C. Will water bubble into the va phase under these circumstances?

2.72 The vapor pressure of water at 100°C is 101 kN/m² bec water boils under these conditions. The vapor pressure of wa decreases approximately linearly with decreasing temperatu a rate of 3.1 kN/m²/°C. Calculate the boiling temperature of water at an altitude of 3000 m, where the atmospheric pressu 69 kN/m² absolute.